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DATE 19 April 1944

SUBJECT

Hydrophone Calibration by Explosion Waves

NAVAL RESEARCH LABORATORY

BELLEVUE, D. C.

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NAVY DEPARTMENT

Report on

HYDROPHONE CALIBRATION BY EXPLOSION WAVES

NAVAL RESEARCH LABORATORY
ANACOSTIA STATION
WASHINGTON, D. C.

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ABSTRACT

The use of explosion waves in the relative calibration of a hydrophone offers a new method of approach to the problems of response measurement. Application of this method to two standard hydrophones in the frequency range 5-100 kc gives a relative calibration which agrees with the relative calibration obtained by conventional continuous wave methods.

The method may be applied to the calibration of projectors and other linear underwater sound transducers. More rapid analysis of the data is possible through the use of appropriate mechanical or electrical harmonic analyzers. Restriction to the use of explosion waves is not necessary as the method is applicable with any transient.

The method also gives information concerning the explosion wave, if the absolute calibration of the hydrophone is known.

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I. INTRODUCTION

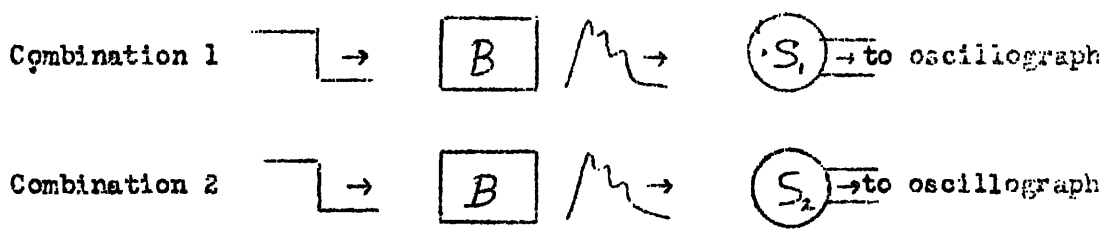
1. The use of transients to calibrate hydrophones is based on the mathematical theorem that the response of a linear system to any transient determines completely its response to any other transient as well as its steady state or continuous wave response. In this report this principle is used to obtain the relative calibration curve of two standard tourmaline hydrophones, the results being compared with those obtained by the conventional CW method. The explosion wave from a blasting cap was used as a convenient transient since it is strong, consistent, and free of electromagnetic disturbances.

2. The method can not yet be claimed to be as rapid as the CW comparison method unless mechanical or electronic harmonic analyzers are used. However, oscillograms of the responses of hydrophones struck by explosion waves show many of their characteristics qualitatively, just as the responses of electrical networks to square waves or Heaviside pulses - with which the explosion wave has much in common - disclose circuit qualities (Bibliog. 1). In this sense the explosion wave can be considered a tool of analysis, as is an electrical square wave. The method outlined in this report can be extended to projectors or to any electromechanical system which is linear.

II. THEORY

3. The response of a linear system to any type of wave can be determined from Duhamel's integral if its response to a Heaviside unit function, or its indicial admittance, is known. Bedford and Fredenhall (Bibliog 2.) give a method for determining, by a graphical evaluation of this integral, the steady state sensitivity and phase shift of a given system from its observed response to a square wave. This graphical procedure suggests the following method for determining the relative sensitivity and phase vs frequency curves of two systems, S_1 and S_2, by observing their respective responses to an arbitrary and unknown transient.

4. Assume that an additional structure B having the property of transforming an incident unit function into an arbitrary transient immediately precedes each system S (Sketch 1). The response and phase of each such combination, $B + S_1$ or $B + S_2$, to an incident square wave,



Sketch 1

as determined by the nature of Bedford and Fredenhall, may be plotted as a curve in the complex plane, the frequency appearing as a parameter. Any one point on such a curve will be the product of two factors, $B(\nu) \times S_1(\nu)$ or $B(\nu) \times S_2(\nu)$, where $B(\nu)$, $S_1(\nu)$ and $S_2(\nu)$ are functions of the frequency and represent the effect on sensitivity and phase that the additional structure and each system respectively have on an incident sinusoid of frequency ν. Hence a point by point ratio of the curves for the two combinations will give a curve representing the relative response and phase difference of the two systems, since the effect of the additional structure cancels. One can thus dispense with the artifice of the additional structure B and merely specify that the same transient be incident on both systems.

5. The mathematical expression for Duhamel's integral yielding the response to a sinusoid may be written in the form

$$i_{(T)} = \int_0^T \epsilon^{i\omega(t-\lambda)} \frac{dA_{(\lambda)}}{d\lambda} d\lambda \qquad (1)$$

when $A_{(0)} = C$. Here $i_{(T)}$ is the response to the sinusoid of frequency $\nu(=\frac{\omega}{2\pi})$ at the time T, and $A_{(\lambda)}$ is the indicial admittance. In the present case, $A_{(\lambda)}$ is the oscillogram of the response of the hydrophone to the explosion wave. Only the absolute magnitudes and phase differences of the two systems are of interest here and the factor ϵ^{iwt} can be eliminated.

$$\left| i_{(T)} \right| = \left| \int_0^T \epsilon^{-i\omega\lambda} \frac{dA_{(\lambda)}}{d\lambda} d\lambda \right| \qquad (2)$$

6. The method of Bedford and Fredenhall is to carry out this integral graphically by vectorial addition of the ΔA's at intervals Δt, the angle between successive ΔA's being $360^\circ \times \nu \Delta t$. Theoretically the integration should be carried out to infinity, or until the tail of the curve returns permanently to the t axis, if one wishes to obtain from equation (2) the steady state response. From a practical viewpoint this condition may be difficult to fulfill, and sufficient accuracy can be obtained by integrating until the tail approaches a constant value near zero. In practice, also, T must be less than the duration of the sweep. If the oscillogram has not "settled down" by the end of the sweep, then equation (2) obviously can not give a good estimate of the steady state response in such a short interval.

7. Integration of equation (2) by parts gives

$$\left| i_{(T)} \right| = \left| A_{(T)} \epsilon^{-i\omega T} + i\omega \int_0^T A_{(\lambda)} \epsilon^{-i\omega\lambda} d\lambda \right| \qquad (3)$$

If $A_{(T)} = 0$, an equation of the same ~~from~~ form as (2) is obtained, with the ordinates themselves replacing the difference in ordinates. If $A_{(T)} \neq 0$, a corrective term is necessary if one is to obtain the same result as with equation (2). In most of the mechanical or electronic methods of harmonic analysis (Bibliog. 3), an integral of the form appearing in equation (3) is evaluated.

III. APPARATUS

8. The two hydrophones which were compared were the NRL standard tourmaline hydrophones OL-A-2 and OL-A-3. Absolute calibrations of both hydrophones from 5 to 100 kc, based on a weighted absolute calibration of the standard OL-A-4 (Bibliog. 4 and 5), are shown in Plate 1. The weighted calibration of OL-A-4 is obtained from data taken at the Naval Research Laboratory and at the Mountain Lakes station of the Underwater Sound Reference Laboratories, and is considered accurate on the average within ± 2/3 db, with any point within ± 1.5 db. In addition to the comparison obtained from the above absolute calibrations, a direct CW comparison of the two hydrophones was made as an additional check during the course of this investigation.

9. A DuMont Type 172A experimental model single-sweep cathode-ray oscillographic assembly was employed to record the transient voltage response of the hydrophone to the explosion wave. The time scale on the oscillograms was obtained by modulating the spot intensity with a voltage of known frequency. (See Plate 5). The voltage scale was obtained from frequent calibration photographs of a sinusoid of known amplitude from a standard signal generator.

10. A Dupont #8 electric blasting cap was used for the explosion source. The sweep circuit of the oscillograph was triggered by the impulse from a hydrophone consisting of a single Rochelle salt crystal mounted a few inches above and in front of the hydrophone to be calibrated. This distance corresponds to the initial straight part of the oscillograms (Plates 5, 6) or the delay between the triggering of the sweep circuit and the arrival of the explosion wave at the hydrophone.

11. A mechanical harmonic analyzer (Bibliog. 6) was employed as one means of analyzing the oscillograms. Its distinguishing characteristic is its ability to analyze a curve of arbitrary base length.

IV. TECHNIQUE OF OBSERVATION AND MEASUREMENT

12. The hydrophone to be calibrated was mounted on the end of a length of standard 2" pipe and was located approximately midway between bottom and surface in water the depth of which varied from 17 to 20 feet with the tide. The blasting cap was located 18 feet from the hydrophone. At this distance the echoes from bottom and surface did not arrive until at least one millisecond had elapsed after the arrival of the direct wave.

13. Two series of oscillograms were made for each hydrophone, each series consisting of two groups of three oscillograms each, two modulated and one unmodulated. One group was made using a sweep of approximately 200 μsec duration and the other using a sweep of either 400 or 1000 μsec duration. The 1000 μsec sweep was used in one series of oscillograms (OL-A-2) while the 400 μsec sweep was used in the remaining series of oscillograms.

14. A modulation frequency of 200 kc, producing points at 5 μsec intervals, was used on the 200 and 400 μsec sweeps, and a modulation frequency of 50 kc giving 20 μsec intervals was used on the 1000 μsec sweep. As a further aid to measurement a time (x) axis was applied to each oscillogram before the camera shutter was closed. Amplitude calibration oscillograms were taken following each explosion.

15. The effect of orientation of the hydrophone relative to the direction of the explosion was investigated for angles up to $\pm$ 30°. Typical oscillograms are shown in Plate 8.

16. The oscillograms to be analyzed were projected upon the screen of a Recordax microfilm reader and traced upon thin paper from which the measurements were made. The start of the shock wave was located in the time coordinate by measuring back from a well established modulation point on the oscillogram. Measurements of the ordinates in the case of the 200 μsec sweep were made every μsec through approximately the first 40 μsec, wherein the most rapid fluctuations in amplitude occurred. Thereafter the ordinates were measured at 5 μsec intervals and also at any maxima or minima. In the cases of the 400 μsec or 1000 μsec sweeps, measurements were made at 5 or 20 μsec intervals, respectively, after the passage of the main peak. In the region of the peak, amplitude changes occurred too rapidly with these sweeps for accurate measurements to be made.

17. The observed measurements were corrected to mid-screen position by application of experimentally determined correction factors. These corrections were found by applying calibrating sinusoidal voltages to the oscillograph and measuring the resultant deflection amplitudes at the various positions on the screen. It should be remarked that these corrections should always be determined experimentally, since they are asymmetric in x, and in addition are in a direction opposite

to that to be expected on the basis of a spherical screen. Measurements from one oscillogram of 200 μsec sweep and from one of 400 or 1600 microsec sweep were combined into one curve to be analyzed, the shorter sweep furnishing the details of the peak region, and the longer showing the behavior in the tail of the curve. Two such curves, one from each series of oscillograms, were obtained for each hydrophone. Thus two independent relative calibrations were available.

16. Three methods were used in analyzing the curve; a) graphical, b) numerical, and c) mechanical.

a) Graphical - Differences in the ordinates were determined from the curve for every μsec through the first 200 μsec. For a given frequency ν, at which the response was to be found, these differences were added vectorially according to the method of Redford and Fredenhall. One μsec intervals were used over the entire region graphed at frequencies above 50 kc and over the first 40 μsec at frequencies from 10 to 50 kc. Intervals greater than 1 μsec were used in the regions where the ΔA's were small, though the time intervals were kept small enough so that the angle between successive vectors was never greater than 36°. For ease in plotting, values of ν giving angles of 360°/n, n an integer, were used.

b) Numerical - At the higher frequencies where there were 20 or less vector additions per 360°, all the vectors in a given direction were added algebraically and the resultant of the components thus obtained was determined by trigonometry. This method was much more rapid than the graphical method and also eliminated errors due to graphing. At lower frequencies, however, it became cumbersome because of the number of components involved.

c) Mechanical - A harmonic analyzer was used which evaluates an integral of the form $\int_0^T A_{(\lambda)} \epsilon^{-i\omega\lambda} d\lambda$, the integral in equation (3) of paragraph 7. In the oscillograms analyzed, the curve had not returned to the t axis at the time T to which integration was carried, and the correction term $A_{(T)} \epsilon^{-i\omega T}$ was included. As a check and to reduce any errors occurring because of the choice of the direction of tracing, each curve was traced by the analyzer both clockwise and counter-clockwise at each frequency. Since this type of analyzer (6) is independent of the baseline, it was possible to determine the response at frequencies which were non-integral harmonics of the "fundamental" frequency of period T. This property of the analyzer, along with the saving in time, made this method superior to the other methods employed.

19. The comparison calibration of the two hydrophones by the transient method is shown in Plate 2. Also included are the direct CW comparison calibration and the comparison obtained from the absolute calibrations. The calibrations agree very well except for occasional points. The agreement between the transient method and the direct comparison is as good as the agreement between the direct comparison and the comparison of the absolute calibrations.

20. Oscillograms of both OL-A-2 and OL-A-3 response for 200 μsec and 400 μsec sweeps are shown in Plates 5 and 6. The resonances which appear as irregularities in the calibration curves of the hydrophones appear as oscillations in the oscillograms. The period of the most prominent oscillations in the OL-A-2 oscillograms is approximately 8 μsec, corresponding to a frequency of 125 kc, above the range of the present analysis. The most prominent oscillations in the OL-A-3 oscillograms have periods between 13 and 16 μsec, corresponding to frequencies between 60 and 77 kc. This is the region of violent fluctuation in the OL-A-3 calibration curve (Plate 1). There is also a very long period (approximately 600 μsec) oscillation in the OL-A-2 oscillogram, visible in the longer sweep, corresponding to a frequency of about 1.6 kc. A resonance has been found in this region in other OL-A type hydrophones tested at the lower audio frequencies. Interference phenomena or beats of the oscillations will be noted where multiple resonances are present.

21. The graphical method of analysis exhibited characteristics of the analysis which were not apparent from the other methods. Plate 7 shows a typical graphical plot. The numbers on the graph indicate the time interval in μsec from the arrival of the shock front. The change of sign of the ΔA's at each maximum or minimum results in the formation of cusps in the graph. A criterion on how far the integration should be carried is given by the convergence of the vectors in the tail of the oscillogram. Integration over 200 μsec was found to be sufficient at most frequencies and 200 μsec therefore was chosen as the standard time for use in this investigation. At frequencies of pronounced resonance the cusps occurred approximately every 180° of rotation and resulted in a series of semicircular segments the diameters of which were additive in the same direction. Convergence was less rapid in these cases and the uncertainty of the resulting response was greater than at other frequencies.

22. As absolute calibrations for both hydrophones were known, the absolute pressure of the explosion wave was calculated from the hydrophone response. The "calibrations" of the explosion wave, obtained from each hydrophone, are shown in Plates 3 and 4. These curves can be interpreted in two ways. Mathematically, except for a constant factor, $1/\sqrt{2\pi}$, each gives the absolute magnitude of the

Fourier transform of the derivative of the explosion wave. Or if the blasting cap is considered as a linear projector which emits the actual explosion wave when excited by a unit function, then each curve gives, as a function of frequency, the amplitude of the CW wave developed at the reference point by the "projector" when excited by a CW voltage of constant amplitude. The value of the pressure approached at high frequencies by this curve is the value of the peak pressure in the explosion wave; it is about 5.7×10^6 dynes/cm^2 at 18 feet, a value in agreement with the peak pressure measured by more direct methods.

23. On the assumption that the explosion wave obeys the exponential law of decay ϵ^{-t/T_c}, where T_c is the time constant of the exponential, the pressure should be falling off rapidly below a frequency $\nu \cong 1/2\pi T_c$ The theoretical pressure vs frequency curves (the pressure at high frequencies being assumed correct) are also shown in Plates 3 and 4 for different values of T_c near the correct value ($\sim$20 μsec) for a #8 cap. It is seen that the pressure does not fall off at the lower frequencies as the theoretical curves would indicate. Actually the pressure in the tail is believed to follow a $t^{-4/5}$ instead of an exponential law, so that the deviation of the observations from the values predicted by the exponential approximation is in the right direction.

24. The oscillograms of Plate 8 show that the effects of rotating the hydrophone relative to the direction to the explosion are a) to round off the main peak and increase the time of rise, and b) to smooth out the oscillations in the tail. The graphical analysis of an oscillogram taken at an angle of $\sim 15^\circ$ showed the expected decrease in the high frequency sensitivity.

25. The transient method of comparison also gives the relative phase shifts of the hydrophones; and, if the shape of the explosion wave is known, the absolute phase shift with frequency can be determined. The accuracy is not very high, $\pm 10^\circ$, but this additional datum is mentioned since relative phase shift is not easily determined by CW methods of comparison.

Determination of Ind. Admittance.

VI. SUMMARY AND CONCLUSIONS

26. The use of a transient in the relative calibration of a hydrophone is feasible and gives results which are in agreement with relative calibrations obtained with continuous waves. The use of explosion waves, or other types of transients, can be extended to the relative calibration of projectors and other linear underwater sound transducers.

27. The calibration of an explosion wave or other transient by means of a standard hydrophone is illustrated, and this suggests the possibility of utilizing a standard transient rather than a standard

hydrophone for calibrations. As much qualitative information concerning the performance of a unit can be obtained from an examination of an oscillogram of its response to an explosion wave, it is suggested that explosion waves can be used as tools of analysis in underwater sound research in the same way that square waves are used in investigations of electrical systems. The method may be adaptable to the multiple testing of similar units by comparison of the uniformity of their response oscillograms.

28. A determination of the absolute pressure in the explosion wave from a #8 blasting cap has been obtained from the responses of standard hydrophones. This pressure does not decrease in the tail of the wave as much as the simple exponential theory indicates. The peak pressure so determined agrees well with that obtained by other methods.

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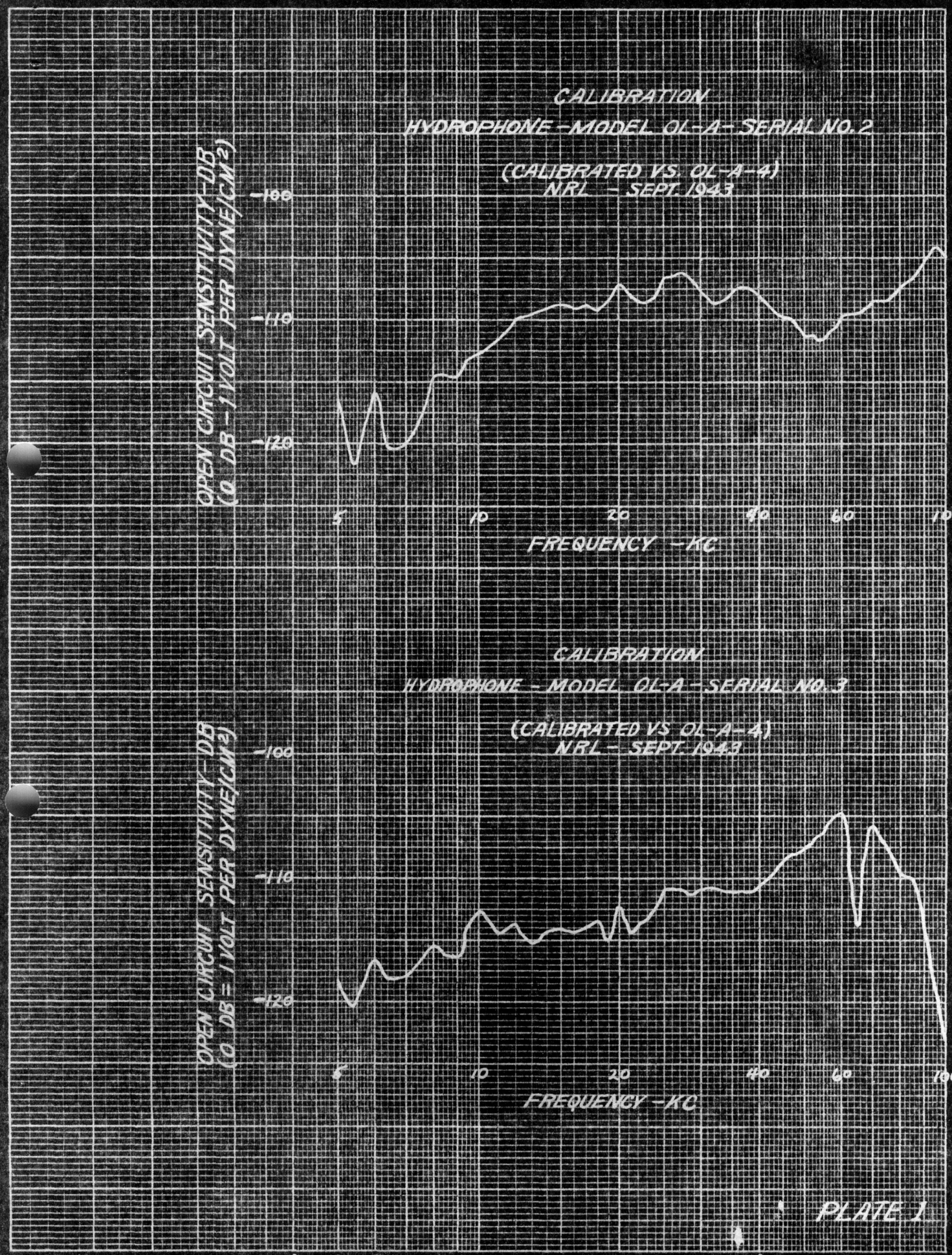


PLATE 1

NO. 3115. 20 DIVISIONS PER INCH (200 DIVISIONS) BY 2 3/4-INCH CYCLES RATIO RULING.
CODEX BOOK COMPANY, INC., NORWOOD, MASSACHUSETTS.
PRINTED IN U.S.A.

NO. 31,193. 20 DIVISIONS PER INCH (120 DIVISIONS) BY TWO 4 1/2-INCH CYCLES RATIO RULING. CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS.

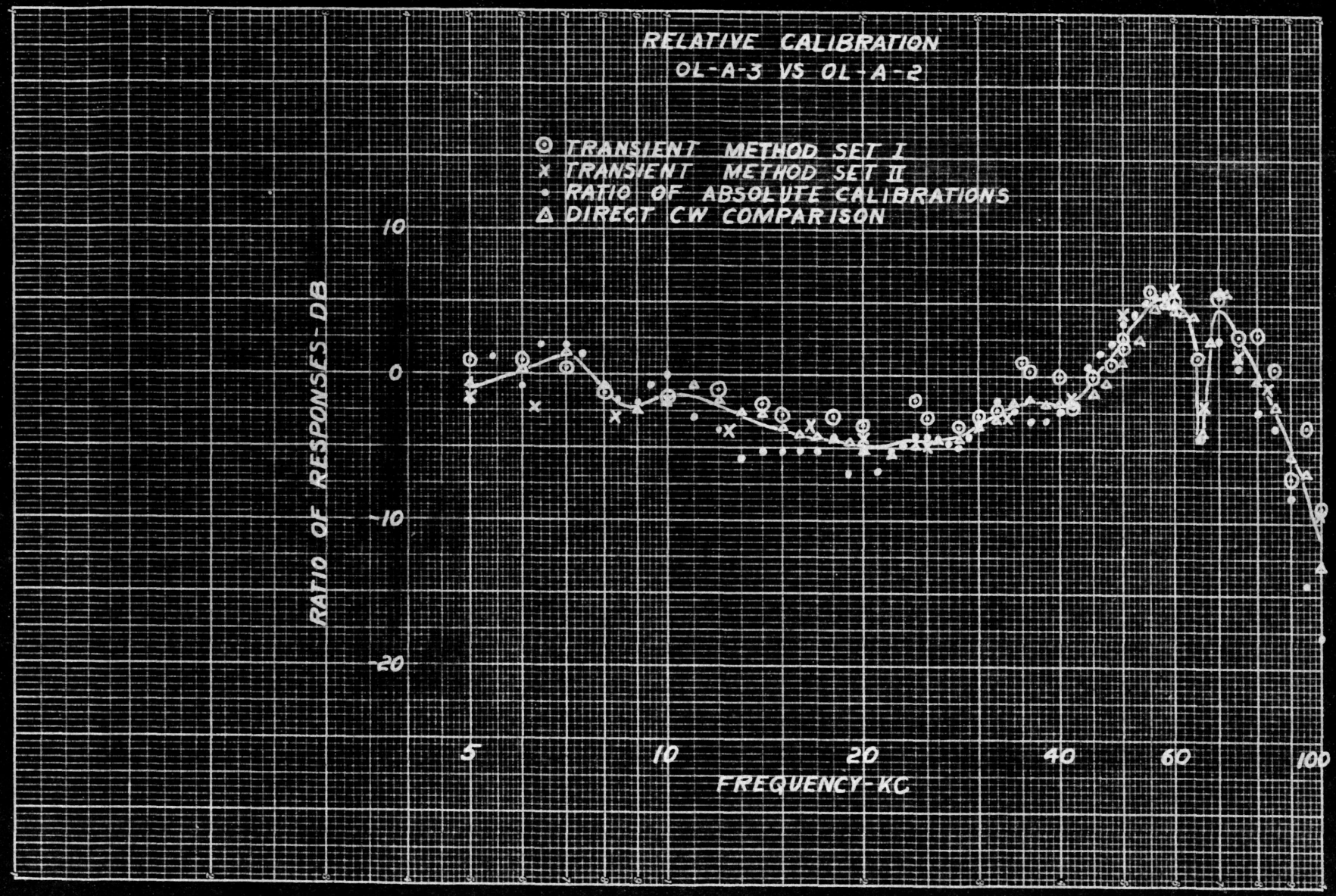


PLATE 2

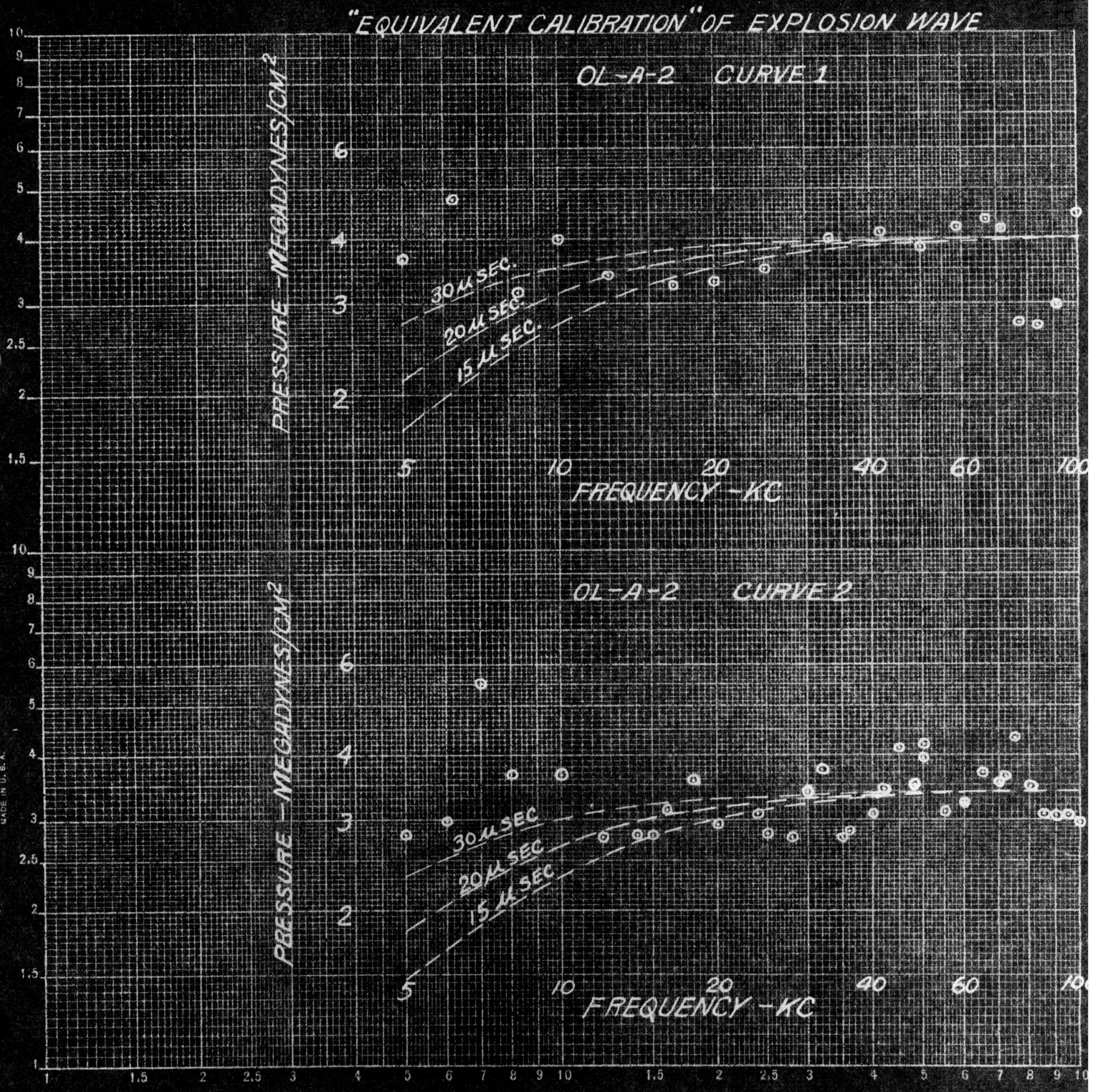


PLATE 3

KEUFFEL & ESSER CO., N.Y.
Logarithmic, 2 X 2
MADE IN U.S.A.

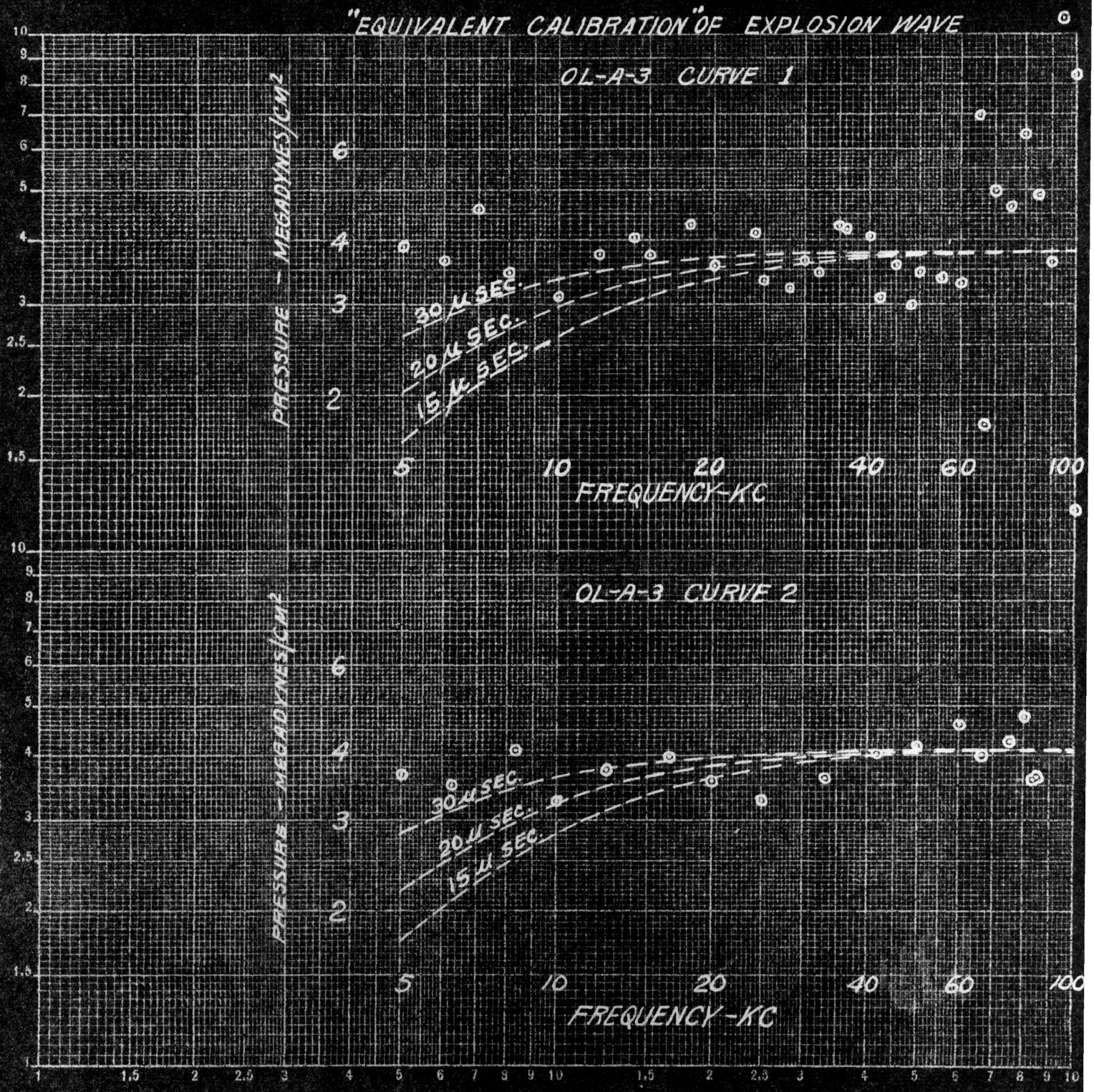
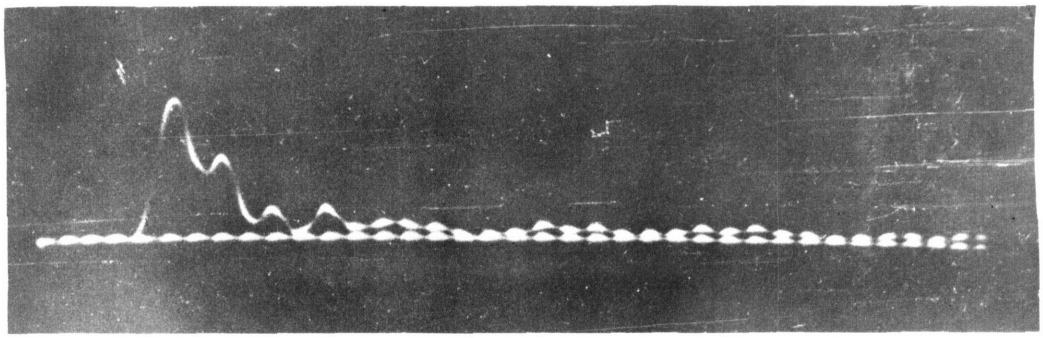
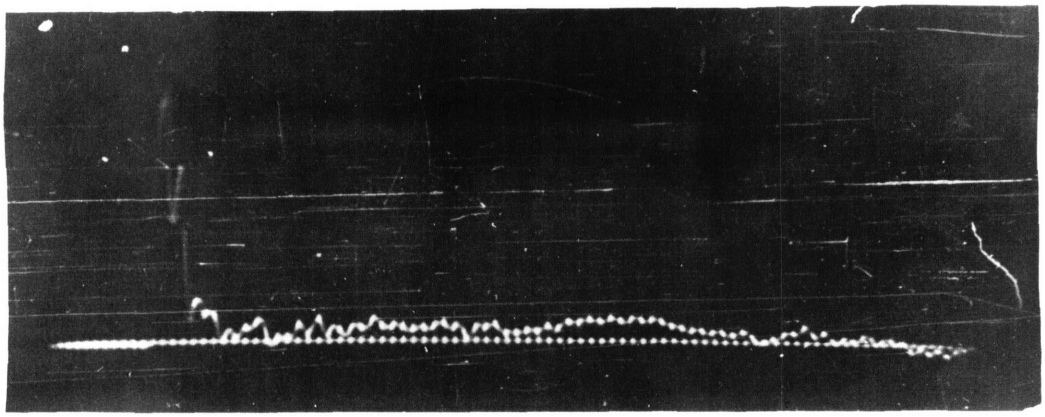

"EQUIVALENT CALIBRATION" OF EXPLOSION WAVE
OL-A-3 CURVE 1
PRESSURE - MEGADYNES/CM²
30 μ SEC.
20 μ SEC.
15 μ SEC.
FREQUENCY-KC
OL-A-3 CURVE 2
PRESSURE - MEGADYNES/CM²
30 μ SEC.
20 μ SEC.
15 μ SEC.
FREQUENCY-KC

PLATE 4

TYPICAL OSCILLOGRAMS
OL-A-2

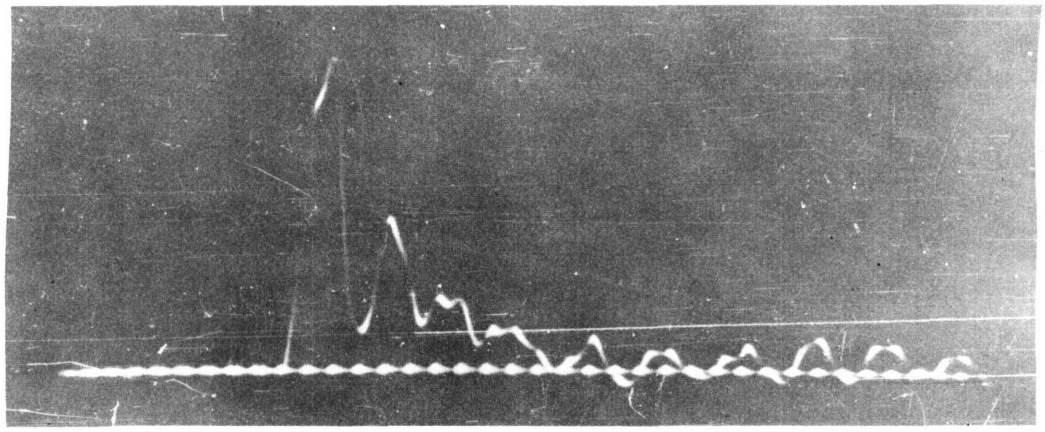


200μ SEC SWEEP
MODULATION FREQUENCY 200 KC

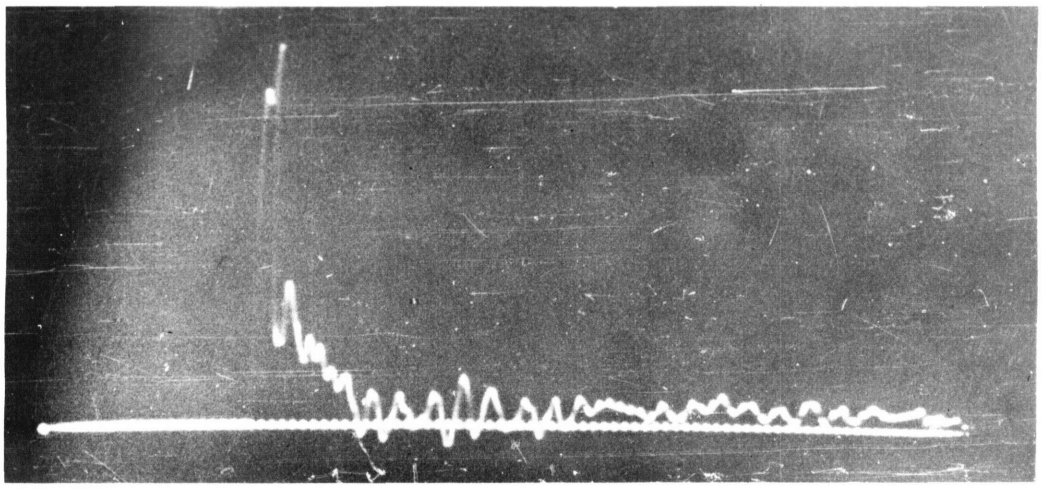


400μ SEC SWEEP
MODULATION FREQUENCY 200 KC

TYPICAL OSCILLOGRAMS
OL-A-3



200 μ SEC SWEEP
MODULATION FREQUENCY 200 KC



400 μ SEC SWEEP
MODULATION FREQUENCY 200 KC

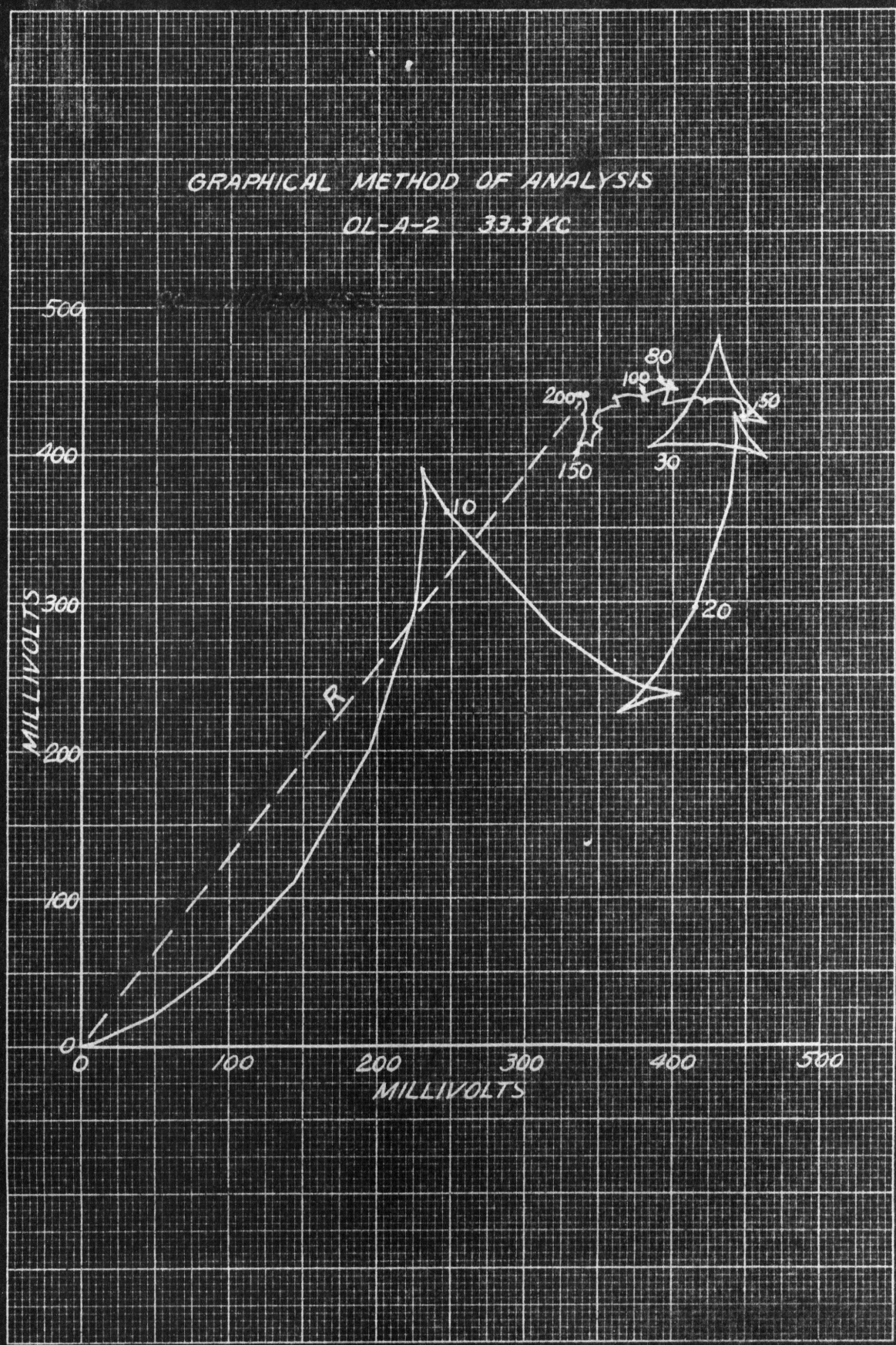
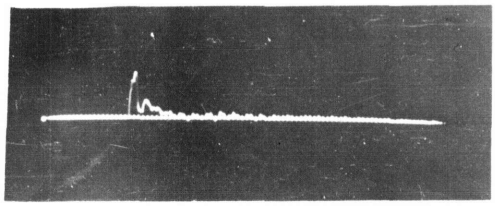


PLATE 7

NO. 311O. 20 DIVISIONS PER INCH BOTH WAYS. 120 BY 180 DIVISIONS.
CODEX BOOK COMPANY, INC., NORWOOD, MASSACHUSETTS.
PRINTED IN U.S.A.



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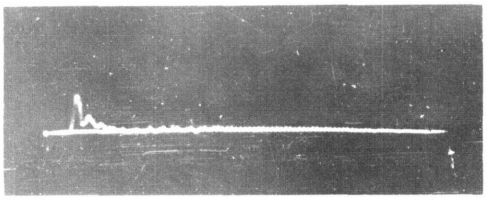
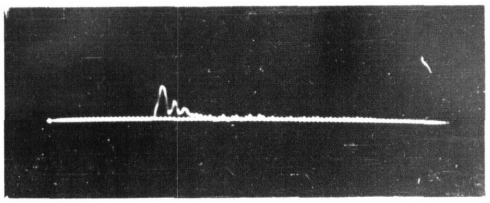
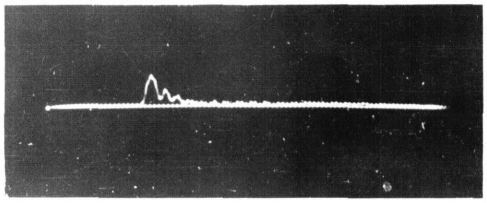


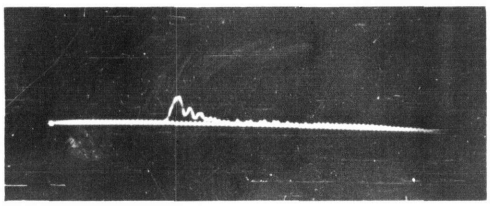
+ 10°



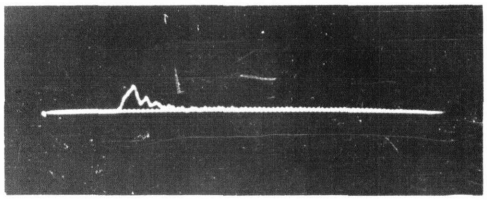
- 10°



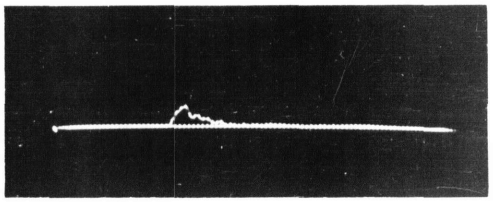
+ 20°



- 20°



+ 30°



- 30°

400 μ SEC SWEEP

RESPONSE TO TRANSIENT VS. ANGLE OF INCIDENCE
OL-A-3

PLATE 8