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A NELIAC-N Computer Program to Calculate the Components of the Earth's Magnetic Field

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A NELIAC-N Computer Program to Calculate the Components of the Earth's Magnetic Field

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Abstract: Experiments with the Lofti series of satellites have demonstrated that propagation can take place in the ionosphere at very low frequencies. In this frequency range the nature of the wave propagation depends critically upon the orientation of the wave with respect to the earth's magnetic field; therefore the orientation of both the magnetic field and the satellite, relative to the earth, must be known before satellite data can be analyzed in detail. Since the steady-state magnetic field of the earth at a given point in space can be calculated as the gradient of a potential function that satisfies Laplace's equation, this potential can be expressed as a spherical harmonic expansion and its gradient computed analytically. A computer program, written in NELIAC-N language, has been developed that calculates the magnitude and direction of the earth's magnetic field lines corresponding to a given spatial location of the satellite, using a spherical harmonic expansion with Jensen and Cain 1960 Schmidt-normalized coefficients.

INTRODUCTION

The earth's magnetic field, subsequently referred to as the terrestrial magnetic or geomagnetic field, can be viewed as the superposition of a steady-state, or permanent, field and a transient field. The latter component is the aggregate of short-term fluctuations in the terrestrial magnetic field that stem from many different sources; among these fluctuations are the daily variation, lunar-cycle variation, and variations caused by magnetic storms or changes in sunspot activity. The steady-state component is the portion of the terrestrial magnetic field that is quiescent in nature and changes only gradually over a long period of time. About every five years, magnetic measurements are made at stations and observatories throughout the world and then are processed statistically to isolate the steady-state component. Changes in the steady-state field that occur between times of measurement are usually linearly approximated.

The steady-state field also can be calculated by theoretical methods, specifically by the relation

$$\mathbf{H} = \nabla V,$$

where $\mathbf{H}$ is the magnetic intensity vector and V is the geomagnetic potential, both being functions of position relative to the earth. The potential V is found s the solution of Laplace's equation

$$\nabla^2 V = 0,$$

which is evaluated most conveniently as a spherical harmonic expansion (to be given later). Questions arise as to which boundary conditions to use and how many of them, with the latter question possibly being rephrased: where should one truncate the harmonic expansion?

A computer program that calculates the components of $\mathbf{H}$ for a given spatial position has been developed for the NAREC (Naval Research Electronic Computer). It is written as a function, *i.e.*, subroutine with input and output parameters, in the NELIAC-N compiler language. Although the program contains a fixed set of boundary conditions, provisions have been made for the programmer, at his option, to replace these with a different set.

NRL Problem R01-36; Project SF 019-02-02-7447. This report completes one phase of Lofti data analysis. Other phases are continuing. Manuscript submitted August 19, 1966.

THEORETICAL DISCUSSION

Coordinate Systems Used

Figure 1 indicates the relationship between the Cartesian and spherical coordinates of a point P in space. All coordinates refer to the terrestrial system, in which the origin of coordinates O coincides with the center of the earth. The z axis is colinear with the rotation axis, positive in the direction of the North Pole; the x axis is colinear with the intersection of the equatorial plane and the Greenwich meridian plane, positive in the direction of Greenwich; the position of the y axis is determined immediately by the requirements of orthogonality and right-handedness.

The Cartesian coordinates (x, y, z) of P are the lengths of the projections of the radius vector OP along the x, y, and z axes respectively. The spherical coordinates (r, θ, ϕ) of P are defined such that r is the length of the radius vector OP and such that θ and ϕ represent the geocentric colatitude and east longitude of P, respectively.

We may refer also to the orthogonal right-handed triad of unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_\phi$ originating at P. In this triad $\mathbf{e}_r$ is parallel to OP and points away from the origin O; $\mathbf{e}_\theta$ lies in the meridian plane of P, is perpendicular to $\mathbf{e}_r$, and points in the direction of increasing θ; and $\mathbf{e}_\phi$ is perpendicular to the meridian plane of P and points in the direction of increasing ϕ.

General Relations

The magnetic intensity $\mathbf{H}$ at a point (r, θ, ϕ) is the gradient of a magnetic potential $V = V(r, \theta, \phi)$ that satisfies Laplace's equation $\nabla^2 V = 0$. In spherical coordinates this magnetic potential is expressed (1) as

$$V = a \sum_{n=1}^{\infty} \left[\left(\frac{r}{a}\right)^n T_n^e + \left(\frac{a}{r}\right)^{n+1} T_n^i \right] = V^e + V^i, \tag{1}$$

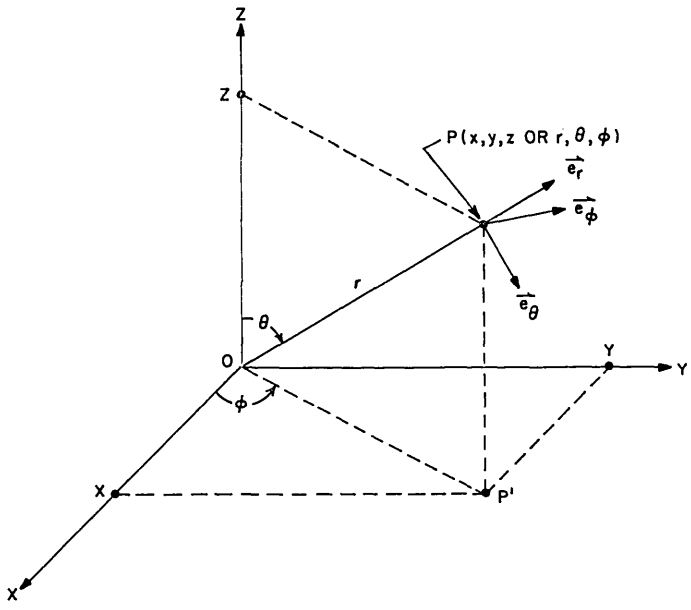

Fig. 1 – Relationship between Cartesian coordinates (x, y, z) and spherical coordinates (r, θ, ϕ)

where a denotes the equatorial radius of the earth, and the superscripts e and i refer respectively to parameters produced by sources external and internal to the earth. Recent spherical harmonic analysis of the earth's field indicates that most of the field is of internal origin; hence we can neglect V^e and the T_n^e terms. The quantity T_n^i is computed from the relation

$$T_n^i = \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi)\, P_n^m(\theta),$$

where g_n^m and h_n^m are the boundary coefficients and $P_n^m(\theta)$ is a spherical harmonic expression.

The coefficients g_n^m and h_n^m result from magnetic field measurements at several locations on (or possibly above) the earth's surface. The choice of the units of these coefficients determines the units of the computed magnetic intensity **H**, since they both have the same physical dimensions. Because of the diversity of scientific groups that make these measurements and the different techniques each group employs to obtain and process them, several sets of such coefficients are available, some having more elements than others, depending on the maximum value of n used. Furthermore, the epochs (particular times for which each set of coefficients can be used for computing **H**) of the sets differ from one another. The particular coefficients used in this program are the Jensen and Cain 1960 Schmidt-normalized coefficients (Table 1). They are available for $n = 1, 2, \ldots, 6$ and are stored in units of gammas. Thus the present NELIAC function truncates the summation given by Eq. (1) at $n = 6$.

TABLE 1
Jensen and Cain 1960
Schmidt-Normalized Coefficients*

n	m	g_n^m	h_n^m
1	0	30411.2050	0
1	1	2147.36855	−5798.90501
2	0	1602.35780	0
2	1	−2959.11288	1912.41919
2	2	−1545.12754	−182.320082
3	0	−1260.71461	0
3	1	2029.16029	485.642189
3	2	−1285.73407	−210.427010
3	3	−821.643102	26.6033864
4	0	−955.299735	0
4	1	−818.548632	−213.671410
4	2	−556.966078	255.698165
4	3	335.058632	20.5761409
4	4	−276.393247	187.290901
5	0	206.420660	0
5	1	−338.429996	−7.82856536
5	2	−253.044420	−26.0297301
5	3	−12.9235846	97.6833999
5	4	125.097951	109.109674
5	5	99.3217552	−173.622198
6	0	−135.225442	0
6	1	−25.6743872	−30.4621753
6	2	21.4913967	−58.4480602
6	3	214.927897	−34.1875112
6	4	19.2512242	−2.16539201
6	5	9.74850023	−47.9724735
6	6	165.955739	−48.3601505

*Furnished by J. C. Cain, of NASA.

It should be noted that some recent analyses of the earth's magnetic field include the oblateness of the earth (2), and the resulting equations require boundary coefficients that differ from those that would be obtained assuming a spherical earth. The program described in this report uses equations based upon the assumption of a spherical earth. Therefore, when substituting a new set of coefficients in this program, one should ascertain that the new set has been derived for a spherical and not an oblate earth.

The spherical harmonic expression $P_n^m(\theta)$ is defined as

$$P_n^m(\theta) = \begin{cases} P_{n,m}(\theta) & m = 0 \\ \dfrac{2(n-m)!}{(n+m)!}\, P_{n,m}(\theta) & m > 0, \end{cases}$$

where

$$P_{n,m}(\theta) = \frac{(2n)!}{2^n n!\,(n-m)!} \sin^m \theta \cos^{n-m} \theta \left[1 + \sum_{k=1}^{\infty} \frac{(-1)^k \cos^{-2k} \theta}{k!\, 2^k} \frac{(n-m)(n-m-1)\ldots(n-m-2k+1)}{(2n-1)(2n-3)\ldots(2n-2k+1)}\right].$$

The latter summation terminates, since all terms after $k = [(n - m)/2]$ are zero (where the brackets indicate greatest integer). (Note that the entire sum is zero when $n = m$ or $n = m + 1$.)

Spherical Components of Magnetic Intensity

The spherical components of **H** are computed from the relation

$$\mathbf{H} = \nabla V = H_r \mathbf{e}_r + H_\theta \mathbf{e}_\theta + H_\phi \mathbf{e}_\phi,$$

where H_r, H_θ, and H_ϕ are the components of **H** in the directions of $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_\phi$, respectively. Thus

$$H_r = \frac{\partial V}{\partial r},\ H_\theta = \frac{1}{r}\frac{\partial V}{\partial \theta},\ \text{and}\ H_\phi = \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi}.$$

These partial derivatives may be evaluated analytically, thus eliminating the need for numerical techniques.

Cartesian Components of Magnetic Intensity

The components H_x, H_y, and H_z of **H**, parallel to the positive x, y, and z axes, respectively, can be found by means of the matrix equation

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} H_r \\ H_\theta \\ H_\phi \end{pmatrix}.$$

PROGRAM DESCRIPTION

A typewritten copy of the program tape is given in Appendix A, and a sample program is given in Appendix B. A summary description of the program in a standard in-house format is the following.

1. **Name of Routine:** Computation of Geomagnetic Field.
2. **Class:** C-3. **Polynomials and Special Functions,** Evaluation of Special Functions.
3. **Purpose:** Given the three spherical coordinates of a point P in space, the function computes the spherical and Cartesian components of the magnetic intensity at that point. This is accomplished by use of a spherical harmonic expansion which incorporates a given set of boundary conditions adjustable at the option of the programmer. The particular boundary conditions currently installed in the function are the Jensen and Cain 1960 Schmidt-normalized coefficients.
4. **Language Used:** NELIAC-N.
5. **Authors:** Gerald Chayt, Gary Flenner, and Maurice Brinkman, NRL Code 6012, Building 47, Room 117.
6. **Flowchart Label and Name:** F5248 GEOMAG POTENTIAL.
7. **Information for Use:**
   Form of function call

GEOMAG POTENTIAL (R, COL, E LONG; HX, HY, HZ, HR, HTHET, HPHI),

where all parameters are dimensioned or expressed in NELIAC floating point and are defined as follows:

R contains (or is) the distance in kilometers between the center of the earth and point *P*;

COL contains (or is) the geocentric colatitude in radians of point *P*;

ELONG contains (or is) the east longitude, in radians of point *P*;

HX, *HY*, *HZ*, *HR*, *HTHET*, and *HPHI* are the names of the respective locations where the parameters H_x, H_y, H_z, H_r, H_θ, and H_ϕ are stored by the function, each parameter having units of gammas. (The preceding theoretical discussion gives the meaning of the symbols.)

8. **Use of Machine-Dependent Coding:** None

9. **Other Information:**

(a) A colatitude of 0° or 180° is not permitted, since the east longitude is undefined for these values. If the programmer, however, furnishes one of these values (in radians) for *COL* in the function call, the function will halt the computer with the printout "A COLATITUDE OF .0000000000 + 000 (or .3141592654 + 001) IS INCOMPATIBLE WITH SOME OF THE EQUATIONS USED IN THE FUNCTION GEOMAG POTENTIAL."

(b) The nouns GTABLE and HTABLE refer to tables, each of 27 elements, containing the coefficients g_n^m and h_n^m, respectively. These tables are dimensioned globally so that the programmer may, at his option, substitute a set of coefficients other than the Jensen and Cain 1960 coefficients, provided that the new set is for $n = 1, 2, \ldots, 6$ and thus has the same number of elements as the Jensen and Cain set. To substitute the new set the programmer stores the new coefficients into GTABLE and HTABLE prior to calling on the function. Each table is stored in a sequence of groups, each group containing coefficients with a fixed subscript n, the groups being arranged in increasing order of n. Within a group the coefficients are arranged in increasing order of m. That is, g_n^m is stored in GTABLE [I] and h_n^m is stored in HTABLE [I], where $I = (n(n+1)/2) + m - 1$. The physical dimensions of the new coefficients determine the dimensions of all computed magnetic intensity components. Once a set of new coefficients is stored in the tables, the function should be tested to ensure that the values of **H** computed for regions near the North Pole tend to point downward, *i.e.*, that H_r tends toward negative values as θ approaches 0°. If not, the signs of all the elements in the GTABLE and HTABLE should be reversed.

ACKNOWLEDGMENT

The authors are indebted to Dr. J. C. Cain, of NASA, who furnished the Jensen and Cain 1960 coefficients and offered many helpful suggestions.

REFERENCES

1. Chapman, S., and Bartels, J., "Geomagnetism," Oxford:Clarendon Press, Vol. II, p. 639, 1940
2. Cain, J. C., *et al.*, "An Evaluation of the Main Geomagnetic Field, 1940–1962," *J. Geophys. Res.* **70(No. 15)**:3647-3652 (Aug. 1965)

Appendix A
TYPEWRITTEN COPY OF TAPE F5248

```
7
(:PAGE F5248-1)
V|IRT ZER = 1 * - 19,
T|HETA.
P|HI.
S|INTHETA.
C|OSTHETA.
S|INMPHI (7) = 0.0,
C|OSMPHI (7) = 1.0,
S|UMTABLE (27).
S|UMTABLE 2K (27).
GTABLE (27) = 30411.205, 2147.36855, 1602.3578, -2959.11288, -1545.12754,
   -1260.71461, 2029.16029, -1285.73407, -821.643102, -955.299735, -818.548632,
   -556.966078, 335.058632, -276.393247, 206.42066, -338.429996, -253.04442,
   -12.9235846, 125.097951, 99.3217552, -135.225442, -25.6743872, 21.4013967,
   214.927897, 19.2512242, 9.74850023, 165.955739,
HTABLE (27) = 0.0, -5798.90501, 0.0, 1912.41919, -182.320082, 0.0, 485.642189,
   -210.42701, 26.6033864, 0.0, -213.67141, 255.698165, 20.5761409, 187.290901,
   0.0, -7.82856536, -26.0297301, 97.6833999, 109.109674, -173.622198, 0.0,
   -30.4621753, -58.4480602, -34.1875112, -2.16539201, -47.9724735,
   -48.3601505,
F|LINTEGERS (7) = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0,
M|NMAX = 6,
A|DRTAB = {SUMTABLE},
A|DRTAB2K = {SUMTABLE2K};
```

```
(:PAGE F5248-2)
GEOMAG POTENTIAL (RADIUS. COLAT = 0*0, ELONG. HSUBX. HSUBY. HSUBZ.
HSUBR. HSUBTHETA.
HSUBPHI. A = 6378.388, R. M, N, NN. AOVERR. QUOT. ANS. INA.):
{RADIUS → R, COLAT → THETA, ELONG → PHI,
SIN (THETA; SINTHETA),
SINTHETA = 0:
   ,{<>,<<A | COLATITUDE | OF | > COLAT < | IS | INCOMPATIBLE |
   WITH | SOME | OF | THE | EQUATIONS | USED | IN | THE |
   FUNCTION | GEOMAG | POTENTIAL.>>} XITGEOPOT.;
COS (THETA; COSTHETA), COSTHETA = 0: VIRT ZER → COSTHETA;;
M = 1 (1) MNMAX {FLINTEGERS[M] * PHI → INA,
SIN (INA; SINMPHI [M]), COS (INA; COSMPHI [M])}
SET SUM TABLES (MNMAX, COSTHETA, ADRTAB, ADRTAB2K;),

A/R → AOVERR → QUOT,
0 → HSUBR → NN,
N = 1 (1) MNMAX {NN + 1.0 → NN, AOVERR * QUOT → QUOT, TSUBN (N; ANS),
(NN + 1.0) * QUOT * ANS + HSUBR → HSUBR}
- AOVERR * HSUBR → HSUBR,
```

```
(:PAGE F5248-3)
AOVERR → QUOT,
0 → HSUBTHETA,
N = 1 (1) MNMAX {AOVERR * QUOT → QUOT,
PARTIALTSUBNTHETA (N; ANS),
QUOT * ANS + HSUBTHETA → HSUBTHETA}
AOVERR * HSUBTHETA → HSUBTHETA,

AOVERR → QUOT,
0 → HSUBPHI,
N = 1 (1) MNMAX {AOVERR * QUOT → QUOT, PARTIALTSUBNPHI (N; ANS),
QUOT * ANS + HSUBPHI → HSUBPHI}
AOVERR / SINTHETA * HSUBPHI → HSUBPHI,

SINTHETA * COSMPHI [1] * HSUBR + COSTHETA * COSMPHI [1] *
HSUBTHETA - SINMPHI [1] * HSUBPHI → HSUBX,
SINTHETA * SINMPHI [1] * HSUBR + COSTHETA * SINMPHI [1] *
HSUBTHETA + COSMPHI [1] * HSUBPHI → HSUBY,
COSTHETA * HSUBR - SINTHETA * HSUBTHETA → HSUBZ}
```

```
(:PAGE F5248-4)
T|SUBN (N¹, SUM. M¹, POFTHETA. I, SCR, SCRIPT,)
:{0 → SCR → SUM, I = N¹ (-¹) 2 {I + SCR → SCR}
M¹ = 0 (¹) N¹ {SCR + M¹ → SCRIPT, PMNFUNCTIONTHETA (M¹, N¹, SCRIPT; POFTHETA),
(GTABLE [SCRIPT] * COSMPHI [M¹] + HTABLE [SCRIPT] * SINMPHI [M¹] ) *
POFTHETA + SUM → SUM}}

P|ARTIALTSUBNTHETA (N2, TNTHETA. M2, PARPOFTHETA. I, SCR, SCRIPT,)
:{0 → SCR → TNTHETA, I = N2 (-¹) 2 {I + SCR → SCR}
M2 = 0 (¹) N2 {SCR + M2 → SCRIPT, PARTIALPMNOFTHETA (M2, N2, SCRIPT; PARPOFTHETA),
(GTABLE [SCRIPT] * COSMPHI [M2] + HTABLE [SCRIPT] * SINMPHI [M2]) *
PARPOFTHETA + TNTHETA → TNTHETA}}

P|ARTIALTSUBNPHI (N3, TNPHI. M3, POFTHETA. I, SCR, SCRIPT,)
:{0 → SCR → TNPHI, I = N3 (-¹) 2 {I + SCR → SCR}
M3 = 1 (¹) N3 {SCR + M3 → SCRIPT, PMNFUNCTIONTHETA (M3, N3, SCRIPT; POFTHETA),
(- GTABLE [SCRIPT] * SINMPHI [M3] + HTABLE [SCRIPT] * COSMPHI [M3]) *
FLINTEGERS [M3] * POFTHETA + TNPHI → TNPHI}}
```

```
(:PAGE F5248-5)
P|MNFUNCTIONTHETA (M4, N4, INDEX, ANS. A. B. C. Q,)
:{FRACFACT (M4, N4; A),
N4 - M4 → Q,
SINTOXCOSTOY (M4, Q; B),
M4 > 0:
   CORRECTION FACTOR (M4, N4; C), A * B * C * SUMTABLE [INDEX] → ANS;
   A * B * SUM TABLE [INDEX] → ANS;}

P|ARTIALPMNOFTHETA (M5, N5, INDEX, ANS. A. B. C. D. P, Q,)
:{FRACFACT (M5, N5; A),
M5 - 1 → P,
N5 - M5 - 1 → Q,
SINTOXCOSTOY (P, Q; B),
FLINTEGERS [M5] - (FLINTEGERS [N5] * SINTHETA * SINTHETA) → D,
B * D * SUMTABLE [INDEX] → C;
B * SINTHETA * SUMTABLE2K [INDEX] → D,
A * (C + D) → ANS, M5 > 0:
   CORRECTION FACTOR (M5, N5; C), C * ANS → ANS;;}
```

```
(:PAGE F5248-6)
F|RACFACT (M, N, ANSFF. NN. TWONFACT. WLOC. I, TWOTON.
NFACT. NMINUSMFACT.)
:{FLINTEGERS [N] → NN,
2.0 * NN → TWONFACT,
TWONFACT -1.0 → WLOC,
PT1.
L|P1:  WLOC -1.0 → WLOC,
P|T1:  WLOC = 1.0:
   PART2.
   TWONFACT * WLOC → TWONFACT, LP1.
P|ART 2:  1.0 → TWOTON,
I = N (-1) 1 {2.0 * TWOTON → TWOTON}
NN → NFACT,
NFACT - 1.0 → WLOC,
WLOC = 0:
   PART 3.
   PT2.
```

```
(:PAGE F5248-7)
L|P2:  WLOC - 1.0 → WLOC,
WLOC < 1.0:
   PART 3.;
P|T2:  NFACT * WLOC → NFACT, LP2.
P|ART3:  NN - FLINTEGERS [M] → NMINUSMFACT,
NMINUSMFACT - 1.0 → WLOC,
WLOC < 0:
   1.0 → NMINUSMFACT, PART4.
   PT3.
L|P3:  WLOC - 1.0 → WLOC,
WLOC < 1.0:
   PART 4.;
P|T3:  NMINUSMFACT * WLOC → NMINUSMFACT, LP3.
P|ART4:  TWONFACT / TWOTON / NFACT /
NMINUSMFACT → ANSFF}
```

```
(:PAGE F5248-8)
S|INTOXCOSTOY (X, Y, PRODUCT. I,)
:{⍞.0 → PRODUCT,
X = 0 ∩ Y = 0:
   EXIT.;
X = 0:
   SCPT.;
-X = ⍞:
   PRODUCT / SINTHETA → PRODUCT,
   Y = 0:
      EXIT.
      SCPT.;
   I = X (-⍞) ⍞ {SINTHETA * PRODUCT → PRODUCT}
   Y = 0:
      EXIT.
      SCPT.;
S|CPT: -Y = ⍞:
   PRODUCT / COSTHETA → PRODUCT, EXIT.;
I = Y (-⍞) ⍞ {COSTHETA * PRODUCT → PRODUCT} E|XIT:}
```

```
(:PAGE F5248-9)
C|ORRECTION FACTOR (M6, N6, ANS. MM. NN. WLOC.
NMINUSMFACT. MPLUSNFACT.)
:{FLINTEGERS [M6] → MM,
FLINTEGERS [N6] → NN,
NN - MM → NMINUSMFACT,
NMINUSMFACT - 1.0 → WLOC,
WLOC < 0:
   1.0 → NMINUSMFACT, PRT2.
   PNT1.
L|UP1:  WLOC - 1.0 → WLOC,
WLOC < 1.0:
   PRT2.;
P|NT1:  NMINUSMFACT * WLOC → NMINUSMFACT, LUP1.
P|RT2:  MM + NN → MPLUSNFACT,
MPLUSNFACT - 1.0 → WLOC,
L|UP2:  WLOC = 1.0:;
   MPLUSNFACT * WLOC → MPLUSNFACT, WLOC - 1.0 → WLOC, LUP2.
2.0 * NMINUSMFACT / MPLUSNFACT → ANS,
SQRT (ANS; ANS)}
```

```
(:PAGE F5248-70)

S|ET SUM TABLES (NMAX, COSTHETA: CT. A, A7, NFX, NFL. MFX, MFL. S2. C2.SUM. SUM2K.):
|0 → NFX → NFL → MFX, CT * CT → C2, 7.0 - C2 → S2,
SQRT (S2; S2), 7.0 / C2 → C2,

NFX = 7(7)NMAX |0→MFL, MFX  = 0(7)NFX |SUMMATIONS (NFX, MFX, NFL, MFL, C2, S2; SUM, SUM2K),
SUM + 7.0 → [A], SUM2K → [A7], A+7→A, A7+7→A7,
MFL + 7.0 → MFL} NFL + 7.0 → NFL}}

S|UMMATIONS (N, M, NN. MM. CTM2. SINTHETA. ANS. ANS2K. KK. KMAX, IN. ID. ITH TERM. INDEX):
|(N-M) / 2 → KMAX, NN - MM → IN, 2.0 * NN - 7.0 → ID, 7.0 → ITH TERM → KK,
0 → ANS → ANS 2K,
INDEX = 7(7) KMAX | -ITH TERM * CTM2 * IN * (IN - 7.0) / (KK * 2.0 * ID) → ITH TERM
* 2.0 * KK * SINTHETA + ANS2K → ANS2K, ITH TERM + ANS → ANS,
IN - 2.0 → IN, ID - 2.0 → ID, KK + 7.0 → KK}} X|ITGEOPOT:..
```

Appendix B
SAMPLE PROGRAM

This example of the computation of the total magnetic intensity and the magnetic inclination illustrates the use of the GEOMAG POTENTIAL function, and it produces results which can be compared with those of Vestine,* thereby providing a check of the correctness of the program.

The first computation performed is that of the total magnetic intensity $H = \sqrt{H_x^2 + H_y^2 + H_z^2}$ (denoted as F by Vestine*). This is followed by the computation of the magnetic inclination $I = \arctan\,(-H_r/\sqrt{H_\theta^2 + H_\phi^2})$. These computations are performed for values of geocentric latitude ranging from −80° to +80° in 10° increments and values of east longitude ranging from 0° to 300° in 60° increments, for each possible combination of latitude and longitude.

Included are typewritten copies of the program and results of the computer run, showing a reasonable agreement in magnitude and variation with the tabular values of Vestine* (one cannot expect close agreement because of the difference in epochs and in the means of obtaining results).

```
7
(:PAGE F 5248TEST-1)
TEST GP,,..
7
DEG PER RAD = 57.29577951, LATITUDE = 00,
FLOATING LAT. COLAT. EAST LONG. EL. HX.
HY. HZ. HR. HTHET. HPHI. HTOT. TOTAL = 000,
HRZ. TAN INC. INCL RAD. INCL (9, 6).
INCL PR = 000.0;

TOTAL COMP: PR HEADING H, 80. → FLOATING
LAT,
I = 0 (1) 8 |0 → EAST LONG, (90. - FLOATING LAT) /
DEG PER RAD → COLAT, FL TO FX (FLOATING
LAT; LATITUDE), |<LATITUDE|>:|
J = 0 (1) 5 |EAST LONG / DEG PER RAD → EL,
GEOMAG POTENTIAL (6378.388, COLAT, EL;
HX, HY, HZ, HR, HTHET, HPHI),
(:EQUATORIAL RADIUS OF EARTH = 6378.388 KM)
HX * HX + HY * HY + HZ * HZ → HTOT,
```

*E. H. Vestine, *et al.*, "Description of the Earth's Main Magnetic Field and Its Secular Change, 1905-1945," Washington:Carnegie Institution, Publication 578, pp. 454-469, 1947.

```
(:PAGE F 5248TEST-2)
SQRT (HTOT; HTOT), HTOT / ¬100. → HTOT,
FL TO FX (HTOT; TOTAL), {<|| TOTAL >:}
HTHET * HTHET + HPHI * HPHI → HRZ,
SQRT (HRZ; HRZ), -HR / HRZ → TANINC,
ARCTAN(TANINC; INCL RAD), INCL RAD *
DEG PER RAD → INCL [I, J], EAST LONG + 60.
→ EAST LONG}, {<>} FLOATING LAT - 20.
→ FLOATING LAT}

INCLINATION PRNT: PR HEADING I,
80 → LATITUDE, I = 0 (¬) 8 {, {<LATITUDE|>:}
J = 0 (¬) 5 {INCL [I,J] → INCL PR, {<|| INCL PR>:}},
{<>} LATITUDE - 20 → LATITUDE} STOP.

PR HEADING H: {, {<>,,,,, << TOTAL| MAGNETIC|
FIELD| IN| HUNDREDS | OF | GAMMAS>>,,,<< LATI-
ωω | EAST | LONGITUDE >><< TUDE ∪ 0 |||| 60 ||| ¬120
||| ¬180 ||| 240 ||| 300>>,}}
```

```
(:PAGE F 5248TEST-3)
PR HEADING I: {, {<>,,,,, << MAGNETIC |
INCLINATION | IN | DEGREES >>,,,<< LATI-
ωω | EAST | LONGITUDE>> << TUDE ∪ 0 ∪ | 60
∪ | ¬120 ∪ ¬180 ∪ 240 ∪ 300>>,}}
STOP:..
```

TESTGP

NELIAC-N COMPILER MOD 6.0 10 AUGUST 1965

NR	ROUTINE NAME	FIRST	LAST
01	TOTALCOMP	2700	29e0
02	GEOMAGPOTENTIAL	29e1	2db2
03	IOPACKAGE	2db3	30e7
04	LIBRARYPACKAGE	30e8	331b

TESTGP

NAME LIST DUMP

ARCTAN	323f
COLAT	271e
COMCOS	31a8
COMSIN	3226
COS	30e9
DEGPERRAD	271b
DOWNLINE	2e8b
DIVIDE	2d8a
EASTLONG	271f
EL	2720
ENDOFFILE	2ef2
ERROR	2976
FLOATINGLAT	271d
FLTOFX	3295
FLTOFXFR	317a
FXFRTOVL	3204
FIADD	288e
FIDIV	28ba
FIMUL	28a2
FISUB	2898
GEOMAGPOTENTIAL	2a6b
GTABLE	2a2b
HPHI	2726
HR	2724
HRZ	2729
HTABLE	2a46
HTHET	2725
HTOT	2727
HX	2721
HY	2722
HZ	2723
I	2701
INCL	272d
INCLINATIONPRNT	27bb
INCLPR	2763
INCLRAD	272b
IOPACKAGE	2e2c
J	2702
K	2703
L	2704
LATITUDE	271c
LIBRARYPACKAGE	30e8
M	2705
MULTIPLY	2d71
N	2706
PRHEADINGI	2800
PRHEADINGH	27de
PRINTOUT	2e31
SIN	31c2
SPLIT	3136
SQRT	32cc
STOP	2823
SUBSCR1P	29c5
SUBSCR2P	2859
TANINC	272a
TOTAL	2728
TOTALCOMP	2764
TOPOFFORM	2ef7

TOTAL MAGNETIC FIELD IN HUNDREDS OF GAMMAS

LATI-TUDE	EAST LONGITUDE 0	60	120	180	240	300
80	489	510	538	539	543	517
60	448	525	584	505	588	540
40	418	510	542	399	545	528
20	345	422	430	326	422	451
0	268	329	401	346	327	333
-20	225	302	503	449	344	242
-40	215	329	606	562	447	250
-60	317	429	643	646	557	378
-80	514	559	631	651	611	542

MAGNETIC INCLINATION IN DEGREES

LATI-TUDE	EAST LONGITUDE 0	60	120	180	240	300
80	81.5	83.7	87.2	85.5	88.0	84.1
60	73.1	76.8	77.5	69.1	80.3	79.8
40	58.6	62.0	59.0	49.8	65.6	70.6
20	32.5	35.7	29.0	25.1	44.2	55.8
0	-4.6	-3.8	-14.5	-10.6	11.9	34.1
-20	-36.8	-40.5	-51.0	-43.9	-26.1	5.6
-40	-47.2	-54.4	-72.7	-65.3	-52.3	-23.4
-60	-51.0	-60.9	-82.7	-79.3	-67.0	-47.9
-80	-65.6	-70.6	-79.1	-81.1	-74.2	-66.9

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A NELIAC-N COMPUTER PROGRAM TO CALCULATE THE COMPONENTS OF THE EARTH'S MAGNETIC FIELD

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A final report on one phase of Lofti data analysis.

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13. ABSTRACT

Experiments with the Lofti series of satellites have demonstrated that propagation can take place in the ionosphere at very low frequencies. In this frequency range the nature of the wave propagation depends critically upon the orientation of the wave with respect to the earth's magnetic field; therefore the orientation of both the magnetic field and the satellite, relative to the earth, must be known before satellite data can be analyzed in detail. Since the steady-state magnetic field of the earth at a given point in space can be calculated as the gradient of a potential function that satisfies Laplace's equation, this potential can be expressed as a spherical harmonic expansion and its gradient computed analytically. A computer program, written in NELIAC-N language, has been developed that calculates the magnitude and direction of the earth's magnetic field lines corresponding to a given spatial location of the satellite, using a spherical harmonic expansion with Jensen and Cain 1960 Schmidt-normalized coefficients.

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Partial differential equations						
Ionospheric propagation						
Low frequency						
Communication systems						
Satellites (artificial)						