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A GUIDE TO BASIC PULSE-RADAR MAXIMUM-RANGE CALCULATION

PART 1 - EQUATIONS, DEFINITIONS, AND AIDS TO CALCULATION

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ABSTRACT

The equations and other information required for calculation of "basic" radar maximum range are presented. The term "basic" here refers to the range for somewhat idealized conditions, assuming no losses due to clutter of other targets, rain, sea return, jamming, interference, or anomalous propagation effects. Several conventions for a standardized basic range calculation are proposed, relating to such factors as the system noise temperature, atmospheric absorption loss, atmospheric refraction, rough-sea reflection coefficient, and "visibility factor" for cathode-ray-tube displays. An attempt is made to provide a set of standardized, unambiguous, and mutually compatible range-factor definitions. An appendix presents a work-sheet for range calculation, together with curves, tables, and auxiliary equations needed for evaluating some of the range-equation quantities. This is Part 1 of a two-part report, and is intended primarily to provide the basic information needed for range calculation. Part 2, to be published later, will treat topics that are important in some but not all applications and will present detailed derivations of some of the propositions stated without proof in Part 1. Additional details on some topics are contained in three previously published reports, NRL Reports 5601, 5626, and 5668.

PROBLEM STATUS

The work described in this report is part of a continuing project. This is an interim report.

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PART 1 - EQUATIONS, DEFINITIONS, AND AIDS TO CALCULATION

INTRODUCTION

In a Memorandum Report (1) published in November 1960, reference was made to a more comprehensive report in preparation, on the subject of basic pulse-radar maximum range calculation. Subsequently, it was decided that some of the subtopics of this subject should be treated in separate reports, both to make the information available more quickly, and to avoid publishing a single report of excessive bulk. Three such subtopic reports have been published (2-4), on the subjects of atmospheric absorption loss, atmospheric refraction, and noise temperature calculation. (None of these reports are intended as general treatises on the subjects, but are attempts to provide the bases needed for radar range calculation.) Brief accounts of these matters have also been published in journals of the Institute of Radio Engineers (5-7).

A considerable demand for additional copies of the Memorandum Report led to a reprinting, in June 1961 (with a 3-page addendum), and at the present time it is again out of print. Engineers of the Bureau of Ships have requested an additional quantity of this report, to be used as a guide for radar-system contractors. Instead of reprinting it again, however, it was decided to publish essentially the same material, with some minor revisions and additional material, as Part 1 of the "comprehensive" report.

This report therefore is partly a reprinting of the "interim" memorandum report, but is now interim only in the sense that it does not contain full explanations of some matters which will be more thoroughly treated in Part 2, along with some details of the range-calculation problem not treated in Part 1.

There has been some updating, compared to the Memorandum Report, in the areas of noise-temperature calculation and atmospheric absorption. Additional information has been included on calculation of pattern-propagation factor, on radar cross section, blip/scan ratio, cumulative probability of detection, noise jamming, and range-calculation accuracy. The emphasis is on the essential information required for basic range calculation. Part 2 will treat some less-fundamental topics (less fundamental only in the sense that they are not encountered in every range-calculation problem). It will discuss in more detail the calculation of various losses that occur, and extension of the range-calculation technique to some special cases.

Several "conventions" are proposed in the report. It is necessary to adopt conventions if standardized range calculations of competing systems are to be compared, on a common basis. If the conventions do not exist, every engineer who attempts to calculate the range of a radar faces a number of difficult decisions as to the environmental assumptions he should make, and different engineers will make them differently. Standardized and unambiguous definitions of range-equation quantities are also needed, formulated to be mutually compatible. Otherwise the same physical effect may be incorrectly included in two or more different parts of the equation — for example, antenna dissipation losses, which may be included in the definition of antenna gain or in transmission-line loss but should not be in both. Numerous other examples could be cited.

Care has been taken to preserve compatibility and avoid ambiguity. At the same time it is realized that in an engineering approach to radar range calculation absolute rigor is not feasible, and numerous departures from it could be pointed out. However, these departures are of no consequence in the great majority of practical cases.

Conventions are proposed for antenna noise temperature, atmospheric absorption losses, atmospheric refraction, the reflection coefficient of a rough sea, and "visibility factor" for cathode-ray-tube displays of the A-scope and PPI type. The conventions proposed are not completely arbitrary. They attempt to be representative of typical or average conditions, or results encountered in practice. They are so devised that their use will always represent a condition that would be well within the range of variation that actually occurs in practical and ordinary experience. Yet, the entire convention may not represent a set of conditions which would simultaneously exist at any one time for all values of parameters involved (e.g., frequency, number of pulses integrated, elevation angle, etc.). But this is an acceptable aspect of conventions.

The type of range calculation to which most of the material of this report applies is termed "basic" because, while it is a starting point for range calculations of more complete operational significance, it does not take into account many of the factors that are operationally important, such as "clutter" echoes, jamming signals, and various fluctuating-signal factors, except in a rudimentary way. At the same time, the range calculated by the methods to be presented will have some operational significance. The probabilistic aspect of detection range is recognized, and the equations are written in terms of the range for 0.5 probability. This has acquired some status as a convention for comparative range calculation, partly because of its convenience from several points of view. The choice of a convenient basis for such comparison is justifiable, because any value adopted as a standard for system comparison is basically arbitrary.

The full operational performance of the radar can only be described in terms of range vs probability curves. At present, such curves may be obtained, with reliability, only by experiment, and the additional information that they convey is related as much as to the characteristics of the target and possibly the propagation medium as to the radar itself. The primary emphasis here is on calculation of the range performance of the radar per se. This is not wholly possible, of course. The environment cannot be ignored. However, a "simple" and "standard" environment is assumed, one which is as realistic as possible without excessive complication.

The report applies primarily to radars in the frequency range 100 to 10,000 Mc. In particular, the antenna-noise-temperature and atmospheric-absorption-loss conventions are restricted to this range. At lower frequencies the extremely variable effect of the ionosphere, and at higher frequencies the effects of variation of atmospheric water-vapor content, preclude the establishment of acceptable conventions for these factors. However, the basic calculation technique and much of the auxiliary material are applicable over a greater frequency range. (In fact, absorption-loss curves are given for frequencies well above 10,000 Mc, but are not proposed as conventions because of the variability that will occur from place to place, day to day, and season to season.)

A note on various meanings of the words "detect," "detector," and "detection" is desirable prior to some of the discussion contained in this report. There are two distinct meanings and several shades of meaning of these terms. The two definite categories of meaning of "detector" are: (a) the device that "demodulates" an rf or i-f signal (as exemplified by the "second detector" of a superheterodyne receiver, which is often simply a rectifier) and (b) the "decision making" device that usually follows the demodulator. This may be an automatic threshold device, automatic processing equipment of more complicated nature, or the eye-brain combination of a human observer of a cathode-ray-tube display. Similar distinctions apply to the terms "detect" and "detection." It will be assumed in most of the report that the meaning to be understood will be evident from the context. In some cases where confusion might otherwise result, the phrase "detection-decision" will be used in place of detection alone when the second category of meaning applies.

There is another distinction that may sometimes be important in operational analysis of radar detection. A target may present a detectable echo and yet not be detected. Thus, in this type of analysis, detection is said to have occurred only when the presence of a target is realized and reported by a human being, as distinct from the fact that an echo signal of a certain strength has been received. The problem to which this report is addressed is to determine the range at which a detectable signal will be received from a target of known statistical cross-section value under standardized propagation conditions. This is of course only a first step in constructing a complete operational theory of radar performance, but it is a necessary step.

The emphasis of the report is on calculation of the range of pulse radars, and especially those whose antennas are not at a high elevation. Much of the material is useful, however, for calculation of the range of other classes of radars.*

RANGE EQUATIONS

The fundamental radar transmission equation is found in standard texts such as that of Kerr (Ref. 8, p. 35, Eq. (28)), and is reproduced here in Kerr's notation:

$$\frac{P_r}{P_t} = \frac{G^2 \lambda^2 \sigma F^4}{(4\pi)^3 R^4} \,. \tag{1}$$

The symbols have the following definitions:

P_r - received-signal power

P_t - transmitted power

G - antenna power gain (relative to an isotropic radiator)

λ - wavelength

σ - radar cross section of target

F - pattern-propagation factor; ratio of actual field strength, E, at target, to field strength that would exist, E_o, for a free-space propagation path of the same distance and direction

R - radar-to-target distance (range).

The quantities of this equation are to be expressed in any consistent set of units, e.g., watts and meters. Also, P_r, P_t, and G refer to power actually radiated by the antenna and received by the antenna aperture (ahead of any ohmic losses). It is assumed that the same antenna is used for transmitting and receiving.

*For a discussion of the calculation of range on a more general basis, see J. J. Bussgang et al., "A Unified Analysis of Range Performance of CW, Pulse, and Pulse Doppler Radar," Proc. I.R.E. 47:1753 (Oct. 1959).

A General Radar Equation

For practical calculation, it is more convenient to employ "mixed" units, to speak of radar frequency rather than wavelength, and to define P_t at least, and possibly P_r, in terms of quantities measured at transmitter and receiver terminals rather than at the antenna aperture. In many practical cases, different antennas are used for transmission and reception. Also, it is more convenient and customary to deal with signal-to-noise ratio, S/N, than with absolute received power level, through the relationship

$$S/N = P_r/P_N, \tag{2}$$

where P_N is the effective input noise power to the receiver, i.e., the output noise referred to some point in the predetection portion of the receiving system, at which the received signal is to be measured or calculated. One may then derive the following equation from Eq. (1) in a direct and simple manner:

$$R = 726.8 \left[\frac{P_{t(kw)}\, G_t\, G_r\, \sigma F_t^2 F_r^2}{f_{Mc}^2\, T_N (S/N)\, B_{kc} L} \right]^{1/4} . \tag{3}$$

The symbols in this equation are defined as follows:

R - range of target from radar, nautical miles (ray-path distance)

$P_{t(kw)}$ - transmitter output power, kilowatts (average power of cw radar, pulse power of pulse radar)

G_t, G_r - transmitter and receiver antenna directive power gains; power gain relative to an isotrope with the same total radiated power

σ - radar cross section of target, square meters

F_t, F_r - pattern-propagation factors for transmission and reception; ratio of field strength (e.g., electric intensity, at the target for F_t, or at the receiving antenna for F_r) to the value that would exist at the same range in free space, in the maximum gain direction of the antenna beam, but with the same absorption loss as in the actual case (see Ref. 8, pp. 34-41)

f_{Mc} - radar frequency, megacycles

T_N - system noise temperature, degrees Kelvin, representing total receiver output noise power referred to some point in the receiving system ahead of the selective circuits (receiver input terminals); the total noise is composed of antenna noise, transmission-line thermal noise, and internal noise of the receiver

S/N - ratio of signal power at reference point chosen for T_N to noise power referred to same point, $kT_N B$;* the signal power is on a cw basis or a pulse basis, consistent with the basis used for P_t

*The noise power is $kT_N B_N$ watts, where k is Boltzmann's constant, 1.38×10^{-23} watt-seconds per cycle-per-second, and B_N is the receiving-system overall noise bandwidth, cycles per second.

B_{kc} - receiver predetection bandwidth, kilocycles; this is actually the noise bandwidth,* but in practice the half-power bandwidth is sufficiently accurate in most cases

L - power loss factor, expressing total system losses; it will usually be the product of numerous specific loss factors; power loss factor is here defined as the ratio of power input to power output of the lossy element of the system; hence, $L \geqq 1$.

If the range is desired in other units, the following numerical factors may be used in place of the factor 726.8:

Range Units	Numerical Factor	Log_{10}
Statute miles	836.4	2.922
Kilometers	1346	3.129
Thousands of yards	1472	3.168
Thousands of feet	4416	3.645

This equation is not a "maximum range" equation in the usual sense. It expresses the range at which a signal-to-noise power ratio of value S/N will appear at the receiver input terminals when the target cross section is σ. For pulse radar, it applies to individual pulses. This equation is quite general, in that it applies to cw as well as pulse radars. For "modulated carrier" systems it refers to the signal-to-noise ratio at an instant of time, delayed with respect to the instant that the transmitter power was P_t by a time equal to $2R/c$, where c is the velocity of electromagnetic propagation (strictly, it is the average velocity over the actual propagation path). Or, if P_t is the average transmitter power, then S/N refers to the average received signal-to-noise ratio when the range is R.

Caution is necessary in using this equation because of the limited significance of the signal-to-noise ratio at the receiver input. In most applications, it is the output signal-to-noise ratio that determines the success or failure of the reception. The "noise" in the power ratio S/N of Eq. (2) is the output noise power, referred to the input; but, the "signal" is the receiver input signal power, which is the quantity that is usually known or readily calculable.

To write the equation in terms of output signal-to-noise ratio, $(S/N)_{out}$, it is necessary to know the effect of the receiver passband on the signal power (from the pure filter-theory point of view, apart from any amplification that occurs). This is a problem that requires consideration of the signal waveform, the filter transfer characteristic, and the nature of the use to be made of the output of the receiver, i.e., the characteristics of the intelligence-extracting process.

*D. O. North, The Absolute Sensitivity of Radio Receivers, RCA Review, Jan. 1942, p. 334. As therein defined, the noise bandwidth is

$$B_N = \frac{1}{G_o} \int_0^\infty G(f)\, df ,$$

where G_o is the receiver power gain at the nominal radar frequency and $G(f)$ is the power gain at frequency f. "Predetection" bandwidth means the overall bandwidth of the receiving system, including the antenna, up to the detector (demodulator). It is assumed that the postdetection (video) bandwidth is equal to at least half the predetection bandwidth.

From these considerations it is in general (or in principle) possible to determine a value of $(S/N)_{out}$ that is required for successful operation, and to express it in terms of a required value of S/N, the input signal-to-noise ratio. This expression may (or generally will) contain a factor accounting for the effect of the receiver tuned circuits on the required value. Equation (3) simply assumes that this required value is known, by separate analysis of the problem, and is thus available to "plug in" to the equation.

As written, Eq. (3) applies to "monostatic" radars (transmitter and receiver at approximately the same location). Only a minor change is required to make it apply to bistatic radars: the range R is replaced by $\sqrt{R_t R_r}$, where R_t is the distance from transmitter to target and R_r the distance from target to receiver; and the monostatic radar cross section, σ (or σ_m) is replaced by σ_b, the bistatic radar cross section of the target.

Equations will now be shown in which the "required value" of S/N is expressed in terms of quantities specifically adapted to pulse radar.

Pulse-Radar Equation for Cathode-Ray-Tube Displays

The foregoing equation can be modified to give a "maximum" detection range for the case of a human observer and a cathode-ray-tube display of signals and noise, taking into account (implicitly) the effect of integration of a train of received pulses (an effect that automatically occurs due to the characteristics of the human eye and brain, as well as persistence of the cathode-ray-tube phosphor).

The following equation is a modification of the one presented by Norton and Omberg (9). It is here written for a specific probability of detection, or, in the case of a scanning radar, blip/scan ratio:

$$R_{50} = 129.2 \left[\frac{P_{t(kw)}\ \tau_{\mu sec}\ G_t\ G_r\ \sigma_{50\,(sq\,m)}\ F_t^2\ F_r^2}{f_{Mc}^2\ T_N\ V_{o(50)}\ C_B\ L} \right]^{1/4} . \tag{4}$$

The quantities that differ from those of the preceding equation are:

R_{50} - range, nautical miles, for 0.5 probability of detection, or 0.5 blip/scan ratio in the case of a scanning radar

$\tau_{\mu sec}$ - radar pulse length, microseconds; ordinarily, the pulse duration between half-power points

$\sigma_{50(sq\,m)}$ - the median value of the target cross section, square meters

$V_{o(50)}$ - "visibility factor" for 0.5 probability of detection, optimum bandwidth; ratio of minimum detectable pulse energy (watt-seconds) at the receiver input terminals to noise power per unit bandwidth (watts per cycle-per-second) referred to the same terminals; also, ratio of minimum detectable signal power to noise power in a bandwidth equal to the reciprocal of the pulse length* (see Figs. 1 and 2)

C_B - bandwidth correction factor, equal to one when the bandwidth is optimum, otherwise greater than one (see Fig. 3).

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*That these two definitions are equivalent is shown later, by Eq. (7). The first form of definition is the one used originally by Norton and Omberg (9). The second form is used by Lawson and Uhlenbeck (10). Though it is not there called visibility factor, the curves of Figs. 1 and 2 are based on data published in Refs. 10 and 11.

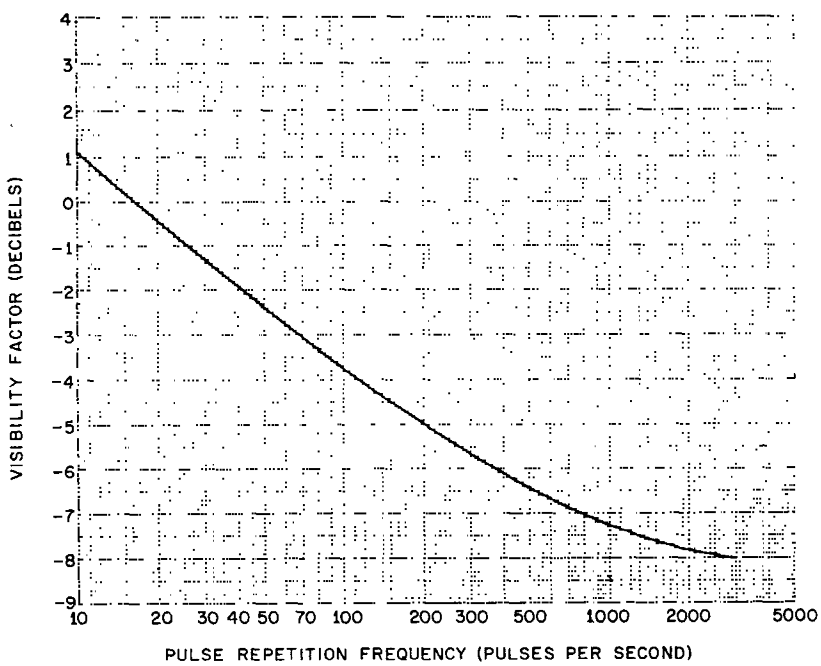

Fig. 1 - Visibility factor $V_{o(50)db}$ for type-A cathode-ray-tube display; based on Figs. 8.2 and 8.23 of Ref. 10, or Figs. 1 and 4 of Ref. 11. The values given in Fig. 8.23 for $B\tau \cong 1.2$ have been adjusted to 0.5 probability in accordance with Fig. 8.2.

If the range is desired in other units, the following numerical factors may be used in place of the factor 129.2:

Range Units	Numerical Factor	Log_{10}
Statute miles	148.7	2.172
Kilometers	239.3	2.379
Thousands of yards	261.7	2.418
Thousands of feet	785.0	2.895

The product $P_t\tau$ appearing in Eq. (4) will be recognized as the transmitter output pulse energy. In the Norton-Omberg equation this was condensed to a single symbol, E_t, but the separate symbols are used here because radar system parameters are customarily specified in this form.

The pulse energy appears in the visibility-factor definition, although the definition can also be stated in power terms. Either is equally correct, and for some purposes the power representation is more convenient. The energy formulation is particularly useful for radars of the so-called "chirp" or pulse-compression type, where some ambiguity of definition of pulse power and pulse length could arise. However, for such radars the pulse power and pulse length may be used if care is taken to avoid inconsistency from the energy standpoint.

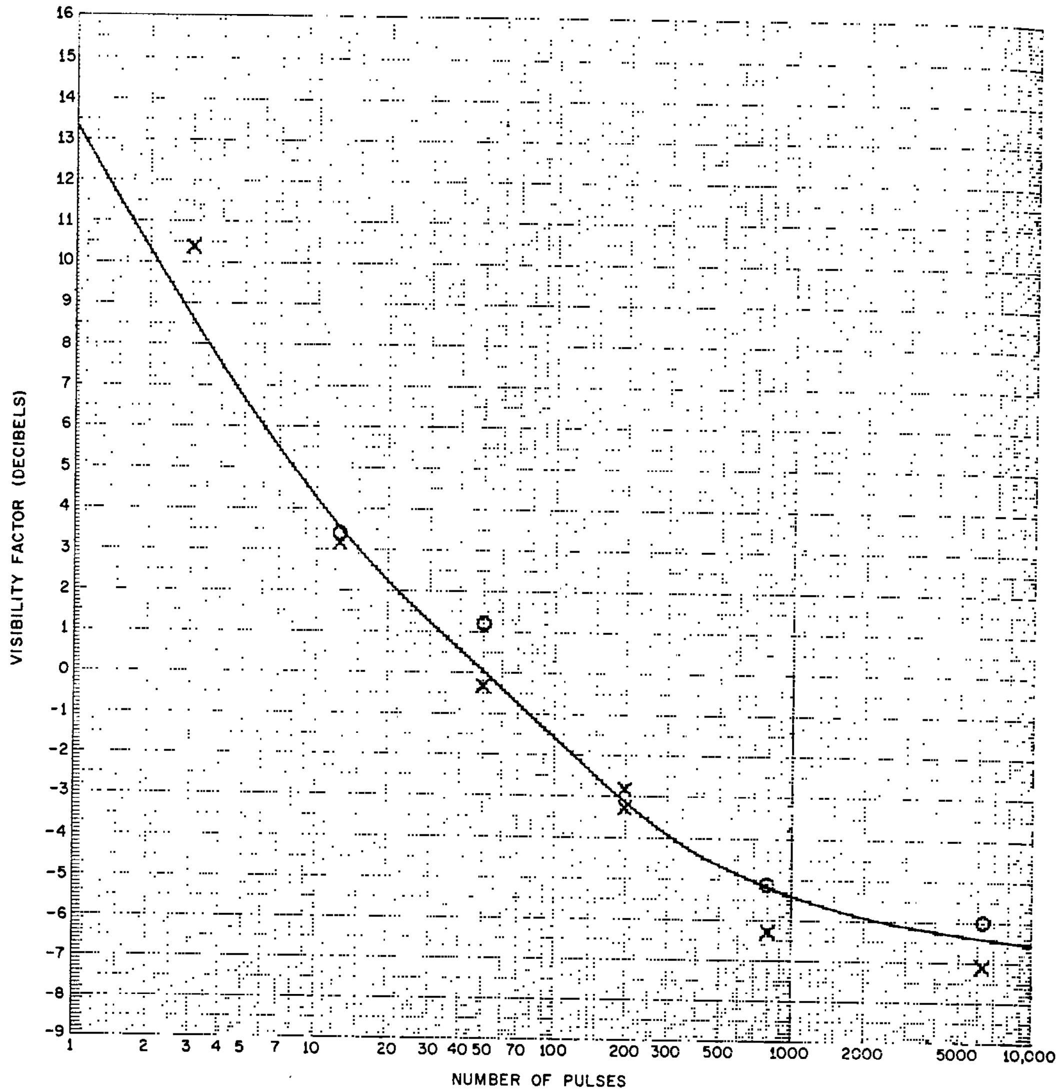

Fig. 2 - Visibility factor $V_{o(50)db}$ for PPI cathode-ray-tube display (applicable to intensity-modulated displays generally); based on Figs. 8.2 and 9.2 of Ref. 10 or Figs. 1 and 21 of Ref. 11, adjusted to 0.5 probability and extrapolated to single-pulse detection, with slight revision of slopes at ends of curve

Pulse-Radar Equation for Automatized-Detection Radars

The preceding equation is not actually restricted to radars with cathode-ray-tube displays, but is more specifically adapted to them. The curves for V_o as a function of number of pulses integrated, Figs. 1 and 2, are experimental curves, applicable only to cathode-ray-tube displays with human observers. They are characterized by a probability of detection (0.5), but not by an explicit false-alarm probability.

It is possible, however, to calculate a signal-to-noise ratio required at the input terminals of the receiver detector (demodulator, rectifier) for specified probability of detection and false-alarm probability. Such curves are often associated with the work of J. I. Marcum (12), who probably was the first to compute and publish them, in 1947,

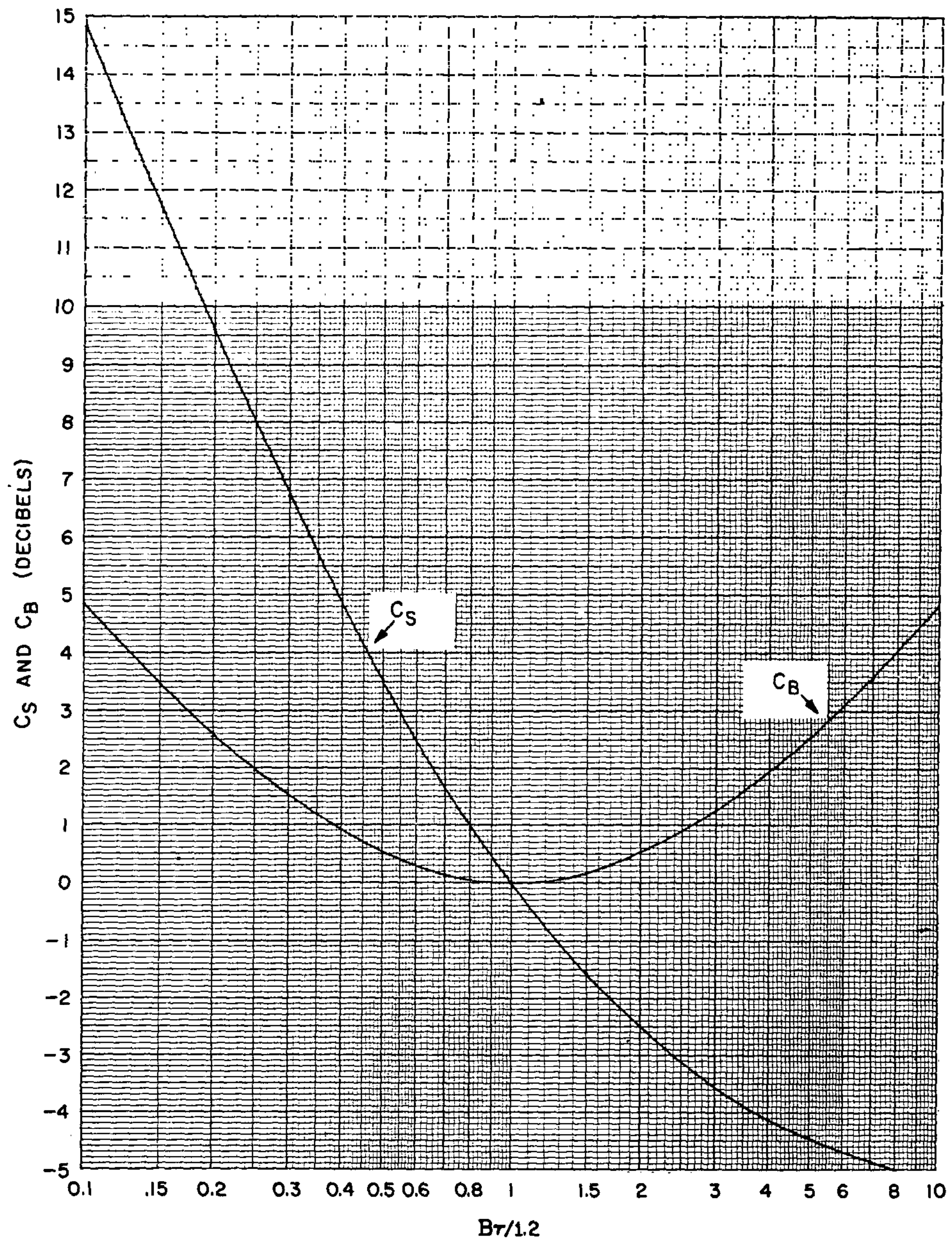

Fig. 3 - Bandwidth correction factors for cathode-ray-tube indicator and human observer. The factor C_B (expressed as a power ratio) applies to $V_{o(50)}$ in Eq. (4), and C_S (power ratio) applies to $(S/N)_{o(50)}$ as in Eq. (9).

although the concepts and basic techniques of calculation were described by D. O. North in 1943 (13).

Curves of this type, calculated by the author using the methods described by North for a fixed-threshold decision-making device, are given by Fig. 4. The details of the calculation will be included in Part 2. They are calculated for a linear-rectifier detector — the type commonly used in radar receivers.*

The independent variable for these calculations is the signal-to-noise power ratio at the detector (linear rectifier) input corresponding ordinarily to the output of the i-f amplifier. This is not the same quantity previously defined as the visibility factor. Therefore it cannot be used directly in Eq. (4). To distinguish it from the visibility factor it is here called "detectability factor," D. It is related to the visibility factor by a constant factor, m, for any specific radar system; that is, $V_o C_B = D m$.† The range equation in which D may be used is therefore

$$R_{50} = 129.2 \left[\frac{P_{t(kw)} \tau_{\mu sec} G_t G_r \sigma_{50(sq\ m)} F_t^2 F_r^2}{f_{Mc}^2 T_N D_{50} m L} \right]^{1/4} \text{naut mi.} \tag{5}$$

The factor m is a "matching factor" depending on the relative shapes of the pulse and the receiver passband. When these are Fourier transforms of each other, $m = 1$.‡ Otherwise $m > 1$. It is similar but not the same as the bandwidth correction factor of Eq. (4). For some specific pulse shapes and passband characteristics its value may be obtained from Fig. 5.

Equation (5) is in a sense only trivially different from Eq. (4). The latter may be used with the values plotted in Fig. 4 by converting them to visibility-factor values. In the matched-filter case even this is not necessary since $m = 1$. (Otherwise an appropriate

*Most of Marcum's calculations were for a square-law detector, and are plotted in a form that is not directly adaptable to the range equations of this report. Marcum calculated that the results for a square-law detector differ from those of a linear detector by at most 0.2 db. This is confirmed by Ref. 10, p. 205. According to Marcum, the two detectors give identical results for single pulses; the linear detector is superior by about 0.1 db for 10 pulses integrated, and the square-law detector is superior, by an amount asymptotic to 0.2 db, for more than 70 pulses integrated.

†The introduction here of additional "signal-to-noise ratio" notation and terminology was done with considerable reluctance, because the author believes strongly in minimizing the number of terms, symbols, and definitions that apply to essentially a single quantity or concept. However, it does seem necessary to distinguish between this signal-to-noise ratio and those which have been previously defined, to emphasize the fact that they are not directly interchangeable in the range equations. Moreover, curves of the type given in Fig. 4 can only be given in terms of the detector-input signal-to-noise ratio, unless complicated stipulations are made concerning filter matching or correction for non-matching. In the discussion following Eq. (3), the notation $(S/N)_{out}$ was used to apply to essentially the same quantity that is here called D. But, this seemed to be a somewhat clumsy notation, especially when it becomes necessary to add more subscripts, e.g., $(S/N)_{out(50)db}$! Similarly the term detectability factor (an extension of the idea expressed by visibility factor) is shorter and simpler than "detector-input (or filter output) signal-to-noise power ratio." In Refs. 1 and 5, the relation $V_o C_B = m D C_B'$ was employed. The change to the above relation was made to avoid two symbols where one will do. The earlier notation was adopted to separate the two effects of nonoptimum bandwidth (expressed by C_B') and nonoptimum passband shape (expressed by m). Here m accounts for both considerations.

‡This relationship between pulse shape and passband characteristic is optimum according to a theorem due independently to North, Van Vleck, Wiener, and Hansen. See Refs. 10 and 13.

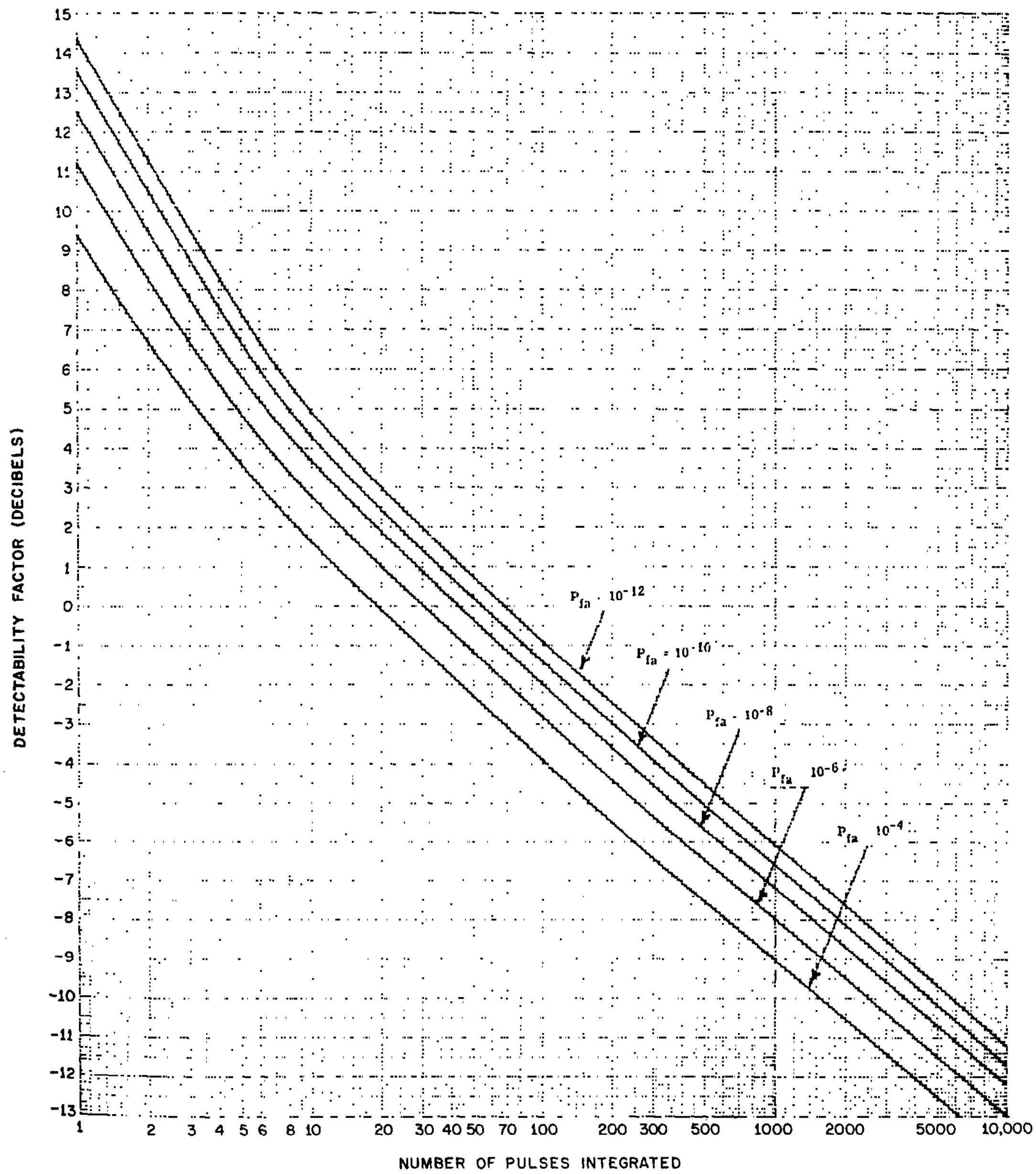

Fig. 4 - Detectability factor $D_{50(db)}$; calculated signal-to-noise power ratio at input of linear-rectifier detector followed by perfect-memory linear video integrator and a fixed-threshold-level automatic-decision device, for 0.5 probability of detection and several values of false-alarm probability

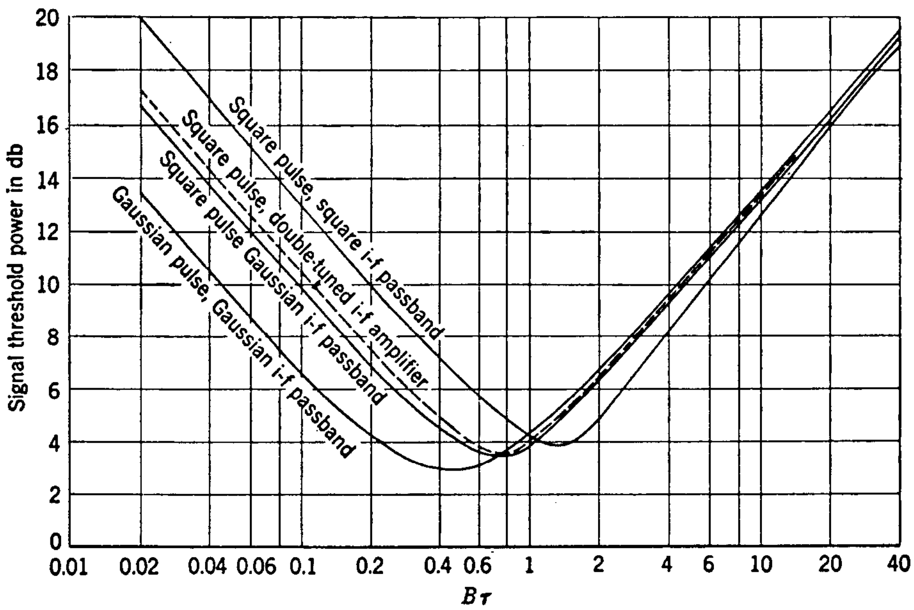

Fig. 5 - Effect of bandwidth and passband shape, calculated for several pulse shapes, assuming automatized detection (Fig. 8.11 of Ref. 10). In terms of the matching factor m of Eq. (5), the ordinates are equal to $3 + 10 \log m$, or $m_{db} + 3$. Note that $m = 1$ at the minimum of the Gaussian-pulse Gaussian-passband curve, which represents a matched-filter condition; also note that at this point $(B\tau)_{opt} = 0.44$ (but, this result for the Gaussian case does not necessarily apply to other matched-filter conditions).

curve must also be used in place of Fig. 3, which is for cathode-ray-tube displays with human observers.) However, Eq. (5) is a useful formulation, since it allows employing results of the type of Fig. 4 directly.

Values of m for certain pulse shapes and passband characteristics that are not "matched" can be obtained from Fig. 5. (The ordinate values of Fig. 5 are equal to $3 + 10 \log m$.) For the derivation of these results, see Ref. 10, pp. 204-210.* In particular, for a rectangular pulse and optimum-width receiver passband of the transitionally-coupled-circuit type, $m = 1.12$. Also, for this case, it is indicated that theoretically the optimum bandwidth is 0.7 times the reciprocal of the pulse length, for half-power definition of bandwidth and pulse length. This is in contrast to the value 1.2 found experimentally. The difference is ascribed to characteristics of the human observer, not taken into account in the theoretical analysis.† A similar explanation applies to the difference in shape of the bandwidth correction factor C_B of Fig. 3 and the curves of Fig. 5. These matters will also be discussed in greater detail in Part 2.

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*In Eq. (14b), p. 207, Ref. 10, the numerical constant apparently should be 1.21 rather than 1.10.

†In recent correspondence with the author (Oct. 30, 1962), D. K. Barton of RCA (Moorestown, N.J.) has explained the difference as arising from the fact that automatic detection is assumed to be based on peak instantaneous signal voltage (Refs. 10, 13, et al.), while the human observer probably responds to the signal averaged over the pulse length. The average signal-to-noise ratio changes more slowly as $B\tau$ is varied around the optimum value than does the peak-signal-to-noise ratio. Barton's analysis of this matter will appear in a book "Radar Systems Analysis" that he is preparing for publication by Prentice-Hall.

Interrelationship of Various Signal-to-Noise-Ratio Definitions

Three different quantities in the general category of "signal-to-noise power ratio" have been employed in the foregoing range equations. A discussion of their interrelationship* will help in understanding the distinctions between these equations.

In Eq. (3), the quantity S/N is the signal-to-noise power ratio measured at the receiver input. (The effective portions of the spectrum of broad-band noise at the receiver input or antenna terminals are therefore those contained within the receiver passband.)

In Eq. (4), the corresponding quantity is $V_{o(50)}$, defined as "ratio of minimum-detectable-signal pulse energy, at a reference point in the receiving system ("input terminals"), to the predetection noise power per unit bandwidth referred to the same point." An equivalent definition is "ratio of minimum-detectable-signal pulse power, at the reference point, to noise power in a bandwidth equal to the reciprocal of the pulse length." The subscripts qualify the definition further as the value of this quantity for optimum bandwidth and 0.5 probability of detection.

The relationship between S/N and V is best understood in terms of the mathematical definitions,

$$S/N = \frac{P_r}{k\, T_N\, B_N} \tag{6}$$

$$V = \frac{P_r\, \tau}{k\, T_N} = \frac{P_r}{k\, T_N\, (1/\tau)}\ , \tag{7}$$

where P_r is the received signal pulse power at a reference point in the receiving system, and $k T_N B_N$ is the effective receiver noise power (bandwidth B_N†) referred to the same point. As the second form of definition of V indicates, if $B = 1/\tau$, S/N and V are equal to each other numerically, but otherwise they are not. Strictly, the subscript "min" or some similar notation should be appended to P_r in Eq. (7), but not in Eq. (6), since V is defined in terms of "minimum visible" signal, while S/N is not.

To express S/N and P_r in terms of their values for 0.5 probability of detection and optimum bandwidth, an appropriate notation is $(S/N)_{o(50)}$ and $P_{r(o)(50)}$. From (6) and (7) it may be deduced that the relationship between $(S/N)_{o(50)}$ and $V_{o(50)}$ is

$$(B\tau)_{opt}\ (S/N)_{o(50)} = V_{o(50)}\ . \tag{8}$$

That is, they differ in definition by the factor $(B\tau)_{opt}$, which is the value of $B\tau$ corresponding to optimum bandwidth. The value of $(B\tau)_{opt}$, as previously mentioned, has been found experimentally to be about 1.2 for rectangular pulses and cathode-ray-tube displays with human observers.

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*Acknowledgment is due to Lee E. Davies of Stanford Research Institute for correspondence which stimulated thinking that led to clarification of these relationships, and for pointing out the desirability of formulating the signal-to-noise ratio at the receiver in terms of the signal pulse energy and the noise power density, as Norton and Omberg had done in their visibility-factor definition. This formulation was also used by North (13). Lawson and Uhlenbeck (10) employ an equivalent formulation stated in power-ratio terms. The ratio of signal-pulse energy to noise power per cycle also occurs naturally in the information-theory approach to the problem of signal detection. See for example, P. A. Woodward (14).

†Henceforth, the symbol B with no subscript will signify the noise bandwidth or an approximation of it, such as the half-power bandwidth.

Equation (3) would become a pulse-radar maximum-range equation if the notation $(S/N)_{o(50)}$ were introduced and if also a bandwidth-correction factor were added. This would not be the same factor, C_B, that appears in Eq. (4). It may be denoted C_S. That is,

$$(S/N)_{50} = (S/N)_{o(50)} C_S \tag{9}$$

just as

$$V_{(50)} = V_{o(50)} C_B . \tag{10}$$

From Eqs. (6) through (10), it may be deduced that

$$C_S = \frac{(B\tau)_{opt}}{B\tau} C_B . \tag{11}$$

A plot of C_S is also shown in Fig. 3, for comparison. It is evident that C_S is not a slowly varying function of B in the vicinity of the optimum point, as is C_B. Hence, if Eq. (3) modified in this way were to be used as a pulse-radar maximum range equation, a knowledge of the exact receiver bandwidth and a precise evaluation of C_S would be important, whereas C_B in Eq. (4) can be omitted with small error for most ordinary values of bandwidth. However, both equations are equally correct.

It is also possible to manipulate the various quantities in such a way that the resulting equation contains $(S/N)_{o(50)}$ together with the C_B bandwidth-correction factor; in fact, this results by making the substitution indicated by Eq. (8) in Eq. (4). This provides a perfectly workable range equation having all the advantages of Eq. (4) but in terms of signal-to-noise power ratio instead of visibility factor. However, it contains the extra factor $(B\tau)_{opt}$.

It has already been stated that the relationship between $V_{o(50)}$ and D_{50} is

$$V_{o(50)} \cdot C_B = D_{50}\, m . \tag{12}$$

From (8) and (12) it is evident that

$$(S/N)_{o(50)} = \frac{V_{o(50)}}{(B\tau)_{opt}} = \frac{D_{50}\, m_{opt}}{(B\tau)_{opt}} , \tag{13}$$

where m_{opt} refers to the value of m when the bandwidth is optimum but the passband shape is not necessarily optimum.

It is apparent that $(S/N)_{o(50)}$ must always be equal to or greater than D_{50}, since the noise power at the detector input is equal to the noise power at the receiver input terminals, $k\, T_N\, B$, multiplied by the receiver predetection gain, while the signal power at the detector is in some cases less than the input signal multiplied by the gain, because of the action of the bandpass filter (see Ref. 10, p. 209). This may be the case even with a matched filter of optimum width, because $(B\tau)_{opt}$ may be less than one for a matched-filter passband (see Ref. 10, p. 207).

Range Eqs. (3), (4), and (5), as the foregoing discussion implies, may be shown to be completely equivalent to one another by making appropriate substitutions based on Eqs. (6) to (13).

DECIBEL-LOGARITHMIC EQUATIONS

All of the range equations can be expressed in a decibel-logarithmic form which is very convenient when many of the quantities are given in decibels, as they often are. In these forms of the equations, quantities to be expressed in decibels have the subscript "db." All other quantities are defined as in the original form of the equation. The equations that follow have the same numbers as the parent (nondecibel) equations followed by the letter "a." All logarithms are to the base 10. It is assumed for simplicity that $F_t = F_r = F$; when this is not the case, F should be replaced in the following equations by $\sqrt{F_t F_r}$. Also, the equations may all be applied to the bistatic radar case by further substituting $\sqrt{R_t R_r}$ for R, and σ_b for σ, as previously mentioned.

The decibel-logarithmic signal-to-noise-power-ratio formulation of the general radar range equation is

$$R = F \text{ antilog} \left\{ 2.861 + \frac{1}{40} \left[10 \log P_{t(kw)} + G_{t(db)} + G_{r(db)} + 10 \log \sigma_{sq\,m} - 20 \log f_{Mc} - 10 \log T_{N(Kelvin)} - (S/N)_{db} - 10 \log B_{kc} - L_{db} \right] \right\} \text{ naut mi.} \tag{3a}$$

The decibel-logarithmic visibility-factor formulation of the pulse-radar maximum-range equation is

$$R_{50} = F \text{ antilog} \left\{ 2.111 + \frac{1}{40} \left[10 \log P_{t(kw)} + 10 \log \tau_{\mu sec} + G_{t(db)} + G_{r(db)} + 10 \log \sigma_{50(sq\,m)} - 20 \log f_{Mc} - 10 \log T_{N(Kelvin)} - V_{o(50)(db)} - C_{B(db)} - L_{(db)} \right] \right\} \text{ naut mi.} \tag{4a}$$

The decibel-logarithmic detectability-factor formulation of the pulse-radar maximum-range equation is

$$R_{50} = F \text{ antilog} \left\{ 2.111 + \frac{1}{40} \left[10 \log P_{t(kw)} + 10 \log \tau_{\mu sec} + G_{t(db)} + G_{r(db)} + 10 \log \sigma_{50(sq\,m)} - 20 \log f_{Mc} - 10 \log T_{N(Kelvin)} - D_{50(db)} - 10 \log m - L_{(db)} \right] \right\} \text{ naut mi.} \tag{5a}$$

In Appendix A, a range calculation work-sheet based on a slight modification of Eq. (4a) is given, together with a collection of the various auxiliary equations, curves, and tables ordinarily required for range calculation.

EVALUATION OF RANGE-EQUATION QUANTITIES

The following sections of the report have two primary objectives. The first is to provide sufficiently clear and precise definitions of the quantities that occur in the range equation to avoid ambiguities and misunderstanding. In various range equations that have been published, certain physical phenomena are sometimes taken into account by one of the range-equation factors, sometimes by another. Generally the choice is somewhat arbitrary, but it is extremely important to make such choices for all the range-equation quantities in a compatible way. For example, the antenna gain may be defined in terms of power radiated (and intercepted by the effective aperture on reception), or it may be defined in terms of power at the actual electrical terminals or ports of the structure denoted "antenna." These two choices are sometimes given different designations, the former being called the "directivity" of the antenna, but in less formal usage both may be called "antenna gain." The choice that is made, in a particular case, dictates the way in which the system noise temperature is defined and calculated, and affects the calculation of receiving-system loss factor L_r. Similar considerations apply to other terms in the equation.

The second objective is to provide auxiliary equations, curves, and tables which allow calculation of appropriate values to use for the various quantities in the equation, when only the basic characteristics of the radar are given. Ordinarily, for example, the quantities S/N, $V_{o(50)}$, or D_{50}, T_N, C_B, L, and F are not directly given, and sometimes even G or σ are not given. Usually, however, these quantities can be calculated or estimated from the information that is available. Where needed, conventions or "standard conditions" for range calculation are proposed.

The factors in the equation are not discussed in the order of their occurrence. Those whose evaluation is most crucial to the calculation — such factors as signal-to-noise ratio and system noise temperature — are discussed first, and those which are ordinarily determined by simple measurement, such as transmitter power, pulse length, and frequency, are discussed last, primarily for the purpose of giving definitions that are precise and compatible with the definitions given for the other quantities of the equations. Some of the topics discussed relate only indirectly to the evaluation of the range-equation factors, such as number of pulses integrated, probabilistic aspects of detection, and the elevation-angle parameter.

Signal-to-Noise Ratio (S/N)

In the formulation of Eq. (3), the quantity S/N, receiver-input signal-to-noise power ratio, occurs; it is defined formally by Eq. (2). The range given by Eq. (3) simply applies to whatever value of S/N is chosen. This may be done somewhat arbitrarily, although even such arbitrary choosing of an S/N value is usually based on some kind of a feeling for the practical significance of various S/N values. For example, it is generally recognized that radars in which there is some integration of pulses will permit detection of targets for which S/N is about one (zero decibels). However, this is of course a very rough estimation. It is also generally recognized that radars displaying only single received pulses permit detection of targets for which S/N is about 13 db. But, this figure also lacks precision, since one must actually specify a probability of detection and a false-alarm probability, and possibly other factors, in order to arrive at a precisely meaningful value of S/N. If the aim is to compute a detection range, a formulation in terms of required signal-to-noise ratio, for specified probability, is appropriate. Either Eq. (4) or (5) should be used, depending on whether human observers or automatized detection are to be employed.

There is a situation, however, for which the formulation of Eq. (3), in terms of S/N values, unrelated to detection requirements, is applicable. Radars are sometimes used as measurement devices, to study the radar-reflecting properties of an object (e.g., an astronomical body, or a man-made "space" target such as an artificial earth's satellite). The radar cross section of such an object may be calculated from measurements of the received signal power. The accuracy of such a measurement depends upon the signal-to-noise ratio (among other things). In calculating the range capability of such a radar, the criterion employed is the degree of accuracy required, or the degradation of accuracy, due to noise, which will be permitted. This requirement determines the value of S/N that should be employed in Eq. (3).

The relationship is shown in Fig. 6, in terms of the measurement error statistics. The detailed calculation of these curves is given in an appendix to a previous NRL report (15). As the curves indicate, a typically "acceptable" value for S/N is about 100 (20 db) for reasonable accuracy (probability about 0.5 that the measured receiver output voltage differs from the calculable mean value by more than 5%). For really good accuracy (probability about 0.05 for an error in excess of 5%), an S/N of 30 db is required.

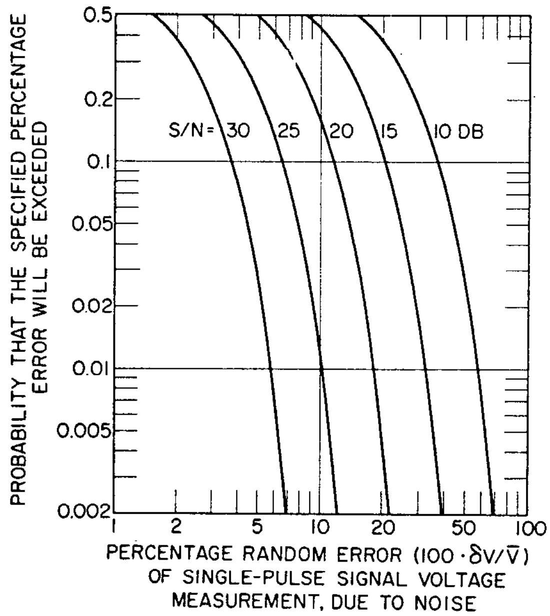

Fig. 6 - Probability of occurrence of specified error percentages in receiver signal-voltage measurement, as a function of predetection signal-to-noise ratio

Generally, a radar used for this purpose will employ adequate or more-than-adequate receiver bandwidth, so that the signal-to-noise ratio at the detector input, to which Fig. 6 refers, is the same as that at the receiver input, to which Eq. (3) refers. If this is not the case, a correction factor must be applied to the S/N values of Fig. 6, for use in Eq. (3), to account for the difference. In contrast with the factors denoted C_B and m in Eqs. (4) and (5), this correction factor applies only if the bandwidth B is too small for the radar transmitted waveform employed or for the sampling-time characteristic of the measurement instrumentation. The effect of too large a bandwidth is accounted for by the presence of the factor B in the equation.

The results of Fig. 6 are for a sample time equal to or less than the reciprocal of the receiver bandwidth or, in the pulse-radar case, for single pulses. The measurement error can be reduced greatly by averaging the results of several single-sample measurements, or by averaging (integrating) before measurement, in accordance with well-known statistical principles. However, in computing this effect where postdetection averaging or integration is employed, if the improvement considered is sufficient to permit operating with small values of S/N, the percentage errors in terms of receiver input voltage or power cannot be directly determined by simple calculation; it is necessary to take into account the somewhat complicated statistics of signal and noise combination in the detection (rectification) process.

Visibility Factor (V) and Bandwidth Correction Factor (C_B)

The visibility factor is a function of the number of pulses integrated. In the case of a human observer and cathode-ray-tube display, values have been determined by experiment, and are given by Figs. 1 and 2 for an A-scope display and a PPI display, respectively.

The subscripts on the visibility-factor symbol, $V_{o(50)}$, indicate the value that applies for optimum receiver bandwidth and for 0.5 probability of detection. Curves for other probabilities are of similar form but lie above or below the 0.5-probability curve — above for higher probability, below for lower.

Those who are familiar with the probabilistic aspects of signal detection may inquire as to the false-alarm probability to which these curves correspond. The experiments were not designed to determine this quantity. However, it may be regarded as a value applicable to average human observers.

Curves could be obtained for higher or lower probabilities of false reporting of signals, lying below or above the curves shown. Selection of a particular one of these curves as a basis for general range calculation is somewhat arbitrary, just as is the choice of 0.5 probability of detection. There is a need, however, for agreement on a particular curve or curves as a convention for purposes of comparative range calculation, and the curves of Figs. 1 and 2 seem to be as good as any for that purpose, for radars that employ cathode-ray-tube indicators and human observers. There does not seem to be any better basis for such curves than the Radiation Laboratory data. Experience has shown that these visibility-factor curves result in calculated ranges that agree reasonably well with experimental results (16). As seen by comparison with Fig. 4, they also conform in a general way to curves calculated on a theoretical basis.

The changes in slope of these curves in the regions of very few pulses and very large numbers of pulses will be discussed in some detail in Part 2. Briefly, the slope change in the region of few pulses (at about $N = 10$, Figs. 2 and 4) is the result of the statistics of signal and noise combination in the linear-rectification process. (A similar result is obtained with a square-law detector.) The leveling off that occurs for very large numbers of pulses integrated (e.g., 1000 pulses) is the result of the limited contrast discernment of the human eye and brain (see Ref. 10, p. 223). This effect does not occur when a physical integrating device is employed in place of the cathode-ray tube and human being (see Fig. 4).

If the bandwidth is other than optimum, the actual visibility factor is given by the optimum-bandwidth value multiplied by a correction factor C_B. For cathode-ray-tube displays and human observers, and for approximately rectangular pulses and typical receiver passband characteristics, the optimum bandwidth has been found to be 1.2 times the reciprocal of the pulse length (10). The bandwidth correction factor is approximately given by the following empirical formula* (17):

$$C_B = \frac{B\tau}{4.8}\left(1 + \frac{1.2}{B\tau}\right)^2. \tag{14}$$

As this formula indicates, when $B\tau = 1.2$, $C_B = 1$. Decibel values of C_B are plotted in Fig. 3 as a function of the $B\tau$ parameter. As the curve indicates, C_B changes very slowly in the vicinity of $B\tau = 1.2$, so that for general range-calculation purposes when the exact value of B is not known, it is usually satisfactory to assume $C_B = 1$ ($C_{B(db)} = 0$).

The numerical value of the optimum bandwidth, in relation to the pulse length, depends on the way in which these quantities are defined. Customarily, the half-power definitions are employed — the pulse length between half-power points of the pulse waveform, and bandwidth between half-power points of the overall frequency response curve of the receiver predetection circuits and amplifiers. Figure 3 is plotted on the basis of these definitions. The statement that the optimum bandwidth is 1.2 times the reciprocal of the pulse length is thus based on the half-power definitions of pulse length and bandwidth, and on the further conditions that the pulse is approximately rectangular and the receiver has a conventional double-tuned i-f amplifier, or one with reasonably similar passband shape. It is also restricted to the case of detection by a human observer of a conventional cathode-ray-tube indicator.

As discussed in Ref. 10, p. 177, the true noise bandwidth may differ considerably from the half-power bandwidth in some cases, but ordinarily the difference is not great. The half-power width is much easier to measure and is therefore usually specified as

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*The formula of Ref. 17 is based on the assumption that optimum bandwidth is exactly the reciprocal of the pulse length rather than 1.2 times the reciprocal. The formula given here has been modified accordingly.

the bandwidth. The flatness of the C_B curve in the vicinity of the optimum, for radars with cathode-ray-tube indicators and human observers, is a further reason for the adequacy of half-power bandwidth specification.

As indicated by the discussion following Eq. (4), the primary requirement on the definition of pulse length is with respect to the energy significance of the product $P_t \tau$. The half-power definition conforms to this requirement. Of course, with approximately rectangular pulse shapes the significance of the particular definition employed for pulse length is further minimized.

Number of Pulses Integrated (N)

As the foregoing paragraphs indicate, the number of pulses integrated by the observer, or by an automatic detection device as will be subsequently discussed, is an important factor in the range equation, although an implicit one. The basic equation for the number of pulses integrated by a nonscanning radar is

$$N = \overline{PRF}\, t_i , \tag{15}$$

where $\overline{PRF}$ is the radar pulse rate, pulses per second, and t_i is the effective integration time, seconds. The characteristics of the integrator determine t_i. Its value for electronic storage or delay devices can usually be assessed readily, but it is difficult to assign a numerical value for the combination of a human observer and cathode-ray tube. This problem is by-passed in Fig. 1 by employing the $\overline{PRF}$ directly as the variable.

Figure 2 may be used for scanning radars without exact knowledge of t_i when it can be assumed that t_i is shorter than the scan period and longer than the interval required for the beam to traverse the target. In this case the number of pulses integrated is taken to be the number occurring while the target is within the half-power limits of the (one-way) antenna pattern during a single scan. When this assumption can be made, the following formulas are useful for computing the number of pulses N illuminating the target per azimuth scan of a scanning radar, as required for use with Figs. 2 and 4. For azimuth-only scanning,

$$N = \frac{\theta_h\, \overline{PRF}}{6\,(\cos\,\theta_e)\, \overline{RPM}} , \tag{16}$$

where θ_h is the horizontal beamwidth in degrees (the value applicable at the target elevation angle), $\overline{PRF}$ is the pulse repetition frequency in pulses per second, $\overline{RPM}$ is the scanning speed in revolutions per minute, and θ_e is the target elevation angle. For an azimuth-and-elevation-scanning radar (assuming that there are many elevation scans per azimuth scan) the formula is

$$N = \frac{\theta_h\, \theta_v\, \overline{PRF}}{6\,(\cos\,\theta_e)\, \omega_v\, t_v\, \overline{RPM}} , \tag{17}$$

where θ_v is the vertical beamwidth in degrees, ω_v is the vertical scanning speed in degrees per second (at the elevation angle of the target), and t_v is the vertical scanning period in seconds (including dead time, if any). These formulas apply as long as $\theta_h/\cos\,\theta_e$ is not greater than about 90 degrees. For greater values, more complicated formulas are required.

These formulas assume that the target is either stationary or moving with a speed and direction such that during the time interval equal to θ/ω the angular distance moved is small compared to θ, where θ is the beamwidth in either scanning direction and ω is the angular velocity of the beam in the same direction. If this condition on angular target motion is not fulfilled, a correction to the number-of-pulses result must be made, to take the target motion into account. For radial target motion, correction may have to be made if the distance moved during the time θ/ω is appreciable compared to $c\tau/2$, where c is the velocity of electromagnetic propagation and τ is the pulse length; this correction will be necessary if a stationary range gate of length comparable to the radar pulse length is employed. Ordinarily, for standard range-calculation purposes, target velocity is assumed to be such that these corrections are not necessary.

When the number of pulses given by $\overline{PRF}\ \theta/\omega$ is less than one, this number may be interpreted as a new statistical factor in the detection problem, namely, the probability that a radar pulse will be transmitted during the time the antenna beam is aimed at the target, assuming a nonintegral relationship between the radar pulse rate and the scan rate. Also, for targets at great ranges a pulse or pulses may be transmitted while the target is in the beam, but by the time the echo pulses are received the beam has moved angularly an appreciable fraction of a beamwidth, or possibly even a full beamwidth or more. These results occur when the time $2R/c$ becomes comparable to, equal to, or greater than the time θ/ω, where R is the target range. In all of these cases special analyses must be made to determine the effective number of pulses, the pattern loss factor, and the probability of detection.

MIT Radiation Laboratory experiments during World War II indicate (10,11) that t_i may be as great as 6 to 10 seconds for highly trained observers, but many radar engineers feel that a somewhat shorter time is probably characteristic of the average observer. It is suggested that a more conservative value, $t_i = 2$ seconds, be assumed for conventional range-calculation purposes. Thus, for example, if a radar scans in azimuth at a rate faster than 30 revolutions per minute ($\overline{RPM}$), some scan-to-scan integration should be assumed, and the number of pulses integrated would be the number occurring per azimuth scan, as determined by Eqs. (16) or (17), multiplied by $\overline{RPM}/30$. For radars that scan in more complicated fashion these formulas may not be applicable, but the same principles apply.

The number of pulses thus computed may be used in connection with Fig. 2 to determine a value of $V_{o(50)}$, in decibels, applicable to Eq. (4) when a PPI (or any similar intensity-modulated cathode-ray-tube display) is used with a scanning radar and a human observer, without other integrating devices. (Generally, supplementary integrators improve the visibility factor only if they have a longer effective integration time than that of the human observer, or if they operate as predetection integrators.)

Detectability Factor (D) and Matching Factor (m)

For automatized radar detection, curves of minimum-detectable signal-to-noise power ratio, defined at the detector input terminals, may be calculated, as exemplified by the work of Marcum (12). Curves of this type, calculated by the author following methods described by North (13), are shown in Fig. 4. They are for a fixed-threshold-level decision-making device, preceded by a linear-rectifier detector and a perfect-memory linear video integrator. The N pulses integrated are assumed to be of constant amplitude. The quantity represented by these curves may be called detectability factor D to distinguish it from the visibility factor V and from the signal-to-noise ratio at the receiver input S/N. The factor D is a power ratio defined at the detector terminals rather than at the receiver input terminals. The selective circuits of the receiver intervene. North (13) has shown that if the receiver passband transfer characteristic "looks like the conjugate of the spectrum of the echo at the antenna," V and D are equal, but otherwise they are not. In general, as stated earlier,

$$V_{o(50)}\, C_B = D_{50}\, m\,, \tag{12}$$

where $m \geq 1$ may be called a matching factor. The values of C_B given by Eq. (14) and Fig. 3 do not apply in general for automatized detection. Lawson and Uhlenbeck (Ref. 10, pp. 204-210) have analyzed these matters for certain specific cases (though not in the terminology employed here).

Their results are summarized for several pulse shapes and passband characteristics in Fig. 5 (their Fig. 8.11). The signal level is shown on a scale such that the minimum for the matched filter case ($m = 1$) occurs at the +3-db point. The curves may therefore be used to evaluate 10 log m by subtracting 3 db. The curve for Gaussian pulse shape and Gaussian passband is of course a matched-filter case.

The calculation of the curves of Fig. 4 is based on the statistics of the noise-only and signal-and-noise voltages after detection (linear rectification) corresponding to the statistics of the envelopes of the predetection voltages. If the probability-density functions of these envelopes are denoted respectively by $p_{n(N)}$ and $p_{sn(N)}$, where the subscript (N) refers to the form of these probability-density functions after integration of N pulses, then the false-alarm probability is calculated, for a given signal-to-noise ratio D, from the equation

$$P_{fa} = \int_{V_t}^{\infty} p_{n(N)}(V)\, dV , \tag{18}$$

where V is the integrated rectifier (detector) output voltage and V_t is the threshold setting of the automatic detection device. The probability of detection is calculated from

$$P_d = \int_{V_t}^{\infty} p_{sn(N)}(V)\, dV . \tag{19}$$

The function $p_{sn(N)}(V)$ is dependent upon the predetection signal-to-noise ratio D. The basic procedure of calculation is to choose an acceptable false-alarm probability P_{fa} and then use Eq. (18) to determine the correct threshold-level setting V_t. Then Eq. (19) may be used to calculate P_d for a given $p_{sn(N)}$; or, as it turns out, the necessary value of D can be computed to give $P_d = 0.5$ (this value is denoted D_{50}). There are mathematical difficulties but it is possible to make some approximations which do not lead to appreciable errors.

The details of the calculations will be given in an appendix to Part 2. Comparison with results calculated by others using different techniques has indicated general agreement. Many of the published curves of this type, however, are for a square-law detector rather than for a linear rectifier. Although the maximum difference in threshold level for the square-law and linear detectors has been shown (10,12) to be only about 0.2 db, the linear rectifier is actually the kind ordinarily employed in radar receivers; therefore it seems more appropriate to calculate the detection curves applicable to it (although, as it turns out, it is mathematically more difficult to do).

As previously stated, the calculations assume video (postdetection) integration, which is ordinarily employed in practical radars. Under some circumstances, however, it is possible to employ predetection integration. As North first showed (13), a considerable improvement in signal detectability then results. If perfect predetection integration were assumed each curve of Fig. 4 would have the same value shown for $N = 1$, denoted $D_{50(db)}(1)$, but would follow the law

$$D_{50(db)}(N) = D_{50(db)}(1) - 10 \log N . \tag{20}$$

That is, the curves would be perfectly straight lines (on the $D_{50(db)}$ vs $\log N$ plot) with a negative slope of 10 decibels per decade, whereas for the video-integration case the slope has initially this value, at $N = 1$, but becomes asymptotic, for large N, to -5 decibels per decade.

Since predetection integration requires phase coherence of successive echo pulses, an unusual degree of transmitter and receiver oscillator stability is necessary. Also, compensation for any target motion must be provided. In short, a highly sophisticated and complex radar system is needed, so that for ordinary radar applications postdetection integration is employed.

Probabilistic Aspects of Signal Detection

The false-alarm probability P_{fa} is the probability that the threshold level V_t will be exceeded at a particular instant by the integrated voltage output of the receiver when no signal is present. If there were no integration, and if the receiver output were being observed continuously, then the average time between false alarms would be

$$t_{fa} = \frac{1}{B_N P_{fa}}, \tag{21}$$

where B_N is the predetection (e.g., i-f) bandwidth. (The time between "independent noise samples" in the receiver output is $1/2B_v$, where B_v is the video bandwidth. It can be shown that for the purpose of this type of calculation, $B_N \cong 2B_v$, assuming that the video-amplifier passband is adequate so that the noise output spectrum is determined by the i-f passband.) When the integrator adds N independent noise samples (on successive range sweeps) and delivers only one output voltage corresponding to every successive group of N input samples, then the average false-alarm time is N times as great as that given by Eq. (21), assuming that the threshold voltage V_t has been adjusted to give the same false-alarm probability P_{fa}.

If the individual range sweeps are observed by means of automatic detection devices through a set of nonoverlapping range gates, each of length $t_g = 1/B_N$, and if further there is a dead time on each range sweep during which no range gates are active, the formula becomes

$$t_{fa} = \frac{\eta N t_g}{P_{fa}}, \tag{22}$$

where η is the ratio of the interpulse period to the active (gated) sweep time. Many other possible arrangements exist. These examples are given to illustrate the principles and to indicate that the practical effect of the false-alarm probability depends heavily upon the nature of the detecting and integrating apparatus. Also, definitions vary with different authors. For example, Marcum (12) defined false-alarm time as the interval during which the probability of a false alarm is 0.5. For this definition the equation corresponding to Eq. (21) would be

$$t_{fa(0.5)} = \frac{\log 0.5}{B_N \log (1 - P_{fa})}. \tag{21a}$$

In practice, the desired value of P_{fa} is determined by first deciding what average interval between false alarms will be acceptable (or what interval for which the probability of a false alarm is 0.5, using Marcum's definition). Then P_{fa} can be calculated from an appropriate equation such as Eq. (21) or (22). This permits calculating V_t from Eq. (18) and P_d from Eq. (19).

The probability of detection P_d is the probability that the threshold level V_t will be exceeded by the receiver output when there is a signal actually present. In the particular case of an azimuth-scanning radar for which it can be safely assumed that no scan-to-scan integration occurs, this probability corresponds to the so-called blip/scan ratio, the ratio of average number of scans on which detections occur to the total number of scans observed, for a target at a given range with constant values of all radar-equation parameters. The blip/scan ratio and a related or derived quantity, the cumulative probability of detection, will be discussed in a later section.

The foregoing discussion has tacitly assumed that the probabilistic aspects of the detection process are introduced solely by the randomness of the receiver noise. It is also possible, however, that random fluctuations of the target cross section, or propagation conditions, or in principle any of the radar-equation quantities, may contribute to the statistical aspect of the problem. As will be discussed in subsequent sections of the report, the complete analysis of radar range performance can become very difficult and complicated for these cases; however, some significant statements about them can be made with very little additional complication, as will be done.

The treatment of the detection process in this report is elementary. Very sophisticated mathematical treatments exist in the unclassified literature.

System Noise Temperature (T_N)

The receiving-system noise temperature T_N is a fictitious temperature that expresses noise power available at the receiver output as an equivalent available power density at some reference point in the receiving system cascade. Since the receiver gain and other factors are variable over the total passband, this spectral density is in general a function of frequency; in the language of receiver noise factor definitions, it is a "spot" noise temperature. The power density is kT_N watts per cycle of bandwidth, where k is Boltzmann's constant, 1.38×10^{-23}, and the total noise power is the frequency integral of this density. However, just as receivers are often characterized by an average noise factor, so also it is customary to characterize a receiving system (or component) by an average noise temperature (4) $\bar{T}_N$ defined so that the total noise power is $k\bar{T}_N B_N$, where B_N is the receiver noise bandwidth. In the discussion that follows, the statements and equations are generally applicable to either the spot or the average temperatures, for single-response systems. In the range equations this average noise temperature is meant, although the bar is omitted. Similarly, the average antenna, transmission-line, and receiver noise temperatures are used for calculating the average system noise temperature in the equations that follow.

The noise temperature T_N is the sum of contributions from external radiating sources, thermal noise due to receiving-transmission-line losses, and internal receiver noise. Each of these three sources is ascribed a noise temperature, termed respectively antenna noise temperature T_a, receiving-transmission-line output noise temperature T_r, and effective receiver input noise temperature T_e. Each of these temperatures is referred to the input or output of the device with which it is associated, and to obtain a total system noise temperature by addition, they must first be referred to a common point in the system. If this common or reference point is chosen as the receiver input terminals, the equation for the system noise temperature becomes

$$T_N = T_a/L_r + T_r + T_e , \tag{23}$$

where L_r is the loss factor for the portion of the system that precedes the receiver input terminals.

It is for some purposes preferable to choose as reference point the system input terminals (ahead of all transmission-line losses). The temperature thus computed may be called the system-input noise temperature and is given by

$$T_{NI} = T_a + L_r(T_r + T_e) = T_a + T_{r(I)} + L_r T_e \tag{24}$$

in which $T_{r(I)}$ is the transmission-line input noise temperature ($= L_r T_r$). It is apparent that T_{NI} is obtained by simply multiplying T_N by L_r. The advantage of using the T_{NI} rating is that it is a meaningful index of system noise performance, a figure of merit for comparing different systems, whereas comparison of the merit of different systems by means of the T_N rating is not valid except when $L_r = 1$. From the range-calculation point of view, however, it makes no difference except in the evaluation of the system loss factor L. If the system noise temperature is defined as T_N, then L contains the factor L_r. If T_{NI} is used in the range equation, L_r is omitted in evaluating the system loss factor L. Since the system input noise temperature does have the advantage mentioned, it will be used in the range-equation work sheet given in Appendix A, although the equations previously written have used the somewhat more general notation T_N.*

The antenna noise temperature T_a is dependent in a somewhat complicated way on the effective noise temperatures of various radiating sources within the receiving pattern of the antenna (including side lobes and back lobes). However, T_a is not directly dependent on the antenna beamwidth and gain. Therefore, it is possible to calculate an antenna temperature which is approximately applicable to any typical radar antenna as a function of frequency. In the microwave region, however, where the thermal noise due to atmospheric absorption is dominant, T_a is also a function of the length of the path in the atmosphere traversed by the beam center, and hence of the beam elevation angle (but not of the target elevation angle, per se).

Curves of antenna temperature are shown in Fig. 7, calculated for the following conditions judged to be typical: (a) average cosmic noise (which actually varies greatly with beam direction, but not in a manner expressible in geocentric coordinates); (b) sun noise temperature 10 times the quiet level, with the sun assumed to be viewed in a side lobe of unity gain; (c) a cool temperate-zone atmosphere; (d) a contribution of 36°K from ground radiation, which would result (for example) if a ground of blackbody temperature 290°K were viewed over a π-steradian solid angle by side lobes and back lobes averaging 0.5 gain (-3 db). This ground-noise contribution, independent of frequency, elevation angle, and beamwidth, is the most arbitrary of the assumptions. It can be justified as a general assumption, but if in a specific case it is not justifiable, the value of T_a given by the curve may be corrected by adding or subtracting an appropriate amount. The atmospheric noise contribution is based on the one-way absorption values corresponding to the maximum-range two-way absorption values given by Figs. 13-18 and Table 1. The dashed horizontal line at $T_a = 36°K$ indicates the assumed ground-noise level. The dashed curves are for maximum and minimum cosmic and atmospheric noise. Although the solid curves may thus not be correct for every operational condition, they are believed to be suitable as a convention for general range calculation.

The receiving-transmission-line output noise temperature T_r is related to the power loss factor L_r and to the thermal (kinetic) temperature of the line T_t by the formula

$$T_r = T_t(1 - 1/L_r) \tag{25}$$

and therefore the input temperature is

$$T_{r(I)} = L_r T_r = T_t(L_r - 1)\,. \tag{25a}$$

The loss factor L_r represents all available losses† preceding the receiver input terminals, including those in the antenna system. It has a multiple role in the range calculation. As indicated by Eq. (23), it attenuates the antenna noise, and in accordance with Eq. (25), it results in generation of thermal noise. It also attenuates the radar echo

*T_N is more general because it allows any reference point to be chosen, including the system input terminals, for which case $T_N = T_{NI}$.

†For a discussion of the "available loss" concept, see Ref. 4, pp. 17-20.

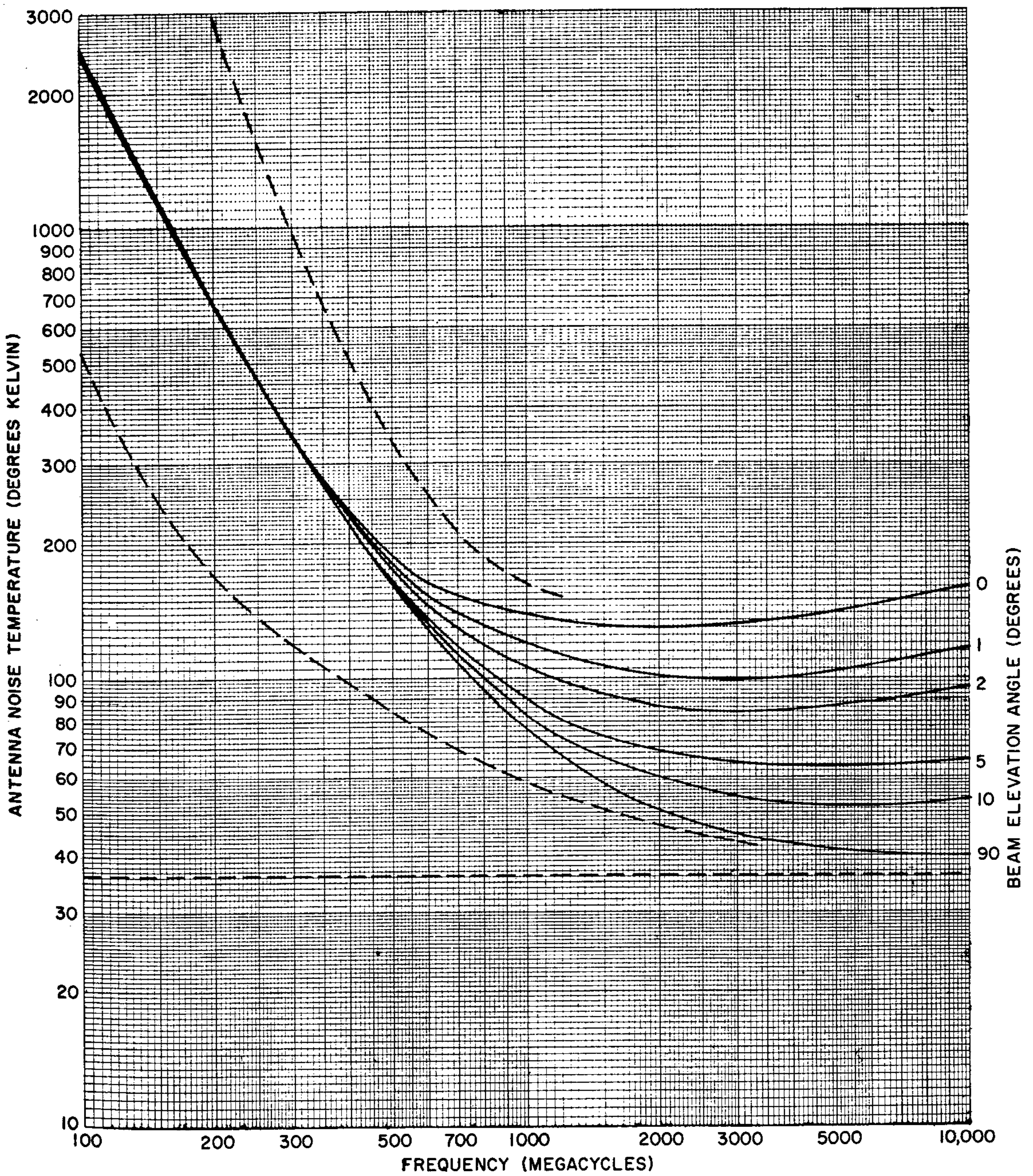

Fig. 7 - Antenna noise temperature for typical conditions of cosmic, solar, atmospheric, and ground noise. The dashed curves indicate the maximum and minimum levels of cosmic and atmospheric noise likely to be observed. The horizontal dashed line is the assumed level of ground-noise contribution (36°K).

signal. The definitions given for G_r, T_r, and T_N are designed to allow a single definition of L_r in these three roles.

If various lossy components of the transmission-line system operate at appreciably different thermal temperatures, T_r or $T_{r(I)}$ must be computed from a cascade formula. A suggested conventional value for T_t is the value already standard for receiver noise-factor rating, 290°K, applicable except when the transmission line or some of its components may be expected to operate at considerably above or below ambient temperature.

The effective receiver input noise temperature T_e is related to the receiver noise factor $\overline{NF}$ (IRE Standard 59 IRE 20.S1) by the formula

$$T_e = (\overline{NF} - 1)\, T_0 , \tag{26}$$

where T_0 = 290°K, the reference temperature for noise-factor measurement.

This formula is strictly applicable to single-response receivers, as are also Eqs. (23) and (24). The somewhat more complicated expressions applicable to multiple-response receivers are given in a previous NRL report (4), which also contains more detailed discussion of all the foregoing material, together with the detailed calculation of the curves of Fig. 7.

Listed below are some points concerning the use of the noise-temperature concept which are not always recognized, although some of them may seem trivial to those who are experienced in its use. For details see Ref. 4.

1. Antenna noise temperature, as represented by the curves of Fig. 7 and as usually defined, represents only the effect of external radiating noise sources, and does not include noise generated by any dissipative elements of the antenna itself. Therefore, noise of the latter type must be accounted for by including antenna dissipation losses in the loss factor L_r. (This is also consistent with the usual way of computing received signal power.)

2. The factor L_r must be defined as the available loss, which is the ratio of the available power at the line input to the available power at the line output. This loss will not in all cases be the actual dissipation loss that occurs; it is the loss that would occur with a matched load on the line, whether the load is matched in the actual case or not. A distinction must sometimes be made here between a load matched to the line in the Thévenin Theorem sense, and one which is matched to the characteristic impedance of the line; the former meaning is applicable in this case. However, ordinarily the distinction is not necessary.

3. The average noise temperatures ordinarily used in signal-noise or radar range calculations are engineering approximations which are usually accurate enough for practical purposes, but a rigorous treatment requires noise temperature to be viewed as a point function of frequency, so that total noise power is computed by integrating over the receiver passband rather than multiplying by an arbitrarily defined bandwidth. Complications arise especially in the case of multiple-response receivers; however, the average-noise-temperature concept can be extended to this case (4).

4. The noise temperature of a system, transducer, generator, or load must always be referred to a particular point, pair of terminals, or port, to be meaningful. A transducer has both an input and an output noise temperature, and noise temperature in a cascade system may be referred to any point in the cascade. Output noise temperature is a more basic quantity, since noise temperature actually always describes output noise, although it may do so in terms of an equivalent noise power referred to the input terminals (or port). On the other hand, as a figure of merit for comparing the signal-to-noise performance of different transducers or systems, only the input noise temperatures have significance.

5. As is well known, the noise temperature specification of a transducer is meaningless unless the impedance of the input termination is also specified. However, it is a common and acceptable practice to omit specification of the input termination on the assumption that the optimum termination is meant (the one which results in minimum noise temperature).

6. Existing IRE transducer noise temperature definitions at the time of writing, based on Eq. (26), are not satisfactory for multiple-response transducers. Definitions that are suitable are given in Ref. 4. Improved official IRE definitions are under consideration. The definitions that result may or may not coincide with those given in Ref. 4; however, those of Ref. 4 are consistent with the range equations of the present report and will therefore be valid for range calculation. Moreover, no changes in the definitions applicable to the single-response receivers ordinarily used in radar systems are contemplated. (It is understood that the new definitions are to be published by the IRE in March 1963.)

Pattern-Propagation Factor (F)

The pattern propagation factor F, as defined by Kerr (8), is the ratio of the actual field strength at the target to that which would be observed in free space at the same range, in the beam maximum. As used in the range equations of this report, the further provision is made that the field strengths in this definition shall be those that would exist in the absence of any propagation-medium absorption. This is because such losses are taken into account in the system loss factor L. Strictly speaking, this "separating out" of absorption loss from other propagation effects may not be a valid procedure in some cases of multipath propagation, but it is ordinarily permissible and results in great simplification of formulas.

The principal factors taken into account by the pattern-propagation factor are then the antenna pattern, reflection-interference effects, refraction, shadowing, and diffraction.

Separate factors F_t and F_r are used for the transmission and reception pattern-propagation effects. This is only necessary when the transmitting and receiving antennas are not identical in location and pattern. In the discussion that follows, the subscripts will be omitted, but it should be realized that the computation of F_t and F_r must in some cases be performed separately for the two propagation paths.

The pattern factor of the antenna beam $f(\theta,\phi)$ is the ratio of the radiated field strength (electric intensity) in the angular direction θ,ϕ to that in the beam maximum. In free space, $F = f(\theta,\phi)$, and in the beam maximum, $f(\theta,\phi) = 1$; hence, in free space in the beam maximum, $F = 1$. If this value is used in the radar equation, the resulting range is called the "free space range" of the radar. This range, when corrected for atmospheric absorption, is applicable, under idealized atmospheric conditions, to a radar whose antenna beam is vertically narrow and directed at the target elevation angle, provided further that the target elevation is more than a half beamwidth - or, more precisely, that there is no appreciable energy reaching the target by a reflected path.

Under some conditions, however, F may be practically zero, or as great as 2, due to interference of direct and reflected waves. Under special conditions, values of F greater than 2 are possible. Since the radar maximum range is directly proportional to F, when $F = 2$ the range is double the free-space value. Therefore the reflection-interference effect is sometimes a very important factor in radar performance.

The slight refraction that occurs normally in the atmosphere (standard refraction) does not generally affect F, but it does affect the range-height-angle relationships of targets, as will be discussed. No formulas are given here for computing F when anomalous refractive effects, variously called superrefraction, trapping, and ducting, occur, because these effects are so variable and complicated. They are discussed in detail by Kerr et al. (8). While they are of fairly common occurrence, they primarily affect detection of surface or very low altitude targets, although high-altitude ducts or refracting layers may also occur.

The presence of trapping layers in the atmosphere may create radio holes, or regions in which $F \cong 0$, and therefore also $R_{50} \cong 0$. The importance and prevalence of such regions as factors in radar detection of air targets is at present a matter of some controversy, requiring additional experimental study for its resolution (18). However, in a large proportion of practical cases it seems quite certain that such effects do not occur to any significant extent in ground-to-air propagation paths, especially below 1000 Mc.

The factor F is of primary importance in the range calculation when there is reflection from the earth or sea. This effect is treated extensively by Kerr (8), for both the flat-earth approximation and the more general spherical-earth case. The latter case is generally important only for radars located well above the earth (e.g., airborne). Readers interested in this case should consult Kerr, pp. 112 ff. Here only the flat-earth case, applicable for most surface-based radars (antenna height less than a few-hundred feet) will be considered. (Even for this case, however, the so-called divergence effect of the earth's curvature will be considered for low-angle rays.)

Reflection from the earth or sea creates an interference lobe pattern in the vertical plane parallel to the propagation direction, analogous to the Lloyd's mirror effect of optics. This is usually, or often, the principal effect that has to be taken into account in computing F. Formulas for variable possible conditions may be derived. The general equation is

$$F = |f(\theta_1) + \rho D f(\theta_2)\, e^{-i\alpha}|, \tag{27}$$

where $f(\theta)$ is the pattern factor at the angles of the direct ray from antenna to target (θ_1) and of the reflected ray (θ_2), ρ is the magnitude of the reflection coefficient of the surface, D is the divergence factor that accounts for "spreading" of the reflected rays due to curvature of the surface, and the angle α is the phase difference, at the target, of the direct and reflected rays. When the pattern factors of the transmitting and receiving antennas are not the same, and including for generality the dependence of F on azimuth as well as elevation angle (although the azimuth angle will ordinarily be the same for the direct and reflected rays), this leads to

$$\begin{aligned} F_t F_r = \; & |f_t(\theta_1,\phi_1)\, f_r(\theta_1,\phi_1) + \rho D \big[f_t(\theta_1,\phi_1)\, f_r(\theta_2,\phi_2) \\ & + f_t(\theta_2,\phi_2)\, f_r(\theta_1,\phi_1)\big]\, e^{-i\alpha} + \rho^2 D^2 f_t(\theta_2,\phi_2)\, f_r(\theta_2,\phi_2)\, e^{-2i\alpha}|. \end{aligned} \tag{28}$$

In ordinary trigonometric notation, Eq. (27) may be written

$$F = |\sqrt{f^2(\theta_1) + 2\rho D\, f(\theta_1)\, f(\theta_2) \cos\alpha + \rho^2 D^2 f^2(\theta_2)}|. \tag{27a}$$

The angle α may be written as the sum of two terms:

$$\alpha = \beta + \gamma, \tag{29}$$

where β expresses the phase difference due to the path-length difference of the direct and reflected rays ΔR, and γ is the phase difference resulting from the process of reflection.

The phase change β due to a path difference ΔR is equal to 2π radians multiplied by the number of wavelengths in ΔR, which is $2\pi\, \Delta R/\lambda$. For the flat-earth case, trigonometric analysis indicates that when the target range is very large compared to the radar antenna height,

$$\Delta R \cong 2h \sin\theta = \frac{2hH}{R}, \tag{30}$$

where h is the antenna height above the reflecting surface, θ is the target elevation angle, H is the target height, and R is the target slant range. Therefore, for this case

$$\beta = \frac{4\pi h \sin\theta}{\lambda} \text{ radians} \tag{31}$$

which can also be written

$$\beta = \frac{4\pi h H}{\lambda R} \text{ radians.} \tag{31a}$$

For most range-equation purposes, Eq. (31a) is inconvenient because it results in a transcendental equation in R. One exception occurs, however, when β is very small, as in the analysis of detecting a target well below the first-lobe maximum of the interference pattern; in this case $\sin\beta$ can be approximated by β, and a useful range equation results, involving the target height H instead of the target elevation angle θ.

For sea water and horizontal polarization, γ is virtually a constant, of value 180 degrees (π radians). Actually it is 180 degrees at zero grazing angle, increasing very slightly to less than 184 degrees at normal incidence within the normal range of radar frequencies. For vertical polarization, at zero grazing angle γ has the value 180 degrees, the same as for horizontal polarization, but at greater grazing angles it is a complicated function of grazing angle and frequency, as shown in Fig. 8.

The identification of the ordinates of these curves with the angle γ in Eq. (29) is restricted to the case in which the direct and reflected rays are practically parallel to each other. This requirement will be satisfied if the target distance is large compared to the antenna height (a condition that the formulas given for F require for other reasons also), and if the target is at a positive elevation angle with respect to the antenna position. For analyzing the interference of vertically polarized waves without this restriction, the behavior of the purely vertical and purely horizontal (longitudinal) components of the direct ray, the incident ray, and the reflected ray, and the definition of "reflection coefficient" for each component, must be separately considered (Ref. 8, p. 397).

When a single antenna is used and the beam is symmetrical with respect to the horizon, Eq. (27) becomes

$$F = F_t = F_r = \left| f(\theta)\sqrt{1 + \rho^2 D^2 + 2\rho D \cos\alpha} \right|. \tag{32}$$

Here θ is the target elevation angle.

If the antenna polarization is horizontal ($\gamma \cong \pi$), the antenna height is low, the target elevation angle is such that earth's curvature can be neglected, and the target range is great compared to the antenna height, then

$$\cos\alpha \cong -\cos\beta = -\cos\left(\frac{4\pi h \sin\theta}{\lambda}\right). \tag{33}$$

In many practical cases — e.g., for low-frequency radar with a moderately smooth sea — it is permissible to assume that $\rho = 1$ and $D = 1$. Applying Eq. (33) to Eq. (32) and performing some trigonometric manipulation* then results in the simplified formula

*Using the relation $\sqrt{2 - 2\cos\beta} = 2\sin(\beta/2)$.

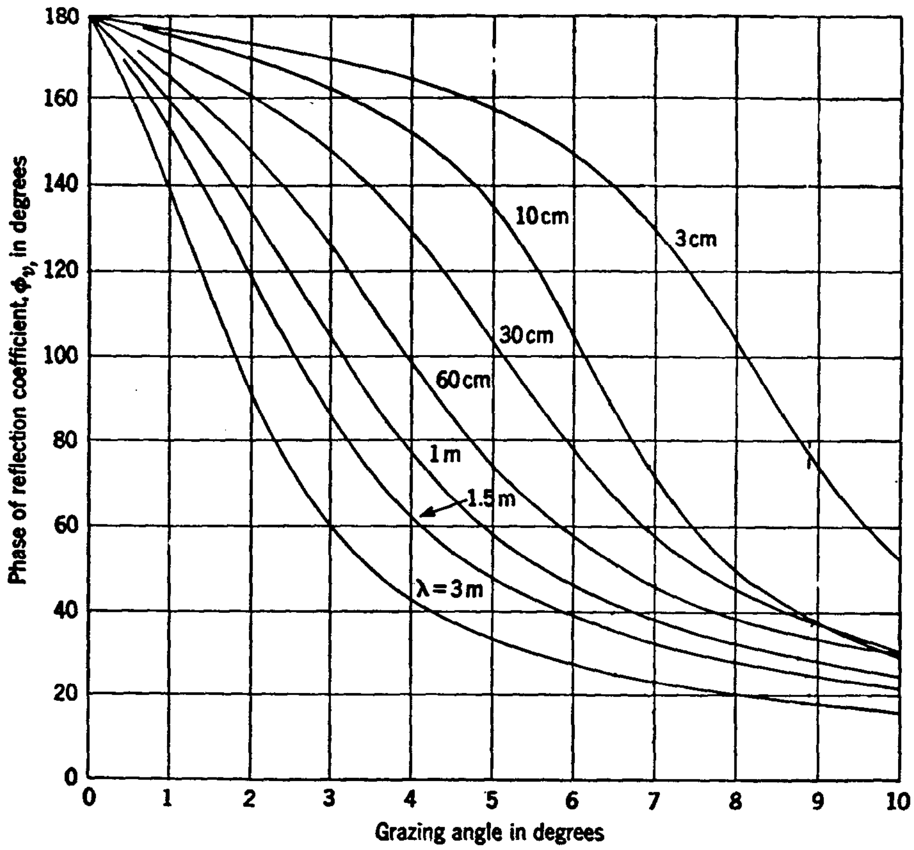

Fig. 8 - Phase angle of the reflection coefficient as a function of grazing angle for a smooth sea and vertical polarization, for several radar wavelengths from 3 meters (frequency 100 Mc) to 3 cm (10,000 Mc); Fig. 5.5 of Ref. 8. Ordinates may be identified with angle γ in Eq. (29) when the radar target is at a positive elevation angle and the range is much greater than the radar antenna height.

$$F = 2\ f(\theta) \left| \sin\left(\frac{2\pi\ h \sin\theta}{\lambda}\right)\right|, \tag{34}$$

which may also be written

$$F = 2\ f(\theta)\ |\sin(0.366\ h_{ft}\ f_{Mc} \sin\theta)^\circ|, \tag{35}$$

where h_{ft} is antenna height in feet and f_{Mc} is the radar frequency in megacycles. The angle in large parentheses in Eq. (34) is in radians, and in Eq. (35) it is in degrees.

Thus F oscillates as θ increases, with $F = 0$ (nulls or minima) when

$$\sin\theta_{min} = \frac{n\lambda}{2h} = \frac{492\ n}{f_{Mc}\ h_{ft}}, \qquad n = 0, 1, 2, 3, \ldots \tag{36}$$

and $F = 2$ (maxima, or lobe centers) when

$$\sin\theta_{max} = \frac{(2n-1)\lambda}{4h} = \frac{246\,(2n-1)}{f_{Mc}\ h_{ft}}, \qquad n = 1, 2, 3, \ldots. \tag{37}$$

These formulas may be used with good accuracy at frequencies below about 300 Mc up to moderate elevation angles, and with smooth seas and at elevation angles below about a degree they are applicable up to considerably higher frequencies. However, for more accurate calculation, Eq. (32) (and, when applicable, Eq. (33)) must be used, taking into account the effects of the surface reflection coefficient ρ and the divergence factor D. Moreover, if the antenna beam is not symmetrical with respect to the horizon, then Eq. (27) or (27a) must be used rather than Eq. (32).

When any one of the factors ρ, D, $f(\theta_1)$, or $f(\theta_2)$ is less than unity, the value of F at the maxima will be less than 2, and in the minima it will be greater than zero except for the special case in which $f(\theta_1) = \rho D f(\theta_2)$. However, in these cases the maxima and minima will still occur at the angles given by Eqs. (36) and (37). The values of F at the maxima and minima will be

$$F_{max} = f(\theta_1) + \rho D f(\theta_2) \tag{38}$$

$$F_{min} = f(\theta_1) - \rho D f(\theta_2). \tag{39}$$

Divergence Factor (D)

The divergence factor expresses the weakening of the reflected field that occurs because the reflecting surface is slightly spherically convex rather than truly flat. This effect is of importance only when the antenna is quite distant from the reflection point, so that for the low antenna heights considered here, it is important only at quite low grazing angles (small values of θ). Kerr* gives the following approximate formula for D in terms of θ, applicable when the radar antenna height is moderate (less than 1000 feet) and the target range is much greater than the antenna height:

$$D \cong \left[\frac{1}{3}\left(1 + \frac{2x}{\sqrt{x^2+3}}\right)\right]^{1/2}. \tag{40}$$

The parameter x is given by

$$x = 3.734 \times 10^3 \left(\frac{\tan\theta}{\sqrt{h}}\right), \tag{41}$$

where h is the radar antenna height in feet, and θ is the target elevation angle. Equation (40) is quite accurate for $\theta \gtreqqless 0.5$ degree and is fairly accurate to $\theta = 0$ degree; however, for $x < 0$ (target elevation angle negative), Eq. (40) is not accurate. Kerr also gives a correction formula (Ref. 8, p. 138) and curves for improving the value of D computed from Eq. (40) when θ is small (e.g., less than 0.5 degree), as well as exact formulas (Ref. 8, pp. 114 and 404-406). Figure 9 is a plot of D as a function of x.

Reflection Coefficient of the Sea (ρ)

The sea is the most common reflector that is considered in computing F by the formulas that have been given. At low or moderate frequencies and elevation angles, the sea surface behaves like a smooth flat reflector of radio waves, but the roughness of the surface seriously reduces the reflection as the elevation angle of the target and the frequency increase. Land may also be a good reflector when it is moist and smooth, and the formulas that have been given for computing F may often be applied for land reflection,

*Reference 8, pp. 137 ff. The method is attributed to R. A. Hutner et al., as originally published in MIT Radiation Laboratory Report No. 23, Sept. 28, 1943, pp. 31-33.

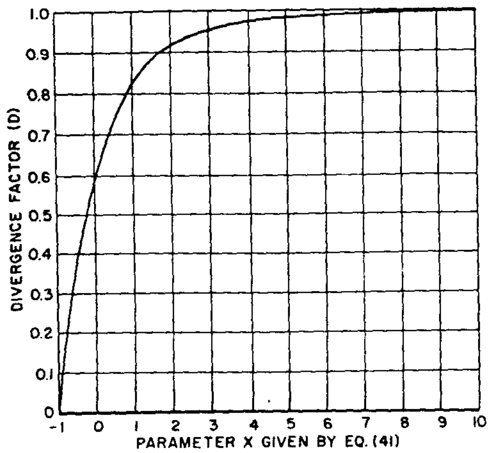

Fig. 9 - Divergence factor D as a function of parameter x defined by Eq. (41); Fig. 2.25 of Ref. 8. Values are valid for moderate antenna height (< 1000 feet), target range much greater than antenna height, and positive elevation angles, with some error below about 0.5 degree. For more accurate values see Ref. 8, pp. 137-138, Fig. 2.26.

especially at low angles and low frequencies. At microwave frequencies and at high angles, however, land is ordinarily a poor reflector, and $\rho = 0$ is a good approximate assumption to make.

Analyses of reflection by land and by water are given by Norton and Omberg (9) and Kerr (8). The detailed analysis of land reflection is too complicated for the scope of this report, but the sea surface, even when moderately rough, will be considered in some detail because it is of great importance, and because it is more readily analyzable, although the case of the rough sea can only be treated very approximately.

The analysis of the variation of smooth-sea reflection coefficient, ρ_0, for vertically and horizontally polarized radio waves, is given by Kerr (8) and the results are summarized in Figs. 10 and 11.

When the sea is rough (as it virtually always is), it may nevertheless behave as a smooth reflector at low grazing angles and low frequencies. Rayleigh's criterion of optical roughness states that a surface will reflect essentially specularly if the following relation holds (19):

$$\delta h \sin \theta < \lambda/16 , \tag{42}$$

where δh is the maximum height difference between high and low points of the surface and θ is the grazing angle of the ray of wavelength λ. Actually the $\lambda/16$ value is somewhat arbitrary; it does not define a precise distinction between "smooth" and "rough" surfaces. It defines a transition region between purely specular (smooth surface) and purely diffuse (rough surface) reflection.

Even when the roughness of a surface (in terms of its physical configuration or statistics) can be specified, the computation of the degree of specularity of the reflection is a formidable problem. In the practical case, sea roughness varies so greatly with the wind and other factors that it would be useless to attempt precise calculation of the value of ρ at any given time. A statistical description of ρ for the range of sea states typically encountered would be useful, but complete information does not exist. Therefore it seems justifiable to devise some arbitrary convention, conforming to the applicable boundary conditions and to the approximate knowledge that is available. An attempt to do this has been made, by the author, based generally on Rayleigh's criterion. Burrows and Attwood (19) state that "experience has shown that when the differences in level that constitute roughness are of the order indicated (by Rayleigh's criterion), the reflection coefficient is reduced to ... (about one-fifth) of the value calculated for an ideal surface."

To devise the convention for calculating ρ, a sea of 6-foot wave height was assumed, corresponding to moderate roughness. In applying Rayleigh's criterion, the effect of "shadowing" was considered. That is, a ray of small grazing angle cannot be reflected from the lowest parts of a surface viewed at right-angles to the waves because the troughs of the waves are shadowed by the crests, therefore the effective height-difference δh is less than the full geometric height difference, and is a function of the grazing angle, the

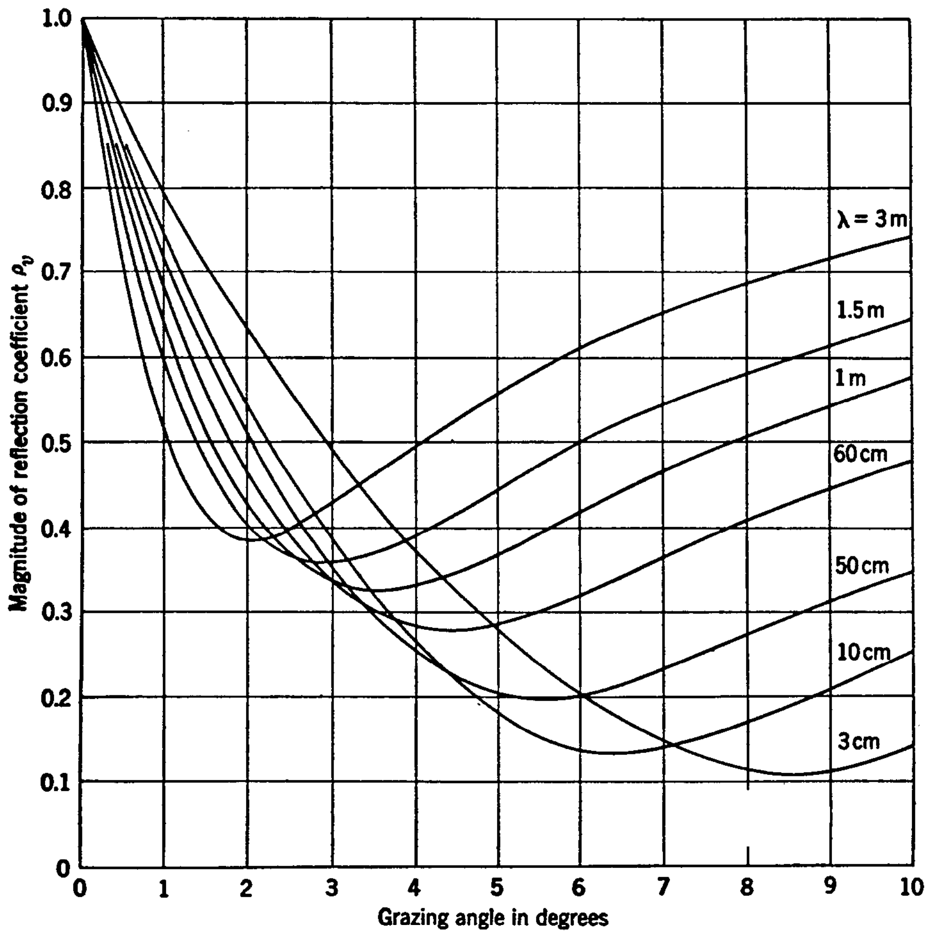

Fig. 10 - Magnitude of the reflection coefficient as a function of grazing angle for a smooth sea and vertical polarization, for several radar wavelengths from 3 meters (100 Mc) to 3 cm (10,000 Mc); Fig. 5.4 of Ref. 8

aspect (azimuth) with respect to the direction of the sea waves, and the (sea) wavelength. According to data on sea waves, the average length of waves of 6-foot height is about 50 feet.

A formula for the effect of roughness on reflection coefficient should express its reduction relative to the smooth-sea value; that is, it should be in terms of the ratio ρ/ρ_0. The known boundary conditions are that $\rho/\rho_0 \to 1$ as the product of the frequency and the grazing angle approaches zero, and $\rho/\rho_0 \to 0$ as the product of the frequency and the grazing angle becomes very large. These conditions are met by an equation of the form

$$\rho/\rho_0 = \frac{k}{k + A(f,\theta)}, \tag{43}$$

where k is a suitable constant and $A(f,\theta)$ is a positive and monotonically increasing function of frequency f and grazing angle θ. A constant and function that meet these requirements, and conform quite well to Burrows and Attwood's statement that $\rho/\rho_0 \cong 0.2$ when the equality sign holds in Eq. (42), are $k = 25$ and

$$A = 0.011\ f_{Mc}^2 \sin^2\theta \exp(0.05\ \theta°). \tag{44}$$

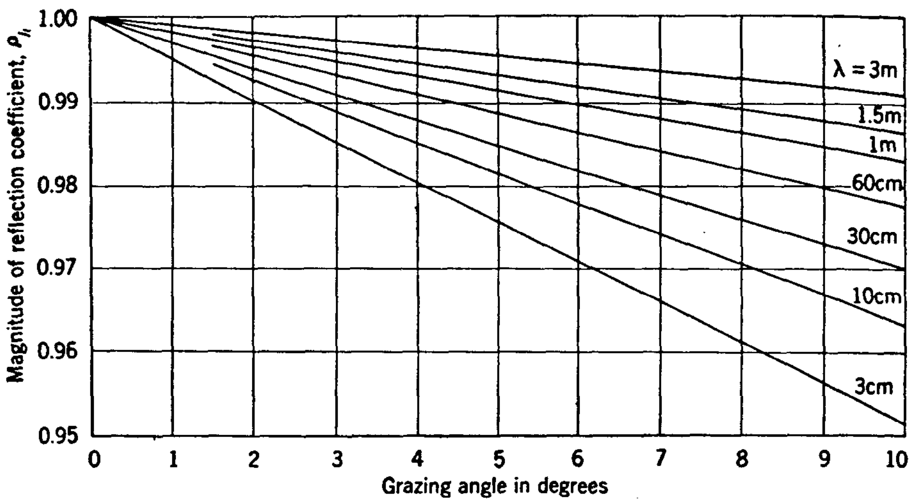

Fig. 11 - Magnitude of the reflection coefficient as a function of grazing angle for a smooth sea and horizontal polarization, for several radar wavelengths from 3 meters (frequency 100 Mc) to 3 cm (10,000 Mc); Fig. 5.6 of Ref. 8

In this expression, f_{Mc} denotes frequency in megacycles, and $\theta°$ denotes grazing angle in degrees.

Figure 12 is a plot of Eq. (43) for $k = 25$ and the form of A given by Eq. (44). It has been found to be in general agreement with experiment at a few points where experimental data are available, in addition to giving values that are generally thought to be "reasonable." Therefore Eqs. (43) and (44) are suitable as a convention or standard for calculating radar range when sea-reflection is a factor, until improved statistical data or calculations permit devising a better one. Details of the basis of Eq. (43), and discussion of the work of others on this topic, will be given in Part 2.

Transmitter Power (P_t) and Pulse Length (τ)

The product $P_t \tau$ represents the pulse energy, which is the time integral of the pulse power envelope, at the transmitter output terminals. The pulse power is

$$P_t = \frac{1}{\tau} \int_{-T/2}^{T/2} W(t)\, dt\,, \tag{45}$$

where t is time, T is the pulse period, and $W(t)$ is the pulse power envelope, excluding any nonuseful portions such as spikes and tails.

The pulse power P_t and the pulse length τ must be defined in such a way that their product is the pulse energy. It is evident that any definition of τ will give correct results if the same definition is used in Eq. (45) and in the range equation. The customary definition is the duration of the pulse between half-power points of the envelope. (This definition is also used in connection with evaluating the bandwidth-correction factor C_B. There also, the basis of definition is arbitrary, subject only to rules of consistency.)

Ordinarily pulse power is measured by measuring average power and dividing this quantity by the duty factor, which is the product of pulse length and pulse rate. The basis of definition of τ used in this method of pulse-power determination must of course also be the same one that is used in the range equation.

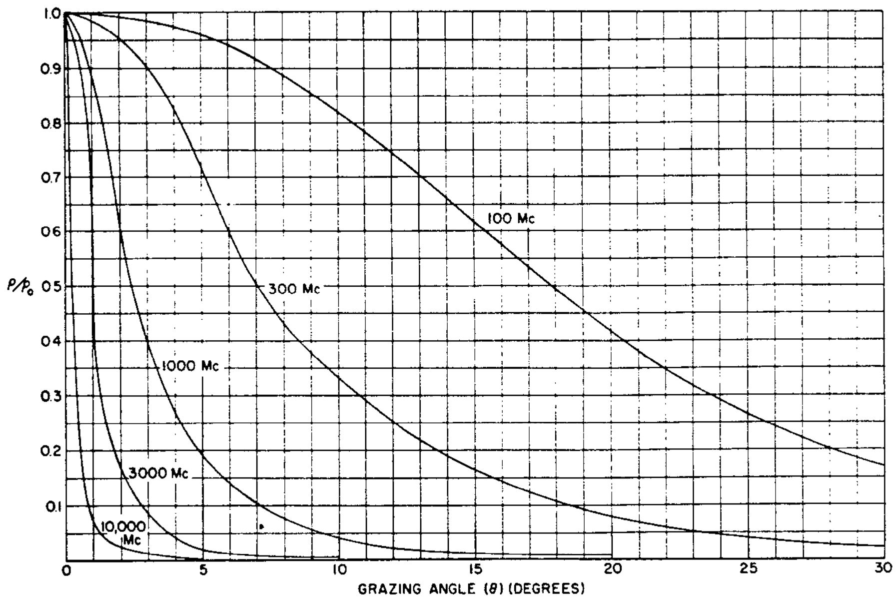

Fig. 12 - Ratio of the magnitude of the reflection coefficient (ρ) to the smooth-sea value (ρ_0) as a function of grazing angle θ and frequency f_{Mc}, calculated from Eq. (43) with k = 25 and A as given by Eq. (44)

For some purposes — e.g., analyzing the range resolution or accuracy of the radar as a function of pulse length — arbitrary definition of the pulse length is not permissible. In fact the half-power definition is not always a good one for this purpose. The problem of formulating a suitable definition for such purposes is not within the scope of this report (but it is in order to mention the limited applicability of arbitrary pulse-length definition).

Antenna Gain (G)

The antenna gains G_t and G_r are both defined, on a transmitting basis, as the ratio of the power density radiated in the maximum direction to that, at the same range, of an isotropic antenna radiating the same total power. If the antenna gain is measured in terms of power input rather than power radiated, the resulting figure must be increased by the ratio of the input power to the radiated power to preserve compatibility with the definitions of the loss factor L and the noise temperature T_N. If the antenna gain is not known by direct measurement, it may be estimated by two different methods. These are not exact gain formulas, but in the absence of more precise information they are useful. In the case of antennas characterized by a large plane "aperture" (arrays, reflectors, lenses), the formula is

$$G = k_1 \left(\frac{4\pi A}{\lambda^2} \right) = k_1 \left(1.3 \times 10^{-5} A_{sf} f_{Mc}^2 \right), \tag{46}$$

where the symbols are defined as follows:

A - actual area of aperture (subscripts sf denote square feet)
λ - wavelength, same units as $A^{1/2}$
f_{Mc} - frequency, megacycles
k_1 - aperture efficiency factor.

Generally, k_1 ranges from about 0.6 to 0.9 for well-designed antennas of conventional beam shape. For a horn-fed paraboloidal reflector it is typically about 0.65. For a linear-array-fed parabolic cylinder, more than twice as long as it is wide (or high), $k_1 \cong 0.7$. For a Dolph-Tchebyscheff-tapered dipole array, $k_1 \cong 0.85$.

Gain may also be estimated for narrow-beam antennas, if the beamwidths are known, from the formula

$$G = \frac{k_2}{\theta_h \, \theta_v}, \tag{47}$$

where θ_h and θ_v are the horizontal and vertical half-power beamwidths, in degrees. The constant k_2 is about 27,000 for a horn-fed paraboloidal reflector, about 30,000 for a linear-array-fed long parabolic cylinder, and about 41,000 for a Dolph-Tchebyscheff array.

Antenna Beamwidth

Antenna beamwidth directly affects range calculation through its effect on number of pulses integrated for a scanning radar. The general formula relating beamwidth to antenna size is

$$\theta_i = k_3 \frac{\lambda}{d_i} = \frac{984 \, k_3}{d_{i(ft)} \, f_{Mc}}, \tag{48}$$

where θ_i is the beamwidth in degrees, in the direction of the i-dimension of the antenna, d_i is the antenna dimension, λ is the wavelength, and f_{Mc} is the frequency in megacycles. For paraboloidal or parabolic-cylinder antennas, k_3 has values ranging from about 60 to 75, for half-power definition of the beamwidth. Equations (47) and (48) assume "conventional" antenna designs and beam shapes, and do not apply to antennas having "cosecant-squared" or other specially shaped patterns.

For a plane reflector and plane array of the Dolph-Tchebyscheff type of beamwidth less than 10 degrees, Eq. (48) applies if the width d is measured between the centers of the end dipoles of the array, and if the following values of k_3 given by Stegen (20)* are used for various design values of the side-lobe level:

Side-Lobe Level (db)	k_3
-20	51.1
-25	56.0
-30	60.6
-35	65.0
-40	68.7

System Loss Factor (L) and Principal Component Losses

Loss factor is defined as the ratio of the power input to the power output of the lossy element of the system (i.e., reciprocal of gain). The general loss factor L is the product of numerous specific loss factors, certain ones of which are generally present. (L_{db} is of course the sum of the component loss factors expressed in decibels.)

*The beam direction is assumed to be normal to the array. Formulas are also given in this paper for the beamwidth of squinted arrays and for arrays of beamwidth greater than 10 degrees.

In terms of the losses that are generally present, with an added factor to account for miscellaneous additional losses, L may be expressed as

$$L = L_t L_r L_p L_a L_x . \tag{49}$$

The factor L_t is the transmission-line loss that occurs during transmitting. It includes all losses between the transmitter output terminals and the radiating surfaces of the antenna. Thus, duplexer loss, losses in joints and couplers, and ohmic losses in the antenna itself are included. (Antenna losses are included because the antenna gain definition used is the directive gain, in terms of radiated power rather than power at the input terminals of the antenna.)

The factor L_r is the transmission-line loss on reception, analogously defined (but not necessarily equal to L_t). However, L_r is the so-called available loss,* or ratio of available power at the antenna to available power at the receiver input terminals, whereas L_t is the actual loss (ratio of actual transmitter output power to power radiated by the antenna). Available power is that which would be delivered to a matched load impedance (complex conjugate of source impedance).

The factor L_p, the antenna-pattern loss, accounts for the fact that the gain of a scanning antenna, in the target direction, varies from pulse to pulse in accordance with the antenna pattern, while the antenna gain factors in the equation are applicable to targets in the beam maximum. It also takes into account the arbitrary designation of beamwidth, for the purpose of counting the number of pulses integrated, as the half-power value. Analysis of these matters (21) indicates that a loss factor of 1.45 (1.6 db) is appropriate in the case of unidirectionally scanning radar. For a bidirectionally scanning radar, the problem of analyzing the loss is more complicated. It probably depends on the particular scanning pattern employed. In the absence of a more accurate analysis of a specific case, this factor may be estimated to be the square of the loss factor for a unidirectional scan — i.e., 2.1 (3.2 db).

For a nonscanning radar, $L_p = 1$, $(L_{p(db)} = 0)$. If the target is not in the beam maximum for such a radar, appropriate correction should be made in the pattern-propagation factor F. In the case of a unidirectionally scanning radar, if the target is displaced from the beam maximum in the direction orthogonal to the scanning direction, this should also be taken into account by the pattern-propagation factor.

The term scanning loss has been used to mean many different things. Most commonly it has referred to the reduction of radar sensitivity that results when the antenna beam is scanned instead of remaining fixed (searchlighting) on a target. This loss is a function of the scanning speed and the antenna beamwidth. It is primarily the result of the reduced number of pulses received during the integration time, and is automatically taken into account when the minimum detectable signal-to-noise ratio, $V_{o(50)}$ or D_{50}, is determined, from Fig. 2 or 4, on the basis of number-of-pulses integrated as computed from Eq. (16) or (17). Hence no additional scanning loss need be introduced into the calculation. Sometimes the antenna pattern loss L_p has been called a scanning loss. This is not an inappropriate label, but it is not used here because of the confusion that might result.

L_a in Eq. (49) represents the loss due to absorption in the propagation medium. L_x represents miscellaneous further losses that may occur in some applications. Among the possibilities are collapsing loss, sweep-speed loss, video mixing loss, video-bandwidth loss, pulse-length loss (due to finite excitation time of certain types of array antennas),

*The necessity for this definition of L_r was called to the author's attention by L. E. Davies of Stanford Research Institute.

and polarization angle loss (due to polarization rotation by the ionosphere). These losses will be discussed in detail in Part 2.

Propagation Absorption Loss (L_a)

The factor L_a is for loss due to absorption in the propagation medium. Curves for this loss in the atmosphere as a function of target range and elevation angle are given by Figs. 13-24 for several elevation angles up to 10 degrees. These are similar to curves previously published (1,2,5,7), but have subsequently been recalculated using slightly improved values of some of the quantities involved in the calculation. Also, the original calculations were manually performed, but the recalculation was done using the NRL NAREC electronic digital computer. The new results are in good agreement with the original calculations, differing very slightly at some frequencies and some elevation angles due to the changed values of certain factors in the calculations.

As discussed in the original report presenting these curves (Ref. 2, pp. 6, 11), it was realized after the original calculations were made that probably certain values employed were not the best choices, although the differences in computed results would not be great if improved values were used. Because of the labor of the manual calculations, recomputation was not done at that time.

For the recent calculation by machine, the line-breadth constant for the oxygen attenuation was taken to be Van Vleck's originally proposed value, $\Delta\nu = 0.02\ \text{cm}^{-1}$ (instead of the two different values, $(\Delta\nu)_1 = 0.018$ and $(\Delta\nu)_2 = 0.05$, used previously). The value of the water-vapor line-breadth constant $(\Delta\nu)_3$ was taken to be 0.1 as it was before, but $(\Delta\nu)_4$ was taken to be 0.27 instead of 0.1. The sea-level value of water-vapor density was

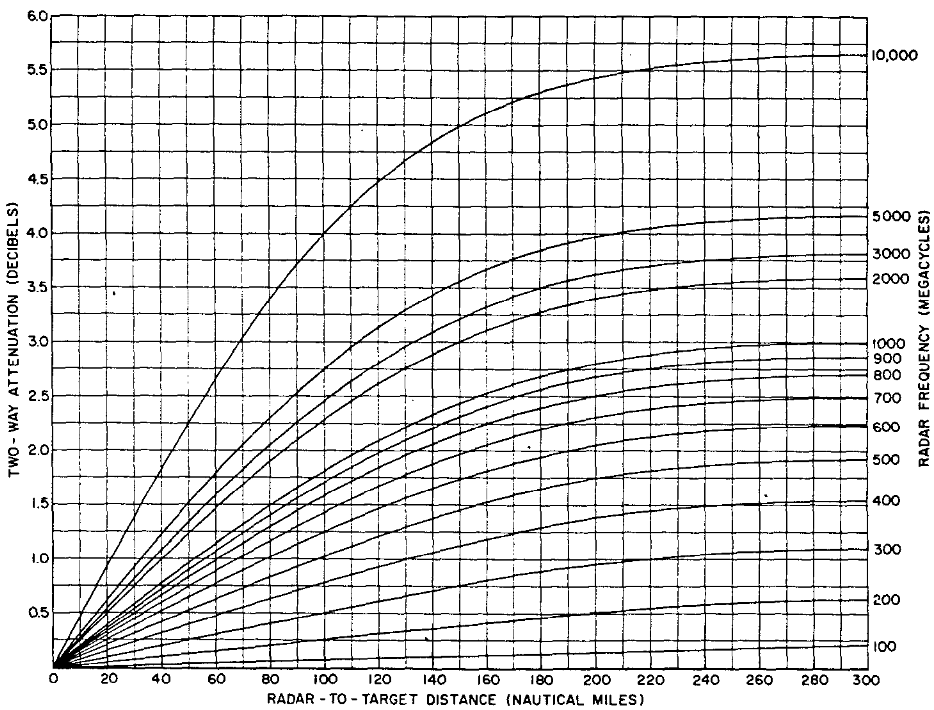

Fig. 13 - Radar atmospheric attenuation, 0-degree ray elevation angle, 100-10,000 Mc

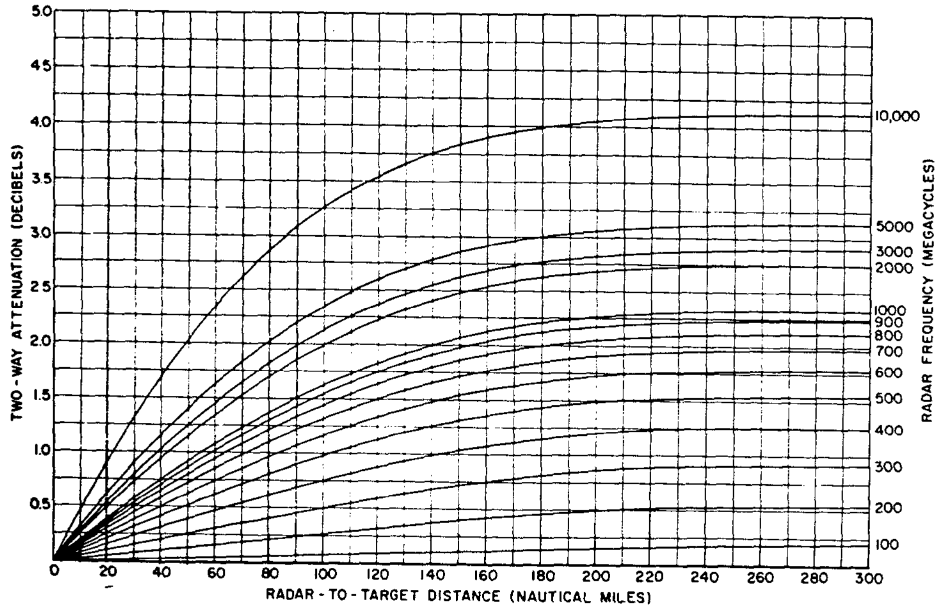

Fig. 14 - Radar atmospheric attenuation, 0.5-degree ray elevation angle, 100-10,000 Mc

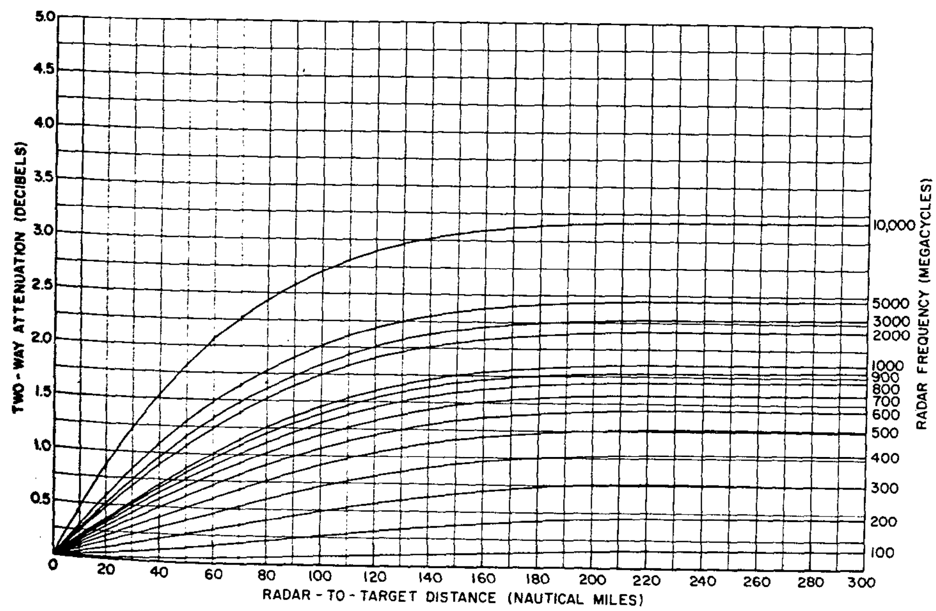

Fig. 15 - Radar atmospheric attenuation, 1.0-degree ray elevation angle, 100-10,000 Mc

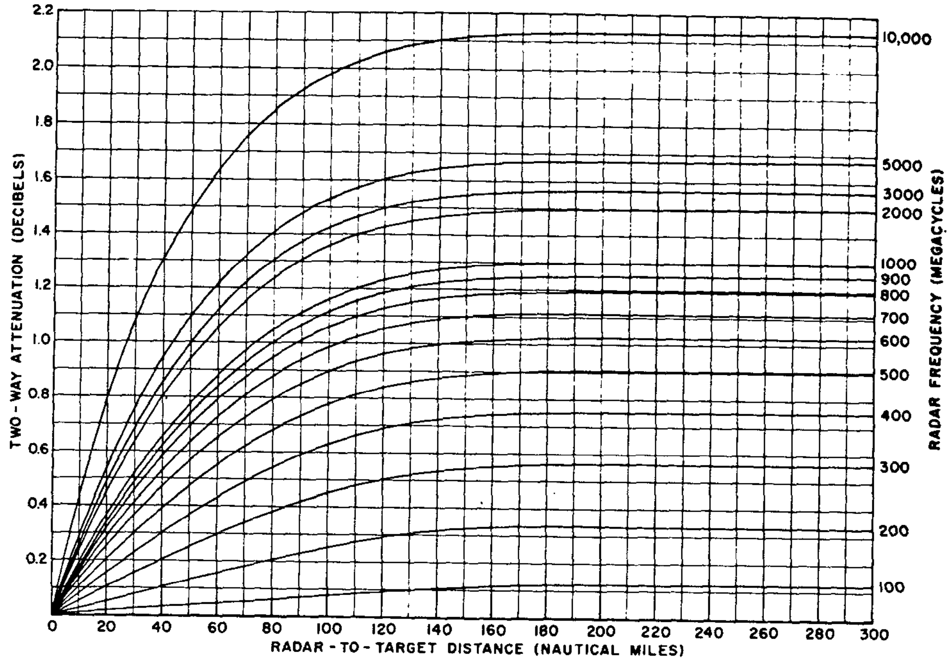

Fig. 16 - Radar atmospheric attenuation, 2.0-degree ray elevation angle, 100-10,000 Mc

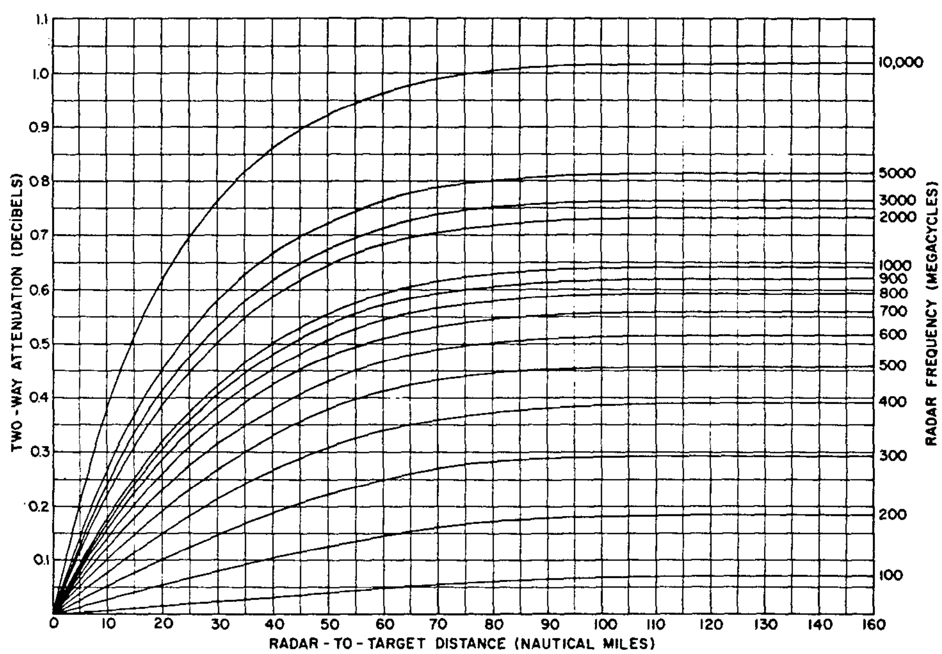

Fig. 17 - Radar atmospheric attenuation, 5.0-degree ray elevation angle, 100-10,000 Mc

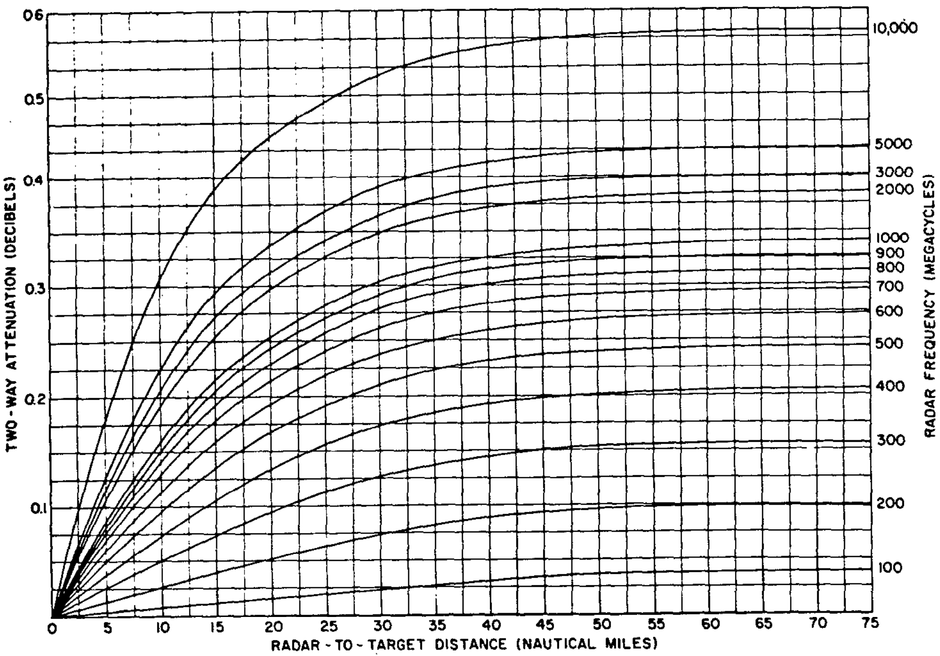

Fig. 18 - Radar atmospheric attenuation, 10-degree ray elevation angle, 100-10,000 Mc

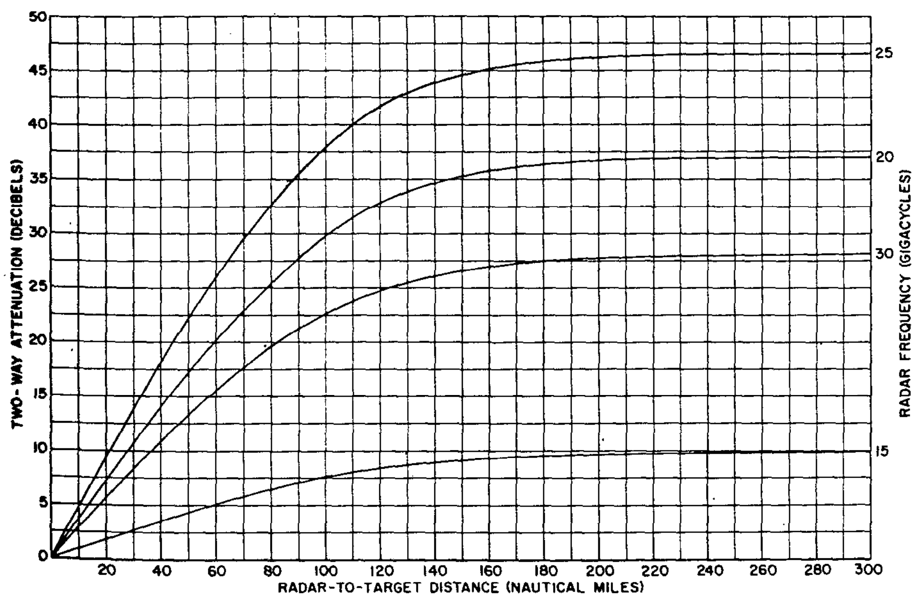

Fig. 19 - Radar atmospheric attenuation, 0-degree ray elevation angle, 15-30 Gc

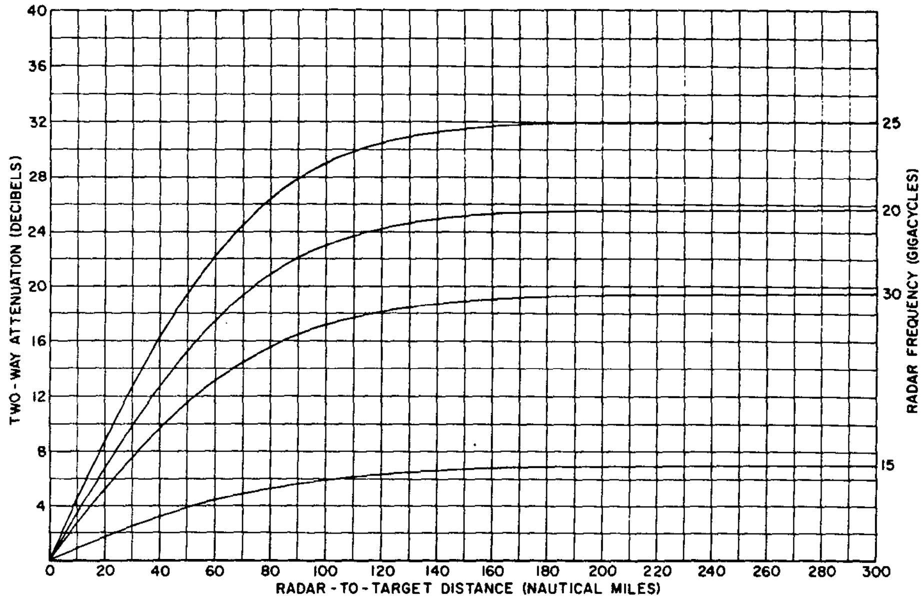

Fig. 20 - Radar atmospheric attenuation, 0.5-degree ray elevation angle, 15-30 Gc

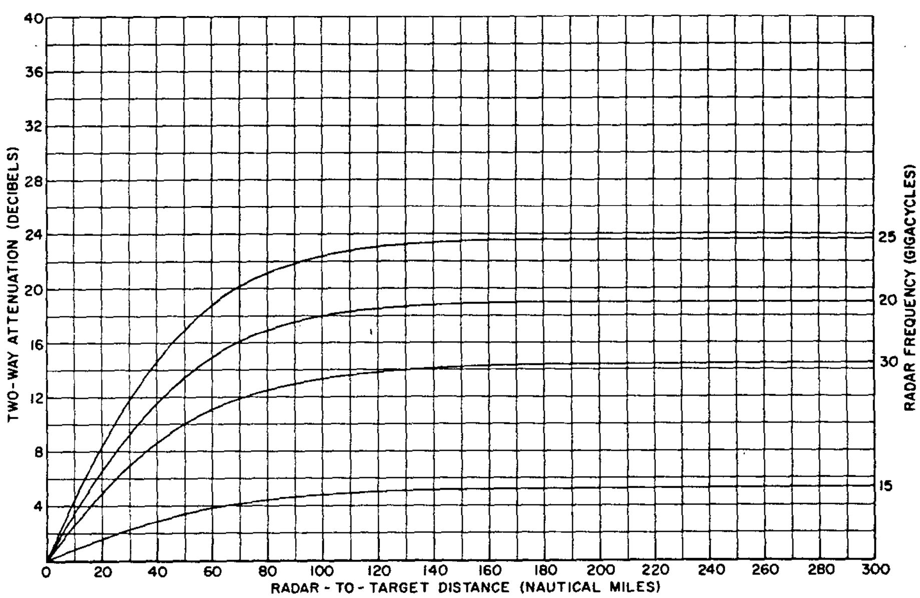

Fig. 21 - Radar atmospheric attenuation, 1.0-degree ray elevation angle, 15-30 Gc

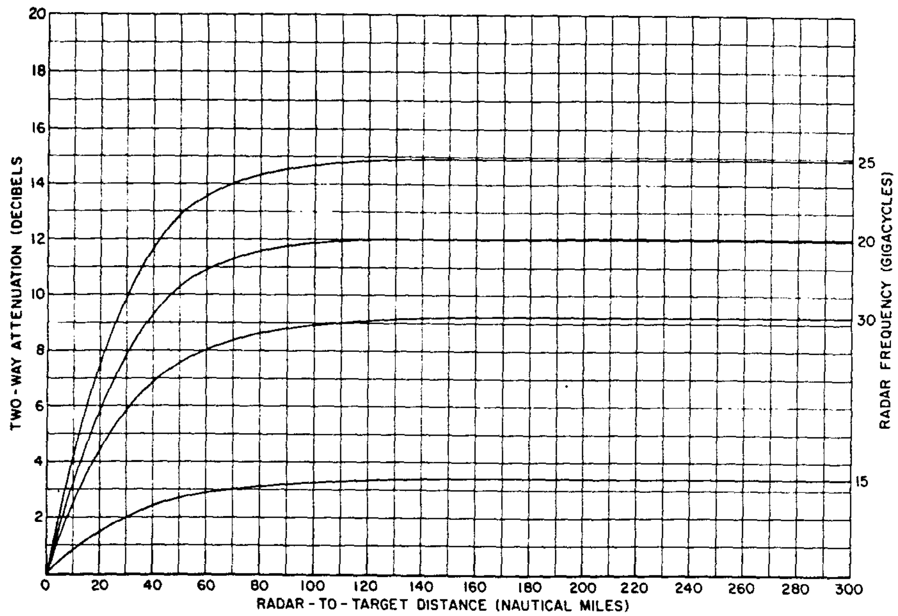

Fig. 22 - Radar atmospheric attenuation, 2.0-degree ray elevation angle, 15-30 Gc

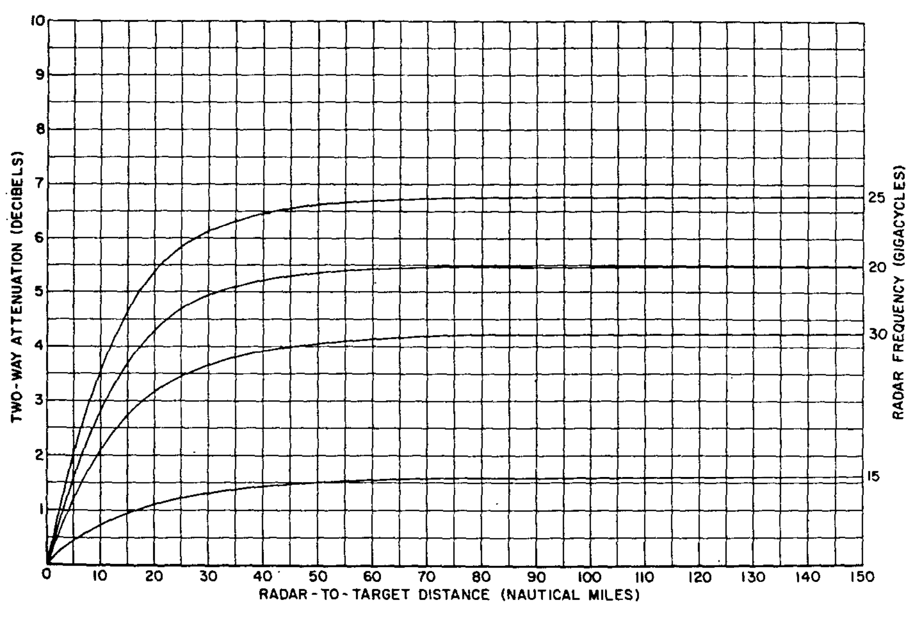

Fig. 23 - Radar atmospheric attenuation, 5.0-degree ray elevation angle, 15-30 Gc

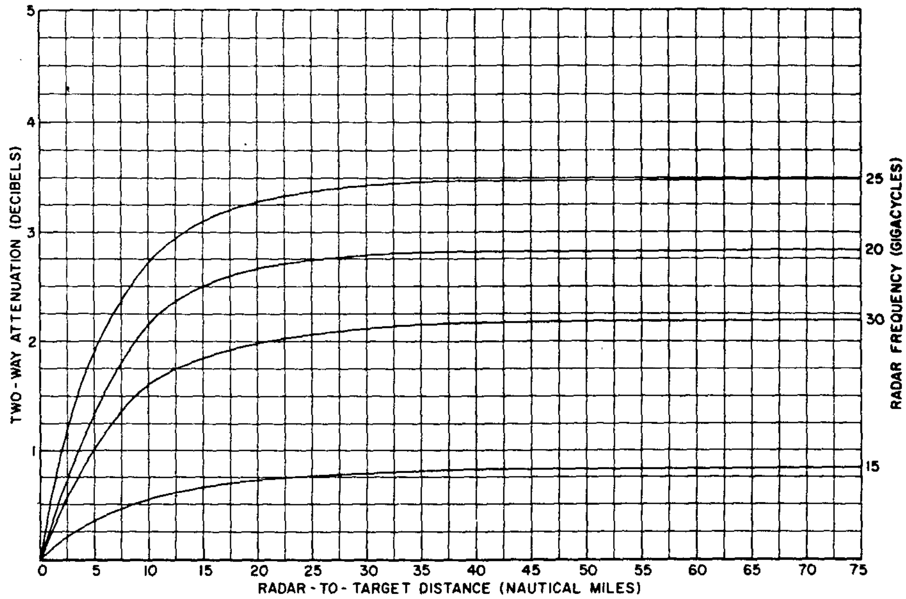

Fig. 24 - Radar atmospheric attenuation, 10-degree ray elevation angle, 15-30 Gc

taken to be 7.5 grams per cubic meter instead of 6.18. The atmospheric pressure and temperature values were taken from the ICAO Standard Atmosphere (22). The ray-path range-height-angle values were based on the CRPL Exponential Reference Atmosphere (a refractive-index model) (3,23) with a surface refractivity $N_s = 313$. The result of all these slight modifications is to give very slightly smaller attenuation values at the lowest frequencies and slightly greater values at the highest frequencies. For most practical purposes the differences are negligible.

The calculations were carried out for a greater range of initial ray angles and frequencies than before, including additional frequencies within the original range of 100-10,000 Mc, and four frequencies above it — 15, 20, 25, and 30 Gc. Inasmuch as the absorption at these higher frequencies is so sensitive to water vapor, the values shown in Figs. 19-24 should be taken as only a rough guide to the losses above 10,000 Mc. It is important to note that the frequencies 20 and 25 Gc lie on either side of the water-vapor absorption resonance at 22.2 Gc. As indicated in Fig. 25, the absorption at this resonance is much higher than it is at either 20 or 25 Gc, therefore interpolation for absorption values between these two frequencies is not possible. Figure 25 is a plot of the attenuation values for two-way transit of the entire atmosphere, corresponding to the maximum-range values shown in Figs. 13-24. Since the absorption at exactly the water-vapor resonance frequency, 22.2 Gc, was not calculated, the curves are left broken in that region. (The absorption at the resonance frequency, though large, is finite.)

Values have also been calculated for elevation angles of 30 and 90 degrees. Since at these angles the entire atmosphere is traversed in a relatively short distance and the computer was programmed to give attenuation values at 10-mile intervals, these results are tabulated rather than plotted, in Table 1. (Although the calculated values are given to three significant figures, the results do not actually have that degree of absolute physical significance.)

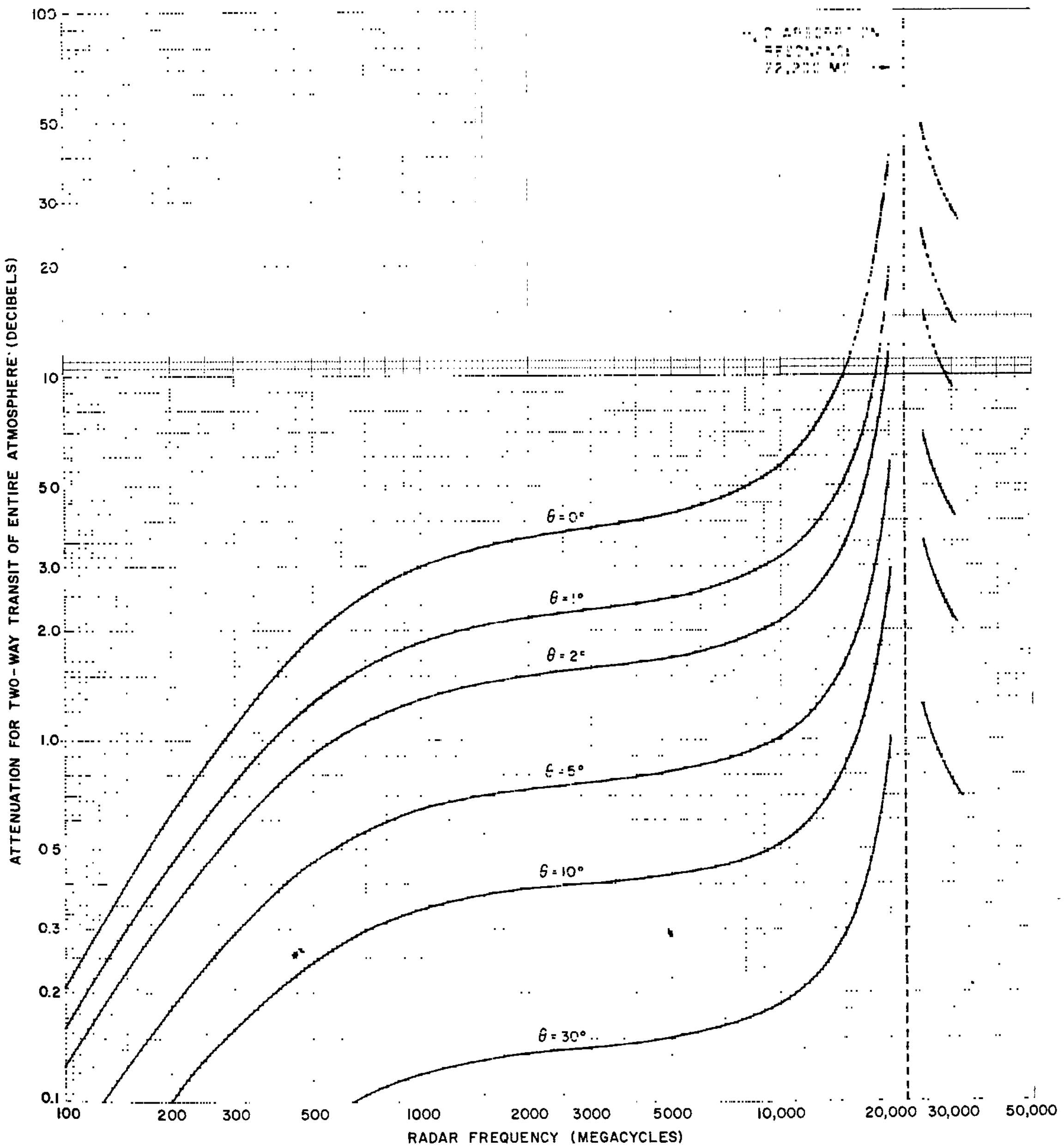

Fig. 25 - Radar atmospheric attenuation for targets outside the atmosphere, not including ionospheric attenuation which may occur below 500 Mc

Since the absorption loss depends on the range, it is necessary first to calculate the range for no loss ($L_a = 1$), and then to apply a correction factor based on the loss determined for this range, from curves of the type given in Figs. 13-24. Further correction may be made, if necessary, by finding a revised loss value for the corrected range, and then correcting for the difference between the initial and revised losses. (Range-correction factors for losses in tenth-decibel steps are given in Appendix A, Table A3.) In general, the procedure need not be carried beyond this point to achieve negligible error compared to the indefinitely continued iterative procedure that would in principle give the "true" range.

Rain may cause greatly increased absorption losses at the higher frequencies, principally above 3000 Mc. However, rain is too variable a phenomenon to include as part of a "standard" range calculation. The atmospheric absorption losses given by Figs. 13-25 assume no rain. Nevertheless, in some parts of the world, at some seasons, absorption by rain may be quite important.* This subject is discussed by Kerr (8).

The maximum-range loss values plotted in Fig. 25 are applicable to radar targets that lie completely outside the atmosphere, provided they are not in or beyond the ionosphere. However, loss due to ionospheric absorption is not significant except at the very lowest frequencies considered in Fig. 25, and then only in the daytime. Nevertheless, in some applications it may be significant, for targets such as missiles, artificial satellites, space probes, or astronomical bodies. This absorption loss has

Table 1
Atmospheric Absorption Losses for 30- and 90-Degree Elevation Angles†

Frequency (Mc)	Two-Way (Radar) Attenuation (db)				
	$\theta = 30°$			$\theta = 90°$	
	R = 10	R = 20	R = 30	R = 10	R = 20
100	0.0077	0.0131	0.0138	0.0066	0.0070
200	0.0249	0.0347	0.0355	0.0174	0.0178
300	0.0432	0.0551	0.0559	0.0276	0.0281
400	0.0591	0.0720	0.0728	0.0361	0.0366
500	0.0719	0.0854	0.0862	0.0428	0.0432
600	0.0819	0.0958	0.0966	0.0480	0.0484
700	0.0898	0.104	0.105	0.0520	0.0524
800	0.0958	0.110	0.111	0.0551	0.0555
900	0.101	0.115	0.116	0.0575	0.0580
1,000	0.104	0.119	0.120	0.0593	0.0597
2,000	0.121	0.135	0.136	0.0677	0.0681
3,000	0.126	0.141	0.142	0.0705	0.0710
5,000	0.135	0.150	0.151	0.0750	0.0754
10,000	0.171	0.187	0.187	0.0934	0.0939
15,000	0.275	0.291	0.292	0.146	0.147
20,000	0.970	0.989	0.990	0.495	0.496
25,000	1.20	1.22	1.22	0.611	0.611
30,000	0.735	0.765	0.768	0.384	0.384

† θ is the elevation angle; R is the radar range, naut. mi.

*Since this material was written, the need has arisen, in connection with a specific problem, to provide attenuation data for a "standard light rain" and a "standard heavy rain" for radar range-calculation purposes. This work will be reported in Part 2.

been computed for average conditions by Millman (24), for one-way propagation through the entire ionosphere. The loss is a function of elevation angle and also varies greatly between daytime and nighttime. In the daytime the loss at 100 Mc may be as great as 1.3 db (2.6 db for two-way radar propagation), while at night it is less than 0.1 db, one-way, for the worst case (small elevation angle). For high elevation angle the loss at 100 Mc is about 0.2 db daytime and 0.01 db nighttime, one-way. The decibel loss is proportional to the inverse square of the frequency. Thus, at frequencies above a few hundred megacycles ionospheric attenuation is ordinarily negligible.

Integration Loss and Operator Loss

The concept of "integration loss" is sometimes encountered in the literature of radar range calculation (12,25). In this approach, the quantity V_o (or some similar factor) is evaluated on the assumption that coherent integration of the N pulses has occurred. Then a loss factor is applied, accounting for the difference in value of V_o for coherent and noncoherent integration of the N pulses.

This approach is not used here. The values of $V_{o(50)}$ or D_{50} given by Figs. 2-4 are those for noncoherent integration, which is the form of integration ordinarily employed. For special cases in which coherent integration is possible, the appropriate modification of Fig. 4 has been described (Eq. (20)). The "integration loss" approach is deprecated by the author because it requires an additional factor and an additional step in the calculation, and (perhaps primarily) because of the implication that integration is a process that results in loss. Integration is of course a gainful process, and the term "integration loss" really means "the loss incurred by integrating noncoherently instead of coherently." "Noncoherent-integration loss" would be more appropriate, but more cumbersome. Presumably the intent of the concept is to emphasize the improvement that results with coherent integration, and to formulate the range equation for this "ideal" case, with a loss factor for departure from the ideal. Here the approach has been, following Norton and Omberg (9), to formulate the equation so that it applies directly to either the practical case or to the ideal case.

The concept of "operator loss" is also sometimes employed, to describe the increase in V_o required by a typical operator compared to an ideal operator. However, here again the approach has been to express V_o directly, in Figs. 1 and 2, as the value applicable to an actual human operator. The "operator loss" tends in practice to become an arbitrary factor to account for observed discrepancies between computed and observed radar performance, and while in some cases it may be a valid explanation, it may in other cases tend to be misused. In any case it is too vague a concept to employ in a range calculation aimed at evaluating the merit of a particular radar design, or for other engineering purposes.

System-Degradation Loss

Inclusion of a system-degradation loss factor in range calculations, as is sometimes done, is deprecated, for reasons similar to those just discussed. The inclusion of such a factor tends to discourage attempts to evaluate other range-equation factors as precisely as possible. There is little point in expressing other factors to the nearest tenth of a decibel when the "system degradation factor" cannot possibly be specified that closely, except arbitrarily. It may be argued that some of the other range equation factors are often not known very precisely either — notably, the target cross section and the pattern-propagation factor. However, these quantities at least have precise values in principle, which by improved measurement or theoretical techniques might be determined, sometimes statistically. But system degradation loss does not even have stationary statistical properties, and cannot ever be evaluated precisely.

Of course, the fact that "system degradation" exists cannot be ignored. The approach recommended here, however, is the use of values for system parameters in the range equation that are the most realistic possible values, not "laboratory peak" values but nevertheless representative of a properly maintained and adjusted system in the field or aboard ship. If specific components are known to deteriorate with time in a predictable way, it is appropriate to use a mean operating value for the recommended component lifetime. (Presumably the range of values thus permitted would not be great.) But, inclusion of a factor for deterioration due to poor maintenance is inappropriate in a range calculation in which an attempt is made to evaluate the physical factors as accurately as possible.

When the range has been calculated on the basis thus recommended, it is a simple matter, of course, to apply arbitrary degradation factors to it, if it is desired to note the effect that possible amounts of degradation would have. (The range factors of Table A3, Appendix A, are especially convenient for this purpose.)

If military or naval agencies wish to allow for an arbitrary amount of system performance degradation in system performance specifications, it is recommended that they allow for this by upgrading the nominal range requirement rather than by stipulating inclusion of a degradation factor in the contractor's calculation of expected range of a proposed system. The latter practice may make it almost impossible to determine whether the delivered system complies with the specification.

Coverage Diagrams

For surface-based radar systems, to which the material of this report directly applies, the target elevation angle θ is an important parameter in the range calculation. Although it does not appear explicitly in the range equation it enters into the calculation of the system noise temperature T_N, the pattern-propagation factor F, and the absorption loss L_a. In certain types of elevation-scanning radars the antenna gain G, the number of pulses per beamwidth per scan N (which affects V_o), and even the frequency f_{Mc} may be functions of θ. Therefore the range of a surface-based radar generally varies with the target elevation angle, and accordingly a full description of a radar's maximum-range capability can in general only be given in terms of a coverage diagram, plotted for a vertical plane extending from $\theta = 0$ degrees to $\theta = 90$ degrees. In some instances, of course, the range is of interest primarily for a particular angle, or small range of angles, for example at or near $\theta = 0$ degrees. In such cases a coverage diagram is not necessary; a single range figure will suffice. But in general the diagram is required.

As is well known, radio rays are bent slightly downward in the earth's atmosphere, and this fact must be taken into account in plotting coverage diagrams, since it affects the range-height-angle relationship. The refraction of rays by the atmosphere is ordinarily very slight, but appreciably affects the altitude of low angle rays at practical radar detection ranges. The effect varies with the condition of the atmosphere, primarily the water-vapor content. Therefore it is necessary to specify a particular refractive condition of the atmosphere as standard for plotting radar coverage diagrams. The refractivity model suggested (3) is the CRPL Exponential Reference Atmosphere (23), with the surface refractivity value $N_s = 313$ (corresponding to the index of refraction 1.000313). The height dependence of the index according to this model is

$$n(h) = 1 + 313 \times 10^{-6} \exp(0.04385\,h), \tag{50}$$

where h, the height, is in thousands of feet. Given this dependence, the ray paths for various initial elevation angles may be calculated (3,23).

For radar coverage diagrams, a range-height-angle chart on which the rays are straight lines is desirable, to facilitate plotting the range as a function of the initial ray angle. Such a chart has been constructed for the refractivity model of Eq. (50), and is shown in Fig. 26. The method of constructing such a chart is described in an NRL Memorandum Report (26).

The range plotted in this chart is the "slant range" of the target (distance along the ray path from radar to target), not the ground range (distance along earth's surface). The heights and angles are plotted with respect to the position of the radar antenna. Therefore the actual target height above the earth's surface, for the ranges and angles indicated, would be the height shown on the chart plus the height of the antenna. In practical cases this will be a negligible correction, since the antenna height must be assumed to be low in using this chart. Moreover, it must be kept in mind that this chart is meant solely as a "convention," representing a rough approximation to the average results that would be obtained for a wide range of practical conditions, including various land altitudes as well as various atmospheric conditions. It should never be used as a basis for precision target altitude measurement from radar angle and range data. For this purpose charts based on the known altitude at the radar site should be used, and separate charts should be used for different seasons of the year and also for different conditions of the atmosphere during a given season. (Ideally, of course, a chart based on the measured refractivity profile should be used.) Details of this subject are contained in the literature (27-29) but are beyond the intended scope of this report.

A convention employed for determining the range-height-angle relationship when the antenna and target altitudes are both low is the so-called 4/3-earth's-radius principle. Actually this amounts to assuming that the atmospheric refractive index as a function of height has a linear negative gradient given by

$$\frac{dn}{dh} = -\frac{1}{4a}, \tag{51}$$

where a is the earth's radius. On a chart in which the geometry is distorted so that the earth's surface has a radius of curvature 4/3 as great as its true radius, the ray paths for this assumption plot as straight lines. The range-height-angle relationship for this convention is given by

$$H = h + 6076\, R \sin\theta + 0.6624\, R^2 \cos^2\theta, \tag{52}$$

where H is the target height and h is the antenna height, both in feet, R is the target slant range in nautical miles, and θ is the target elevation angle. This expression is based on an assumed true earth's radius of 3440 nautical miles, or a 4/3-earth-radius of 4587 nautical miles.

The limits of usefulness of this expression are about H = 10,000 feet and R = 100 miles at low angles. It is a useful formula for plotting ranges and heights that are too small to be plotted on Fig. 26. For ranges and heights that are too large for Fig. 26, the values of Table A4, Appendix A, may be used. This table includes the values used in plotting Fig. 26, but extends them to a height of 1,000,000 feet (165 nautical miles) and a slant range of 1120 miles at θ = 0 degree. When the ray height is in or beyond the ionosphere (which begins at about 250,000 feet), these values are valid only at frequencies high enough to be unaffected by ionospheric refraction, above about 500 Mc. Reference 24 discusses the refractive effect of the ionosphere at lower frequencies.

Sometimes it is of interest to determine the "horizon distance" of a radar, for a given antenna height h. In terms of the 4/3-earth's-radius convention this is given by

$$R_h = 1.23\sqrt{h}, \tag{53}$$

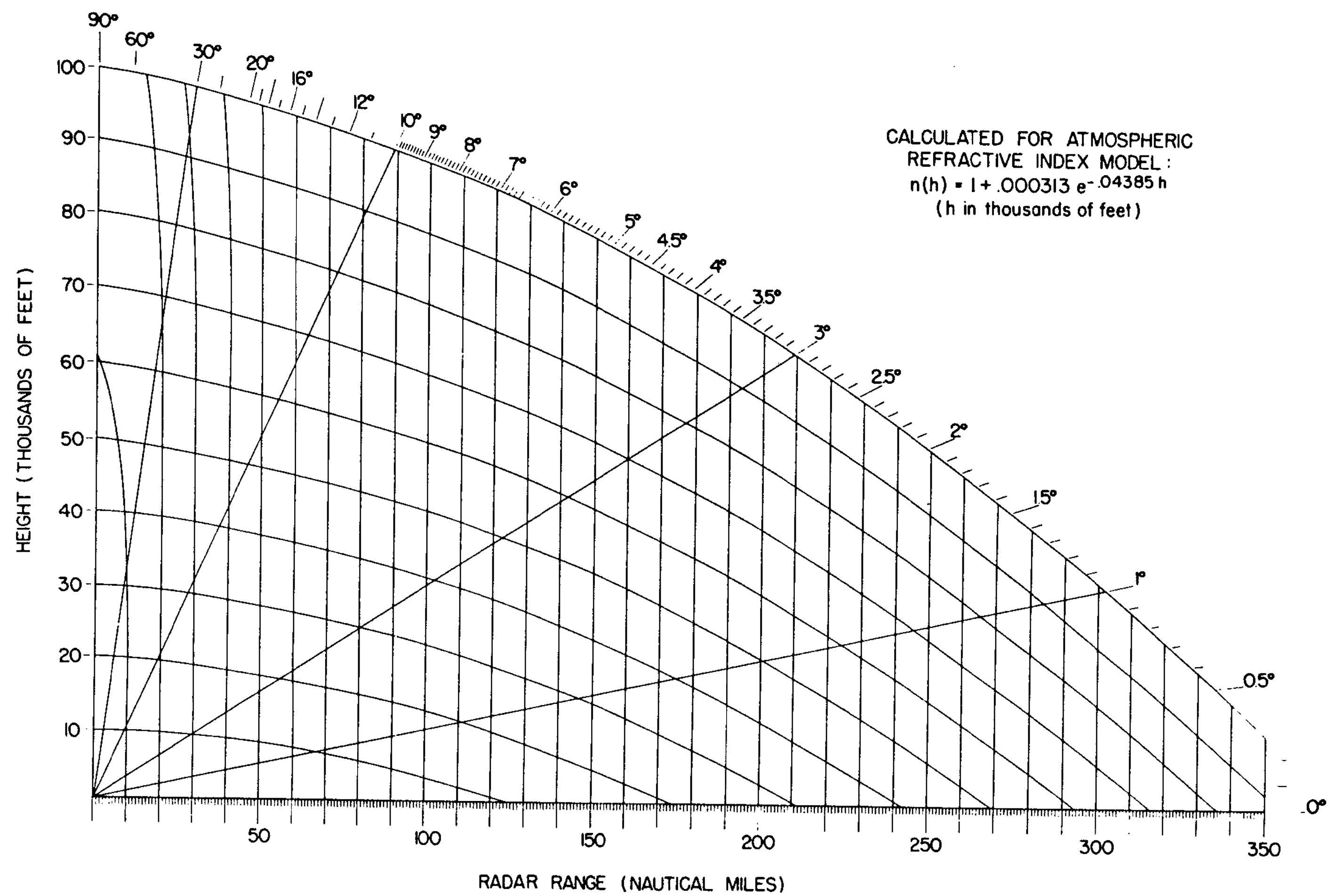

Fig. 26 - Radar range-height-angle chart calculated for an exponential model of the atmospheric refractive index. ("Radar range" means "distance along the ray path." Elevation angles are angles of rays with respect to horizontal at radar antenna.)

where R_h is the horizon distance in nautical miles and h is the antenna height in feet. Thus a target at height H would be on the radar horizon when its range is that given by the intersection of this H curve with the $\theta = 0$ degree line in Fig. 26, plus the horizon range given by Eq. (53). This result is based on a mixed set of conventions, but for the usual values of h in Eq. (53) the result is insignificantly different from the one that would be obtained on the sole basis of the CRPL refractivity model.

Frequency (f_{Mc})

Questions seldom arise concerning the proper value of frequency to employ in the range equation, because the bandwidth of the transmitted signal is usually a small fraction of the nominal or center frequency of the signal spectrum. A question can arise, however, whenever the radar frequency is varied from one pulse to the next, for whatever reason, if the variation is appreciable.

One situation of this type that is becoming more common is found in frequency-scanning radars, in which the antenna beam direction is changed by shifting the frequency. When the frequency scanning is in the vertical direction, the radar frequency becomes still another factor contributing to the elevation-angle dependence of the range capability. When the scanning is in azimuth, then in principle the range is an azimuth-dependent quantity. In practice, the frequency variation is usually small enough, over the entire scanned sector, so that using the average or median frequency for range calculation is sufficiently accurate. The point is mentioned here, however, both for completeness and because it is entirely conceivable that a radar might employ frequency variations large enough to affect the range appreciably even if the transmitter power, receiver noise temperature, and dissipative losses remained constant.

It must not be overlooked, moreover, that the range equation contains factors that are frequency-dependent in addition to the frequency term itself. The antenna gain is strongly frequency-dependent, while the system noise temperature, some types of losses, some propagation effects, and in some cases the target cross section are dependent upon frequency in varying degrees. System noise temperature is a decreasing function of frequency at vhf and low uhf, while in the microwave region it is increasing. For constant antenna aperture and scanning speed, the number of echo pulses returned during the traverse of the beam across the target will decrease as the frequency increases, since the beamwidth decreases with frequency (Eq. (48)). Accordingly, for this situation, the visibility factor V_o becomes an increasing function of frequency. Therefore it is not a simple matter to state the total dependence of radar range upon the frequency of operation. The appearance of the square root of the frequency in the denominator of the equation can be entirely misleading, since the product $(G_t G_r)^{1/4}$ in the numerator is directly proportional to frequency if the effective aperture sizes of the antennas are held constant.

If the frequency were to vary appreciably for the individual pulses during the group of pulses integrated prior to the detection decision, and if the variation were appreciable so that simply using an average frequency value in the range equation would not be acceptable, then a very involved analysis would have to be made to calculate the range correctly. The techniques used for analyzing the fluctuating-target cross section would be applicable, although the noise level as well as the signal level would in this case be varying from pulse to pulse, because of the frequency dependence of the system noise temperature.

Radar equations may of course be expressed either in terms of the frequency f or the wavelength λ, which is equal to c/f, where c is the velocity of electromagnetic propagation, 2.99793×10^9 meters or 161,875 nautical miles per second. The following relation between frequency in megacycles and wavelength in feet or meters is useful:

$$f_{Mc} = \frac{983.573}{\lambda_{ft}} = \frac{299.793}{\lambda_m} . \tag{54}$$

The frequency regions considered in this report are designated by the following customary abbreviations:

vhf (very-high frequency) – 30-300 Mc
uhf (ultra-high frequency) – 300-3000 Mc
shf (super-high frequency) – 3000-30,000 Mc (3-30 Gc).

The following radar frequency-band terminology is also commonplace:

P-band – 225-390 Mc
L-band – 390-1550 Mc
S-band – 1550-5200 Mc
X-band – 5200-10,900 Mc.

These designations originated during World War II and were employed for reasons of secrecy, but are now unclassified. X-band was also divided into subbands. The designation C-band is sometimes used to denote a region encompassing roughly the 5000-6000-Mc region – but this notation is apparently not officially recognized by the military services. The term "microwaves" generally refers to wavelengths less than 30 cm (frequencies greater than 1000 Mc).

Target Cross Section (σ)

The radar cross-section definition is the standard one given by Kerr (8) and others. The dependence of the cross section on geometric properties of the target is discussed by Kerr (8), Norton and Omberg (9), and numerous others. As previously stated, σ_{50} denotes the median value when the target cross section fluctuates. Measured cross sections are sometimes quoted as the median and sometimes as the mean value, and occasionally other percentile values have been used. It is recommended that the median be adopted as standard. The value 1 square meter is conventional for assessing the relative range performance of radar systems when no specific target is stipulated. Results of measurement and calculation of the cross sections of targets of military interest are available in the classified literature, for example Ref. 18, pp. 53-151. The target fluctuation characteristics (probability distribution, spectrum) are of importance in operational analysis of radar range performance, but are not needed for the basic type of range calculation considered here. Radar cross section is in general a function of frequency, aspect angle, and polarization. Hence the complete cross-section properties of a complex target cannot ordinarily be expressed by a single number.

The radar cross section of a target is a fictitious area σ such that when the target is in a field of transmitted power density S_t, and the reflected power density at the radar receiving antenna is S_r, then

$$\sigma = 4\pi R_r^2 S_r/S_t F_r^2 , \tag{55}$$

where R_r is the distance from the target to the receiving antenna. Radar cross section may be measured by using a calibrated radar, whose transmitted power P_t and antenna gains G_t and G_r are accurately known, and measuring the received signal power P_r at the receiver input terminals. Since

$$P_r = \frac{S_r G_r \lambda^2}{4\pi L_r} \tag{56}$$

and

$$S_t = \frac{P_t\, G_t\, F_t^2}{4\pi\, R_t^2\, L_t}, \tag{57}$$

where the symbols here have the same definitions as in Eqs. (2)-(4). Equation (55) in terms of the known or measurable radar quantities then becomes

$$\sigma = \frac{(4\pi)^3\, R_t^2\, R_r^2\, P_r\, L_t\, L_r}{P_t\, G_t\, G_r\, \lambda^2\, F_t^2\, F_r^2}, \tag{58}$$

which is an equation for radar-cross-section measurement. (The equation as written assumes a consistent set of units, e.g., watts, meters.) In the monostatic case, of course, $R_t = R_r = R$.

It is possible to measure radar cross section also by a comparison method, whereby the only quantity on the right-hand side of Eq. (58) that is needed is P_r. That is, the equation may be written

$$\sigma = K\, P_r, \tag{59}$$

and then the value of K may be determined by measuring P_r when the target has a known value of σ (a so-called standard target). In fact, only the ratio of the values of P_r for the standard target and the unknown target is needed, since

$$\sigma = \frac{\sigma_s\, P_r(\sigma)}{P_r(\sigma_s)} = r\,\sigma_s, \tag{60}$$

where σ_s is the standard-target cross section.

Standard targets are often spherical reflectors of known diameter. The sphere is an ideal standard target because its cross section can be accurately calculated (8) and the cross section is independent of aspect angle. If the radius of the sphere a is larger than the radar wavelength, the radar cross section is asymptotically (as a increases) equal to πa^2, the projected geometric area of the sphere. Figure 27 shows the exact behavior of σ as a function of the ratio a/λ for a sphere. Reference 8 also gives the theoretical results for the cross sections of targets of other simple shapes. A few of these are of sufficient interest and importance to include here.

The monostatic cross section of a large flat plate viewed perpendicularly to the surface is

$$\sigma = \frac{4\pi\, A^2}{\lambda^2}, \tag{61}$$

where A is the area of the plate, assuming that both the width and length of plate are large compared to the wavelength; the exact shape is immaterial, except that the edges must not be wildly irregular. (There must not be projecting portions whose width is small compared to a wavelength.)

The cross section of a large cylindrical object viewed perpendicularly to its axis is

$$\sigma = \frac{2\pi\, a\, \ell^2}{\lambda}, \tag{62}$$

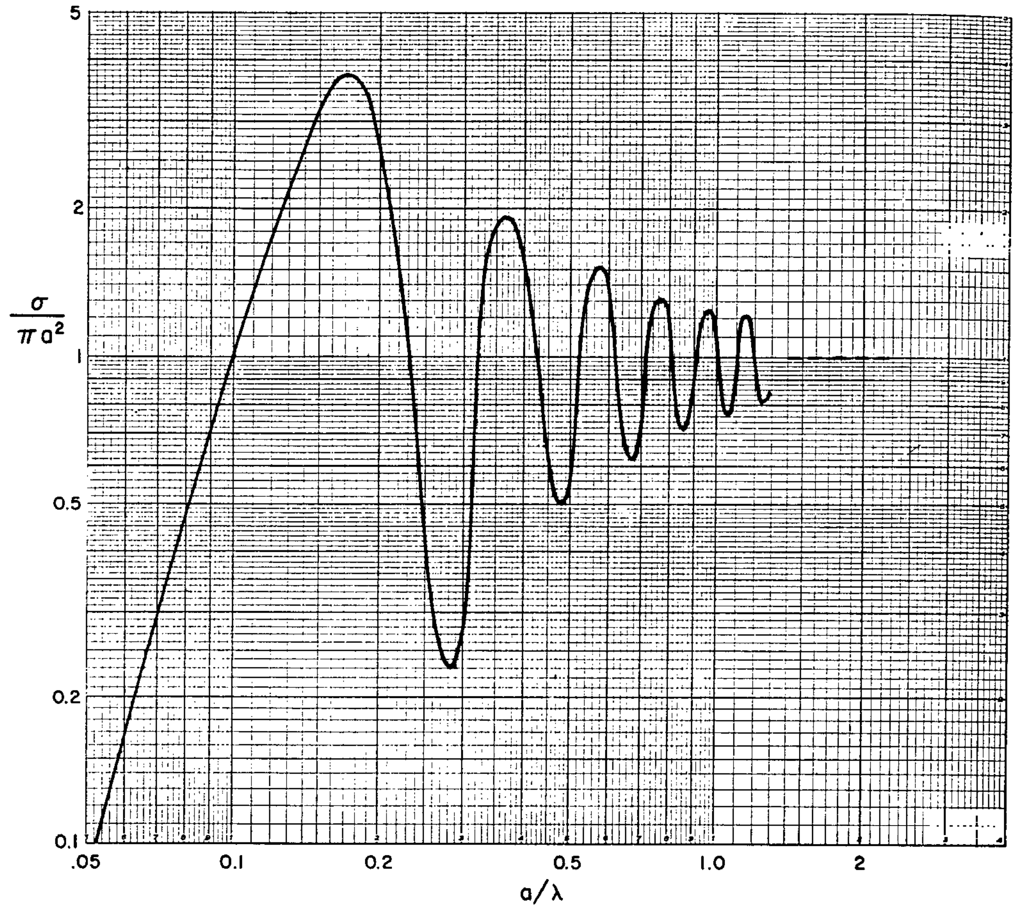

Fig. 27 - Radar cross section of a sphere, normalized relative to projected geometric cross section πa^2, as a function of its radius a relative to the wavelength λ, from Fig. 6.1, Ref. 8. As a/λ increases beyond the values shown, the oscillations of σ become progressively smaller and converge to the value πa^2.

where a is the radius of the cylinder and ℓ is its length, both assumed large compared to λ.

The cross section of a thin unloaded half-wave dipole viewed perpendicularly to its axis and with the polarization optimum is approximately $0.88\,\lambda^2$. Its average cross section over all orientations and polarizations is approximately $0.11\,\lambda^2$.

The targets thus far discussed were assumed to be small compared to the lateral dimensions of the radar antenna beam and the range dimension of the radar pulse ("point" targets). For area-extensive, range-extensive, and volume-extensive targets, the effective value of σ to be entered into the range equation becomes a function of the beamwidth, or the pulse length, or both. Such targets are characterized by a quantity σ^0 called* "radar cross section per unit area (or length or volume)," and the total effective cross section is obtained by integrating σ^0 over the area covered by the radar beam, or over the range depth of the radar pulse, or both, if the echo signals from the elemental scattering areas or volumes add noncoherently in the radar receiver. For the coherent case, the integration must be performed on a signal-voltage basis, taking phase into account;

*The notation σ^0 was originally used in analysis of sea and ground return ("clutter"). Its application here to extended targets in general is a somewhat loose use of the notation.

this result is then converted into an effective cross section. In either case, appropriate weighting factors must be applied to account for the variation of F and R, if these quantities vary appreciably over the region of integration.

In the case of the area-extensive target, in addition to this dependence of σ upon the beamwidth, there will also be an increase in the number of echo pulses received as the beam traverses the target (for a scanning radar). The effective number may be calculated by analyzing the amplitude or power pattern of the echo pulses received as a function of angle, as the beam scans, and taking the half-power angular width of this pattern as the effective beamwidth (θ_h or θ_v) in Eqs. (16) and (17). For targets of width considerably greater than the actual antenna beamwidth, the effective beamwidth will be approximately equal to the target angular width, provided that the detection device or observer is capable of fully integrating the resulting number of pulses.

In the case of the range-extensive target, there will be an enhancement of detectability of the echo, compared to a point target of the same cross section, due to the lengthened received pulse. This effect may be taken into account, in Eqs. (4) and (5), by using the received rather than the transmitted pulse length τ in addition to calculating σ in the manner described. However, unless the receiver passband is tailored in width and shape to this received pulse, then an appropriate value of C_B or m must be used, based on the relationship of the received pulse to the receiver passband.

In the case of the volume-extensive target, both the beam-widening and pulse-lengthening effects occur simultaneously. These effects must be taken into account in calculating maximum detection range, as with Eqs. (4) and (5). When a calculation of range for a given signal-to-noise ratio is being made, as with Eq. (3), then σ must be calculated in the manner described (if the target is extensive), but the increased number of pulses per beamwidth and increased received pulse length do not affect the calculation.

Cross-Section Fluctuation

The typical complex point target, unlike the sphere, has a very complicated pattern of reradiation, even for the purely monostatic case, so that the cross section is usually a strong function of the aspect from which it is viewed by the radar. Thus for aircraft targets the nose-aspect, tail-aspect, and broadside cross sections are often given. When the aspect is not specified for an airplane, usually the nose aspect is meant, since this is the one of greatest interest both militarily and for many (but not all) civilian radar applications. The suggested standard value of 1 square meter for calculating radar performance corresponds approximately to the nose aspect of a small or medium-sized fighter aircraft, although variation by a factor of at least 10 in either direction may be observed for different fighter-type airplanes at different frequencies.

Especially at the higher frequencies, even when a target maintains a constant nominal aspect, slight changes of aspect may occur, as well as vibration or other motion of the reflecting elements. The result is that typically the echo from any moving target fluctuates in amplitude and in phase. This fact complicates the statistical analysis of the echo detection problem. However, as has been shown (16), if the target cross section is specified to be the median value, then the range as given by Eqs. (4) and (5) is approximately the 0.5-probability-of-detection value whether the target is steady or fluctuating. This convenient result hinges upon adoption of 0.5 probability as a convention for standard range calculation, as well as upon specification of target cross section as the median value. In order to calculate the range for other probabilities it is necessary to know the complete probability distribution, or density function, for the target. Moreover, the procedure is quite complicated. Swerling (30) has published results of fluctuating-target analysis.

An approach often taken to the calculation of radar range for a fluctuating target is to use for σ in the range equation the average value (or the median, perhaps) and to include (as a component of the system loss factor L) an additional loss called "fluctuation loss" to account for the difference in range that would result when the target is fluctuating compared to that for the steady target of the same cross section. The necessary loss factor depends upon the probability of detection for which the range is desired as well as upon the percentile value of the cross section entered into the equation, upon the probability distribution of the target fluctuation, and upon the number of pulses integrated. (At low probability of detection, target fluctuation may actually produce a gain rather than a loss.) Swerling analyzed several assumed fluctuation distributions, and his results indicate less than 0.3 db difference from the nonfluctuating case at the 0.5-probability point, depending somewhat on the probability density function assumed for the target fluctuation. For "Rayleigh" fluctuation, the difference is less than 0.1 db. Kaplan (31) has also analyzed the detection of fluctuating signals.

It is often assumed that the received voltage amplitude (predetection) for a fluctuating target has a Rayleigh distribution. This is equivalent to assuming that the two-dimensional probability distribution, taking into account phase angle, is Gaussian, an assumption based on the central limit theorem of probability theory. This theorem assumes that the total received signal is the (vector) sum of voltages from a number of reflecting elements within the total target complex, that the phases of these voltages are varying with respect to one another randomly and independently of all the others, and that the individual voltage amplitudes are all of comparable magnitude — i.e., that no one or a few predominate. These assumptions are satisfied for some large complex targets, including the older airplanes of less rigid constructions, at high enough frequencies for vibration of the airplane parts to be significant. Examples of cases which ideally satisfy the requirements are the "clutter" echoes received from sea waves, the echo from rain at frequencies such that the individual drops reflect, and the echo from aggregations of the small artificial aluminum reflectors called "Chaff." However, many modern aircraft are characterized by distinctly non-Rayleigh distributions, which suggest that the total echo is the result of a rather few predominant reflecting surfaces, especially in the vhf and low uhf regions.

Because of its historical importance and also because it is actually applicable to some targets, the Rayleigh probability density function will be stated. The mathematical formulation in terms of the median voltage as the parameter is

$$p_1(v) = 2(\ln 2)\frac{v}{v_{50}^2}\,2^{-(v/v_{50})^2}, \tag{63}$$

where v is the voltage amplitude and v_{50} is its median value. Since the target cross section fluctuates proportionally to the square of the received signal voltage, it is deducible that the corresponding density function for the cross section σ is

$$p_2(\sigma) = \frac{\ln 2}{\sigma_{50}}\,2^{-\sigma/\sigma_{50}}, \tag{64}$$

which is a "negative exponential" distribution, although because of the above-described relationship to a Rayleigh-distributed signal voltage, targets having this cross-section distribution are called "Rayleigh" targets.

Another important factor in discussion of target fluctuation characteristics is the spectrum of the fluctuation — its frequency composition, or the fluctuation rate — in relation to the radar pulse rate, integration time, scanning speed, and beamwidth. This factor determines the statistical dependence or independence of the signal amplitudes separated by various amounts of time, e.g., by the pulse period or by the scan period.

Pulse-to-pulse fluctuations within a group of integrated pulses are much less important in the detection statistics than the lower frequency fluctuations that affect the average (integrated) amplitudes of successively observed pulse groups.

Short-period aircraft-echo fluctuations may be caused by propeller or jet-turbine-blade modulation and by vibration of structural parts. Slightly longer period fluctuations may be due to roll, pitch, and yaw, which cause aspect variation.

Sometimes very long-term fluctuations of aircraft echoes (with periods measured in minutes) are observed. These may in some cases be due to fluctuating propagation factors, but it is also conceivable that they are due to small course corrections of the aircraft made at frequent intervals. These would cause slight aspect changes, with resultant cross-section changes which may be quite large. The question of which of these explanations is more generally applicable was discussed at a Naval Research Laboratory symposium (18) on radar detection theory in 1956. The experiments needed to settle the question are difficult, and so far as is known by the writer, have yet to be performed.

Thus there are many matters still to be resolved before the analysis of the effect of target fluctuation on radar range may be considered to be satisfactory, and therefore this report does not attempt to present methods of handling the problem in practical range calculation. The foregoing discussion indicates why it seems desirable, at present, to avoid attempting any sophisticated consideration of target fluctuations in "basic" radar range calculation. Nevertheless, in military-operational analysis of radar-system capability, the subject cannot be avoided. The following section is therefore a brief sketch of some of the theory and practice that have been evolved.

BLIP/SCAN RATIO AND CUMULATIVE PROBABILITY OF DETECTION

The concepts of blip/scan ratio and cumulative probability of detection were developed by the Operational Evaluation Group of the Office of the Chief of Naval Operations during the latter part of World War II and in the early postwar years (32). It had been noted that with scanning radars the echo (blip) strength fluctuated from scan to scan, and when the target was near maximum range, the echo would appear on some scans and be absent on others. The fraction of scans on which a blip was observed, averaged over a small range interval, was named the blip/scan (B/S) ratio. It is apparent that this quantity is a function of target range, and corresponds to the probability of detection that has been discussed, when the observer's integration time (t_i, Eq. (15)), is less than the scan period of the radar.

The concept of cumulative probability of detection was developed to express the operational effectiveness of scanning radars against approaching targets. It answers the question, "What is the probability that an approaching target (e.g., aircraft) will have been detected by the time it reaches a given range?" Evidently this question requires a knowledge of the target speed and the radar scan rate as well as the variation of the blip/scan ratio as a function of range. It is a military-operational concept, primarily.

Both the blip/scan ratio and cumulative probability of detection concepts are limited in their applicability, especially as new scanning techniques are gaining acceptance. However, it is of some historical interest at least, to present a brief review of these ideas; also, they are still of practical value in some cases.

Range-Dependence of the Blip/Scan Ratio

The variability of blip strength, and of presence or absence of a blip on a particular scan, would in principle be observed even for a target of steady cross section (e.g., a sphere) because of the combining of the signal with the noise in the detector. If the only factor operating, the range dependence of blip/scan ratio would be readi culated, and moreover, the ratio would go from very small (near zero) to very unity) values of blip/scan ratio fairly steeply as a function of range, especially if detection-decision is based on integration of only a few pulses so that the signal-to ratio required for detection is fairly large. (The foregoing language is not precise, should be adequate for conveying the general idea.)

Therefore, especially at large signal-to-noise ratios, the factor that generally dominates in determining blip/scan behavior is the target cross-section fluctuation. fluctuation "spreads out" the curve of blip/scan ratio as a function of range, making transition from low to high ratios more gradual. This function can be computed if target fluctuation characteristics (amplitude distribution and spectrum) are known. calculation is difficult when the combined effects of noise and signal fluctuation have be considered. But where the signal fluctuation predominates, the effect of the nois be ignored, and the calculation is much simpler. For this case, if the target is at a ticular range R, the blip/scan ratio $\psi(R)$ is equal to the probability that the fluctuati σ causes it to exceed a threshold value σ_t; that is,

$$\psi(R) = \int_{\sigma_t(R)}^{\infty} p(\sigma)\, d\sigma ,$$

where $p(\sigma)$ is the probability density function for the cross-section fluctuation. The threshold value is given by

$$\sigma_t(R) = \sigma_{50} \left[\frac{R}{F\, R'_{50}} \right]^4 ,$$

where R'_{50} is the range computed by means of Eq. (4) or (5) assuming $F = 1$. It is ily deduced that when $p(\sigma)$ is given by Eq. (64), i.e., for a Rayleigh target,

$$\psi(R) = 2^{-(R/FR'_{50})^4}$$

When $F = 1$ (free-space propagation) the resulting curve for $\psi(R)$ is easily com puted. When F is a function of target elevation angle, however, as described by Eqs. et seq., the range-dependence of F must be obtained by assuming a specific target al tude, and a much more complicated $\psi(R)$ function results. Moreover, Eqs. (66) and (assume that such factors as L_a and T_N do not vary appreciably as the target (presum ably at a fixed altitude) changes range; hence these are essentially low-frequency (vhf low uhf) equations.

Another consideration at higher frequencies is the "fineness" of the sea-reflection interference lobe structure. A high-speed aircraft may fly through an appreciable po tion of a lobe during the scan period of a radar. F is no longer a slowly varying func of time and range, in relation to the usual radar scan rates, and the statistical nature ψ no longer permits F to be considered as a quasi-stationary parameter; rather, it comes part of the statistics, contributing "fluctuation" due to the random part of the structure in which the target is observed from scan to scan. The analysis of this lobe structure" case has been made by Alderson (33), for the Rayleigh target, negle the fluctuation contribution of the receiver noise. Alderson also has analyzed the of roll and pitch of the radar platform (ship) for an unstabilized antenna, with the sa simplifying assumptions and the further assumption that the roll and pitch periods not integrally or nearly integrally related to the scan period.

[redacted]

Cumulative Probability and "Operator Factor"

In principle, if the probability of detecting the target on a single scan at range R_i is P_i, assuming that the target fluctuation is independently random from scan to scan, the cumulative probability $\mathbb{P}(R)$ that the approaching target will be detected at least once by the time it reaches range R is

$$\mathbb{P}(R) = 1 - \prod_{i=1}^{n} (1 - P_i), \tag{68}$$

where the scans occurring prior to the target reaching range R are numbered 1, 2, 3, ...

The assumption that the fluctuation is independently random from scan to scan may not be justified in all cases (18), so caution must be used in applying this formula. Moreover, evaluation of the P_i's may be very difficult. If they are known as a tabulation of values from experimental data, calculation of $\mathbb{P}(R)$ from this formula will require excessive labor unless a digital computer is employed. However, for certain assumptions concerning the form of P_i as a function of range, the product term of Eq. (68) can be represented as an integral, and solutions have been obtained by OEG analysts (32).

There are so many questions concerning the validity of assumptions necessary for computing cumulative probability that it is not the intention here to present the concept as a practically useful one, but rather to mention and describe it briefly as a matter of historical interest, primarily. However, under some circumstances the necessary assumptions may be realized, and calculations of cumulative probability may then be of value.

The probability of detection on the i^{th} scan, at range R_i, would be $\psi(R_i)$ ideally. Generally, however, analysts have postulated that the human operator suffers from fatigue, boredom, etc., so that P_i is somewhat less than ψ_i, the latter being taken as the ideal or theoretical value that would apply with an alert or alerted observer (operator). An "operator factor" is defined to express this relationship:

$$P_i = p_o \psi_i . \tag{69}$$

The operator factor is generally defined as the probability that the operator will see the echo assuming that it appears (is detectable). In practice, however, p_o has tended to be used as a curve-fitting parameter to account for all differences between theory and experiment! Thus its usefulness as an engineering quantity based on extensive experimental data is limited. Moreover, it was originally assumed that p_o was a "constant" for a given experiment or a given operator and environment. Later it was realized that operator factor would certainly be a function of the signal strength, and hence of ψ itself, and possibly of other factors. Although some analysis and experiments have been performed to explore this more sophisticated viewpoint (34), it must be said that the role of operator factor in radar range theory remains in a somewhat nebulous and unsatisfactory state. It has also been proposed (32) that an operator recognizes the presence of a target only if it is observed for a succession of k scans, where k may be 1, 2, or even more. The probability of successful outcome for this sequence of events is $p_o \psi^k$, on the assumption that p_o applies only on the first scan of the sequence, the operator thereafter being assumed alert and attentive ($p_o = 1$). (It is also assumed that the k scans all occur within a small range interval, so that ψ remains constant.)

It is sometimes asserted that the military operational effectiveness of a radar should be expressed in terms of the range for a stated cumulative probability of detection, with a realistic assumption for the operator factor (i.e., $p_o < 1$). While this statement undoubtedly has merit, the fact remains that cumulative probability can only be given for a target of specified speed and specified fluctuation characteristic, and a good operator-factor theory is not available. Therefore it seems preferable, to the author, to evaluate the relative merits of radars on the basis of range for a stated single-scan or single-observation probability of detection, e.g., $P_d = 0.5$, assuming an alerted or attentive operator. Admittedly this is not a complete "figure of merit" for the radar, but it is an essential ingredient of the full merit rating. The coverage diagram, expressing R_{50} as a function of angle, together with a statement of scanning speed (information rate), constitute a fairly complete description of the radar's capabilities, without the complication of assumptions about target fluctuation characteristics, operator factor, and target speed.

RADAR RANGE EQUATIONS FOR A NOISE-JAMMING ENVIRONMENT

Although it was stated in the introduction that equations would be presented only for "basic" range calculation, not taking into account such factors as "clutter" echoes or jamming, the case of wide-band noise jamming is of special interest, and equations will be given that apply to this case, in terms of the basic equations and other environmental assumptions already described.

The situation first assumed is that the target to be detected by the radar (usually an aircraft) is carrying a jamming transmitter which radiates a noise signal having an effective spectral power density of $\mathcal{P}_j$ watts per megacycle. The jammer is also assumed to have an antenna gain G_j in the direction of the radar. The noise is assumed to be of the same nature as the noise already present in the radar from natural sources. Its bandwidth is assumed sufficiently larger than the radar receiver bandwidth so that the effective noise power radiated within the receiver passband is $\mathcal{P}_j B_N$, where B_N is the receiver noise bandwidth.

The equations to be given are derived from Eq. (4) by assuming that the noise power at the receiver input due to the jamming is considerably greater than the noise power represented by the quantity $k T_N B_N$, and therefore the factor $k T_N$ in the equation* is replaced by an expression representing the received jamming power density. This expression contains the factors G_r, L_r, F_r, and f_{Mc} in such a way that they cancel the corresponding factors in the original equation. The equation thus obtained for R_{50} in nautical miles is

$$R_{50} = 4.817 \times 10^{-3} F_t \left[\frac{P_{t(kw)} \tau_{\mu sec} G_t \sigma_{50\,(sq\,m)}}{\mathcal{P}_{j(w/Mc)} G_j V_{o(50)} C_B L} \right]^{1/2} . \tag{70}$$

The loss factor L, as given by Eq. (49), must be modified in Eq. (70) by deleting L_r and by redefining L_a as the one-way absorption loss (half the decibel values given by Figs. 13-25). Moreover, for scanning radars L_p should be reduced to about half the decibel value that would apply for the no-jamming case, because the received jamming power is reduced during the part of the scan when the antenna-beam maximum is not pointed directly at the jammer. Another equation, applicable when the jammer is not at the target position, can be similarly derived, but it contains additional terms. In this expression, Eq. (71), the range of the jammer, R_j (nautical miles), appears in the numerator, and a pattern-propagation factor for the jamming-signal propagation path, F_j, appears in the denominator. (This factor accounts for propagation effects and also for the pattern factor of the radar receiving antenna in the jammer direction.) The factor F_r does not cancel out of the equation, and the exponent 1/4 is retained. The frequency term f_{Mc}^2, the receiving antenna gain G_r, and the receiving line loss L_r cancel as in the self-screening case. Therefore L_r is again deleted in evaluating L. The decibel value of the absorption loss, L_a, is in Eq. (71) equal to the two-way radar loss minus the one-way loss for the

*The factor k is Boltzmann's constant, which does not appear explicitly in Eq. (4) because it has been incorporated into the constant numerical factor on the right-hand side.

jamming-signal path. L_p is the same value as in the no-jamming case; any side-lobe "pattern loss" may be accounted for by suitable evaluation of F_j. The equation is

$$R_{50} = 6.940 \times 10^{-2} \left[\frac{P_{t(kw)}\, \tau_{\mu sec}\, G_t\, \sigma_{50(sq\,m)}\, F_t^2\, F_r^2\, R_j^2}{\mathcal{P}_{j(w/Mc)}\, G_j\, F_j^2\, V_{o(50)}\, C_B\, L} \right]^{1/4} \tag{71}$$

Note that if in Eq. (71) $R_j = R_{50}$ and $F_j = F_r$, Eq. (70) results. In using both Eqs. (70) and (71) it is apparent that by making $\mathcal{P}_j\, G_j$ very small, or in the case of Eq. (71) by making R_j very large or F_j very small, very large values of R_{50} may be obtained. If the value of R_{50} thus obtained is larger than the value that would have been calculated without jamming, i.e., from Eq. (4), then of course the range calculated with Eq. (70) or (71) is false and is so because the assumption that the received jamming-signal spectral power density is considerably greater than kT_N has been violated. In order for Eqs. (70) and (71) to be valid, the ranges calculated using them must be appreciably less than the range calculated by Eq. (4).

The question arises, how shall the correct range be calculated when this result is not obtained? If the range calculated by Eq. (70) or (71) is appreciably greater than that calculated by Eq. (4), it means that the jamming is not powerful enough to reduce the radar range very much, and Eq. (4) may be used without much error — i.e., the jamming may be ignored. If, however, the ranges calculated by Eqs. (4) and (70) or (71) are comparable, the true range may be quite difficult to calculate for the self-screening case, and for the fixed-range jammer the equation is more complicated than Eq. (71). In each case the correct equation is obtained by replacing $k\,T_N$ in Eq. (4) by the sum of the jamming and natural noise-power spectral densities at the receiver input. In the self-screening case this results in a quadratic equation in R_{50}^2. Fortunately this case is not of much practical importance.

The full derivation of Eqs. (70) and (71), as well as the equations applicable when the kT_N term must be retained, will be given in Part 2. Jamming-range equations applicable to automatic detection, comparable to Eq. (5) for the nonjamming case, are obtained by simply substituting $D_{50}\, m$ for $V_{o(50)}\, C_B$ in Eqs. (70) and (71).

ACCURACY OF RADAR RANGE CALCULATIONS

Calculations of radar maximum range were, in the early days of radar, notoriously unreliable. The reasons were various. The dependence of signal detectability on number of pulses integrated was not explicitly recognized. Range calculations were often made on the assumption that a signal was detectable if it was of about the same power level as the noise. This was roughly true for many of the early "searchlighting" radars, but with the advent of scanning radars it was often far from true. As seen in Figs. 1-3, an error of 10-15 db may readily be incurred by making this assumption.

The probabilistic aspect of radar detection was also not well understood until it was first expounded by North (13) in 1943, who also first made clear the role of pulse integration in the detection process, including a precise analysis of the different results obtainable with predetection and postdetection integration. These considerations have now been well understood for many years, and are taken into account in the equations of this report, though in as elementary and simple a manner as possible. Therefore the ranges calculated with these equations are based on a moderately sophisticated analysis of the problem, and it can be claimed that if the correct values are used for all the factors of the equation, the result will be accurate, and will be supported by statistical analysis of experimental results. That this is true within reasonable tolerances was shown by the author (16) in a classified report on results obtained with some experimental naval radars at several frequencies ranging from vhf to shf.

Nevertheless, a precise agreement between calculation and experiment cannot be expected, partly because some deviation is inherent in all probabilistic processes, and partly because seldom are all of the factors in the range equation known precisely. Possibly the least well known quantity in most observations of complex target structures is the radar cross section σ. (Different workers often disagree by 10 db or more in measurements of σ for aircraft targets. This is probably due largely to the extreme aspect sensitivity of σ, although calibration errors probably play some part.) In some cases significant error in calculation of F may be made. Because of the strong dependence of the range on the factor F compared to most of the other factors, this error is more significant than comparable errors in some of the other factors. It may arise especially through incorrect estimation of the magnitude of the reflection coefficient ρ. In some cases superrefractive effects may also cause unexpectedly large or small values of F. At microwave frequencies, excessive atmospheric moisture, or precipitation, may cause absorption losses much higher than those predicted by Figs. 13-24. Also, numerous unrecognized losses may occur within the radar system.

Some of the quantities in the equation are not easy to measure or calculate precisely, notably the antenna gain and the system noise temperature. The gain formulas given, Eqs. (46) and (47), are approximate expressions, valid only for certain classes of antennas. A measured antenna gain figure should be used in radar range calculation whenever possible; however, accurate gain measurement is sometimes fairly difficult to accomplish. The system noise temperature contains two components that may be subject to appreciable error; the sky noise varies greatly over the celestial sphere (4), and the receiver noise temperature is not always accurately known. It is therefore of some interest to consider the relative effects of errors in the individual range-equation factors upon the total calculated-range error.

The effect of definite increments of the independent variables is well known. For example, the range is proportional to the fourth root of the transmitter power and of several other quantities in the equation. Hence a change in one of these quantities by the factor x changes the range by the factor $x^{1/4}$. Table A3 shows the relationship between the range change and decibel changes in range factors on which the range has this fourth-root dependence. However, the range is directly proportional to the pattern-propagation factor F and proportional to the square root of the frequency f (except that, as discussed previously, this square-root dependence on f is only the explicit part of the dependence, and assumes that all other factors in the equation are constant as the frequency changes, or that their variation with frequency is taken into account separately).

In considering the effect of sky-noise variations upon the accuracy of the range calculation, it is necessary to realize that the fourth-root dependence is upon the system noise temperature, of which the sky noise is an additive component. Therefore, the sensitivity of the range to the value of the sky noise temperature will depend partly upon the relative magnitudes of the sky noise temperature and the other components of system noise temperature; and of course similar statements are true of the other temperature components, such as the receiver noise temperature.

The foregoing discussion has been concerned with the relationship between exactly known variations in the individual range-equation factors and the corresponding range variations. It is also of interest to consider the range-calculation error that results when it may be assumed that each range-equation factor is subject to an error that can be estimated statistically but is not known exactly — that is, it can be specified or estimated in terms of a standard deviation. It is further assumed that the errors of the various quantities are statistically independent. Generally this assumption will be approximately correct even though there may be some interdependence as discussed in relation to the radar frequency.

The following equation, based on Eq. (4), has been derived* for this relationship, where the symbol $\mathcal{E}$ denotes the fractional standard error of the quantity that follows in parentheses: for example $\mathcal{E}(P_t)$ is the ratio of the standard deviation of the transmitter power to its nominal or assumed value:

$$\mathcal{E}(R_{50}) = \left\{\mathcal{E}^2(F) + \frac{1}{4}\,\mathcal{E}^2(f_{Mc}) + \frac{1}{16}\left[\mathcal{E}^2(P_t) + \mathcal{E}^2(\tau) + \mathcal{E}^2(G_t) + \mathcal{E}^2(G_r) + \mathcal{E}^2(\sigma_{50}) + \mathcal{E}^2(T_N) + \mathcal{E}^2(V_{o(50)}) + \mathcal{E}^2(C_B) + \mathcal{E}^2(L)\right]\right\}^{1/2} \qquad (72)$$

In practice, radar engineers usually estimate the uncertainty of the assigned values of the range equation quantities which have the dimensions of power, or power ratio, as decibel values. Such estimates usually do not have the statistical precision implied by the "standard deviation" definition of $\mathcal{E}$ in Eq. (72). For purposes of approximate calculation, however, decibel errors thus estimated, designated E_{db}, may be converted to values of $\mathcal{E}$ by means of the formula

$$\mathcal{E} = \left[\text{antilog}\ |0.1\ E_{db}|\right] - 1\,. \qquad (73)$$

This formula implies that the decibel error value E_{db} is actually 10 times the logarithm of the ratio of the sum of the assigned value and the standard deviation to the assigned value of the range-equation factor.

Equation (72) gives the range error (standard deviation) in terms of the symbols of Eq. (4). Similar error equations in terms of the symbols of Eqs. (3) and (5) may be obtained by substituting the terms of these equations comparable to those of Eq. (4) into Eq. (72). That is, D_{50} and m of Eq. (5) substitute directly for $V_{o(50)}$ and C_B. Similarly, B and S/N of Eq. (3) substitute for τ and $V_{o(50)}$, and C_B drops out.

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The information presented has been drawn from many sources. Credit has been given where possible, but in many cases the original source of information is not known. Apology is therefore offered to those to whom due credit has not been given. Special recognition is due to the importance of the original papers by Norton and Omberg (9) and North (13), and of Volumes 13 and 24 of the MIT Radiation Laboratory Series (8,10).

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*The derivation was made at the author's request by John Wood under the direction of S. F. George of the NRL Radar Division Mathematics Staff. The details of this derivation will be given in Part 2.

REFERENCES

1. Blake, L.V., "Interim Report on Basic Pulse-Radar Maximum-Range Calculation," NRL Memorandum Report 1106, Nov. 1960

2. Blake, L.V., "Curves of Atmospheric-Absorption Loss for Use in Radar Range Calculation," NRL Report 5601, March 1961

3. Blake, L.V., "A Note on Selection of an Atmospheric Refractivity Model for Radar Range-Height-Angle Charts," NRL Report 5626, Apr. 1961

4. Blake, L.V., "Antenna and Receiving-System Noise-Temperature Calculation," NRL Report 5668, Sept. 1961

5. Blake, L.V., "Recent Advancements in Basic Radar Range Calculation Technique," IRE Trans. Mil. Elec. MIL-5 (No. 2):154-164 (Apr. 1961)

6. Blake, L.V., "Antenna and Receiving-System Noise-Temperature Calculation" (Correspondence), Proc. IRE 49:1568 (Oct. 1961)

7. Blake, L.V., "Tropospheric Absorption Loss and Noise Temperature in the Frequency Range 100-10,000 Mc" (Correspondence), IRE Trans. Antennas and Propagation, AP-10:101 (Jan. 1962)

8. Kerr, D.E., "Propagation of Short Radio Waves," MIT Radiation Laboratory Series Vol. 13, New York:McGraw-Hill, 1951

9. Norton, K.A., and Omberg, A.C., "The Maximum Range of a Radar Set," Proc. IRE 35:4 (Jan. 1947)

10. Lawson, J.L., and Uhlenbeck, G.E., "Threshold Signals," MIT Radiation Laboratory Series Vol. 24, New York:McGraw-Hill, 1950

11. Ashby, R.M., Josephson, V., and Sydoriak, S.G., "Signal Threshold Studies," NRL Report 3007, Dec. 1946 (report based on work done by authors at MIT Radiation Laboratory, later published in Ref. 10)

12. Marcum, J.I., "A Statistical Theory of Target Detection by Pulsed Radar," RAND Research Memorandum RM-754, Dec. 1947, and "Mathematical Appendix," RM-753, July 1948 (reprinted in IRE Trans. IT-6, April 1960)

13. North, D.O., "An Analysis of the Factors Which Determine Signal/Noise Discrimination in Pulsed Carrier Systems," RCA Labs., Princeton, N. J., Tech. Report PTR-6C, June 1943

14. Woodward, P.M., "Probability and Information Theory," New York:McGraw-Hill, 1953

15. Search Radar Branch and Equipment Research Branch, Radar Division, NRL, "A VHF-UHF Missile and Space-Research Radar with a 150-Foot Steerable Antenna," NRL Report 5801, Aug. 1962

[redacted]

16. Blake, L.V., "Comparison of Predicted and Experimental Ranges of Some Developmental Navy Search Radars," title unclassified, NRL Memorandum Report 611 (Confidential), June 1956

17. Haeff, A.V., "Minimum-Detectable Radar Signal and Its Dependence Upon Parameters of Radar Systems," Proc. IRE 34:857 (Nov. 1946)

18. "Radar Detection Theory," ONR Symposium Report ACR-10 (Secret), Mar. 1-12, 1956. See esp. Foreword and pp. 7, 39, 187. (This report also contains Ref. 16.)

19. Burrows, C.R., and Attwood, S.S., "Radio Wave Propagation," Consolidated Summary Technical Report of the Committee on Propagation of the National Defense Research Committee, New York:Academic Press, p. 466, 1949

20. Stegen, R.J., "Excitation Coefficients and Beamwidths of Tchebyscheff Arrays," Proc. IRE 41:1671 (Nov. 1953)

21. Blake, L.V., "The Effective Number of Pulses per Beamwidth for a Scanning Radar," Proc. IRE 41:770 (June 1953), and "Addendum," Proc. IRE 41:1785 (Dec. 1953)

22. Minzner, R.A., Ripley, W.S., and Condron, T.P., "U.S. Extension to the ICAO Standard Atmosphere," U.S. Dept. of Commerce, Weather Bureau, and USAF ARDC Cambridge Research Center, Geophysics Research Directorate; U.S. Govt. Printing Office, 1958

23. Bean, B.R., and Thayer, G.D., "Models of the Atmospheric Radio Refractive Index," Proc. IRE, 47:740 (May 1959); also, "CRPL Exponential Reference Atmosphere," National Bureau of Standards Monograph 4, U.S. Govt. Printing Office, Oct. 1959

24. Millman, G.H., "Atmospheric Effects on VHF and UHF Propagation," Proc. IRE, 46:1492 (Aug. 1958)

25. Hall, W.M., "Prediction of Pulse Radar Performance," Proc. IRE, 44:224 (Feb. 1956); also "General Radar Equation," Space/Aeronautics R&D Handbook, 1962-1963

26. Clarke, F.D., and Blake, L.V., "A Radar Range-Height Chart Representing Refracted Rays as Straight Lines," NRL Memorandum Rept. 879, Dec. 1958

27. Bauer, J.R., et al., "Radio Refraction in a Cool Exponential Atmosphere," MIT Lincoln Laboratory Report 186, ASTIA No. 202331, Aug. 1958

28. Bean, B.R. and Cahoon, B.A., "The Use of Surface Weather Observations to Predict the Total Atmospheric Bending of Radio Rays at Small Elevation Angles" (Correspondence), Proc. IRE 45:1545 (Nov. 1957)

29. Bean, B.R., and Thayer, G.D., "Models of the Atmospheric Refraction Index," Discussion by P. Misme and by the authors (Correspondence), Proc. IRE 48:1498 (Aug. 1960)

30. Swerling, P., "Probability of Detection for Fluctuating Targets," RAND Research Memorandum RM-1217, Mar. 1954 (reprinted in IRE Trans. IT-6, Apr. 1960)

31. Kaplan, E.L., "Signal Detection Studies, with Applications," Bell Sys. Tech. Jour. 34:403 (Mar. 1955) (reprinted as Bell Telephone System Monograph 2394)

32. Crane, R., "A Method for Prediction of the Operational Performance of Search Radars," Office of Chief of Naval Operations, Operational Evaluation Group Study 416 (Confidential), 1950; also OEG Study 313 (Confidential), 1947

33. Alderson, W.S., "Calculation of Radar Performance and the Effect of Roll and Pitch," NRL Report 4584, Aug. 1955

34. Varela, A.A., Alderson, W.S., and Ellis, R.E., "The Operator Factor in the Detection of Radar Signals," NRL Report 4268 (Confidential), Dec. 1953 (summarized in Ref. 18, pp. 153-163)

Roll and Pitch,"

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APPENDIX A

A WORK-SHEET FORM FOR RANGE CALCULATION

In this appendix, a work sheet based on a slight modification of Eq. (4a) is presented, along with tables, curves, and auxiliary equations needed for the calculation.

The modified equation is

$$R_{50} = 100\ F\ \text{antilog}\ \frac{1}{40}\Big[4.45 + 10\log P_{t(kw)} + 10\log \tau_{\mu sec} + G_{t(db)} + G_{r(db)} + 10\log \sigma_{50\,(sq\,m)} - 20\log f_{Mc} - 10\log T_{NI} - V_{o(50)(db)} - C_{B(db)} - L_{t(db)} - L_{p(db)} - L_{a(db)} - L_{x(db)}\Big]\ . \tag{A1}$$

This equation differs from Eq. (4a) in that some of the constants have been manipulated to place the factor 100 on the right-hand side. This is done for convenience in using Table A3 as an aid to the calculation. Also, the equation is written using the system-input noise temperature T_{NI} so that the L_r term does not explicitly appear in the equation. Full-page curves for the "visibility factor" $V_{o(50)db}$, for the bandwidth-correction factor C_B, for the antenna noise temperature T_a, and the atmospheric absorption loss L_a, in the frequency range 100 to 10,000 Mc, are included for ready reference in this appendix, along with the work-sheet form and a full-page-size sample of the coverage-diagram chart. The curve for the detectability factor D_{50} is also included. The work sheet may be used for computing range on the basis of Eq. (5) by using D_{50} in place of $V_{o(50)}$, and M (see text) in place of C_B. It may also be used for computing on the basis of Eq. (3) by using S/N in place of $V_{o(50)}$, replacing C_B by B_{kc}, changing the range-equation decibel constant from 4.45 to 34.46 (40 log 7.268), and deleting the $\tau_{\mu sec}$ entry.

Removable (perforated-edge) copies of the work sheet and of the range-height-angle (coverage diagram) chart are included at the end of this appendix. These may be used as masters for quantity reproduction if desired.

PULSE-RADAR RANGE-CALCULATION WORK SHEET

For use with NRL Report 5868

1. Compute system input noise temperature, T_{NI}, following outline in section (1) below.

2. Enter range factors known in other than decibel form in section (2) below, for reference.

3. Enter logarithmic and decibel values in section (3) below, positive values in plus column, negative in minus. (Example: If $V_{o(50)(db)}$ as given by Fig. A1 or A2 is negative, then $-V_{o(50)(db)}$ is positive, goes in plus column.) To convert range factors to decibel values, use Table A2. For $C_{B(db)}$ use Fig. A3.

Radar antenna height: h = ft. Target elevation angle: θ = °. (See Fig. A12).

(1) COMPUTATION OF T_{NI}:

$$T_{NI} = T_a + T_{r(I)} + L_r T_e$$

(a) For general range computation, use Figure A5 for T_a.

(b) To find L_r, given $L_{r(db)}$, use first and second columns of Table A2.

(c) Also in Table A2, opposite $L_{r(db)}$ in first column, read $T_{r(I)}$ in third column.

Note: If thermal temperature (T_t) of transmission line is appreciably different from 290°K, multiply Table A2 values of $T_{r(I)}$ by $T_t/290$.

(d) Opposite $\overline{NF}_{db}$ in first column, read T_e in third col.

T_t		T_a	
L_r		$T_{r(I)}$	
$\overline{NF}_{db}$		$L_r T_e$	
T_e		T_{NI}	

(2) RANGE FACTORS		(3) DECIBEL VALUES	PLUS (+)	MINUS (-)
$P_{t(kw)}$		$10 \log P_{t(kw)}$	.	.
$\tau_{\mu sec}$		$10 \log \tau_{\mu sec}$	.	.
G_t		$G_{t(db)}$	.	.
G_r		$G_{r(db)}$	.	.
$\sigma_{50(sq.m.)}$		$10 \log \sigma_{50}$	.	.
f_{Mc}		$-20 \log f_{Mc}$	.	.
T_{NI}, °K		$-10 \log T_{NI}$	.	.
$V_{o(50)}$		$-V_{o(50)(db)}$	.	.
C_B		$-C_{B(db)}$		.
L_t		$-L_{t(db)}$		.
L_p		$-L_{p(db)}$		.
L_x		$-L_{x(db)}$		.
Range-equation constant (40 log 1.292)			4 . 45	
4. Obtain column totals →			.	.
5. Enter smaller total below larger →			.	.
6. Subtract to obtain net decibels →			+ .	- .

7. In Table A3, find range ratio corresponding to this net decibel value, taking its sign (±) into account. Multiply this ratio by 100. This is R_o →

8. Multiply R_o by the pattern-propagation factor.

F = See Eqs. 27 - 44, and Figs. 8 - 12.

$R_o \times F = R'$ →

9. On the appropriate curve of Figures A6 - A11, determine the atmospheric-absorption loss factor, $L_{a(db)}$, corresponding to R'. This is $L_{a(db)(1)}$ →

10. In Table A3, find the range-decrease factor corresponding to $L_{a(db)(1)}$, δ_1 →

11. Multiply R' by δ_1. This is a first approximation of the range, R_1 →

12. If R_1 differs appreciably from R', on the appropriate curve of Figures A6 - A11 find the new value of $L_{a(db)}$ corresponding to R_1. This is $L_{a(db)(2)}$ →

13. In Table A3, find the range-increase factor corresponding to the difference between $L_{a(db)(1)}$ and $L_{a(db)(2)}$. This is δ_2. →

14. Multiply R_1 by δ_2. This is the radar range in nautical miles, R_{50} →

Note: If the difference between $L_{a(db)(1)}$ and $L_{a(db)(2)}$ is less than 0.1 db, R_1 may be taken as the final range value, and steps 12 - 14 may be omitted. If $L_{a(db)(1)}$ is less than 0.1 db, R' may be taken as the final range value, and steps 9 - 14 may be omitted. (For radar frequencies up to 10,000 megacycles, correction of the atmospheric attenuation beyond the $L_{a(db)(2)}$ value would amount to less than 0.1 db.)

The following equations from the text of the report are repeated below for reference purposes. For additional details and equations the text should be consulted. For definitions of the symbols see Table A1.

NUMBER OF PULSES PER SCAN, UNIDIRECTIONAL (AZIMUTH) SCANNING

$$N = \frac{\theta_h^\circ \, \overline{PRF}}{6 \, (\cos \theta_e) \, \overline{RPM}} \tag{16}$$

NUMBER OF PULSES PER SCAN, ORTHOGONAL BIDIRECTIONAL SCANNING

$$N = \frac{\theta_h^\circ \, \theta_v^\circ \, \overline{PRF}}{6 \, (\cos \theta_e) \, \omega_v \, t_v \, \overline{RPM}} \tag{17}$$

(above equations) valid if $\theta_h / \cos \theta_e < 90^\circ$

PATTERN-PROPAGATION FACTOR FOR SEA-REFLECTION INTERFERENCE

If $f(+\theta) = f(-\theta)$: $$F = \left| f(\theta_e) \sqrt{1 + \rho^2 D^2 + 2 \rho D \cos \alpha} \right| \tag{32}$$

If also $\gamma = \pi$: $$\cos \alpha = - \cos \left[\frac{4\pi \, h \sin \theta_e}{\lambda} \right] \tag{33}$$

(for calculating ρ and D, see Figs. 8-12).

FOR $\rho = 1$ AND $D = 1$

$$F = 2 \, f(\theta_e) \, \left| \sin \, (0.366 \, h_{ft} \, f_{Mc} \sin \theta)^\circ \right| . \tag{35}$$

ANGLES OF MINIMA

$$\sin \theta_{min} = \frac{492 \, n}{f_{Mc} \, h_{ft}} , \qquad n = 0, 1, 2, 3, \ldots . \tag{36}$$

ANGLES OF MAXIMA

$$\sin \theta_{max} = \frac{246 \, (2n - 1)}{f_{Mc} \, h_{ft}} , \qquad n = 1, 2, 3, \ldots . \tag{37}$$

GAIN OF ANTENNA OF AREA A (SQ FT)

$$G = k_i \left(1.3 \times 10^{-5} \, A_{sf} \, f_{Mc}^2 \right) \tag{46}$$

($k_i = 0.6$ to 0.9 for efficient designs).

GAIN FOR KNOWN BEAMWIDTHS

$$G = \frac{k_2}{\theta_h^\circ \, \theta_v^\circ} \tag{47}$$

$$(k = 27{,}000 \text{ to } 41{,}000).$$

BEAMWIDTH FOR KNOWN APERTURE, d_i

$$\theta_i = \frac{984 \, k_3}{d_{i(ft)} \, f_{Mc}} \tag{48}$$

$$(k_3 = 50 \text{ to } 75).$$

TARGET ALTITUDE VS ELEVATION ANGLE, SMALL HEIGHT AND RANGE

$$H = h + 6076 \, R \sin\theta + 0.6624 \, R^2 \cos^2\theta \tag{52}$$

$$(H, h, \text{ feet}; R \text{ naut mi}).$$

HORIZON RANGE OF LOW ANTENNA

$$R_{hor} = 1.23 \sqrt{h_{ft}} \tag{53}$$

$$(\text{naut mi, ft}).$$

FREQUENCY VS WAVELENGTH

$$f_{Mc} = \frac{983.573}{\lambda_{ft}} = \frac{299.793}{\lambda_{meters}} . \tag{54}$$

RADAR CROSS SECTION OF LARGE FLAT PLATE, AREA A

$$\sigma = \frac{4\pi A^2}{\lambda^2} \tag{61}$$

RADAR CROSS SECTION OF LARGE CYLINDER, LENGTH ℓ, RADIUS a

$$\sigma = \frac{2\pi \, a \, \ell^2}{\lambda} . \tag{62}$$

RADAR CROSS SECTION OF LARGE SPHERE, RADIUS $a \gg \lambda$

$$\sigma = \pi a^2$$

(for $a < 2\lambda$, see Fig. 26).

[redacted]

Table A1
List of Range-Equation and Auxiliary-Equation Symbols with Brief Definitions
(For Use with Range-Calculation Work Sheet)

(47)	R_{50}	- for scanning radar, range of 0.5 blip/scan ratio; for searchlighting radar, range of 0.5 probability of detection during observer's integration time
	$P_{t(kw)}$	- transmitter pulse power output, kilowatts, measured at transmitter output terminals
(48)	$\tau_{\mu sec}$	- radar pulse length, microseconds, between half-power points of pulse waveform
	G_t, G_r	- directive antenna gains on transmission and reception, in beam maxima; ratio of radiated power density in beam maximum relative to that of an isotrope radiating same total power, at same range
	$\sigma_{50\,(sq\,m)}$	- median radar cross section of target, square meters
	f_{Mc}	- radar frequency, megacycles
(52)	T_{NI}	- effective system input noise temperature (referred to system input terminals), degrees Kelvin
	T_a	- effective noise temperature of antenna, degrees Kelvin (see Fig. A5)
	$T_{r(I)}$	- effective input noise temperature of receiving transmission line, degrees Kelvin (see Table A2)
(53)	T_e	- effective input noise temperature of receiver, degrees Kelvin (see Table A2)
	$\overline{NF}$	- receiver noise factor (IRE Standard 59 IRE 20.S1); IRE Proc. 48:60 (Jan. 1960)
	$V_{o(50)}$	- visibility factor for optimum bandwidth, 0.5 probability of detection (see Figs. A1 and A2)
(54)	C_B	- bandwidth correction factor (see Fig. A3).
	$B\tau$	- product of receiver predetection bandwidth, cycles per second, and pulse length, seconds (as used in connection with Fig. A3)
	L_t	- power loss factor for transmission-line system (including antenna ohmic losses) during transmitting; ratio of power delivered at transmitter output terminals to power radiated
(61)	L_r	- power loss factor for transmission-line system (including antenna ohmic losses) during reception; ratio of available power captured by receiving antenna to available power at the receiver input terminals
	L_p	- antenna-pattern loss factor for scanning radar; for unidirectional scan, L_p = 1.45 (1.6 db); for orthogonal bidirectional scan, an estimated value is L_p = 2.1 (3.2 db); for searchlighting (nonscanning) radar L_p = 1 (0 db)
(62)	L_a	- loss factor for absorption by propagation medium (for atmospheric loss, see Figs. A6 to A11
	L_x	- loss factor for "other" losses that may occur in special cases

Table continues

Table A1 (Continued)

F	-	pattern-propagation factor; ratio of electric field intensity, radar-to-target and target-to-radar, in absence of propagation-medium absorption losses, to that which would exist in free space in beam maximum at same range; see Eqs. (27) to (44)
N	-	number of pulses integrated, or number per scan on target within half-power beamwidth
θ_h°	-	azimuth beamwidth of antenna, degrees
θ_v°	-	vertical beamwidth of antenna, degrees
θ_e°	-	target elevation angle, degrees
$\overline{PRF}$	-	radar pulse rate, pulses per second
$\overline{RPM}$	-	antenna rotation rate, revolutions per minute
ω_v	-	vertical scan speed, degrees per second, at the target elevation angle
t_v	-	vertical scanning period, seconds, including dead time if any
ρ	-	reflection coefficient of earth, sea $(0 \leqq \rho \leqq 1)$
D	-	divergence factor $(0 \leqq D \leqq 1)$
h	-	antenna height
$f(\theta_e)$	-	antenna vertical pattern factor, for $f(0) = 1$, $f(+\theta) = f(-\theta)$ $f(0) = 1$
λ	-	radar wavelength
H	-	target altitude

Table A2
Transmission Line and Receiver Input Noise Temperatures

Opposite the decibel value of the transmission-line available loss L_r, in the first column, find in the second column the corresponding power-ratio value of L_r. In the third column, find the corresponding value of transmission-line input noise temperature $T_{r(I)}$, assuming that the thermal temperature T_t is approximately equal to T_o = 290°K, according to the formula

$$T_{r(I)} = T_o (L_r - 1).$$

If in the actual case T_t has an appreciably different value, multiply these values of $T_{r(I)}$ by $T_t/290$.

Opposite the decibel value of receiver noise factor $\overline{NF}$ in the first column, find in the third column the corresponding value of receiver input noise temperature T_e, according to the formula

$$T_e = T_o (\overline{NF} - 1).$$

$\overline{NF}$ L_r decibels	$\overline{NF}$ L_r power ratios	T_e $T_{r(I)}$ ° Kelvin	$\overline{NF}$ L_r decibels	$\overline{NF}$ L_r power ratios	T_e $T_{r(I)}$ ° Kelvin	$\overline{NF}$ L_r decibels	$\overline{NF}$ L_r power ratios	T_e $T_{r(I)}$ ° Kelvin
0	1.0000	0.00	2.2	1.660	191	6.2	4.169	919
0.01	1.0023	0.67	2.3	1.698	202	6.3	4.266	947
0.02	1.0046	1.33	2.4	1.738	214	6.4	4.365	976
0.03	1.0069	2.00	2.5	1.778	226	6.5	4.467	1005
0.04	1.0093	2.70	2.6	1.820	238	6.6	4.571	1036
0.05	1.0116	3.36	2.7	1.862	250	6.7	4.677	1066
0.06	1.0139	4.03	2.8	1.905	262	6.8	4.786	1098
0.07	1.0162	4.70	2.9	1.950	276	6.9	4.898	1130
0.08	1.0186	5.39	3.0	1.995	289	7.0	5.012	1163
0.09	1.0209	6.06	3.1	2.042	302	7.1	5.129	1197
0.10	1.0233	6.76	3.2	2.089	316	7.2	5.248	1232
0.15	1.0351	10.2	3.3	2.138	330	7.3	5.370	1267
0.20	1.0471	13.7	3.4	2.188	345	7.4	5.495	1304
0.25	1.0593	17.2	3.5	2.239	359	7.5	5.623	1341
0.30	1.0715	20.7	3.6	2.291	374	7.6	5.754	1379
0.35	1.0839	24.3	3.7	2.344	390	7.7	5.888	1418
0.40	1.0965	28.0	3.8	2.399	406	7.8	6.026	1458
0.45	1.1092	31.7	3.9	2.455	422	7.9	6.166	1498
0.50	1.1220	35.4	4.0	2.512	438	8.0	6.310	1540
0.55	1.1350	39.2	4.1	2.570	455	8.1	6.457	1583
0.60	1.1482	43.0	4.2	2.630	473	8.2	6.607	1626
0.65	1.1614	46.8	4.3	2.692	491	8.3	6.761	1671
0.70	1.1749	50.7	4.4	2.754	509	8.4	6.918	1716
0.75	1.1885	54.7	4.5	2.818	527	8.5	7.079	1763
0.80	1.2023	58.7	4.6	2.884	546	8.6	7.244	1811
0.85	1.2162	62.7	4.7	2.951	566	8.7	7.413	1860
0.90	1.2303	66.8	4.8	3.020	586	8.8	7.586	1910
0.95	1.2445	70.9	4.9	3.090	606	8.9	7.762	1961
1.00	1.2589	75.1	5.0	3.162	627	9.0	7.943	2013
1.1	1.288	83.5	5.1	3.236	648	9.1	8.128	2067
1.2	1.318	92.2	5.2	3.311	670	9.2	8.318	2122
1.3	1.349	101	5.3	3.388	693	9.3	8.511	2178
1.4	1.380	110	5.4	3.467	715	9.4	8.710	2236
1.5	1.413	120	5.5	3.548	739	9.5	8.913	2295
1.6	1.445	129	5.6	3.631	763	9.6	9.120	2355
1.7	1.479	139	5.7	3.715	787	9.7	9.333	2417
1.8	1.514	149	5.8	3.802	813	9.8	9.550	2480
1.9	1.549	159	5.9	3.890	838	9.9	9.772	2544
2.0	1.585	170	6.0	3.981	864	10.0	10.000	2610
2.1	1.622	180	6.1	4.074	891			

Temperature conversion relations: $T_{Kelvin} = 273.16 + T_{Centigrade} = 255.38 + (5/9)\, T_{Fahrenheit}$

290° Kelvin = 16.84° Centigrade = 62.32° Fahrenheit

Table A3
Radar Range Factors for System Power Change from 0 to 40 Decibels (in Steps of 0.1 Decibel)

The table is intended for use with an equation of the type:

$$R = k\left[\frac{P_t G^2 \lambda^2 \sigma F^4}{P_r L}\right]^{1/4} = k\,P^{1/4};\ \text{i.e., } R \propto P^{1/4}$$

where R is the radar range and P may be regarded as an equivalent system power variable. The table is based on the relation:

$$R/R_o = \text{antilog}\left[\frac{1}{40}\,(10 \log P/P_o)\right]$$

where R/R_o is the range factor, and $10 \log P/P_o$ is the power change in decibels. P_t is transmitter power, G antenna gain, λ wavelength, σ target cross section, L loss factor, F pattern-propagation factor, and P_r received power.

Range factors for power changes greater than 40 db can be obtained from the table by the following procedure: (1) Subtract from the absolute value of the power change in db the integral multiple of 40 which results in a positive remainder less than 40; (2) Look up the range factor corresponding to the remainder; (3) Shift the decimal point one place for each 40 db subtracted; for range increase, shift to the right, for decrease shift to the left. For example, the range increase for a power change of 47.3 db is 15.22, and for 87.3 it is 152.2, because for 7.3 db it is 1.522. The decrease factor for 47.3 db is 0.06569, and for 87.3 it is 0.006569, etc.

Power Change, Decibels ±	Range Increase Factor (Decimal Point)	Range Decrease Factor (Decimal Point)	
0.0	1 0000	1 0 000	40.0
0.1	1 0058	9 943	39.9
0.2	1 0116	9 886	39.8
0.3	1 0174	9 829	39.7
0.4	1 0233	9 772	39.6
0.5	1 0292	9 716	39.5
0.6	1 0351	9 661	39.4
0.7	1 0411	9 605	39.3
0.8	1 0471	9 550	39.2
0.9	1 0532	9 495	39.1
1.0	1 0593	9 441	39.0
1.1	1 065	9 386	38.9
1.2	1 072	9 333	38.8
1.3	1 078	9 279	38.7
1.4	1 084	9 226	38.6
1.5	1 090	9 173	38.5
1.6	1 096	9 120	38.4
1.7	1 103	9 068	38.3
1.8	1 109	9 016	38.2
1.9	1 116	8 964	38.1
2.0	1 122	8 913	38.0
2.1	1 129	8 861	37.9
2.2	1 135	8 810	37.8
2.3	1 142	8 760	37.7
2.4	1 148	8 710	37.6
2.5	1 155	8 630	37.5
2.6	1 161	8 610	37.4
2.7	1 168	8 561	37.3
2.8	1 175	8 511	37.2
2.9	1 182	8 463	37.1
3.0	1 189	8 414	37.0
3.1	1 195	8 366	36.9
3.2	1 202	8 318	36.8
3.3	1 209	8 270	36.7
3.4	1 216	8 222	36.6
3.5	1 223	8 175	36.5
3.6	1 230	8 128	36.4
3.7	1 237	8 082	36.3
3.8	1 245	8 035	36.2
3.9	1 252	7 989	36.1
4.0	1 259	7 943	36.0
4.1	1 266	7 898	35.9
4.2	1 274	7 852	35.8
4.3	1 281	7 807	35.7
4.4	1 288	7 763	35.6
4.5	1 296	7 718	35.5
4.6	1 303	7 674	35.4
4.7	1 311	7 630	35.3
4.8	1 318	7 586	35.2
4.9	1 326	7 542	35.1
	Range Decrease Factor	Range Increase Factor	± Decibels Power Change

Power Change, Decibels ±	Range Increase Factor	Range Decrease Factor	
5.0	1 334	7 499	35.0
5.1	1 341	7 456	34.9
5.2	1 349	7 413	34.8
5.3	1 357	7 371	34.7
5.4	1 365	7 328	34.6
5.5	1 372	7 286	34.5
5.6	1 380	7 244	34.4
5.7	1 388	7 203	34.3
5.8	1 396	7 162	34.2
5.9	1 404	7 120	34.1
6.0	1 413	7 080	34.0
6.1	1 421	7 039	33.9
6.2	1 429	6 998	33.8
6.3	1 437	6 958	33.7
6.4	1 445	6 918	33.6
6.5	1 454	6 879	33.5
6.6	1 462	6 839	33.4
6.7	1 471	6 800	33.3
6.8	1 479	6 761	33.2
6.9	1 488	6 722	33.1
7.0	1 496	6 683	33.0
7.1	1 505	6 645	32.9
7.2	1 514	6 607	32.8
7.3	1 522	6 569	32.7
7.4	1 531	6 531	32.6
7.5	1 540	6 494	32.5
7.6	1 549	6 457	32.4
7.7	1 558	6 420	32.3
7.8	1 567	6 383	32.2
7.9	1 576	6 346	32.1
8.0	1 585	6 310	32.0
8.1	1 594	6 273	31.9
8.2	1 603	6 237	31.8
8.3	1 612	6 202	31.7
8.4	1 622	6 166	31.6
8.5	1 631	6 131	31.5
8.6	1 641	6 095	31.4
8.7	1 650	6 061	31.3
8.8	1 660	6 026	31.2
8.9	1 669	5 991	31.1
9.0	1 679	5 957	31.0
9.1	1 689	5 923	30.9
9.2	1 698	5 888	30.8
9.3	1 708	5 855	30.7
9.4	1 718	5 821	30.6
9.5	1 728	5 788	30.5
9.6	1 738	5 754	30.4
9.7	1 748	5 721	30.3
9.8	1 758	5 689	30.2
9.9	1 768	5 656	30.1
	Range Decrease Factor	Range Increase Factor	± Decibels Power Change

Power Change, Decibels ±	Range Increase Factor (Decimal Point)	Range Decrease Factor (Decimal Point)	
10.0	1 778	5 623	30.0
10.1	1 789	5 591	29.9
10.2	1 799	5 559	29.8
10.3	1 809	5 527	29.7
10.4	1 819	5 495	29.6
10.5	1 830	5 464	29.5
10.6	1 841	5 433	29.4
10.7	1 851	5 401	29.3
10.8	1 862	5 370	29.2
10.9	1 873	5 340	29.1
11.0	1 884	5 309	29.0
11.1	1 895	5 278	28.9
11.2	1 905	5 248	28.8
11.3	1 916	5 218	28.7
11.4	1 928	5 188	28.6
11.5	1 939	5 158	28.5
11.6	1 950	5 129	28.4
11.7	1 961	5 099	28.3
11.8	1 972	5 070	28.2
11.9	1 984	5 041	28.1
12.0	1 995	5 012	28.0
12.1	2 007	4 983	27.9
12.2	2 018	4 954	27.8
12.3	2 030	4 926	27.7
12.4	2 042	4 898	27.6
12.5	2 054	4 870	27.5
12.6	2 065	4 842	27.4
12.7	2 077	4 814	27.3
12.8	2 089	4 786	27.2
12.9	2 101	4 759	27.1
13.0	2 113	4 732	27.0
13.1	2 126	4 704	26.9
13.2	2 138	4 677	26.8
13.3	2 150	4 650	26.7
13.4	2 163	4 624	26.6
13.5	2 175	4 597	26.5
13.6	2 188	4 571	26.4
13.7	2 200	4 545	26.3
13.8	2 213	4 519	26.2
13.9	2 226	4 493	26.1
14.0	2 239	4 467	26.0
14.1	2 252	4 441	25.9
14.2	2 265	4 416	25.8
14.3	2 278	4 390	25.7
14.4	2 291	4 365	25.6
14.5	2 304	4 340	25.5
14.6	2 317	4 315	25.4
14.7	2 331	4 290	25.3
14.8	2 344	4 266	25.2
14.9	2 358	4 241	25.1
	Range Decrease Factor (Decimal Point)	Range Increase Factor (Decimal Point)	± Decibels Power Change

Power Change, Decibels ±	Range Increase Factor (Decimal Point)	Range Decrease Factor (Decimal Point)	
15.0	2 371	4 217	25.0
15.1	2 385	4 193	24.9
15.2	2 399	4 169	24.8
15.3	2 413	4 145	24.7
15.4	2 427	4 121	24.6
15.5	2 441	4 097	24.5
15.6	2 455	4 074	24.4
15.7	2 469	4 050	24.3
15.8	2 483	4 027	24.2
15.9	2 497	4 004	24.1
16.0	2 512	3 981	24.0
16.1	2 526	3 958	23.9
16.2	2 541	3 936	23.8
16.3	2 556	3 913	23.7
16.4	2 570	3 890	23.6
16.5	2 585	3 868	23.5
16.6	2 600	3 846	23.4
16.7	2 615	3 824	23.3
16.8	2 630	3 802	23.2
16.9	2 645	3 780	23.1
17.0	2 661	3 758	23.0
17.1	2 676	3 737	22.9
17.2	2 692	3 715	22.8
17.3	2 707	3 694	22.7
17.4	2 723	3 673	22.6
17.5	2 738	3 652	22.5
17.6	2 754	3 631	22.4
17.7	2 770	3 610	22.3
17.8	2 786	3 589	22.2
17.9	2 802	3 569	22.1
18.0	2 818	3 548	22.0
18.1	2 835	3 528	21.9
18.2	2 851	3 508	21.8
18.3	2 867	3 487	21.7
18.4	2 884	3 467	21.6
18.5	2 901	3 447	21.5
18.6	2 917	3 428	21.4
18.7	2 934	3 408	21.3
18.8	2 951	3 388	21.2
18.9	2 968	3 369	21.1
19.0	2 985	3 350	21.0
19.1	3 003	3 330	20.9
19.2	3 020	3 311	20.8
19.3	3 037	3 292	20.7
19.4	3 055	3 273	20.6
19.5	3 073	3 255	20.5
19.6	3 090	3 236	20.4
19.7	3 108	3 217	20.3
19.8	3 126	3 199	20.2
19.9	3 144	3 181	20.1
20.0	3 162	3 162	20.0
	Range Decrease Factor (Decimal Point)	Range Increase Factor (Decimal Point)	± Decibels Power Change

Table A4
Ray Paths Calculated for Proposed Standard Model of Atmospheric Refractive Index (CRPL Exponential Reference Atmosphere for $N_S = 313$)

Values of range in nautical miles, for ray of specified initial elevation angle, at selected heights

Height (feet)	Initial Elevation Angle (degrees)								
	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
1,000	39.80	15.86	8.952	6.137	4.651	3.740	3.125	2.684	2.351
2,000	56.20	28.19	17.10	12.00	9.179	7.414	6.212	5.343	4.686
3,000	68.73	38.61	24.63	17.61	13.59	11.03	9.262	7.978	7.004
4,000	79.25	47.80	31.65	23.01	17.89	14.58	12.27	10.59	9.306
5,000	88.49	56.09	38.25	28.21	22.09	18.07	15.25	13.18	11.59
6,000	96.82	63.70	44.48	33.22	26.19	21.51	18.19	15.74	13.86
7,000	104.4	70.77	50.41	38.07	30.21	24.89	21.10	18.28	16.12
8,000	111.5	77.40	56.07	42.77	34.13	28.22	23.97	20.80	18.36
9,000	118.1	83.64	61.49	47.33	37.97	31.50	26.82	23.30	20.58
10,000	124.4	89.57	66.70	51.76	41.73	34.73	29.63	25.78	22.79
20,000	174.1	137.6	110.5	90.59	75.80	64.67	56.11	49.41	44.05
30,000	211.3	174.1	145.2	122.6	105.0	91.15	80.13	71.24	63.98
40,000	242.2	204.6	174.5	150.4	130.9	115.1	102.2	91.56	82.74
50,000	269.2	231.3	200.4	175.1	154.2	137.0	122.6	110.6	100.5
60,000	293.3	255.2	223.8	197.6	175.7	157.3	141.8	128.6	117.4
70,000	315.3	277.0	245.2	218.4	195.7	176.3	159.8	145.7	133.5
80,000	335.6	297.3	265.1	237.8	214.4	194.3	176.9	161.9	148.9
90,000	354.7	316.2	283.8	256.0	232.0	211.3	193.2	177.5	163.7
100,000	372.6	334.1	301.4	273.3	248.8	227.5	208.8	192.5	178.1
200,000	516.1	477.2	443.2	413.1	386.0	361.5	339.2	318.9	300.5
300,000	625.5	586.4	551.9	520.9	492.6	466.6	442.6	420.4	399.8
400,000	717.7	678.4	643.6	612.1	583.1	556.2	531.1	507.7	485.8
500,000	799.0	759.7	724.7	692.8	663.2	635.7	610.0	585.7	562.9
600,000	872.6	833.3	798.1	766.0	736.1	708.1	681.8	656.9	633.3
700,000	940.6	901.2	865.9	833.5	803.3	775.0	748.2	722.9	698.8
800,000	1004	964.5	929.1	896.6	866.2	837.6	810.5	784.7	760.1
900,000	1064	1024	988.8	956.1	925.5	896.6	869.2	843.1	818.1
1,000,000	1120	1081	1045	1013	981.8	952.7	925.0	898.6	873.3

(Table Continues)

Table A4 (Continued)
Ray Paths Calculated for Proposed Standard Model of Atmospheric Refractive Index
(CRPL Exponential Reference Atmosphere for $N_S = 313$)

Values of range in nautical miles, for ray of specified initial elevation angle, at selected heights

Height (feet)	Initial Elevation Angle (degrees)								
	4.5	5.0	6.0	7.0	8.0	9.0	10	15	20
1,000	2.092	1.884	1.572	1.349	1.182	1.051	0.947	0.636	0.481
2,000	4.172	3.760	3.139	2.695	2.361	2.101	1.893	1.271	0.962
3,000	6.241	5.627	4.701	4.037	3.538	3.150	2.839	1.906	1.443
4,000	8.298	7.486	6.259	5.377	4.714	4.197	3.783	2.541	1.924
5,000	10.34	9.336	7.811	6.713	5.887	5.242	4.726	3.176	2.404
6,000	12.38	11.18	9.358	8.047	7.058	6.286	5.667	3.810	2.885
7,000	14.40	13.01	10.90	9.377	8.227	7.328	6.608	4.444	3.365
8,000	16.41	14.84	12.44	10.70	9.393	8.369	7.548	5.077	3.845
9,000	18.41	16.65	13.97	12.03	10.56	9.409	8.486	5.710	4.326
10,000	20.40	18.46	15.50	13.35	11.72	10.45	9.424	6.343	4.805
20,000	39.69	36.09	30.49	26.37	23.22	20.74	18.73	12.65	9.597
30,000	57.97	52.93	45.00	39.07	34.50	30.87	27.92	18.93	14.37
40,000	75.33	69.05	59.04	51.46	45.55	40.84	37.00	25.16	19.13
50,000	91.89	84.52	72.64	63.54	56.39	50.65	45.95	31.36	23.88
60,000	107.7	99.40	85.84	75.34	67.03	60.31	54.79	37.52	28.61
70,000	122.9	113.8	98.67	86.87	77.46	69.82	63.51	43.65	33.32
80,000	137.6	127.6	111.1	98.15	87.72	79.19	72.12	49.73	38.02
90,000	151.7	141.1	123.3	109.2	97.79	88.43	80.63	55.79	42.70
100,000	165.3	154.1	135.2	120.0	107.7	97.53	89.04	61.80	47.37
200,000	283.6	268.2	241.1	218.3	199.0	182.4	168.2	120.1	93.20
300,000	380.7	363.0	331.2	303.6	279.6	258.6	240.2	175.5	137.6
400,000	465.3	446.1	411.0	380.1	352.7	328.4	306.8	228.4	180.8
500,000	541.3	521.0	483.6	450.1	420.1	393.2	369.0	279.1	222.9
600,000	611.0	589.8	550.6	515.1	483.1	454.0	427.7	328.0	263.9
700,000	675.8	653.9	613.2	576.1	542.4	511.6	483.5	375.1	303.9
800,000	736.7	714.2	672.3	633.9	598.7	566.4	536.7	420.8	343.1
900,000	794.3	771.4	728.4	688.8	652.4	618.8	587.8	465.1	381.5
1,000,000	849.1	825.8	782.0	741.4	703.9	669.1	637.0	508.2	419.1

Table A4 (Continued)
Ray Paths Calculated for Proposed Standard Model of Atmospheric Refractive Index (CRPL Exponential Reference Atmosphere for N_s = 313)

Values of range in nautical miles, for ray of specified initial elevation angle, at selected heights

Height (feet)	Initial Elevation Angle (degrees)							
	25	30	40	50	60	70	80	90
1,000	0.389	0.329	0.256	0.215	0.190	0.175	0.167	0.165
2,000	0.779	0.658	0.512	0.430	0.380	0.350	0.334	0.329
3,000	1.168	0.987	0.768	0.644	0.570	0.525	0.501	0.494
4,000	1.557	1.316	1.024	0.859	0.760	0.701	0.669	0.658
5,000	1.946	1.645	1.280	1.074	0.950	0.876	0.836	0.823
6,000	2.335	1.974	1.536	1.289	1.140	1.051	1.003	0.987
7,000	2.724	2.303	1.792	1.504	1.330	1.226	1.170	1.152
8,000	3.113	2.632	2.048	1.719	1.520	1.401	1.337	1.317
9,000	3.502	2.961	2.304	1.933	1.710	1.576	1.504	1.481
10,000	3.891	3.290	2.560	2.148	1.900	1.751	1.671	1.646
20,000	7.775	6.576	5.118	4.296	3.800	3.503	3.342	3.292
30,000	11.65	9.858	7.675	6.443	5.700	5.254	5.013	4.937
40,000	15.52	13.14	10.23	8.589	7.600	7.005	6.685	6.583
50,000	19.38	16.41	12.78	10.74	9.499	8.756	8.356	8.229
60,000	23.24	19.68	15.34	12.88	11.40	10.51	10.03	9.875
70,000	27.08	22.94	17.89	15.02	13.30	12.26	11.70	11.52
80,000	30.92	26.20	20.43	17.17	15.20	14.01	13.37	13.17
90,000	34.75	29.46	22.98	19.31	17.09	15.76	15.04	14.81
100,000	38.57	32.71	25.53	21.45	18.99	17.51	16.71	16.46
200,000	76.35	64.97	50.89	42.83	37.95	35.01	33.42	32.92
300,000	113.4	96.82	76.09	64.15	56.88	52.50	50.13	49.37
400,000	149.8	128.3	101.1	85.39	75.79	69.97	66.83	65.83
500,000	185.5	159.3	126.0	106.6	94.66	87.44	83.53	82.29
600,000	220.6	190.0	150.8	127.7	113.5	104.9	100.2	98.75
700,000	255.1	220.4	175.4	148.7	132.3	122.3	116.9	115.2
800,000	289.2	250.4	199.8	169.7	151.1	139.8	133.6	131.7
900,000	322.7	280.1	224.1	190.7	169.9	157.2	150.3	148.1
1,000,000	355.7	309.5	248.3	211.5	188.6	174.6	167.0	164.6

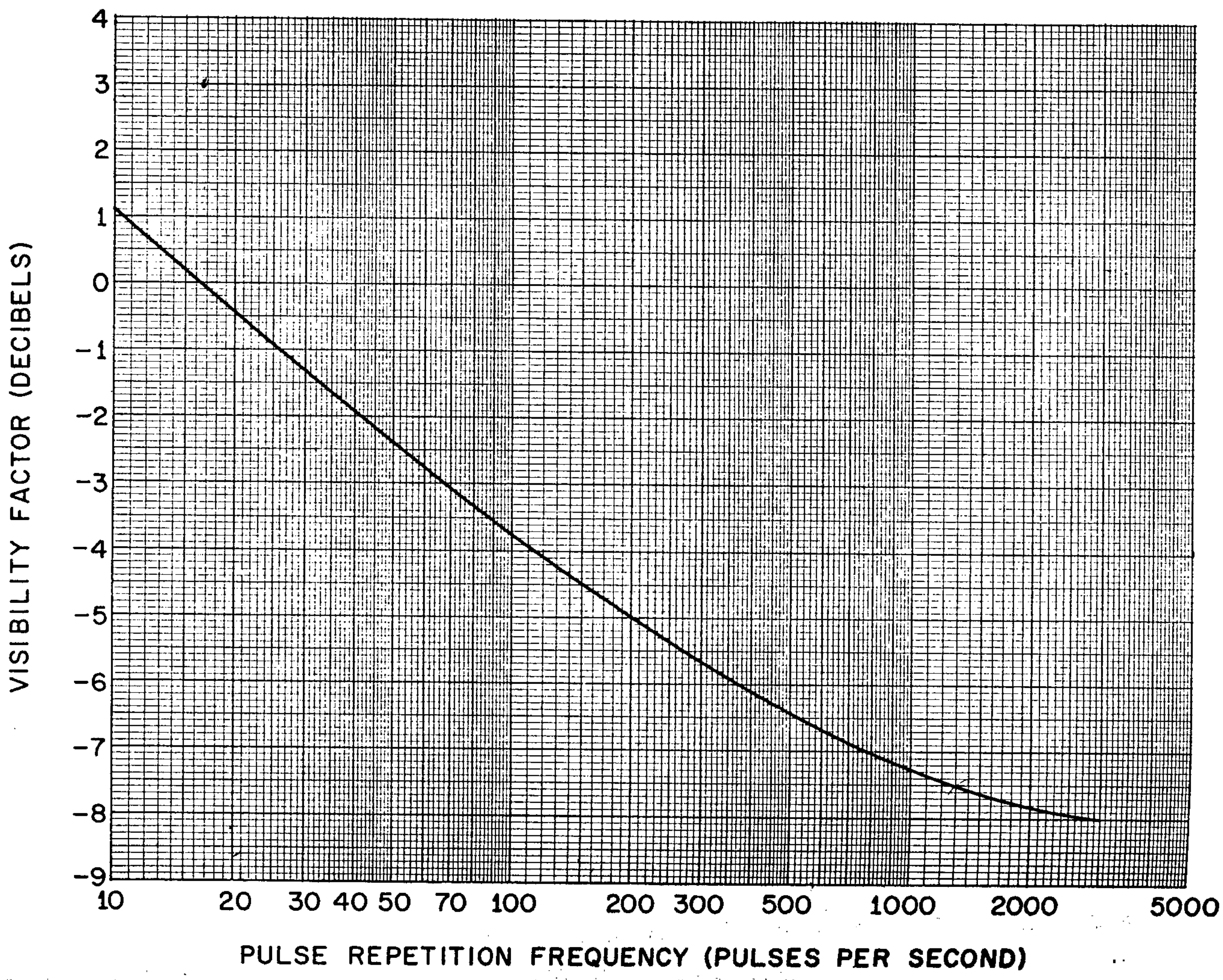


Fig. A1 - Visibility factor $V_{o(50)db}$ for type-A cathode-ray-tube display; based on Figs. 8.2 and 8.23 of Ref. 10, or Figs. 1 and 4 of Ref. 11. The values given in Fig. 8.23 for $Br \geq 1.2$ have been adjusted to 0.5 probability in accordance with Fig. 8.2

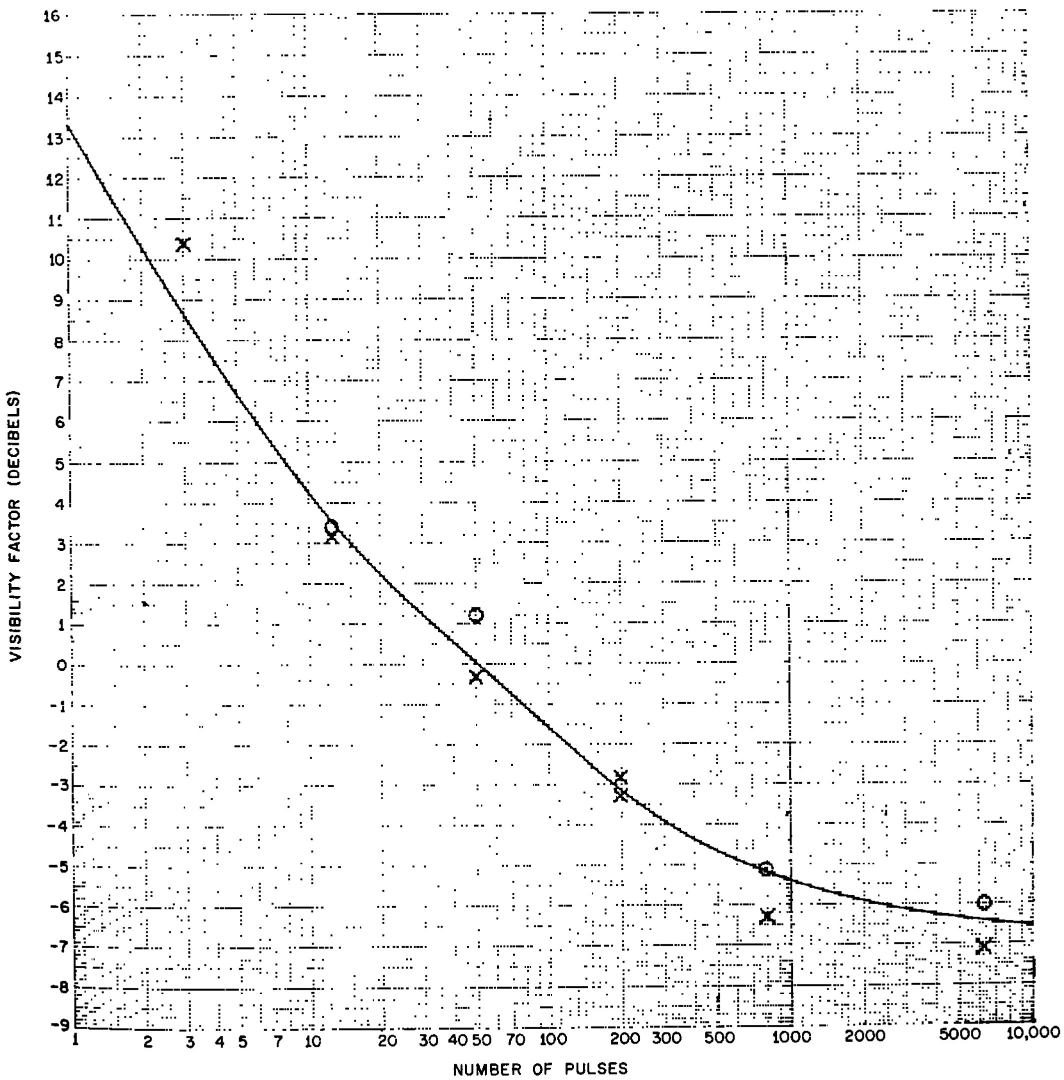


Fig. A2 - Visibility factor $V_{o(50)db}$ for PPI cathode-ray-tube display (applicable to intensity-modulated displays generally); based on Figs. 8.2 and 9.2 of Ref. 10 or Figs. 1 and 21 of Ref. 11, adjusted to 0.5 probability and extrapolated to single-pulse detection, with slight revision of slopes at ends of curve.

Fig. A1 - Visibility factor $V_{o(50)db}$ for type-A cathode-ray-tube display; based on Figs. 8.2 and 8.23 of Ref. 10, or Figs. 1 and 4 of Ref. 11. The values given in Fig. 8.23 for $B\tau \approx 1.2$ have been adjusted to 0.5 probability in accordance with Fig. 8.2

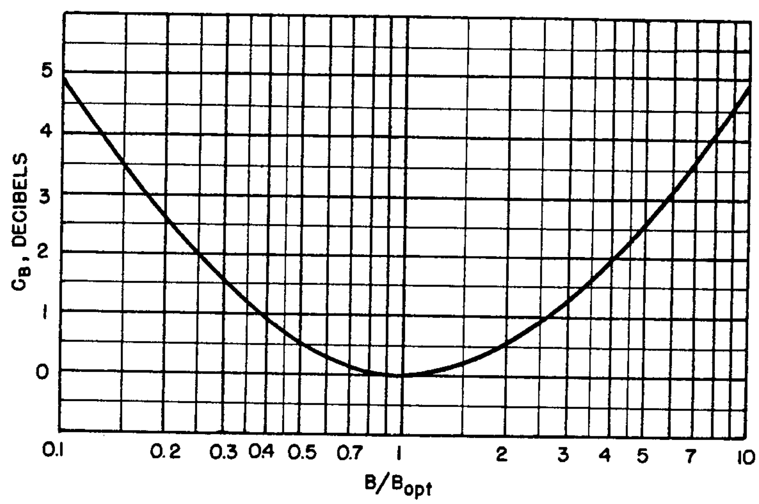


Fig. A3 - Bandwidth correction factor $C_{B(db)}$ as a function of the ratio of the actual predetection bandwidth B to the optimum value $B_{opt} = 1.2/\tau$, for cathode-ray-tube indicator and human observer

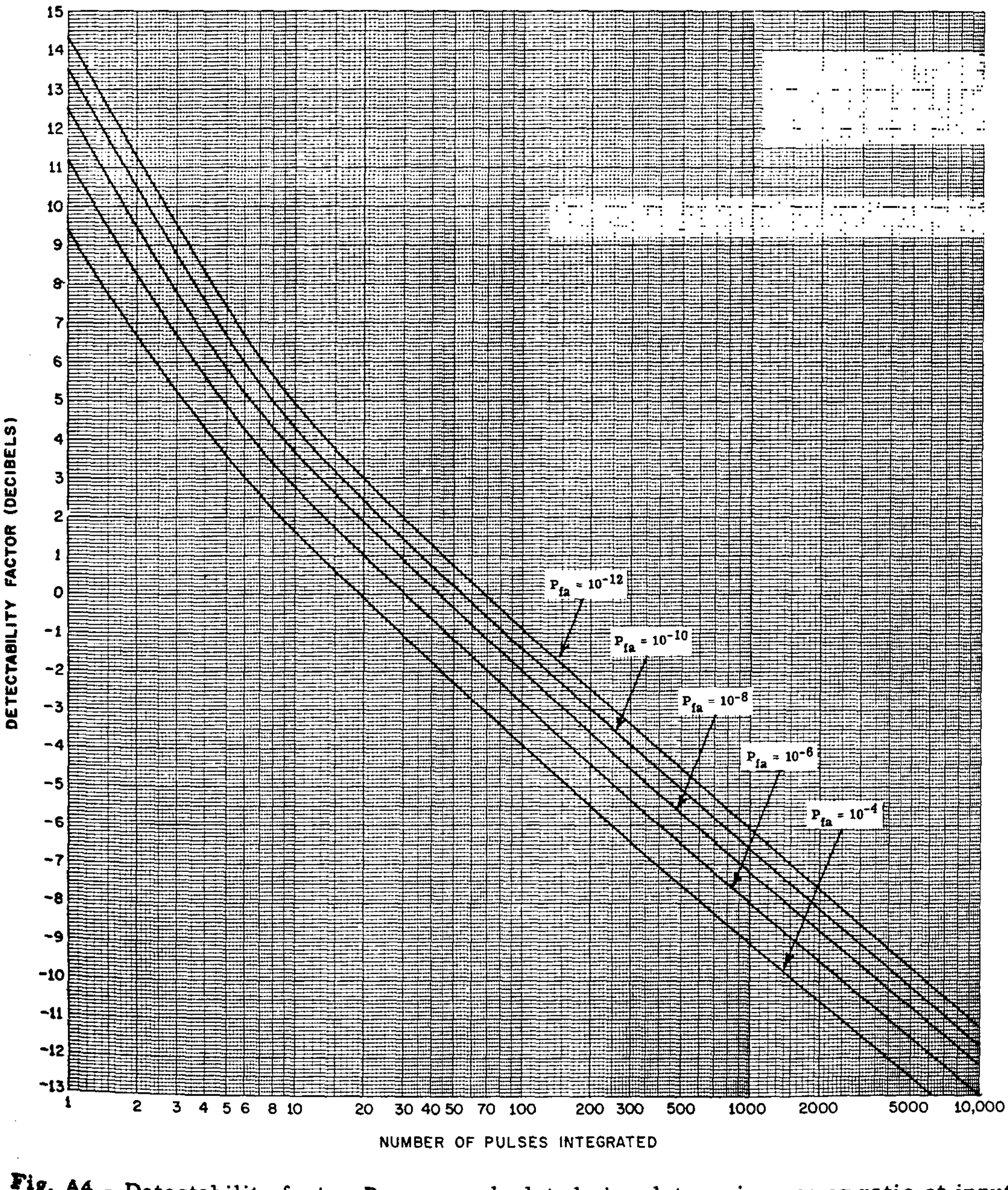


Fig. A4 - Detectability factor $D_{50(db)}$; calculated signal-to-noise power ratio at input of linear-rectifier detector followed by perfect-memory linear video integrator and a fixed-threshold-level automatic-decision device, for 0.5 probability of detection and several values of false-alarm probability

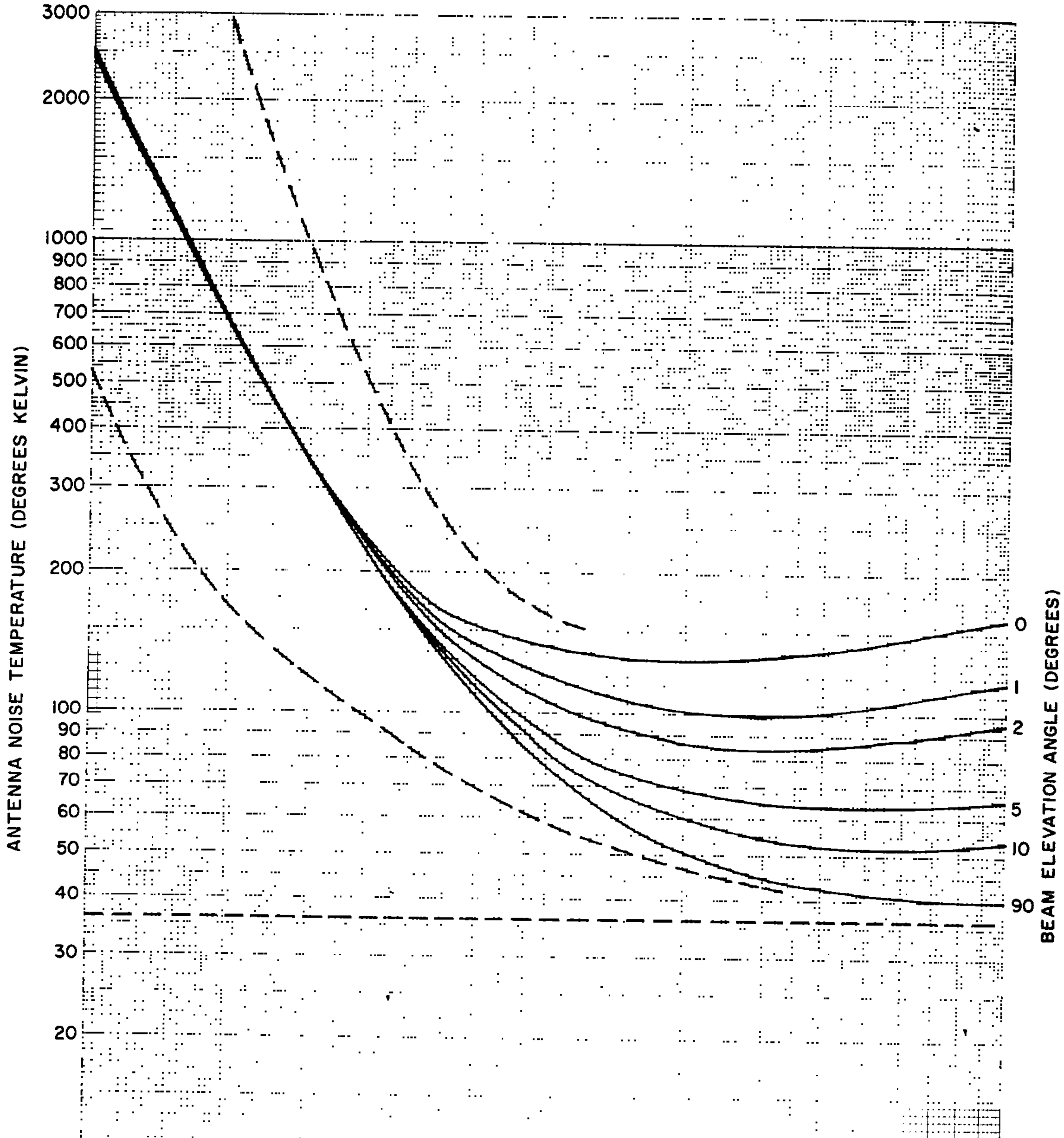


Fig. A5 - Antenna noise temperature for typical conditions of cosmic, solar, atmospheric, and ground noise. The dashed curves indicate the maximum and minimum levels of cosmic and atmospheric noise likely to be observed. The horizontal dashed line is the assumed level of ground-noise contribution (36°K).

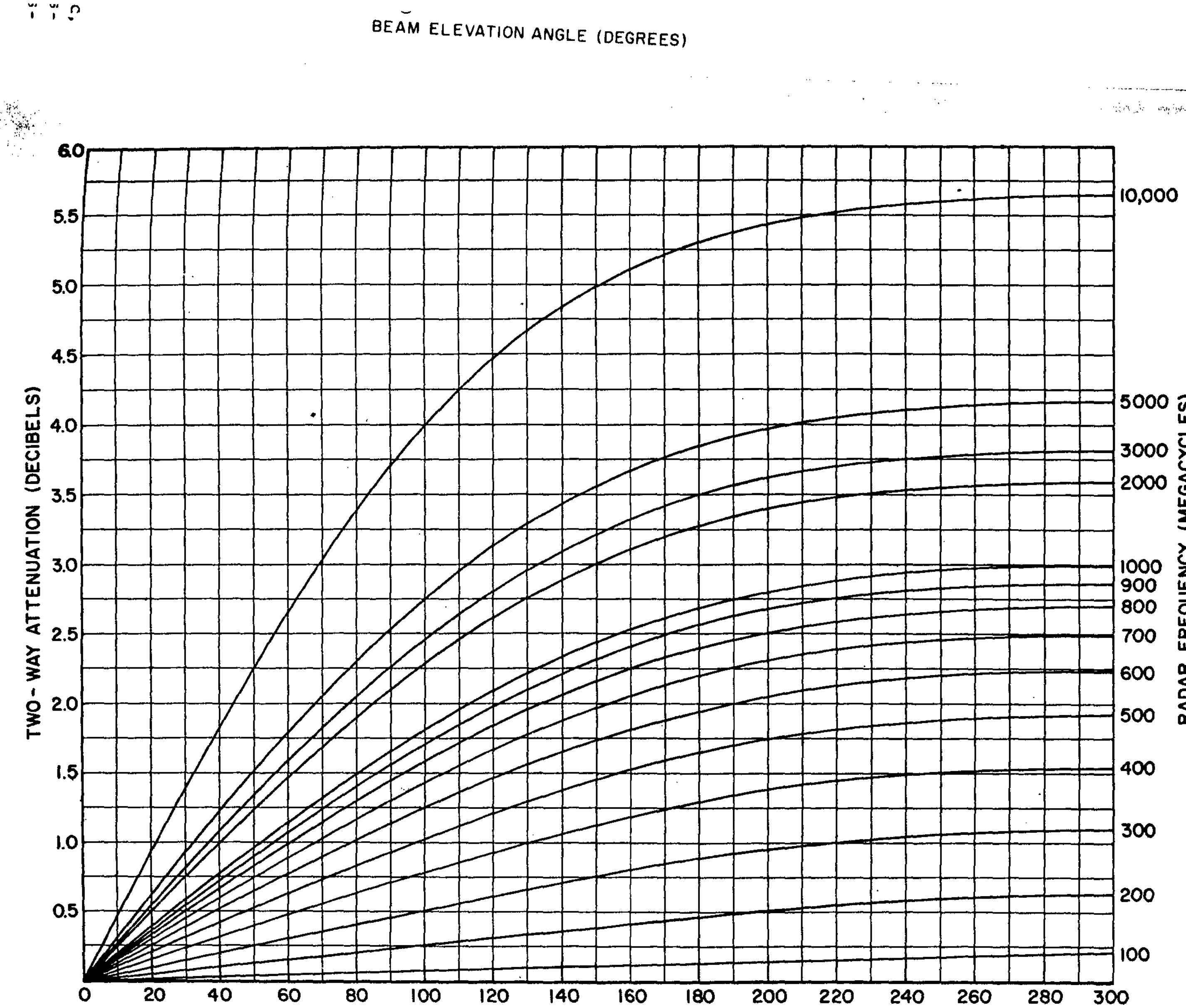


Fig. A6 - Radar atmospheric attenuation, 0-degree ray elevation angle, 100-10,000 Mc

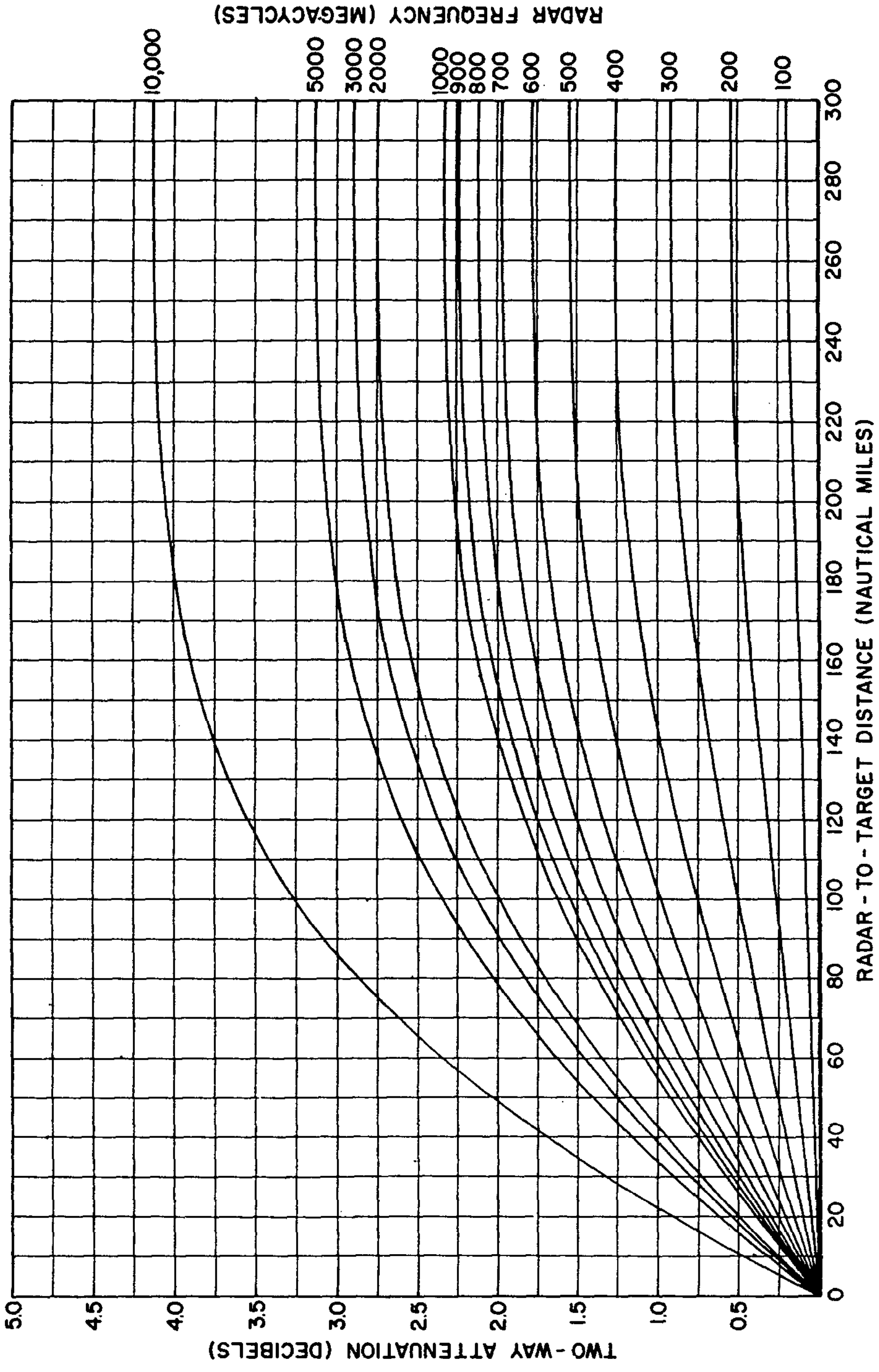


Fig. A7 - Radar atmospheric attenuation, 0.5-degree ray elevation angle, 100-10,000 Mc

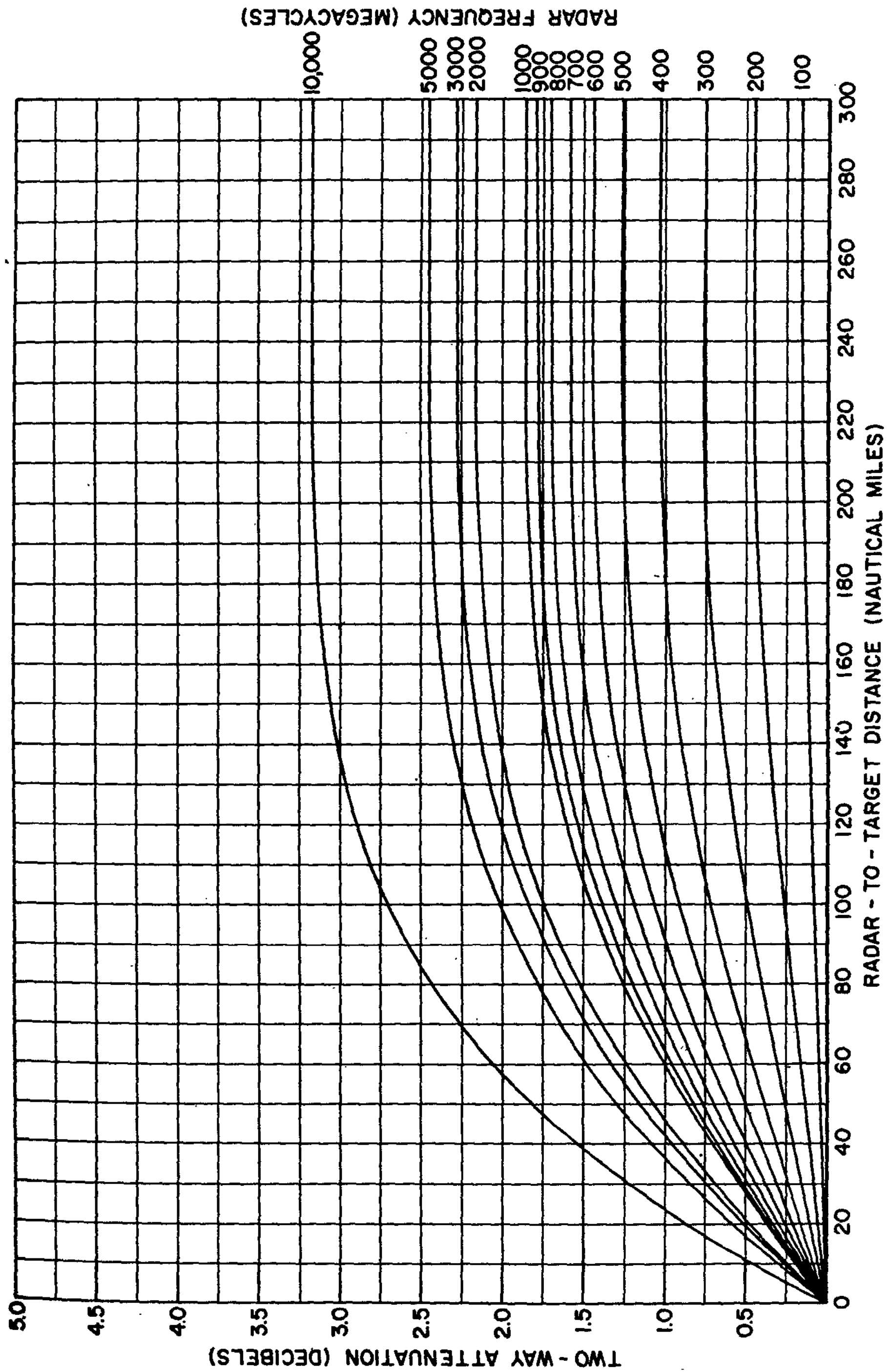


Fig. A8 - Radar atmospheric attenuation, 1.0-degree ray elevation angle, 100-10,000 Mc

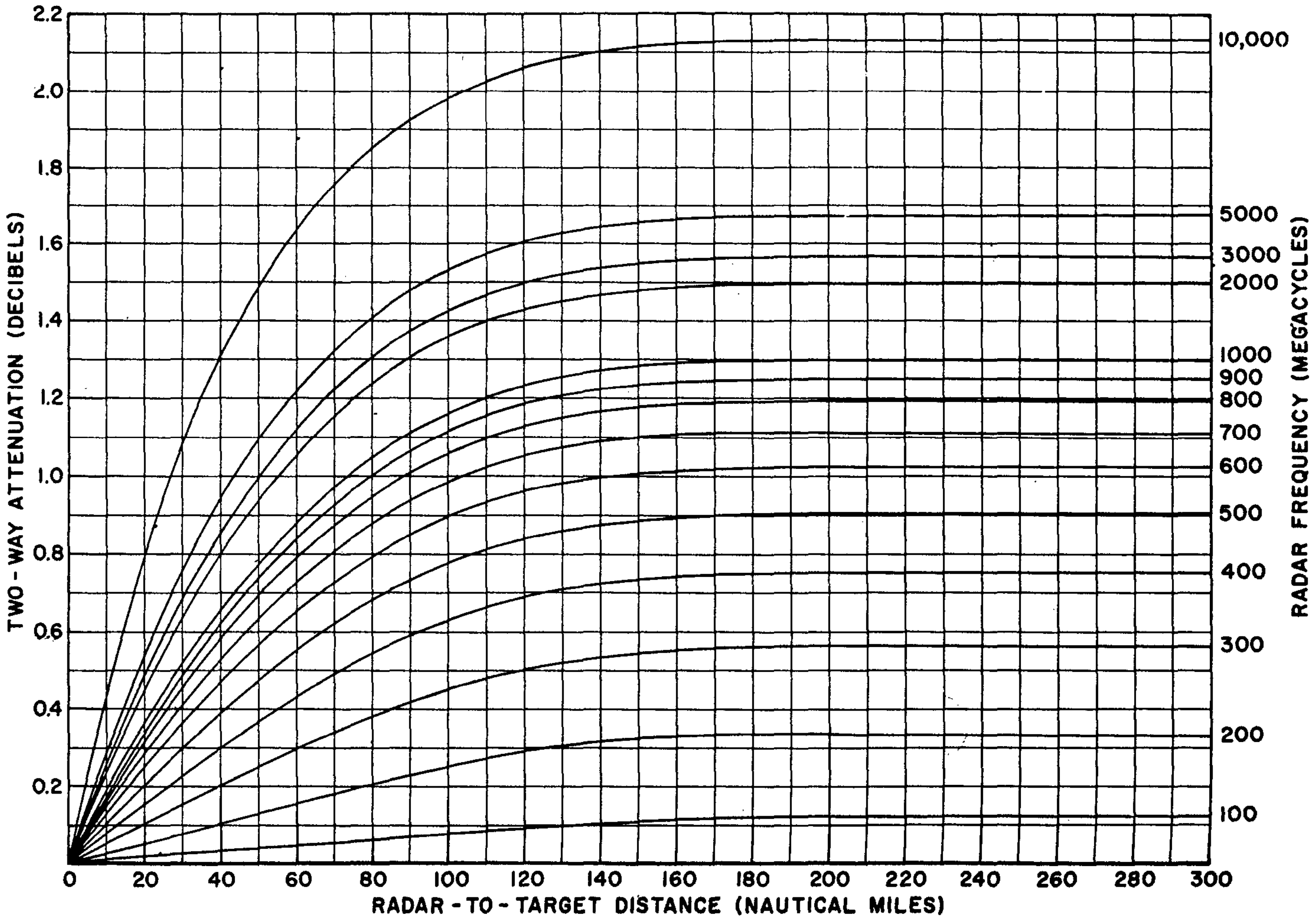


Fig. A9 - Radar atmospheric attenuation, 2.0-degree ray elevation angle, *100-10,000 Mc*

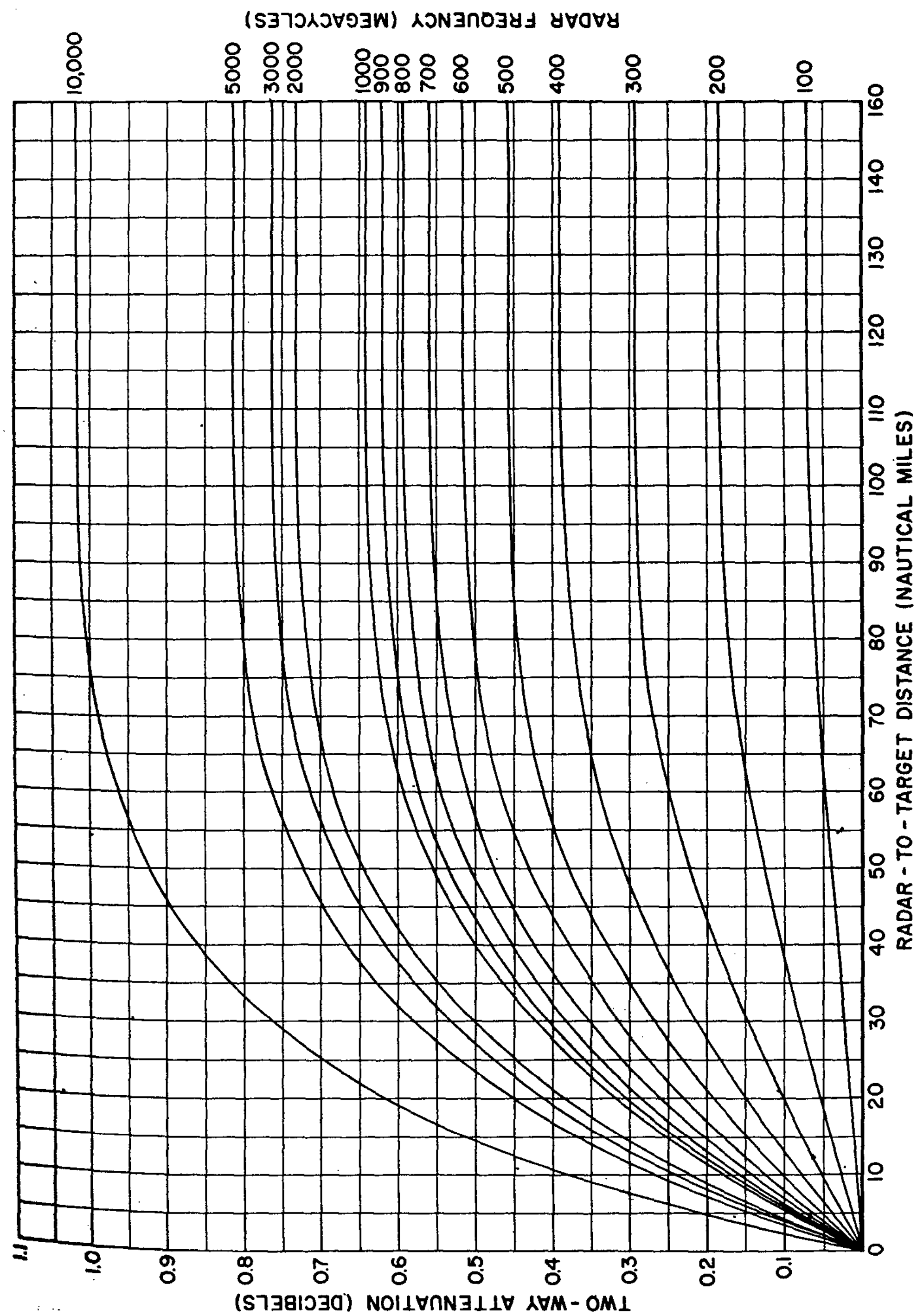


Fig. A10 - Radar atmospheric attenuation, 5.0-degree ray elevation angle, 100-10,000 Mc

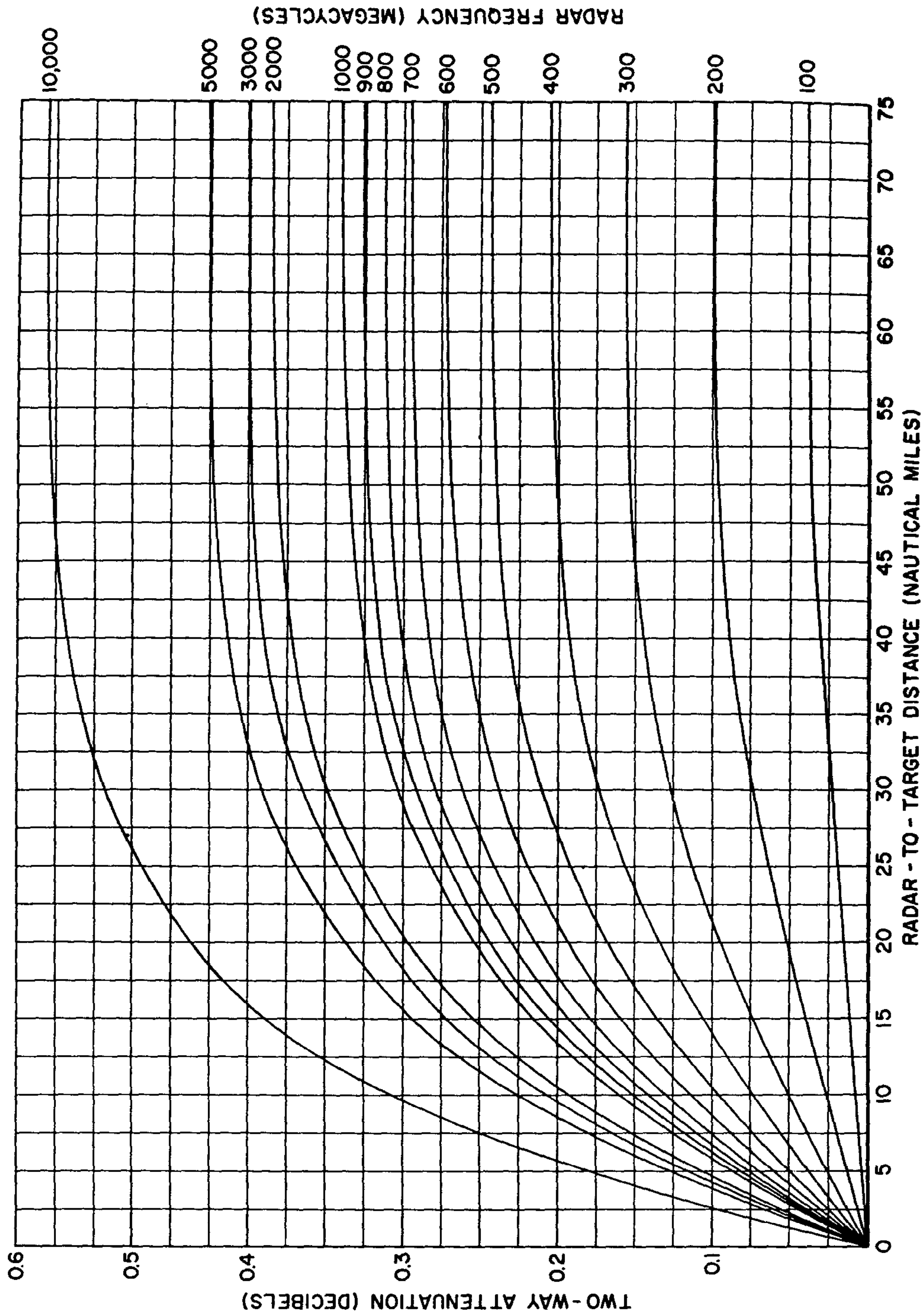
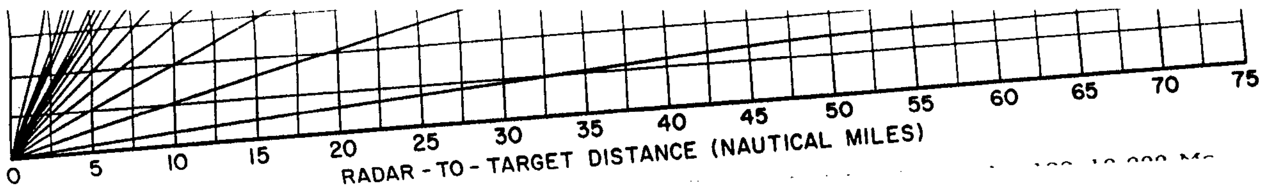


Fig. A11 - Radar atmospheric attenuation, 10-degree ray elevation angle, 100-10,000 Mc



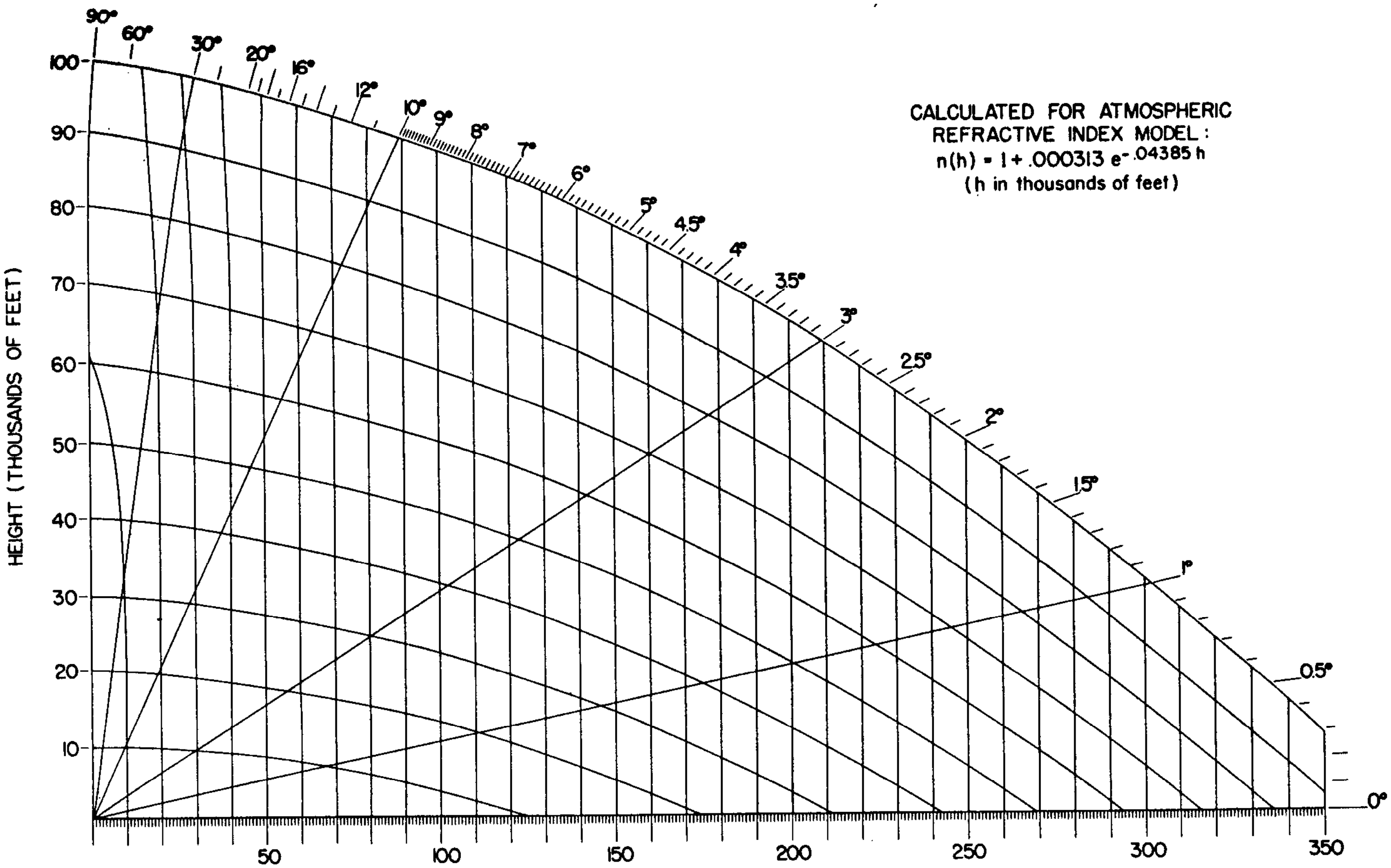


Fig. A12 - Radar range-height-angle chart calculated for an exponential model of the atmospheric refractive index. ("Radar range" means "distance along the ray path." Elevation angles are angles of rays with respect to horizontal at radar antenna.)

PULSE-RADAR RANGE-CALCULATION WORK SHEET

For use with NRL Report 5868

1. Compute system input noise temperature, T_{NI}, following outline in section (1) below.

2. Enter range factors known in other than decibel form in section (2) below, for reference.

3. Enter logarithmic and decibel values in section (3) below, positive values in plus column, negative in minus. (Example: If $V_{o(50)(db)}$ as given by Fig. A1 or A2 is negative, then $-V_{o(50)(db)}$ is positive, goes in plus column.) To convert range factors to decibel values, use Table A2. For $C_{B(db)}$ use Fig. A3.

Radar antenna height: $h =$ ft.	Target elevation angle: $\theta =$ °. (See Fig. A12).

(1) COMPUTATION OF T_{NI}:

$$T_{NI} = T_a + T_{r(I)} + L_r T_e$$

(a) For general range computation, use Figure A5 for T_a.

(b) To find L_r, given $L_{r(db)}$, use first and second columns of Table A2.

(c) Also in Table A2, opposite $L_{r(db)}$ in first column, read $T_{r(I)}$ in third column.

Note: If thermal temperature (T_t) of transmission line is appreciably different from 290°K, multiply Table A2 values of $T_{r(I)}$ by $T_t/290$.

(d) Opposite $\overline{NF}_{db}$ in first column, read T_e in third col.

T_t		T_a	
L_r		$T_{r(I)}$	
$\overline{NF}_{db}$		$L_r\ T_e$	
T_e		T_{NI}	

(2) RANGE FACTORS		(3) DECIBEL VALUES	PLUS (+)	MINUS (-)
$P_{t(kw)}$		$10 \log P_{t(kw)}$	.	.
$\tau_{\mu sec}$		$10 \log \tau_{\mu sec}$	.	.
G_t		$G_{t(db)}$	.	.
G_r		$G_{r(db)}$	.	.
$\sigma_{50(sq.m.)}$		$10 \log \sigma_{50}$	.	.
f_{Mc}		$-20 \log f_{Mc}$	.	.
T_{NI}, °K		$-10 \log T_{NI}$	.	.
$V_{o(50)}$		$-V_{o(50)(db)}$	.	.
C_B		$-C_{B(db)}$		.
L_t		$-L_{t(db)}$		.
L_p		$-L_{p(db)}$		.
L_x		$-L_{x(db)}$		.
Range-equation constant (40 log 1.292)			4 .45	
4. Obtain column totals →			.	.
5. Enter smaller total below larger →			.	.
6. Subtract to obtain net decibels →			+ .	- .

7. In Table A3, find range ratio corresponding to this net decibel value, taking its sign (±) into account. Multiply this ratio by 100. This is R_o → ☐

8. Multiply R_o by the pattern-propagation factor. $\boxed{F =}$ See Eqs. 27 - 44, and Figs. 8 - 12.
$R_o \times F = R'$ → ☐

9. On the appropriate curve of Figures A6 - A11, determine the atmospheric-absorption loss factor, $L_{a(db)}$, corresponding to R'. This is $L_{a(db)(1)}$ → ☐

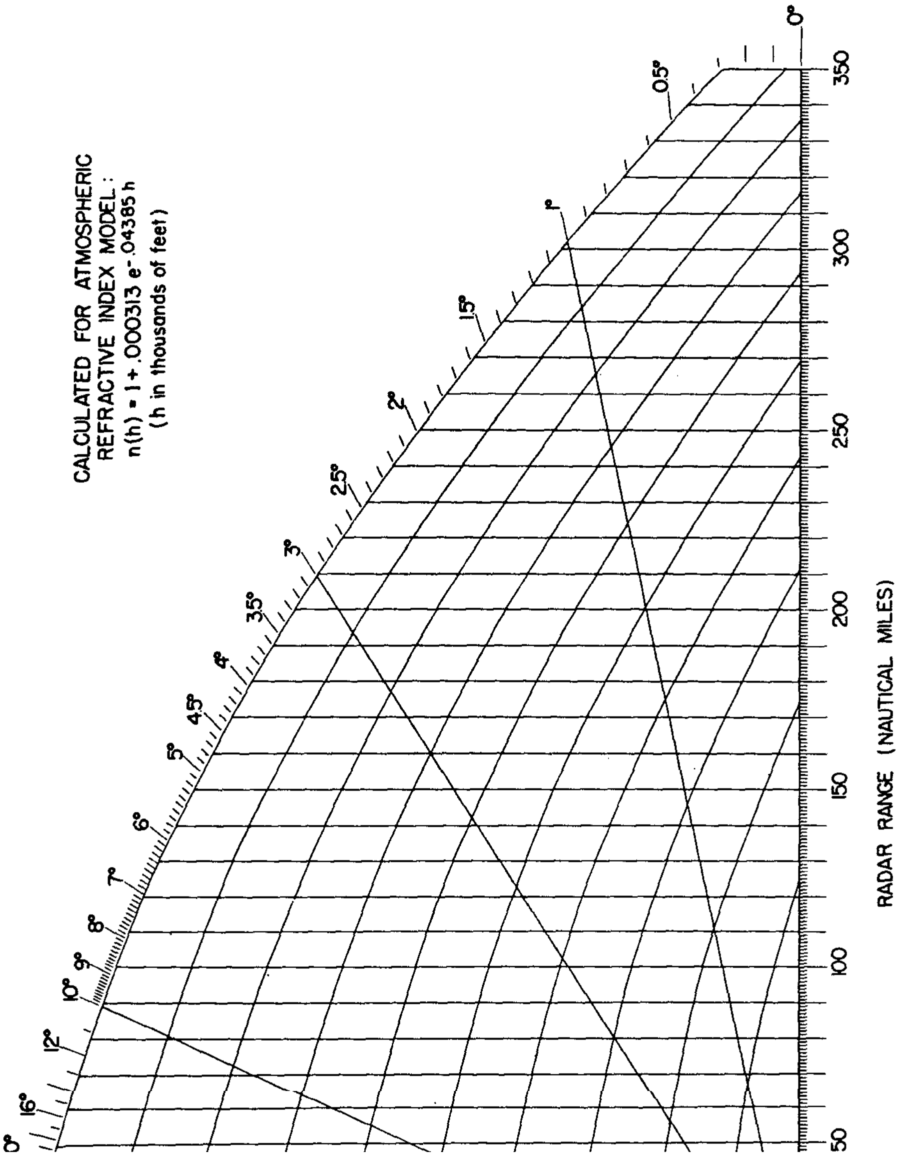
10. In Table A3, find the range-decrease factor corresponding to $L_{a(db)(1)}$. δ_1 → ☐

11. Multiply R' by δ_1. This is a first approximation of the range, R_1 → ☐

12. If R_1 differs appreciably from R', on the appropriate curve of Figures A6 - A11 find the new value of $L_{a(db)}$ corresponding to R_1. This is $L_{a(db)(2)}$ → ☐

13. In Table A3, find the range-increase factor corresponding to the difference between $L_{a(db)(1)}$ and $L_{a(db)(2)}$. This is δ_2. → ☐

Note: If the difference betw


CALCULATED FOR ATMOSPHERIC
REFRACTIVE INDEX MODEL:
n(h) = 1 + .000313 e^-.04385 h
(h in thousands of feet)
RADAR RANGE (NAUTICAL MILES)
50
100
150
200
250
300
350
0°
0.5°
1°
1.5°
2°
2.5°
3°
3.5°
4°
4.5°
5°
6°
7°
8°
9°
10°
12°
16°