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GREAT CIRCLE CALCULATIONS BY DIGITAL COMPUTER

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1. The following two lines should be inserted at the bottom of the table on page 5 of the subject report:

1944	t2003	u2433	b1748	t2013	u1940	b1423
1950	m1415	c2023	c2024	b1420	s1748	m2236

M. E. Jansson

M.E. Jansson
By direction

ABSTRACT

A 12-track program (excluding standard subroutines) which performs great circle calculations, required in connection with direction finding, radio propagation, and navigation problems, has been written for the LGP-30, a small, general-purpose digital computer. The program is characterized by simplified input and concise, unambiguous output.

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

AUTHORIZATION

NRL Problem R07-12

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GREAT CIRCLE CALCULATIONS BY DIGITAL COMPUTER

Great circle bearings and distances between places on the surface of the earth are often required in connection with radio-wave propagation, direction-finding, and navigation problems. These quantities may be obtained from navigation tables published by the Navy Hydrographic Office or by trigonometric calculation.* However, both of these methods are time consuming and subject to human error, particularly when computations involving many sites are required. A more advantageous method is that of using a digital computer which has been programmed to produce as output information the desired solutions of the great circle calculation problem. Such a program, based on a spherical earth, has been written for the LGP-30, a small, general-purpose digital computer, manufactured by Librascope and distributed by the Royal-McBee Corporation.

The LGP-30 is a serial, single-address, fixed-point, binary, stored-program digital computer with a 4096-word drum memory. There are 16 commands in the order code. Operating times for most arithmetic instructions are of the order of 2 and 19 milliseconds, respectively, for optimally and nonoptimally located operands. Multiplication and division times are of the order of 19 and 36 milliseconds for optimal and nonoptimal locations, respectively. An electric typewriter with paper-tape punch and reader and standard keyboard for manual entries is the normal input-output device.

The symbols and commands are listed below.

B mmmm Replace the contents of the accumulator with the contents of memory location mmmm.

A mmmm Add contents of mmmm to the contents of the accumulator and retain the sum in the accumulator.

S mmmm Subtract the contents of mmmm from the contents of the accumulator and retain the difference in the accumulator.

M mmmm Multiply the number in the accumulator by the number in memory location mmmm, terminating the result at 30 binary places, and retain the most significant half of the product in the accumulator.

N mmmm Multiply the number in the accumulator by the number in memory location mmmm and retain the least significant half of the product in the accumulator.

D mmmm Divide the number in the accumulator by the number in memory location mmmm and retain the quotient (rounded to 30 bits) in the accumulator.

H mmmm Store the contents in the accumulator in mmmm and retain the number in the accumulator.

*See, for example, "Reference Data for Radio Engineers," 3rd ed., New York: Federal Telephone and Radio Corporation, p. 419, 1949.

C mmmm Store the contents of the accumulator in memory location mmmm and clear the accumulator.

R mmmm Add one to the address held in the control counter register and record it in the address portion of the instruction in memory location mmmm.

E mmmm Extract a portion of the contents of the accumulator by performing a Boolean multiplication with the contents of memory location mmmm and retain the result in the accumulator.

U mmmm Transfer control to location mmmm unconditionally.

T mmmm Transfer control to memory location mmmm only if the number in the accumulator is negative.

I mmmm This command fills the accumulator from the Flexowriter. If tape input is being used, tape reading continues until a stop code is read.

P mmmm Print an electric typewriter symbol "." indicated by six of the bits in memory location mmmm.

Z mmmm Stop.

Y mmmm Store only the address portion of the word in the accumulator in memory location mmmm, leaving the rest of the word in mmmm undisturbed.

THE COMPUTATION

Given the latitude and longitude of two places on the surface of the earth, the solutions of the great circle calculation are the bearing of the first site with respect to the second, the bearing of the second site with respect to the first (inverse bearing), and the great circle distance between the two sites. The program, as presently written, performs these computations for a number of sites ($0 < n \leq 999$) with respect to a reference site.

Because maximum advantage of a digital computer is realized when the input and output data are retained in a form as nearly as possible like that in which the original problem was formulated, a special data input subroutine was designed, as part of the general program, to enable the user to enter data without special code words or the necessity of transformation. Latitude and longitude, which are ordinarily expressed in degrees, minutes, and seconds, are entered directly in this form. Also printout is in a form which is directly readable, concise, and unambiguous: site latitudes, longitudes, and bearings are given in decimal degrees and distances, in nautical miles.

The program is entered by means of an electric typewriter or, preferably, by punched tape via tape reader as an exchange between user and computer in the form of questions and answers as follows:

Computer: Coordinates of reference site, please.

Response: 0453327′1352218′

The response, entered as two groups of seven digits (plus sign if negative), each group followed by a stop code, represents a reference site located at 45°33′27″ north latitude and 135°22′18″ west longitude. North latitudes and west longitudes are entered as positive; south latitudes and east longitudes are negative.

Computer: Number of sites?

Response: 3′.

Any integer from 1 to 999 is entered, followed by a stop code, representing the number of sites (in this case 3) for which calculations are to be made with respect to the given reference site.

Computer: Coordinates of sites

Response: -0372437′-1281519′
0253015′ 0900000′
0450000′ 0103000′

Two seven-digit groups followed by stop codes are entered for each of the sites. The pair of words for the first site shown above represents a site located at $37^{\circ}24'37''$ south latitude and $128^{\circ}15'19''$ east longitude.

The computer then performs the great circle calculation in the following sequence.

Each word of site data (in degrees, minutes, and seconds) is converted to decimal degrees, binarized, scaled, and stored. The reference data are examined to determine that they fall within allowable limits and if not, the computer prints error and calls for new reference data. The sine and cosine of the reference latitude are then computed and stored. The computer next prints the reference latitude and longitude and begins to analyze the data for the computation of the first site. The coordinates of the reference and other site are examined for existence of particular solutions (north-south, east-west bearings, same site, or antipodal site). If any of these bearings exist, the computer sets up the proper direct and inverse bearings and great circle distance and prints the result. If the same site or antipodal site condition exists, the computer prints this result. If the locations of the sites are such that none of the particular solutions obtains, the computer solves the following spherical trigonometric equations:

$$a = \arccos\left(\sin L_r \sin L_s + \cos L_r \cos L_s \cos A\right)$$

$$B = \arccos\left[\frac{\sin L_r - \sin L_s \cos a}{\cos L \sin a}\right]$$

$$C = \arccos\left[\frac{\sin L_s - \sin L_r \cos a}{\cos L_r \sin a}\right]$$

where L_r is the latitude of the reference site,

L_s is the latitude of the other site,

A is the difference in longitude between the two sites,

B is the bearing at the reference site,

C is the bearing at the other site (inverse bearing), and

a is the great circle distance between the sites.

Finally, the computer converts bearings to module $360^{\circ} \pm 0.01^{\circ}$ and great circle distances to nautical miles ±0.01 and prints the results in the following form:

Reference

Lat.	Long.
45.56	135.37

	Site		Bearing	Bearing	Distance
	Lat.	Long.	(at reference)	(at other site)	(nautical miles)
001	-37.41	-128.26	245.34	053.24	07182.10
002	25.50	090.00	103.24	310.96	02477.45
003	45.00	010.50	036.51	323.90	04631.23

When the computations for all sites have been printed, a re-entry is made and a new series of question and answers is initiated as before. The computing and printout time is of the order of 30 seconds per site.

The complete coding in machine language appears in Appendix A; a program flow diagram, in Appendix B. The program utilizes several subroutines. Their initial locations are tabulated below.

Subroutine	Initial Location
Alphanumeric (α#)	0500
Decimal printout	0600
Sine-cosine	0900
Arc sine	1000
Square root	1100
Binarizing	1300

SUMMARY

It has been shown that a small, general-purpose digital computer may be programmed to perform accurate great circle calculations and produce the required output in concise, unambiguous form. The input and output data are retained in the form in which the original problem was formulated, thus eliminating a prime source of error by insuring continuity of thought on the part of the problem solver.

APPENDIX A

GREAT CIRCLE CODING FOR THE LGP-30

```
1500  r0500 u0500 ,2020106f ,0846461f ,2f223272 ,5f4f7f06
1506  ,465406lf ,4f544flf ,4f326f4f ,067f225f ,4f360642 ,0j4f727f
1512  ,4f2q207q b1408 c1410 b1419 y1348 u1518
1518  r1359 u1300 b1420 h6300 t1527 b1422
1524  s6300 t1531 u1539 c6301 s6301 c6300
1530  u1523 r0500 u0500 ,20100j08 ,725f225f ,522f4f06
1536  ,4flf1f46 ,1f2q207q u1500 b1421 h6300 t1546
1542  b1423 s6300 t1550 u1558 c6301 s6300
1548  c6300 u1542 r0500 u0500 ,20100j08 ,46325j22
1554  ,5f522f4f ,064flflf ,461f2q7q u1500 r0500 u0500
1560  ,20103208 ,523f0f4f ,1f064654 ,067f225f ,4f7f207q c6300
1602  p0000 i0000 n1662 h6300 h6301 e1417
1608  m1403 a6301 c6301 b6300 e1418 m1402
1614  a6301 h1620 m1415 s1408 c1410 u1621
1620  ,wwwwwwwj b1620 m1416 s1408 c1620 r0500
1626  u0500 ,20106f08 ,46461f2f ,2232725f ,4f7f0646 ,54067f22
1632  ,5f4f7f7q b2443 y1348 u1636 r1359 u1300
1638  r0500 u0500 ,2020101f ,084f544f ,1f4f326f ,4f200610
1644  ,0j08725f ,2q300606 ,100j0846 ,325j2q06 ,207q0000 b1420
1650  n1408 r0609 u0600 z0002 b1421 m1700
1656  r0609 u0600 z0003 p1600 z0000 u1703
1662  z0004 z1500 ,40000000 z1500 z1701 b1420
1704  r0949 u0900 c1424 b1420 r0949 u0904
1710  c1425 b1408 ,800g18g0 b2443 y1717 a1408
1716  y1728 b2720 h6301 t1726 b1422 s6301
1722  t2405 u1728 c6301 s6301 c6301 u1722
1728  b2721 h6301 t1735 b1423 s6301 t2415
1734  u1739 c6301 s6301 c6301 u1731 b1717
1740  y1743 a1408 y1745 b2720 c1748 b2721
1746  c1749 u1750 ,09fqg852 ,13wq61k8 b1420 t1800
1752  s1422 t1814 b1748 t1759 s1422 t2029
1758  u2425 c6300 s6300 s1422 t2029 u2433
1800  c6300 s6300 s1422 t1814 b1748 t1809
1806  s1422 t2038 u2433 c6300 s6300 s1422
1812  t2038 u2425 b1748 t1819 s1422 t2525
1818  u2047 c6300 s6300 s1422 t1824 z0500
1824  b1421 t1849 b1749 t1836 c6300 s6300
1830  a1421 t1907 c6300 s6300 t1907 u1852
1836  a1421 t1907 c6300 s6300 t1907 b1423
1842  a1749 s1421 t2108 c6300 s6300 t2108
1848  u2433 b1749 t2517 u1836 b1420 t1862
1854  b1748 t1949 s1420 t1949 c6300 s6300
1860  t1958 u2425 b1748 t1901 u1958 s1420
1902  t1949 c6300 s6300 t1958 u2425 b1421
1908  t1926 b1749 t1918 s1421 t2618 s1423
1914  t2108 u1930 ,wwjw3j00 z1500 b1421 s1749
1920  s1423 t2108 c6300 s6300 t2108 u1930
1926  b1749 t2108 s1421 u1920 b1420 t1946
1932  b1748 t1940 s1420 t2003 c6300 s6300
1938  t2003 u2623 a1420 t2013 c6300 s6300
```

```
1956  c2025 u2325 b1423 m1415 c2024 c2023
1962  b1748 s1420 m2236 c2025 u2325 b1420
2004  a1748 c6300 b1423 s6300 m2236 c2025
2010  c2023 c2024 u2325 b1423 m1415 h2023
2016  c2824 b1423 a1420 a1748 m2236 c2025
2022  u2325 ,247g604f ,0kqk80qj ,0008j1j2 z2023 z2024
2028  z2025 b1423 m1415 c2023 c2024 b1422
2034  s1748 m2236 c2025 u2325 b1423 m1415
2040  c2024 c2023 b1422 a1748 m2236 c2031
2046  u2325 b1423 m1415 c2024 c2023 b1422
2052  s1420 m2236 c2025 u2325 b1423 m1415
2058  c2023 c2024 b1422 a1420 m2236 c2025
2100  u2325 b1421 s1749 t2108 c6300 s6300
2106  t2108 u1852 b1748 r0949 u0900 c1426
2112  b1748 r0949 u0904 c1427 b1421 s1749
2118  r0949 u0904 m1425 m1427 c2231 b1424
2124  m1426 m1415 a2231 n1408 h2232 r1021
2130  u1211 h2233 r0949 u0900 h2234 m1425
2136  c2231 b1424 m2232 c2235 b1426 m1415
2142  s2235 m1415 d2231 r1021 u1211 m1415
2148  c2023 b1427 m2234 c2231 b1426 m2232
2154  c2235 b1424 m1415 s2235 m1415 d2231
2160  r1021 u1211 m1415 c2024 b2233 m2236
2202  c2025 b1421 t2215 b1749 t2210 s1421
2208  t2227 u2223 b1421 s1749 s1423 t2227
2214  u2223 b1749 t2221 s1421 s1423 t2223
2220  u2227 s1421 t2227 b1423 s2023 c2023
2226  u2325 b1423 s2024 c2024 u2325 ,00411jwf
2232  ,3www2648 ,00255jk6 ,00f6qw58 ,1405118q ,1q000000 u2238
2238  b2321 c2322 b2318 h2245 h2247 a2319
2244  c2249 p0200 z0000 p0600 z0000 p0600
2250  z0000 p2400 z0000 b2322 s2321 h2322
2256  t2259 b2323 y2237 u2327 b2249 a2319
2262  h2249 s2320 t2245 b2318 c2249 b2247
2304  a2319 h2247 s2320 t2245 b2318 c2247
2310  b2245 a2319 h2245 s2320 t2245 b2324
2316  y2237 u1500 p0200 z0400 p4200 z0001
2322  ,wwwwwwk8 z2260 z2238 r2259 u2237 b1748
2328  n2321 c2363 b1749 m1700 c2400 b2363
2334  r0609 u0600 z0002 b2400 r0609 u0600
2340  z0003 b2023 r0609 u0600 z0003 b2024
2346  r0609 u0600 z0003 b2025 r0609 u0600
2352  z0005 b1620 s2321 h1620 t2315 b1728
2358  a2321 y1717 a2321 y1728 u2402 ,26gfq148
2400  ,09ww30qj z2363 p1600 z0000 u1717 r2259
2406  u2237 r0500 u0500 ,100j0872 ,5f225f52 ,2f4f064f
2412  ,1f1f461f ,7q000000 u2353 r2259 u2237 r0500
2418  u0500 ,100j0846 ,325j225f ,522f4f06 ,4f1f1f46 ,1f7q0000
2424  u2353 r2259 u2237 r0500 u0500 ,107f0872
2430  ,3f4f067f ,225f4f7q u2353 r2259 u2237 r0500
2436  u0500 ,10720832 ,5f224246 ,2f720j06 ,7f225f4f ,7q000000
2442  u2353 z2700 r0500 u0500 ,20203006 ,06060606
2448  ,0606107f ,08225f4f ,3030100f ,084f721f ,22325j06 ,30100f08
2454  ,4f721f22 ,325j3010 ,2f08227f ,5f72326f ,4f203006 ,06100j08
2460  ,725f2q06 ,3006100j ,0846325j ,2q302828 ,104j0872 ,5f061f4f
2502  ,544f1f4f ,326f4f10 ,04302828 ,4j08725f ,06465f62 ,4f1f067f
2508  ,225f4f10 ,04302828 ,4j083272 ,525f226f ,720j063f ,220j4f7f
2514  ,10040806 ,20207q00 u1713 c6300 s6300 a1421
2520  t1907 c6300 s6300 t1907 u1852 b1420
2526  c6300 s6300 t1824 b1748 c6300 s6300
2532  t1824 b1421 t2552 c6300 s6300 t2604
2538  b1749 t2632 c6300 s6300 t2628 u2425
2544  b1749 m2236 c2025 b2617 c2023 b2616
```

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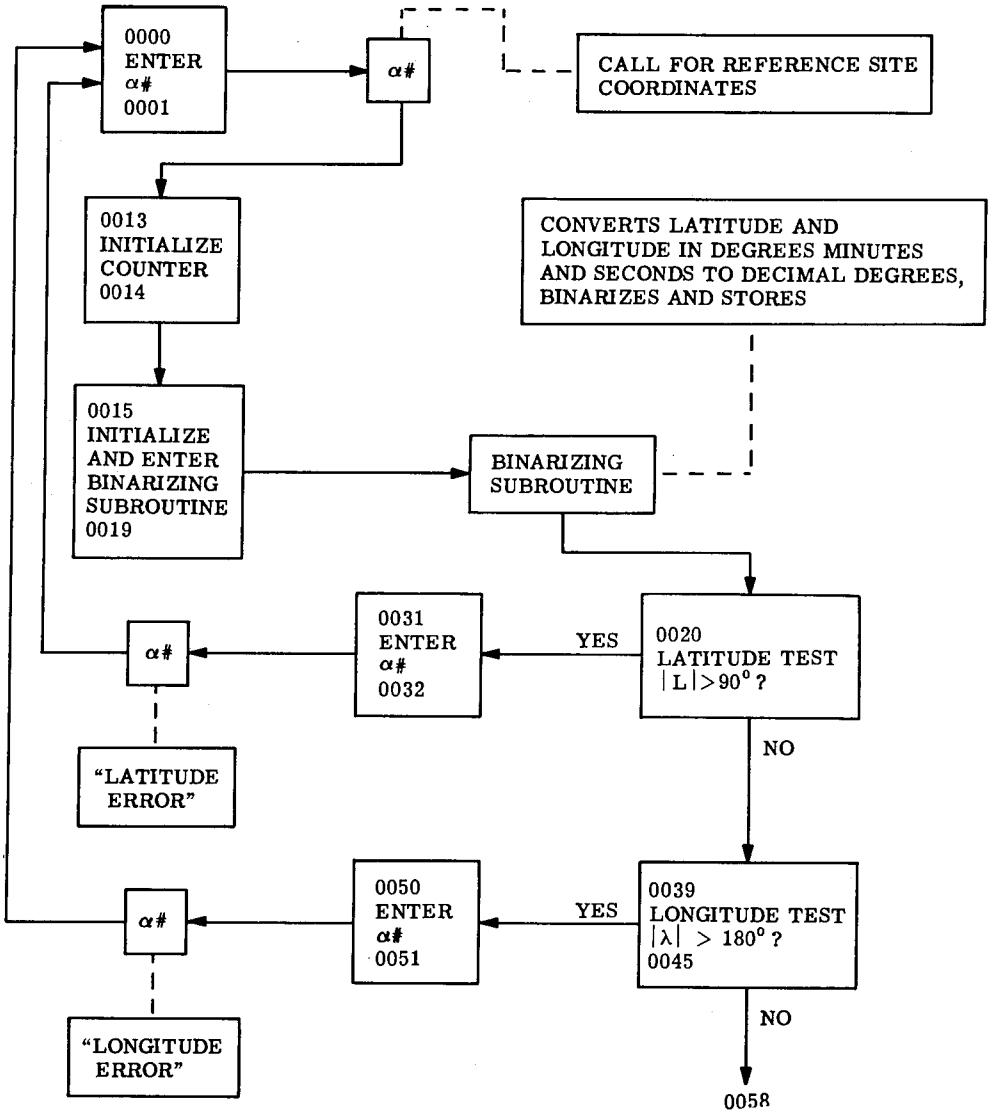
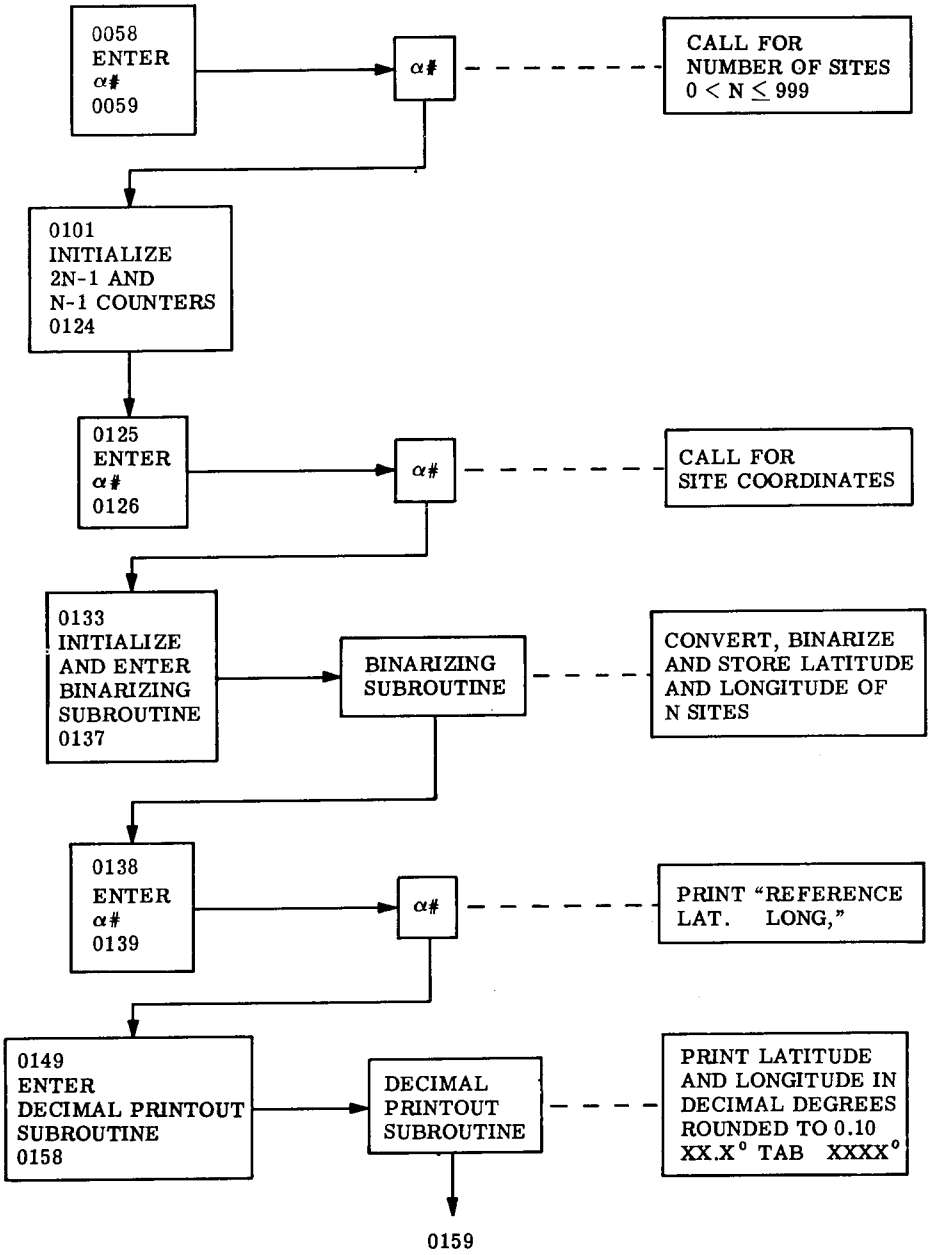
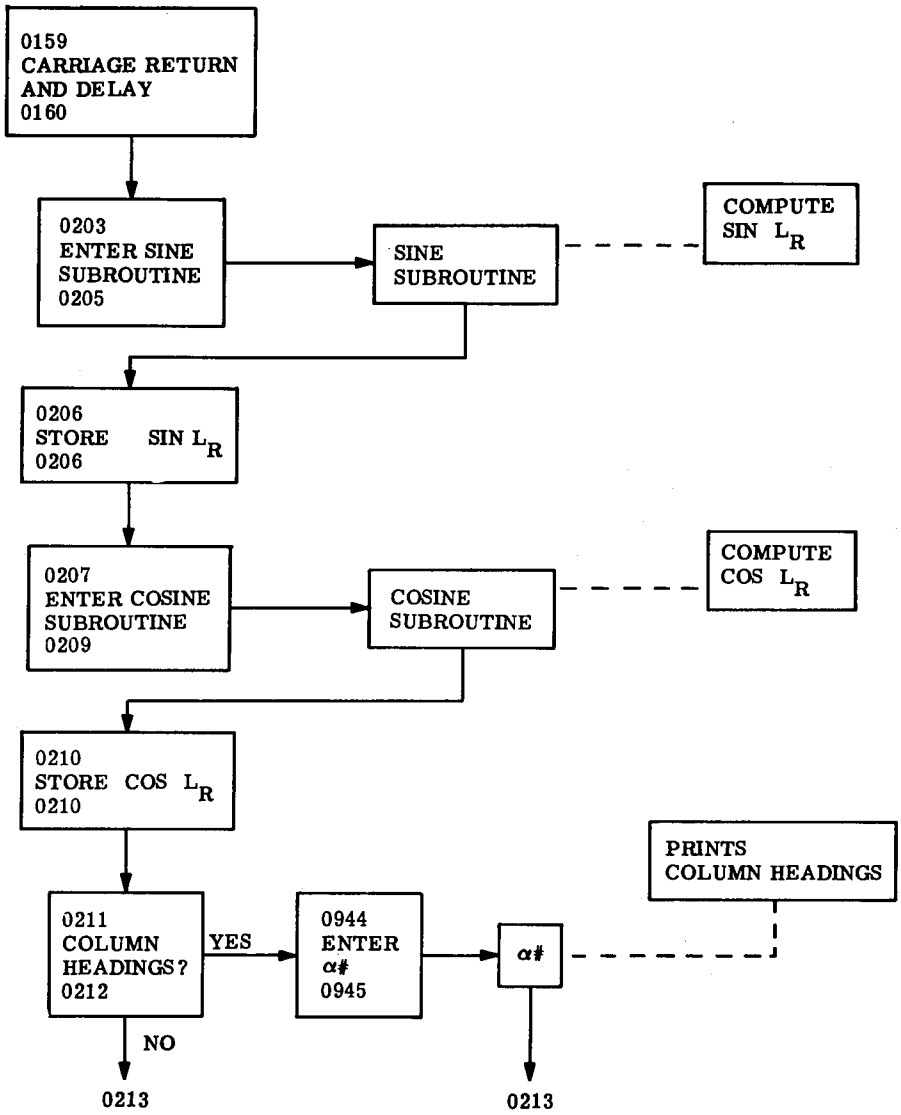
2550  c5224 u2325 b1749 t2556 s1421 u2545
2556  s1421 t5759 u2545 c6300 s6300 m2236
2562  c2025 b2616 c2023 b2617 c2024 u2325
2604  b1749 t2609 s1421 t2559 u2545 c6300
2610  s6300 a1421 u2561 c6300 s6300 u2561
2616  ,0g400000 ,21j00000 c6300 s6300 s1423 t2108
2622  u1930 b1748 c6300 s6300 t2003 u2433
2628  b1749 s1423 t2544 u2433 c6300 s6300
2634  h6300 s1423 t2638 u2433 b6300 u2561
```

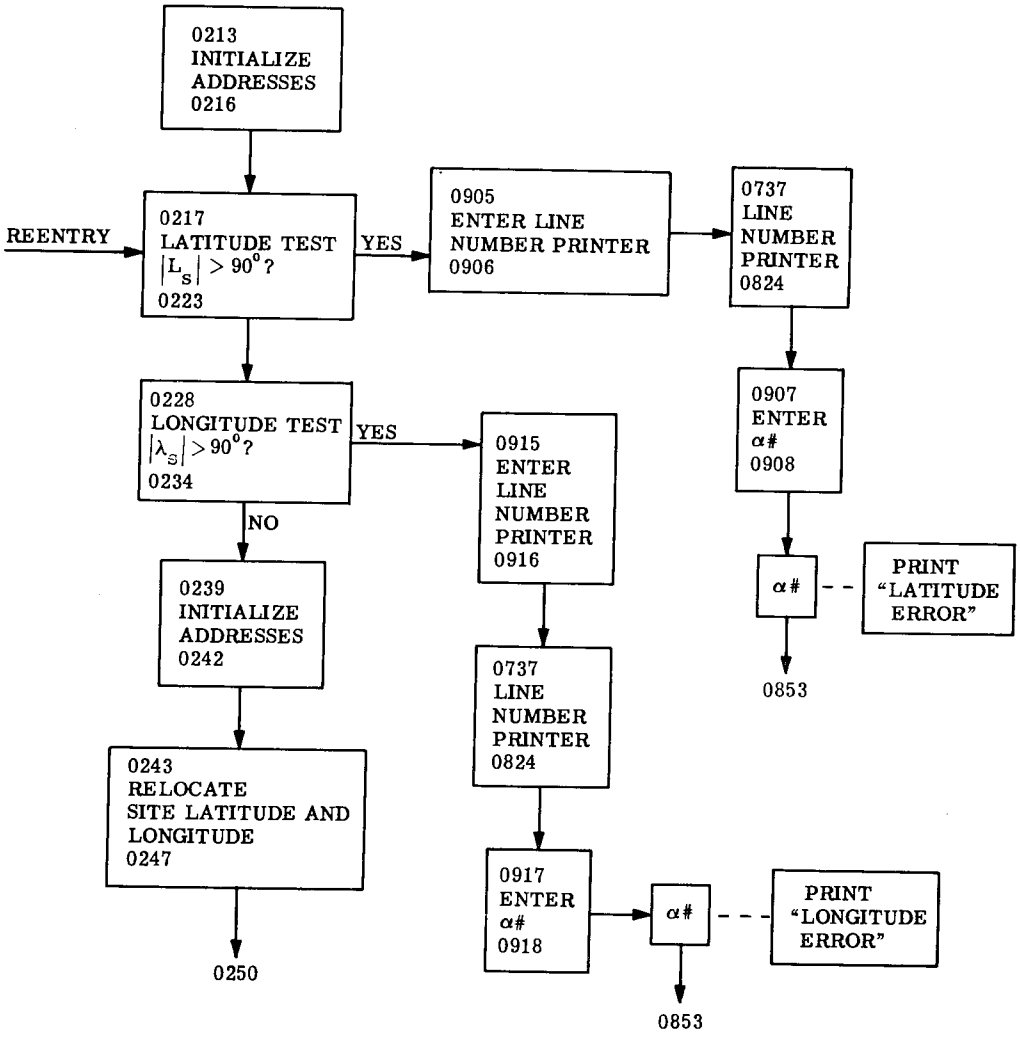
APPENDIX B

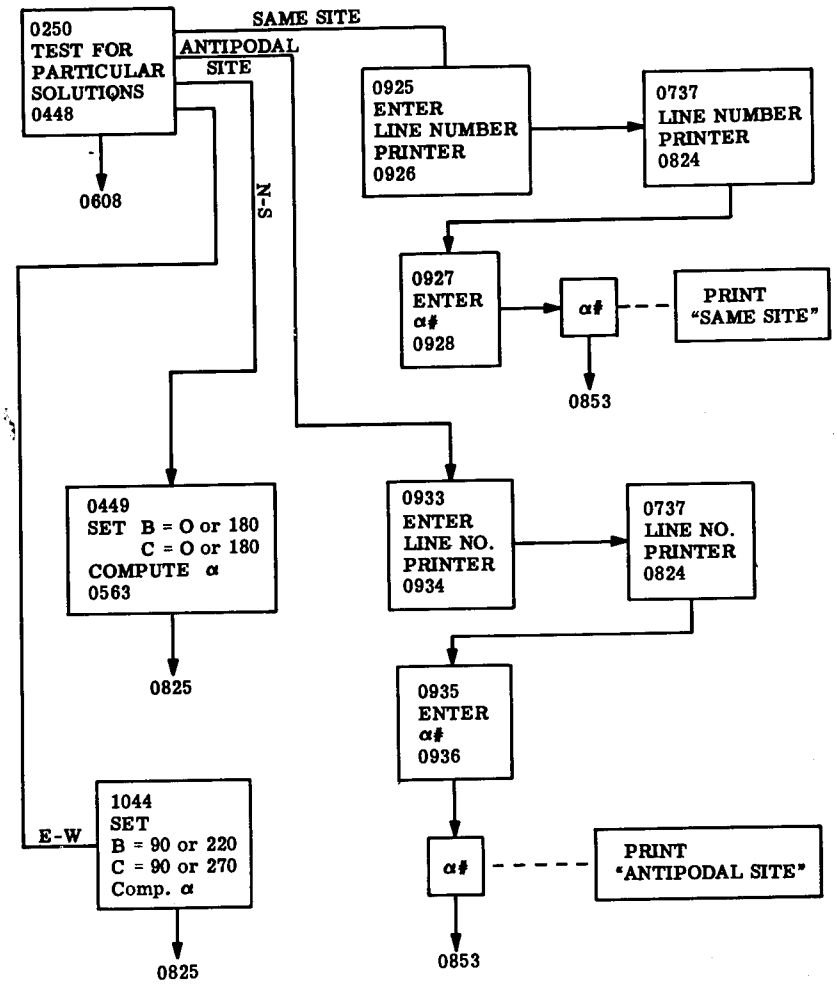
FLOW CHART OF GREAT CIRCLE CALCULATION

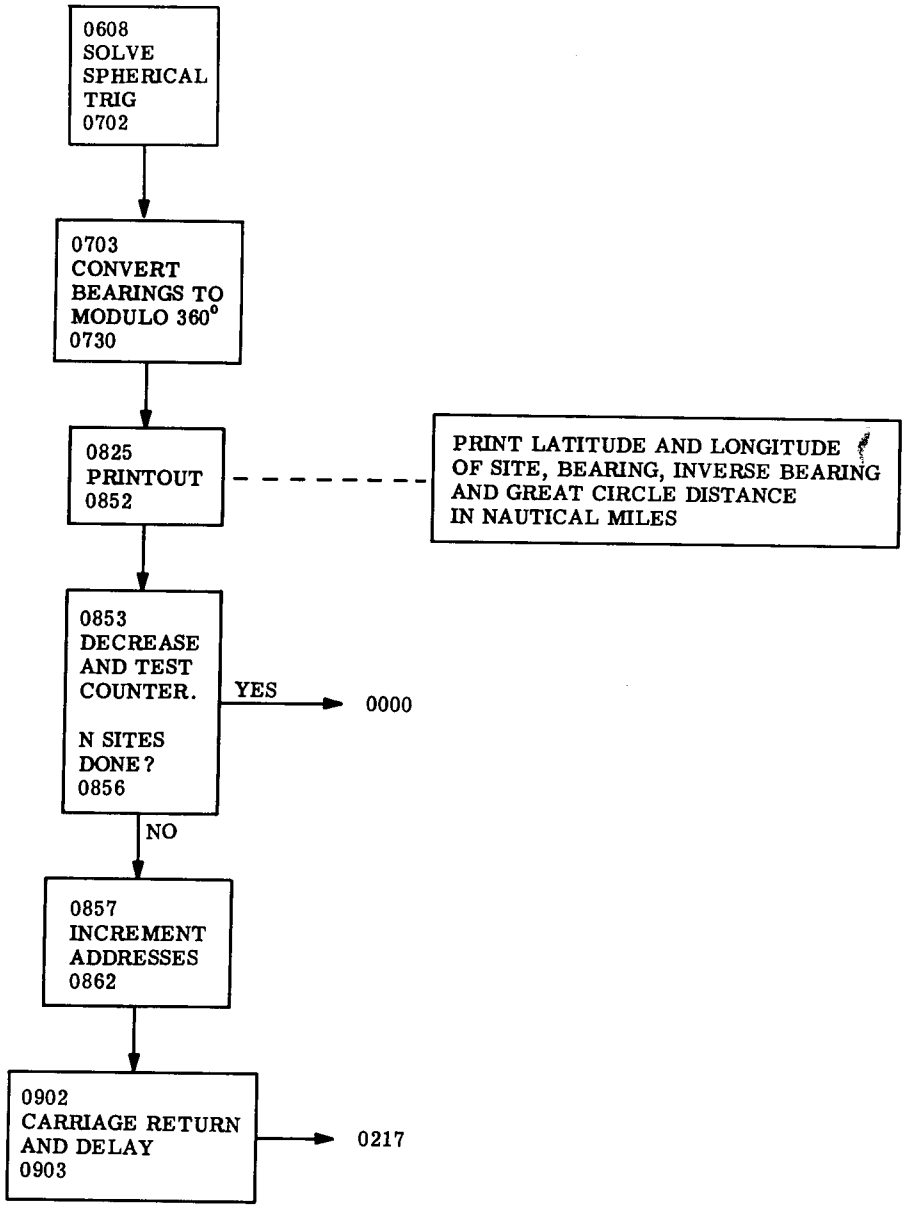



0058
ENTER
α#
0059
α#
CALL FOR
NUMBER OF SITES
0 < N ≤ 999
0101
INITIALIZE
2N-1 AND
N-1 COUNTERS
0124
0125
ENTER
α#
0126
α#
CALL FOR
SITE COORDINATES
0133
INITIALIZE
AND ENTER
BINARIZING
SUBROUTINE
0137
BINARIZING
SUBROUTINE
CONVERT, BINARIZE
AND STORE LATITUDE
AND LONGITUDE OF
N SITES
0138
ENTER
α#
0139
α#
PRINT "REFERENCE
LAT. LONG,"
0149
ENTER
DECIMAL PRINTOUT
SUBROUTINE
0158
DECIMAL
PRINTOUT
SUBROUTINE
PRINT LATITUDE
AND LONGITUDE IN
DECIMAL DEGREES
ROUNDED TO 0.10
XX.X° TAB XXXX°
0159


0159
CARRIAGE RETURN
AND DELAY
0160
0203
ENTER SINE
SUBROUTINE
0205
SINE
SUBROUTINE
COMPUTE
SIN L_R
0206
STORE SIN L_R
0206
0207
ENTER COSINE
SUBROUTINE
0209
COSINE
SUBROUTINE
COMPUTE
COS L_R
0210
STORE COS L_R
0210
PRINTS
COLUMN HEADINGS
0211
COLUMN
HEADINGS?
0212
YES
0944
ENTER
α#
0945
α#
NO
0213
0213


0213
INITIALIZE
ADDRESSES
0216
REENTRY
0217
LATITUDE TEST
$|L_s| > 90^\circ$?
0223
YES
0905
ENTER LINE
NUMBER PRINTER
0906
0737
LINE
NUMBER
PRINTER
0824
0907
ENTER
α#
0908
α#
PRINT
"LATITUDE
ERROR"
0853
0228
LONGITUDE TEST
$|\lambda_s| > 90^\circ$?
0234
YES
0915
ENTER
LINE
NUMBER
PRINTER
0916
NO
0239
INITIALIZE
ADDRESSES
0242
0737
LINE
NUMBER
PRINTER
0824
0243
RELOCATE
SITE LATITUDE AND
LONGITUDE
0247
0917
ENTER
α#
0918
α#
PRINT
"LONGITUDE
ERROR"
0853
0250


0250
TEST FOR
PARTICULAR
SOLUTIONS
0448
0608
SAME SITE
ANTIPODAL
SITE
N-S
E-W
0925
ENTER
LINE NUMBER
PRINTER
0926
0737
LINE NUMBER
PRINTER
0824
0927
ENTER
α#
0928
α#
PRINT
"SAME SITE"
0853
0449
SET B = O or 180
C = O or 180
COMPUTE α
0563
0825
0933
ENTER
LINE NO.
PRINTER
0934
0737
LINE NO.
PRINTER
0824
0935
ENTER
α#
0936
1044
SET
B = 90 or 220
C = 90 or 270
Comp. α
0825
α#
PRINT
"ANTIPODAL SITE"
0853


0608
SOLVE
SPHERICAL
TRIG
0702
0703
CONVERT
BEARINGS TO
MODULO 360°
0730
0825
PRINTOUT
0852
PRINT LATITUDE AND LONGITUDE
OF SITE, BEARING, INVERSE BEARING
AND GREAT CIRCLE DISTANCE
IN NAUTICAL MILES
0853
DECREASE
AND TEST
COUNTER.
N SITES
DONE?
0856
YES
0000
NO
0857
INCREMENT
ADDRESSES
0862
0902
CARRIAGE RETURN
AND DELAY
0903
0217