

LIBRARY  
**SECRET**  
 NAVAL RESEARCH LABORATORY  
 RESTRICTED

22 June 1943

NRL Report No. S-2097

Classification changed from **Secret**  
 to **RESTRICTED**

By authority of *Lab. Memo. 117-451b.*  
 File No. *Dated 7/1/46*

NAVY DEPARTMENT

Report on

**THE GERMAN SOUND-CONTROLLED TORPEDO**

An Analysis of its Probable Mode and Range  
 of Operation and of Suitable Countermeasures

NAVAL RESEARCH LABORATORY  
 ANACOSTIA STATION  
 WASHINGTON, D.C.

APPROVED FOR PUBLIC  
 RELEASE - DISTRIBUTION  
 UNLIMITED

Number of Pages:

Text - 13

Plates - 10

Reference: BuShips ltr S-S75-1 (347V) Ser. 4570  
 to NRL dated 12 May 1943.

Prepared by:

Dr. Harvey C. Hayes, Head Physicist,  
 Superintendent, Sound Division.

Approved by:

A. H. Van Keuren, Rear Admiral, USN,  
 Director.

Distribution: BuShips  
 Cominch  
 CR&D  
 EuORD (Re6)  
 VCNC (For Naval Attaché, London)  
 ONI  
 Files - 6

~~SECRET~~**RESTRICTED**

TABLE OF CONTENTS

|  | <u>PAGE</u> |
|--|-------------|
| ABSTRACT   |             |
| I. INTRODUCTION . . . . .                        | 1           |
| II. SOUND CONTROL FREQUENCY. . . . .             | 1           |
| III. COURSE OF TORPEDO. . . . .                  | 2           |
| IV. OPERATION OF SOUND CONTROL . . . . .         | 5           |
| V. THE CHANCES OF ESCAPE. . . . .                | 8           |
| VI. SUMMARY OF TORPEDO CHARACTERISTICS . . . . . | 9           |
| VII. SUGGESTED COUNTERMEASURES. . . . .          | 11          |

PLATES

1. Relation between Acoustic Operating Range  
and Frequency
2. Sample Torpedo Paths
3. Limits for Successful Attack
4. Conditions for Stern Misses
5. Conditions for Bow Misses
6. Forward Directed Sound Control
7. Broadside Rudder Setting Control
8. Formulae for Avoiding Hits
9. Graphic Illustration of Formulae
10. Sound Field Proposed for Countermeasures

~~SECRET~~

RESTRICTED

ABSTRACT

This report, analyzes the German Sound-Controlled Torpedo in the light of the scant information at hand in order to determine the conditions that must be met by countermeasures, and outlines the character and development of methods and means for meeting these conditions.

SECRET

RESTRICTED

UNCLASSIFIED

## I. INTRODUCTION

1. Reports from abroad indicate that the torpedo normally runs at 30 knots until the target is brought within range of the sound control equipment, and then slows to about 20 knots. These reports further state that sound control is accomplished through two series of hydrophones, eight or twelve on a side, which operate at about 5 kc. A report of the interrogation of the survivors of the "U660" gives the following additional information: -- The torpedo has been given the extra long range of 17,500 yards to enable it to overhaul its target. The radius of its minimum turning circle is 104 yards. The amount of helm required is given to the rudders by the microphones, which set the rudder to a given number of degrees according to the microphone that is nearest to the noise. More helm is given when the after microphones are affected. Information regarding the geometrical arrangement and spacing of the several microphones of each group is wholly lacking.

## II. SOUND CONTROL FREQUENCY

2. It may be noted at the start that the choice of a frequency band centered at 5 kilocycles for directing the torpedo was well considered. The sound spectrum of a torpedo proceeding at 20 knots reaches a maximum intensity within the band 100-2500 cps and drops to relatively low value at 5 kc. The local noise background, which limits the range of operations, becomes favorable within this frequency band. The 5 kc band also imposes difficulties from the standpoint of countermeasures since it limits the range within which countermeasures can be effectively used. This can be understood by reference to Plate 1, which serves to demonstrate that the range D to which the sound equipment should be expected to work in deep water is given by the expression

$$D \leq \frac{8h^2}{\lambda}$$

where h is the depth of the torpedo and  $\lambda$  is the sound wavelength. Taking 10' as the normal running depth and 1' as the average wavelength in the frequency band centered about 5 kc gives

$$D \leq 800'$$

This figure assumes a low noise background such as would not obtain on the torpedo. The practical range to which the torpedo will respond to acoustical control should be expected to fall somewhat short of 800 feet. Probably 600 feet comes nearer to representing the practical limiting range to which the torpedo will respond to countermeasures.

3. A survey of the sound generated by merchant ships leads to the conclusion that the homing point of a sound-directed torpedo employing a frequency band centered at approximately 5 kc must be the propellers of the target. Since the purpose of countermeasures is to prevent the torpedo from reaching the target, a consideration of the character and limitations of the sound-controlled path of the torpedo becomes of prime interest.

### III. COURSE OF TORPEDO

4. Local noise background must limit the sound control to relatively short ranges. Therefore, the torpedo must be launched and gyro-directed along a collision course to within some roughly predetermined range where the intensity of the target propeller sound is sufficient to operate the acoustical controls. From this point on, the torpedo may be expected to follow a sinuous course along a theoretical smooth path directed continuously at the propellers of the moving target. The character and limitations of the theoretical path are obviously dependent upon the velocity  $V_t$  of the torpedo, the velocity  $V_s$  of the target ship, and the relative bearing of the origin of the sound-controlled path with respect to the target. Such paths are drawn to scale in Plate 2, wherein the paths originate on a circle of 800' radius centered on the target's propellers at the instant the torpedo crosses this circle. The speed and fixed course of the target are represented by the vector  $V_s$ ; and the speed, course, and origin of the path of the torpedo are indicated by vector  $V_t$ .

5. Four paths are shown in Plate 2. The two with origins specified by vectors  $V_t$  and  $V_t'$  are located off the bow symmetrically with respect to the target, and those with origins specified by vectors  $V_t''$  and  $V_t'''$  are similarly located off the quarter. The two starboard paths shown are for  $V_t = 20$  knots and  $V_s = 15$  knots, while the two port side paths shown are for  $V_t = 20$  knots and  $V_s = 7.5$  knots.

~~SECRET~~  
RESTRICTED

A consideration of these paths and of their underlying theory, leads to the conclusions:-

- a) The curved paths are concave toward the forward direction of the target.
- b) The curvature increases as the angular bearing  $\phi$  of the path-origin, measured from the sternward projected course of the target, becomes greater.
- c) The curvature increases as the ratio  $V_s/V_t$  decreases.
- d) The torpedo can reach the target only along paths where the radius of curvature does not become less than the minimum turning radius of the torpedo, and hence only along paths having their origin within some limiting values of  $\phi$ , depending upon the ratio  $V_s/V_t$ .
- e) There is, thus, a region forward of the target of angular spread  $2(180^\circ - \phi)$  that does not need to be safeguarded by countermeasures. This region is represented by the cross-hatched areas on Plate 2. A torpedo following a path within these areas must miss the target, and continue on, presumably in a circular path of radius R equal to the minimum turning radius of the torpedo. The small unshaded area forward of the target theoretically is exposed to attack, but practically need not be guarded.

SECRET  
RESTRICTED

6. The shaded "safe sectors" of Plate 2 were determined with the aid of Plate 3, the mathematical theory of which is too involved to be given here. It is assumed that the target ship is rectangular in shape, and 30 feet wide. The minimum turning radius of the torpedo is taken as 300 feet.

7. It is of interest to consider the conditions necessary for avoiding hull contact when the torpedo traverses a circular path of minimum turning radius. The circle with radius  $R$  of Plate 4 represents the minimum turning circle of the torpedo. This path has just missed the target's propeller at point  $p$ . The course of the target may lie along any chord of the circle starting at  $p$ , such as  $p-q$ , depending on the direction at which the torpedo crossed the track of the target. The target will avoid a hit by causing the torpedo to pass astern when it traverses the chord  $p-q$  in less time than the torpedo traverses the arc. Therefore, one condition for avoiding the torpedo becomes:-

$$\frac{2R \sin \theta}{V_s} \leq \frac{2R\theta}{V_t}$$

or  $\frac{\sin \theta}{\theta} \leq \frac{V_s}{V_t}$  (1)

Substituting the two values  $15/20$  and  $7.5/20$  gives for  $\theta$  respectively the values  $73^\circ$  and  $125^\circ$ . The complements of these angles,  $17^\circ$  and  $-35^\circ$ , give the courses of the target with respect to the diameter  $p-s$  of the turning circle. Therefore the 15 knot target will escape if its course lies within  $17^\circ$  of the diameter  $p-s$  of the turning circle of the torpedo. The 7.5 knot ship, however, cannot escape unless its course lies outside, (i.e., to the right of) the chord making  $-35^\circ$  with the diameter  $p-s$  of the turning circle and on the side opposite to the torpedo. This results in directing the torpedo along the major instead of the minor arc  $p-q$ . This condition can arise only when the approach path of the torpedo crosses the ship's course at an angle less than  $90^\circ$ , and hence should rarely occur in practice. Therefore, the slow ship should not be expected to escape when the approaching torpedo misses close astern.

8. The target will escape a hit due to passage of the torpedo forward of the bow when the time required for the torpedo to travel the arc  $p-q$  is less than the time re-

quired for the bow of the target to reach the end of the chord at q and, in practice, the chance of escape becomes greatest when chord p-q becomes the diameter. Therefore, in this case, as shown on Plate 5, the most favorable conditions for escape become:-

$$\frac{\pi R}{V_t} < \frac{2R-L-a}{V_s} \quad \text{or} \quad \pi R \frac{V_s}{V_t} < 2R-L-a \quad (2)$$

Substituting in equation (2) the following practical values:-

- R = 300', turning radius of the torpedo.
- L = 400', length of the target (average freighter).
- a = the distance the approach course passed astern.
- $V_s = 7.5$  knots - Since slow target speed favors a forward miss,

gives:-  $\pi \cdot 300 \frac{7.5}{20} < 600 - 400 - a$

and,  $a < -150.$

This means that the bow of the target, even when it steams at the slow speed 7.5 knots, will have passed 150' beyond the point q by the time the torpedo arrives at this point. Therefore the target cannot escape a hit by virtue of the torpedo crossing ahead of its bow.

#### IV. OPERATION OF SOUND CONTROL

9. In the light of the above analysis the chances of a hull contact by the subject torpedo, after it misses contacting the propellers as a result of its somewhat considerable turning radius, is so high as to suggest that the design may well have been directed primarily to this end. It will be seen that a consideration of possible modes of operation of the two groups of microphones tends to support this suggestion, as do the quoted statements of paragraph (2), and a more recently received statement from a survivor of the torpedoed freighter S.S. MATTHEW LUCKENBACH which reads as follows:

At 1054 GCT, one torpedo was sighted approximately 3 points aft the port beam at approximately 400 to 500 feet and another a few seconds

later 2 points aft the port beam at about 200 feet. A third was also sighted at this time approximately broad on the port beam at approximately 600 feet. Wakes were noted commencing approximately 10' astern of the torpedoes which were traveling almost entirely submerged. No estimate was made as to their length. The torpedoes appeared to be coming from astern on a course parallel to the MATTHEW LUCKENBACH at a speed not more than 2 or 3 knots in excess of ship's speed. When approximately abeam, the first two torpedoes dived, momentarily exposing fin and rudder assembly which was noted to be black. The third torpedo continued on ahead of the ship and was not seen to dive. The first two torpedoes were not sighted again, one explosion occurring shortly after diving and a second possibly some seconds later. The first explosion occurred outboard abreast of port side of #2 hatch which contained general cargo, ripping a hole about 8' to 10' fore and aft along waterline, also damaging the port boatdeck superstructure. Sulphur smell was noted. The second explosion occurred outboard abreast of portside of #4 hatch which contained grain.

10. The acoustical control as reported is dependent on two like groups of 8 or 12 microphone units symmetrically mounted on the port and starboard sides, respectively. It is presumed they will be located well forward to minimize noise disturbance from the propellers. These groups may therefore be expected to center about 10' forward of the torpedo's propellers. If the torpedo is designed primarily to contact the target's propellers, it might be expected that the several receivers of each group would be spaced along an element of the cylindrical surface with the response of the receivers of each group brought into like phase along lines 1 and 2 making the angles  $\theta$  and  $\theta'$  equal, acute, and symmetrical with respect to the forward direction as depicted in Plate 6. Such an arrangement provides the double lobe type of reception that has long since proved effective for homing purposes, and at the same time makes the microphone response to the local propeller noise relatively weak. Moreover, the use of multiple receivers with properly designed retardation lines for achieving directivity is in accordance with German practice.

11. But this seemingly optimum arrangement and use of the receivers of each group lacks agreement with the prisoners' statements quoted in paragraph 2. Moreover a torpedo directed by this arrangement would not agree in behavior with the torpedoes that sank the MATTHEW LUCKENBACH. It therefore appears that the several microphones of each group are utilized in some other way. However, the number involved, 8 or 12, is such as to argue convincingly that at least a part of the microphone units of each group are directionally employed.

12. If, in accordance with ONI Report, Ser. 800, from Naval Attaché, London, dated February 26, 1943, it is assumed that when sound is received on the forward microphones only, small amounts of rudder are applied, but that the rudder angle is increased when the after microphones are affected, then it must also be assumed that the after microphones are used directionally as a group and that the group is directed well away from ahead. Otherwise on approach these after microphones would respond as readily to the target sound as would the forward ones. And since broadside directivity requires no retardation units, thereby greatly simplifying the equipment, it may be expected that the rear 7 or 11 microphones of each group are connected in parallel or in series to give such broadside receiving sensitivity as is indicated by the solid line contours of Plate 7. With such an arrangement the two forward receivers, whose sensitivity contours are indicated by the broken lines of Plate 7, will prevail during the approach until the torpedo tends to pass by the target, when the vigorous response of the broadside directed group facing the target will prevail and turn the rudder to a greater angle. The limit of this angle may be expected to depend upon the range to the target's propellers when the torpedo passes, and to become smaller as the range becomes greater.

13. If it is assumed that the rudder is locked at the angle given when the torpedo passes the target, then its behavior will conform in many respects to the behavior of the two torpedoes striking the MATTHEW LUCKENBACH. These two torpedoes were described as approaching from astern at the respective ranges of about 200' and 400' off the port quarter and on courses substantially parallel with that of the ship. This would bring them roughly to ranges 140' and 280' respectively when they came along side and were reported to dive, presumably as a result of intense broadside reception of the target propeller sounds. Shortly thereafter one torpedo hit the ship alongside port hatch #2, and slightly later

~~SECRET~~

the other hit alongside port hatch #4. Therefore, the intense broadside sound reception that presumably served to work the diving rudders appears also to have effected the horizontal rudder control that carried both torpedoes to the ship's hull. The third torpedo appears to have passed beyond the operating range as discussed in paragraph (3).

14. The fact that all three torpedoes approached on the port side of the MATTHEW LUCKENBACH, suggests that the sound control on approach is deliberately offset to avoid a direct propeller contact. This suggestion is emphasized by the fact that the relatively slow speed required to reduce local noise background to a level that permits sound control may well prevent the torpedo from making headway directly against the propeller wash - and by the further fact that the chances of a hull contact are surprisingly high for a torpedo that approaches from astern on a course roughly parallel with, and at a range  $R$  from, that of the target, and then proceeds toward the target along a circular course of radius  $R$  from the point where the target's propellers bear broadside to the torpedo.

#### V. CHANCES OF ESCAPE

15. The conditions and limits under which a torpedo controlled in accordance with the above assumption may be expected to contact the hull of a target ship can be understood by reference to Plate 8, wherein the target, of length  $L$ , is assumed to proceed on a fixed course at speed  $V_s$ . The torpedo, with speed  $V_t$ , has reached its position along some sound-controlled course that has become substantially parallel with and at range  $R$  from the course of the target. At this point the torpedo, through broadside directive reception of the propeller sound, takes on a rudder setting that directs the course toward the target along a circle of radius  $R$ . The circular course of the torpedo and the straight course of the ship intersect at point  $p$ . The conditions and limitations sought are those that bring this crossing point  $p$  between the bow and stern of the target.

16. The distance the torpedo must travel to strike the ship is  $2\pi R/4$  or  $\pi R/2$ . The distances the bow and stern of the target must travel to reach point  $p$  are respectively  $R-L$  and  $R$ . The time required for the torpedo to reach point  $p$  is  $\pi R/2V_t$ , while the times required for the bow and stern

of the target to reach this point are, respectively,  $(R-L)/V_s$  and  $R/V_t$ . Therefore the torpedo will cross forward of the target when

$$\frac{\pi R}{2V_t} \leq \frac{R-L}{V_s} \quad , \text{ or when } \frac{V_s}{V_t} \leq \frac{2}{\pi} \left( \frac{R/L-1}{R/L} \right)$$

and will cross astern of the target when

$$\frac{\pi R}{2V_t} \geq \frac{R}{V_s} \quad , \text{ or when } \frac{V_s}{V_t} \geq \frac{2}{\pi}$$

These conditions are represented graphically in Plate 9, for a significant range of the ratio  $R/L$ . As has been noted  $R$  can not reasonably exceed 800 feet and the length of ship,  $L$ , is not likely to be less than 250 feet. In the case of the German torpedo  $V_t$  is 20 knots. Clearly a ship making more than  $2 \times 20 / \pi$ , or 12.75 knots, regardless of its length,  $L$ , or the range,  $R$ , will outrun the torpedo and cause it to miss. Quite as clearly a ship making less than 12.75 knots will very probably be hit by the torpedo, for the conditions for the torpedo to pass around the bow are satisfied only for values of  $V_s/V_t$  and  $R/L$  within a restricted region. Assuming a minimum normal convoy speed of 7.5 knots,  $V_s/V_t$  will rarely be less than .375. Accepting this as an additional condition, the chance that the torpedo will pass around the bow of a target ship making less than 12.75 knots becomes very remote indeed. It follows that the probability of a torpedo hit for targets within the speed range of all but the fastest convoys approximates unity.

## VI. SUMMARY OF TORPEDO CHARACTERISTICS

17. The character and potentialities of the German Acoustically-Controlled Torpedo as conceived through the above analysis can be summarized as follows:

- a) The range to which the German torpedo will respond to the propeller sound of the target, and hence to acoustical countermeasures, may be expected to approximate 600' while operating in depths beyond about 100 fathoms.

~~SECRET~~

RESTRICTED

UNCLASSIFIED

- b) The torpedo is not designed for use against fast ships or convoys but should prove especially effective against ships or convoys proceeding at speeds less than about 12 knots.
- c) The torpedo is designed to home on the propellers of the target through the difference in intensity of the propeller sound as received by two substantially non-directive receivers symmetrically located on opposite sides and designed for receptions of a band of frequencies centered about 5 kilocycles.
- d) The intensity differential of the propeller sound of the target picked up by the two broadcast receivers must result from differential shielding by the intervening body of the torpedo. And since the diameter of the shield is only of the order of twice the wave length, it may be expected that the resulting relatively weak homing tendency of the torpedo will favor missing rather than directly hitting the target's propellers.
- e) The chances of a hull hit by a torpedo that misses the propellers on an approach from the rear and that on passing takes a circular course toward the target are so high as to encourage the belief that the Germans have deliberately designed the sound control of these acoustically operated torpedoes for such action. They should prove ideal for firing into a slow moving convoy from a considerable distance.
- f) Most of the microphones of both the port and starboard groups are believed to be connected for directive reception, with the probability that the direction is broadside to the torpedo.

The energy received from any sound source that has a fairly intense frequency component adjacent to 5 kc and is presented broadside to the torpedo should be expected to work the rudder controls. Therefore, countermeasures for the subject torpedo must work to prevent its approach from the rear to a broadside bearing from the propellers of the target.

#### VII. SUGGESTED COUNTERMEASURES

10. These conclusions suggest the following countermeasures:-

- a) Eliminate or greatly weaken the propeller sound-sources of the target by stopping the engines whenever torpedoes are seen or heard to approach slowly from astern similarly to those that sank the MATTHEW LUCKENRACH, and by coasting without change of course until the torpedoes are well past. This move must be taken before the torpedoes have approached to broadside bearing of the propellers of the target.

Note 1: While this procedure might increase the hazard in case of a gyro-controlled torpedo, there appears to be no reason for mistaking the two types of shots, since it appears improbable that gyro-controlled shots would approach from astern along paths tending to parallel the course of the target or that the subject torpedo would approach on any other but a course from well astern. Thus it would seem that this simple procedure can always be used with safety unless

or until other and better countermeasures are at hand.

- b) Provide a predominating sound source by trailing behind the target one or more sound generators of such frequency and intensity and so located that the torpedo will be acoustically intercepted before it can reach a position broadside from the target's propellers.

Note 1: Due to the limiting range to which the torpedo can be actuated by sound, it becomes necessary to trail the sound source or sources employed for countermeasures at a distance substantially equal to that range. Five hundred feet would seem to be a reasonable distance.

Note 2: The USS DELTA, a 490 foot turbine-driven, single-screw, freighter of 9,700 tons displacement generates approximately 0.03 watts in the frequency band 4000-6000 cps, when steaming at 14.5 knots. A sound source for countermeasures should generate a minimum of about 30 watts within the frequency band 4500-5500 cps. The figure falls well within practical limits, and if service tests prove that somewhat less power will suffice, then the driving power can be lowered and the life of the equipment correspondingly lengthened.

Note 3: If, as appears probable, countermeasures are primarily concerned with guarding against directions from well astern, then a single projector trailed about 500' astern might serve, but two such projectors separated a distance about equal to the minimum turning circle of the torpedo should provide greater protection. The sound field resulting from the use of a single projector is shown in Plate 10 where the

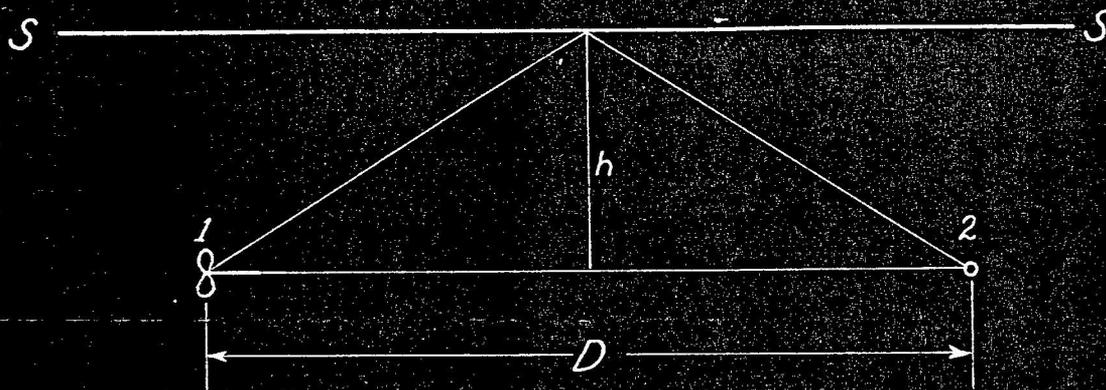
cross-hatched area represents the region in which the sound field from the propellers of the target will predominate.

Note 4: Possibly a stream of machine gun projectiles fired into the water off the bow of the torpedo can serve to change its course.

- c) Interpose a sound shield between the torpedo and the target in the form of a cloud of air bubbles surrounding the propellers of the target.

Note 1: The amount of air required is not prohibitive. It can be tank stored and hence provided by any one of several small compressors on the market.

19. The Sound Division of the Naval Research Laboratory is working to perfect countermeasures as outlined in (b) and (c). The sound generator employs two opposed radiating diaphragms one foot in diameter, each of which is driven by magnetostriction elements that are weighted on the free end to permit varying the resonant frequency to meet changes that may be made in the operating frequency band of the torpedo. The bubble shield promises to be a vertical sheet on both port and starboard side of the propellers supplied from vertically mounted perforated pipes. Meantime it appears advisable to determine promptly the possibilities of the parallel pipes for use as an acoustic interceptor at least as a stop gap.



At point 2, for reasonable strength of signal emanating from point 1,

$$2\sqrt{\left(\frac{D}{2}\right)^2 + h^2} - D \geq \frac{\lambda}{4} .$$

In practice,  $D \gg \lambda$ ; hence the above expression reduces to

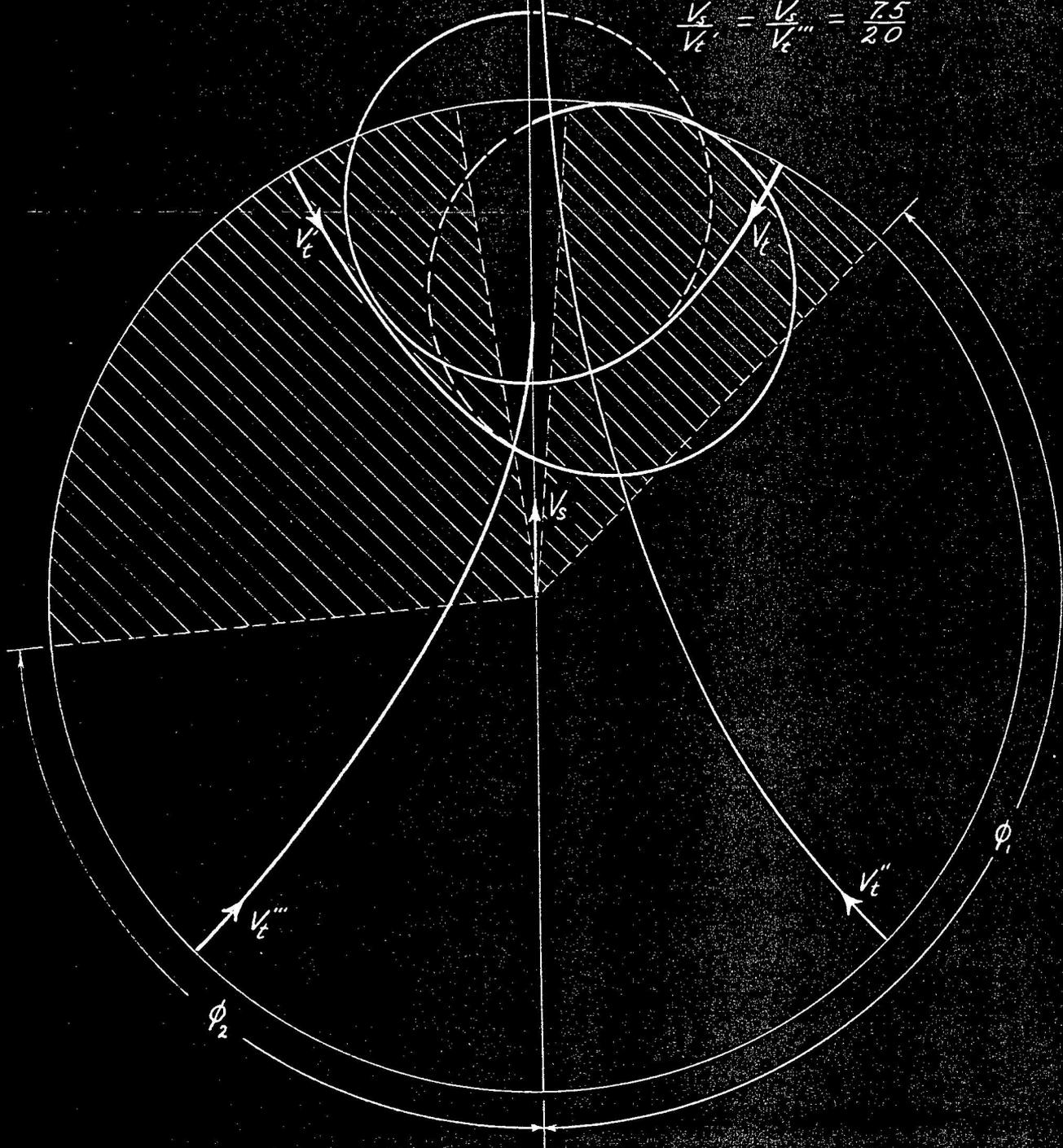
$$D \leq \frac{8h^2}{\lambda} .$$

For  $h = 10$  feet,  $\lambda = 1$  foot,  $D \leq 800$  feet

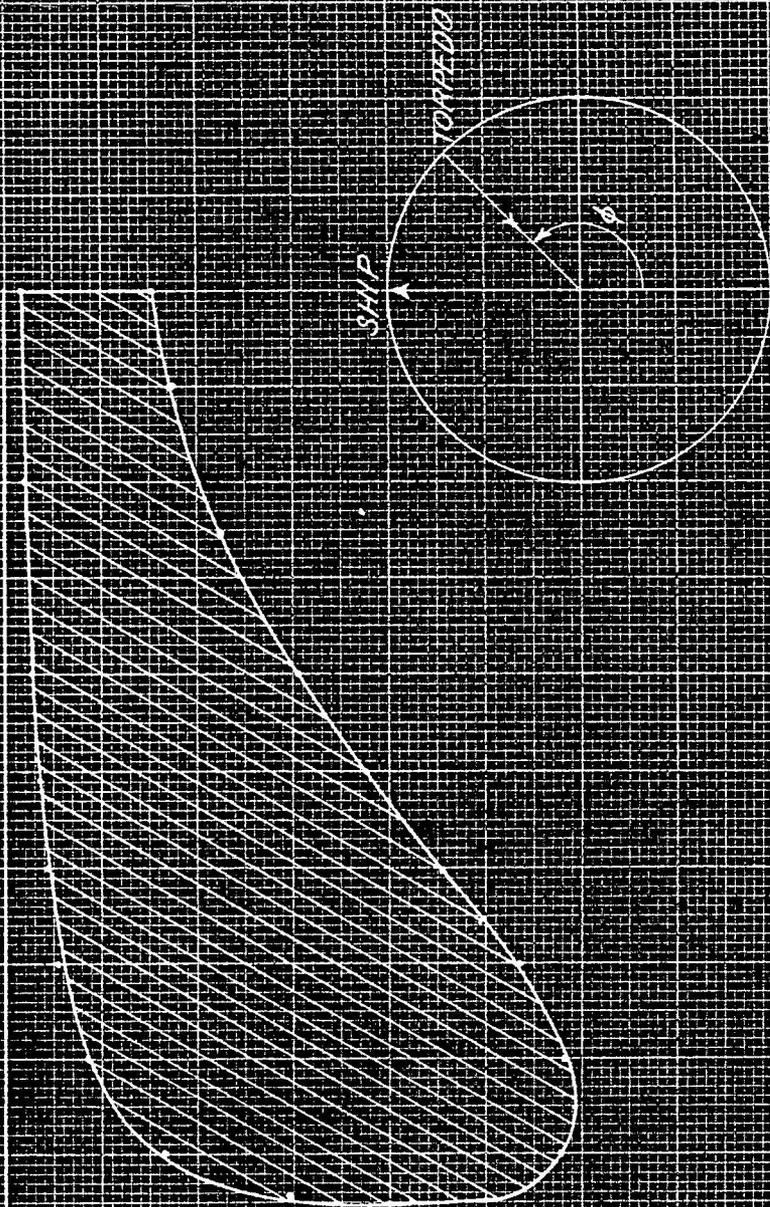
Assumed radius of circle, 800 ft.  
 Assumed speed ratios:

$$\frac{V_s}{V_t} = \frac{V_s}{V_t''} = \frac{15}{20}$$

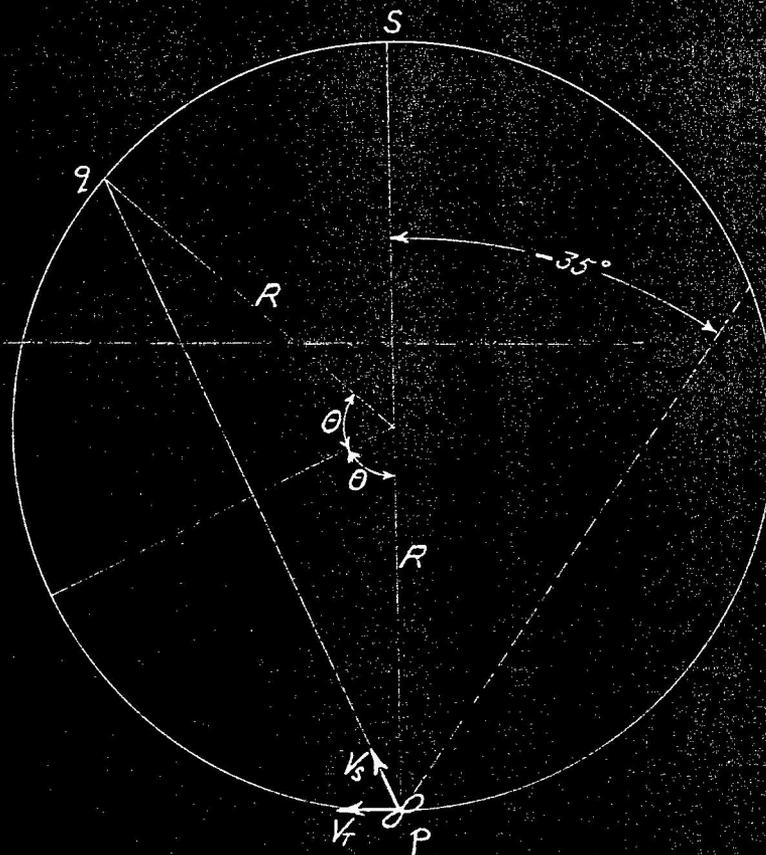
$$\frac{V_s}{V_t'} = \frac{V_s}{V_t'''} = \frac{7.5}{20}$$



IN SHADDED REGION TORPEDO CANNOT HIT SHIP (ASSUMED 30 FT. SHIP)  
 ASSUMING THAT 300 FT. IS THE MINIMUM POSSIBLE RADIUS OF CURVATURE  
 OF THE TORPEDO'S PATH



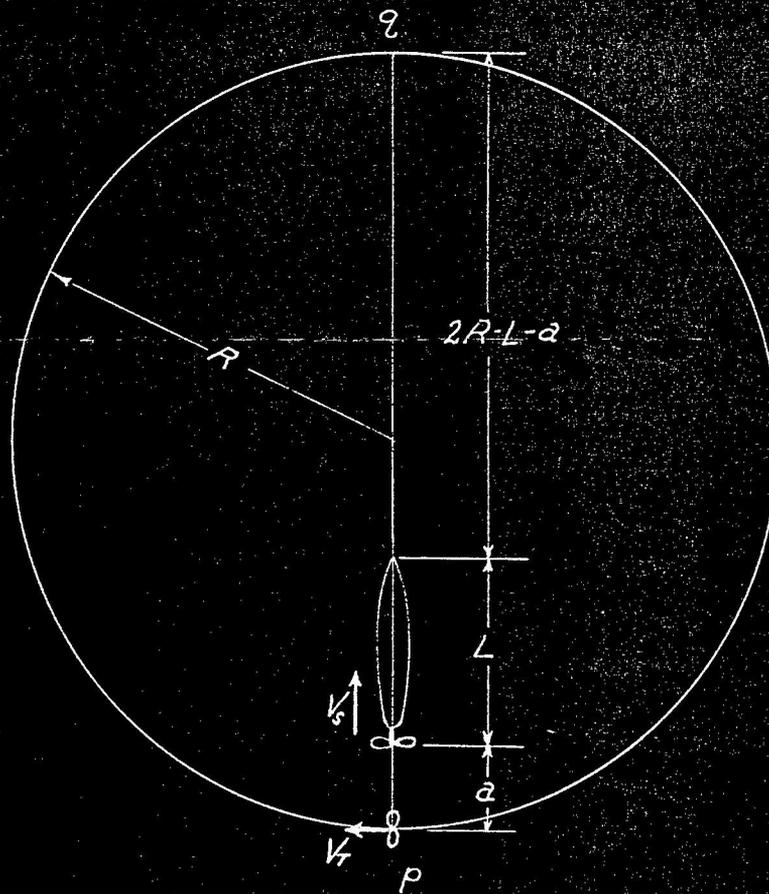
$$A = \frac{V_5}{V_7}$$



For a stern miss,

$$\frac{2R\theta}{V_T} \geq \frac{2R \sin \theta}{V_s},$$

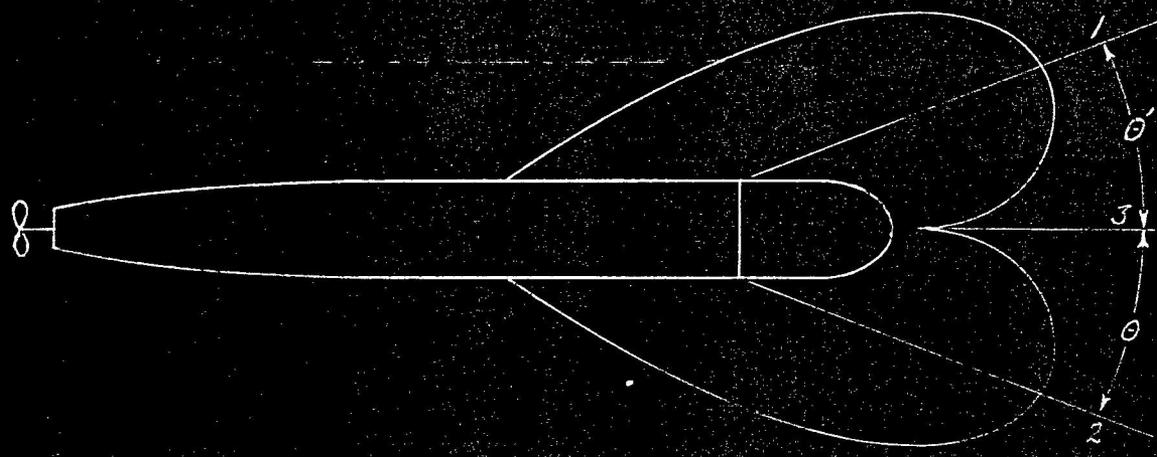
or 
$$\frac{\sin \theta}{\theta} \leq \frac{V_s}{V_T}.$$

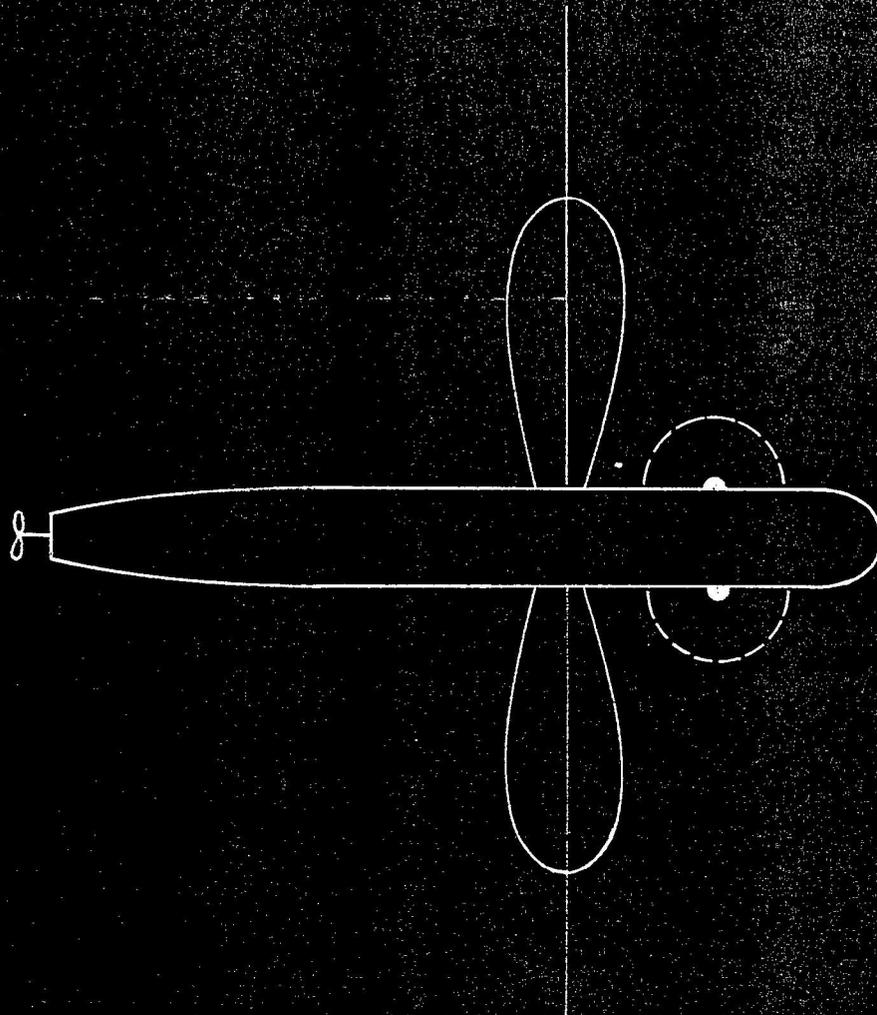


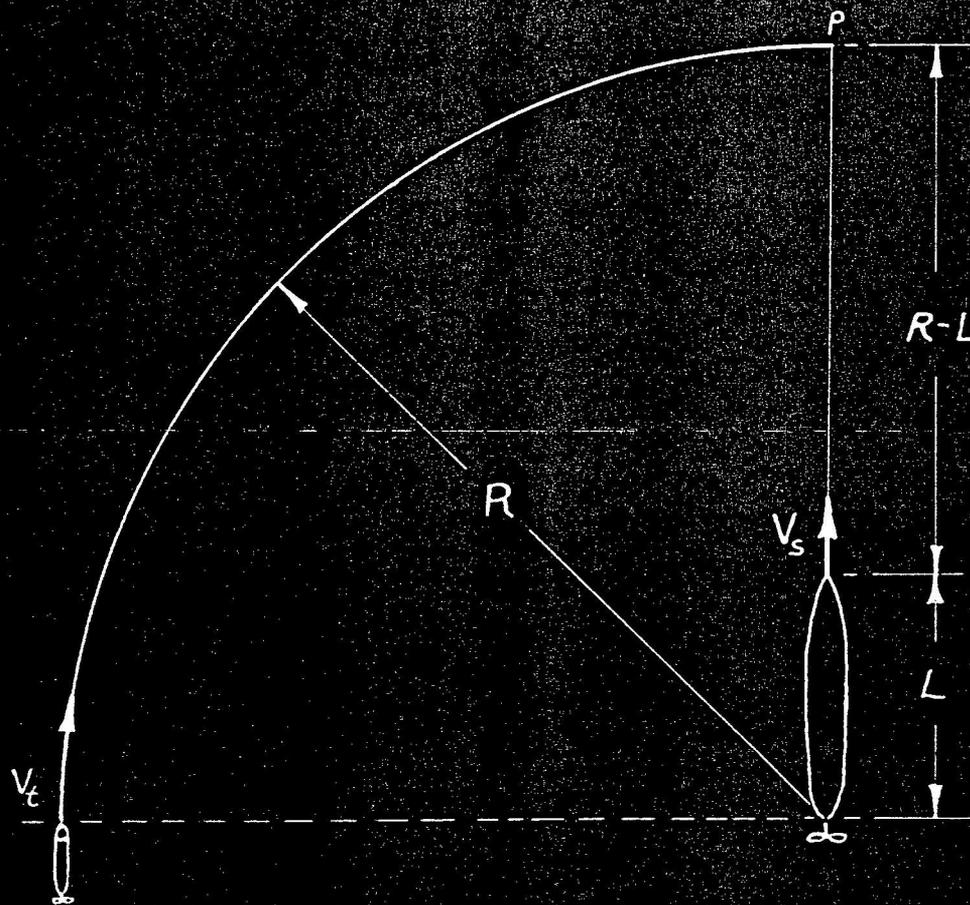
For a bow miss,

$$\frac{\pi R}{V_T} \leq \frac{2R-L-a}{V_S} ,$$

or  $\pi R \frac{V_S}{V_T} \leq 2R-L-a .$







To avoid bow hit :

$$\frac{\frac{\pi}{2} R}{V_t} \leq \frac{R-L}{V_s},$$

or

$$\frac{V_s}{V_t} \leq \frac{2}{\pi} \frac{\frac{R}{L} - 1}{\frac{R}{L}}$$

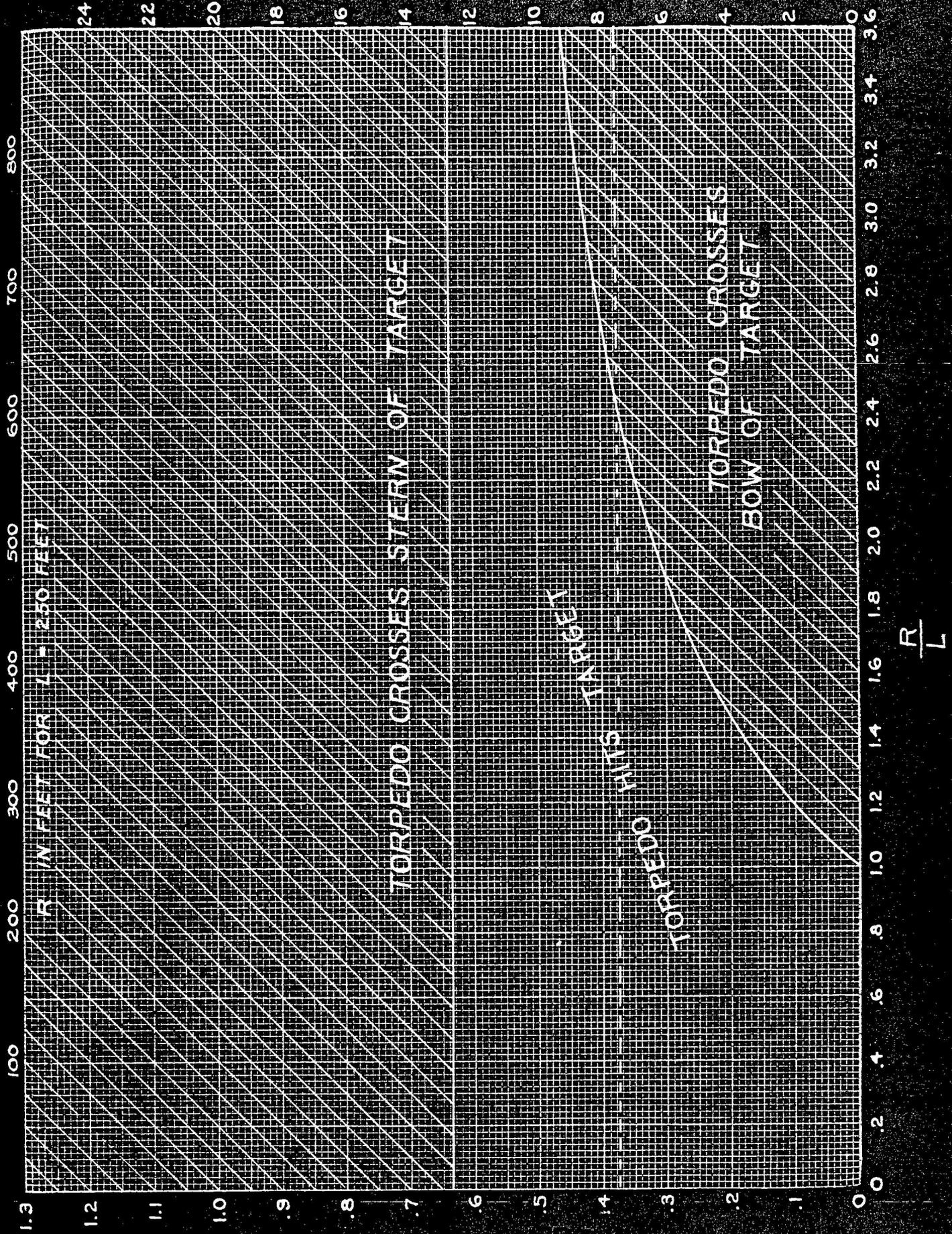
To avoid stern hit :

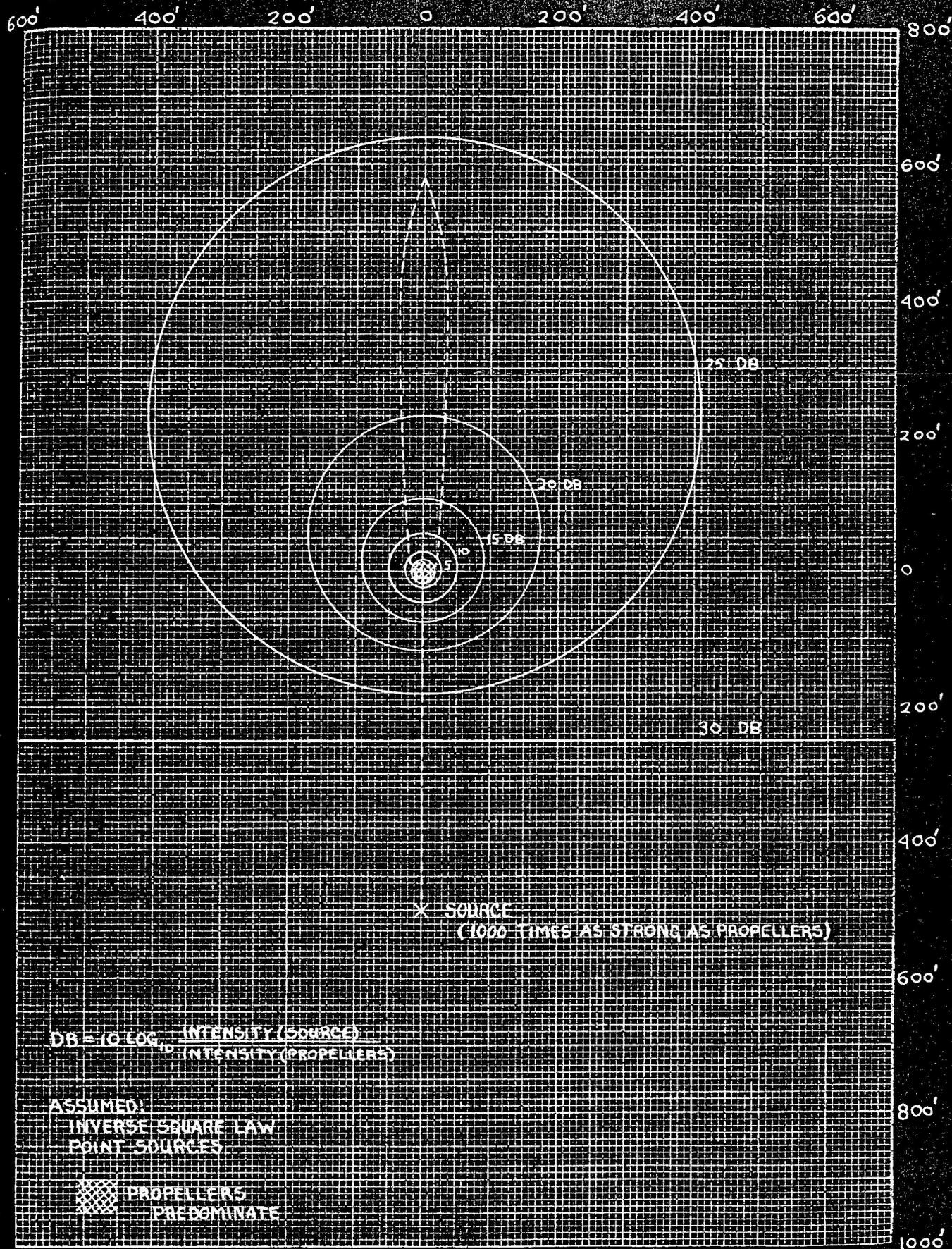
$$\frac{\frac{\pi}{2} R}{V_t} \geq \frac{R}{V_s},$$

or

$$\frac{V_s}{V_t} \geq \frac{2}{\pi}$$

$V_s$  IN KNOTS FOR  $V_t = 20$





UNCLASSIFIED