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EXACT AND APPROXIMATE TREATMENTS OF THE ONE-DIMENSIONAL BLAST WAVE

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ABSTRACT

A completely analytic solution is presented for a one-dimensional strong blast wave in a perfect γ-law gas. On the basis of qualitative information obtained from this exact solution, approximate expressions are derived for the dependence of the shock radius on time, total energy, and ambient density. These approximate expressions are applicable in any number of dimensions. The effects of ionization and dissociation of the properties of the blast wave are considered in a semi-quantitative manner.

PROBLEM STATUS

This is a final report on one phase of the problem; work on the problem is continuing.

AUTHORIZATION

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INTRODUCTION

The following problem is considered. A given amount of energy W is deposited instantaneously in a very narrow slab of fluid in a tube of unit cross section. We wish to find the subsequent motion of the fluid. This situation is approximately realized in a shock tube of the Fowler type in which the energy is transferred to the gas through an electric arc. It is hoped that the analyses presented will be of use in interpreting the data from such a shock tube.

If the relation between internal energy (per unit mass), pressure, and density can be written in the form

$$E = \frac{1}{\gamma - 1} \frac{p}{\rho}, \tag{1}$$

where γ is a constant, then it is possible to find a completely analytic solution to the problem. The method for doing this parallels closely that used by Latter* in treating the spherical blast wave. From the exact solution thus obtained a simple approximate method for finding the dependence of the shock radius on time which is applicable in any number of dimensions is derived. This method gives the correct dependence of the shock radius on time, total energy, and density of the undisturbed fluid. In addition, the influence of dissociation and ionization on the velocity of the shock front is considered.

EXACT SOLUTION FOR A γ-LAW GAS

The hydrodynamic equations to be solved are conservation of mass,

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x} (\rho u), \tag{2}$$

conservation of momentum,

$$\frac{\partial}{\partial t} (\rho u) = - \frac{\partial}{\partial x} (\rho u^2 + p), \tag{3}$$

and conservation of energy,

$$\frac{\partial}{\partial t}\left(\frac{1}{2} \rho u^2 + \rho E\right) = - \frac{\partial}{\partial x} \left(\frac{1}{2} \rho u^3 + \rho E u + p u\right). \tag{4}$$

These equations must be supplemented by the equation of state

$$E = \frac{1}{\gamma - 1} \frac{p}{\rho} . \tag{5}$$

*R. Latter, RM 1435, Rand Corp., Santa Monica, Cal., 1955

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The above four equations must be solved for the four dependent variables: u, ρ, p, and E. In addition to these equations, we have the equation for the conservation of entropy,

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + u\frac{\partial}{\partial x}\left(\frac{p}{\rho^\gamma}\right) = 0; \tag{6}$$

this equation is not independent but may be derived from the others.

At the shock front derivitives do not exist, and the above equations must be replaced by the Hugoniot equations. These equations are

$$\rho_L(V-u_L) = \rho_R(V-u_R) \tag{7}$$

for the conservation of mass,

$$\rho_L\, u_L(V-u_L) - \rho_R\, u_R(V-u_R) = p_L - p_R \tag{8}$$

for the conservation of momentum, and

$$\rho_L\left(\frac{1}{2}u_L^2 + E_L\right)(V-u_L) - \rho_R\left(\frac{1}{2}u_R^2 + E_R\right)(V-u_R) = p_L\, u_L - p_R\, u_R \tag{9}$$

for the conservation of energy. Here the subscripts ~~L and R denote the left (strong)~~ and right (weak) sides of the shock wave, and V is the velocity of the shock front. We shal be concerned only with shock waves running into stationary and "cold" fluids (the temper-~~ature on the~~ weak side of the wave is negligible compared with that on the strong side). Therefore we will set u_R, p_R and E_R equal to zero.

In addition to the above we have the condition

$$\frac{1}{2}W = \int_0^\infty dx\left(\frac{1}{2}\rho\, u^2 + \rho E\right) = \text{constant}. \tag{10}$$

That is, we assume that there is no energy loss from the tube, and that one half of the energy goes into the right half of the tube.

We will now use dimensional analysis to reduce the partial differential equations to ordinary differential equations. Our dependent ~~variables are functions of~~ x, t, W, ρ_0 the ~~density of the~~ cold gas, and γ. They cannot depend on any other variables or parameters, since no others enter the equations or boundary conditions. Therefore we may write

$$\begin{aligned} \rho &= \rho(x,t,W,\rho_0), \\ u &= u(x,t,W,\rho_0), \\ p &= p(x,t,W,\rho_0), \\ E &= E(x,t,W,\rho_0). \end{aligned} \tag{11}$$

We have omitted γ since it is dimensionless and will not enter into our present considerat

Let us assume that the solutions may be represented by power series such as

$$\rho = \sum_{\alpha,\beta,\gamma,\delta} C_{\alpha,\beta,\gamma,\delta}\, x^\alpha\, t^\beta\, \rho_0^\gamma\, W^\delta. \tag{12}$$

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Since each term in the sum must have the same dimensions, α, β, γ, and δ cannot be independent. Equating the dimensions of the left hand side to any term of the right hand side gives

$$(ML^{-3}) = L^{\alpha}\, T^{\beta}\, (ML^{-3})^{\gamma}\, (MT^{-2})^{\delta}, \tag{13}$$

hence

$$1 = \gamma + \delta$$
$$-3 = \alpha - 3\gamma$$
$$0 = \beta - 2\delta.$$

These three equations in four unknowns can be solved for three of them in terms of the remaining one. Finally

$$\rho = \rho_0 \sum_{\lambda} C_{\lambda} \left(\frac{x\, \rho_0^{1/3}}{t^{2/3}\, W^{1/3}}\right)^{\lambda} \tag{14}$$

or

$$\rho = \rho_0 H(\chi), \tag{15}$$

where

$$\chi = \frac{x\, \rho_0^{1/3}}{t^{2/3}\, W^{1/3}}, \tag{16}$$

and the function $H(\chi)$ is still undetermined.

Similarly we can show that

$$u = \frac{W^{1/3}}{t^{1/3}\, \rho_0^{1/3}}\, G(\chi) = \frac{W^{1/2}}{\rho_0^{1/2}\, x^{1/2}}\, \chi^{1/2}\, G(\chi), \tag{17}$$

$$p = \frac{\rho_0^{1/3}\, W^{2/3}}{t^{2/3}}\, F(\chi) = \frac{W}{x}\, \chi\, F(\chi), \tag{18}$$

$$E = \frac{W^{2/3}}{t^{2/3}\, \rho_0^{2/3}}\, K(\chi) = \frac{W}{\rho_0\, x}\, \chi\, K(\chi). \tag{19}$$

If the relations

$$\frac{\partial}{\partial t} = \frac{\partial \chi}{\partial t}\, \frac{d}{d\chi} = -\frac{2}{3} \cdot \frac{\chi}{t}\, \frac{d}{d\chi} \tag{20}$$

and

$$\frac{\partial}{\partial x} = \frac{\partial \chi}{\partial x}\, \frac{d}{d\chi} = \frac{\chi}{x}\, \frac{d}{d\chi} \tag{21}$$

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are now used in the partial differential equations (2), (3), (4), and (6) to reduce them ordinary differential equations, we find

$$\frac{2}{3}\chi H' = (HG)'$$

$$\frac{2}{3}\chi^{1/2}(\chi^{1/2}HG)' = (HG^2+F)'$$

$$\frac{d}{d\chi}\left(\frac{1}{3}H\chi G^2+\frac{2}{3}H\chi K - \frac{1}{2}HG^3 - \gamma HGK\right) = 0,$$

$$-\frac{2}{3}(\chi F H^{-\gamma})' + G(FH^{-\gamma})' = 0,$$

the prime indicating differentiation with respect to χ. The equation of state becomes

$$K = \frac{1}{\gamma-1}\frac{F}{H}.$$

The Hugoniot relations become

$$H_L\left(\frac{2}{3}\chi_1 - G_L\right) = \frac{2}{3}\chi_1,$$

$$G_L\left(\frac{2}{3}\chi_1 - G_L\right) = (\gamma-1)K_L,$$

$$\frac{1}{3}H_L\chi_1 G_L^2 + \frac{2}{3}\chi_1 H_L K_L - \frac{1}{2}H_L G_L^3 - \gamma H_L K_L G_L = 0.$$

In these equations

$$\chi_1 = \frac{x_1 \rho_0^{1/3}}{t^{2/3} W^{1/3}}$$

and

$$V = \dot{x}_1 = \frac{2}{3}\chi_1 \frac{W^{1/3}}{\rho_0^{1/3} t^{1/3}},$$

where x_1 is the position of the shock front at time t. The Hugoniot relations may b for H_L, G_L, K_L, and F_L in terms of χ_1. The results are

$$H_L = \frac{\gamma+1}{\gamma-1},$$

$$G_L = \frac{4}{3(\gamma+1)}\chi_1,$$

$$K_L = \frac{8}{9(\gamma+1)^2}\chi_1^2,$$

$$F_L = \frac{8}{9(\gamma+1)}\chi_1^2.$$

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Equation (10) becomes

$$\frac{1}{2} = \int_0^\infty d\chi \left(\frac{1}{2} H G^2 + H K\right)$$
$$= \int_0^{\chi_1} d\chi \left(\frac{1}{2} H G^2 + H K\right). \tag{36}$$

It is convenient now to make the following changes of variables

$$\begin{aligned} \chi &= \chi_1 z, \\ F &= \chi_1^2 f(z), \\ K &= \chi_1^2 k(z), \\ G &= \chi_1 g(z), \\ H &= h(z). \end{aligned} \tag{37}$$

The Hugoniot relations become

$$\begin{aligned} f(1) &= \frac{8}{9(\gamma+1)} \\ k(1) &= \frac{8}{9(\gamma+1)^2}, \\ g(1) &= \frac{4}{3(\gamma+1)}, \\ h(1) &= \frac{\gamma+1}{\gamma-1}. \end{aligned} \tag{38}$$

The remaining equations become

$$\frac{2}{3} z h' = (h g)', \tag{39}$$

$$\frac{2}{3} z^{1/2} (z^{1/2} h g)' = (h g^2 + f)', \tag{40}$$

$$k = \frac{1}{2} g^2 \frac{\frac{2}{3} z - g}{\gamma g - \frac{2}{3} z}, \tag{41}$$ *

$$\frac{2}{3} (z f h^{-\gamma})' = g (f h^{-\gamma})', \tag{42}$$

$$k = \frac{1}{\gamma - 1} \frac{f}{h}, \tag{43}$$

$$\frac{1}{2} = \chi_1^3 \int_0^1 d z \left(\frac{1}{2} h g^2 + h k\right). \tag{44}$$

The solution to the problem is now the solution of (39) through (43) subject to the boundary conditions in (38). From (44) we then find χ_1, which gives us the position of the shock front.

*Obtained by integrating (24) taking into account (29).

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The remainder of the solution then follows from the definitions of f, g, h, k, F, G, H, K, and χ.

We begin by writing (42) in the form,

$$\left(\frac{2}{3}z-g\right)f' - \gamma\left(\frac{2}{3}z-g\right)f\,\frac{h'}{h} = -\frac{2}{3}f,$$

and (39) in the form,

$$\left(\frac{2}{3}z-g\right)h' = h\,g'.$$

After dividing (45) by $(2/3\,z-g)f$ and (46) by $(2/3\,z-g)h$, and then adding the equation we find

$$\frac{f'}{f} - (\gamma-1)\frac{h'}{h} = \frac{-\frac{2}{3}+g'}{\left(\frac{2}{3}z-g\right)} = -\frac{\left(\frac{2}{3}z-g\right)'}{\left(\frac{2}{3}z-g\right)}.$$

This can immediately be integrated with the result

$$\frac{f}{h^{\gamma-1}}\left(\frac{2}{3}z-g\right) = A$$

where the constant A is found from the boundary conditions in (38) to be

$$A = \frac{16}{27}\,\frac{(\gamma-1)^{\gamma}}{(\gamma+1)^{\gamma+1}}. \qquad (4$$

Since (41), (43), and (48) are three algebraic equations relating f, g, h, and k, they c be solved for h in terms of g. The result is

$$h^{\gamma-2} = \frac{\gamma-1}{2A}\,g^2\,\frac{\left(\frac{2}{3}z-g\right)^2}{\left(\gamma g-\frac{2}{3}z\right)}. \qquad (4$$

This can be used in (46) to get a first order differential equation containing only g. Th result is

$$g' = \frac{g}{z}\,\frac{\frac{2}{3}(2\gamma-1)\left(\frac{g}{z}\right)-\frac{4}{9}}{\gamma(\gamma+1)\left(\frac{g}{z}\right)^2-\frac{4}{3}(\gamma+1)\left(\frac{g}{z}\right)+\frac{8}{9}}. \qquad (4$$

This equation is of a standard form and may be integrated to give

$$z = B\,\frac{\left|\frac{g}{z}-\frac{2}{3\gamma}\right|^{\frac{\gamma-1}{2\gamma-1}}}{\left|\frac{g}{z}\right|^{2/3}\left|\frac{g}{z}-\frac{2}{\gamma+1}\right|^{\frac{5\gamma-4}{6\gamma-3}}}. \qquad (4$$

The constant of integration B is to be chosen so that the boundary condition on g is satisfied.

We will now relate the functions f, g, h, and k to our original functions p, u, ρ, and E. In terms of V, the velocity of the shock front, it is easily shown that

$$u = \frac{3}{2}\,V\,g(z), \qquad (5$$

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$$p = \frac{9}{4} V^2 \rho_0 f(z), \tag{54}$$

$$\rho = \rho_0 h(z), \tag{55}$$

$$E = \frac{9}{4} V^2 k(z). \tag{56}$$

Also

$$V = \dot{x}_1 = \frac{2}{3} \chi_1 \frac{W^{1/3}}{\rho_0^{1/3} t^{1/3}}, \tag{57}$$

$$x_1 = \chi_1 \frac{W^{1/3}}{\rho_0^{1/3}} t^{2/3}, \tag{58}$$

$$z = \frac{x}{x_1}, \tag{59}$$

where x_1 is the position of the shock front.

The numerical work has been carried out for a monoatomic gas $(\gamma = 5/3)$, and the results are presented in Table 1 and Fig. 1

TABLE 1

z	g	h	f	k
0.000	0.000	0.000	0.0000	∞
0.566	0.232	0.167	0.0464	0.416
0.661	0.278	0.320	0.0631	0.298
0.728	0.313	0.585	0.0900	0.231
0.768	0.338	0.960	0.124	0.194
0.821	0.369	1.19	0.140	0.177
0.861	0.396	1.52	0.165	0.163
0.898	0.422	1.95	0.197	0.151
0.934	0.449	2.45	0.230	0.141
0.970	0.475	3.10	0.275	0.133
1.00	0.500	4.00	0.333	0.125

APPROXIMATE SOLUTION OF BLAST WAVE PROBLEMS

A characteristic of blast waves in one, two, and three dimensions is that most of the mass is concentrated very close to the shock front. This is more pronounced when $(\gamma - 1)$ is small. This suggests that we treat all the mass as being concentrated at the shock front and having the velocity and internal energy which prevail there. That is, we will put

$$\begin{aligned} W &= \int \left(\frac{1}{2} u^2 + E \right) \rho \, d\tau \\ &\approx \left(\frac{1}{2} u_L^2 + E_L \right) \rho \, d\tau \\ &\approx \left(\frac{1}{2} u_L^2 + E_L \right) \rho_0 \, \Omega(R). \end{aligned} \tag{60}$$

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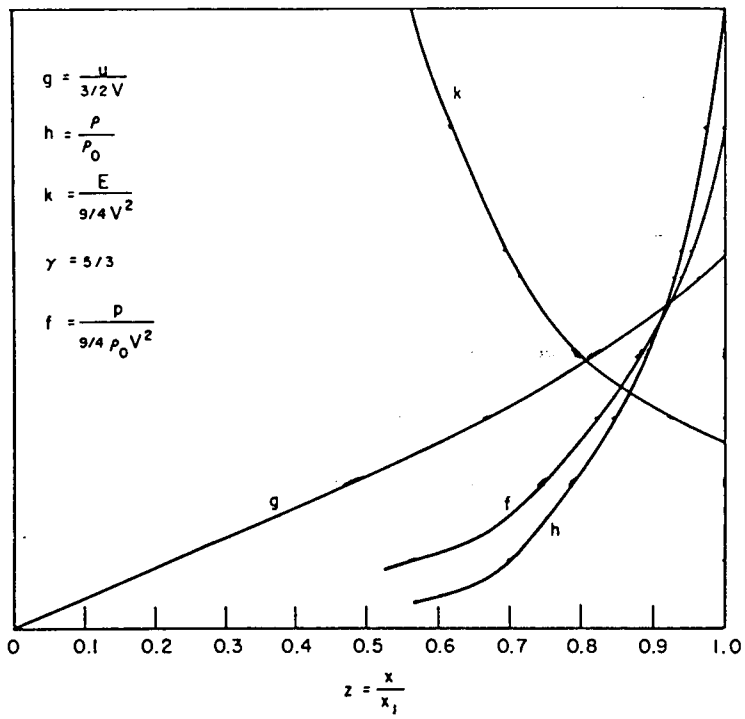

Figure 1

Here, u_L and E_L are the flow velocity and internal energy at the shock front, $d\tau$ is th element of volume, and $\Omega(R)$ is the volume enclosed by the shock radius. The Hugonic equations, (7), (8), and (9), are easily solved for the strong shock case with the result

$$u_L = \frac{2}{\gamma + 1} V, \qquad ($$

$$E_L = \frac{2}{(\gamma+1)^2} V^2, \qquad ($$

$$p_L = \frac{2}{\gamma + 1} V^2 \rho_0, \qquad ($$

$$\rho_L = \frac{\gamma + 1}{\gamma - 1} \rho_0, \qquad ($$

where $V = \dot{R}$ is velocity of the shock front. Thus

$$\frac{1}{2} u_L^2 = E_L = \frac{2}{(\gamma+1)^2} \dot{R}^2 \qquad ($$

and

$$W = \frac{4}{(\gamma+1)^2} \dot{R}^2 \rho_0 \Omega(R). \qquad ($$

This becomes

$$\frac{W}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^2 \dot{R}^2 (2R). \qquad ($$

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in one dimension,

$$\frac{W}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^2 \dot{R}^2 \, (\pi R^2) \tag{68}$$

in two dimensions, and

$$\frac{W}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^2 \dot{R}^2 \left(\frac{4\pi}{3} R^3\right) \tag{69}$$

in three dimensions. These equations can be integrated with the results

$$R = \left(\frac{3}{2}\right)^{2/3} \left(\frac{\gamma+1}{2}\right)^{2/3} \left(\frac{W}{2\rho_0}\right)^{1/3} t^{2/3} \tag{70}$$

in one dimension,

$$R = \frac{2^{1/2}}{\pi^{1/4}} \left(\frac{\gamma+1}{2}\right)^{1/2} \left(\frac{W}{\rho_0}\right)^{1/4} t^{1/2} \tag{71}$$

in two dimensions, and

$$R = \left(\frac{5}{2}\right)^{2/5} \left(\frac{3}{4\pi}\right)^{1/5} \left(\frac{\gamma+1}{2}\right)^{2/5} \left(\frac{W}{\rho_0}\right)^{1/5} t^{2/5} \tag{72}$$

in three dimensions.

In all cases the correct dependence on t, W, and ρ_0 is given. The numerical factor in front is probably not in error by more than a factor of 2.

EFFECT OF IONIZATION AND DISSOCIATION

To be specific we will think in terms of hydrogen gas although the results derived will be more general. We assume that the temperature on the strong side of the shock is sufficiently high that the hydrogen on the strong side is completely dissociated and ionized. This will be an important limitation on the validity of the treatment. A hydrogen molecule passing through the shock front must be given the energy to dissociate it (E_D = 4.85 ev) plus the energy to ionize it ($2E_I$ = 2 × 13.5 = 27 ev). In addition each proton and electron will have a kinetic energy of $(3/2)kT$. Taking the energy of a hydrogen molecule at zero temperature as the zero of energy, we may write the internal energy of the gas on the strong side of the shock as

$$E = \frac{4 \times \frac{3}{2} kT + E_D + 2E_I}{2M_H} \tag{73}$$

$$= \frac{3}{2}\frac{p}{\rho} + \frac{1}{2M_H}(E_D + 2E_I) \tag{74}$$

$$= \frac{1}{\gamma - 1}\frac{p}{\rho} + U \tag{75}$$

where M_H is the mass of a hydrogen atom and $\gamma = 5/3$. It is this expression for E that must be used in the Hugoniot relations. For the strong shock case these relations become

$$\rho_L (V - u_L) = \rho_R V \tag{76}$$

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$$\rho_L\, u_L\, (V-u_L) = p_L = (\gamma-1)\, \rho_L\, (E_L-U) \tag{77}$$

$$\rho_L \left(\frac{1}{2} u_L^2 + E_L\right) (V-u_L) = p_L\, u_L = (\gamma-1)\, \rho_L\, (E_L-U) u_L. \tag{78}$$

From these equations it is easily shown that

$$E_L = \frac{1}{2}\, u_L^2 \tag{79}$$

and

$$V = \frac{\gamma+1}{\sqrt{2}}\, E_L^{1/2} - \frac{\gamma-1}{\sqrt{2}}\, E_L^{-1/2}\, U. \tag{80}$$

We now make the same assumption that was made in the last section, that is,

$$W = \left(\frac{1}{2} u_L^2 + E_L\right) \rho_0\, 2R, \tag{81}$$

or from (79),

$$W = 4\, E_L\, \rho_0\, R, \tag{82}$$

from which

$$E_L = \frac{W}{4\, \rho_0\, R}. \tag{83}$$

Substituting this into (80) we find a differential equation for R,

$$V = \dot{R} = \frac{\gamma+1}{2\sqrt{2}} \left(\frac{W}{\rho_0}\right)^{1/2} R^{-1/2} - \frac{2(\gamma-1)}{\sqrt{2}} \left(\frac{\rho_0}{W}\right)^{1/2} U R^{1/2}, \tag{84}$$

From which

$$\int_0^R \frac{R^{1/2}\, dR}{1 - \frac{\gamma-1}{\gamma+1}\, \frac{4 U \rho_0}{W}\, R} = \frac{\gamma+1}{2\sqrt{2}} \left(\frac{W}{\rho_0}\right)^{1/2} \int_0^t dt. \tag{85}$$

It is convenient to introduce the dimensionless variables,

$$x = \frac{\gamma-1}{\gamma+1}\, \frac{4 U \rho_0}{W}\, R \tag{86}$$

and

$$\tau = 2\sqrt{2}\, \frac{(\gamma-1)^{3/2}}{(\gamma+1)^{1/2}}\, \frac{U \rho_0}{W}\, U^{1/2}\, t, \tag{87}$$

in terms of which (85) becomes

$$\int_0^x \frac{x^{1/2}\, dx}{1-x} = \tau. \tag{88}$$

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This can be immediately integrated to obtain

$$\tau = -2x^{1/2} + \log\left|\frac{1 + x^{1/2}}{1 - x^{1/2}}\right|, \tag{89}$$

or when x is small,

$$\tau = -2x^{1/2} + 2\left(x^{1/2} + \frac{1}{3}x^{3/2} + \ldots\ldots\right)$$

$$= \frac{2}{3}x^{3/2} \tag{90}$$

or

$$x = \left(\frac{3}{2}\right)^{2/3} \tau^{2/3} \tag{91}$$

and

$$R = \left(\frac{3}{2}\right)^{2/3}\left(\frac{\gamma+1}{2}\right)^{2/3}\left(\frac{W}{2\rho_0}\right)^{1/3} t^{2/3} \tag{92}$$

in agreement with previous results.

We now consider the limits of validity of this theory. We see from (75) that when $E_L = U$ the pressure and temperature on the strong side of the shock become zero. If this is true ~~the assumption of~~ complete dissociation and ionization becomes nonsensical. From (82) we see that this corresponds to a shock radius of

$$R = \frac{W}{4\rho_0 U} \tag{93}$$

and from (86)

$$x = \frac{\gamma - 1}{\gamma + 1} = \frac{1}{4}. \tag{94}$$

For such a small value of x, (92) is a good approximation to (89). We conclude then that within the limits of validity of this theory the shock front ~~radius is adequately given by~~ (70).

We need scarcely remark that no account has been taken here of radiation losses or of wall effects. The actual conditions within the shock tube may be profoundly modified by these effects.

We conclude by estimating the energy that a shock wave must give to a hydrogen molecule to raise it to a temperature $kT \approx 3$ ev. At this temperature hydrogen is almost completely ionized, and from (73)

$$4 \times \frac{3}{2}kT + E_D + 2E_I \approx 50 \text{ ev}. \tag{95}$$

From (79) we see that the same energy must go into hydrodynamic motion. Therefore, about 100 ev is required to raise each molecule to a temperature of about 3 ev. However, it would seem that when the shock was reflected so that $u_L^2/2$ was reduced to zero this

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energy would go into thermal energy. Writing the energy balance equation after the tion we find

$$100 = \left(4 \times \frac{3}{2} kT + 32\right)_{\text{AFTER}}$$

from which

$$(kT)_{\text{AFTER}} = \frac{68}{6} \approx 11 \text{ ev},$$

showing that the temperature has more than tripled.

* * *