

Range Resolution of Targets Using Automatic Detectors

G. V. TRUNK

*Radar Analysis Staff
Radar Division*

November 28, 1977



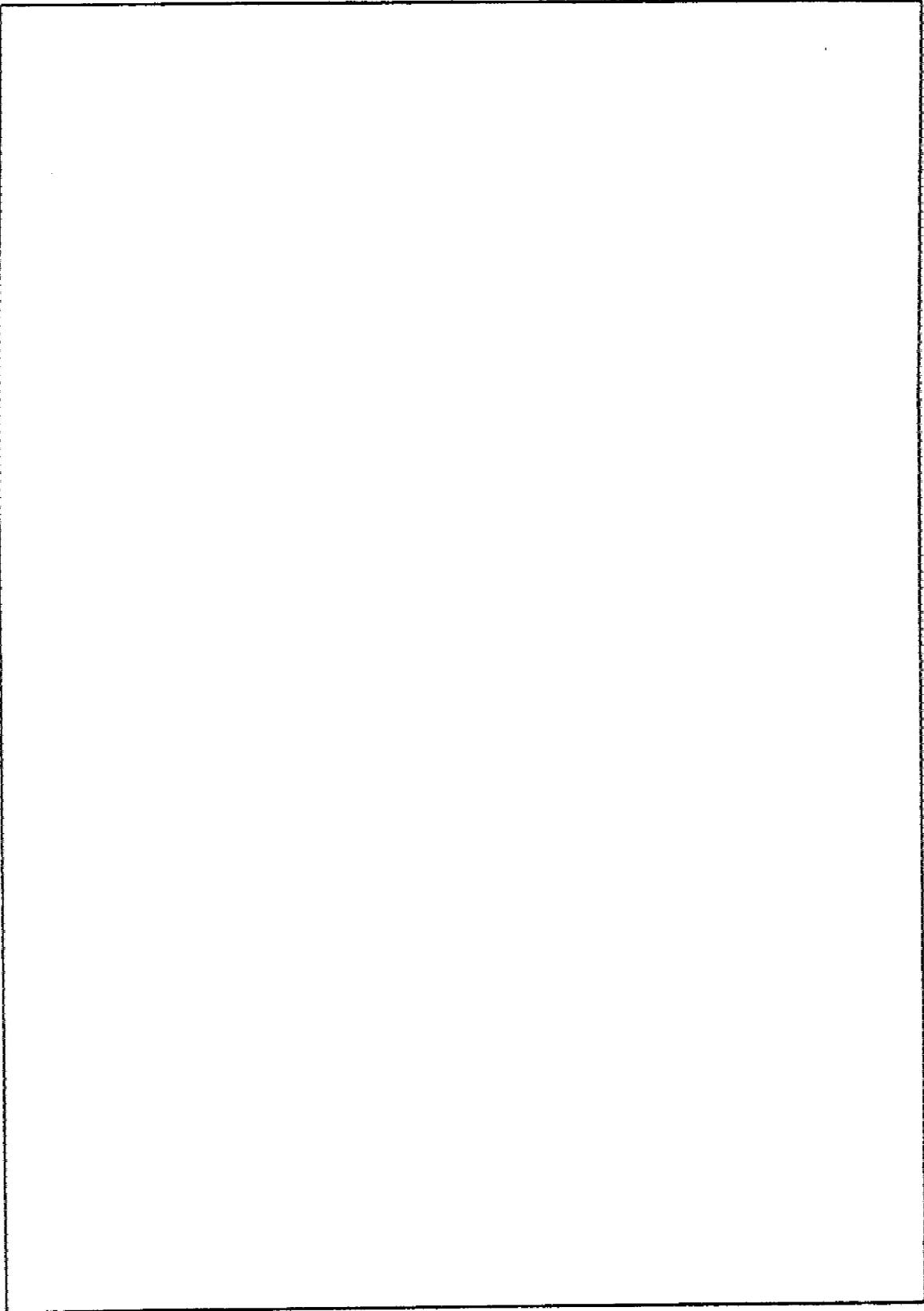
NAVAL RESEARCH LABORATORY
Washington, D.C.

Approved for public release; distribution unlimited.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Report 8178	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RANGE RESOLUTION OF TARGETS USING AUTOMATIC DETECTORS	5. TYPE OF REPORT & PERIOD COVERED Preliminary Report on one phase of the NRL Problem	
	6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) G. V. Trunk	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem R12-51 Element 61153N Project RR014-09-41	
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Office of Naval Research Arlington, VA 22217	12. REPORT DATE November 28, 1977	
	13. NUMBER OF PAGES 19	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Range resolution Target suppression Automatic detectors		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) When two targets are closely separated in range, automatic detectors will declare the presence of only one target. To increase the probability of resolving targets in range, log video should be used and the threshold should be of the form $T = \mu + F$, where the mean μ is the smaller of the two means calculated from a number of reference cells on either the greater range side of the test cell or the lesser range side and F is a fixed number. When the adjacent-detection merging algorithm (which decides that detections at the same azimuth and elevation in adjacent range-resolution cells are from the same target) is used, the probability of resolving targets does not rise above 0.9 until the targets are separated in range by 2.5 pulse widths.		

DD FORM 1473
1 JAN 73EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)



CONTENTS

INTRODUCTION	1
RESOLUTION ALGORITHM	1
DETECTION CLASS	4
SIMULATION	6
MONTE CARLO RESULTS	6
CONCLUSION	8
ACKNOWLEDGEMENT	9
REFERENCES	9
APPENDIX A — Subroutine DET3D	11



RANGE RESOLUTION OF TARGETS USING AUTOMATIC DETECTORS

INTRODUCTION

There are two adverse effects of closely separated targets on automatic detectors. First, a target can lie within the reference cells of the other target, causing the estimated threshold to be too large and resulting in the other target not being detected (target suppression). Second, if the targets are close, the detections from both targets are merged and a single target is reported (targets are not resolved). The problem of target suppression has received some attention, and the papers of Urkowitz and Perry [1], Trunk et al. [2], and Rickard and Dillard [3] propose solutions by removing large returns from the reference cells. The problem of range resolution of targets has received little attention and is the subject of this report.

The probability of resolving targets is not only a function of target separation but also a function of the signal strengths. However, it has usually been assumed that if targets are large enough to be detected, they can be resolved if they are separated by a pulsewidth or, equivalently, lie within different range cells [4]. In practice this resolution criterion is rarely achieved. The basic problem is that the shape of the returned pulse from a target is rarely known in a realistic radar environment. There are several reasons for this. First, the targets are not point scatterers but are complex scatterers. Second, even from a point scatterer the shape of the returned pulse is seldom known because of uncertainties in the transmitted pulse and receiver bandwidth in addition to spreading of the pulse due to reflections from nearby objects such as buildings, towers, and masts.

The purpose of this report is to investigate and recommend the type of thresholding that should be used in automatic detection systems so that targets can be resolved. Since there is uncertainty about the shape of the returned pulse, the optimal resolution system will not be considered. Instead an algorithm that is presently being used in several systems will be used in this study to merge multiple detections from a single target and to resolve multiple detections from different targets. The algorithm for a 3D radar is discussed in the next section.

RESOLUTION ALGORITHM

The antenna beam positions for a typical 3D surveillance radar are shown in Fig. 1. Consequently, depending on the target size and location, the target can be detected in several azimuth-elevation beam positions. Furthermore the target can be detected in adjacent range cells. Thus it is not unusual for a large-cross-section target to be detected in as many as 10 range-azimuth-elevation cells. Consequently an algorithm (decision rule) is required to merge these detections (threshold crossings) into a single centroided detection.

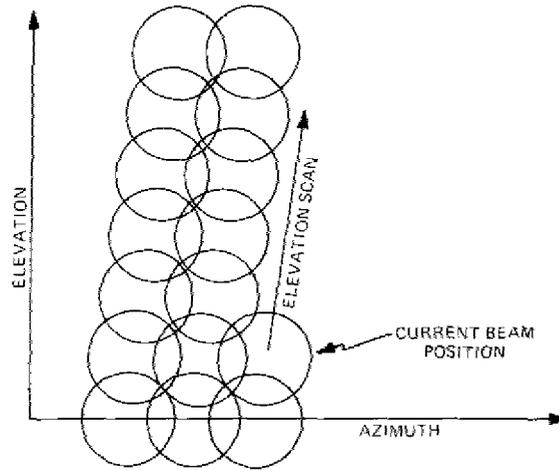


Fig. 1 — Antenna beam positions for a typical 3D radar. The circles represent 3-dB contours of the antenna beam at different scanning positions.

The algorithm selected for this study decides whether a new detection is adjacent to any of the previously determined set of adjacent detections. If the new detection is adjacent to any detection in the set of adjacent detections, it is added to the set. If it is not adjacent to any detection in the set, the next detection is considered. Two detections are adjacent if two of their three parameters (range, azimuth, and elevation) are the same and the other parameter differs by the resolution element: range resolution cell ΔR , azimuth beamwidth θ , or elevation beamwidth γ . To avoid roundoff errors, the new detection is added to the previous set of adjacent detections if any of three conditions are satisfied for any k th detection in the set of K detections. The first condition is

$$|R - R_k| \leq 1.2\Delta R,$$

$$|A - A_k| \leq 0.1\theta,$$

$$|E - E_k| \leq 0.1\gamma,$$

the second condition is

$$|R - R_k| \leq 0.1\Delta R,$$

$$|A - A_k| \leq 1.2\theta,$$

$$|E - E_k| \leq 0.1\gamma,$$

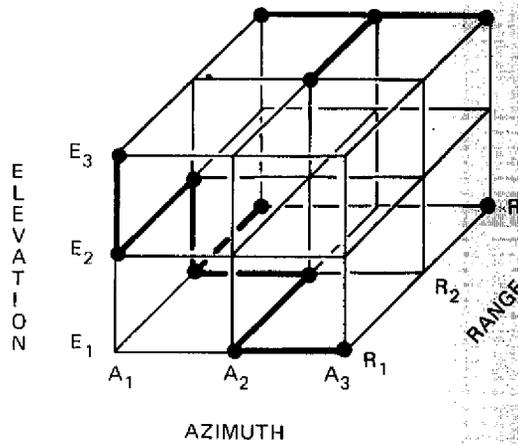
and the third condition is

$$|R - R_k| \leq 0.1\Delta R,$$

$$|A - A_k| \leq 0.1\theta,$$

$$|E - E_k| \leq 1.2\gamma,$$

Fig. 2— Example of merging of adjacent detections. The groupings that are merged are indicated by heavy connecting lines.



where R , A , and E and R_k , A_k and E_k are the range, azimuth, and elevation of the detection being tested and the k th detection in the set of adjacent detections respectively. Therefore the 13 detections in Fig. 2 are merged into three centroided detections. The detections in the largest grouping of adjacent detections are $(R_1 A_1 E_3)$, $(R_1 A_1 E_2)$, $(R_1 A_2 E_1)$, $(R_1 A_3 E_1)$, $(R_2 A_1 E_2)$, $(R_2 A_1 E_1)$, $(R_2 A_2 E_1)$, and $(R_3 A_1 E_1)$. The second largest grouping is $(R_2 A_2 E_3)$, $(R_3 A_1 E_3)$, $(R_3 A_2 E_3)$, and $(R_3 A_3 E_3)$. The final grouping contains the single detection $(R_3 A_3 E_1)$.

The estimates of range R , azimuth A , and elevation E for the K adjacent detections are

$$R = \frac{\sum_{k=1}^K S_k R_k}{\sum_{k=1}^K S_k},$$

$$A = \frac{\sum_{k=1}^K S_k A_k}{\sum_{k=1}^K S_k},$$

and

$$E = \frac{\sum_{k=1}^K S_k E_k}{\sum_{k=1}^K S_k},$$

where S_k is the signal power associated with the k th adjacent detection.

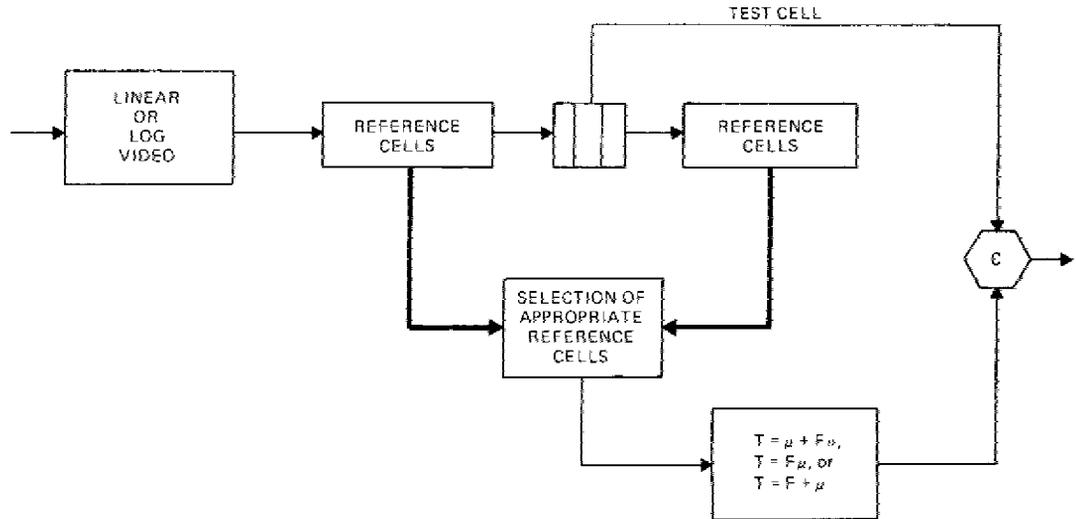


Fig. 3 — Variations of the basic detector. T is the threshold, μ either is the mean of all the reference cells or is the smaller (the minimum) of the means of the reference cells at only the right and of the reference cells at only the left, F is a fixed number, and σ is the variance.

Many types of detection schemes are possible. The collection of detectors considered is discussed in the next section.

DETECTION CLASS

The basic detector uses the reference cells surrounding the test cell to generate a threshold to which the test cell is compared. Variations of this basic detector are shown in Fig. 3. The set of detectors uses either linear or log video, either all the reference cells or the half with the minimum mean value, and either a one-parameter (mean) threshold or a two-parameter (mean and variance) threshold.

Generally M pulses are integrated in each range cell and beam position, yielding the integrated signals

$$Z_j = \sum_{i=1}^M X_{ij}, \tag{1}$$

where X_{ij} is the returned envelope-detected signal from the i th pulse in the j th range cell. For 3D radars, M usually equals one or two. Then the mean value (depending on whether the number of reference cells N_R on one side of the test cell are used or the total number of reference cells $2N_R$ are used) is given by

$$\bar{Z}_j = \frac{1}{2N_R} \sum_{i=1}^{N_R} (Z_{j+1+i} + Z_{j-1-i}), \tag{2}$$

$$\bar{Z}_j = \frac{1}{N_R} \sum_{i=1}^{N_R} Z_{j+1+i}, \quad (3)$$

or

$$\bar{Z}_j = \frac{1}{N_R} \sum_{i=1}^{N_R} Z_{j-1-i}. \quad (4)$$

The corresponding mean squares are

$$\bar{Z}_j^2 = \frac{1}{2N_R} \sum_{i=1}^{N_R} (Z_{j+1+i}^2 + Z_{j-1-i}^2), \quad (5)$$

$$\bar{Z}_j^2 = \frac{1}{N_R} \sum_{i=1}^{N_R} Z_{j+1+i}^2, \quad (6)$$

and

$$\bar{Z}_j^2 = \frac{1}{N_R} \sum_{i=1}^{N_R} Z_{j-1-i}^2, \quad (7)$$

and the standard deviation is

$$\sigma_j = \left[\bar{Z}_j^2 - (\bar{Z}_j)^2 \right]^{1/2}, \quad (8)$$

where \bar{Z}_j^2 and \bar{Z}_j use the same reference cells. The two-parameter threshold is [5]

$$T_j = \bar{Z}_j + F_\sigma \sigma_j, \quad (9)$$

regardless of whether linear or log video is being used. The parameter F_σ is used to set the false alarm rate. (The two-parameter threshold for log video has dubious meaning, since the threshold can be dominated by the shape of the density function near zero.) The one-parameter threshold is

$$T_j = F_\sigma \bar{Z}_j \quad (10)$$

for linear video and

$$T_j = F_\sigma + \bar{Z}_j \quad (11)$$

for log video.

Finally, detections are declared by comparing Z_j to T_j , with a target being declared if $Z_j \geq T_j$. Since it is difficult to analytically evaluate the resolution properties of the various detectors, simulation techniques were used.

SIMULATION

Before considering the simulation of the detector, let us consider the target geometry. Two targets at the same azimuth and elevation are separated in range by a distance ΔD , where $1.5\Delta R \leq \Delta D \leq 3.5\Delta R$, with ΔR being the width of the range-resolution cell. The signal-to-noise ratio for each target is 20 dB per pulse when the target is in the center of a range-azimuth-elevation cell. Finally, in one example, a third target is placed in the reference cells, seven range cells away from the first target. A detailed description of the simulation of the detector subroutine is given in Appendix A.

In this simulation the target was uniformly distributed in range and azimuth in the center range-azimuth cell. The range cell dimension was the 3-dB pulse width, and adjacent azimuth beam positions crossed at the 1.6-dB point. The targets were placed in the center of the elevation beam. Since the signal energy is down 36 dB in adjacent elevation beam positions crossing at the 3-dB point, only one elevation beam was simulated. Finally, only one pulse was used (no integration), and ten reference cells were used on either side of the test cell.

The detector subroutine declares target detections by generating pulse-to-pulse video returns, integrating the returned signal, and comparing it to an adaptive threshold which is generated from the surrounding reference cells. In addition to the range-azimuth cell in which the target is present, signal energy is placed in adjacent range cells (only one on either side) according to a $[(\sin x)/x]^2$ pulse shape and in adjacent azimuth beams (again only one on either side) according to a $[(\sin x)/x]^4$ antenna pattern. Thus, in each elevation beam position the target can be detected in nine range-azimuth cells (three range cells times three azimuth cells).

MONTE CARLO RESULTS

The first case simulated consisted of the two targets separated by 1.5, 2.0, 2.5, 3.0, and 3.5 range cells. For each separation ten repetitions were used. The results using all the reference cells or using the half of the reference cell with the smaller mean value are given in Table 1. When the target separation is $1.5\Delta R$, almost always only one target is declared regardless of the video, reference cells, or thresholding employed. However, when all the reference cells are used and the target separation is larger, the one-parameter thresholds correctly identify the presence of two targets, but the two-parameter threshold generally declares no targets. The targets are suppressed when two-parameter thresholding is used because a target in the reference cells has a great effect on the estimation of the noise variance but has little effect on the estimation of the mean. When the reference cells with the smaller mean are used and the target separation is $2.5\Delta R$ or greater, all detectors correctly identify the presence of two targets.

The second case simulated consisted of two targets separated by 1.5, 2.0, 2.5, 3.0, and 3.5 range cells and a third target 7.0 range cells from the first target. Table 2 shows

Table 1 -- Number of Targets Detected in the Case of Two Targets Separated by 1.5, 2.0, 2.5, 3.0, or 3.5 Range-Cell Dimensions ΔR

Target Separation	No. of Times a Given No. of Targets were Detected Out of Ten Opportunities									
	Linear Detector, $T = \mu + F\sigma$			Linear Detector, $T = F\mu$			Log Detector, $T = F + \mu$			
	Zero Targets	One Target	Two Targets	Zero Targets	One Target	Two Targets	Zero Targets	One Target	Two Targets	Three Targets
Threshold Based on All Reference Cells										
1.5 ΔR										
2.0 ΔR										
2.5 ΔR										
3.0 ΔR										
3.5 ΔR										
Threshold Based on Minimum of Reference Cells										
1.5 ΔR										
2.0 ΔR										
2.5 ΔR										
3.0 ΔR										
3.5 ΔR										

that the log detector using the reference cells with the smaller mean value has the best performance and thus should be used.

To determine more accurately the resolution capability of the log detector using the minimum of the reference-cell means, the distance between the two targets was increased from 1.0 ΔR to 3.5 ΔR in steps of 0.1 ΔR , and for each separation 40 repetitions were run. The results of this simulation are given in Fig. 4. The probability of correctly resolving the two targets appears fairly linear from 1.7 ΔR to 2.6 ΔR . Search techniques corroborated that the function

$$\begin{aligned}
 P(N = 2|\Delta D) &= 0, & \Delta D &\leq 1.7\Delta R, \\
 &= \frac{\Delta D - 1.7\Delta R}{0.9\Delta R}, & 1.7\Delta R &\leq \Delta D \leq 2.6\Delta R, \\
 &= 1, & 2.6\Delta R &\leq \Delta D,
 \end{aligned}$$

where $P(N = 2|\Delta D)$ is the probability that two targets are detected when the two targets are separated by distance ΔD , is the best fit whether the mean-square error, maximum error, or absolute error is used as the goodness-of-fit criterion. The goodness-of-fit error can be reduced by a factor of 2 if the following (more complicated) expression is used:

Table 2 — Number of Targets Detected in the Case of Two Targets Separated by 1.5, 2.0, 2.5, 3.0, and 3.5 Range-Cell Dimensions ΔR and a Third Target 7.0 Range Cells From the First.

Separation of First Two Targets	No. of Times a Given No. of Targets Were Detected Out of Ten Opportunities					
	Linear Detector, $T = F\mu$			Log Detector, $T = F + \mu$		
	Zero Targets	One Target	Two Targets	Zero Targets	One Target	Two Targets
	Threshold Based on All Reference Cells					
1.5 ΔR						
2.0 ΔR						
2.5 ΔR						
3.0 ΔR						
3.5 ΔR						
	Threshold Based on Minimum of Reference Cells					
1.5 ΔR						
2.0 ΔR						
2.5 ΔR						
3.0 ΔR						
3.5 ΔR						

$$\begin{aligned}
 P(N = 2|\Delta D) &= 0, & \Delta D &\leq 1.66\Delta R, \\
 &= 0.88(\Delta D - 1.66 R)/\Delta R, & 1.66\Delta R &\leq \Delta D \leq 2.16\Delta R, \\
 &= 1 - 0.56 \exp[-5.87(\Delta D - 2.16)/\Delta R], & 2.16\Delta R &\leq \Delta D.
 \end{aligned}$$

CONCLUSION

For resolving closely spaced targets, log video should be used and the threshold should be of the form $T = \mu + F$ where the mean μ is the smaller of the two means calculated from the reference cells on either side of the test cell and F is a fixed number. This procedure does not allow a few large returns in the reference cells to raise the detection threshold appreciably. Thus this procedure will not suppress spiky clutter and should not be used without an MTI in the clutter regions.

When the adjacent-detection merging algorithm is used, the probability of resolving targets does not rise above 0.9 until the targets are separated by 2.5 pulse widths. If this resolution is not adequate, it can be improved by using other resolution algorithms which

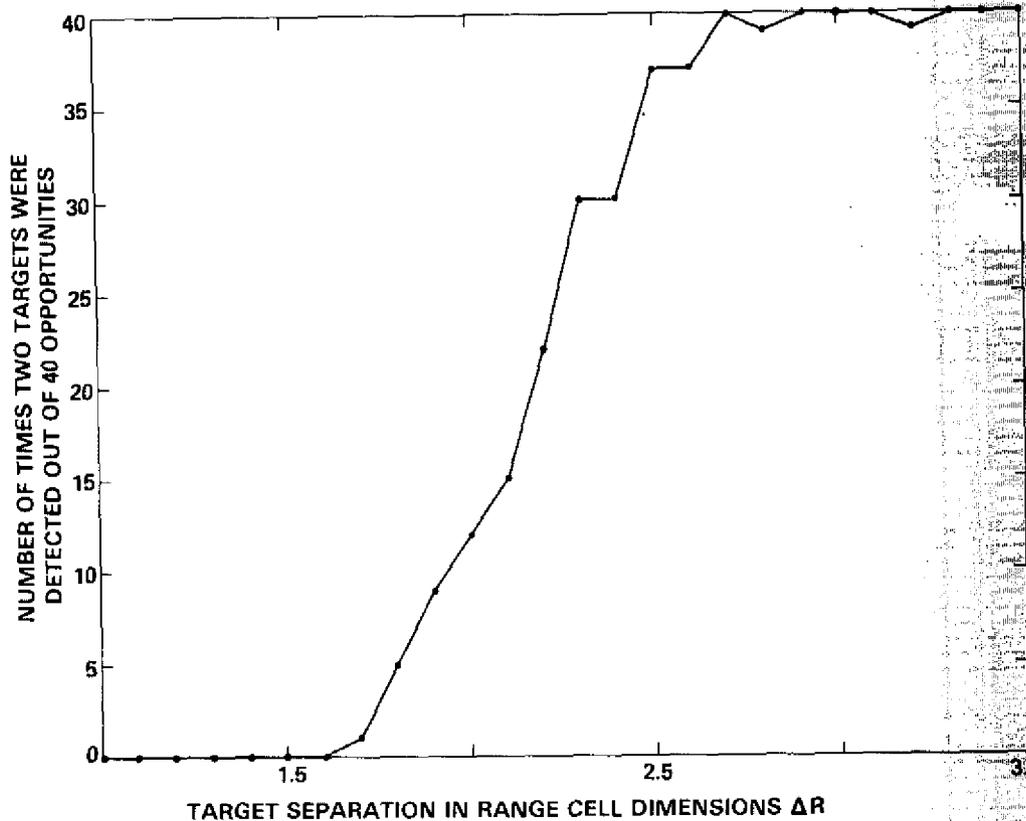


Fig. 4 — Resolution capability of log detector which uses reference cells with the minimum mean

depend on assuming a returned pulse shape. If the pulse-shape assumption is correct, one should be able to resolve targets to within (approximately) a pulse width.

ACKNOWLEDGMENT

I thank Mr. Prengaman and Mr. Bath of the Applied Physics Laboratory of the Johns Hopkins University for discussions about resolution algorithms.

REFERENCES

1. H. Urkowitz and R. P. Perry, "A CFAR Technique Insensitive to Nearby Targets," RCA System Technology Memo STM-11011, May 18, 1973.
2. G. V. Trunk, B. H. Cantrell, and F. D. Queen, "Modified Generalized Sign Test Processor for 2-D Radar," *IEEE Trans. on Aerospace and Electronic Systems* AES-10 (No. 5), 574-582 (Sept. 1974).
3. J. T. Rickard and G. M. Dillard, "Adaptive Detection Algorithms for Multiple-Target Situations," *IEEE Trans. on Aerospace and Electronic Systems* AES-13 (No. 4), 338-343 (July 1977).

G. V. TRUNK

4. D. K. Barton, *Radar System Analysis*, Prentice-Hall, 1964.
5. G. V. Trunk, "Survey of Radar Signal Processing," NRL Report 8117, June 21, 1977.

Appendix A

SUBROUTINE DET3D

Subroutine DET3D should be called for every target and elevation mode as long as the difference between the center of the elevation beam of the radar mode and the target elevation angle is less than the 3-dB elevation beamwidth. The purpose of the routine is to declare target detections by generating pulse-to-pulse video returns, integrating the returned signal, and comparing it to an adaptive threshold which is generated from the surrounding reference cells. Detections can be made in either adjacent range cells or adjacent azimuth beam position in addition to the range-azimuth cell in which the target is present. Thus, in theory a radar mode can report nine detections of a single target.

The subroutine initially tests whether appropriate input parameters are less than pre-assigned values: the number of pulses integrated M is less than 32, number of reference cells N_R on each side of the test cell is less than 10, and the absolute value of the clutter correlation coefficient ρ is less than 1.0. If any parameter exceeds its limit, an error message is printed, and one exits from the subroutine.

Next M times the signal-to-clutter-plus-noise ratio is compared to 2. If the value is less than 2, one declares that the target is not detected, the number of detections of the I th target by the J th mode [NDET3(I, J)] is set to zero, and one exists from the subroutine.

The detail simulation is begun by finding all the targets which lie within the reference cells of the NTARth target. The list of interfering targets (INF) is initialized by setting $INF(1) = NTAR$, and the corresponding signal power $SNREF(1)$ is set equal to the target power S . The remaining interfering targets are found by calling subroutine RESOL which calculates the number of interfering targets (NI), lists the index of the interfering targets in the array INF, and lists the corresponding signal powers in P .

The main azimuth do loop, which generates the azimuth beam positions on either side of the azimuth beam position with maximum gain, first generates the azimuth beam position θ proceeding the maximum-gain beam position. This beam position is given by

$$\theta = \theta_B(K_\theta + IAZ - 2), \quad (A1)$$

where $IAZ = 1$, θ_B is the azimuth angle between complete elevation scans, and K_θ is the integer defined by

$$K_\theta = \text{integer} [(A - \theta)/\theta_B + 0.5], \quad (A2)$$

in which A is the target azimuth and θ is the azimuth angle of the first elevation scan.

In the simulation the video return is generated in only the reference cells surrounding the target. Thus, to save computer storage, only 25 range cells are saved, and the target is always placed in the 13th range. Consequently the range to the start of the first range cell is

$$R_S = \Delta R(K_{RS} - 13), \quad (\text{A3})$$

where ΔR is the range-cell dimension and K_{RS} is the integer defined by

$$K_{RS} = \text{integer } [R/\Delta R], \quad (\text{A4})$$

with R being the target range.

The first step in generating the radar video return is to generate the signal (target) return in the appropriate range cells. Specifically, the index of the first range cell is

$$N_F = 11 - N_R, \quad (\text{A5})$$

and the index of the last range cell is

$$N_L = 15 + N_R. \quad (\text{A6})$$

Thus, if $N_R = 10$, the signal must be placed in all 25 range cells. The signal return for the i th pulse and the j th range cell (S_{ij}) and an indicator of signal in the j th range cell $I(j)$ are initially set to zero for all i and j . Then, the signal return from each of the N_I interfering targets lying within the reference cells is generated. For the k th target this is accomplished by first generating the appropriate fluctuating amplitudes F_{ik} for the i th pulse, if the valid fluctuating amplitudes have not been calculated previously: $\text{IKEY}(\text{NTAR}) = 1$ indicates that F_{ik} has been calculated previously for Swerling cases 1 and 3 (scan-to-scan fluctuations) for either a previous azimuth or elevation beam position. If F_{ik} needs to be calculated (either the first time for Swerling cases 1 and 3 or every time for Swerling cases 2 and 4), F_{ik} is given for the appropriate Swerling case (NSW) by the following, in which the U_i are independent random variables uniformly distributed between 0 and 1:

- For $\text{NSW} = 0$ (nonfluctuating target)

$$F_{ik} = 1, \quad i = 1, \dots, M; \quad (\text{A7})$$

- For $\text{NSW} = 1$ (scan-to-scan fluctuations, Rayleigh density)

$$F_{ik} = -\log(U_1), \quad i = 1, \dots, M; \quad (\text{A8})$$

- For $\text{NSW} = 2$ (pulse-to-pulse fluctuations, Rayleigh density)

$$F_{ik} = -\log(U_i), \quad i = 1, \dots, M; \quad (\text{A9})$$

- For $\text{NSW} = 3$ (scan-to-scan fluctuations, chi-square density)

$$F_{ik} = -0.5[\log(U_1) + \log(U_2)], \quad i = 1, \dots, M; \quad (\text{A10})$$

- For NSW = 4 (pulse-to-pulse fluctuations, chi-square density)

$$F_{ik} = -0.5[\log(U_i) + \log(U_{M+i})], \quad i = 1, \dots, M. \quad (A11)$$

The signal is next placed in the appropriate range cell and the adjacent range cells by reducing the returned signal by a $[(\sin x)/x]^2$ pulse shape and a $[(\sin x)/x]^4$ antenna pattern. The return signal from the k th target is centered in range cell K_R .

$$K_R = \text{integer} [(R_k - R_S)/\Delta R], \quad (A12)$$

where R_k is the range of the k th target. The signal-return reduction (F) in the adjacent range cell K_T ,

$$K_T = K_R + I, \quad I = -1, 0, 1 \quad (A13)$$

(because of the $(\sin x)/x$ pulse shape), is given by

$$F = [(\sin F_d)/F_d]^2, \quad (A14)$$

where

$$F_d = 2.7832(R_k - R_T)/\Delta R; \quad (A15)$$

and

$$R_T = (K_T + 0.5)\Delta R + R_S. \quad (A16)$$

At this time the indicator $I(K_T)$ is set equal to 1. Similarly the signal reduction (G) due to the $(\sin x)/x$ antenna is given by

$$G = [(\sin G_d)/G_d]^4, \quad (A17)$$

where

$$G_d = 2.7832(A_k - \theta)/\theta_{3dB}, \quad (A18)$$

in which θ_{3dB} is the antenna 3-dB azimuth beamwidth and A_k is the azimuth of the k th target. Finally the signal (normalized by the clutter power C and noise power N) due to the k th target in the K_T range cell (for the azimuth beam position specified by (A18)) is

$$S_{i,K_T}^{(\text{new})} = S_{i,K_T}^{(\text{old})} + (G)(F)(F_{ik})P_k/(C + N). \quad (A19)$$

The calculation indicated by (A13) through (A19) is first repeated for the adjacent range cells indicated in (A13). Then the calculation indicated by (A7) through (A19) is repeated for all NI targets in the reference cells. Thus, at the end of all the above repetitions, $S_{i,j}$ is the signal power in the i th pulse and j th range cell due to all the targets in the reference cells.

Next Rayleigh noise (and possibly correlated clutter) is added to the signal to produce the total video return X_{ij} . The video return (because of computer speed considerations) is generated for three distinct cases: clutter insignificant and no signal present in the range cell, clutter insignificant and signal present in the range cell, and clutter significant. The significance of clutter is indicated by $(C)(M)$ being greater than N , and signal present in the j th cell is indicated by $I(j) = 1$. Thus the i th return in the j th cell (x_{ij}) is given by the following:

- When clutter is insignificant ($(C)(M) \leq N$) and no signal is present in the j th cell ($I(j) = 0$),

$$x_{ij} = (-2 \log U_i)^{1/2}, \quad i = 1, \dots, M, \quad (\text{A20})$$

where the U_i are independent uniform random numbers (0.1), different for each j ;

- When clutter is insignificant ($(C)(M) \leq N$) and signal is present in the j th cell ($I(j) = 1$),

$$x_{ij} = \{[\alpha_i \cos \theta_i + \sqrt{2} (S_{ij})^{1/2}]^2 + (\alpha_i \sin \theta_i)^2\}^{1/2}, \quad (\text{A21})$$

where

$$\alpha_i = [-2 \log U_i]^{1/2} \quad (\text{A22})$$

and

$$\theta_i = 2\pi U_{i+M}; \quad (\text{A23})$$

- When clutter is significant ($(C)(M) > N$),

$$x_{ij} = \{[CX_i + \alpha_i \cos \phi_i + (2S_{ij})^{1/2}]^2 + (CY_i + \alpha_i \sin \phi_i)^2\}, \quad (\text{A24})$$

where the Rayleigh noise ($\alpha_i \cos \phi_i$ and $\alpha_i \sin \phi_i$) is given by

$$\alpha_i = N'(-2 \log U_{i+M})^{1/2}, \quad (\text{A25})$$

$$\phi_i = 2\pi U_{i+3M}, \quad (\text{A26})$$

and

$$N' = [N/(C + N)]^{1/2}, \quad (\text{A27})$$

the clutter is given by

$$CX_i = \rho CX_{i-1} + (1 - \rho^2)^{1/2} \alpha'_i \sin \theta'_i \quad (\text{A28})$$

and

$$CY_i = \rho CY_{i-1} + (1 - \rho^2)^{1/2} \alpha'_i \cos \theta'_i, \quad (\text{A29})$$

in which

$$\alpha'_i = C' [-2 \log U_i]^{1/2} \quad (\text{A30})$$

and

$$\theta'_i = 2\pi U_{i+2M} \quad (\text{A31})$$

with

$$C' = [C/(C + N)]^{1/2} \quad (\text{A32})$$

and the initial clutter values are

$$CX_0 = C'\alpha_0 \cos \theta'_0 \quad (\text{A33})$$

and

$$CY_0 = C'\alpha_0 \sin \theta'_0 \quad (\text{A34})$$

Next a decision is made whether to use linear or log video. Then the M pulses are integrated in each range cell, yielding the values

$$Z_j = \sum_{i=1}^M X_{ij} \quad (\text{A35})$$

which equation was given in the main text as (1). The detection threshold T_j for the j th range cell ($j = 12, 13, \text{ and } 14$) uses either all the reference cells, the minimum of the reference cells, or the maximum of the reference cells. Furthermore, the threshold may be either a one-parameter (mean) threshold or a two-parameter (mean and variance) threshold. Thus the mean is given by (2), (3), or (4). The corresponding mean squares, the standard deviation, the two-parameter threshold, and the one-parameter thresholds are given by (5) through (11).

Finally, detections are declared by comparing Z_{12} , Z_{13} , and Z_{14} to T_{12} , T_{13} , and T_{14} respectively. If Z_j is greater than T_j , a detection is declared in the j th range cell, the counter for number of detections (parameter labeled II) is increased by one, and the following detection parameters are saved:

II	the number of detections,
$R_S + (j + 0.5)\Delta R$	the range of the detection,
θ	the azimuth of the detection,
$Z_j(C + N)$	the signal amplitude of the detection,
Time	the time of the detection.

Also, if any target lies within three range cells and 2.4 azimuth beamwidths of the NTARth target, MER3(NTAR) is set to -1, noting this target interfering condition.

After the detection tests have been performed for the initial azimuth beam position (IAZ = 1), all calculations are repeated for the other two beam positions. If there are target interfering conditions and if the target has not been detected on this mode or previous modes of this radar scan, MER3(NTAR) is set to -1. Then one returns to the main program.