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Regularities in Thermal Neutron Absorption Cross Sections

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ABSTRACT

Thermal neutron absorption cross sections have generally not been considered to show much systematics. However, some regularities are brought out in this report which we have not seen stated before. Each stable odd Z_i-even N_j nucleus has a higher cross section than the corresponding even $(Z_i - 1)$-even N_j nucleus, with very few exceptions. The odd Z nuclei show a pronounced cyclic variation in cross section, while the corresponding even Z nuclei have considerably less variation. Justifications for these rules are presented on the basis of the existence of a pairing force in the nucleus. Suggestions are given for additional cross section measurements to be made. These involve the determination of cross sections of stable, as well as, radioactive nuclei.

PROBLEM STATUS

This is an interim report on one phase of a continuing problem.

AUTHORIZATION

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REGULARITIES IN THERMAL NEUTRON ABSORPTION CROSS SECTIONS

INTRODUCTION

It has generally been considered that thermal neutron absorption cross sections of nuclei do not exhibit a great deal of systematics. The accidental presence of resonances in the compound nucleus at the neutron binding energy would cause the cross sections to be erratic. However, in general, it is known (1) that odd A nuclei have higher cross sections than even Z-even N nuclei. Also, nuclei with a closed shell in either neutrons or protons have (2, 3) lower values than neighboring nuclei. Those with closed shells in both neutrons and protons have considerably lower values.

In the course of working with nuclear data for reactors, certain regularities were noticed which we have not seen specifically stated before. It was thought that it would be worthwhile to write these out.

COMPARISON OF THERMAL σ_a FOR ODD Z-EVEN N AND EVEN (Z-1)-EVEN N NUCLEI

The following comparison was made: The thermal neutron absorption cross section σ_a of each odd Z_i-even N_j nucleus was compared with that of the corresponding even (Z_i-1)-even N_j nucleus (that is, one proton less). The odd Z nuclei considered actually include all the stable isotopes as well as those long-lived radioactive ones for which cross sections have been measured.

The comparison is shown in Table 1 and in Fig. 1, where the cross section σ_a is plotted against the neutron number N of each nucleus. In most instances, the cross-section values that were used are the ones call "absorption" cross sections (4). When these were not available, or as in the case of P^{31} (0.40 ± 0.40 barn) the errors seem out of line, the activation values are taken. The vertical lines on the graph indicate closed neutron shells. The numbers near the points are the appropriate proton or Z numbers.

With only four exceptions out of 70 pairs of nuclei, the odd-even nucleus of each pair has a higher cross section than the even-even nucleus. The rule cannot be applied to four nuclei listed, H^1, Li^7, B^{11}, and Ag^{107}, because in the first two cases the corresponding even-even nuclei do not exist and in the latter two, the σ_a of the even-even nuclei are not available. If we look at the 66 pairs of nuclei for which the regularity appears to hold and consider the errors of the measurements in the most unfavorable way, that is, take the smallest probable value of the odd Z nucleus and the largest probable one of the even Z one, then the rule that the cross section is higher for an odd Z-even N nucleus than for the even-even nucleus with one less proton still holds with perhaps one exception.

A qualitatively similar statement holds for the even Z_i-odd N_j nuclei compared with the even Z_i-even (N_j-1) ones (that is, one neutron less). However, there are many more exceptions to the rule. Also, not as many values are available for comparison, since it is, generally, more difficult to obtain good determinations of the σ_a values of even Z isotopes.

Table 1
Thermal Neutron Absorption Cross Sections–
Comparison of $\sigma_a(Z, N)$ and $\sigma_a(Z-1, N)$, Where Z Is Odd and N Is Even

Odd Z Nucleus	Z	N	$\sigma_a(Z, N)$	$\sigma_a(Z-1, N)$	Even (Z-1) Nucleus
H^{1}	1	0	0.332	–	–
Li^{7}	3	4	0.036	–	He^{6}
B^{11}	5	6	0.005	–	Be^{10}
N^{15}	7	8	0.00024	$<10^{-6}$	C^{14}
F^{19}	9	10	0.009	0.00021	O^{18}
Na^{23}	11	12	0.525	0.036	Ne^{22}
Al^{27}	13	14	0.241	0.030	Mg^{26}
P^{31}	15	16	0.20	0.11	Si^{30}
Cl^{35}	17	18	45.	0.26	S^{34}
Cl^{37}		20	0.56	0.14	S^{36}
K^{39}	19	20	1.94	0.8	Ar^{38}
K^{41}		22	1.24	0.53	Ar^{40}
Sc^{45}	21	24	22.3	0.72	Ca^{44}
V^{51}	23	28	4.5	0.14	Ti^{50}
Mn^{55}	25	30	13.2	0.38	Cr^{54}
Co^{59}	27	32	38.0	1.0	Fe^{58}
Cu^{63}	29	34	4.51	15.	Ni^{62} *
Cu^{65}		36	2.2	1.52	Ni^{64}
Ga^{69}	31	38	2.1	1.1	Zn^{68}
Ga^{71}		40	5.0	0.085	Zn^{70}
As^{75}	33	42	4.3	0.62	Ge^{74}
Br^{79}	35	44	10.4	0.4	Se^{78}
Br^{81}		46	3.3	0.61	Se^{80}
Rb^{85}	37	48	0.91	0.16	Kr^{84}

*Exceptions to stated rule.

Table continues

Table 1 (cont'd)
Thermal Neutron Absorption Cross Sections:
Comparison of $\sigma_a(Z, N)$ and $\sigma_a(Z-1, N)$, Where Z Is Odd and N Is Even

Odd Z Nucleus	Z	N	$\sigma_a(Z, N)$	$\sigma_a(Z-1, N)$	Even (Z-1) Nucleus
Rb^{87}	37	50	0.12	0.06	Kr^{86}
Y^{89}	39	50	1.31	0.005	Sr^{88}
Y^{91}		52	1.07	1.0	Sr^{90}
Nb^{93}	41	52	1.16	0.25	Zr^{92}
Tc^{99}	43	56	22.	0.51	Mo^{98}
Rh^{103}	45	58	149.	1.44	Ru^{102}
Rh^{105}		60	20,000.	0.7	Ru^{104}
Ag^{107}	47	60	31.	–	Pd^{106}
Ag^{109}		62	87.	10.	Pd^{108}
In^{113}	49	64	58.	0.03	Cd^{112}
In^{115}		66	207.	1.24	Cd^{114}
Sb^{121}	51	70	5.9	0.14	Sn^{120}
Sb^{123}		72	4.1	0.16	Sn^{122}
I^{127}	53	74	7.0	0.8	Te^{126}
I^{129}		76	32.	0.3	Te^{128}
I^{131}		78	50.	0.5	Te^{130}
Cs^{133}	55	78	28.	0.2	Xe^{132}
Cs^{135}		80	8.7	0.2	Xe^{134}
Cs^{137}		82	0.11	0.15	Xe^{136} *
La^{139}	57	82	8.9	0.7	Ba^{138}
Pr^{141}	59	82	11.3	0.66	Ce^{140}
Pr^{143}		84	89.	0.94	Ce^{142}
Pm^{147}	61	86	60.	10.	Nd^{146}
Eu^{151}	63	88	7800.	100.	Sm^{150}
Eu^{153}		90	440.	216.	Sm^{152}

*Exceptions to stated rule.

Table continues

Table 1 (cont'd)
Thermal Neutron Absorption Cross Sections:
Comparison of $\sigma_a(Z, N)$ and $\sigma_a(Z-1, N)$, Where Z Is Odd and N Is Even

Odd Z Nucleus	Z	N	$\sigma_a(Z, N)$	$\sigma_a(Z-1, N)$	Even (Z-1) Nucleus
Eu^{155}	63	92	14,000.	5.5	Sm^{154}
Tb^{159}	65	94	46.	3.9	Gd^{158}
Ho^{165}	67	98	64.	2800.	Dy^{164} *
Tm^{169}	69	100	127.	2.0	Er^{168}
Lu^{175}	71	104	23.	60.	Yb^{174} *
Ta^{181}	73	108	21.	14.	Hf^{180}
Re^{185}	75	110	104.	2.24	W^{184}
Re^{187}		112	69.	35.	W^{186}
Ir^{191}	77	114	960.	8.	Os^{190}
Ir^{193}		116	130.	1.6	Os^{192}
Au^{197}	79	118	98.8	0.87	Pt^{196}
Au^{199}		120	30.	4.0	Pt^{198}
Tl^{203}	81	122	11.4	3.8	Hg^{202}
Tl^{205}		124	0.8	0.43	Hg^{204}
Bi^{209}	83	126	0.034	0.0006	Pb^{208}
Ac^{227}	89	138	795.	20.	Ra^{226}
Pa^{231}	91	140	200.	27.	Th^{230}
Pa^{233}		142	51.	7.56	Th^{232}
Np^{237}	93	144	170.	7.	U^{236}
Np^{239}		146	60.	2.71	U^{238}
Am^{241}	95	146	630.	286.	Pu^{240}
Am^{243}		148	74.	30.	Pu^{242}
Bk^{249}	97	152	500.	6.	Cm^{248}
Es^{253}	99	154	300.	28.	Cf^{252}
Es^{255}		156	40.	<2.	Cf^{254}

*Exceptions to stated rule.

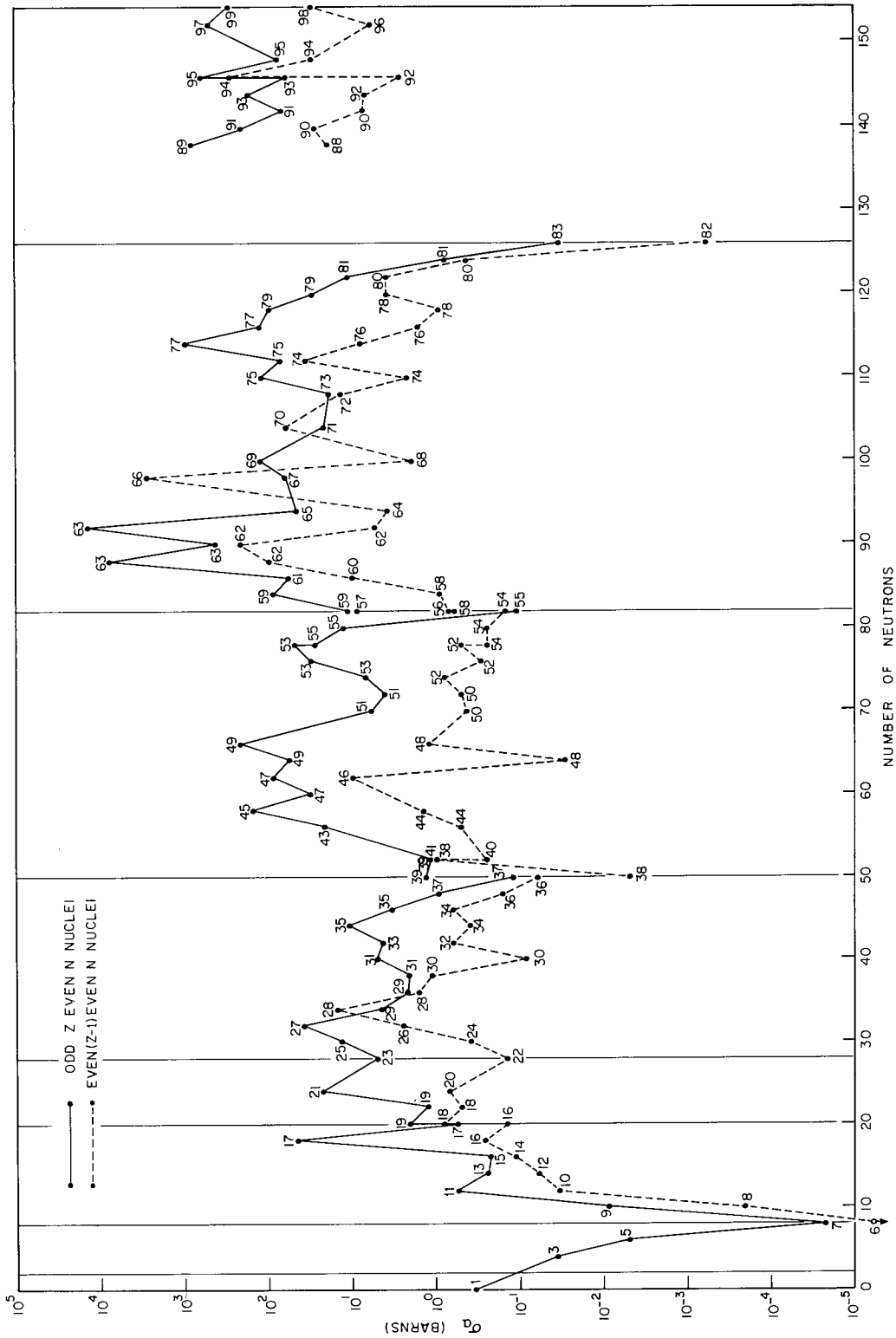

Fig. 1 - Thermal neutron absorption cross sections: comparison of odd Z-even N nuclei and even (Z-1)-even N nuclei

DISCUSSION OF EXCEPTIONS TO THE RULE FOR ODD Z VERSUS EVEN Z NUCLEI

The activation cross section of ${}_{55}Cs^{137}_{82}$ is 0.11 barn, whereas that of ${}_{54}Xe^{136}_{82}$ is 0.15 ± 0.08 barn. Therefore the errors overlap, and better measurements might reverse the comparison in favor of our rule. However, it is to be noted, both isotopes have a magic number of neutrons and consequently low absorption cross sections.

Again in the case of ${}_{71}Lu^{175}_{104}$ versus ${}_{70}Yb^{174}_{104}$ the errors overlap and could reverse the comparison. The Lu^{175} cross section has been recently remeasured to be 23 ± 3 barns while the Yb^{174} is quoted as 60 ± 40 barns. This isotope is 31% abundant, and the measured value is highly inaccurate and seems out of line with neighboring even-even nuclei.

The case of ${}_{29}Cu^{63}_{34}$ versus ${}_{28}Ni^{62}_{34}$ is more interesting and quite unexplained. The absorption cross section of Cu^{63} is 4.5 ± 0.1 barns in comparison with the higher 15 ± 2 barns of the even-even Ni^{62}. Neither isotope exhibits any "1/v" nature near the thermal energies. From plots of cross sections of other even-even nuclei in this region 2 barns would appear more reasonable for Ni^{62}, which would reduce its cross section below the value of Cu^{63}, which appears reasonable for neighboring odd-even nuclei.

The low-energy absorption cross section of dysprosium is approximately "1/v," which normally indicates contributions from a bound level below the binding energy of a neutron to one of the nuclides involved. ${}_{66}Dy^{164}$ (28.18% abundant) has a thermal activation cross section of 2800 barns, which means this isotope accounts for 7/8 of the thermal absorption cross section of elemental Dy. The analysis of the low-energy cross-section data of Dy indicates a bound level (4) in Dy^{164}, and the first known resonance is at 146 ev. Our comparison of ${}_{66}Dy^{164}$ is made with ${}_{67}Ho^{165}$, whose low-energy total cross section appears to be "1/v" also but not nearly as high, even though the first level is at 3.92 ev with a level separation of approximately 6 ev. Therefore, this exception to the stated rule is explainable by the presence of a bound level in Dy^{165}.

CYCLIC CHARACTER OF THE THERMAL σ_a OF ODD Z-EVEN N NUCLEI

If the odd Z nuclei are considered separately, and the three longer neutron periods are superimposed on one graph as shown in Fig. 2, then a cyclic appearance which was evident before, is brought out more clearly. The σ_a values are plotted for the three neutron periods 20-50, 50-82, and 82-126. The abscissa is the number of neutrons beyond the closed shells: 20, 50, and 82. The numbers near the points are again the proton numbers.

The three curves seem to have the same general shape. They each have a broad primary peak and a narrower secondary one. The cross section starts off low initially, reaches the primary maximum in the first half of the period, and the secondary peak in the last half. The primary maxima go up successively with shell number, similarly the secondary peaks, and also the minima or valleys. The valley in the third period, N = 82 to 126, appears to be rather broad. In general, it almost seems as if one can transform the lowest curve into each of the others by an expansion in two dimensions. The highest curve goes down to a lower value at ${}_{83}Bi^{209}_{126}$ because it has reached closed shells at N = 126 and Z = 82.

If the nuclei beyond bismuth are plotted on the same graph, they go just above the highest curve shown.

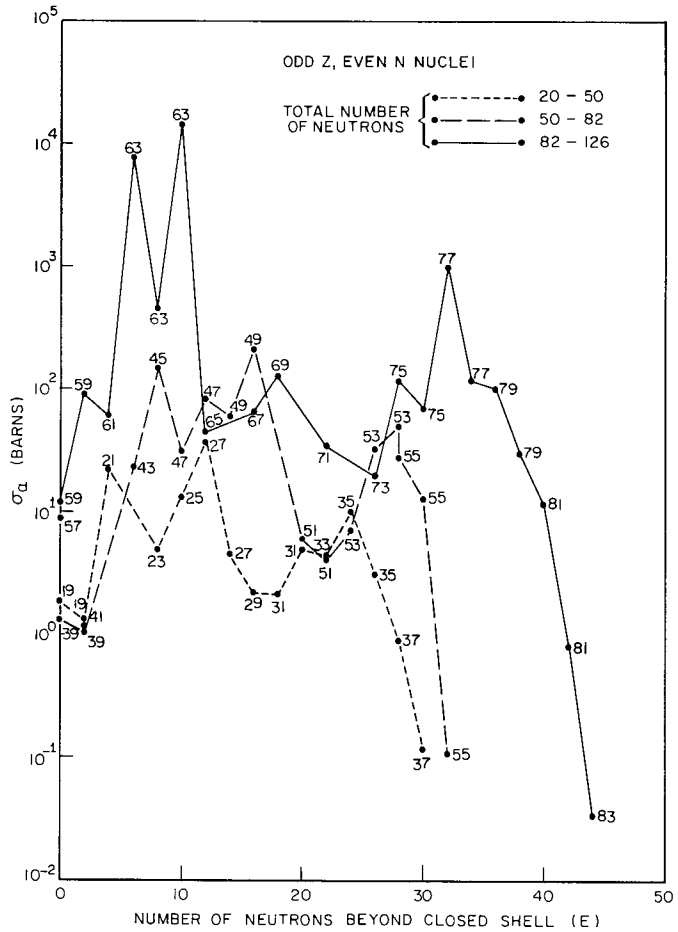

Fig. 2 - Thermal neutron absorption cross sections of the three longer neutron periods of odd Z-even N nuclei

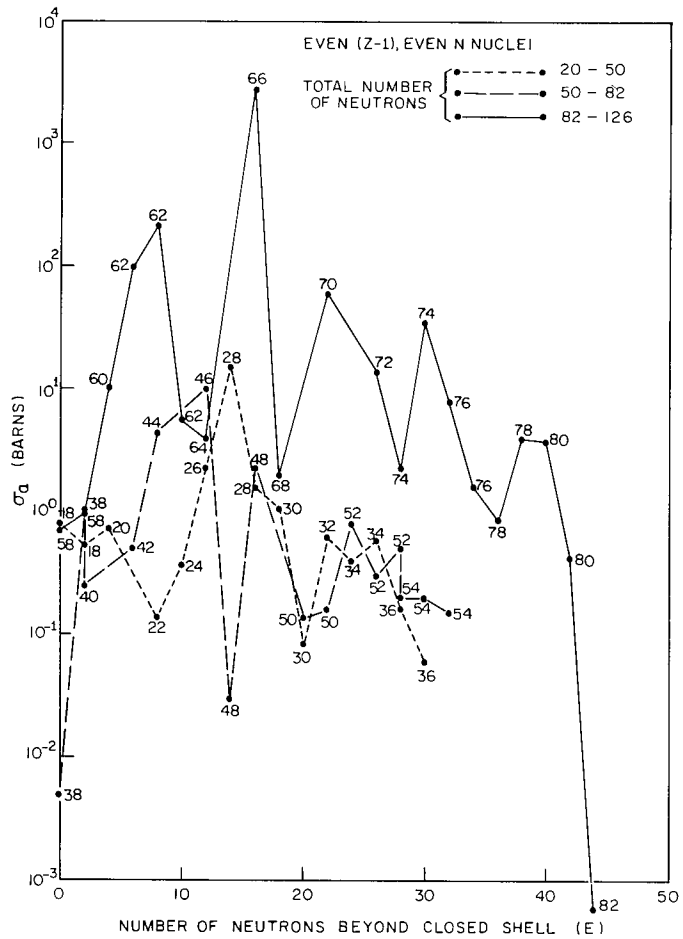

Fig. 3 - Thermal neutron absorption cross sections of the three longer neutron periods of even (Z-1)-even N nuclei

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THE THERMAL σ_a OF THE CORRESPONDING EVEN-EVEN NUCLEI

On the other hand, the σ_a of the corresponding even-even nuclei seem to act quite a bit differently. The three longer periods of even (Z-1)-even N nuclei of Fig. 1 were plotted in the same way as the odd Z nuclei were in Fig. 2. This is shown in Fig. 3, where one can see that the curves for the first two periods, 20-50 and 50-82, fall close together, while that for the higher period, 82-126, is considerably above those two. If the values for the alpha radioactive nuclei above lead, 126-154, are also plotted on the same graph, they fall on this higher level. In general, it might be said that the cross sections of the corresponding even-even nuclei do not show as much variation as the odd Z ones.

DISCUSSION

The relationships that have been brought out in this report are that:

1. Adding a proton to an even-even nucleus produces a nucleus with a higher thermal σ_a than the original one.

2. The cross sections of the stable odd Z nuclei tend to have a cyclic character relative to neutron shells.

3. The corresponding even-even nuclei do not show as much variation in σ_a as the odd Z ones.

An explanation of these trends can be made to depend mainly on the pairing force in the nucleus. The neutron absorption cross section is here considered to be basically a function of two effects: the amount of binding energy that the neutron brings into the nucleus, and the level structure of the compound nucleus at that particular excitation energy.

It is well known that a neutron brings in more binding energy when added to an odd-even nucleus of type (Z_i+1, N_j) than when added to an even-even nucleus of type (Z_i, N_j), where Z_i and N_j are both even. Also, the level structure of the nucleus of type (Z_i+1, N_j+1) that results when the neutron is added to the odd-even nucleus is more complex (5) than the level structure of type (Z_i, N_j+1) that results when the neutron is added to the even-even nucleus. This is known to be true near the ground state and is presumably so at higher excitation energies. These are two effects that cause the cross section of an odd (Z_i+1)-even N_j nucleus to be higher than that of the corresponding even Z_i-even N_j one.

The situation at closed shells is somewhat analogous to the even-even case, but everything favors lower cross sections. The binding energy of a neutron to a closed neutron shell nucleus is less (6) by approximately 2 Mev from the binding to the average even-even nucleus. Also, the levels of a closed shell nucleus are farther apart than those of the nonclosed shell nucleus. If a nucleus has a few neutrons more than a closed shell, then the binding energy of an additional neutron goes up and also the level structure becomes more complex. This agrees with the higher cross sections near the beginning of a neutron shell. As the neutron shell comes within a few neutrons of closing, the binding energy of an additional neutron does not go down, but the levels tend to be spread further apart, and the cross section starts going down.

These effects would explain qualitatively the cyclic nature of the odd Z thermal absorption cross sections.

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TENTATIVE CONSIDERATIONS

In view of the preceding discussion, one could perhaps think of the absorption cross sections of nuclei in the following manner. The even-even nuclei that we have considered can be thought of as having basic cross sections; the corresponding odd-even ones have additional cross sections (superimposed on these basic values) due to the additional odd proton. This would be analogous to the way other properties of the nuclei vary with the addition of an odd nucleon to an even-even core. For one, the level structure becomes more complex when an odd nucleon is added. Also, the binding energy of an additional neutron is greater when there is an odd particle originally. These two effects, of course, have been used previously in our discussion to explain this variation in cross section.

Further, as we have stated before, the above even-even nuclei do not vary as greatly in cross section as the odd-even ones. For $N \leq 82$, most of the even-even nuclei, with the exception of the lightest ones, have values of σ_a in the range of 0.1 to 1.0 barn, or of the order of the cross section of the proton. For higher values of N, the cross sections of the even-even nuclei go to a higher level.

SUGGESTIONS FOR MEASUREMENTS OF CROSS SECTIONS

In light of these previous considerations, it was thought that thermal neutron absorption measurements of the following nuclei would be both interesting and useful: new determinations of Be^{10} and Pd^{106}, and additional determinations of C^{14}, Si^{30}, Ca^{48}, Ni^{62}, Zn^{70}, Sr^{88}, Cd^{112}, Cs^{137}, Yb^{174}, and Tl^{205}. Then one would see how the new data fit the regularities considered in this report. Tl^{205} is included because it is an example of a nuclide where there is a large discrepancy between the activation and absorption measurements. More generally, it might be said that more definitive determinations of the cross sections of even Z isotopes would be of assistance in a better understanding of nuclear interrelations.

Other possibilities for measurements are long-lived isotopes of odd Z elements, particularly where the corresponding even Z isotopes have been measured. Some favorable examples are Mn^{53}, Co^{57}, Rb^{83}, Nb^{95}, Rh^{101}, Ag^{111}, Sb^{125}, Ho^{163}, Tm^{171}, and Lu^{173}. In these cases the odd Z cross sections are expected to be large enough and the ratios of initial half-life to product half-life great enough to result in a measurable product activity after a reasonable irradiation near the core of a thermal reactor.

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