

NRL Report 8304

# RAYSTC: A Computer Code for Calculating Single Ray-Path Statistics, Assuming the Garrett-Munk Model of Internal Waves

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## RAYSTC: A COMPUTER CODE FOR CALCULATING SINGLE RAY-PATH STATISTICS, ASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES

### I. INTRODUCTION

In a recent paper [1] Munk and Zachariasen used the Rytov approximation to calculate the expressions for the mean-square acoustic phase, phase rate, log-intensity, and their spectra for a single ray in an ocean possessing internal-wave induced sound-speed fluctuations. They expressed their results as integrals over the ray path of quantities associated with the internal-wave spectrum. The purpose of this report is to document a computer program designed to evaluate these integrals for the case in which the internal-wave spectrum is given by the Garrett-Munk internal-wave model [2-4].

In Sec. II we briefly outline the theory developed by Munk and Zachariasen and describe the algorithm used in the computer program. Section III contains a description of the program. In Appendix A we indicate the deck assembly and list in a table all the input parameters. In Appendix B the source language listing is given. Appendix C contains the results of a sample run in which we have assumed that the ray path is quadratic in the range variable. This choice of ray path was dictated solely by convenience, and is not necessarily intended to be representative of an actual physical situation.

### II. SOME BASIC RELATIONS

According to the general Garrett and Munk internal-wave model [2-4], the vertical particle displacement  $\zeta$  at depth  $z$  has the spectrum

$$F_\zeta(\omega, j; z) = \langle \zeta^2(z) \rangle G(\omega, n(z)) H(j), \quad (1a)$$

where

$$G(\omega, n) = 0 \text{ for } \omega < \omega_i \text{ and } \omega > n, \quad (1b)$$

$$\int_{\omega_i}^n d\omega G(\omega, n) = 1, \quad (1c)$$

and

$$\sum_{j=1}^{\infty} H(j) = 1. \quad (1d)$$

Here  $\omega$  and  $j$  are, respectively, the internal-wave radial frequency and the mode number, and  $\omega_i$  is the inertial frequency. The local buoyancy (Brunt-Väisälä) frequency has the form

$$n(z) = n_o \exp(-z/B) \quad (2)$$

in an exponentially stratified ocean. The mean-square displacement is dependent on depth through  $n(z)$ , i.e.,

$$\langle \zeta^2(z) \rangle = \langle \zeta_o^2 \rangle \left( \frac{n(z)}{n_o} \right)^{-1}, \quad (3)$$

where  $\langle \zeta_o^2 \rangle$  is the mean-square displacement extrapolated to the surface. The horizontal and vertical wave numbers are given by the dispersion relations

$$k_H = \frac{j\pi}{Bn_o} (\omega^2 - \omega_i^2)^{1/2}, \quad (4a)$$

and

$$k_V = \frac{j\pi}{Bn_o} n(z). \quad (4b)$$

The analysis in Ref. 1 is based on a version of the Garrett-Munk model [4] GM75 1/2,

where

$$G(\omega, n) = N_G \omega_i \frac{(\omega^2 - \omega_i^2)^{1/2}}{\omega^3}; \quad \omega_i < \omega < n \\ = 0; \quad \text{otherwise,} \quad (5a)$$

and

$$H(j) = N_H \frac{1}{j^2 + j_*^2}. \quad (5b)$$

The mode number parameter  $j_*$  is typically set equal to 3. The quantities  $N_G$ ,  $N_H$  are dimensionless normalization constants determined from Eqs. (1c) and (1d):

$$N_G = \frac{4}{\pi} \left[ 1 + O\left(\frac{\omega_i}{n}\right) \right] \approx \frac{4}{\pi}, \quad (6a)$$

$$N_H = \left[ \sum_{j=1}^{\infty} \frac{1}{j^2 + j_*^2} \right]^{-1} \approx \frac{2j_*^2}{\pi j_* - 1}. \quad (6b)$$

We are interested in the pressure received at a point  $\mathbf{x} = (R_{\max}, 0, z)$  in the ocean due to a source located at  $\mathbf{x}_s = (0, 0, z_s)$  and radiating acoustic energy at frequency  $f$ . Propagation is, therefore, along the  $x$ -axis and  $R_{\max}$  is the range. (Since we consider the manner in which the rms values accumulate as one moves along the horizontal path from source to receiver, we have appended the subscript *max* to indicate  $R_{\max}$  is the maximum range of interest.) The acoustic wavenumber is  $k_o = 2\pi f/c_o$  where, typically,  $c_o = 1.5$  km-Hz.

In the absence of internal waves, the sound speed is equal to the (depth-dependent) mean sound speed:

$$c(\mathbf{x}, t) = \bar{c}(z), \quad (7)$$

and the received pressure is

$$Re [p_o(\mathbf{x}) e^{-2\pi i f t}]. \quad (8)$$

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In Ref. 1 it was assumed  $p_o$  could be approximated using ray acoustics. The ray path of interest,  $z(x)$ <sup>†</sup>, satisfies the eikonal equation

$$\frac{d^2z(x)}{dx^2} + V'(z(x)) = 0, \quad (9a)$$

where

$$V'(z) = -\frac{1}{2} \frac{d}{dx} \left( \frac{c_o}{\bar{c}(z)} \right)^2, \quad (9b)$$

and the end-point conditions

$$z(0) = z_s, \quad z(R_{\max}) = z. \quad (9c)$$

The slope of the path is given by

$$\tan\theta(x) = \frac{dz(x)}{dx}. \quad (10)$$

(The coordinate system is orientated so that the positive  $z$ -direction is downward, hence the ray angle  $\theta(x)$  is positive if the ray is directed toward the bottom.)

Also of interest is the phase curvature function [5]  $A(x)$  defined by the expression

$$[A(x)]^{-1} = \frac{\xi_1(x) \xi_2(x)}{\xi_2(x) \frac{d}{dx} \xi_1(x) - \xi_1(x) \frac{d}{dx} \xi_2(x)}, \quad (11)$$

where  $\xi_{1,2}$  are the linearly independent solutions of the differential equation

$$\left\{ \frac{d^2}{dx^2} + V''(z(x)) \right\} \xi_{1,2} = 0, \quad (12a)$$

with

$$V''(z) = -\frac{1}{2} \frac{d^2}{dz^2} \left( \frac{c_o}{\bar{c}(z)} \right)^2, \quad (12b)$$

that satisfy the boundary conditions

$$\xi_1(0) = \xi_2(R_{\max}) = 0,$$

$$\xi_1(R_{\max}) = \xi_2(0) = 1. \quad (12c)$$

(Because of the Wronskian relation obeyed by  $\xi_{1,2}$ , the denominator in Eq. (11) is actually independent of  $x$ .)

When internal waves are present, the sound speed acquires a small fluctuation  $\delta c(\mathbf{x}, t)$  which is related to the internal-wave vertical displacement by

---

<sup>†</sup>We have assumed the parabolic approximation. Consequently, the  $\sec^2\theta$  term which appears in the expressions for the mean-square values in Ref. 1 has been replaced by unity and the eikonal equation which defines the ray path has a slightly modified form.

$$\frac{\delta c(\mathbf{x}, t)}{c_o} = (\text{constant}) n^2(z) \zeta(\mathbf{x}, t). \quad (13)$$

Therefore, from Eq. (3),

$$\left\langle \left( \frac{\delta c(\mathbf{x}, t)}{c_o} \right)^2 \right\rangle \equiv \left\langle \left( \frac{\delta c(z)}{c} \right)^2 \right\rangle = \left\langle \left( \frac{\delta c}{c} \right)_o^2 \right\rangle \left( \frac{n(z)}{n_o} \right)^3. \quad (14)$$

Here,  $\left\langle \left( \frac{\delta c}{c} \right)_o^2 \right\rangle$  is the mean-square fractional fluctuation extrapolated to the surface. With the presence of  $\delta c$ , the received pressure can be expressed in the form

$$Re \left[ p(\mathbf{x}, t) e^{-2\pi i f t} \right], \quad (15)$$

where

$$p(\mathbf{x}, t) = p_o(\mathbf{x}) \exp [X_1(\mathbf{x}, t) + i X_2(\mathbf{x}, t)] \quad (16)$$

with  $X_{1,2}$  real. This expression can be rewritten as

$$p(\mathbf{x}, t) = p_o(\mathbf{x}) \exp \left\{ \frac{1}{2} [\iota - \langle \iota \rangle] + 2\pi i [\phi - \langle \phi \rangle] \right\}, \quad (17)$$

where anticipating the use of the Rytov approximation we have introduced the definitions

$$\iota \equiv \ln |p(\mathbf{x}, t)|^2, \quad \phi \equiv \arg p(\mathbf{x}, t)/2\pi;$$

$$\langle \iota \rangle \equiv \ln |p_o(\mathbf{x}, t)|^2, \quad \langle \phi \rangle \equiv \arg p_o(\mathbf{x}, t)/2\pi. \quad (18)$$

(The  $2\pi$  is included here because we have chosen to measure phase and phase-rate statistics in cycles and cycles-per-hour, respectively.) Mean-square values are given by the expressions

$$\phi_{\text{rms}}^2 = \langle [\phi - \langle \phi \rangle]^2 \rangle, \quad (19a)$$

$$\dot{\phi}_{\text{rms}}^2 = \langle \dot{\phi}^2 \rangle, \quad (19b)$$

and

$$I_{\text{rms}}^2 = \left( \frac{10}{\ln 10} \right)^2 \langle [\iota - \langle \iota \rangle]^2 \rangle. \quad (19c)$$

Here a dot indicates differentiation with respect to time and  $I_{\text{rms}}$  is the rms intensity in decibels.

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Using the Rytov approximation, Munk and Zachariasen [1] calculated the contribution to these mean-square values from a single ray  $z(x)$ . They found<sup>t</sup>

$$\phi_{\text{rms}}^2 = k_o^2 \int_0^{R_{\text{max}}} dx \left\langle \left( \frac{\delta c}{c} (z(x)) \right)^2 \right\rangle L_p(\theta(x), z(x)), \quad (20a)$$

$$\dot{\phi}_{\text{rms}}^2 = k_o^2 \int_0^{R_{\text{max}}} dx \left\langle \left( \frac{\delta c}{c} (z(x)) \right)^2 \right\rangle V_p(\theta(x), z(x)), \quad (20b)$$

$$I_{\text{rms}}^2 = \left[ 2\pi \frac{10}{\ln 10} \right]^2 k_o^2 \int_0^{R_{\text{max}}} dx \left\langle \left( \frac{\delta c}{c} (z(x)) \right)^2 \right\rangle L_p(\theta(x), z(x)) \frac{|A^{-1}(x)|}{k_o L_V^2(z(x))}. \quad (20c)$$

In these expressions we have introduced

$$L_p(\theta, z) = L(0) f_1 \left( \frac{n(z)}{\omega_i} \tan \theta(z) \right), \quad (21a)$$

$$L(0) \equiv \frac{1}{\pi^4} \langle j^{-1} \rangle \left( \frac{n_o}{\omega_i} \right) B, \quad (21b)$$

$$f_1(\Delta) \equiv \frac{1}{\Delta^2 + 1} + \frac{\Delta^2}{2(\Delta^2 + 1)^{3/2}} \ln \left\{ \frac{(\Delta^2 + 1)^{1/2} + 1}{(\Delta^2 + 1)^{1/2} - 1} \right\}, \quad (21c)$$

with<sup>f</sup>

$$V_p(\theta, z) = \frac{8}{\pi^2} \langle j^{-1} \rangle \omega_i n_o B \ln \left( \frac{n(z)}{\omega_i} \right) f_2 \left( \frac{n(z)}{\omega_i} \tan \theta \right), \quad (22a)$$

$$\ln \left( \frac{n}{\omega_i} \right) f_2(\Delta) \equiv \ln \left( \frac{n}{\omega_i} \right) - \frac{1}{2} \ln \left( \frac{\Delta^2}{4} \right) - \frac{1}{2} \frac{1}{(\Delta^2 + 1)^{1/2}} \ln \left\{ \frac{(\Delta^2 + 1)^{1/2} + 1}{(\Delta^2 + 1)^{1/2} - 1} \right\} \quad (22b)$$

and

$$L_V(z) = \frac{1}{\pi j_*} \left( \frac{\langle j^{-1} \rangle (\pi j_* - 1)}{\pi} \right)^{1/2} \frac{n_o}{n(z)} B. \quad (23)$$

The quantity  $\langle j^{-1} \rangle$  is the average of the reciprocal of the mode number over the internal-wave spectrum<sup>g</sup>:

$$\langle j^{-1} \rangle = N_H \sum_{j=1}^{\infty} \frac{1}{j} \frac{1}{j^2 + j_*^2}. \quad (24)$$

<sup>t</sup>See footnote on p. 3.

<sup>f</sup>Properly, one should write  $f_2$  as a function of both  $\Delta$  and  $n/\omega_i$ .

<sup>g</sup>Using a technique which we believe to be accurate to within 1 part in  $10^6$ , we find  $\langle j^{-1} \rangle = 0.7308, 0.6240, 0.4890, 0.4001, 0.3404, 0.2978$  for  $j_* = 0, 1, 2, 3, 4, 5$ , respectively.

All the other quantities in Eqs. (20)-(23) have been previously defined. The function  $L_p(\theta, z)$  is the sound-speed correlation length measured at depth  $z$  along a line inclined at an angle  $\theta$  with the horizontal, and  $L_v(z)$  is the vertical correlation length at depth  $z$ . The functions  $f_1(\Delta)$  and  $f_2(\Delta)$  are defined in such a way as to be equal to unity when  $\Delta$  is equal to zero. The program assumes  $n_o$  and  $\omega_i$  are given in cycles-per-hour. With this choice,  $\phi_{rms}$  is measured in cycles and  $\phi_{rms}$  is measured in cycles-per-hour. In the program the integrals Eqs. (20) are calculated using the trapezoidal rule. The range of propagation is broken up into  $NR$  slabs of equal width, and the integrands in Eqs. (20) are assumed to be linear within each slab. For all the cases we considered, this simple algorithm gave completely satisfactory results.

### III. PROGRAM DESCRIPTION

The program was written for use on the Texas Instruments Advanced Scientific Computer in the machine-specific language TI-ASC FORTRAN. Since it is a complete program rather than a subroutine, its use does not require a calling sequence.

The total length of the program is 0001A600. The system reserves an additional 8K words of central memory for I/O buffers, etc. It requires no temporary storage and does not use common blocks. The program will compile on the NX compiler at the  $K$  level in 1.60 s. The execution time varies. For the sample computer run recorded in Appendix C, the total central processor time was 5.11 s and the plotter time was 14 min.

The program uses the following external routines: ABS, ALOG, ATAN, EXP, FLOAT, INT, SQRT, TAN, ORIGIN, NXAXIS, NYAXIS, LETTER, NUMBER, PLOTS, PLOT, ENDPLT, R\$TOP.

The required input data are listed in Table A1 of Appendix A. They naturally fall within four categories: acoustic parameters, internal-wave parameters, output-option parameters, and ray characteristics. The acoustic, internal-wave, and output-option parameters are entered on three separate cards which together compose an input file embedded in the job input stream by means of a START/STOP statement pair. This file has the standard Fortran access name FT05F001 and hence is read on logical unit number 5. Distances are given in meters, the acoustic frequency is given in Hertz, and the frequencies associated with the internal-wave model are given in cycles-per-hour.

The present version of the program does not calculate the ray path nor the phase curvature function. The depth of the ray path as a function of horizontal path length is input by means of a card file specified by a START/STOP pair and having the access name FT08F001 (logical unit number 8). This file consists of the depth of the ray path at  $NR+1$  equally-spaced range points extending from the source position to  $R_{max}$  (RMAX). These values are stored in the array RAY in such a way that RAY(I) ( $I = 1, \dots, NR+1$ ) is the depth of the ray path (in meters) at a horizontal distance  $(I-1) RMAX/NR$  from the source. The maximum value of NR allowed by the program is 7000.

It is not the phase curvature function which is required as input but rather the absolute value of its reciprocal. Just as with the ray path,  $NR+1$  values of this function (in meters) compose a card file specified by a START/STOP pair. The file has the access name FT09F001 (logical unit 9). These values are stored in the array ABRECA so that the absolute value at a horizontal distance  $(I-1) RMAX/NR$  is ABRECA(I) where  $I = 1, \dots, NR+1$ .

On encountering an error condition in the transfer of input data, the program will write out the status code of the error message and terminate the job.

The output from the program can logically be divided into three categories; *i.* input parameters, derived quantities of secondary importance, and results, *ii.* optional tables of various arrays, and *iii.* optional plots. In the following three paragraphs we will describe these categories. All output is written on the standard printer (logical unit 6). The standard access name FT59F001 (logical unit 59) is assigned to the plotter output.

The program lists the input parameters from the first two cards of the input file FT05F001 (Table A1) together with their Fortran names and units. In addition, the source and receiver depths are listed. These depths are the values of the first and last elements of the ray-path array RAY. The program lists the values calculated for the range increment  $\Delta r = R_{\max}/NR$  (DELR), the wave number  $k_o$  (WV), the value for  $L(0)$  (LZERO), calculated from Eq. (21b), and the value for  $\langle j^{-1} \rangle$  (AVE). The program then lists the values obtained for the root-mean-square phase, phase-rate, and intensity fluctuations together with the errors associated with the use of the trapezoidal rule.

The user has the option of listing in tables the values of the elements of six arrays for selected values of the indices. These arrays are:

1. RANGE — gives values in meters for the horizontal path length at  $NR+1$  evenly spaced points along the horizontal path of propagation.
2. RAY — contains values for the depth of the ray path.
3. ANGLE — contains the NR values for the ray's grazing angle, in degrees, calculated using a finite difference approximation to the derivative of the ray path.
4. ABRECA — contains values for the magnitude of the phase curvature function. This array is labelled  $|(1/A)|$  in the Appendix C table.
5. PHI — contains values for the calculated rms phase fluctuation along the ray path as a function of horizontal distance from the source.
6. PHIDOT — contains values of the rms phase-rate fluctuation as a function of distance from the source.

The extent to which the tables are constructed is determined by the output-option parameters IARR and NPRNT (see Table A1).

Depending on the values assigned the output-option parameters NPLT and IPLT(I),  $I = 1, \dots, 7$ , the program will construct up to seven plots. Examples of these plots are given in Appendix C and Table A1 contains brief descriptions.

#### IV. ACKNOWLEDGMENTS

I would like to thank D. R. Palmer for pointing out the need for this type of program and for helpful advice during its development and documentation.

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**Appendix A  
DECK ASSEMBLY**

Card Number	Description	JSL Statement Format*
1	JOB	/bJOBbName, Acct. no., User code, OPT = (C, R, D, T)
2	LIMIT	/bLIMITbMIN = 1, BAND = 25
3	PLOT FILE DESCRIPTION	/bFDbFT59F001, FORG=PS, RCFM=U,BKSZ=4000,BAND=1/10/1
4	FORTRAN	/bFTNbIN = SDECK, FTVERS = NX, FTNOPT = (K,U)
5	LINK	/bLNK
6	EXECUTE	/bFXQTbOPT = (I,A)
7	PLOT OUTPUT	/bFOSYSbFT59F001, TYPE = PLOT
8	START SOURCE DECK	/bSTARTbACNM = SDECK : Fortran source deck :
9	STOP SOURCE DECK	/bSTOP
10	START INPUT PARAMETER	/bSTARTbACNM = FT05F001 : Input parameter cards :
11	STOP INPUT PARAMETER	/bSTOP
12	START RAY	/bSTARTbACNM = FT08F001 : ray path data cards :
13	STOP RAY	/bSTOP
14	START PHASE CURVATURE	/bSTARTbACNM = FT09F001 : phase curvature data cards :
15	STOP PHASE CURVATURE	/bSTOP
16	END OF JOB	/bEOJ

\*The lower case letter "b" represents a (necessary) blank.

**Table A1 — Input Data For RAYSTC****FILE ACCESS NAME FT05F001**

<b>CARD 1</b>	<b>Format (F7.2, F8.3, F8.0, I5, F5.0)</b>
CZERO	Reference sound speed $c_o$ (m/sec)
FREQ	Acoustic frequency $f$ (Hz)
RMAX	Maximum range $R_{max}$ (m)
NR	Number of range steps
DEPTH	Ocean depth $D$ (m)
<b>CARD 2</b>	<b>Format (F5.0, F5.2, E9.2, F6.4, I5)</b>
B	Stratification scale $B$ (m)
BVZERO	Buoyancy frequency, extrapolated to the surface, $n_o$ (cph)
DELC	Root-mean-square value of the fractional sound speed fluctuation, extrapolated to the surface, $\langle (\delta c/c)_o \rangle^{1/2}$
FREQIN	Inertial frequency $\omega_i$ (cph)
JSTAR	Mode number parameter $j_*$
<b>CARD 3</b>	<b>Format (I1, I4, 8I1)</b>
IARR	Table output flag — 0; do not print tables — 1; print tables of the arrays RANGE, RAY, ANGLE, ABRECA, PHI, and PHIDOT
NPRNT	Table output count; the tables will be composed of the first, last and every NPRNTth element of the arrays.
NPLT	Plot output flag — 0; construct no plots — 1; construct one or more of the 7 possible plots
IPLT(I)	Individual plot flag — 0; do not construct plot — 1; construct plot
IPLT(1)	— plot of $L_p(\theta, z)$ vs $\theta$ for various values of $z$ (Fig. 1 of the sample output of App. C)
IPLT(2)	— plot of the ray angle $\theta$ vs horizontal path length (Fig. 2 of App. C)
IPLT(3)	— plot of the ray path (Fig. 3 of App. C)
IPLT(4)	— plot of the absolute value of the reciprocal of the phase curvature function (Fig. 4 of App. C)
IPLT(5)	— plot of the rms phase fluctuation vs horizontal path length (Fig. 5 of App. C)
IPLT(6)	— plot of the rms phase-rate fluctuation vs horizontal path length (Fig. 6 of App. C).
IPLT(7)	— plot of $V_p(\theta, z)$ vs $\theta$ for various values of $z$ (Fig. 7 of App. C)

**FILE ACCESS NAME FT08F001**

This file contains a number of cards, format (10F8.1), listing the depth in meters of the ray path at NR+1 evenly spaced points along the horizontal direction of propagation.

**FILE ACCESS NAME FT09F001**

This card file is analogous to FT08F001 but contains values in meters of the reciprocal of the phase curvature function.

## Appendix B

### SOURCE LANGUAGE LISTING

```

0001      PROGRAM RAYSTC
0002      DIMENSION RAY(7001),ABRECA(7001),AVEJ(6),SLOPE(7000),ANGLE(7000),
0003      1PMISQC(2,7001),DOTSQC(2,7001),IOTASQ(2,7001),PHIC(7001),PHIDOT(7001),
0004      2PLTARC(1000),IPLTC(7)
0005      DIMENSION XMAX(7),YMIN(7),YMAX(7),YINC(7),NDIGY(7),BCX1(4),BCX2(4)
0006      1,BCY1(4),BCY2(4),BCY3(2),BCY4(3),BCY5(3),BCY6(3),BCY7(4)
0007      DATA AVEJ/0.7308,0.6240,0.4890,0.4001,0.3404,0.2978/
0008      DATA BCI/2HKM/
0009      DATA NDIGY/-1,1,-1,3,3,3,-1/
0010      DATA BCX1/4HANGL,4HE(0E,4HGRE,4HS) /
0011      DATA BCX2/4HRANG,4HE(KM,4H) /
0012      DATA BCY1/4HLC(P),4H(C-N,4HYCLE,4H**2)/
0013      DATA BCY2/4HANGL,4HE(0E,4HGRE,4HS) /
0014      DATA BCY3/4HRAYC,4HM) /
0015      DATA BCY4/4HABRE,4HCACK,4HM) /
0016      DATA BCY5/4HPHIC,4HCYCL,4HES) /
0017      DATA BCY6/4PHPID,4HOT(C,4HHP) /
0018      DATA BCY7/4HV(P),4H(C-N,4PHH4*,4H2) /
0019      CALL R8STOP
0020      INTEGER ONE
0021      REAL*4 LZERO,LP,LVZERO,IOTASQ,IOTOT,MU
0022      ZERO=0.0
0023      WRITE(6,97)
0024      97   FORMAT(1H1)

```

#### C READ IN ACOUSTIC AND INTERNAL WAVE PARAMETERS

```

0025      1   READ(5,1,ERR=4,STAT=ISTAT)CZERO,FREQ,RMAX,NR,DEPTH
0026      2   FORMAT(F7.2,F8.3,F8.0,I5,F5.0)
0027      3   READ(5,2,ERR=4,STAT=ISTAT)B,BZERO,DELC,FREQIN,JSTAR
0028      4   FORMAT(F5.0,F5.2,E9.2,F6.4,I5)
0029      5   READ(5,3,ERR=4,STAT=ISTAT)IARR,NPRNT,NPLT,(IPLT(I),I=1,7)
0030      6   FORMAT(I1,I4,B11)
0031      7   GO TO 6
0032      8   WRITE(6,7)ISTAT
0033      9   FORMAT(1H ,5HERROR,I10)
0034      STOP

```

#### C CALCULATE EXPRESSIONS BASED ON INPUT PARAMETERS

```

0035      6   DELR=RMAX/FLOAT(NR)
0036      7   PI=3.1415927
0037      8   PISTAR=PI+JSTAR
0038      9   PI2=6.2831853
0039      10  NW=(PI2*FREQ)/CZERO
0040      11  NRP1=NR + 1
0041      12  GAMMA=-1.0/B

```

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```

0039      BETA=BVZERO/FREQIN
0040      AVE=AVEJ(JSTAR + 1)
0041      LZERO=(CAVE*BETAB)/(PI**4)
0042      ALPHA=((WV*DELC)    )**2*LZERO*DELR
0043      VZERO=(32.0+AVE*8+FREQIN*BZERO)/(PI2*PI2)
0044      DELTA=((WV*DELC)    )**2*VZERO*DELR
0045      LVZERO=(B/PISTAR)*SQRT((AVE*(PISTAR-1))/PI)
0046      MU=(((PI2*10)/ALOG(10.))**2)          /(WV*LVZERO*LVZERO)

C READ IN RAY PATH AND ABSOLUTE VALUE OF RECIPROCAL OF PHASE CURVATURE FUNCTION

0047      READ(8,11,ERR=4,STV=ISTAT)(RAY(I),I=1,NRP1)
0048      READ(9,11,ERR=4,STV=ISTAT)(ABRECA(I),I=1,NRP1)
0049      11      FORMAT(10F8.1)
0050      ZSCR=RAY(1)
0051      ZREC=RAY(NRP1)

C CALCULATE SLOPE AND LOCAL ANGLE RAY PATH MAKES WITH RESPECT TO THE HORIZONTAL

0052      DO 50 I=1,NR
0053      SLOPE(I)=(RAY(I+1)-RAY(I))/DELR
0054      ANGLE(I)=ATAN(SLOPE(I))
0055      50      CONTINUE

C CALCULATE PHISQ DOTSQ AND IOTASQ ARRAYS

0056      DO 10 J=1,2
0057      PHISQ(J,1)=0.0
0058      DOTSQ(J,1)=0.0
0059      IOTASQ(J,1)=0.0
0060      DO 10 I=1,NR
0061      T1=GAMMA*RAY(I+J-1)
0062      T2=EXP(3.0*T1)
0063      T3=EXP(T1)
0064      T5=EXP(2.0*T1)
0065      X4=BETA+B3
0066      X=X4*ABS(SLOPE(I))

C CALCULATE F1 AND F2 FUNCTIONS HAVING CALCULATED X

0067      XSQ=X*X
0068      IF(XSQ.NE.ZERO)GO TO 30
0069      FX=1.0
0070      F2=1.0
0071      GO TO 32
0072      30      X1=XSQ+1.0
0073      X2=SQRT(X1)

```

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```

0074      X3=X2**3
0075      X5=XSQ/X3
0076      FX1=1.0/X1
0077      FX2=ALOG(X2+1.0)
0078      FX3=X5*FX2
0079      FX4=.5*X5*ALOG(XSQ)
0080      FX=FX1+FX3-FX4
0081      FX5=ALOG(X4)
0082      72    FX6=.5*ALOG(XSQ/4)
0083      FX7=(ALOG(X2+1.0)/X2)-(ALOG(XSQ)/(2*X2))
0084      71    F2=(FX5-FX6-FX7)/FX5

0085      32    S=ALPHA*T2*FX
0086      U=MU*T5*ABRECA(I+J-1)
0087      PHISQ(J,I+1)=PHISQ(J,I)+S
0088      IOTASQ(J,I+1)=IOTASQ(J,I)+(S+U)
0089      DOTSQC(J,I+1)=DOTSQC(J,I)+(DELTA*T2*ALOG(X4)*F2)
0090      10    CONTINUE

```

## C CALCULATE RESULTS

```

0091      D8 35. I=1,NRP1
0092      D1=0.5*(PHISQ(1,I)+PHISQ(2,I))
0093      D2=0.5*(DOTSQC(1,I)+DOTSQC(2,I))
0094      PHIC(I)=SQR(D1)
0095      PHIDOT(I)=SQR(D2)
0096      35    CONTINUE
0097      PHITOT=PHIC(NRP1)
0098      DOTTOT=PHIDOT(NRP1)
0099      IOTTOT=SQR(0.5*(IOTASQ(1,NRP1)+IOTASQ(2,NRP1)))

```

## C CALCULATE ERROR

```

0100      IF(PHITOT.EQ.ZERO)GO TO 53
0101      EPHI=100.00*ABS(PHISQ(1,NRP1)-PHISQ(2,NRP1))/(PHITOT+PHITOT)
0102      56    IF(DOTTOT.EQ.ZERO)GO TO 54
0103      EDOT=100.00*ABS(DOTSQC(1,NRP1)-DOTSQC(2,NRP1))/(DOTTOT+DOTTOT)
0104      57    IF(IOTTOT.EQ.ZERO)GO TO 55
0105      EIOT=100.00*ABS(IOTASQ(1,NRP1)-IOTASQ(2,NRP1))/(IOTTOT+IOTTOT)
0106      GO TO 52
0107      53    EPHI=0.0
0108      GO TO 56
0109      54    EDOT=0.0
0110      GO TO 57
0111      55    EIOT=0.0

```

## C PRINT OUT INPUT PARAMETERS, CALCULATED QUANTITIES AND RESULTS

```

0112    52  WRITE(6,300)
0113    300  FORMAT(1H1,100HPROGRAM FOR CALCULATING SINGLE RAY PATH STATISTICS
0114                                IASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES/>
0115    301  FORMAT(1H0,25HINPUT ACOUSTIC PARAMETERS/)
0116    302  WRITE(6,302)CZERO
0117    302  FORMAT(1H ,33HREFERENCE SOUND SPEED (CZERO) = ,F7.2,6H M/SEC)
0118    303  WRITE(6,303)FREQ
0119    303  FORMAT(1H ,33HACOUSTIC FREQUENCY (FREQ)      = ,F8.3,6H HERTZ)
0120    304  WRITE(6,304)RMAX
0121    304  FORMAT(1H ,33HMAX RANGE (RMAX)              = ,F8.0,2H M)
0122    305  WRITE(6,305)NR
0123    305  FORMAT(1H ,33HNUMBER OF RANGE STEPS (NR)   = ,I5)
0124    306  WRITE(6,306)DEPTH
0125    306  FORMAT(1H ,33HOCEAN DEPTH (DEPTH)         = ,F5.0,2H M)
0126    307  WRITE(6,307)ZSCR
0127    307  FORMAT(1H ,33HSOURCE DEPTH (ZSCR)        = ,F6.1,2H M)
0128    308  WRITE(6,308)ZREC
0129    308  FORMAT(1H ,33HRECEIVER DEPTH (ZREC)       = ,F6.1,2H M)
0130    309  WRITE(6,309)
0131    309  FORMAT(1H0,30HINPUT INTERNAL WAVE PARAMETERS)
0132    310  WRITE(6,310)B
0133    310  FORMAT(1H0,42HSTRATIFICATION SCALE (B)     = ,F5.0,2H M
1)
0134    311  WRITE(6,311)BVZERO
0135    311  FORMAT(1H ,42HEXTRAPOLATED B-V FREQUENCY (BVZERO) = ,F5.2,4H C
1PH)
0136    312  WRITE(6,312)DELC
0137    312  FORMAT(1H ,41HEXTRAPOLATED FRAC. FLUCTUATION (DELC) = ,E9.2)
0138    313  WRITE(6,313)FREQIN
0139    313  FORMAT(1H ,42HINERTIAL FREQUENCY (FREQIN)      = ,F6.4,4H C
1PH)
0140    314  WRITE(6,314)JSTAR
0141    314  FORMAT(1H ,42HJSTAR (JSTAR)                  = ,I1)
0142    315  WRITE(6,315)
0143    315  FORMAT(1H0,21HCALCULATED QUANTITIES)
0144    316  WRITE(6,316)DELR
0145    316  FORMAT(1H0,21HRANGE INCR. (DELR) = ,F7.2,2H M)
0146    317  WRITE(6,317)WV
0147    317  FORMAT(1H ,21HWAVENUMBER (WV)      = ,F6.4,3H /M)
0148    318  WRITE(6,318)LZERO
0149    318  FORMAT(1H ,21HLZERO                 = ,F8.3,3H M)
0150    319  WRITE(6,319)AVE
0151    319  FORMAT(1H ,21HAVE OF 1/J (AVE) = ,F6.4)
0152    320  WRITE(6,320)
0153    320  FORMAT(1H0,7HRESULTS)

```

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```

0154      WRITE(6,321)PHITOT
0155      321  FORMAT(1H0,41HRMS PHASE FLUCTUATION (PHITOT)      = ,F9.5,7H C
          1CYCLE)
0156      WRITE(6,322)DOTTOT
0157      322  FORMAT(1H ,41HRMS PHASE RATE FLUCTUATION (DOTTOT) = ,F9.5,5H C
          1PH)
0158      WRITE(6,323)IOTOT
0159      323  FORMAT(1H.,41HRMS INTENSITY FLUCTUATION (IOTOT)   = ,F7.3,6H
          1 DB)
0160      WRITE(6,324)EPHI
0161      324  FORMAT(1H0,50ERROR FOR PHASE (EPHI)     = ,F
          16.2,8H PERCENT)
0162      WRITE(6,325)EDOT
0163      325  FORMAT(1H ,50ERROR FOR PHASE RATE (EDOT) = ,F
          16.2,8H PERCENT)
0164      WRITE(6,326)EIOT
0165      326  FORMAT(1H ,50ERROR FOR INTENSITY (EIOT) = ,
          1F6.2,8H PERCENT)

```

C CHECK IF ARRAYS TO BE PRINTED OUT

```

0166      IF(CIARR.EQ.0) GO TO 60

```

C WRITE OUT TABLES

```

0167      WRITE(6,327)
0168      327  FORMAT(1H0,6HTABLES)
0169      WRITE(6,328)
0170      328  FORMAT(1H0,95HINDEX      RANGE-M      RAY(I)-M      ANGLE(I-1)-RAD   IC
          11/A)I(I)-M      PHI(I)-CYCLE      PHIDOT(I)-CPH)
          ONE=1.0
0171      WRITE(6,329)ONE,ZERO,RAY(1),ABRECA(1) ,PHI(1),PHIDOT(1)
0172      329  FORMAT(1H0,I5,4X,F8.0,3X,F8.1,24X,F8.1,7X,F9.5,6X,F9.5)
0173      DO 16 K=NRPNT,NRP1,NPRNT
0174      16   SMR=(K-1)*DELR
0175      WRITE(6,330)K,SMR,RAY(K),ANGLE(K-1),ABRECA(K) ,PHI(K),PHIDOT(K)
0176      330  FORMAT(1H ,I5,4X,F8.0,3X,F8.1,4X,F8.5,12X,F8.1,7X,F9.5,6X,F9.5)
0177      16   CONTINUE
0178      16   IF(K.EQ.NRP1)GO TO 60
0179      SMR=NR*DELR
0180      WRITE(6,330)NRP1,SMR,RAY(NRP1),ANGLE(NRP1-1),ABRECA(NRP1),PHI(NRP1
          1),PHIDOT(NRP1)

```

C CHECK IF PLOTS REQUIRED

```

0182      60   IF(NPLT.EQ.0) GO TO 98
0183      CALL PLOTS(PLTAR,1000,.75)

```

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0184

IFIRST=1

C CALCULATE MINIMUM AND MAXIMUM VALUES ATTAINED BY PLOTS

```

0185      DO 999 K=1,7
0186      YMIN(K)=0.0
0187      IF(K.EQ.2) YMIN(K)=-20.0
0188      IF(K.EQ.3) YMIN(K)=DEPTH
0189      999  CONTINUE
0190      YMAX(1)=10.0*INT((1.1*LZERO)/10.0) + 10.0
0191      YMAX(2)=20.0
0192      YMAX(3)=0.0
0193      TOP=ABRECA(1)
0194      DO 997 K=1,NR
0195      COMP=ABRECA(K+1)
0196      IF((COMP.GT.TOP) TOP=COMP
0197      997  CONTINUE
0198      YMAX(4)=1.1*(TOP/1000.)
0199      YMAX(5)=1.1*PHITOT
0200      YMAX(6)=1.1*DOTTOT
0201      YMAX(7)=10.0*INT(YZERO*(ALOG(BETA)+1)/10.0) + 10.0
0202      DO 996 K=1,7
0203      YINC(K)=(YMAX(K)-YMIN(K))/10.0
0204      IF(K.NE.2) GO TO 996
0205      YINC(2)=5.0
0206      996  CONTINUE
0207      DO 998 K=1,7
0208      XMAX(K)=RMAX/1000.0
0209      IF((K.EQ.1).OR.(K.EQ.7))XMAX(K)=10.0
0210      998  CONTINUE

```

C DRAW PLOTS

```

0211      DO 700 K=1,7
0212      IF(IPLT(K).EQ.0) GO TO 700

```

C DRAW X AND Y AXES

```

0213      BMAX=XMAX(K)
0214      ZMIN=YMIN(K)
0215      ZMAX=YMAX(K)
0216      XIN=BMAX/10.0
0217      YIN=YINC(K)
0218      NX=-1
0219      NY=NDIGY(K)
0220      YSCALE=(ZMAX-ZMIN)/10.0
0221      XSCALE=XIN

```

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```

0222      IPEN=3
0223      XADD=DELR/1000./XSCALE
0224      IF(IFIRST.NE.1) GO TO 15
0225      IFIRST=0
0226      CALL ORIGIN(2.0,0)
0227      15   CALL NXAXIS(0,0,XIN,BMAX,10.,-.07,.07,NX)
0228          IF(K.EQ.1.OR.K.EQ.7) GO TO 13
0229          CALL LETTER(4.0,-.5,.25,BCX2,0,12)
0230      13   CALL NYAXIS(0,0,ZMIN,YIN,ZMAX, 10.0 ,-.07,.07,NY)
0231          CALL NUMBER(-.125,-.125,.07,ZMIN,90.,NY)
0232          CALL NXAXIS(0,10.0,0,XIN,BMAX,10.0,.07,0,NX)
0233          CALL NYAXIS(10.0,0,ZMIN,YIN,ZMAX,10.0,.07,0,NY)
0234          GO TO(801,802,803,804,805,806,807),K

C PLOT 1 L(P)VRS 0

0235      801  ILBL=1
0236          CALL LETTER(3.5,-.50,.25,BCX1,0,16)
0237          CALL LETTER(-.375,3.0,.25,BCY1,90.,16)
0238          RAD=PI/180.
0239          XADD=0.2/XSCALE
0240          DO 63 J=0,2000,250
0241          IF(J.EQ.1250.OR.J.EQ.1750) GO TO 63
0242          ILBL=ILBL + 5
0243          IPEN=3
0244          XAXIS=-XADD
0245          T1=EXP(GAMMA+J)*BETA
0246          DO 25 I=1,51
0247              THETA=(I-1)*.2*RAD
0248              X=TAN(THETA) *: T1

C CALCULATE F1 FUNCTION HAVING CALCULATED X

0249      XSQ=X*X
0250      IF(XSQ.NE.ZERO)GO TO 40
0251      FX=1.0
0252      GO TO 41
0253      40   X1=XSQ + 1.0
0254          X2=SQRT(X1)
0255          X3=X2**3
0256          X5=XSQ/X3
0257          FX1=1.0/X1
0258          FX2=ALOG(X2+1.0)
0259          FX3=X5*FX2
0260          FX4=.5*X5*ALOG(XSQ)
0261          FX=FX1+FX3-FX4
0262      41   XAXIS=(XAXIS+XADD)

```

```

0263      LP=(LZERO+FX)
0264      IF(LP.LT.0.0)LP=0.0
0265      YAXIS=LP/YSCALE
0266      CALL PLOT(XAXIS,YAXIS,IPEN)
0267      IPEN=2
0268      IF(ILBL.EQ.1) GO TO 95
0269      GO TO 25
0270  95  ZNUM=J/1000.
0271      YAX=YAXIS + .03
0272      CALL NUMBER(XAXIS + .03,YAX,.07,ZNUM,0,2)
0273      CALL LETTER(XAXIS + .33,YAX,.07,BCI,0,2)
0274      CALL PLOT(XAXIS,YAXIS,3)
0275  25  CONTINUE
0276  63  CONTINUE
0277  GO TO 61

```

## C PLOT 2 ANGLE(R) VRS R

```

0278      802  XAXIS=0
0279      CALL LETTER(-.375,3.0,.25,BCY2,90.,16)
0280      DO 21 I=1,NR
0281      XAXIS=(XAXIS+XADD)
0282      DEG=180./PI
0283      DEGREE=ANGLE(I)*DEG
0284      IF(DEGREE.GT.20.0)GO TO 12
0285      IF(DEGREE.LT.-20.0)DEGREE=-20.0
0286      GO TO 23
0287  12  DEGREE=20.0
0288  23  YAXIS=(DEGREE-ZNIN)/YSCALE
0289      CALL PLOT(XAXIS,YAXIS,IPEN)
0290      IPEN=2
0291  21  CONTINUE
0292  GO TO 61

```

## C PLOT 3 RAY(R) VRS R

```

0293      803  XAXIS=-XADD
0294      CALL LETTER(-.375,4.0,.25,BCY3,90.,8)
0295      DO 20 I=1,NRP1
0296      XAXIS=(XAXIS+XADD)
0297      IF(RAY(I).LT.0.0)RAY(I)=0.0
0298      IF(RAY(I).GT.DEPTH)RAY(I)=DEPTH
0299      YAXIS=(RAY(I)-ZNIN)/YSCALE
0300      CALL PLOT(XAXIS,YAXIS,IPEN)
0301      IPEN=2
0302  20  CONTINUE
0303  GO TO 61

```

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C PLOT 4 ABRECA(R) VRS R

```
0304    804  XAXIS=-XADD
0305      CALL LETTER(-.375,4.0,.25,BCY4,90.,12)
0306      DO 605 I=1,NRP1
0307      XAXIS=XAXIS + XADD
0308      YAXIS=((ABRECA(I)/1000.)-ZMIN)/YSCALE
0309      CALL PLOT(XAXIS,YAXIS,IPEN)
0310      IPEN=2
0311      605  CONTINUE
0312      GO TO 61
```

C PLOT 5 PHIC(R) VRS R

```
0313    805  XAXIS=-XADD
0314      CALL LETTER(-.375,4.0,.25,BCY5,90.,12)
0315      DO 22 I=1,NRP1
0316      XAXIS=(XAXIS+XADD)
0317      IF(PHIC(I).LT.0.0)PHIC(I)=0.0
0318      YAXIS=(PHIC(I)-ZMIN)/YSCALE
0319      CALL PLOT(XAXIS,YAXIS,IPEN)
0320      IPEN=2
0321      22  CONTINUE
0322      GO TO 61
```

C PLOT 6 PHIDOT(R) VRS R

```
0323    806  XAXIS=-XADD
0324      CALL LETTER(-.375,4.0,.25,BCY6,90.,12)
0325      DO 65 I=1,NRP1
0326      XAXIS=(XAXIS+XADD)
0327      IF(PHIDOT(I).LT.0.0)PHIDOT(I)=0.0
0328      YAXIS=(PHIDOT(I)-ZMIN)/YSCALE
0329      CALL PLOT(XAXIS,YAXIS,IPEN)
0330      IPEN=2
0331      65  CONTINUE
0332      GO TO 61
```

C PLOT 7 V(P) VRS 0

```
0333    807  RAD=3.14159265/180.
0334      CALL LETTER(3.5,-.50,.25,BCX1,0,16)
0335      CALL LETTER(-.375,3.0,.25,BCY7,90.,16)
0336      XADD=0.2/XSCALE
0337      DO 83 J=0,2000,250
0338      IF(J.EQ.1250.OR.J.EQ.1750) GO TO 83
```

```

0339      IPEN=3
0340      XAXIS=-XADD
0341      T1=EXP(GAMMA+J)*BETA
0342      T2=ALOG (T1)
0343      DO 85 I=1,51
0344      THETA=(I-1)*.2*RAD
0345      X=TAN(THETA)*T1

C   CALCULATE F2 FUNCTION HAVING CALCULATED X

0346      XSQ=X*X
0347      IF(XSQ.NE.ZERO)GO TO 90
0348      F2=1.0
0349      GO TO 91
0350      90  X1=XSQ + 1.0
0351      X2=SQRT(X1)
0352      FX5=ALOG(T1)
0353      FX6=.5*ALOG (XSQ/4)
0354      FX7=( ALOG(X2+1.0)/X2)-( ALOG(XSQ)/(2*X2))
0355      F2=(FX5-FX6-FX7)/FX5
0356      91  XAXIS=XAXIS + XADD
0357      VP=VZERO*T2+F2
0358      IF(VP.LT.0.0)VP=0.0
0359      YAXIS=VP/YSCALE
0360      CALL PLOT(XAXIS,YAXIS,IPEN)
0361      IPEN=2
0362      IF(I.NE.4) GO TO 85
0363      ZNUM=J/1000.
0364      TAX=YAXIS + .06
0365      CALL NUMBER(XAXIS + .03,YAX,.07,ZNUM,0,2)
0366      CALL LETTER(XAXIS + .33,YAX,.07,BCI,0,2)
0367      CALL PLOT(XAXIS,YAXIS,3)
0368      85  CONTINUE
0369      83  CONTINUE
0370      61  CALL ORIGIN(12.0,0)
0371      700 CONTINUE
0372      CALL ENDPLT
0373      98  STOP
0374      END

```

## Appendix C SAMPLE RUN

```

/ JOB MARILYN BLOODGETT,           BLOODM1,OPT=(C,R,D,T)
/ LIMIT MIN=1,BAND=25
/ FD FT59F001,FRGRG=PS,RCFM=U,BKSZ=4000,BAND=1/10/1
/ FTN IN=SDECK,FTVERS=NX,FTNOPT=(K,U)
/ LNK
/ FXQT OPT=(I,A)
/ F0SYS FT59F001,TYPE=PL0T
/ START ACNM=SDECK
.
.
.
FORTRAN SOURCE DECK
.
.
.
/ STOP
/ START ACNM=FT05F001
1500.00 100.000 10000. 1004040.
1000. 3.00 0.49E-030.0420   3
1 1011111111
/ STOP
/ START ACNM=FT08F001
 1000.0   960.4   921.6   883.6   846.4   810.0   774.4   739.6   705.6   672.4
   640.0   608.4   577.6   547.6   518.4   490.0   462.4   435.6   409.6   384.4
   360.0   336.4   313.6   291.6   270.4   250.0   230.4   211.6   193.6   176.4
   160.0   144.4   129.6   115.6   102.4   90.0    78.4   67.6   57.6   48.4
    40.0    32.4    25.6    19.6    14.4    10.0    6.4    3.6    1.6    0.4
     0.0     0.4     1.6     3.6     6.4    10.0    14.4    19.6    25.6   32.4
    40.0    48.4    57.6    67.6    78.4    90.0   102.4   115.6   129.6   144.4
   160.0   176.4   193.6   211.6   230.4   250.0   270.4   291.6   313.6   336.4
   360.0   384.4   409.6   435.6   462.4   490.0   518.4   547.6   577.6   608.4
   640.0   672.4   705.6   739.6   774.4   810.0   846.4   883.6   921.6   960.4
 1000.0
/ STOP
/ START ACNM=FT09F001
    0.0    99.0   196.0   291.0   384.0   475.0   564.0   651.0   736.0   819.0
   900.0   979.0   1056.0   1131.0   1204.0   1275.0   1344.0   1411.0   1476.0   1539.0
  1600.0   1659.0   1716.0   1771.0   1824.0   1875.0   1924.0   1971.0   2016.0   2059.0
  2100.0   2139.0   2176.0   2211.0   2244.0   2275.0   2304.0   2331.0   2356.0   2379.0
  2400.0   2419.0   2436.0   2451.0   2464.0   2475.0   2484.0   2491.0   2496.0   2499.0
  2500.0   2499.0   2496.0   2491.0   2484.0   2475.0   2464.0   2451.0   2436.0   2419.0
  2400.0   2379.0   2356.0   2331.0   2304.0   2275.0   2244.0   2211.0   2176.0   2139.0
  2100.0   2059.0   2016.0   1971.0   1924.0   1875.0   1824.0   1771.0   1716.0   1659.0
  1600.0   1539.0   1476.0   1411.0   1344.0   1275.0   1204.0   1131.0   1056.0   979.0
   900.0   819.0    736.0   651.0   564.0   475.0   384.0   291.0   196.0    99.0
    0.0
/ STOP
/ EOJ

```

M. L. BLODGETT

PROGRAM FOR CALCULATING SINGLE RAY PATH STATISTICS ASSUMING THE GARRETT-MUNK MODEL OF INTERNAL WAVES

INPUT ACOUSTIC PARAMETERS

REFERENCE SOUND SPEED (CZERO) = 1500.00 M/SEC  
 ACOUSTIC FREQUENCY (FREQ) = 100.000 HERTZ  
 MAX RANGE (RMAX) = 10000. M  
 NUMBER OF RANGE STEPS (NRP) = 100  
 OCEAN DEPTH (DEPTH) = 4040. M  
 SOURCE DEPTH (ZSRC) = 1000.0 M  
 RECEIVER DEPTH (ZREC) = 1000.0 M

INPUT INTERNAL WAVE PARAMETERS

STRATIFICATION SCALE (B) = 1000. M  
 EXTRAPOLATED B-V FREQUENCY (BVZERO) = 3.00 CPH  
 EXTRAPOLATED FRAC. FLUCTUATION (DELC) = 0.49E-03  
 INERTIAL FREQUENCY (FREQIN) = 0.0420 CPH  
 JSTAR (JSTAR) = 3

CALCULATED QUANTITIES

RANGE INCR. (DELR) = 100.00 M  
 WAVENUMBER (HW) = 0.4189 /M  
 LZERO = 293.387 M  
 AVE OF 1/J (CAVE) = 0.4001

RESULTS

RMS PHASE FLUCTUATION (PHITOT) = 0.19065 CYCLE  
 RMS PHASE RATE FLUCTUATION (DOTTOT) = 0.16132 CPH  
 RMS INTENSITY FLUCTUATION (IOTOT) = 1.886 DB

ERROR FOR PHASE (EPHI) = 0.00 PERCENT  
 ERROR FOR PHASE RATE (EDOT) = 0.00 PERCENT  
 ERROR FOR INTENSITY (EIOT) = 0.00 PERCENT

TABLES

INDEX	RANGE-M	RAY(I)-M	ANGLE(I-1)-RAD	I(I/A)I(I)-M	PHI(I)-CYCLE	PHIDOT(I)-CPH
1	0.	1000.0		0.0	0.00000	0.00000
10	900.	672.4	-0.32055	819.0	0.00381	0.01512
20	1900.	384.4	-0.24686	1539.0	0.00708	0.03089
30	2900.	176.4	-0.17933	2059.0	0.01179	0.05205
40	3900.	48.4	-0.09174	2379.0	0.02075	0.07875
50	4900.	0.4	-0.01200	2499.0	0.06122	0.11076
60	5900.	32.4	0.06790	2419.0	0.09778	0.13730
70	6900.	144.4	0.14693	2139.0	0.09980	0.15093
80	7900.	336.4	0.22417	1659.0	0.10035	0.15756
90	8900.	608.4	0.29878	979.0	0.10056	0.16033
100	9900.	960.4	0.37012	99.0	0.10065	0.16128
101	10000.	1000.0	0.37703	0.0	0.10065	0.16132

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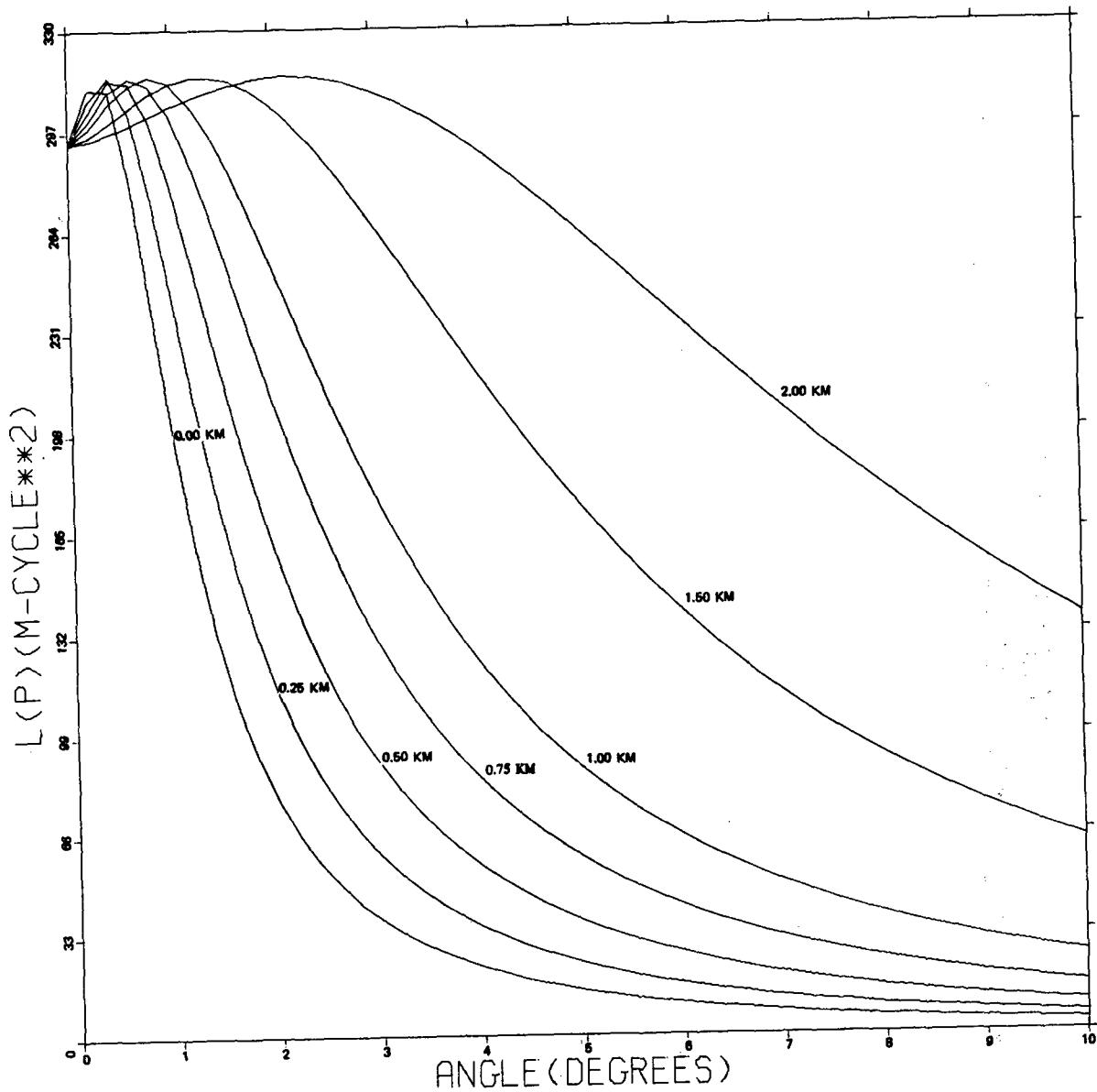


Fig. C1—Plot of  $L_p(\theta, z)$  vs  $\theta$  for various values of  $z$

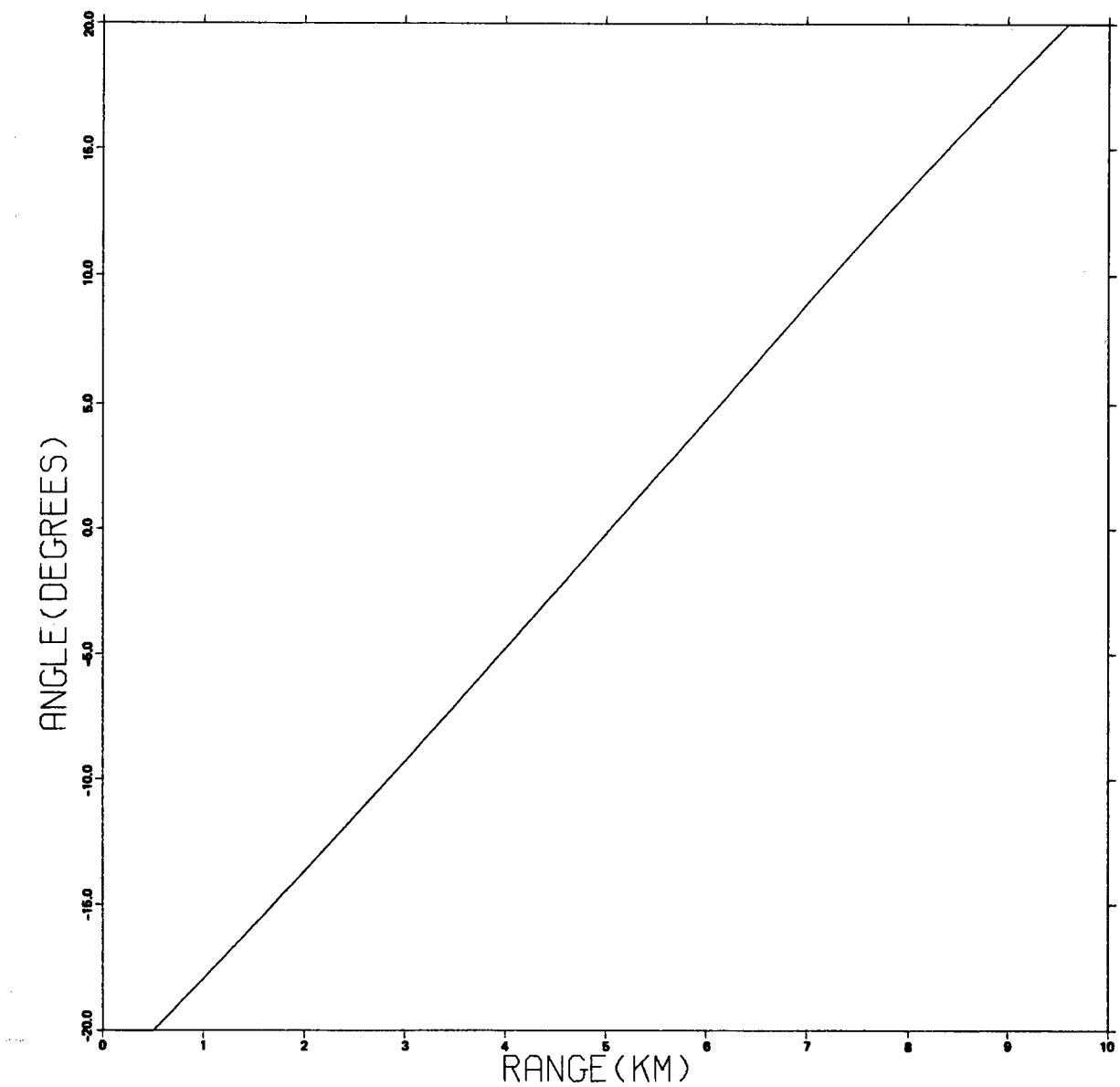


Fig. C2—Plot of the ray angle  $\theta$  vs horizontal path length

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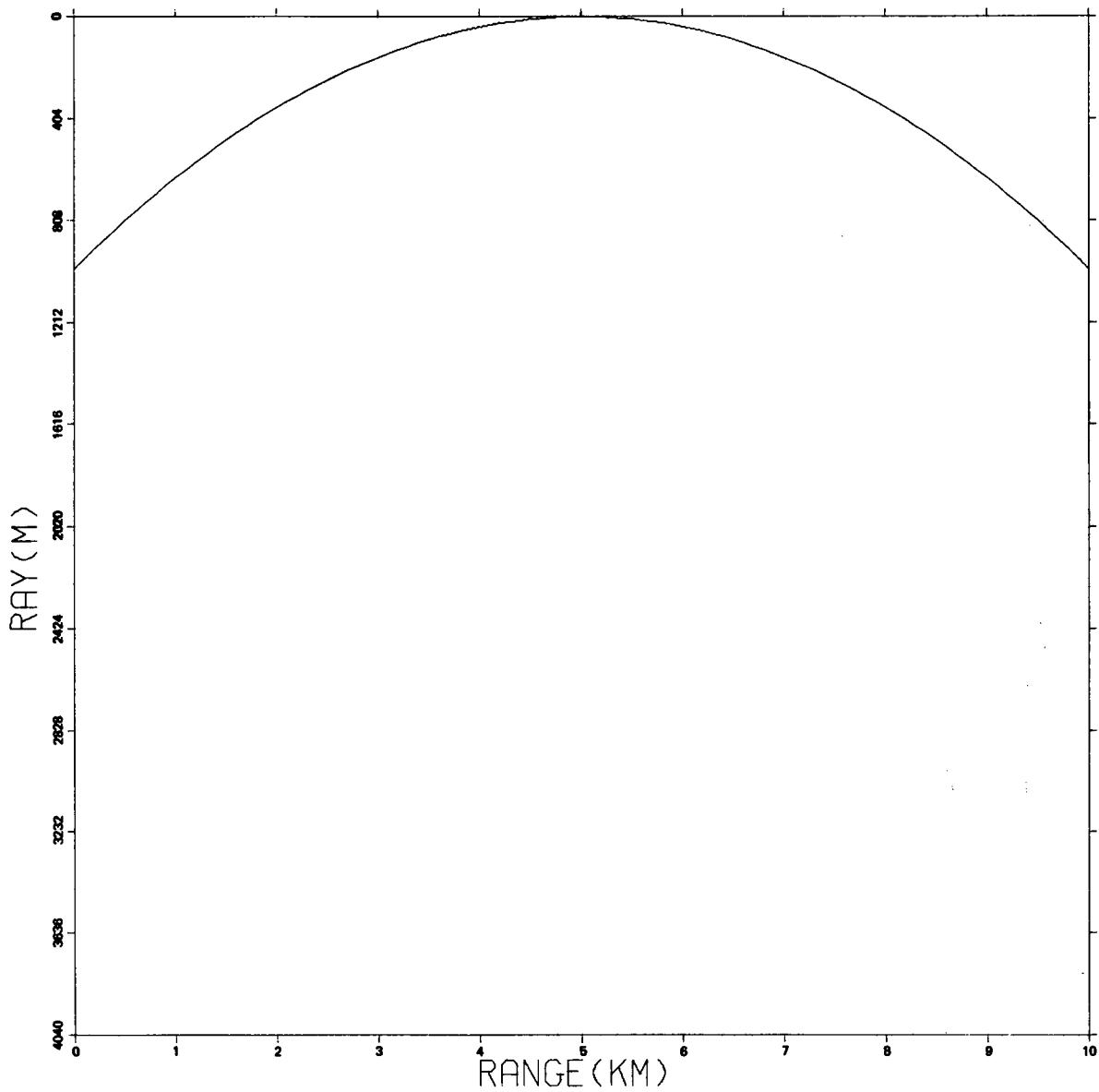


Fig. C3—Plot of the ray path

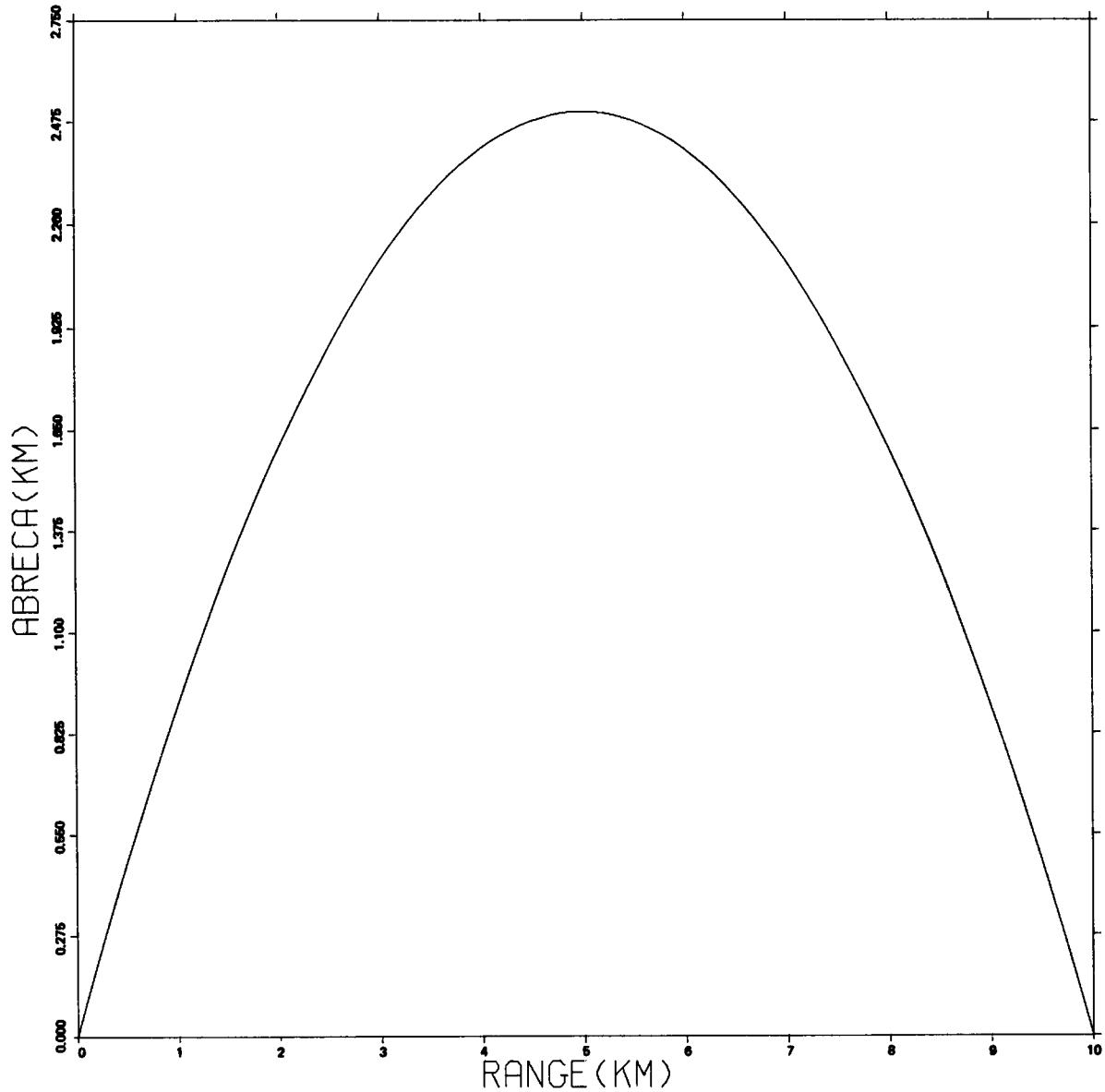


Fig. C4—Plot of the absolute value of the reciprocal of the phase curvature function

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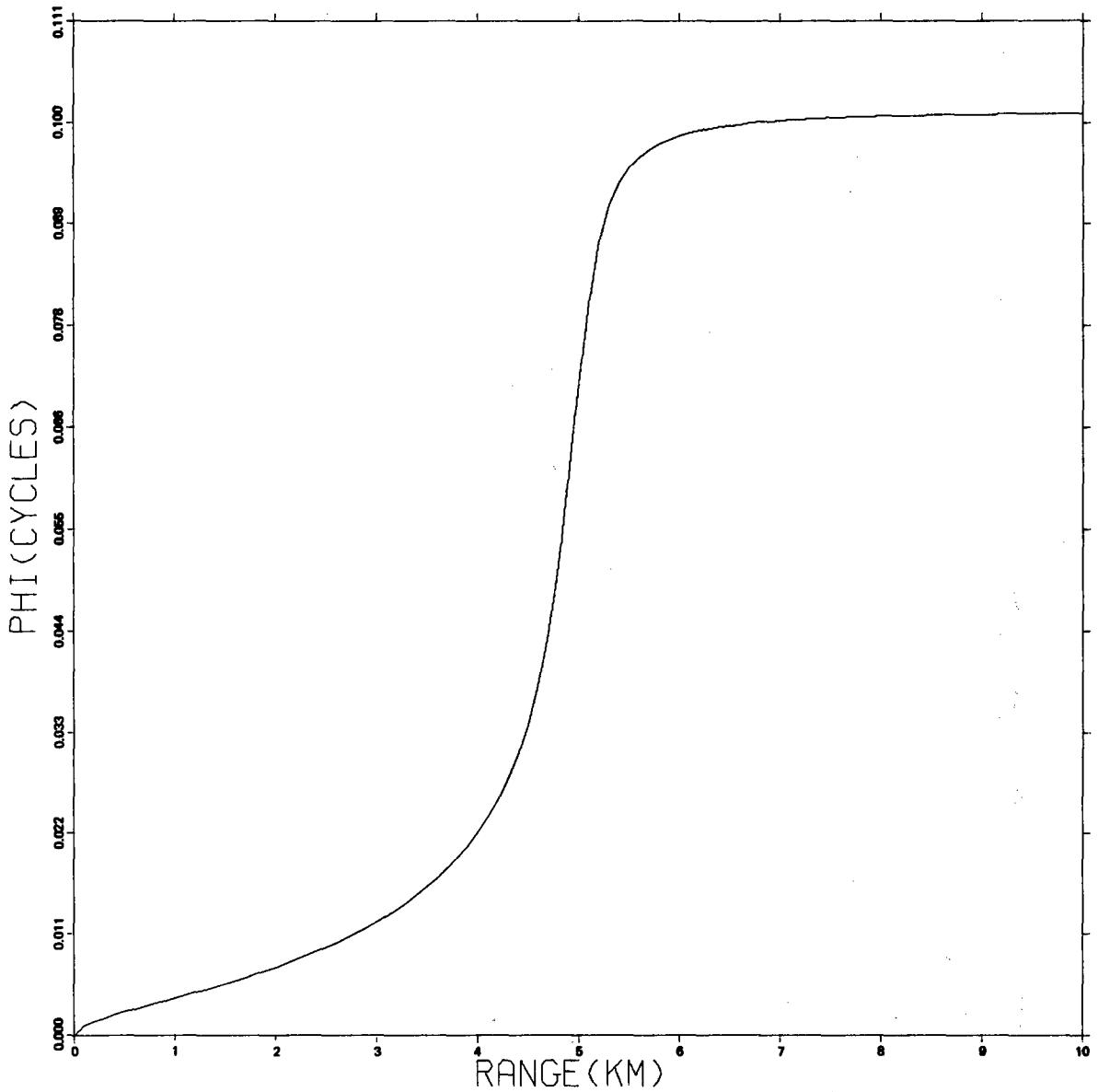


Fig. C5—Plot of the rms phase fluctuation vs horizontal path length

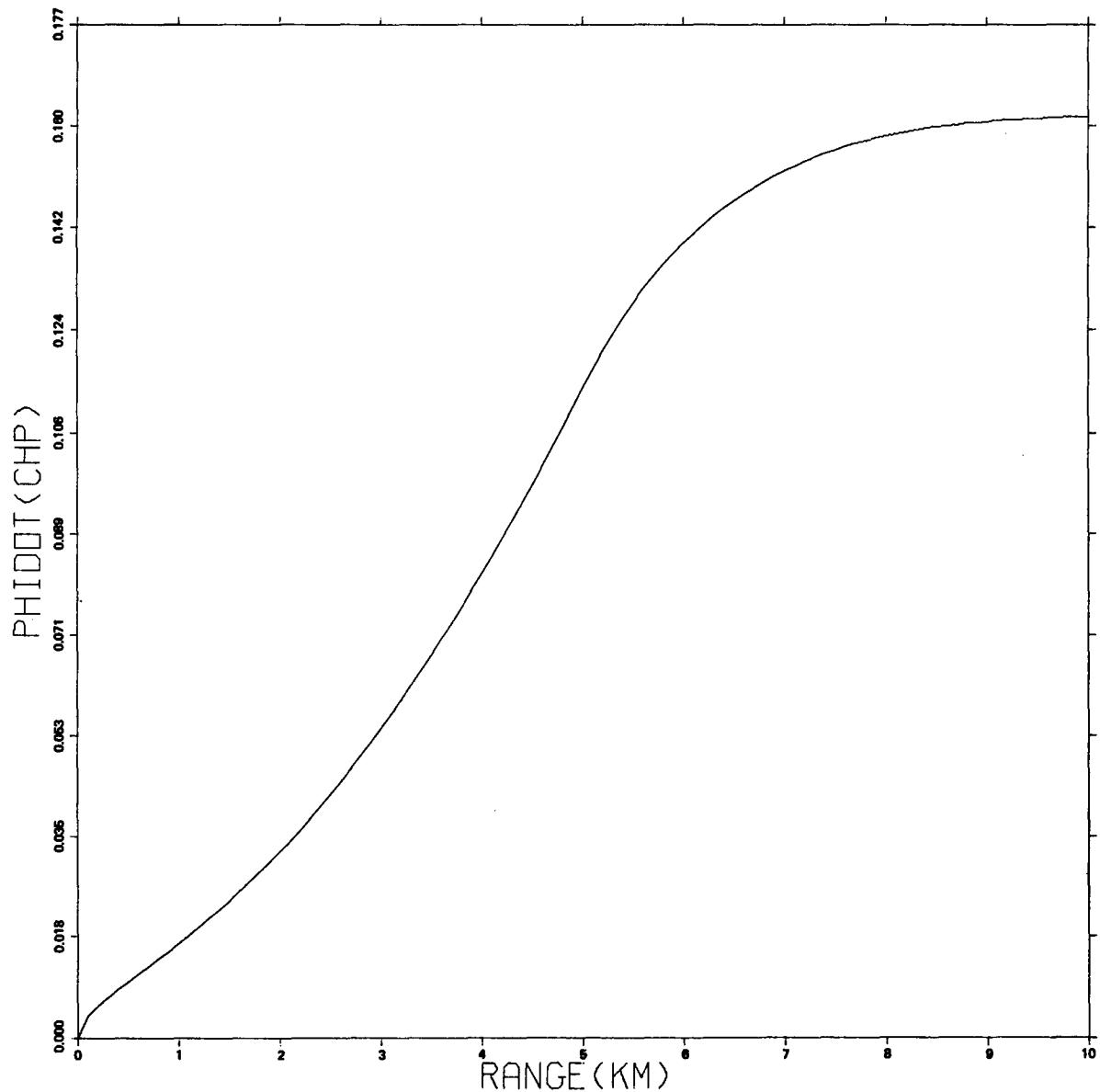
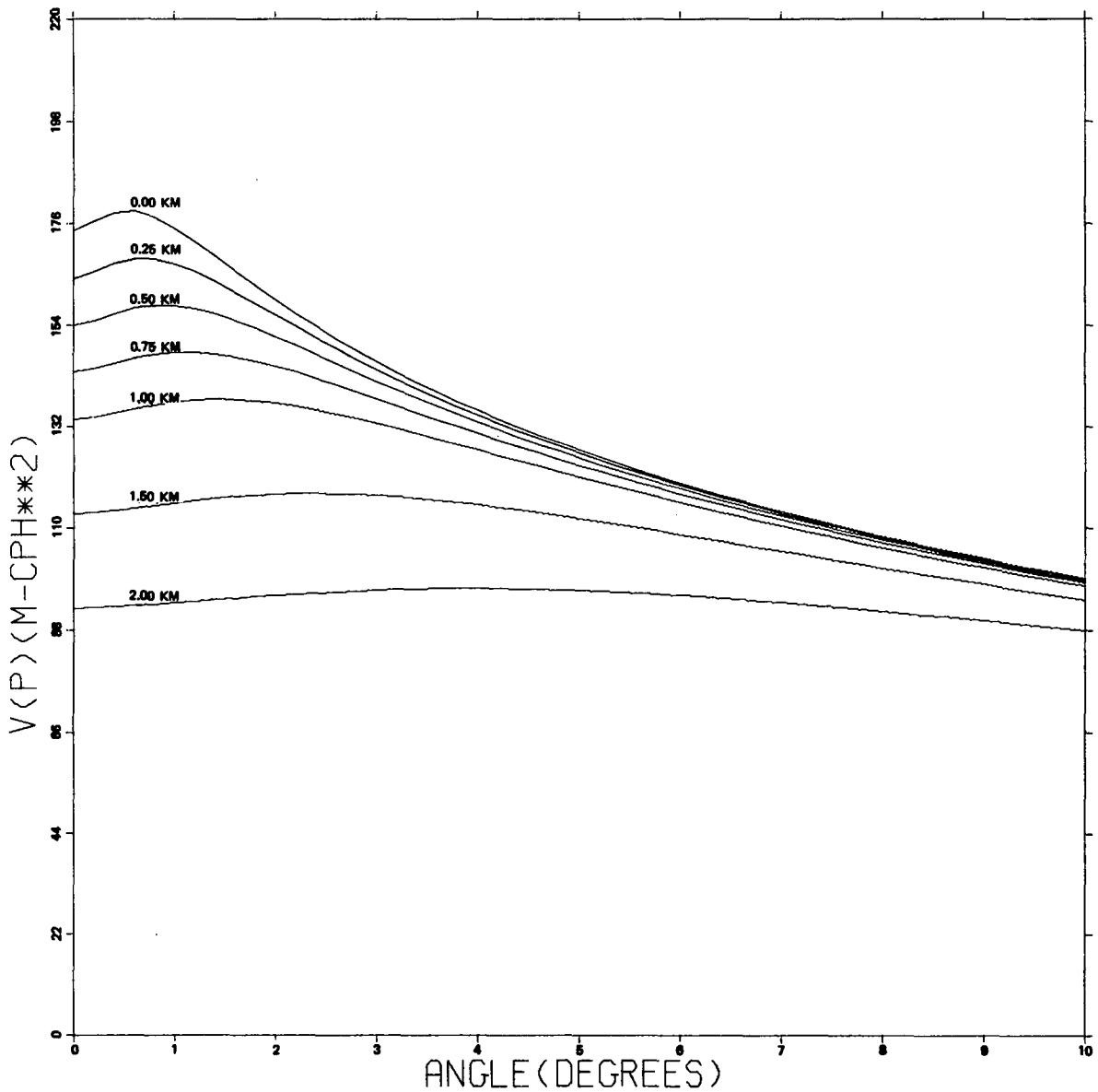


Fig. C6—Plot of the rms phase-rate fluctuation vs horizontal path length

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Fig. C7—Plot of  $V_p(\theta, z)$  vs  $\theta$  for various values of  $z$