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13. ABSTRACT

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CONTENTS

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PROBLEM STATUS

This is an interim report on the problem; work is continuing.

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NOMENCLATURE

a = depth of a surface crack or edge crack

a_0 = initial crack size for fatigue life calculations

a_{cr} = critical crack size for failure

a_p = maximum crack size that can be sustained under proof testing

C_1, C_2, C_3, C_8, C_9 = numerical constant

$2c$ = length of a surface crack

da/dN or $d(2c)/dN$ = fatigue crack growth rate

E = Young's modulus

Δe = nominal strain range in a notched member

K_f = fatigue strength reduction factor

K_T = theoretical stress concentration factor

K = fracture mechanics stress-intensity factor

ΔK = stress-intensity factor range (maximum K minus minimum K)

K_{max} = maximum stress-intensity factor

K_c = critical stress-intensity factor for fracture in plane stress

K_{Ic} = critical stress-intensity factor for fracture in plane strain

K_{Iscc} = critical stress-intensity factor threshold level for stress-corrosion cracking to occur

M, M_B, Q = component geometry and flaw-shape parameter

m_1, m_2, m_3, m_8, m_9 = numerical exponent

N_f = number of cycles to failure

R = stress ratio (minimum stress/maximum stress)

$2r_y$ = crack-tip plastic zone size

$\Delta s, \Delta\sigma$ = nominal stress range in a notched member

t = plate thickness

UTS = ultimate tensile strength

ϵ_T = total strain range (elastic plus plastic)

$\epsilon_{p.l.}$ = proportional limit strain range

σ = stress

σ_{max} = maximum stress

σ_n = endurance limit stress

σ_p = proof stress

σ_{ys} = 0.2% offset yield strength stress

BASIC CONCEPTS FOR DESIGN AGAINST STRUCTURAL FAILURE BY FATIGUE CRACK PROPAGATION

INTRODUCTION

Problems involving structural fatigue are outpacing fatigue-prevention technology, despite a thriving activity in fatigue research. There are basically two reasons for this phenomenon: the application of higher strength materials, and a wave of rising expectations for the structural performance of these new materials. Higher strength alloys are prone to fatigue problems because advances in static yield strength seldom, if ever, are matched by comparable improvements (or any improvements) in fatigue resistance (1). This fundamental inequity in materials properties then results in expensive high-performance structures which are unreliable in service. Throughout aerospace, hydrospace, and high-speed surface-transportation technologies, escalating demands for improved structural performance are on a collision course with marginal improvements currently available in the fatigue resistance of materials. The solution to this dilemma lies not only in continued fatigue research, but also in an understanding of the nature and severity of the structural-fatigue problem by designers and an awareness of advanced design procedures to ward off crack-propagation failures.

THE STRUCTURAL FATIGUE PROBLEM

Although metal fatigue has been recognized and studied as a failure mechanism for more than a century, recognition of structural fatigue as of major importance is a relatively recent phenomenon. Prior to the last two decades, most fatigue failures and virtually all of fatigue research were confined to machinery components (e.g., axles, gears, shafts) rather than entire structures or structural elements (e.g., pressure vessels, airframes, rocket cases). However, within the past two decades, advances in high-strength alloy development combined with advances in fabrication technology have combined to produce massive monolithic structures which are expected to sustain repeated applications of high stresses in service. It is this class of structures which is prone to disastrous failures from metal fatigue. One of the earliest and most highly publicized examples was the ill-fated British Comet aircraft, which were recalled from service following several crashes caused by metal fatigue during the early 1950's (2). The fact that other examples exist and continue to occur is attested to by the formation and continued activities of numerous professional committees devoted to various aspects of the structural-fatigue problem.

The common theme that runs through every facet of the structural-fatigue problem is the unrelenting push to use higher strength materials. A rash of aircraft-fatigue problems followed attempts to substitute higher strength 7075 aluminum for 2024 aluminum alloys previously employed. Fatigue in pressure vessels was seldom a serious problem until higher strength low-alloy steels began to replace mild steel. The substitute higher strength alloys most commonly have similar or inferior fatigue resistance, but nevertheless, are expected to sustain higher working stresses purely on the basis of higher static yield-stress properties. The consequences of such moves, without carefully examining the fatigue aspects, have provided material for countless failure analyses and fatigue-testing programs.

Furthermore, the fatigue analyses employed, when conducted, are most commonly based on past experience with machinery components and bear little relation to actual events in structural fatigue. Machinery components are designed to prevent crack initiation in precisely machined members at long fatigue lives. Structures frequently must be designed to prevent terminal crack propagation resulting from cost-conscious fabrication at much shorter fatigue lives. Approaches to structural fatigue must take these realities into account or else simply become a meaningless exercise conducted to satisfy contract specifications.

THE TRADITIONAL APPROACH TO FATIGUE

Endurance Limit Fatigue

Some of the earliest instances of fatigue failures began to occur with the development of mechanized transportation in the nineteenth century (1,3). These were failures of horse-drawn coach and railway axles. This fatigue technology has now advanced to the point where most automotive and railway components last indefinitely under fatigue loading. For instance, many engine and driveline components in modern heavy commercial vehicles are designed to last 500,000 miles. Thus, the technology now exists to virtually eliminate fatigue in high-quality manufactured articles which must sustain millions of cycles of load at relatively low stresses.

The traditional analytical approach to fatigue is embodied in the S-N (cyclic stress versus number of cycles to failure) diagram, Fig. 1. The strict definition of "cycles to failure" varies from one researcher to another and can either be initiation of a small visible macrocrack or final fracture of the test specimen. At long fatigue lives the difference is irrelevant, but this point becomes more important at short fatigue lives. For ferrous alloys, the salient feature of the S-N diagram is the horizontal portion of the curve at long fatigue lives, indicating an "endurance limit stress" below which fatigue failures will not occur. Nonferrous alloys generally do not exhibit true endurance limit behavior; however, for most alloys the slope of the S-N curve becomes nearly flat beyond 10^7 cycles.

Endurance-limit fatigue involves relatively low elastic stress levels, not more than half of the ultimate tensile strength of the material and frequently much less than this value (Fig. 2). Because endurance-limit fatigue involves low elastic stresses, it is subject to rigorous analysis. S-N fatigue data for a given alloy are sensitive to geometry and surface finish, as well as environment (all fatigue data are sensitive to environment). Mean stress (residual stress) is also an extremely important factor in endurance-limit fatigue.

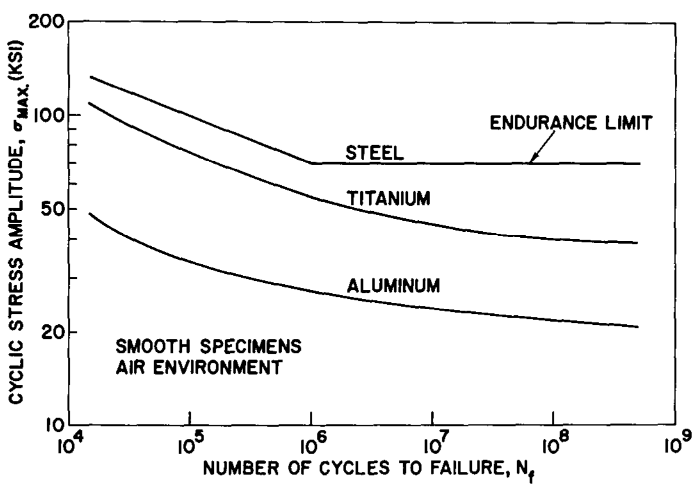


Fig. 1 - Typical S-N fatigue curves for several ferrous and nonferrous high-strength alloys. Note the distinct horizontal endurance limit for the steel vs the gradually sloping curves for the nonferrous alloys at long fatigue lives beyond 10^6 cycles.

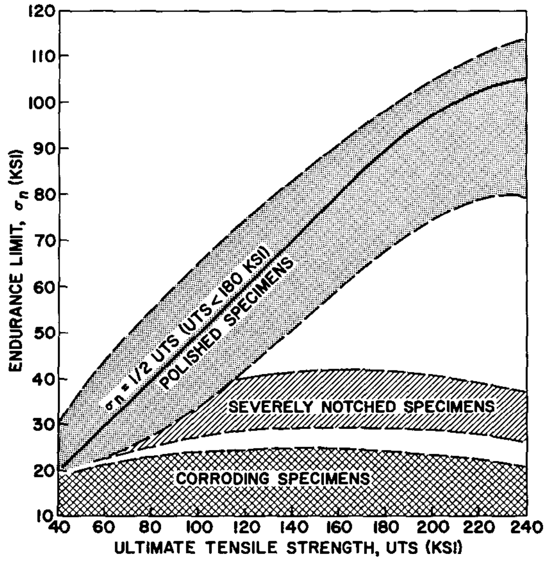


Fig. 2 - Endurance limit characteristics of steels as a function of tensile strength. This shows the effects of notches and corrosion as reported by Dolan (4).

Geometry affects stress concentration in fatigue, and the endurance limit for notched specimens falls well below the curves for highly polished unnotched specimens (Fig. 2). Fatigue strength reduction factors for notches of given geometry are highly useful for estimating the permissible stress levels in components which must contain geometric discontinuities under fatigue loading. Because of the absence of macroscale localized yielding in endurance-limit fatigue, highly accurate analyses can be made of local stresses around notches and other geometric discontinuities of known geometry. A considerable bibliography of such information has been developed over the years (4,5).

Since fatigue-crack initiation is a surface phenomenon, surface finish is of paramount importance (4,5). Not surprisingly, the traditional arsenal of weapons against metal fatigue includes grinding, polishing, shot peening, carburizing, etc. Each of these processes either smooths the surface, thus eliminating stress-concentration sites for crack initiation, or leaves the surface in residual compression, or both at once. Residual compressive stresses are highly beneficial to all aspects of metal fatigue. Conversely, aggressive environmental attack is highly deleterious to all aspects of fatigue, and, with few exceptions, corroded surfaces result in significantly diminished fatigue lives (Fig. 2).

Finite-Life Fatigue

The technology for preventing fatigue-crack initiation indefinitely in service is well developed and widely utilized. However, structural fatigue remains a critical problem because of our inability to apply this technology to large structures. If all stresses were kept down to endurance-limit levels, weight-critical structures such as aircraft, rockets, and submarines could not perform their missions. Also, if such structures were manufactured to the same surface finish and contour tolerances as gear teeth, no nation could afford to build them. However, since high-performance structures commonly become technologically obsolete within a few decades or even much sooner, they need not have infinite fatigue life. Therefore, virtually all structures are built to withstand a finite number of cycles of repeated service loads. Many such situations, though not all, come under the term "low-cycle fatigue," which is arbitrarily defined as fatigue failures which occur within 10^5 cycles.

Finite-life fatigue means that we are looking at failure processes along the sloping portion of the S-N curve. Almost invariably, this means that for structures we are dealing with plastic strains beyond the yield stress of the material, either on a localized scale around notches or at the tips of sharp cracks, or with gross plastic deformation dominating the behavior of an entire region of the structure. Under such conditions fatigue becomes strain controlled, and its analysis is typified by the total strain range (ϵ_T) versus cycles to failure (N_f) plot, as seen in Fig. 3 (Refs. 6-8). However, this is an extension of traditional fatigue design, and this approach continues to rely on crack initiation as a failure criterion for finite-life fatigue.

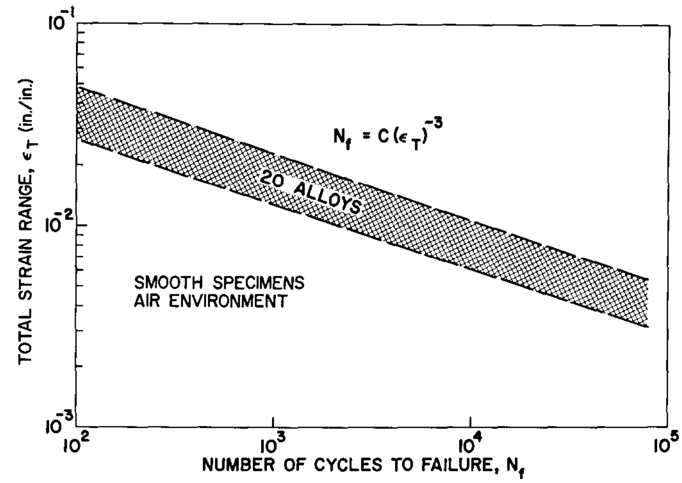


Fig. 3 - Low-cycle fatigue characteristics of structural alloys as a function of total strain range (ϵ_T). Scatterband limits include data from 20 ferrous and nonferrous alloys ranging in yield strength from 40 to 240 ksi as reported by Gross (7).

Strain-controlled fatigue is a fairly recent development and represents one of the first responses to critical structural-fatigue problems which emerged within the past 20 years. Work on this important concept is continuing, primarily in two areas: analysis of localized cyclic plastic strains in the vicinity of notches using Neuber's rule (9) as illustrated in Fig. 4, and the accumulation of fatigue damage resulting from complex load histories (10). This type of crack-initiation analysis is highly important in sophisticated structural design, but it should supplement rather than supersede a crack-propagation analysis. The undesirability of cracks in structures should not blind designers to the probability of their occurrence. Metal will not resist failure simply because a crack was not supposed to exist.

THE CRACK PROPAGATION APPROACH TO FATIGUE

The Significance of Crack Propagation in Structural Fatigue

The traditional approach to fatigue is aimed solely at preventing crack initiation. This is the first line of defense in any fatigue-design situation and should never be totally abandoned, but in many cases involving structural fatigue, prevention of crack initiation is simply not possible. As we begin to closely examine fatigue-failure processes in the finite-life region, we find that a significant portion of what is termed "cycles to failure" in laboratory specimens is actually propagation of cracks. The percentage of the fatigue life of laboratory specimens spent in crack propagation increases as the total number of cycles to failure decreases (11,12). Thus, even when a flaw-free, well-fabricated structure is tested in finite-life fatigue, a significant portion of the total fatigue life of the structure is spent in crack propagation. The commonly used term "fatigue damage" primarily means the existence of fatigue cracks in a structure.

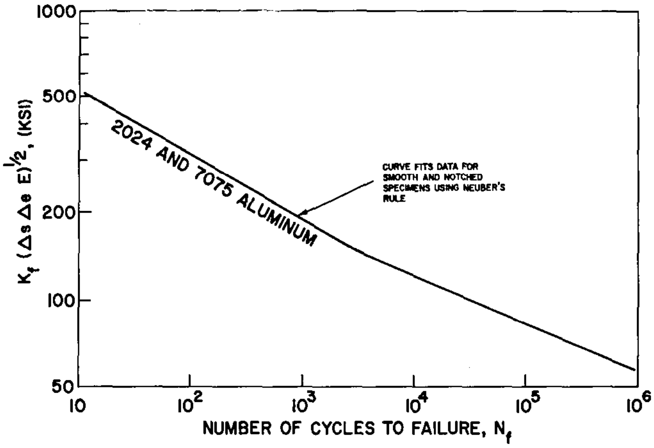


Fig. 4 - Notched-specimen fatigue characteristics of two aluminum alloys fitted to a common curve for specimens of varying notch geometry using Neuber's rule as reported by Topper, et al. (9).

Ideally then, crack-initiation criteria should provide conservative estimates of structural-fatigue life. However, the successful application of the traditional crack-initiation approach to fatigue requires that the manufactured article conform closely to the geometry and specifications called for in design drawings. Such things as removal of tool marks and control of fillet radii tolerances are of paramount importance in preventing fatigue-crack initiation. Such exacting workmanship is either impossible or prohibitively costly in many large, complex structures. Further, the traditional arsenal of surface-treatment weapons against metal fatigue, including surface grinding, polishing, shot peening, carburizing, etc., are much less applicable to large structures than with smaller machined components. Finally, the probability of fabrication defects remaining in welded structures always exists, even in the most carefully inspected welds. For many low-cost commercial welded structures, weld defects are a near certainty. Therefore, it becomes imperative that design engineers recognize that crack propagation will occur in structural fatigue and will in most instances account for 50 to 100 percent of the limiting fatigue life of structures. If considerations of cost and strength-to-density ratios permit conservative design practices, then any remaining fatigue life after crack initiation need not be utilized, providing that no fatigue-sensitive defects occurred in the structure prior to service. However, if the consequences of a hidden defect or prematurely initiated crack propagating to failure in service are severe, such as in aircraft structures, then the initial design considerations should include potential failure by crack propagation. The anticipation of crack propagation should then lead to nondestructive inspection procedures based on safe-life intervals.

Strain Models for Crack Propagation

The successful application of strain parameters to describe fatigue-crack initiation in the finite-life region encouraged early interest in strain models for crack propagation. Strain offers several advantages as an analytical parameter for structural fatigue. It can be measured directly by experimental means, and strain models offer validity in the region of plastic deformation beyond the elastic limit of the material. However, strain models offer one serious disadvantage for structural fatigue. Their translation from laboratory results to structural situations lacks the demonstrated accuracy of models based on linear-elastic theory, and this gap remains to be filled.

Studies conducted by the author (13) have shown that fatigue-crack growth rates (d(2c)/dN) for plate-bend specimens containing an embedded surface flaw correlated with the total strain range (ϵ_T). A single power-law relationship of the form

$$d(2c)/dN = C_1 (\epsilon_T)^{m_1} \quad (1)$$

described crack propagation under both elastic and plastic strain cycling (Fig. 5). This approach was applied to crack propagation in a wide variety of materials, including steels ranging in yield strength from 50 to over 200 ksi. Data for most of these steels fell along a common scatterband (Fig. 6), and this information was the first indication that the intrinsic fatigue-crack-propagation characteristics of structural steels are remarkably insensitive to wide variations in yield strength and fracture toughness. However, when the strain parameter used to describe crack propagation was normalized with respect to the elastic strength of the materials ($\epsilon_T/\epsilon_{p.1.}$) a family of curves developed, with separations occurring on the basis of yield strength (Fig. 7).

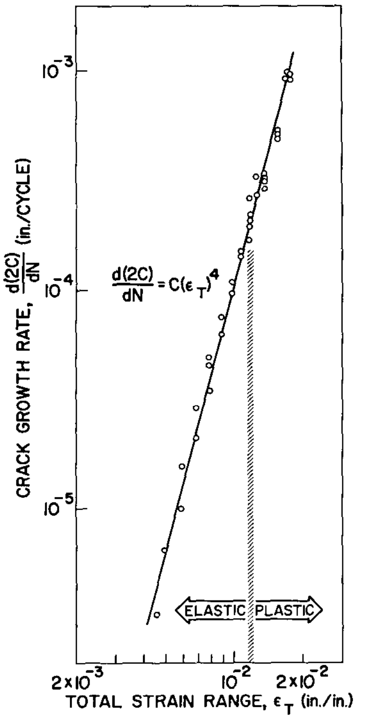


Fig. 5 - Fatigue-crack growth rates as a function of total strain range in a high-strength, 130 ksi yield strength steel using notched plate bend specimens (13). Note that the curve fits data both in the elastic and plastic strain regions.

This plot clearly revealed that structural fatigue failure in low-strength steels is predominantly a plastic fatigue problem, whereas structural fatigue failure in high-strength steels can occur under nominal elastic stress levels which are relatively low in relation to the high yield stress of the material. Similar trends apply to nonferrous alloys as well (13). This fact, combined with the powerful analytical tools offered by linear-elastic theory, has resulted in most subsequent crack-propagation research being primarily aimed at the serious fatigue problems associated with the structural application of higher strength alloys.

Several other applications of strain models to crack propagation have been offered in recent years but have not been widely adopted. Strain models proposed to date lack both the simplicity of application to design and the demonstrated success in actual structural situations that can be claimed by linear-elastic fracture mechanics models for crack propagation. However, a need persists for analyzing crack propagation under high-amplitude conditions involving gross cyclic plastic deformation. Rigorous analyses

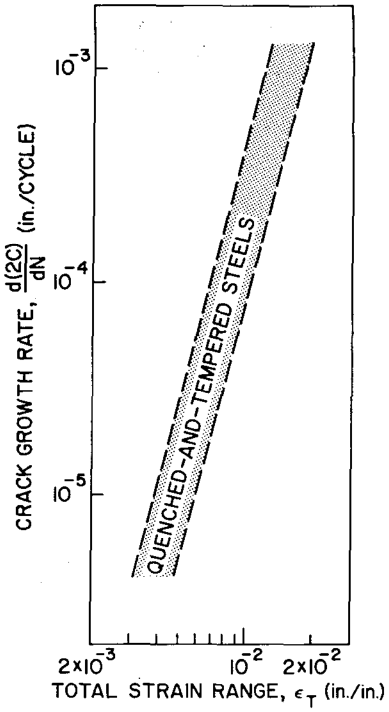


Fig. 6 - Scatterband limits for fatigue-crack propagation data from quenched-and-tempered steels ranging in yield strength from 80 to over 200 ksi (13).

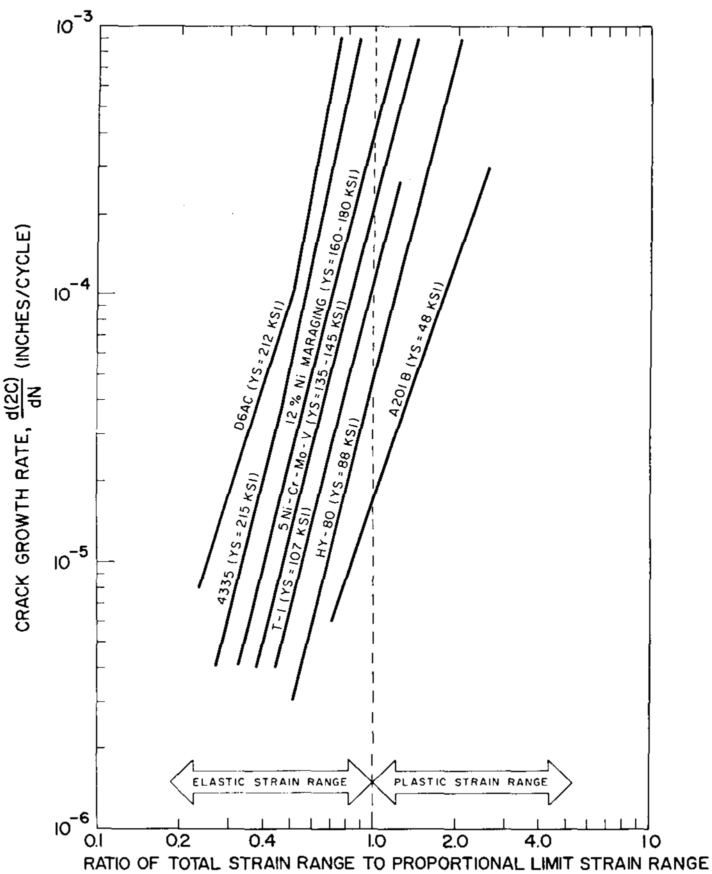


Fig. 7 - Fatigue-crack propagation characteristics of steels as a function of total strain range normalized with respect to the elastic strength of each material (13). Note that the common characteristics illustrated schematically in Fig. 6 separate into a family of curves on a distinct yield-strength basis, with the higher strength materials occupying the least favorable positions.

of such processes are exceedingly difficult, and a semi-empirical approach is more likely to evolve. One such approach (14) has been proposed which describes the crack growth rate (dc/dN) as a function of total strain range (ϵ_T) and flaw size (c) according to a power-law relationship as follows,

$$dc/dN = C_2\left(\epsilon_T \sqrt{c}\right)^{m_2} \tag{2}$$

where the parameter $\epsilon_T \sqrt{c}$ is termed the "strain-intensity factor." Data obtained from this model are limited. However, the concept of using total strain range combined with a geometry parameter is worthy of serious investigation for situations beyond the scope of linear-elastic models.

Fracture Mechanics Models for Crack Propagation

The application of linear-elastic theory to crack propagation has resulted in numerous models for fatigue (15). The simplest and most widely adopted model (16) uses the crack tip stress-intensity factor range (ΔK) as the primary variable in describing crack growth rates (da/dN) according to a power-law relationship of the form

$$da/dN = C_3\left(\Delta K\right)^{m_3}. \tag{3}$$

Since the remainder of this report will deal with the application of this model, some detailed consideration of linear-elastic fracture-mechanics concepts and their application to fatigue would seem to be in order.

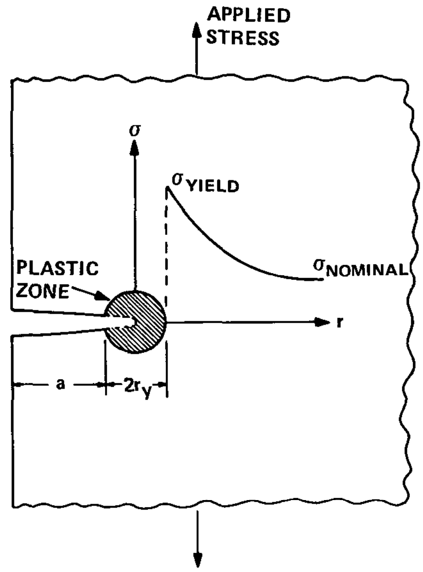
Linear-elastic fracture mechanics provides an analytical basis for dealing with the behavior of load-bearing members containing sharp cracks (17). It is an extension of the traditional mechanics of materials commonly taught to undergraduate engineering students. These traditional courses enable the student to calculate elastic stresses and deformations in smooth bodies, or bodies containing discontinuities, notches, or "stress raisers" of known geometry. Fracture mechanics deals exclusively with the special, but highly important, case of stress fields which exist near the tip of a sharp crack in an elastically stressed material.

For fatigue-crack propagation and brittle fracture, this tiny region is all important. Events in a microscopic region around a single crack tip can control the performance of enormous structures. In fatigue, this fact changes the emphasis altogether away from a surface phenomenon and thereby renders useless most of the best preventive measures available to combat crack initiation.

The traditional graphic illustration of the fracture-mechanics model is shown in Fig. 8. It shows that in metals under stress which contain a sharp crack, a small zone of plastically strained metal occurs at the crack tip. Under fatigue loading this small zone is subjected to repeated plastic strains, even though the nominal stresses in the surrounding material may be elastic and relatively low. Thus, fatigue-crack propagation is a process of localized low-cycle fatigue (18). Small volumes of material undergo plastic strain cycling and fail, causing crack growth and extension of the plastic zone into virgin material.

The crack and its plastic zone at the tip cause an abrupt rise in stresses as this vicinity is approached. Stresses within the plastic zone are assumed to reach the yield strength of the material. The parameter which describes the abrupt rise in stress, from nominal stress levels remote from the crack tip to yield levels at the leading edge of the

Fig. 8 - The fracture-mechanics model for the behavior of sharp cracks in metals. The nominal elastic stresses rise sharply in the vicinity of the crack tip, reaching the yield stress at the leading edge of the plastic zone. The crack length, a, is considered to have an effective length, $a + r_y$, where r_y is half the size of the plastic zone.



plastic zone, is called the stress-intensity factor and is universally denoted as K. This notation immediately leads to confusion among fatigue engineers, since K (K_T or K_f) is also the traditional term for the stress-concentration factor (or fatigue strength-reduction factor).

Obviously there is a strong analogy between the two, but they are not identical in concept or use. The traditional stress-concentration factor describes elastic surface stresses in a smooth body at a point which contains a discontinuity of known geometry. The stress-intensity factor describes the stress fields near the plastically deformed tip of an infinitely sharp crack, either at a free surface or within the material. Henceforth, the term K when it appears in this report will refer to the fracture mechanics stress-intensity factor.

Dimensionally, K is directly proportional to the product of nominal stress (σ) and the square root of flaw size (a)

$$K \propto \sigma \sqrt{a} \quad (4)$$

and has units of psi $\sqrt{\text{in}}$. The exact expression of proportionality is dependent on geometry. The size of the crack-tip plastic zone ($2r_y$) can also be calculated and is proportional to the squared ratio of K to yield strength (σ_{ys})

$$2r_y \propto (K/\sigma_{ys})^2 . \quad (5)$$

The exact proportionality is dependent upon mode of loading (static or fatigue) and stress state (plane strain in thick sections or plane stress in thin sections). A compendium of equations for stress-intensity factors is included in Appendix A.

Plastic-zone size is important for several reasons. Its most critical importance is its size in relation to the dimensions of the cracked body. A linear-elastic fracture-mechanics analysis is valid only if the nominal stresses are below the yield stress, and if the size of the plastic zone is relatively small in proportion to critical dimensions (usually thickness and crack length) of the cracked body. Also it can be useful in some instances to calculate an "effective" crack size, which is arbitrarily defined as the physical crack size (a) plus half the plastic zone size (r_y). This "effective" crack size ($a + r_y$) is then used to compute a K value by iteration techniques which has been "corrected for plasticity." In brittle fracture, the plastic zone size-to-thickness ratio has a powerful effect on material performance because of through-thickness restraint effects. However, this is not nearly so important in fatigue, where crack propagation occurs through localized repeated plastic strains which are relatively unaffected by restraint. Also because of work hardening, plastic-zone sizes in fatigue are generally regarded to be smaller than under static loading.

ESSENTIAL ASPECTS FOR DESIGNING AGAINST FATIGUE USING CRACK-PROPAGATION CRITERIA

Factors Involved in Designing Against Crack Propagation

The application of crack-propagation criteria to fatigue design involves consideration of three factors: initial flaw severity, crack-growth processes, and ultimate flaw tolerance (19). Initial flaw severity and ultimate flaw tolerance can be viewed as boundary conditions linked by the crack-growth process. This concept is illustrated schematically in Fig. 9, which shows the growth of a small initial defect in a structural element to a critical size for failure as a function of cycles of repeated load. In mathematical form, fatigue life in crack propagation is expressed as follows

$$N_f = \int_{a_o}^{a_{cr}} \frac{da}{f(K)} . \tag{6}$$

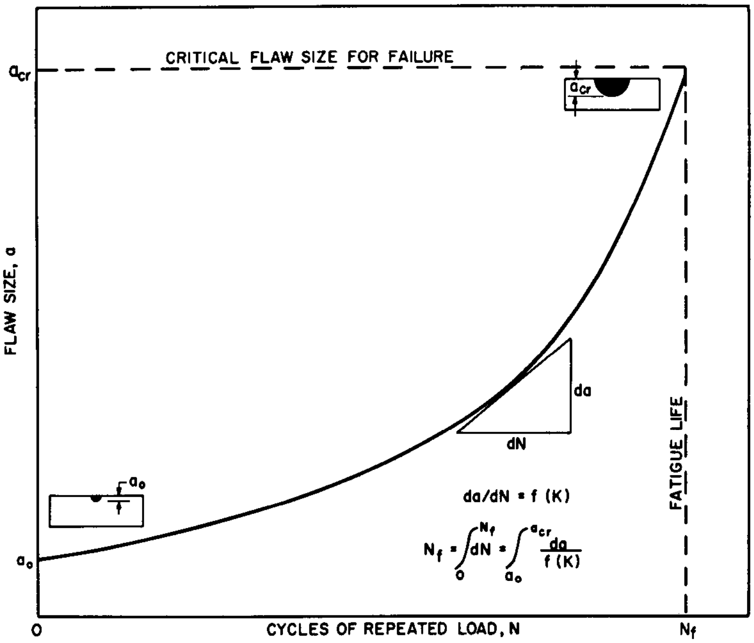


Fig. 9 - Schematic illustration of failure by fatigue-crack propagation. An initial defect whose size is a_0 grows to a critical size for failure, a_{cr}, in N_f cycles. The crack-growth process can be described in terms of the crack tip stress-intensity factor, K, and N_f then becomes the integral of this function between the limits a_0 and a_{cr}.

Obviously this procedure is a good deal more complicated than picking a point off of an S-N diagram. It requires a more thorough stress analysis than simply the surface stress in one critical location. For instance, orientation of principal stresses and stress gradients over entire regions where cracks may propagate is also needed. Also, a knowledge of the nature and location of anticipated fabrication defects and nondestructive inspection capabilities are required. Finally, a knowledge of what will constitute the most critical mode of structural failure (fracture, buckling, collapse, leakage, etc.) is essential in starting a fatigue-crack-propagation analysis. Rational design against failure by crack propagation must include knowledgeable estimates of all these factors.

Initial Flaw Severity

Locating small flaws in large, complex structures is an exceedingly difficult task, and determining initial flaw severity may frequently represent the most elusive aspect of a crack-propagation analysis. However, a reasonably accurate, or at least conservative, estimate of initial flaw severity is necessary for realistic fatigue-life calculations.

Occasionally, nondestructive inspection techniques will reveal a flaw in a critical region of a structure, either following fabrication or during periodic maintenance. The flaw can then be mapped for extent, contour, and orientation using radiographic and ultrasonic techniques. This information, combined with stress analysis, can then be used to determine the potential created by the flaw for fatigue crack growth and failure. Recently this type of approach was utilized in response to a particular type of fabrication defect which had been found to occur in a significant number of power-plant heat exchangers in Britain. However, such instances are relatively rare, and failure-safe design based on crack-propagation criteria cannot rest upon the requirement of locating and analyzing every flaw in every structure. Nevertheless, crack-propagation technology is a powerful tool for answering the serious safety and economic questions which arise when flaws are found in large, expensive structures. In addition, limits for safe operating periods between inspections can be determined from a crack-propagation analysis. Such procedures have great economic potential where large structures are involved.

A second method of determining initial flaw severity is by proof testing the entire structure (20). Again this method has serious limitations, because not all structures are amenable to proof testing, and unsuccessful proof tests frequently damage the structure beyond repair. However, proof testing has found application in pressure-vessel design, and recently the entire fleet of F-111 aircraft underwent proof testing against failure by crack propagation.

The rationale behind proof testing pressure vessels as a means of determining the severity of hidden flaws is as follows. If the fracture toughness of the vessel material is known, and if the vessel is pressurized to a test level beyond normal operating pressure, then the only unknown in the fracture-mechanics equation is the flaw-size parameter. The general form of the fracture-mechanics equation is presented below:

$$K = \sigma \sqrt{Ma} \tag{7}$$

where

K = stress-intensity factor

σ = gross section stress

a = flaw depth

M = component geometry and flaw shape parameter.

If K is chosen to be the fracture toughness of the material (K_c or K_{Ic}) and σ_p is the proof stress, then

$$\sqrt{Ma_p} = K_c/\sigma_p .$$

This procedure yields a value for the most severe flaw which the structure can contain without causing fracture in proof testing. It does not yield an absolute value of flaw size alone, because flaw severity is a function of both flaw size and flaw shape. Knowing the value of $\sqrt{Ma_p}$, the maximum initial K level which can occur in the structure in service is then calculated using the normal operating pressure to define the operating stress level. This procedure defines a conservative estimate of possible initial flaw severity, and therefore a minimum estimate of remaining fatigue life, without actually requiring an exact determination of flaw size and location or even a determination if any flaws actually exist.

However, since the procedure is based on a linear-elastic fracture-mechanics analysis, it requires that the material be brittle at the time of proof testing for its assumptions to remain valid. Since proof testing brittle structures is a hazardous procedure, it is generally limited in application to critical aerospace structures where the disastrous consequence of failure in service far outweighs the undesirable consequence of failure in proof testing. In fact, proof testing is seldom a satisfactory procedure for flaw detection. Failure-safe design through adequate fracture toughness to permit large detectable flaws is a far more desirable approach. However, newly developed techniques for nondestructive inspection utilizing acoustic emission phenomena may eventually alter this situation and greatly improve the potential for proof testing as means of preventing service failures.

The final method for determining initial flaw severity is really a combination of the best nondestructive capability available plus expert knowledge of where to look for flaws and an anticipation of their nature when found (21). Generally, this is difficult to achieve under the best of circumstances, especially in structures of new design and/or utilizing new materials. When all available knowledge and experience have been applied, the safest recourse is to assume that flaws equal to the minimum detection capability of the nondestructive inspection technique exist in critical locations throughout the structure and then base fatigue-life calculations on this assumption. In contrast to the highly detailed analyses available for crack-growth processes and terminal-failure conditions, determination of initial flaw severity inevitably relies heavily on approximations, assumptions, and prior experience.

Crack-Growth Processes

Material-Sensitive Properties — As discussed in a previous section, crack-growth processes in fatigue can be expressed as a function of the crack-tip stress-intensity factor, K. For simple zero-to-tension cycling in the absence of an aggressive environment, crack-growth-rate relationships generally follow the form shown in Fig. 10 (Ref. 19). This schematic curve plotted on logarithmic coordinates is of a sigmoidal (s-shaped) form and consists of three regions. The long, straight portion of the curve is described by the power-law relationship from Eq. (3)

$$da/dN = C_3\left(\Delta K\right)^{m_3} .$$

This power-law region of the curve is bounded by upper and lower inflection points, indicating a region of nonpropagating fatigue cracks at very low ΔK values and a region of accelerated crack growth leading to rapid terminal failure at high ΔK values.

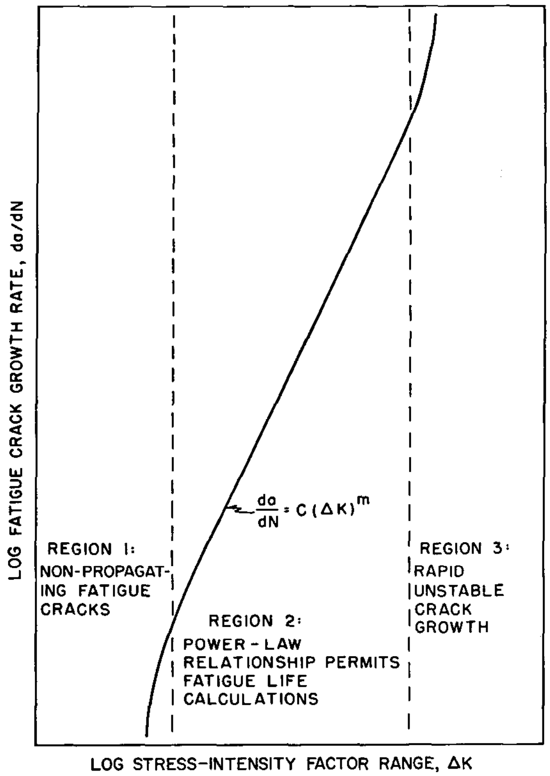


Fig. 10 - Schematic illustration of the typical sigmoidal crack-growth-rate curve for structural alloys as plotted on logarithmic coordinates

The values of the parameters C_3 and m_3, and the ΔK values at which the inflection points occur, are material-sensitive properties. A great deal of crack-propagation research has been aimed at defining these properties, and some generalized conclusions can be summarized. Originally the exponent m was thought to be an invariant (m = 4.0) for all materials (16). A broad sampling of data for steels (Fig. 11) shows that it can vary from 1.5 to 10. However, for most materials of structural significance the values range from approximately 2 to 5 (19). Systematic variations in the value of m have been sought among steels by several researchers (19,22-26). Generally, a minimum value of m is thought to result in optimum fatigue crack propagation resistance, although there can be exceptions to this generalization (26). For a particular alloy, or class of alloys, minimum values for m are commonly associated with metallurgical conditions which result in low or intermediate yield-strength levels and high fracture toughness. However, looking at the broad picture for steels (Fig. 11), it appears that low values of m can be obtained at any yield-strength level from 30 to 300 ksi by careful alloy selection. Similar comprehensive data on a broad sample of aluminum and titanium alloys have not been developed as yet.

The value of the parameter C_3 is most strongly dependent upon Young's modulus (E). Crack-growth-rate data for various alloy families fall along separate scatterbands (Fig. 12). However, values of the parameter C_3 also vary widely in response to changes in m_3. Frequently, curves for competing alloys lie along intersecting paths, and the relative order of merit for the two alloys will depend upon what portion of the crack-growth-rate curve is most crucial for preventing fatigue failure in the particular material and structural system involved. Consideration should be given to the minimum ΔK values which apply to the structural situation involved, and only that portion of the da/dN - ΔK plot above the minimum ΔK value should be used for further comparisons. This type of approach is particularly important in structures where high working stresses and limited flaw-detection capability will result in initial ΔK levels well into the region of moderately fast crack growth rates.

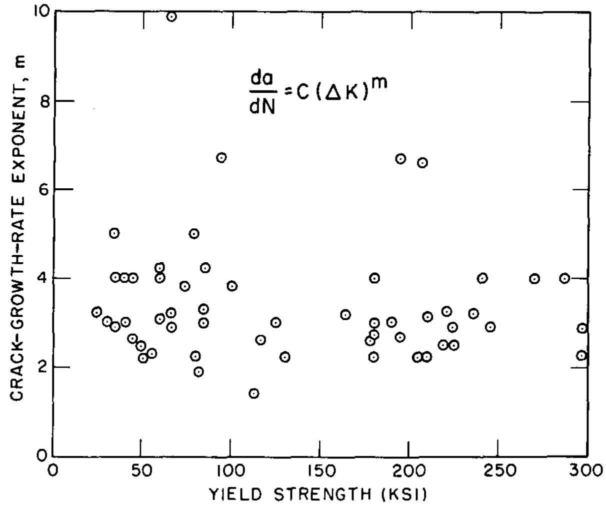


Fig. 11 - Crack-growth-rate exponent, m, as a function of yield strength for structural steels (19). Despite the scatter, note that m is not inherently influenced by yield strength.

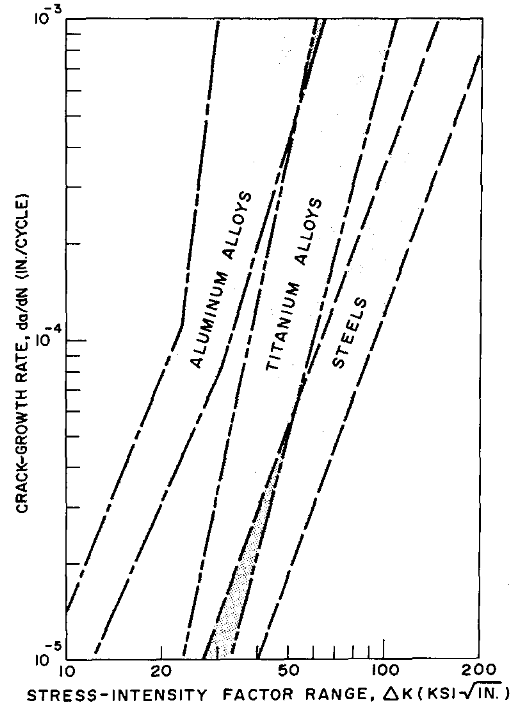


Fig. 12 - Scatterbands for crack-propagation data from steel, titanium, and aluminum alloys (31). The displacement of the scatterbands reflects the strong dependence of the crack-growth-rate constant, C, on Young's modulus for various alloys.

Considerable present research is being directed at defining both the lower and upper inflection points on the da/dN - ΔK sigmoidal curve. Lower inflection points defined to date generally lie well below ΔK = 10 ksi $\sqrt{\text{in.}}$ (Refs. 27,28). This situation involves flaw sizes and stress levels too small to be of general interest to finite-life structural fatigue. However, this phenomenon is of considerable interest in the failure-safe use of large machine components which may inadvertently contain cracks but yet must sustain long-life fatigue cycling.

By way of contrast, the upper inflection point is highly important to finite-life structural fatigue. This point defines the maximum limiting ΔK level which can be sustained in fatigue. For brittle materials, this ΔK level is related to the fracture toughness (as $\Delta K \rightarrow K_c$ (or K_{Ic}), then $da/dN \rightarrow \infty$). For ductile materials, an upper inflection point related to a critical level of crack-opening displacement (COD) exists (26,29-31). This point marks the onset of accelerated crack growth, though it is not necessarily related to imminent fracture. Information of this type is needed to establish realistic and accurate end-points for fatigue-life calculations. Extrapolating power-law relationships to estimated end-points can be a hazardous procedure in finite-life fatigue, because if accelerated crack growth occurs, the calculated fatigue life will not be conservative.

Environmental Effects — Aggressive environments, including aqueous environments and elevated temperatures, can have a strongly deleterious effect on fatigue crack-growth processes. Few materials are totally resistant to environmentally accelerated growth of fatigue cracks. Quantitative analysis of environmental-fatigue crack growth is very difficult, because fatigue is a cycle-dependent phenomenon, and environmental attack is a time-dependent phenomenon. Accurate assessment requires a synthesis of the two aspects. In many structural-fatigue situations, this factor can be resolved by the fact that cycling rates in large structures are generally quite low. Therefore, crack-growth-rate data generated at frequencies of a few cycles per minute or slower are most applicable to a structural design. A few systematic studies of time-dependent effects in corrosion-fatigue crack propagation have been conducted, and these studies generally reveal the problem to be more serious at low frequencies (32-39). A number of studies have been conducted on fatigue-crack growth at elevated temperatures, and the majority of evidence points toward a worsening problem with increasing temperatures (40-44).

However, sufficient attention has been given to this problem that certain salient points can be recognized and brought to the attention of designers. First, careful alloy selection can mitigate environmental effects to a significant degree. An initial step must be to screen alloys for sensitivity to stress-corrosion cracking (SCC). Crack growth rates due to SCC are potentially much greater than growth rates due to fatigue. Although a promising analysis for corrosion-fatigue crack propagation at ΔK levels above K_{Iscc} has been proposed (32), every possible step should be taken through alloy selection to prevent this aspect from threatening structural integrity. At ΔK levels below K_{Iscc}, a number of newer materials, including high-alloy steels, titanium alloys, and some aluminum alloys, are highly resistant to environmentally accelerated fatigue-crack growth (31,45,46).

Second, in the absence of SCC, environmental effects tend to diminish with increasing ΔK and increasing frequency (31,34-39,47,48). Under very-high-amplitude cycling, environmental effects are generally small, regardless of cycling frequency. Environment exerts its greatest effects at low frequencies and low ΔK (Fig. 13). This is why environment has such a powerful effect on finite-life structural fatigue; it eliminates the possibilities of long stages of slow crack growth and accelerates small cracks into large ones very quickly. Therefore, the structure has been denied a period of grace normally occupied by slow crack growth. A 50-percent reduction in structural fatigue life due to environmental effects must be considered minimal, and reductions up to 90 percent are not uncommon.

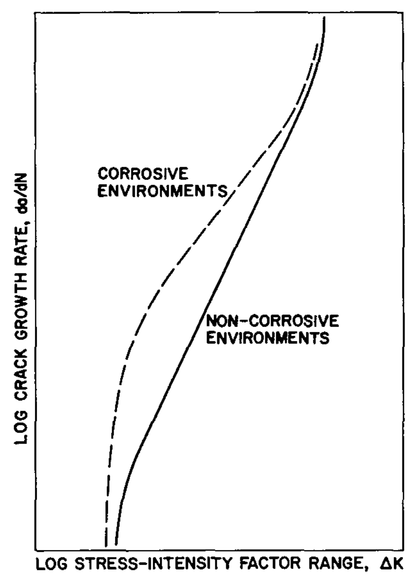


Fig. 13 - Schematic illustration of the typical effect of a corrosive environment on fatigue-crack propagation, with the maximum effect occurring at low ΔK values. The magnitude of these environmental effects tends to decrease with increasing cyclic frequency.

Third, at ΔK levels below K_{Iscc} hold-times under load have little or no effect on crack growth rates (39). The crucial factor is the rate of load application during the rising-load portion of the cycle.

Finally, much crack-propagation data obtained under aggressive environments fails to fit neatly into a single power-law function (34-36,43,47). Either it is nonrectilinear or it has inflection points. However, for purposes of estimating fatigue life, conservative adjustments can usually be made to the parameters C_3 and m_3 to account for environmental effects within the usual crack-growth-rate equation, Eq. (3).

Load-time Profile Effects — Most fatigue-crack-propagation data reported in the literature are obtained under simple zero-to-tension load cycling; however, most structural situations involve far more complex load-time profiles than zero-to-tension. Ideally, crack-propagation data should be gathered using a load-time profile representative of anticipated service conditions. This practice is followed extensively in the aircraft industry. However, this practice is not always possible due to limitations in predicting actual service conditions and in testing capabilities. In situations where a representative load-time profile either cannot be predicted or cannot be reproduced, a set of "worst-case" assumptions can be adopted and applied to ordinary zero-tension data for estimating fatigue life.

Load-time-profile effects generally fall into two categories, stress-ratio effects and load-sequence effects. Stress-ratio effects are fairly well understood, and a number of quantitative expressions for dealing with this phenomenon are available (49-51). However, load-sequence effects are not well understood, especially for fatigue situations involving crack propagation. A good deal of research effort is currently being directed at this problem. A third element in the load-time profile, frequency, has a relatively minor effect on crack propagation, except in situations where an aggressive environment is involved, as discussed in the previous section.

The term stress ratio (R) refers to the ratio of minimum stress (or stress intensity) to maximum stress (or stress intensity). Thus zero-to-tension is $R = 0$, tension-to-tension cycles have positive R values, and tension-to-compression cycles have negative R values. For a given value of ΔK, the crack-growth rate is proportional to R and can vary by as much as an order of magnitude due to variations in R from zero to 0.50 (Ref. 52), as schematically illustrated in Fig. 14 for positive R values. However, increases in crack-growth rates for positive R values over growth rates for $R = 0$ are usually about a factor of 3 for design estimates.

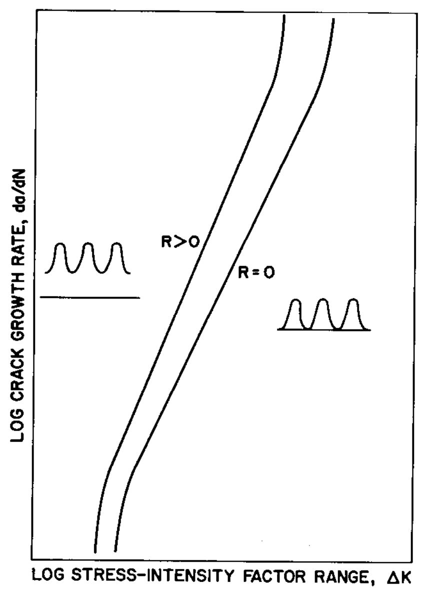


Fig. 14 - Schematic illustration of the typical effect of stress-ratio on fatigue-crack propagation. Positive R values (tension-to-tension cycling) tend to displace the entire curve above the curve for $R = 0$.

Two expressions for describing fatigue crack growth rates under cyclic loads with tensile mean loads $(R > 0)$ are given below:

$$da/dN = C_8 \left(\Delta K\right)^{m_8} \Big/ (1 - R) K_c - \Delta K \tag{8}$$

and

$$da/dN = C_9 \left(K_{max} \Delta K\right)^{m_9} . \tag{9}$$

Both of these equations have been shown to normalize a broad range of data involving positive R values (52). For situations involving negative R values, the preceding equations have not been shown to be applicable. In these situations, the compression portion of tension-to-compression cycling does contribute to fatigue-crack growth, and this effect should be included in design considerations (53). Research has also shown that fatigue cracks do propagate under purely compressive cycling due to residual tensile stresses remaining at crack tips (54). Nevertheless, purely compressive cycling in the region of the crack does not generally have serious fatigue implications for structures where failure is by crack propagation. Compressive loading of complex structures, however, can develop localized tensile stresses in regions where cracks may be present.

The complex interactions between loading cycles of varying amplitude applied in sequence are not as yet subject to a quantitative analysis. Complex loading is one of the newest research areas in fatigue, and the subject is being attacked by both empirical and statistical approaches (55,56). In complex loading, interest is focused on loads at or near peak amplitudes (57). These loads cause the most "fatigue damage" (crack growth) and control events which follow their application. This effect occurs because peak loads (either tensile or compressive) leave residual stresses at crack tips which influence crack growth rates through variations in localized stress-ratios during a period of gradual decay after their application. A high tensile load followed by lower tensile loads leaves a field of residual compression at the crack tip, thus causing a delay in crack growth during successive applications of lower loads. A high compressive load can produce an opposite effect. In addition, recent research has suggested that raising cyclic loads in a low-high sequence can result in a transient acceleration in crack-growth rates (58). Transient effects of load sequence on crack-growth rates are schematically illustrated in Fig. 15.

The number and magnitude of peak load cycles and their frequency of occurrence is probably the most important single factor in fatigue under complex loading. A linear summation of the contribution of each cycle to crack growth cannot provide accurate results without a knowledge of local crack-tip residual stresses and their resulting

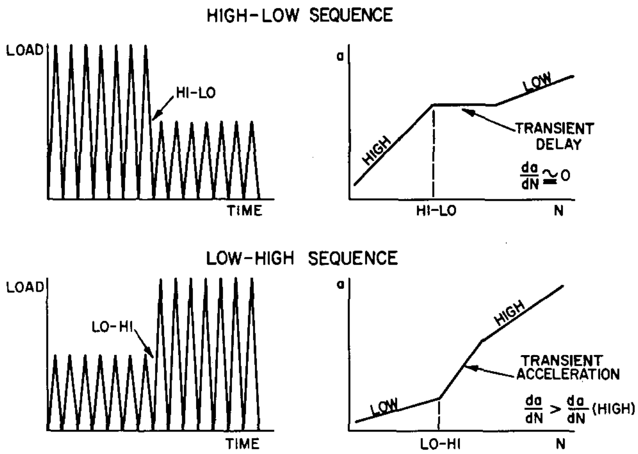


Fig. 15 - Schematic illustration of load-sequence effects on fatigue-crack propagation. Note that a high-low sequence can cause a significant period of delay in crack growth, and a low-high sequence can cause a transient acceleration in crack growth before the crack-growth rate assumes a steady-state value for the high load.

transient effects. No general engineering solution to this problem has yet emerged, and tests must be conducted under representative load-time profiles to provide an expression for fatigue-crack growth rate in a specific case.

Ultimate Flaw Tolerance

Ideally, flaws in structures should be detected prior to ultimate failure. However, this ideal is not always possible to achieve. For this compelling reason, careful attention should be given to the type of failure which will result from undetected crack growth (e.g., plastic collapse, leakage in a pressure vessel, brittle fracture, etc.) and the consequences of ultimate failure (e.g., costly unscheduled repairs, environmental contamination, human disaster, etc.). Consideration of the probable failure mode is a secondary factor in determining the number of cycles to failure in many design situations, so it is frequently ignored for purposes of fatigue-life calculation. However, it remains a primary factor in assuring failure-safe design, because it controls the severity of failure. Fracture toughness per se can influence fatigue life in the low-cycle region (Fig. 16), but this factor becomes less important beyond 10^5 cycles to failure.

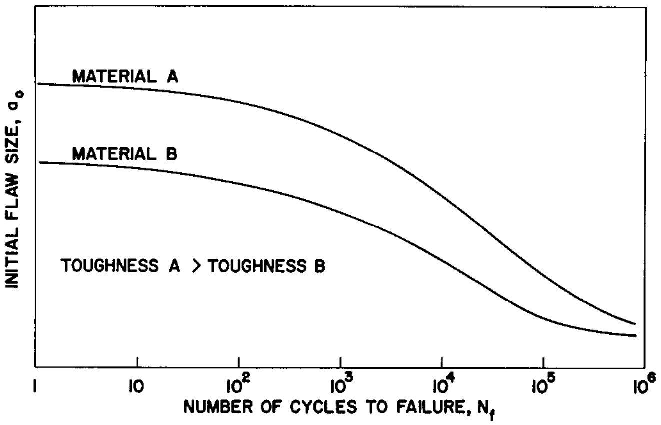


Fig. 16 - Schematic illustration of the typical effect of fracture toughness on fatigue life in crack propagation. For two otherwise similar materials, fracture toughness, per se, can influence fatigue life in the low-cycle life region ($N_f < 10^5$ cycles) but will have a negligible effect at long fatigue lives.

The NRL Ratio Analysis Diagram, or RAD (Fig. 17), has been employed as a means of quantitatively categorizing the ultimate flaw tolerance of materials and for assessing the potential for structural fatigue failure in various materials (19). The parameter employed in this analysis is the K_{Ic}/σ_{ys} ratio, and the following observations can be made on the basis of the value of this parameter for different materials.

Case 1: $K_{Ic}/\sigma_{ys} > 1.5$

The failure mode for steels in this category will not, in most instances, be unstable fast fracture except for service applications below the limit for plane-strain fracture, which is related to the nil-ductility transition (NDT) temperature. Above the temperature-transition region, these steels are capable of tolerating large flaw sizes which are within the detection capability of even the most unsophisticated nondestructive inspection techniques. If allowed to propagate to failure, cracks in these steels will ultimately lead to large inelastic deformations or leakage in pressure vessels.

Further, crack growth rates in lower strength, high-toughness steels will tend to be substantially slower than in higher strength steels. Lower working stresses in lower strength steels will almost invariably result in slower growth rates.

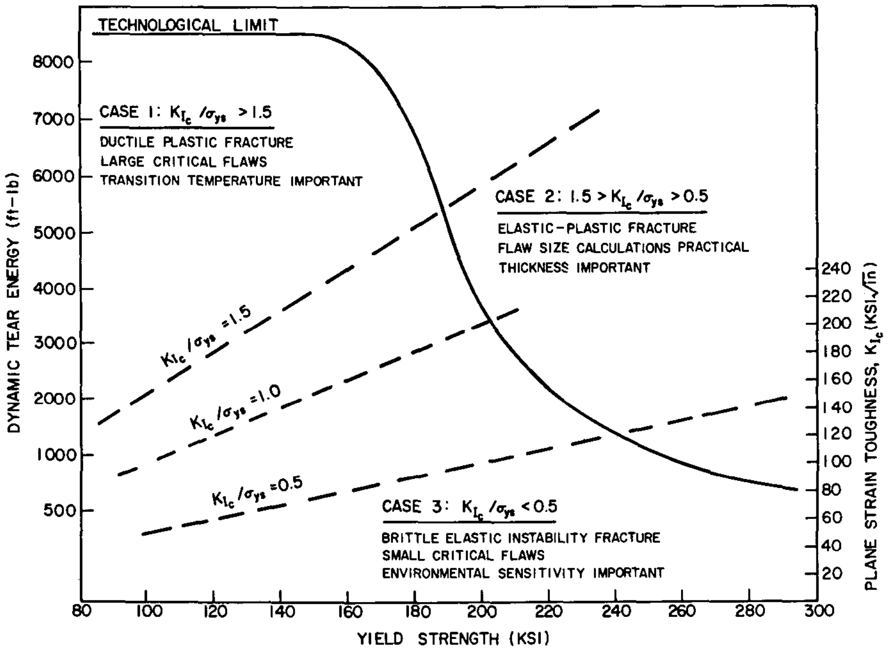


Fig. 17 - NRL Ratio Analysis Diagram for structural steels (19). The diagram is divided into three regions on the basis of the K_{Ic}/σ_{ys} ratio, and the prominent characteristics of steels in each region are listed. Similar generalities can be applied to titanium and aluminum alloys as well.

The lower strength steels, which require heavier sections to maintain low stresses, present the fewest fatigue problem areas. For these steels, there is a large amount of material to contain crack growth; growth rates tend to be slow, and high toughness — which permits large flaw sizes — is easily attainable. Also, these steels present the fewest welding problems to complicate this favorable picture.

Service experience justifies these observations. Barring unusual circumstances, low-strength structures seldom fail by fatigue. Design procedures for common, low-strength steel structures are well documented and are described in detail in building and design codes; problems arise when a critical application requires a departure from code procedures or when unfamiliar higher strength materials are employed.

Case 2: $1.5 > K_{Ic}/\sigma_{ys} > 0.5$

The failure mode for steels in this category will, in most instances, be elastic fracture. The critical flaw size and the degree of localized plasticity can vary widely, however, depending on toughness and thickness. Fatigue problems are likely to become an important consideration for many of the steels in this category, for several reasons. Working stresses are likely to be higher in these steels, thereby substantially increasing crack growth rates. The premium to be sought in using higher strength materials is smaller section sizes; therefore less material is available to contain cracks. The penalty of higher strength materials is lower toughness and smaller critical flaw sizes, thereby severely limiting the upper boundary condition for fatigue failure. New steels in the 100- to 180-ksi yield-strength region are being contemplated in many new designs, which will include a great number of Case 2 steels. Fatigue is likely to take on a new urgency in the failure-safe design of such high-strength structures.

Case 3: $K_{Ic}/\sigma_{ys} < 0.5$

The failure mode for steels in this category will be brittle, elastic-instability fracture. The critical flaw sizes will be very small. The application of this category of steels in large structures poses extreme difficulties from a fatigue and fracture standpoint and should be avoided except where necessity imposes severe strength-to-weight-ratio requirements.

Thus it can be seen that the urgency and required accuracy involved in fatigue design will in many instances be related to ultimate flaw tolerance and the associated penalty for unanticipated failure. It can also be seen that optimization of fracture-toughness properties alone does not solve structural fatigue problems. The past decade has seen enormous progress in the development of high-toughness steels in the 100-180 ksi yield-strength range. Because of their strength and toughness properties, these steels (and many nonferrous alloys possessing similar strength-to-density ratios) will be employed in critical structural applications involving fatigue. It is precisely this class of materials which will pose the most serious fatigue problems, because their high strength and toughness levels will lure designers into utilizing high working stress levels, thus heightening the potential for fatigue failure.

Synthesis Into a Unified Design Procedure

There exists no established handbook procedure for analyzing fatigue-crack propagation in structures. Fatigue life in crack propagation cannot be determined by picking a point off of a published curve. A combination of detailed knowledge and judgment is necessary, together with the application of fracture-mechanics principles to fatigue, which is a well-established textbook subject. However, the synthesis of all the elements of the fatigue-failure process into a unified design procedure remains to be accomplished individually for specific structural situations. Only a few examples of such design procedures have been described in the literature (20,59-64), and most of those have been applied to expensive, highly sophisticated aerospace structures. These limited applications should not be construed as an argument against the broader application of crack-propagation-fatigue technology, but rather they point to potential capabilities for a finer scale analysis of structural fatigue.

To apply crack-propagation technology successfully, a designer must: (a) know his structure, and (b) know his structural material. This is true not only in strict quantitative terms, but also in a vital qualitative sense. As a closing summary to this report, it would be worthwhile to review the various sources of information, both quantitative and qualitative, necessary to develop a structural-fatigue crack-propagation analysis:

1. A knowledge of where to look for flaws in the structure and what type of flaws to look for, and a reliable means for flaw detection.

2. A stress analysis of the critical regions where fatigue is likely to occur.

3. An upper-bound estimate of crack growth rates for the particular combination of material, load profile, and environment.

4. A knowledge of what will constitute failure for the structural and material system.

Thus, it can be seen that the application of fatigue-crack-propagation technology requires not only the traditional disciplines of stress analysis and fatigue, but also a working knowledge of newer areas of structural mechanics, including fracture and nondestructive inspection. However, this emerging approach to fatigue represents an attractive alternative to complete reliance upon full-scale model testing of critical high-performance structures. The application of crack-propagation technology can substantially reduce the necessity for expensive and time-consuming model testing and can be a valuable guide in interpreting the results of model tests. Structural-fatigue design is past the stage where the discovery of fatigue cracks constitutes "failure" per se. The analytical capability now exists to assess this phenomenon in a quantitative manner.

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Appendix A

FORMULAS FOR STRESS-INTENSITY FACTORS AND FATIGUE LIFE

This appendix provides a brief introduction to basic formulas for stress-intensity factors and fatigue life for several common crack-geometry situations. The formulas given are of relatively simple form, and the reader is referred to the references for limitations to their application and for correction factors to be applied for situations where the crack size becomes large in proportion to the dimensions of the cracked member.

EMBEDDED SURFACE CRACK IN TENSION

Figure A1 gives the equation for stress-intensity factors for an embedded surface flaw in tension (A1,A2). As mentioned in a previous section, K for an embedded flaw is a function of both flaw depth (a) and flaw shape (a/2c ratio). A further complication arises from crack-tip plasticity, which results in K also being mildly dependent on the ratio of gross section stress to the yield stress of the material (σ/σ_{ys}). Flaw shape and plasticity effects are defined in the parametric curves given in Fig. A1.

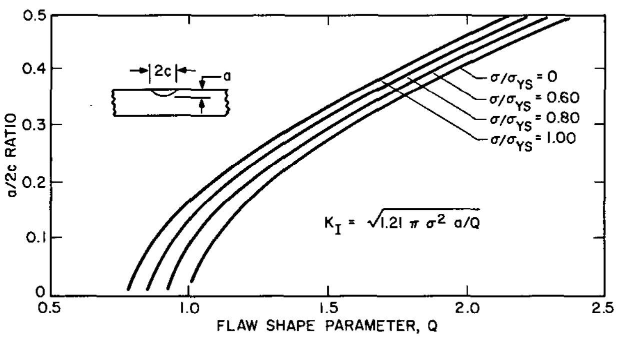


Fig. A1 - Expression for stress-intensity factors in tension-loaded plates containing an embedded surface flaw (A1). Note that K is a function of both flaw shape (a/2c ratio) and degree of plasticity (σ/σ_{ys} ratio).

The equation given in Fig. A1 defines the maximum value of K which occurs at the root of the flaw ($\phi = 0°$ in Fig. A2). Flaw-shape effects result in a significant variation in the value of K around the periphery of an embedded surface flaw, as shown in Fig. A2. This effect is termed "stress-intensity magnification" and is most pronounced for long, shallow flaws. Its effect decreases as the flaw shape approaches a semicircle, and for semicircularly shaped flaws (a/2c ≅ 0.50), K can be regarded as nearly uniform around the periphery of the flaw.

The implications of flaw-shape effects in fatigue are as follows: since fatigue-crack growth rates are exponentially proportional K, cracks with a/2c ratios less than 0.50 will not grow uniformly around their periphery. Such cracks will grow more rapidly in the root region near $\phi = 0°$, where the maximum K value exists. Thus long, shallow cracks with low a/2c ratios tend to grow toward a semicircular shape under tensile fatigue cycling.

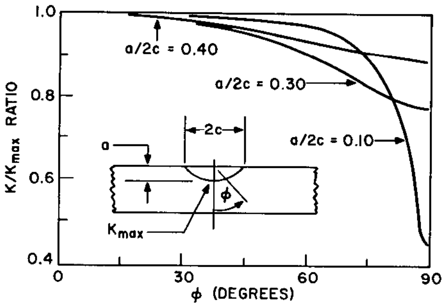


Fig. A2 - Stress-intensity distribution around the periphery of an embedded surface flaw in tension (A1). Note that the distribution becomes more uniform as the crack approaches a semicircular shape (a/2c $\rightarrow$ 0.50).

However, this crack-growth characteristic introduces an additional complication in fatigue-life calculations, because the flaw-shape parameter (Q) becomes a variable, except in situations where the initial flaw is semicircular ($a_0/2c \cong 0.50$). For this reason, flaw severity is defined by the parameter a/Q for fatigue situations involving embedded flaws of changing shape throughout the course of growth (A2,A3).

Additional situations involving stress-intensity magnification arise where the flaw becomes deep with respect to the thickness of the plate, or $a > 1/2$ plate thickness (A4), or where two adjacent flaws are close enough to result in an interaction of their stress fields (A5).

EMBEDDED SURFACE CRACK IN BENDING

The stress-intensity-factor formula and parametric curves for the embedded surface crack in bending are shown in Fig. A3 (A6). The crack-growth behavior of embedded surface cracks is quite different in bending than in tension. In bending the natural behavior is for the crack to grow long and shallow; that is, the a/2c ratio tends to decrease or to remain relatively constant with increasing crack size in bending, as contrasted with the opposite effect in tension cycling. This is because the stress gradient in bending inhibits crack growth in the depth (a) direction. Another important difference, shown by the parametric curves in Fig. A3, is that except for very shallow cracks ($a/t < 0.20$), the point of K_{max} is not at the root of the crack. For a/t values greater than approximately 0.20, K at the free surface ($\alpha = 90°$) is greater than K at the crack root ($\alpha = 0°$).

CENTRAL THROUGH CRACK AND EDGE CRACK IN TENSION

The formulas for stress-intensity factors for a central through crack in tension and an edge crack in tension are shown in Figs. A4 and A5, respectively (A7). Both of these

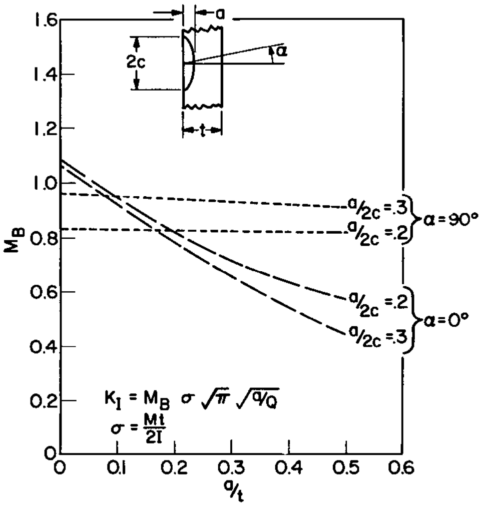


Fig. A3 - Stress-intensity factor formula and parametric curves for the embedded surface flaw in bending (A6). Note that in bending the point of maximum stress intensity is not necessarily at the root of the crack. The term M refers to the bending moment.

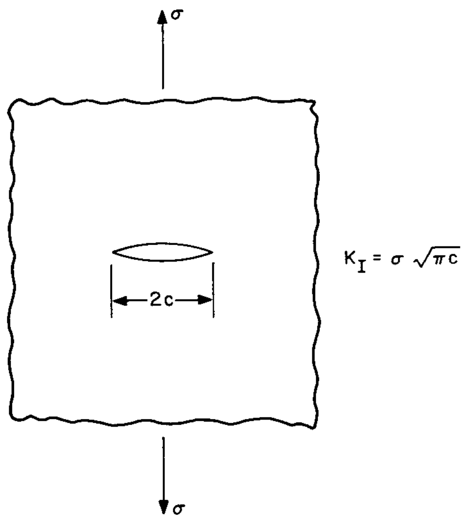


Fig. A4 - Stress-intensity-factor formula for a central through crack in tension in a semi-infinite body (A7). For situations where 2c becomes relatively large with respect to the width of the body, finite width-correction factors should be applied (A7).

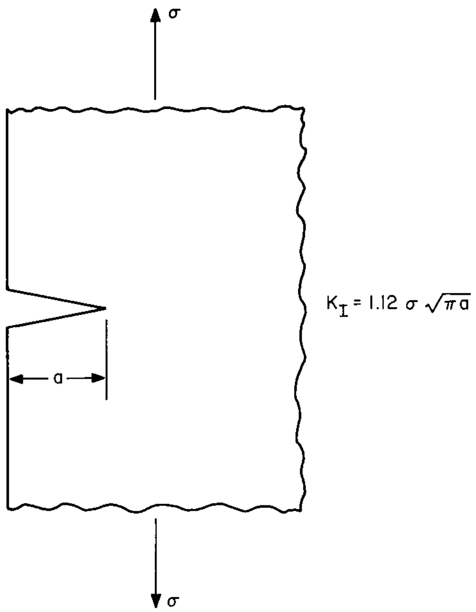


Fig. A5 - Stress-intensity-factor formula for an edge crack in tension in a semi-infinite body (A7). For large cracks, finite width-correction factors should be applied (A7).

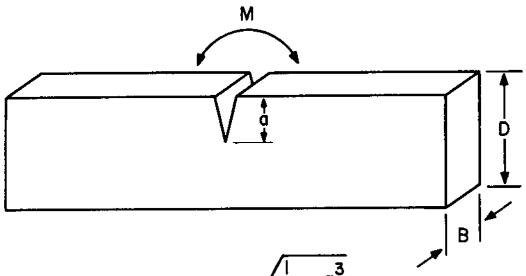
equations apply to semi-infinite bodies where the crack remains relatively small in proportion to the width of the body. For situations where the crack length approaches or exceeds 30 percent of the body width, finite width-correction factors should be applied (A7).

EDGE CRACK IN BENDING

The stress-intensity-factor formula for the edge crack in bending is shown in Fig. A6 (A8). The single-edge-crack bend specimen has proven highly popular and successful for K_{Ic} plane-strain fracture-toughness measurements, and more elaborate stress-intensity formulas have been developed for this purpose. However, for introductory purposes the formula given in Fig. A6 is adequate and is less cumbersome to apply.

FATIGUE-LIFE FORMULA

A general formula for fatigue life in crack propagation has been developed for the situations where the stress-intensity factor can be expressed in the form of $K = \sigma\sqrt{Ma}$ (Ref. A9). This form for K applies to the cases of tension-loaded surface cracks, central through cracks, and edge cracks, as well as surface-cracked plates in bending.



$$K_I = \frac{4.12\, M \sqrt{\frac{1}{\alpha^3} - \alpha^3}}{B D^{3/2}}$$

$$\alpha = 1 - a/D$$

Fig. A6 - Stress-intensity-factor formula for an edge-crack in bending (A8). The term M refers to the bending moment.

However, this form for K does not apply to the case of edge cracks in bending. The general formula for fatigue life is as follows:

$$N_f = \frac{2}{(m - 2) C M^{m/2} (\Delta\sigma)^m} \left[\frac{1}{a_0^{(m-2)/2}} - \frac{1}{a_{cr}^{(m-2)/2}} \right]$$

where

N_f = number of cycles to failure

m, C = parameters from the crack-growth-rate expression (Eq. (3) in the text)

M = geometry and flaw shape parameter ($K = \sigma \sqrt{Ma}$)

$\Delta\sigma$ = cyclic stress range

a_0 = initial flaw size

a_{cr} = critical flaw size for failure.

This relationship can be highly useful for initial estimates of cyclic life in crack propagation. However, it is subject to serious limitations for applications which may require more refined estimates. It is primarily applicable to situations where loading is uniform (no sequence effects) and where stress-ratio effects and environmental effects can be fully accounted for by the parameters C and m in the crack-growth-rate expression (Eq. (3)). Also this expression for fatigue life does not take into account variations in the flaw-shape parameter (Q) or finite width corrections with crack growth. Both of these variables are treated as constants through the parameter M in the equation.

Nevertheless, for those situations where this generalized expression does not apply, a numerical integration of the particular crack-growth-rate expression can readily be conducted with or without the aid of a computer. The principal obstacle in performing a fatigue-crack propagation analysis is obtaining an accurate expression for crack growth rates, not in the subsequent integration of the expression for fatigue life.

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