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Track Initiation in a Dense Detection Environment

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In a dense detection environment present track-while-scan algorithms will inevitably introduce false tracks when processing is performed on a scan-by-scan basis. The maximum likelihood solution involving all detections on all scans is formulated and evaluated for the initiation problem. While the maximum likelihood method cannot be implemented because of the enormous computational requirements, it provides a standard with which to compare algorithms that can be more readily implemented.

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TRACK INITIATION IN A DENSE DETECTION ENVIRONMENT

INTRODUCTION

Track-while-scan systems were first proposed for surveillance radars during the 1950's. If the probability of detection per scan is high, if accurate measurements are made, if the target density is low, and if there are few false detections, the design of the correlation logic and tracking filter is straightforward. However, in a realistic radar environment these assumptions are never valid, and the design problem is difficult. This paper will consider the problem of track initiation in a dense detection environment.

In Fig. 1, there are three tracks, and each track is detected five times. While it is obvious that there are three tracks present, many tracking systems would initiate incorrect tracks because they only associate the nearest detection with the predicted position of a tentative track. Moreover, the situation in Fig. 1 rarely occurs; the situation in Fig. 2 is more common. Figure 2 is of the same three tracks; however, several detections have been merged (i.e., individual targets are not resolved), three detections are missing, and two false alarms have been introduced.

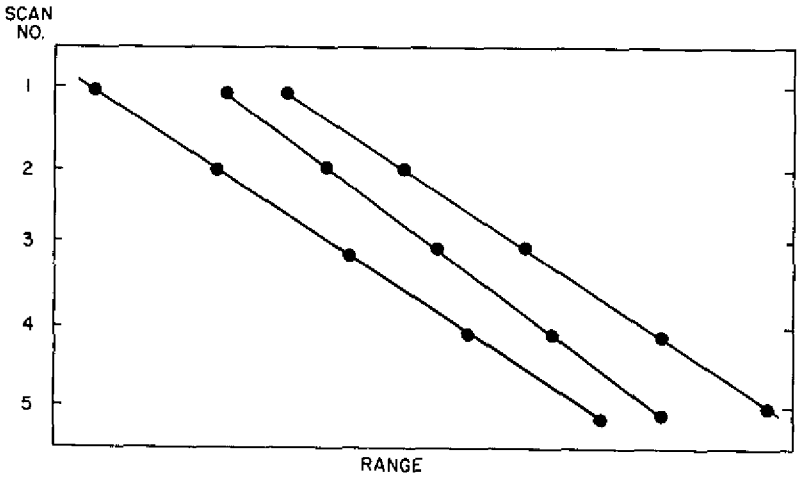

Fig. 1 — History of five scans of three tracks showing all detections present

The optimal solution of such problems has been generated under ideal conditions. Specifically, the maximum likelihood solution has been developed under the assumptions that the probability of detection, the probability of false alarm, the probability of target resolution as a function of target separation, and measurement error characteristics are all known a priori and that all targets are moving in straight lines. (A somewhat similar

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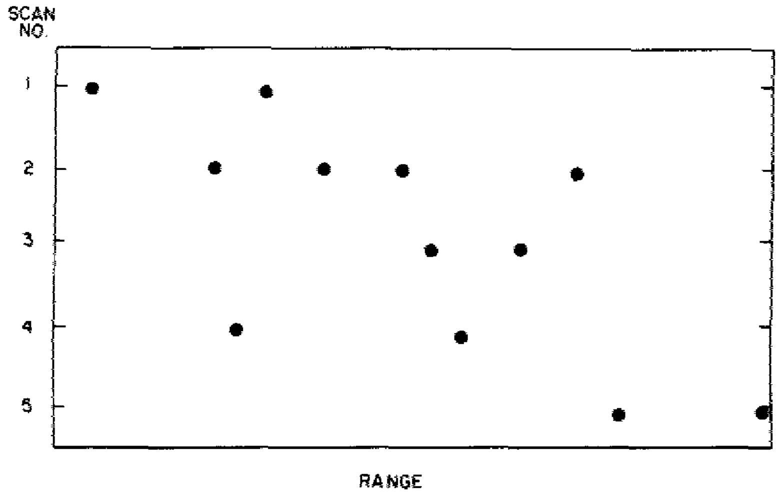

Fig. 2 — History of five scans in which detections were missed, detections were merged, and false alarms occurred

approach was used by Stein and Blackman [1]; however, they did not consider resolution problems.) Even if all the above assumptions were true, the maximum likelihood method cannot be implemented in the foreseeable future because of the enormous computational load. However, it is still useful because it provides a standard with which to compare algorithms that can be more readily implemented.

There are two basic problems with the maximum likelihood method. The first problem is fundamental and concerns the tendency of the maximum likelihood method to indicate two tracks (with many unresolved detections) when a single detection is close to a single track. This problem was solved by penalizing tracks with unresolved targets or missing detections. A detailed description of the maximum likelihood method is given in the next section.

The second problem is the computational load. Since search techniques cannot be used to maximize the likelihood functions because of the large number of local maxima. the concept of a "feasible track" was introduced, a feasible track consisted of a specified number of detections lying within a specified distance of a straight line. Then the maximum likelihood of occurrence of each combination of the feasible tracks was evaluated. If there were N feasible tracks and one is interested in up to M track combinations, $\Sigma_{i=1}^{M} \binom{N}{i}$ likelihood functions would need to be evaluated. For instance, if $N = 30$ and $M = 4$, the number of likelihoods calculated is 31930. This problem is discussed in the section entitled "Calculation of Maximum Likelihood."

A brief description of how the radar data are generated is given in the section entitled "Parameters for Data Generation and Operation of the Simulation," and the results of the maximum likelihood method applied to various target geometries containing one to four tracks and several false alarms are given under "Results." The final section, "Conclusions," summarizes the results and suggests a practical solution that is presently being investigated.

MAXIMUM LIKELIHOOD METHOD

The maximum likelihood method involves calculating the total probability that a given set of detections correctly represents a specified set of tracks. The probability of detection, the probability of false alarm, the probability of target resolution as a function of target separation, and the measurement error characteristics are all taken into account in the calculation of the likelihood. To facilitate the mathematical description of the likelihood method the following terms or definitions are used:

N_S = the number of scans

N_T = the number of tracks

N_D = the total number of detections

N_{FA} = the number of false alarms

N_M = the number of missed detections associated with the N_T tracks

N_{DR} = the number of detections involved in resolution problems (i.e., number of detections used in at least two tracks)

$N_{TR}(k)$ = the total number of tracks using the k-th detection which is used in at least two tracks

x_{ij} = the range of the detection associated with i-th track on the j-th scan. If there is no detection associated (i.e., track has a miss associated), $x_{ij} = 0$

y_{ij} = the predicted range of the i-th track on j-th scan-assuming a straight line trajectory

All of the above variables are not independent. The following relationship holds:

$$N_D = N_S\, N_T - N_M - N_{TR} + N_{DR} + N_{FA}, \tag{1}$$

where

$$N_{TR} = \sum_{k=1}^{N_{DR}} N_{TR}(k).$$

The difference between predicted and measured position is assumed to be Gaussian distributed with zero mean and a variance of σ^2. Thus,

$$p(x_{ij}-y_{ij}) = \frac{1}{(2\pi\sigma^2)^{1/2}}\, e^{-(x_{ij}-y_{ij})^2/2\sigma^2}. \tag{2}$$

For later use, it will be convenient to introduce the expression

$$f(x_{ij}-y_{ij}) = \begin{cases} 1, & x_{ij} = 0 \text{ or } x_{ij} = x_{\ell j}\ i \neq \ell \\ e^{-(x_{ij}-y_{ij})^2/2\sigma^2} & \text{otherwise.} \end{cases} \tag{3}$$

Assuming that the probability of detection P_D is known, the probability of obtaining the specified number of detections is

$$\binom{N_S N_T}{N_M} (P_D)^{N_S N_T - N_M} (1-P_D)^{N_M} \tag{4}$$

The probability of not resolving any $N_{TR}(k) = N_k$ tracks which use a common detection x_k is calculated by first ordering the predicted positions, so that

$$y_{i_1} \leqslant y_{i_2} \leqslant \ldots \leqslant y_{i_{N_k}},$$

where for notational convenience the subscript denoting the scan has been dropped. When one lets $D_\ell = y_{i_\ell} - y_{i_{\ell-1}}$, the probability of not resolving any N_k tracks is given by

$$P_R(x_k) = \prod_{\ell=2}^{N_k} P(D_\ell)\,. \tag{5}$$

In this case the probability of not resolving two tracks separated by distance D (discussed in [2]) is given by

$$P(D) = \begin{cases} 1 & D \leqslant 1.7R \\ (2.6R-D)/(.9R) & 1.7R \leqslant D \leqslant 2.6R, \\ 0 & D \geqslant 2.6R \end{cases} \tag{6}$$

where R is the 3-dB pulse width (range cell dimension). Furthermore, the position of x_k is the sum of two random variables: one uniformly distributed between y_{i_1} and $y_{i_{N_k}}$, and the other Gaussian distributed with mean zero and variance σ^2. In the appendix it is shown that the likelihood of x_k can be approximated by

$$P_e(x_k) = (e^{-\epsilon_k^2/2\sigma^2}) \,/\, \text{Max}(y_{i_{N_k}} - y_{i_1}, \sqrt{2\pi\sigma^2}), \tag{7}$$

where

$$\epsilon_k = \text{Max}\ (0, x_k - y_{i_{N_k}}, y_{i_1} - x_k) \tag{8}$$

is the distance from x_k to the nearest detection if x_k lies outside of the interval defined by the predicted positions; otherwise ϵ_k is zero. Finally, the number and position of false alarms in the range interval of interest R_I is given by the Poisson density

$$P_{FA}(N_{FA}) = \frac{(\lambda R_I)^{N_{FA}} e^{-\lambda R_I}}{(N_{FA})! \; (R_I)^{N_{FA}}} = \frac{\lambda^{N_{FA}} e^{-\lambda R_I}}{(N_{FA})!}, \tag{9}$$

where λ is the false alarm density per unit length and the $(R_I)^{N_{FA}}$ factor in the denominator was due to the fact that the false alarms are uniformly distributed in range.

In terms of the previous expressions the likelihood of an N_T track combination is given by the following:

$$\begin{aligned} L(N_T) = {} & P_{FA}(N_{FA}) \cdot \\ & \binom{N_S N_T}{N_M} (P_D)^{N_S N_T - N_M} (1-P_D)^{N_M} \cdot \\ & \left[\frac{1}{(2\pi\sigma^2)^{1/2}}\right]^{N_S N_T - N_M - N_{TR}} \prod_{i=1}^{N_T} \prod_{j=1}^{N_S} f(x_{ij}-y_{ij}) \cdot \\ & \prod_{k=1}^{N_{DR}} P_R(x_k)\, P_e(x_k). \end{aligned} \tag{10}$$

The first line represents the false alarm probability, the second line represents the detection probability, the third gives the measurement error probability, and the last gives the resolution probability. The maximum likelihood method involves assigning to each i-th track (yielding predicted positions y_{ij}) a sequence of detections (x_{ij}) that maximize the values calculated by formula (10).

As presently formulated, the maximum likelihood method would have trouble with the target geometry shown in Fig. 3. Let the n-tuple $(I_1, I_2, .., I_n)$ represent a track, where I_j is the detection associated with j-th scan of the track. In Fig. 3, there are two tracks, (1,1, 1,1,1) and (2,2,2,2,M); M represents a miss, and detection 3 on scan 1 is a false alarm. However, the maximum likelihood method defined by (10) will yield the solution involving the three tracks (1,1,1,1,1), (2,2,2,2,M), and (3,2,1,M,M). The latter case is more likely (as defined by (10)) for the following reasons: the false alarm likelihood has been increased by 6×10^4 (by eliminating the false alarm), the detection likelihood has been decreased by 0.6 (12 out of 15 detections instead of 9 out of 10 detections), the measurement likelihood is increased by removing a $(1/2\pi\sigma^2)^{1/2}$ factor (two detections declared resolutions but one detection added) and by eliminating two Gaussian errors, and the resolution likelihood is

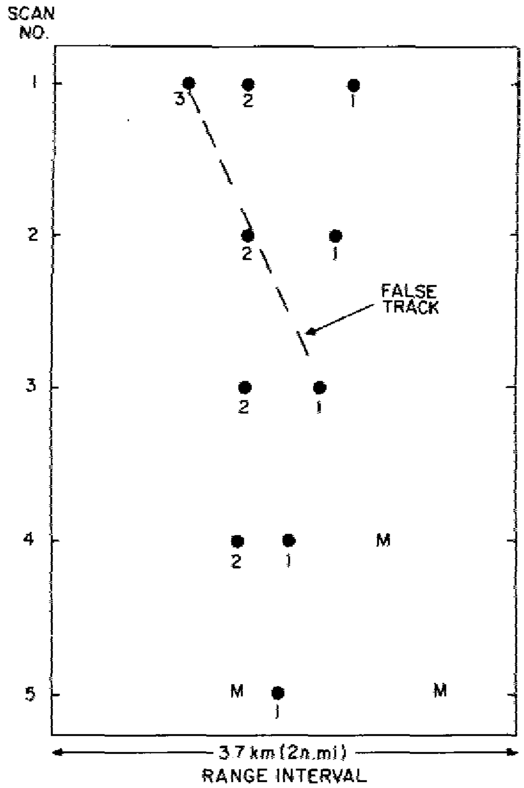

Fig. 3 - Scan history showing a false track created as the result of a false alarm and the incorporation of detections of actual tracks

decreased by $1/2\pi\sigma^2$. Thus, since 6×10^4 is greater than $(1/2\pi\sigma^2)^{1/2}$, the likelihood for three tracks is larger than the likelihood for two tracks. As formulated by (10), the maximum likelihood method will always try to eliminate false alarms by introducing false (extraneous) tracks.

To eliminate this problem, two factors have been introduced. One factor penalizes tracks that have unresolved detections, and the second factor penalizes tracks that have missing detections. Thus, the modified likelihood is given by

$$L_M(N_T) = L(N_T)(F_R)^{N_{TR}-N_{DR}}(F_M)^{N_M} . \tag{11}$$

Usually, we take $1>F_M>F_R$. The values presently being used are $F_M = 0.2$ and $F_R = 0.1$. For the rest of this paper, the maximum likelihood method will be implemented by (11).

CALCULATION OF MAXIMUM LIKELIHOOD

Search techniques cannot be used to maximize the likelihood function (11) because of the large number of local maxima. To solve this computational problem the concept of a feasible track was introduced. Then the maximum likelihood of each combination of feasible tracks was evaluated.

In this study, five scans were considered, and a feasible track required at least three detections. Furthermore, all detections in a feasible track were required to be within 2.6 range cells of the line joining the first and last detections. If two feasible tracks differ only by misses, for instance, $(I_1, I_2, I_3, I_4, I_5)$ and (M, I_2, I_3, I_4, M), the track with additional misses is retained only if its velocity differs from the other track velocity by more than 9.14 m/s (30 ft/s), where velocity is determined from the first and last detections.

Next the maximum likelihood is calculated for all single tracks. A direct search technique is used to determine the target's position on the first and last scan. For each detection associated with the track it is determined whether it is more advantageous to label the detection as coming from the track (with its associated Gaussian error) or whether to declare a false alarm and a missed detection*. At the end of this process, if a track has a detection for each scan, it is called a "perfect" track.

Next, the likelihood is calculated for each possible two-track combination. That is, if there are 30 feasible tracks, there are 30(29)/2 = 435 two-track combinations. However, if the two tracks do not have any common detections, the two tracks are said to be "isolated" and the maximum likelihoods for the single tracks are used. If the two tracks do have common detections, each track is considered to extend from its first to its last detection. For each detection associated with only one track, it is determined whether it is more advantageous to label the detection as coming from the track (with its associated Gaussian error) or whether to declare a false alarm and a missed detection. For detections common to both tracks, one of three actions is determined upon: 1) to declare the tracks unresolved, 2) to associate the detection with the nearest track and declare one target miss, or 3) to declare two missed detections and a false alarm. It should be noted that all of the previous calculations proceed on a scan-to-scan basis. Therefore, it is possible to obtain a slightly different likelihood if the scans were evaluated in a different order (e.g., if one introduced a miss on scan 2, one may not want — or be allowed — to introduce a miss on scan 3).

After all two-track combinations are evaluated, all three-track, four-track, etc., combinations are evaluated. Usually, if the true answer is an n-track combination, all n+1-track combinations are evaluated. Next, the best track combinations (usually the best 5 to 10 are saved) are maximized by the use of direct search techniques in which each target's position on the first and last scan is varied. Finally, the track combination with the maximum likelihood is chosen as the correct series of tracks.

* A miss is never introduced if the miss lowers the number of detections below that required for a feasible track.

When a large number of tracks is present, the computational time on NRL's ASC computor can become exorbitant. For instance, calculation of the likelihoods for a four-track combination of 50 feasible tracks requires over a minute. Thus, to increase computation speed the method was modified to take advantage of "perfect" tracks — those that have detections on each scan. Since it is very likely that the perfect track will be in all the high likelihood track combinations, only track combinations that include the perfect track (or tracks) will be evaluated. For instance, if there are 30 feasible tracks and tracks 2 and 8 are perfect tracks, only one two-track combination, 28 three-track combinations, and (28) (27)/2 four-track combinations will be evaluated. Thus, for this example only 407 (1 + 28 + 28(27)/2) track combinations need be evaluated instead of all the 31 900 possible track combinations: (30(29/2) + 30(29) (28)/6 + 30 (29) (28) (27)/24).

PARAMETERS FOR DATA GENERATION AND OPERATION OF THE SIMULATION

Before the results of several simulations are given the data generation technique will be described briefly. The targets are assumed to be travelling in straight lines at constant speeds. The radar detections are generated on a scan basis in the following manner: A decision is made on whether or not a target is detected. If a target is detected, its position is calculated according to the straight line, and a Gaussian wander is added to its position. Next, false alarms are generated according to a Poisson density, and all the detections are ordered in range. The detections are examined, and it is decided whether adjacent detections should be merged. If a detection is not merged, a Gaussian measurement error is added. If several detections are merged, all merged detections are replaced by a single detection whose range is a Gaussian measurement error added to a detection uniformly distributed between the nearest and farthest merged detections.

Data Input Cards

The data generation and simulation operation are controlled by four or six input cards. The parameters covered by the cards and their formats are as follows.

Card 1 (15 parameters, 15I5 format)

1. The number of targets (N)
2. The number of scans
3. The first repetition case
4. The last repetition case
5. The number of best (high-likelihood) track combinations saved
6. The smallest track combination (minimum of 1)

7. The largest track combination (maximum of 5)

8. Starter for random number generator

9. Track indicator: either tracks inputted (NCAL = 1) or tracks calculated (NCAL. NE. 1)

10. Print control: IPR = 2 (mimimum), IPR = 1 (intermediate), or IPR = 0 (detailed)

11. Number of misses allowed in a feasible track

12. If NREV = 0 or 1, do only 1 power iteration; otherwise do 2 .

13. NOPT = 0 or optimal number of tracks (optimal track groups must be supplied as the last cards)

14. Is a minimum number of perfect tracks required? If negative, do not limit to perfect tracks.

15. IPLOT = 0, no plot; IPLOT = 1, plot ranges; and IPLOT = 2, plot normalized ranges

Card 2 (6 parameters, 6F10.2 format)

1. Probability of detection, actual
2. Average number of false alarms in range interval, actual
3. Variation of true position (in range cells), actual
4. Variation of measured position (in range cells), actual
5. Average track velocity (ft/s), actual
6. Standard deviation of track velocities (ft/s), actual

Card 3 (7 parameters, 7F10.2 format)

1. Far range of interval of interest (ft)
2. Near range of interval of interest (ft)
3. Range cell dimension (ft)
4. Allowable velocity change of each track from average (0 to 1)
5. FSVEL. GE. 0.1: no velocity constraint for individual feasible tracks

6. Average track velocity (ft/s), assumed

7. Standard deviation of track velocities (ft/s), assumed

Card 4 (9 parameters, 9F5.2 format)

1. Probability of detection, assumed

2. Average number of false alarms in range interval, assumed

3. Scan time (s)

4. Variation of true position (in range cells), assumed

5. Variation of measured position (in range cells), assumed

6. Allowable distance between detection and track (in range cells)

7. Allowable velocity difference between feasible tracks (ft/s)

8. Penalty factor for resolutions (F_R)

9. Penalty factor for missed detections (F_M)

If tracks are specified (NCAL = 1, parameter 9 on card 1), as opposed to being random, cards 5 and 6 are used.

Card 5 (N parameters, 8F10.2 format)

1. Range (ft) of first target

2. Range (ft) of second target

.
.
.

N. Range (ft) of N-th target

Card 6 (N parameters, 8F10.2 format)

1. Speed (ft/s) of first target

2. Speed (ft/s) of second target

.
.
.

N. Speed (ft/s) of N-th target

If one desires to evaluate a specific track combination, one can accomplish this by setting NOPT $\neq 0$ (parameter 13, card 1) and by supplying one card for each repetition.

Last Cards; one for each repetition (NOPT parameters, 15I5 format)

1. Feasible track 1

2. Feasible track 2

.

.

NOPT. Feasible track NOPT

RESULTS

Information in Tables

The pertinent parameters, excluding the target parameters, are given in Table 1, and the target parameters are given in Table 2. The maximum likelihood approach was applied to 10 independent realizations of the 5 cases given in Table 2; the results are summarized in Table 3. Of the 50 cases run, 7 were incorrectly identified. However, it was judged (by the authors) that all 7 incorrect solutions were the most reasonable result. In most cases where the number of tracks was underestimated, the true track contained fewer than three detections and these were judged to be false alarms. The two cases where the track had a velocity error greater than 10% occurred because the track either used a false alarm or stole a detection from another track.

Examples — Maximum Likelihood Method

Ten examples of radar detections of a four-track situation will be reviewed to illustrate the maximum likelihood method. The detections made on five scans on each repetition are shown in Figs. 4(a) through 4(j). In each figure the total range interval is 3.7 km or 2 n.mi. Note that for presentation purposes the ranges have been normalized by adding 1524 m (5000 ft) per scan, which corresponds to a velocity of 305 m/s or 1000 ft/s. In the figures, dots represent detections, M's represent missed detections, arcs represent unresolved detections, and FA indicates a false alarm. The dashed lines represent the true tracks. In each scan the detections are numbered from right to left.

Table 1 — Simulation Parameters

Parameter	Value
Number of scans	5
Number of misses allowed in track	2
Probability of detection	0.85
Average number of false alarms per scan	0.3
Gaussian wander, standard deviation	30.5 m (100 ft)
Gaussian measurement error, standard deviation	30.5 m (100 ft)
Range interval	2.0 (n.mi.)
Range cell dimension	152.4 m (500 ft)
Scan time	5.0 s
Penalty factor for resolution (F_R)	0.1
Penalty factor misses (F_M)	0.2

Table 2 — Target Parameters

Case No.	No. of Targets	Initial Ranges (km)	Velocities (m/s)
1R	1	U(181.6, 185.3)*	G(305, 15)†
2	2	183.8, 182.9	305, 282
3R	3	U(181.6 to 185.3)*	G(305, 15)†
3	3	183.6, 183.3, 182.9	305, 305, 305
4	4	183.8, 182.9, 182.7	290, 274, 305, 274

*U(181.6, 185.3) indicates that the initial target positions are uniformly distributed between the two ranges given.

†G(305, 15) indicates that the velocities are Gaussian distributed; the first represents the mean value, and the second gives the standard deviation.

Table 3 — Simulation Results: Number of Times Various Track Combinations Were Selected

Case No.	One Track		Two Tracks		Three Tracks		Four Tracks	
	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
1R	10							
2			10					
3R				3	6	1 *		
3				1	9			
4						1	8	1 *

*At least one track had a velocity error greater than 10%.

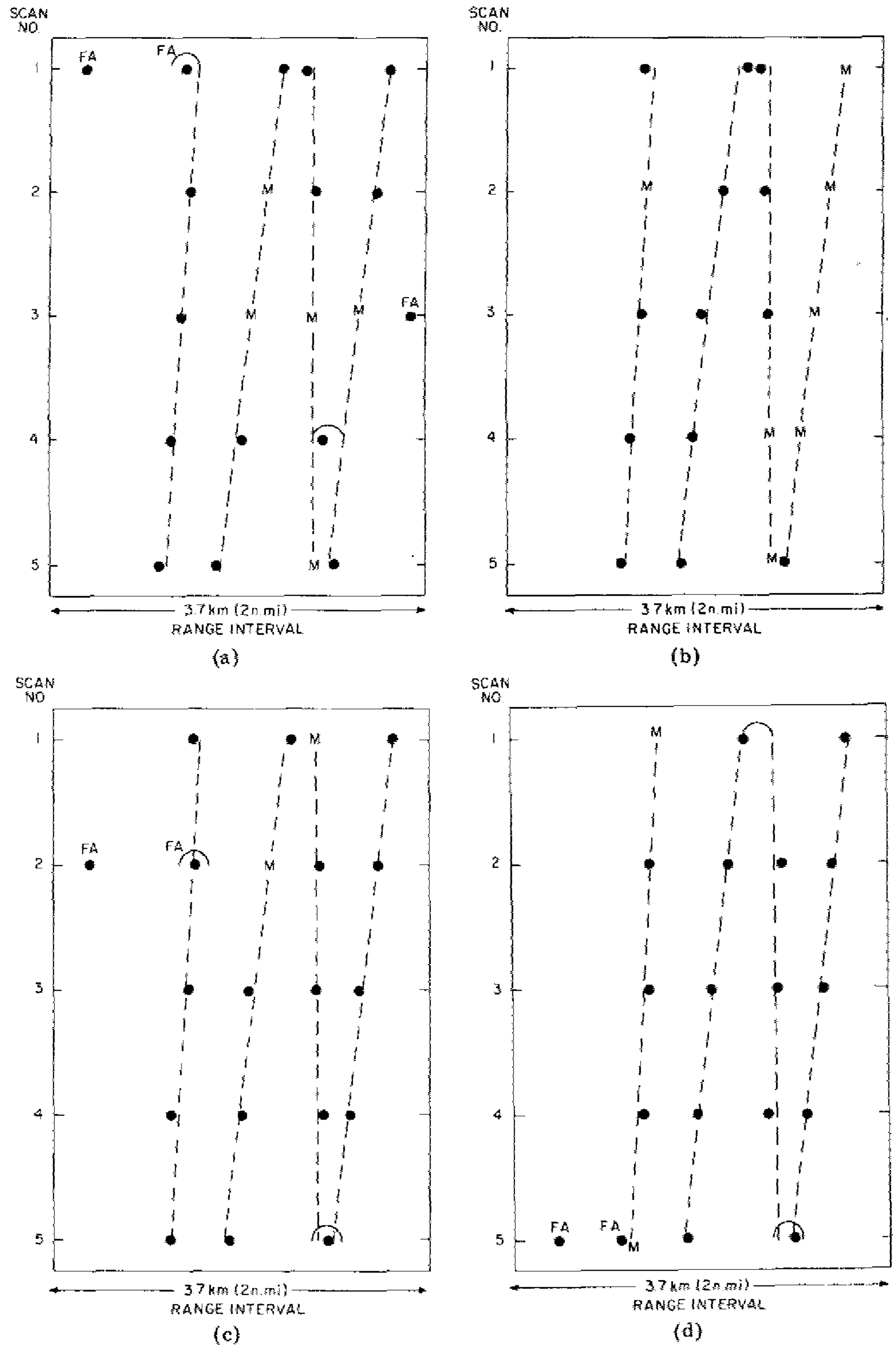

Fig. 4 — The diagrams (a) through (j) making up this figure present 10 iterations of one simulated 4-target raid and the radar responses that occurred. The variations in results are caused by random false alarms that were introduced and by noise and clutter. Dots represent detections, *M's* represent missed detections, arcs represent unresolved detections, and *FA* indicates a false alarm. The dashed lines show the true tracks. In each scan the detections are numbered from right to left.

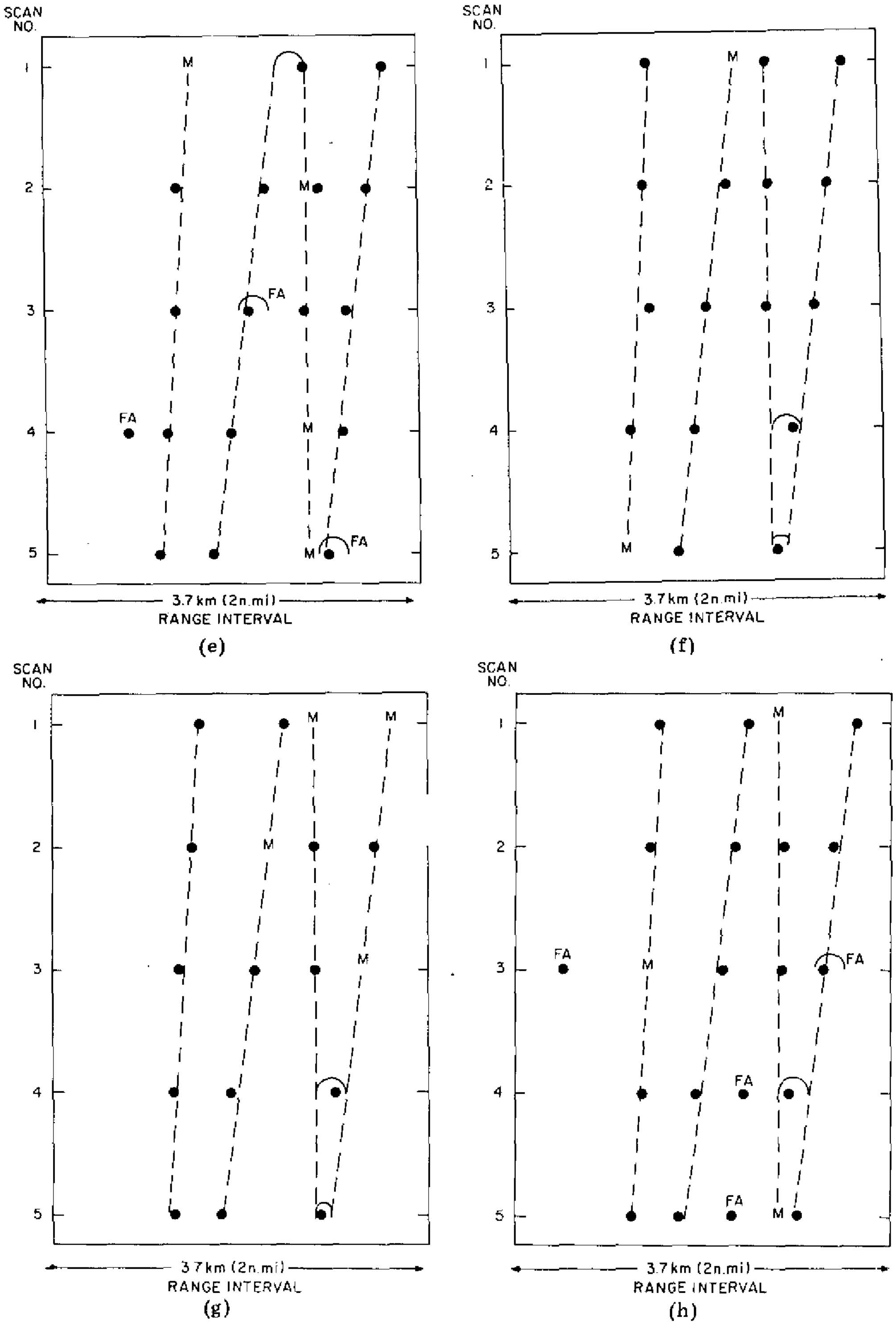

Fig. 4 (Continued) — The diagrams (a) through (j) making up this figure present 10 iterations of one simulated 4-target raid and the radar responses that occurred. The variations in results are caused by random false alarms that were introduced and by noise and clutter. Dots represent detections, *M*'s represent missed detections, arcs represent unresolved detections, and *FA* indicates a false alarm. The dashed lines show the true tracks. In each scan the detections are numbered from right to left.

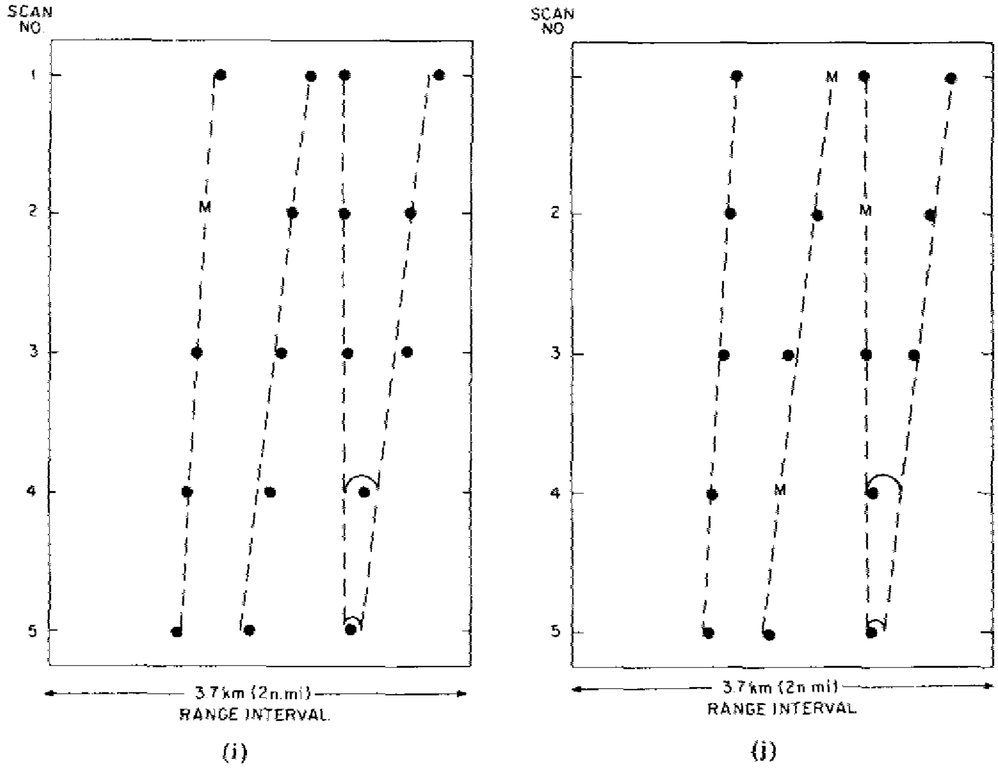

Fig. 4 (Continued) — The diagrams (a) through (j) making up this figure present 10 iterations of one simulated 4-target raid and the radar responses that occurred. The variations in results are caused by random false alarms that were introduced and by noise and clutter. Dots represent detections, *M*'s represent missed detections, arcs represent unresolved detections, and *FA* indicates a false alarm. The dashed lines show the true tracks. In each scan the detections are numbered from right to left.

In Fig. 4(a), the selection in accordance with the maximum likelihood is the following combination of four tracks: $(1,1,M,M,1)$, $(2,2,M,1,1)$, $(3,M,M,2,2)$, and $(4,3,2,3,3)$. The nearest false likelihood involving track $(1,1,1,M,M)$ instead of $(1,1,M,M,1)$, was only five times smaller.

In case 4(b), the three tracks $(1,1,1,M,1)$, $(2,2,2,1,2)$, and $(3,M,3,2,3)$ were selected. Although incorrect, this obviously is what a reasonable person would select. The closest likelihood to this solution differs by a factor of 1000.

In case 4(c), the correct tracks $(1,1,1,1,1)$, $(M,2,2,2,M)$, $(2,M,3,3,2)$, and $(3,3,4,4,3)$ are chosen. All other track combinations considered are simple variations of the above tracks.

In case 4(d), the correct tracks $(1,1,1,1,1)$, $(M,2,2,2,M)$, $(2,3,3,3,2)$, and $(M,4,4,4,3)$ are selected. The closest likelihood, which is not a trivial variation, differs by a factor of 10 000.

In case 4(e), the correct track combination $(1,1,1,1,1)$, $(2,2,2,M,M)$, $(2,3,3,2,2)$, and $(M,4,4,3,3)$ had the largest likelihood. The largest likelihood of a three-track combination, ignoring track 2, differed by 1000.

In case 4(f), the track combination $(1,1,1,1,1)$, $(2,2,2,M,M)$, $(M,3,3,2,2)$, and $(3,4,4,3,M)$ was selected. All other combinations considered were simple variations of this case.

In case 4(g), the correct track combination $(M,1,M,1,1)$, $(M,2,1,M,1)$, $(1,M,2,2,2)$, and $(2,3,3,3,3)$ was selected even though there were only five detections on the first two tracks. The second largest likelihood, which dropped track 2, was 25 times smaller than the maximum likelihood.

In case 4(h), the four-track combination $(1,1,1,1,1)$, $(M,2,2,2,2)$, $(2,3,3,3,3)$, and $(3,4,M,4,4,)$ was selected. The second track selected two false alarms (detections 2) on scans 4 and 5 instead of detections 1. The likelihood of the true track combination differs from the maximum by a factor of five.

In case 4(i), the correct tracks $(1,1,1,1,1)$, $(2,2,2,M,M)$, $(3,3,3,2,2)$, and $(4,M,4,3,3)$ were selected. The likelihood of the three-track combination ignoring track 2 is 100 000 times smaller.

In case 4(j), the correct tracks $(1,1,1,M,M)$, $(2,M,2,1,1)$, $(M,2,3,M,2)$, and $(3,3,4,2,3)$ are selected and the closest three-track combination differs by a factor of 10 000.

In summary, in the 10 repetitions, two false track combinations were selected. However, both of these were very reasonable solutions. That is, with the given detections these are the tracks one would expect any operator or algorithm to deduce.

Since the maximum likelihood solution assumes that the probability of detection (P_D), probability of false alarm (P_{FA}), and Gaussian measurement error (σ), are all known a priori, a sensitivity analysis of the four-track combination was conducted. In the first case, the probability of detection was assumed to be 0.95 instead of the true value of 0.85; in the second case, the average number of false alarms per scan was assumed to be 0.6 instead of the true value of 0.3; in the third case, the Gaussian error was assumed to be 61 m (200 ft) instead of the true value of 30.5 m (100 ft); and in the last case, all the incorrect assumptions were made. The results are shown in Table 4. The three repetitions that produced different results were 1, 7, and 10. In case 1 (referring to Fig. 4(a)) when a larger Gaussian error was assumed, track (1,1,*M*,*M*,1) was replaced by track (1,1,1,*M*,*M*). This had the effect of removing a false alarm. In case 7, different tracks were produced when one assumed P_D = 0.95 and/or σ = 61 m (200 ft). The resulting three tracks (see Fig. 4 (g)) are (*M*,2,1,*M*,1), (1,*M*,2,2,2), and (2,3,3,3,3); that means that track (*M*,1,*M*,1,1) is no longer detected. In case 10, when all the incorrect assumptions were made, track (2,*M*,2,1,1) was dropped, resulting in only the three tracks (1,1,1,1,1), (2,2,3,*M*,2), and (3,3,4,2,3) being detected. In general, the maximum likelihood method is rather insensitive to the assumed parameters. The parameter that it is most sensitive to is the Gaussian error.

Table 4—Number of Tracks Estimated for 4—Track Case When Incorrect Parameters Are Used

Repetition No.	Correct Assumptions	Assumed P_D = 0.95	Assumed No. *FA* = 0.6	Assumed σ = 61 m	All Incorrect Assumptions
1	4	4	4	4*	4*
2	3	3	3	3	3
3	4	4	4	4	4
4	4	4	4	4	4
5	4	4	4	4	4
6	4	4	4	4	4
7	4	3	4	3	3
8	4*	4*	4*	4*	4*
9	4	4	4	4	4
10	4	4	4	4	3

* At least one track had a velocity error greater than 10%.

CONCLUSIONS

The maximum likelihood method of initiating tracks works extremely well. However, the method cannot be implemented because of the enormous computational requirement. For instance, it took 40 seconds on the NRL ASC computer to evaluate all possible four-track combinations of 45 feasible tracks. Thus, a more practical procedure must be considered. Presently, a modification of a raid detector studied by Flad [3] is being pursued. The basic idea is to declare a target raid and estimate the raid velocity and number of targets in the raid.

REFERENCES

1. J. J. Stein and S. S. Blackman, "Generalized Correlation of Multi-target Track Data," *IEEE Trans. Aerospace and Electronic Systems*, Vol. AES-11, pp. 1207-1717, Nov. 1975.

2. G. V. Trunk, "Range Resolution of Targets Using Automatic Detectors," NRL Report 8178, Nov. 28, 1977.

3. E. H. Flad, "Tracking of Formation Flying Aircraft," IEE RADAR-77 Conference, pp. 160-163, Oct. 1977.

Appendix
LIKELIHOOD OF MERGED TARGETS

If several targets are merged, the position of the unresolved detection is given by

$$x = y + z, \tag{1}$$

where y is uniformly distributed between plus and minus A (the nearest and farthest predicted target positions of the merged targets) and z is a Gaussian measurement error with mean 0 and variance σ^2. The density of x is given by the convolution,

$$p(x) = \int_{-A}^{A} \frac{1}{2A} \frac{1}{(2\pi\sigma^2)^{1/2}} \; e^{-(x-y)^2/2\sigma^2} \, dy. \tag{2}$$

Equation (2) will now be evaluated for the two special cases when $A \gg \sigma$ and $\sigma \gg A$. If $A \gg \sigma$ and $|x| < A$ (i.e., detection is between predicted positions), the integral of the Gaussian density is approximately 1, and (2) reduces to

$$p(x) = \frac{1}{2A}. \tag{3}$$

If $A \gg \sigma$ but $|x| > A$ (i.e., detection outside predicted positions), $p(x)$ is approximately given by

$$p(x) = \frac{1}{2A} \, \phi \left(\frac{-\delta}{\sigma}\right), \tag{4}$$

where $|x| = A + \delta$

and

$$\Phi(T) = \int_{-\infty}^{T} \frac{1}{\sqrt{2\pi}} \, e^{-u^2/2} \, du. \tag{5}$$

It should be noted that the situation $|x| > A$ will rarely occur when $A \gg \sigma$.

When $\sigma \gg A$ and $|x| < A$ (which will be very rare), the exponential is essentially one, and $p(x)$ reduces to

$$p\ (x) = \frac{1}{(2\pi\sigma^2)^{1/2}}. \tag{6}$$

On the other hand, when $\sigma >> A$ and $|x| > A$, the exponential is essentially constant and can be pulled outside the integral, resulting in

$$p\ (x) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/2} e^{-\delta^2/2\sigma^2}, \tag{7}$$

where $|x| = A + \delta$. Combining (3) through (7), $p\ (x)$ can be approximated by

$$p\ (x) = \frac{e^{-\delta^2/2\sigma^2}}{Max\{2A,\ (2\pi\sigma^2)^{1/2}\}}, \tag{8}$$

where

$$\delta = Max\{0,\ x-A,\ -A-x\}\ .$$