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The Velocity of Sound in Sea Water

NAVAL RESEARCH LABORATORY  
ANACOSTIA STATION  
WASHINGTON, D. C.

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## ABSTRACT

The purpose of this report is to present the latest available data on the velocity of sound in sea water as modified by changes in temperature, salinity, and depth. These are given in both metric and English units in a series of tables, curves, and equations for depths to 1300 feet.

The basic data have been compiled from the published reports of competent authorities who calculated the velocities from fundamental physical equations whose constants have been experimentally determined with precision. They have been checked with numerous direct measurements, and agree within an experimental error estimated to be less than 0.2%. The soundness of the theoretical basis for the calculations and the consistency of the results indicate that the calculated values are accurate to within less than 0.05% at temperatures below 70° F.

A bibliography of source materials is included.

## The Velocity of Sound in Sea Water

### INTRODUCTION

1. Experimental determination of the velocity of sound in sea water over wide ranges of temperature and salinities presents many difficulties. Consequently several computed tables have been published based on fundamental physical theory and data. These tables are more accurate and comprehensive than direct experimental determinations.

2. The purpose of this report is to present some of the latest available data in a form convenient for use.

### DEFINITION OF SALINITY

3. The salinity is a measure of the salt content of the water in parts per thousand by weight and is written S = ppt. or 0/00.

4. The salinity of the open sea water may range from 30 to 40 ppt. "Standard" salinity is conventionally chosen as 35 ppt.

### CALCULATION OF THE VELOCITY OF SOUND IN WATER

5. The velocity is given by the expression

$$V = \sqrt{\frac{dp}{d\rho}} \quad (1)$$

where  $p$  is the pressure and  $\rho$  is the density. For convenience in calculation from well established physical data the formula used is

$$V = \sqrt{\frac{\gamma}{\rho C}} \quad (2)$$

where  $\gamma$  is the ratio of the specific heats at constant volume and constant pressure and  $C$  is the compressibility. The numerical values of these quantities depend on the temperature, salinity, and depth or pressure. The detailed formulae and the necessary corrections are explained fully in the references to the original data.

### CALCULATED VELOCITY DATA

6. The first comprehensive tables were published by Heck and Service in 1924. They checked their tables by wire soundings, and concluded that the calculated values were probably more accurate than the experimental determinations. In 1927, the British Admiralty published an excellent set of tables for both pure and sea water that were the most extensive and accurate at that time. In 1939, Kuwahara brought out a new set of velocity tables with

added refinements and corrections. In 1939 the British Admiralty published a revised edition of their tables which agree very closely with Kuwahara's. Wood, in his "Textbook of Sound" (1932), gives an empirical equation for the velocity in sea water at "zero" depth.

7. Table 1 shows a comparison of velocities under different conditions as given by the above authorities.

8. Since the tables of Kuwahara are believed to be the most accurate available at this time, they have been used in the preparation of the sea water curves and equations given in later sections of this report. For fresh water and slightly saline water, the values used are those of the British Admiralty tables of 1939.

#### VELOCITY TABLES

9. Because the velocity tables of Kuwahara may not readily be available, a part of his data is given as Table 2. For depths less than 400 meters, a pressure correction of 0.0181 meter per second per meter of depth should be added. For greater depths, the pressure correction is not so simple, and reference should be made to the original source.

#### VELOCITY CURVES

10. Since the effect of pressure is not great, and a correction may easily be made for depths not exceeding about 400 meters or 1300 feet, the velocity curves in this report have been drawn for zero depth.

11. Plate 1 gives the velocity in meters per second in ordinary sea water for Centigrade temperatures. These curves are as precise as the data from which they were drawn - that is, to the nearest 0.1 meter per second.

12. Plate 2 shows in metric units the temperature coefficient of velocity - that is, the change in velocity in meters per second for a change in the temperature of one degree Centigrade. It is to be noted that this coefficient is by no means constant, decreasing from about 4.7 to about 2.0, so that it cannot be used to calculate accurately the velocity at one temperature from a knowledge of the velocity of another temperature, unless the two temperatures are quite close together. Although this curve was drawn for  $S = 35$  ppt., little error will be introduced by using it for any salinity between 30 ppt. and 40 ppt. In case greater accuracy is required, the equations given in Paragraphs 17 to 22 inclusive, may be used.

13. Plate 3 shows approximately the salinity coefficient of velocity - that is, the change in velocity in meters per second for a change in the salinity of one ppt. This coefficient is nearly independent of the salinity, so that for a fixed temperature, the velocity at one salinity may be calculated from a knowledge of the

velocity at some other value of salinity. For greater accuracy, the equations given in Paragraphs 17 to 22 inclusive may be used.

14. Plate 4 gives a summary on one sheet of the data of the preceding plates with some loss of precision due to the smaller scale.

15. Plate 5 shows the velocity of sound in pure and nearly pure water, in metric units, with a precision of about 0.5 meter per second.

16. Plates 6 to 10 give the same data as plates 1 to 5, expressed in English units of feet and Fahrenheit degrees.

#### VELOCITY EQUATIONS

17. Two empirical equations have been set up and the constants calculated to fit the data given in the tables and curves with the precision specified. In order to preserve the accuracy of the equations, the variables should not fall outside the ranges indicated. The following symbols will be employed.

$V_m$  = velocity of sound in sea water, in meters per second

$V_f$  = velocity of sound in sea water, in feet per second

$S$  = salinity in ppt., between 30 and 40 ppt.

$y_m$  = depth in meters below sea surface; between 0 and 400 meters

$y_f$  = depth in feet below sea surface; between 0 and 1300 ft.

$t_c$  = water temperature in  $^{\circ}\text{C}$ ; between 0 and  $32^{\circ}\text{C}$ .

$t_f$  = water temperature in  $^{\circ}\text{F}$ ; between 32 and  $90^{\circ}\text{F}$ .

18. For a first approximation, a quadratic function of the temperature was assumed. Using the Method of Least Squares, an expression for the velocity in meters per second as a function of the Centigrade temperature was obtained for zero depth and standard salinity of 35 ppt. Then this expression was modified to represent the velocity in feet per second for Fahrenheit temperatures. These equations are:

$$V_m = 1445.75 + 4.506 t_c - 0.04235 t_c^2 \quad (3)$$

and

$$V_f = 4436.5 + 10.96 t_f - 0.04288 t_f^2 \quad (4)$$

Values of velocity computed by equation (3) agree with the values given by Kuwahara within a maximum error of 0.4 m/sec, or 0.025%, and an average error of 0.16 m/sec, or 0.010%. However, values for the

difference in velocity per degree temperature difference calculated with the aid of equation (3) may be in error by 1% to 10%. Equation (4) is of the same percentage precision as equation (3).

19. Greater concordance with the data may be obtained from a cubic equation. Using the Method of Least Squares, an equation for the velocity in meters per second as a function of the Centigrade temperature was obtained for zero depth and standard salinity of 35 ppt. Then this equation was modified to represent the velocity in feet per second for Fahrenheit temperatures. These equations are:

$$V_m = 1445.44 + 4.6684 t_c - 0.056,527 t_c^2 + 0.0003151 t_c^3 \quad (5)$$

and

$$V_f = 4742.2 + 8.509 (t_f - 32) - 0.057,239 (t_f - 32)^2 + 0.0001773 (t_f - 32)^3 \quad (6)$$

Values of velocity computed with the aid of equation (5) agree with the values given by Kuwahara to within a maximum error of 0.1 m/sec, or 0.007%, and an average error of 0.04 m/sec, or 0.003%. Moreover, it is believed that the values for the difference in velocity per degree difference in temperature, as calculated from equation (5), are precise to 0.01 m/sec, or 0.5% or better. Equation (6) is of the same percentage precision as equation (5).

20. Having obtained the above cubic expressions for the velocity in terms of the temperature at standard salinity and zero depth, other terms for the salinity and pressure effects were added, yielding

$$V_m = 1445.44 + 4.6684 t_c - 0.056527 t_c^2 + 0.0003151 t_c^3 + [1.304 - 0.0106 t_c + 5.7 \times 10^{-8} t_c^4][(S-35) + 5.2 \times 10^{-5} t_c (S-35)^2] + 0.0181 y_m \quad (7)$$

and

$$V_f = 4742.2 + 8.509 (t_f - 32) - 0.057,239 (t_f - 32)^2 + 0.0001773 (t_f - 32)^3 + [4.278 - 0.0193(t_f - 32) + 1.78 \times 10^{-8}(t_f - 32)^4][(S-35) + 2.9 \times 10^{-5} (t_f - 32) (S-35)^2] + 0.0181 y_f \quad (8)$$

21. The precision of equations (7) and (8) is not as great as the precision of equations (5) and (6). However, in about 100 well-distributed trials over a temperature range of 0° to 30°C, over a salinity range of 30 to 40 ppt. and over a depth range of 0 to 400 meters, the velocities computed from equation (7) agreed with the velocities given by Kuwahara's tables to 0.1 meter per second or better. The larger discrepancies were found at the extremes of temperature, salinity and depth. The percentage precision of equation (8) is the same as that of equation (7).

22. To obtain the rate of change of velocity with respect to

temperature, salinity, or depth, equations (7) and (8) may be differentiated, obtaining:

$$\begin{aligned} \frac{dV_m}{dt_c} = & 4.6684 - 0.113,054 t_c + 0.0009453 t_c^2 \\ & + [-0.0106 + 0.23 \times 10^{-6} t_c^3] (S-35) \\ & + [6.8 \times 10^{-5} - 0.110 \times 10^{-5} t_c + 1.5 \times 10^{-11} t_c^4] (S-35)^2 \end{aligned} \quad (9)$$

(constant salinity and depth)

$$\frac{dV_m}{ds} = [1.304 - 0.0106 t_c + 5.7 \times 10^{-8} t_c^4] [1 + 1.04 \times 10^{-4} t_c (S-35)] \quad (10)$$

(constant temperature and depth)

$$\frac{dV_m}{dy_m} = 0.0181 \quad (11)$$

(constant temperature and salinity)

$$\begin{aligned} \frac{dV_f}{dt_f} = & 8.509 - 0.114,478(t_f - 32) + 0.0005318 (t_f - 32)^2 \\ & + [-0.0193 + 7.1 \times 10^{-8} (t_f - 32)^3] (S-35) \\ & + [12.4 \times 10^{-5} - 0.112 \times 10^{-5} (t_f - 32) + 25.8 \times 10^{-13} (t_f - 32)^4] \\ & (S-35)^2 \end{aligned} \quad (12)$$

(constant salinity and depth)

$$\begin{aligned} \frac{dV_f}{ds} = & [4.278 - 0.0193 (t_f - 32) + 1.78 \times 10^{-8} (t_f - 32)^4] [1 \\ & + 5.8 \times 10^{-5} (t_f - 32) (S-35)] \end{aligned} \quad (13)$$

(constant temperature and depth)

$$\frac{dV_f}{dy_f} = 0.0181 \quad (14)$$

(constant temperature and salinity)

#### EXAMPLES IN THE USE OF THE GRAPHS AND EQUATIONS

##### Example 1

23. To find the approximate velocity in meters per second for a temperature of 16.5°C, a salinity of 34.2 ppt, and a depth of zero.

24. At 16.5°C, the curves on Plate 1 show  $V = 1506.2$  for  $S=33$ , and  $V = 1508.6$  for  $S=35$ . Since equation (7) shows that the salinity correction varies approximately linearly with salinity, we have that the velocity for  $S = 34.2$  is

$$\begin{aligned} V &= 1506.2 + \frac{34.2 - 33.0}{35.0 - 33.0} (1508.6 - 1506.2) \\ &= 1507.6, \text{ meters per second.} \end{aligned}$$

Example 2

25. Using the curves on Plate 4, determine the velocity in meters per second for the same data as in Example 1.

26. At 16.5°C, Curve 1 of Plate 4 shows a velocity of 1508.5 for S=35.0. At 16.5°C, Curve 3 shows a salinity coefficient of 1.132. Hence, the desired velocity is

$$V = 1508.5 - (35.0 - 34.2) \times 1.137 = 1507.6 \text{ meters per second.}$$

Example 3

27. For the same temperature and salinity data as in Example 1, but for a depth of 275 meters, find the velocity.

28. Since the velocity correction is 0.0181 m/sec per meter of depth (see equation 7), the desired velocity is

$$V = 1507.6 + 0.0181 \times 175 = 1510.8 \text{ meters per second.}$$

Example 4

29. At a temperature of 8.8°C, and a salinity of 35 ppt., determine the temperature rate of variation of velocity.

30. From Plate 2, the answer is found to be 3.741 meters per degree Centigrade. From equation (9) the calculated value is 3.747 meters per second per degree Centigrade.

ADDITIONAL DATA

31. The Naval Research Laboratory is collecting data in this field and would appreciate contributions from any source, theoretical or experimental, or suggestions as to convenient forms of presentation.

BIBLIOGRAPHY

1. Velocity of Sound in Sea Water, by N. H. Heck and J. H. Service, USC and GS Special Publication No. 108, 1924.
2. Tables of the Velocity of Sound in Pure Water and Sea Water for Use in Echo-Sounding and Sound Ranging, by D. J. Mathews, British Admiralty Publication H.D. 282, first edition 1927 and second edition 1939.
3. Velocity of Sound in Sea Water and Calculation of the Velocity for Use in Sonic Sounding, by S. Kuwahara, the Hydrographic Review, vol. XVI, No. 2, Nov. 1939, pp. 123 - 140.
4. A Textbook of Sound, by A. B. Wood, G. Bell and Sons, London, 1932, pp 245 - 250.
5. Velocity of Sound in Sea Water, by E. B. Stephenson, Physical Review, vol. 21, pp. 181-185, February, 1923.
6. Recent Acoustic Work of the U. S. Coast and Geodetic Survey, by P. A. Smith, Field Engineers Bulletin No. 8, December, 1934, pp. 60 - 75.
7. Velocity and Ray Paths of Sound Waves in Sea Water, by O. W. Swainson, Field Engineers Bulletin No. 10, December, 1936.

TABLE 1

## VELOCITY OF SOUND IN SEA WATER

(From various tables and formulas)

Depth in Meters	Temp. °C.	Sal. ppt.	Meters per Second				
			Heck & Service	Wood	Br. Adm. 1927	Br. Adm. 1939	Kuwahara
0	0	31	1445	1445	1440.3	1440.2	1440.3
"	10	"	1482	1484	1481.9	1481.9	1482.0
"	20	"	1508	1515	1514.3	1514.3	1514.3
"	30	"	—	1538	1539.0	1538.9	1539.1
0	0	35	1450	1450	1445.3	1445.4	1445.5
"	10	"	1489	1488	1486.6	1486.7	1486.8
"	20	"	1514	1519	1518.6	1518.7	1518.7
"	30	"	—	1543	1543.0	1543.1	1543.2
400	0	35	1454	—	1452.6	1452.7	1452.8
"	10	"	1492	—	1493.8	1493.9	1494.1
"	20	"	1518	—	1525.8	1525.9	1525.9
"	30	"	—	—	1550.3	1550.4	1550.6
3000	0	31	1490	—	1494.7	1494.6	1494.4
"	"	35	1498	—	1499.7	1499.8	1499.8

TABLE 2  
Velocity of Sound in Sea Water - Depth = 0  
(From Kuwahara)

Degrees Cent.	Meters Per Second				
	S=31 ppt	S=33 ppt	S=35 ppt	S=37 ppt	S=39 ppt
0	1440.3	1442.9	1445.5	1448.1	1450.7
1	44.8	47.4	50.0	52.6	55.2
2	49.4	51.9	54.5	57.1	59.6
3	53.8	56.4	58.9	61.4	64.0
4	58.1	60.6	63.1	65.6	68.1
5	1462.3	1464.8	1467.3	1469.8	1472.3
6	66.5	68.9	71.4	73.9	76.3
7	70.5	73.0	75.4	77.9	80.3
8	74.5	76.9	79.3	81.7	84.2
9	78.3	80.7	83.1	85.5	87.9
10	1482.0	1484.4	1486.8	1489.2	1491.6
11	85.7	88.0	90.4	92.8	95.1
12	89.2	91.6	93.9	96.3	98.6
13	92.7	95.0	97.3	99.6	1502.0
14	96.0	98.3	1500.6	1502.9	05.2
15	1499.3	1501.6	1503.9	1506.2	1508.5
16	1502.5	04.7	07.0	09.3	11.5
17	05.6	07.9	10.1	12.3	14.6
18	08.6	10.8	13.0	15.2	17.5
19	11.5	13.7	15.9	18.1	20.3
20	1514.3	1516.5	1518.7	1520.9	1523.1
21	17.2	19.3	21.5	23.7	25.9
22	19.8	22.0	24.1	26.3	28.4
23	22.4	24.6	26.7	28.8	31.0
24	25.0	27.1	29.2	31.3	33.5
25	1527.5	1529.6	1531.7	1533.8	1535.9
26	29.9	32.0	34.1	36.2	38.3
27	32.3	34.3	36.4	38.5	40.6
28	34.6	36.6	38.7	40.8	42.9
29	36.9	39.0	41.0	43.1	45.1
30	1539.1	1541.2	1543.2	1545.3	1547.3

### Appendix

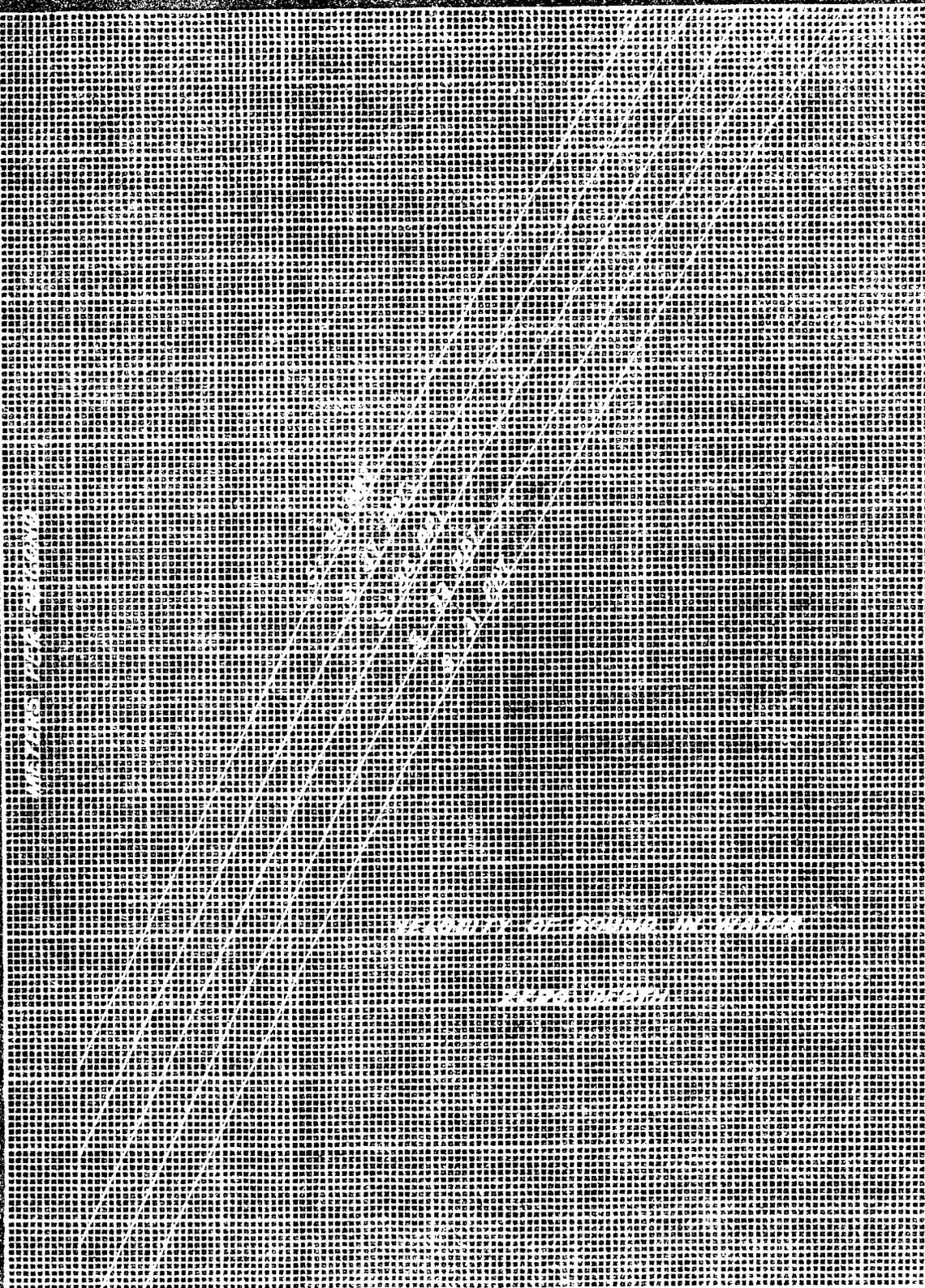
The following table gives approximate values for the velocity of sound in several media. These values have been taken from tables published in handbooks and textbooks, and are believed to be reliable within 1% to 5%.

Temperature = 20° C. = 68° F., unless otherwise noted.

Material	Meters per Second	Feet per Second
Aluminum	5100	16,700
Brass	3500	11,500
Copper	3560	11,700
Iron, cast	4700	15,400
Lead	1230	4,030
Nickel	4970	16,300
Silver	2610	8,560
Steel	5000	16,400
Tin	2500	8,200
Zinc	3700	12,100
Brick	3650	12,000
Cork	430-530	1400-1740
Glass	5000-6000	16,400-19,700
Ice, 0° C.	3200	10,500
Paraffin, 15° C.	1300	4,260
Quartz	5500	18,000
Rubber	30-70	98-230
Wood	1000-4700	3300-15,400
Alcohol, 12.5° C.	1240	4,070
Castor oil	1555	5,100
Benzol, 17° C.	1170	3,840
Mercury	1410	4,625
Air (dry, 760 mm.)	344	1,128
Carbon dioxide, 0° C	258	846
Hydrogen, 0° C.	1270	4,170
Methane, 0° C.	432	1,417
Oxygen, 0° C.	317	1,040
Steam, 100° C	405	1,328
Air, 0°C. (dry, CO <sub>2</sub> free, 760 mm)	331.4	1,087.3



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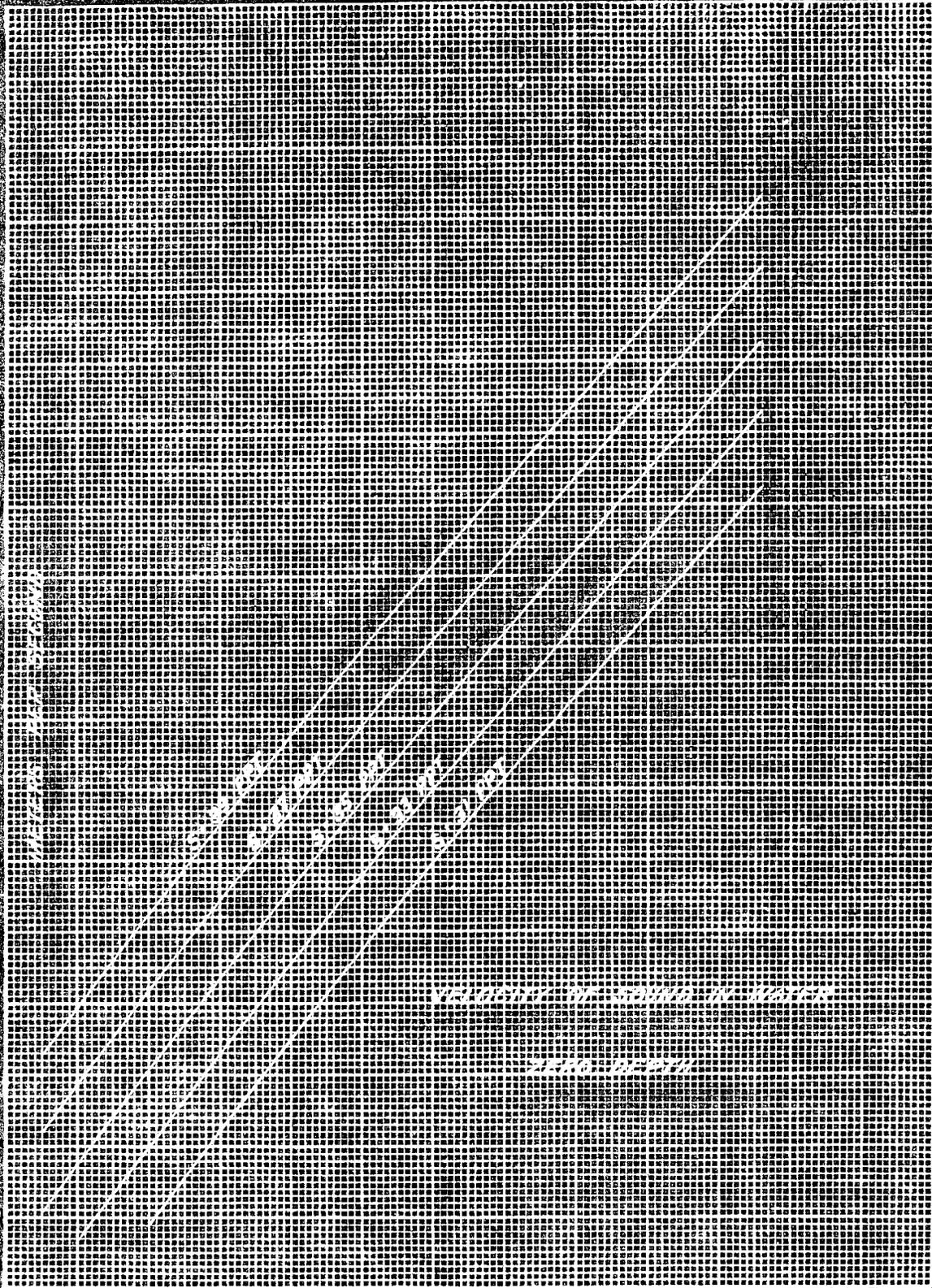
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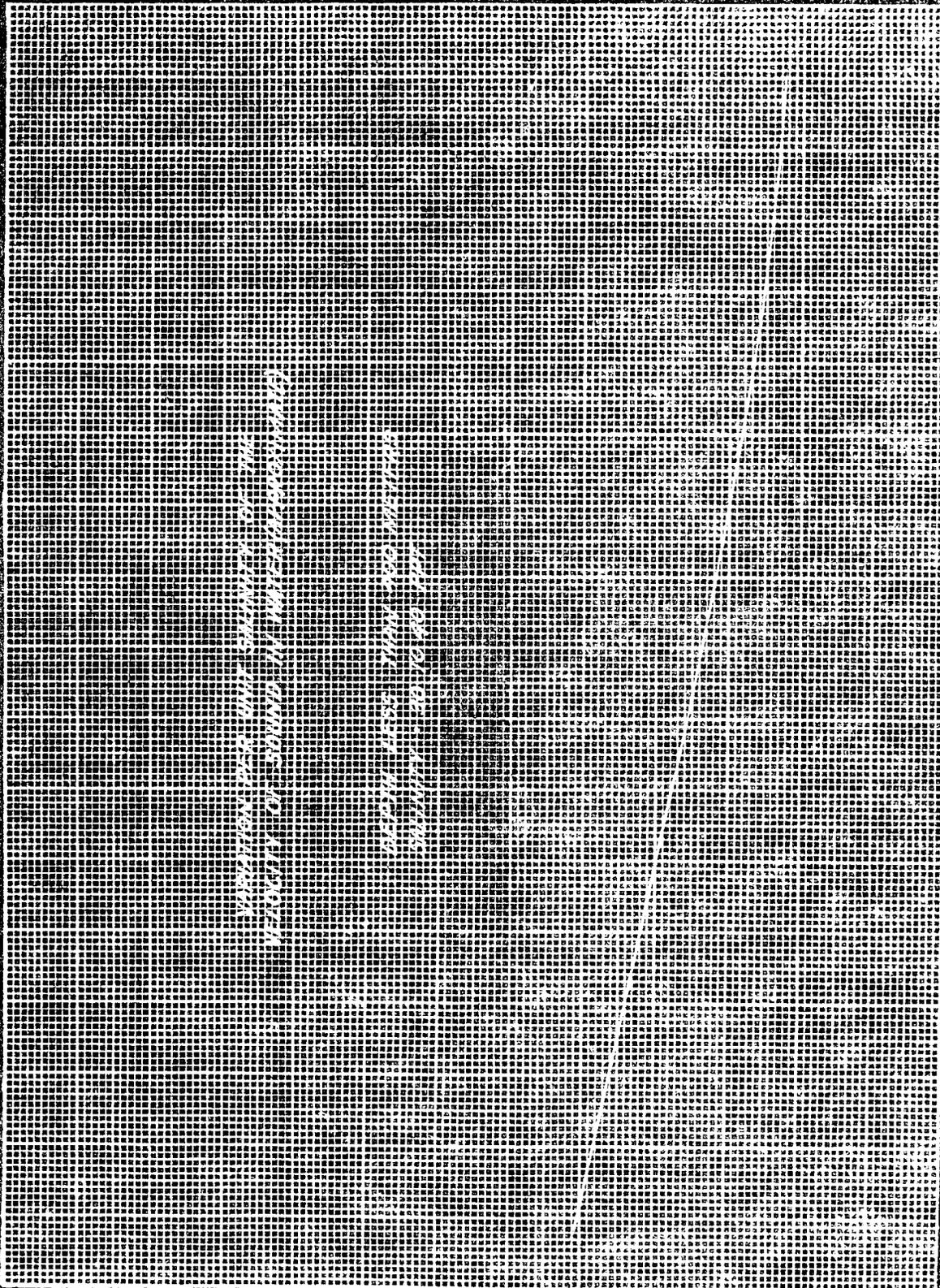
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DEGREES CENTIGRADE

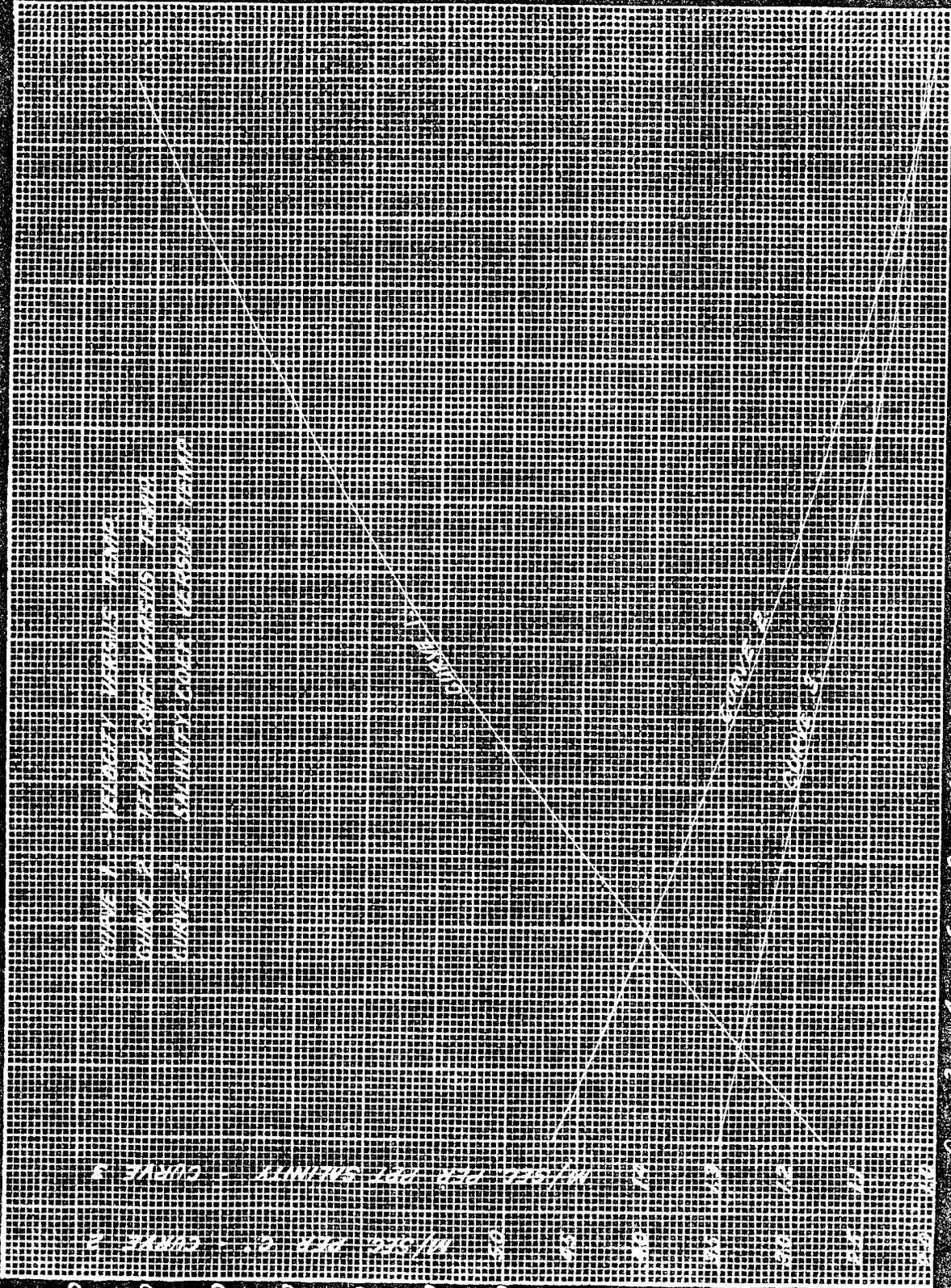


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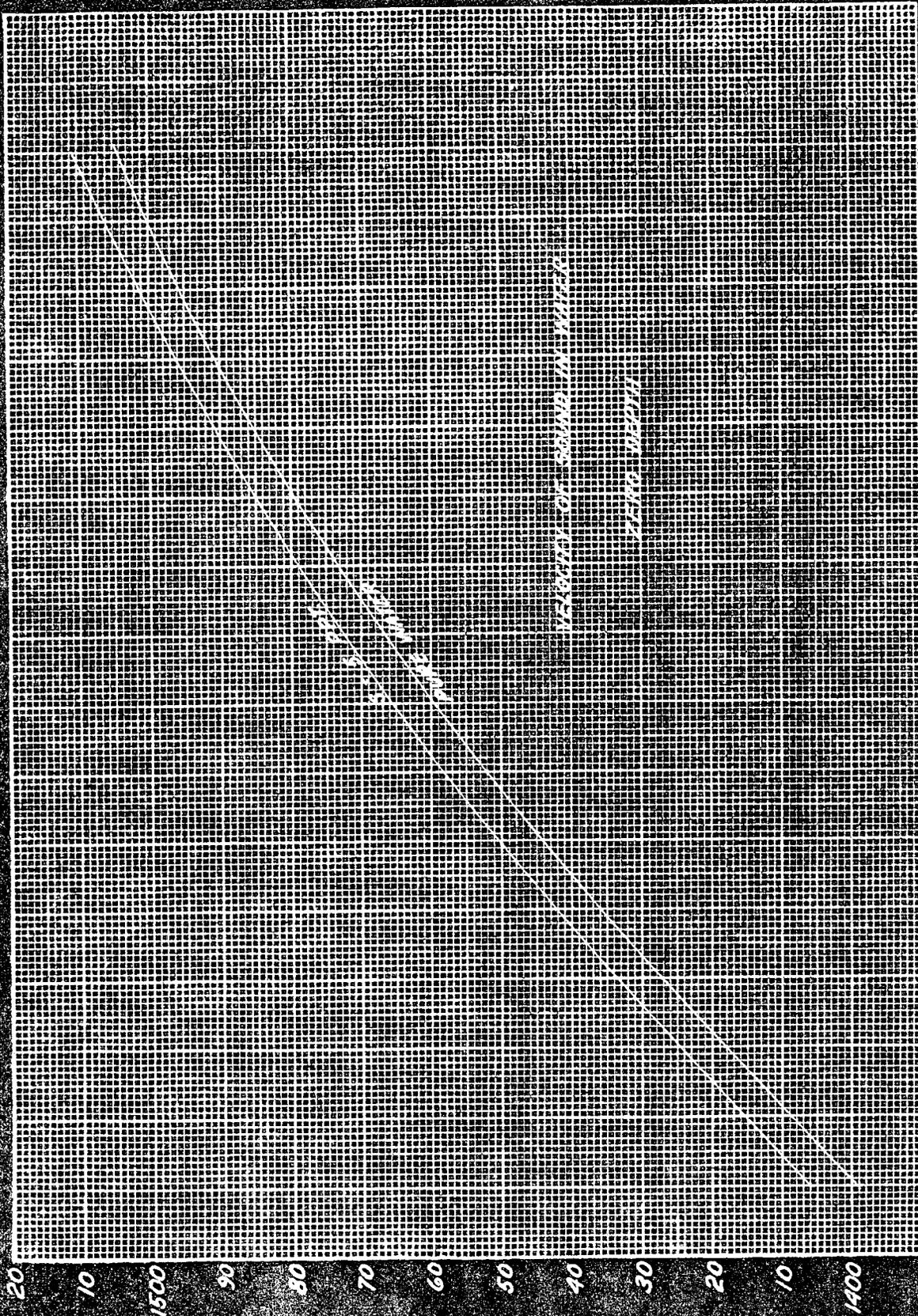


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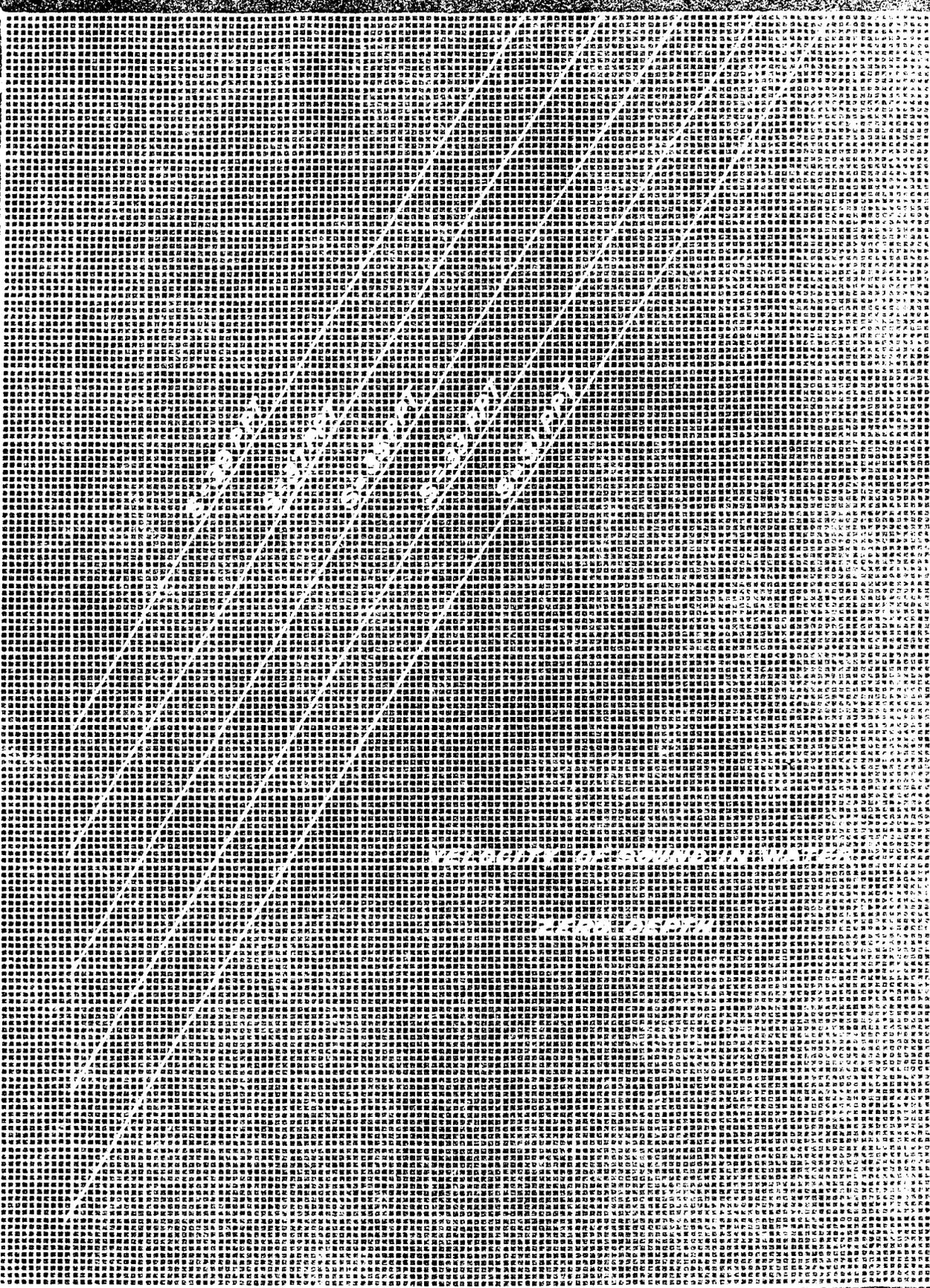


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DEGREES CENTIGRADE

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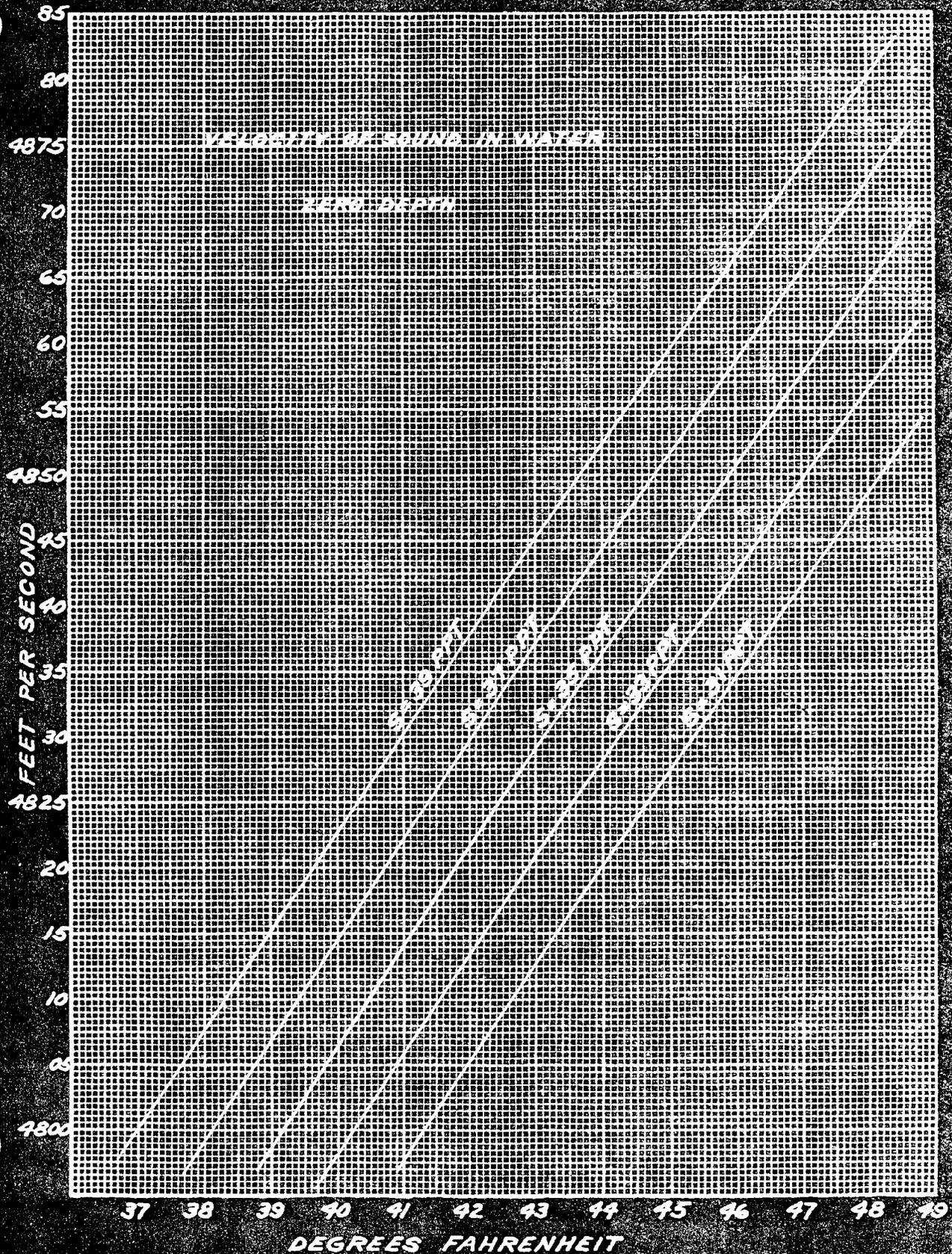


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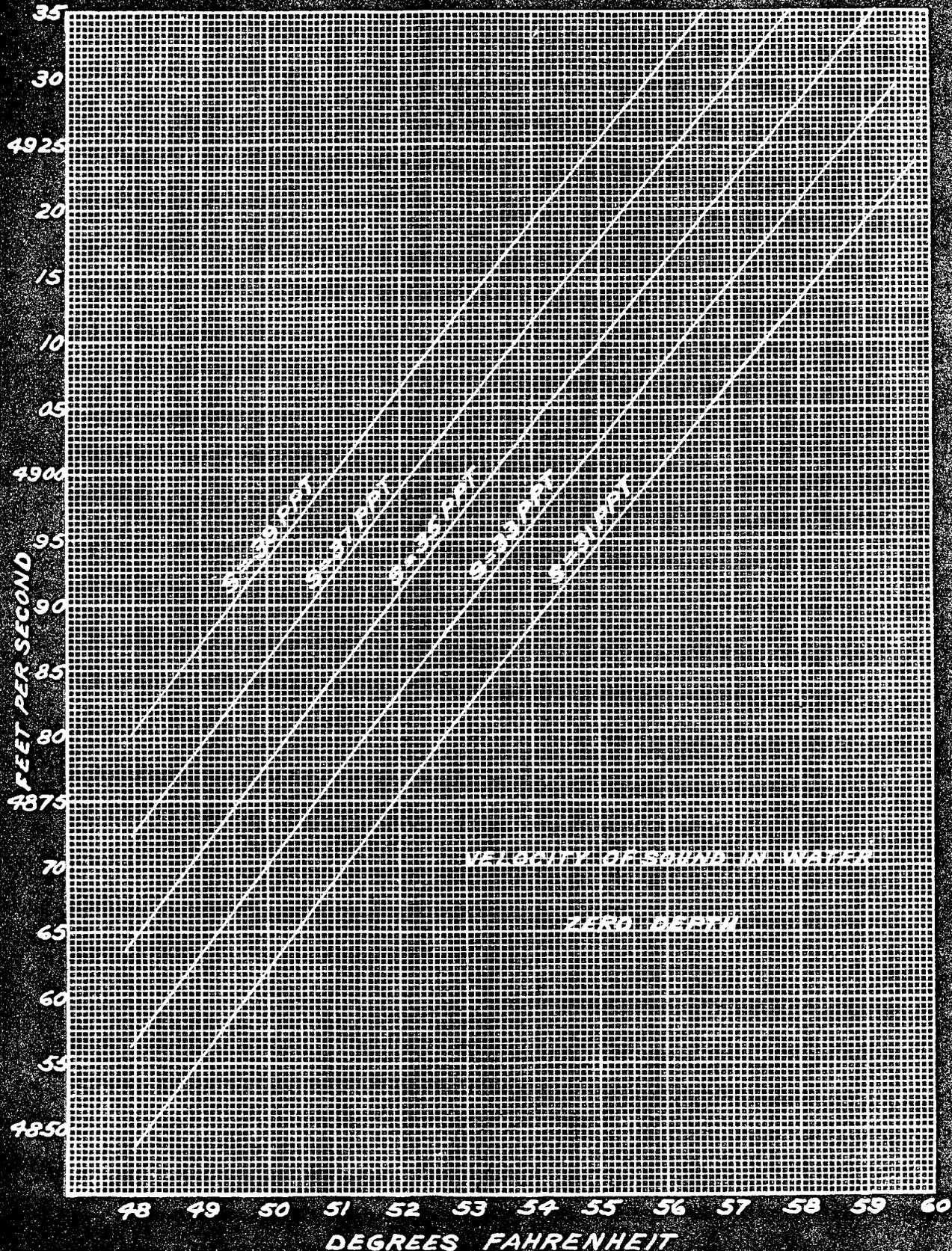
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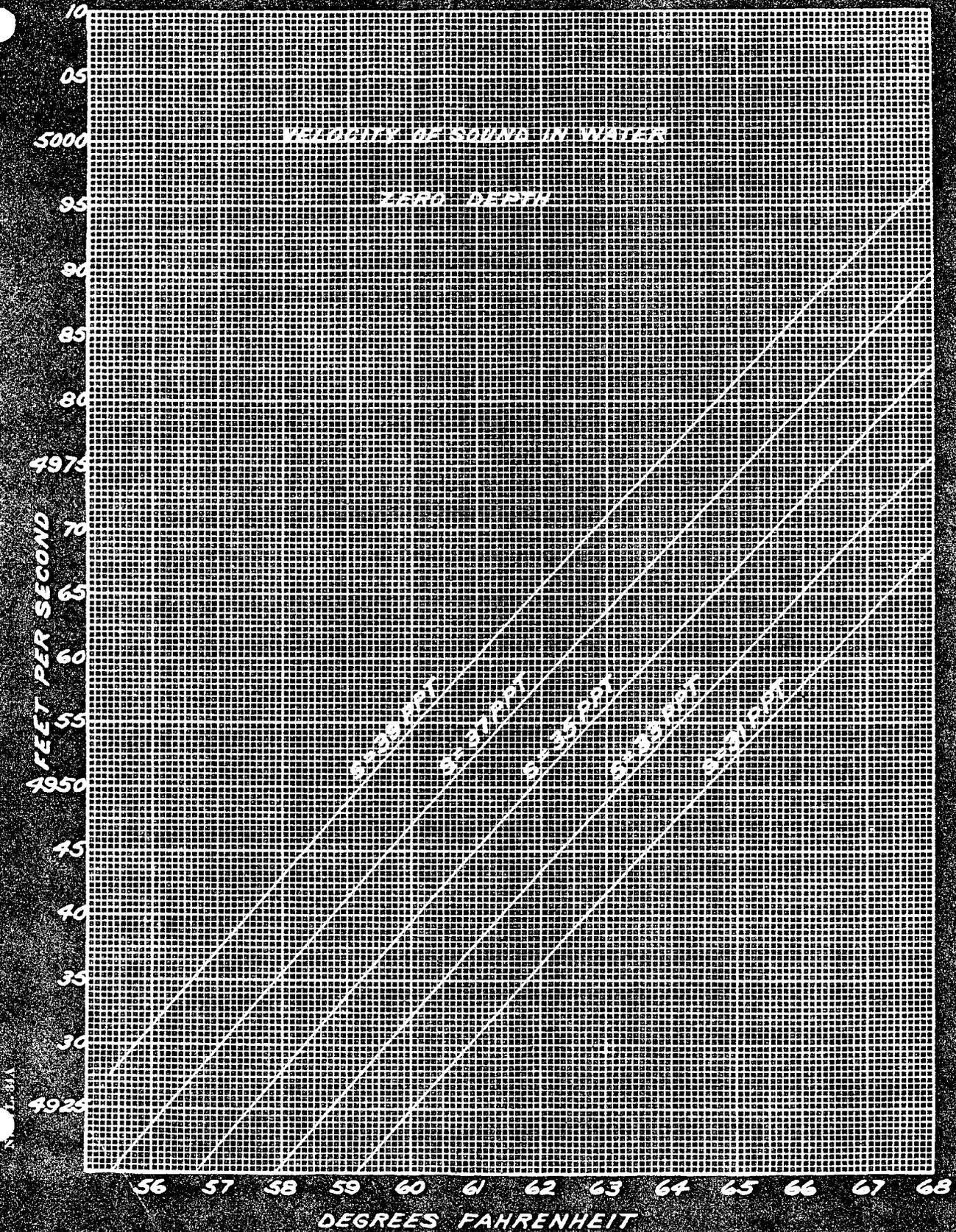
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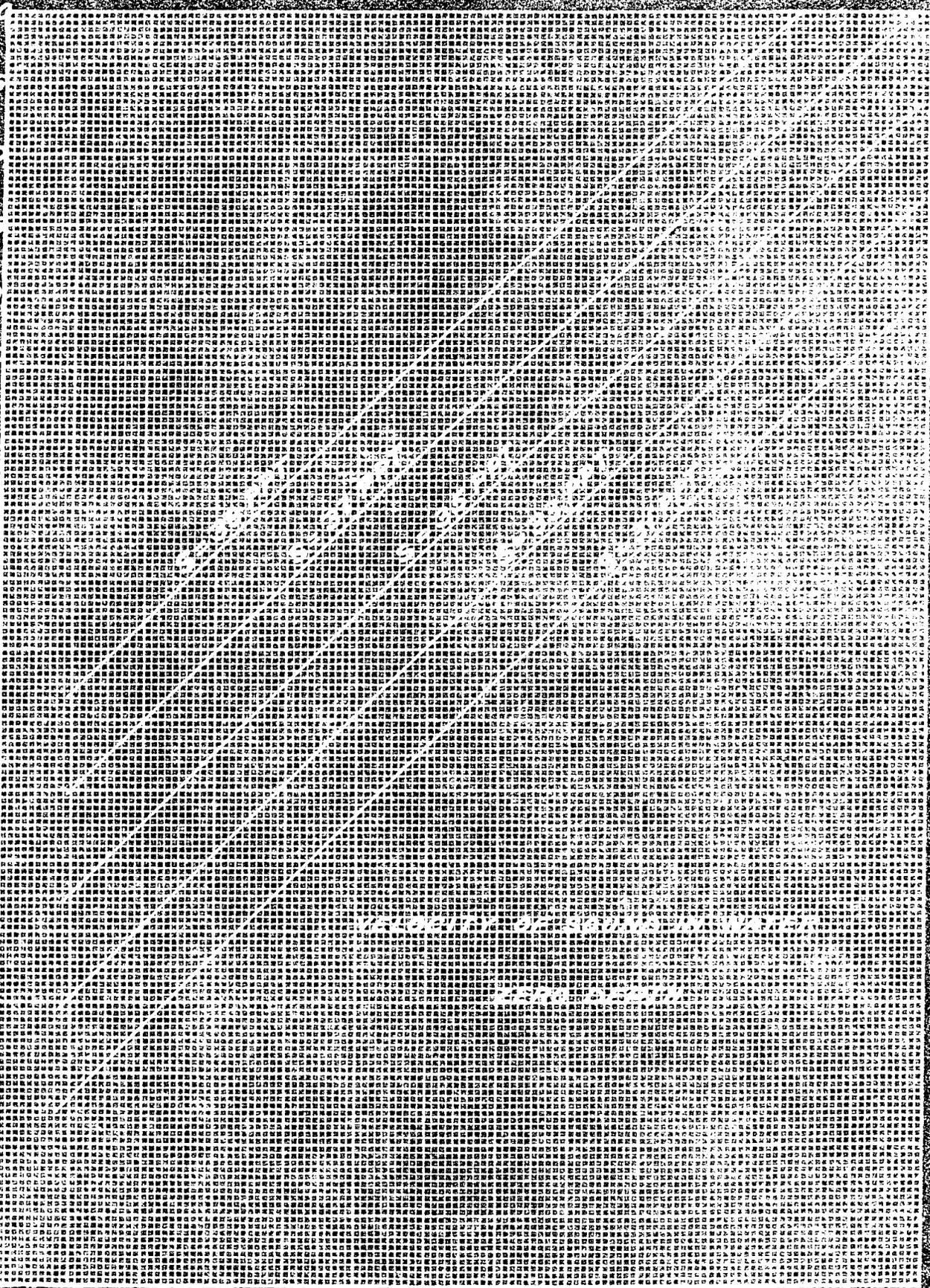
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FEET PER SECOND



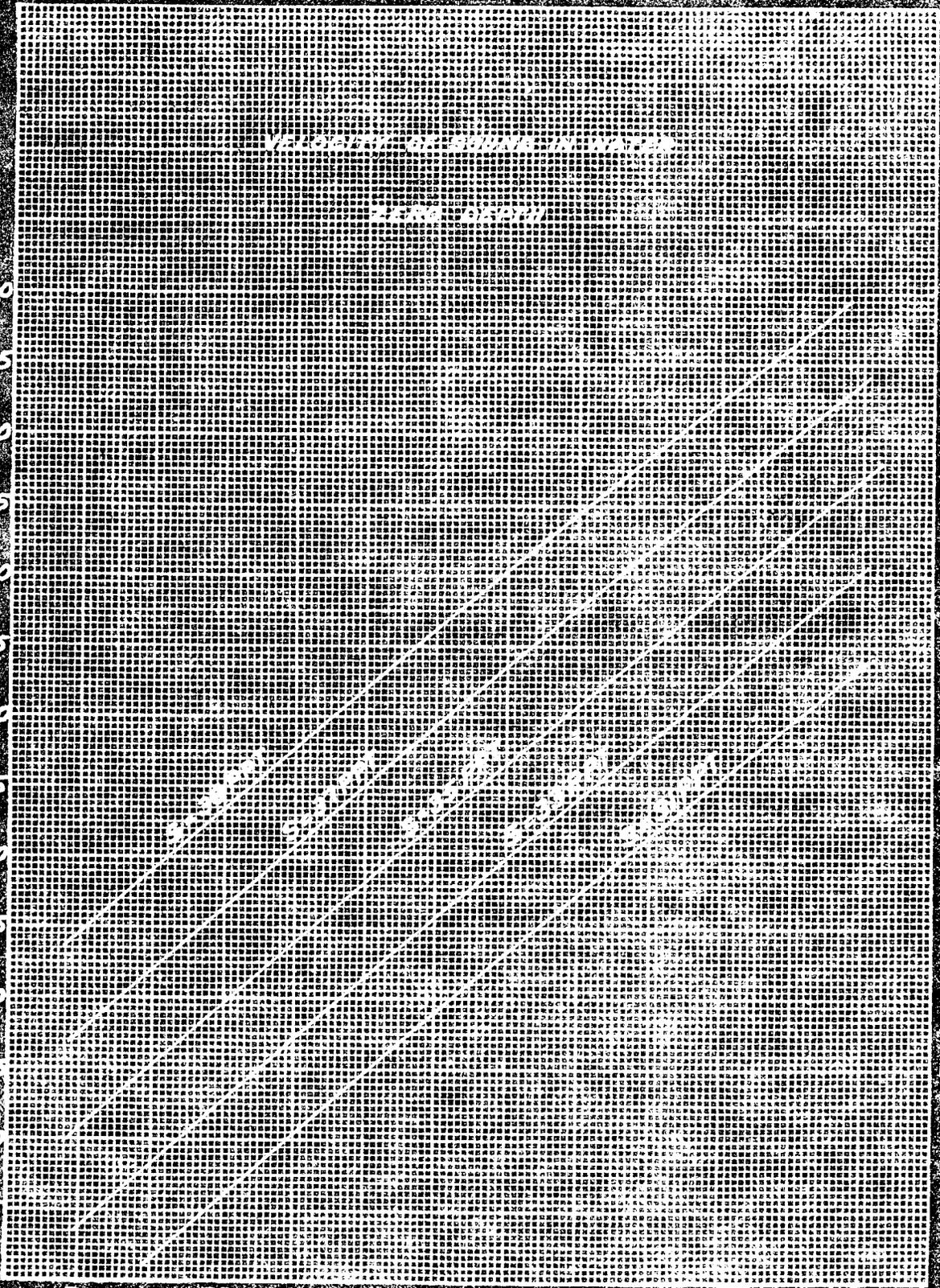
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DEGREES FAHRENHEIT

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15

FEET PER SECOND



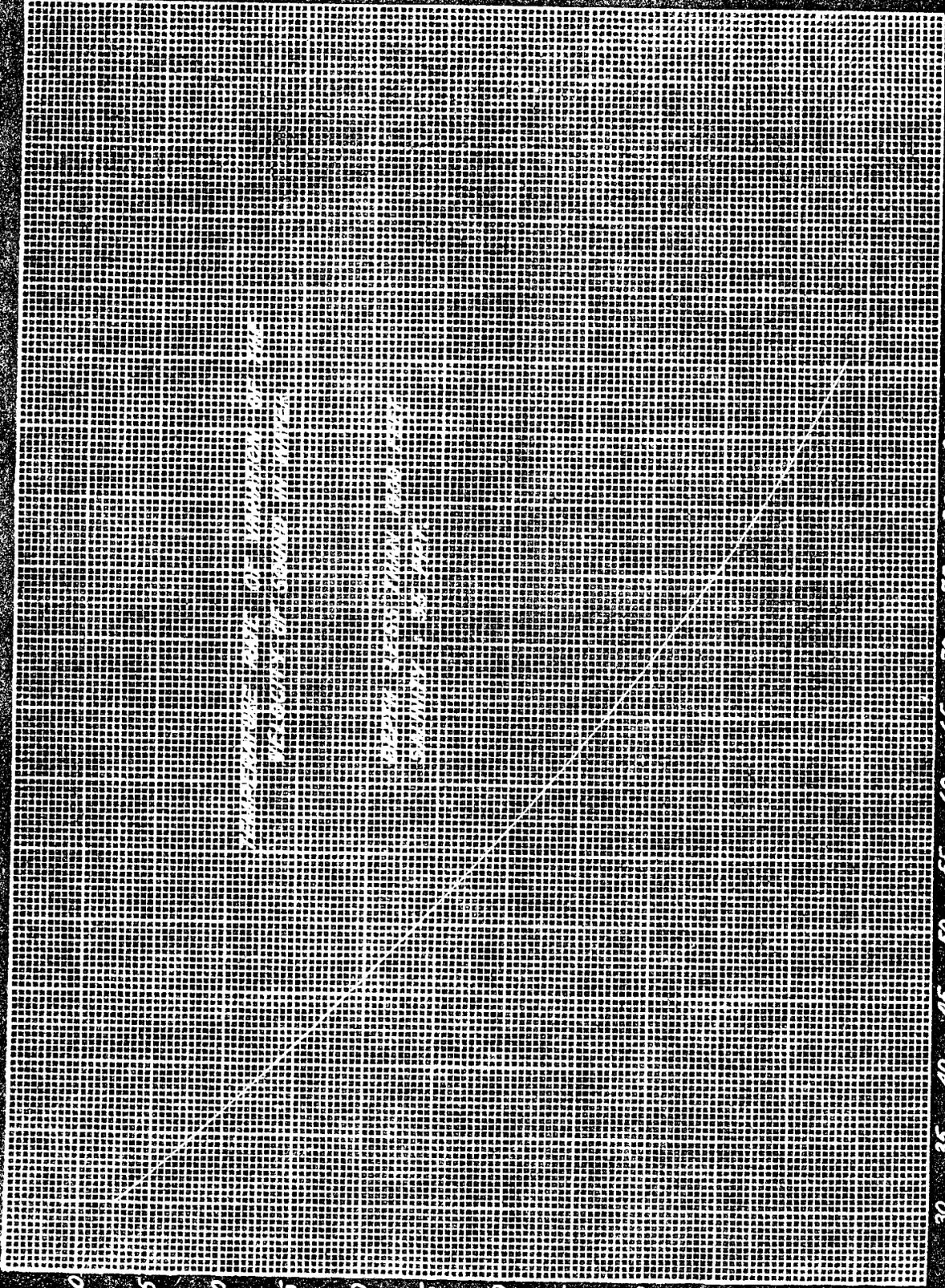
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DEGREES FAHRENHEIT

11-31A

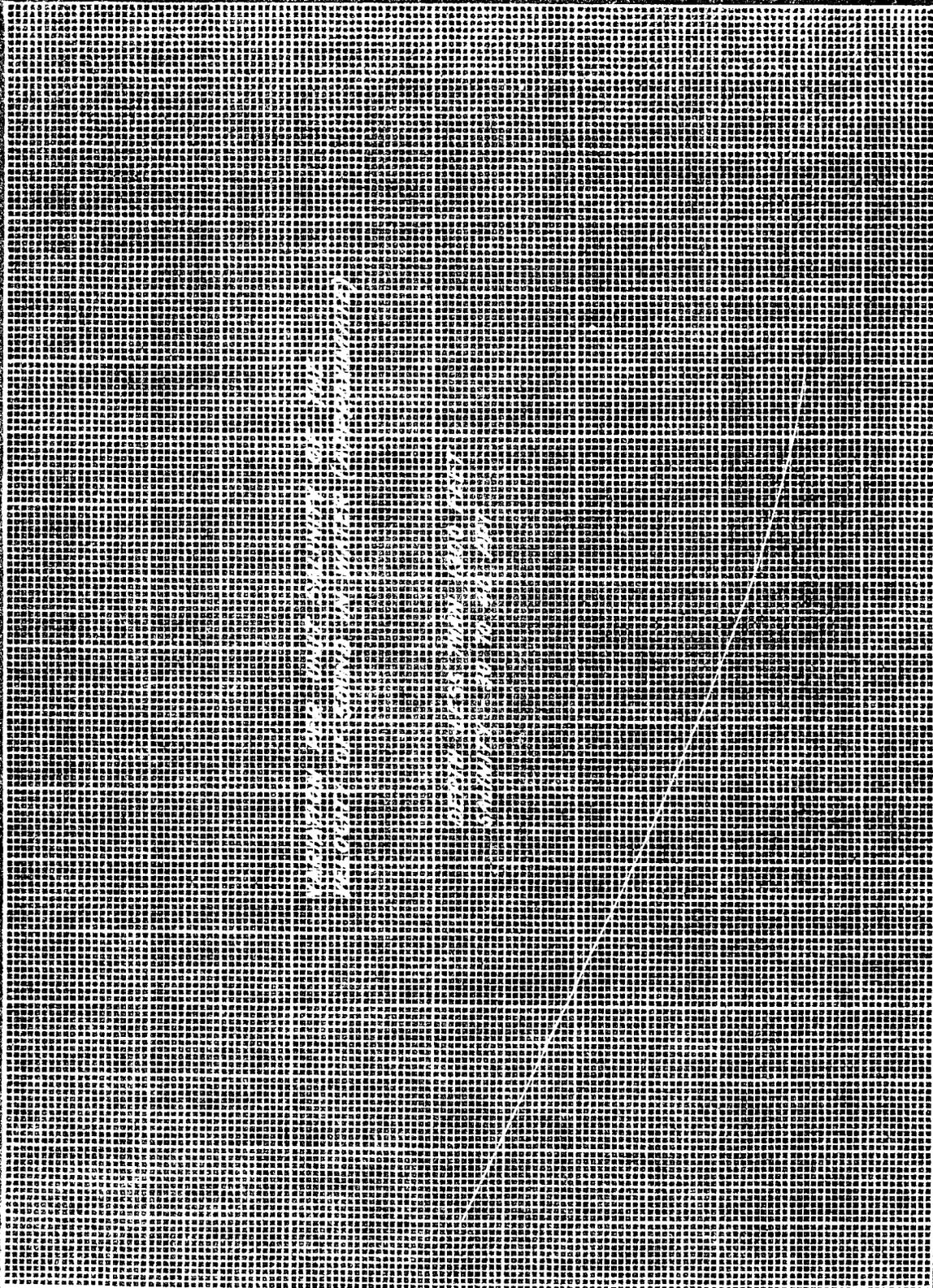
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1-81A



DEGREES FAHRENHEIT

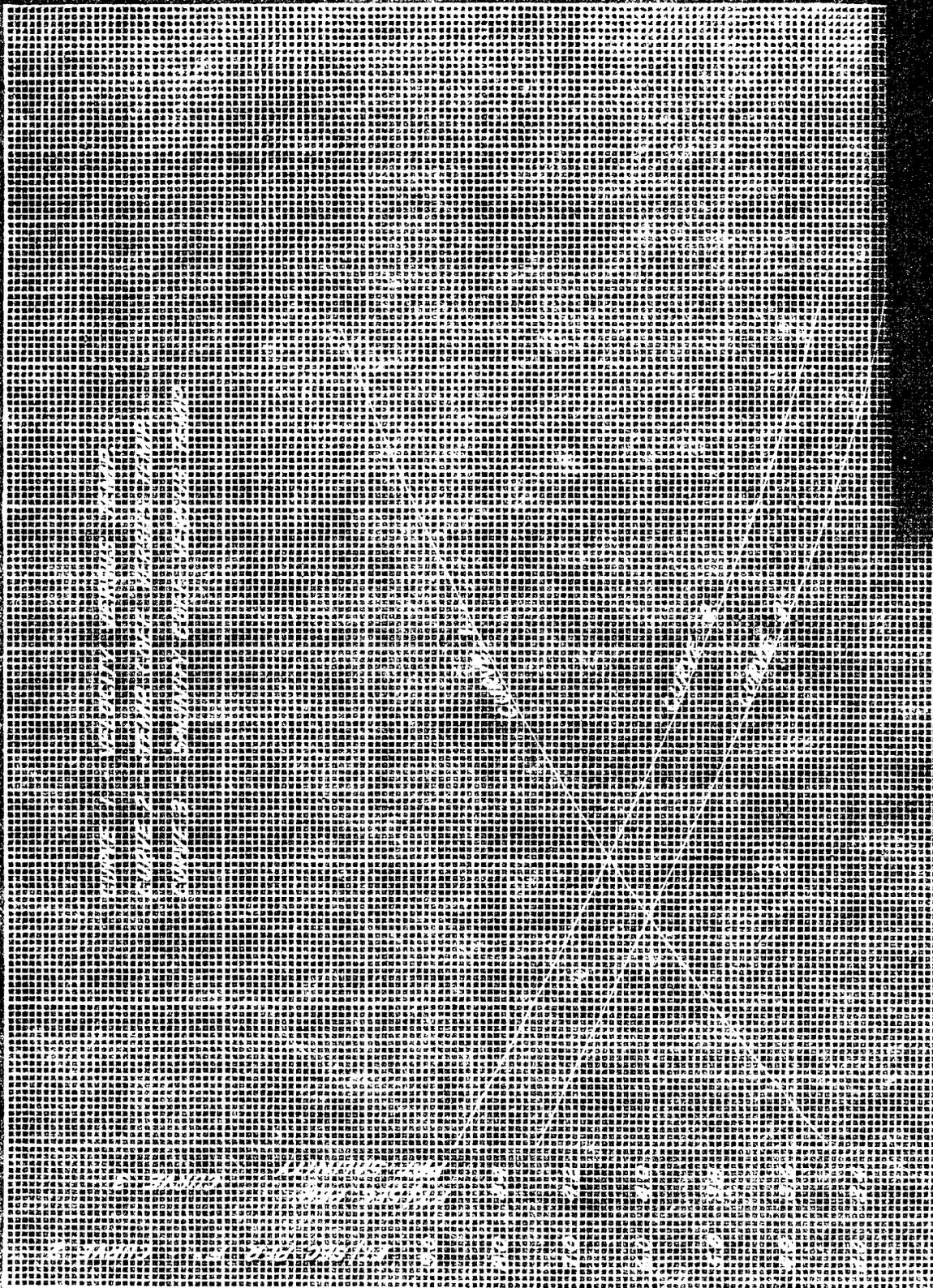
PER SECOND PER DEGREE FAHRENHEIT



FEET PER SECOND PER UNIT SALINITY

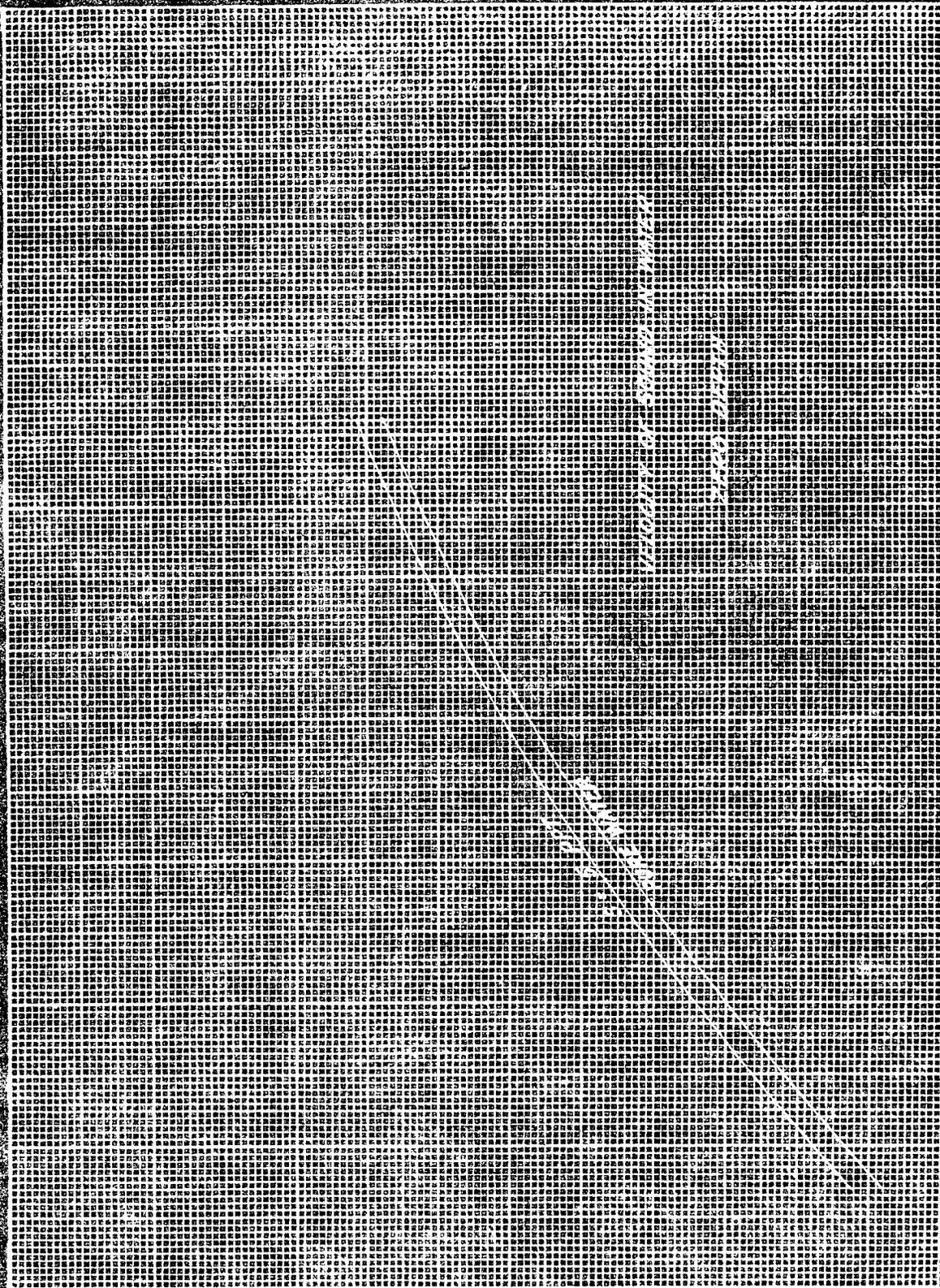
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DEGREES FAHRENHEIT

T-87A



50 55 60 65 70  
DEGREES FAHRENHEIT

5000 50 4900 50 4800 50 4700 50



30 35 40 45 50 55 60 65 70 75 80 85 90

DEGREES FAHRENHEIT

50  
4800  
50  
4800  
50  
4800  
50  
4800