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A Postdetection Method of Measuring Predetection RF Signal-to-Noise Ratio

L.V. Blake

Radar Geophysics Branch
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Lamont V. Blake

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13. ABSTRACT

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A POSTDETECTION METHOD OF MEASURING PREDETECTION RF SIGNAL-TO-NOISE RATIO

INTRODUCTION

The RF signal-to-noise ratio of a received signal is a quantity of considerable practical and theoretical interest in many radio and radar applications, particularly in evaluation of system performance. It is therefore important to be able to measure this quantity. But that is not easy to do by direct methods. A direct approach would be to use a power-sensitive device such as a bolometer or thermocouple directly in the RF circuits of a receiver. If a reading of the noise power in the absence of signal is designated by P_n, and a reading in the presence of signal is $P_s + P_n$, then of course the signal-to-noise ratio P_s/P_n can be calculated. Alternatively, the measurement can be made by comparing the receiver output when a calibrated RF signal generator is connected to the input with that of the actual received signal. However, the apparatus required to make measurements of this kind is expensive and complicated.

This problem recently arose in connection with a project assigned to the Radar Geophysics Branch. The method presented here for solving the problem, although based on a noise measurement technique that was described in the literature many years ago, does not seem to be well known, and the description of it given here may be useful to others.

The readily available output of a receiving system is of course the detected output, which can be in the form of audio, video, or DC current and voltage, depending on the use to be made of the output. It is desirable to be able to make a signal-to-noise ratio measurement using this output. As will be shown, the DC detector output can be used for this purpose. However, the relationship between the noise-only power and the signal-plus-noise power is profoundly altered by the detection process, and therefore the simple arithmetic that can be utilized for predetection measurement no longer applies.

The mathematics of the problem have been worked out and are well known to engineers who specialize in signal-and-noise problems. Moreover, the application of the theory to laboratory noise measurements was reported by North (1) as long ago as 1943. But subsequent application of this knowledge in practical measurements has not been common, probably because of the computational difficulties involved. The purpose of this report is to relate the theory specifically to measurement of predetection signal-to-noise ratio, to present some tabulated and graphical digital-computer results which solve the computational difficulty, and thereby to make practical a simplified procedure for signal-to-noise-ratio measurement.

DESCRIPTION OF THE DETECTOR

The usual form of detector* in practical receivers is a diode rectifier. The diode characteristic is usually linear; that is, the relationship between the instantaneous input voltage E_i and the instantaneous output voltage E_o (developed across a load resistor) is described by the equations:

$$E_o = k\, E_i,\ E_i > 0$$
$$E_o = 0,\ E_i \leq 0 \qquad , \tag{1}$$

where k is a constant depending on the diode and load impedances. Actually, this assumed linearity is only approximately realized; a more nearly square-law characteristic is observed if E_i is very small. However, the usual diode detector is operated at a voltage level that results in practically linear operation. (The linearity depends on the voltage level and not on the signal-to-noise ratio.)

MATHEMATICAL RESULTS

The mathematics of the relationship between the input signal-to-noise ratio and the output current (or voltage) of such a detector have been worked out independently by North (1) and Bennett (2). Rice (3) also published the result. The basic equation, which will not be derived here, is (from Rice's Eq. 4.2-3):

$$\frac{E}{E_o} = e^{-S/2} \left\{ (1 + S)\ I_o(S/2) + S\ I_1(S/2) \right\} , \tag{2}$$

*The term detector here refers to what is sometimes called the "second detector" in superheterodyne receivers. "Demodulator," or simply "rectifier," would be a more precise terminology.

in which E is the DC voltage output of the detector when the input signal-to-noise power ratio is S, and E_0 is the DC output when S = 0 (no signal, or noise only). The symbols I_0 and I_1 denote the modified Bessel functions of orders zero and one respectively.* This result is obtained for a steady sinusoidal signal (CW) and for noise of Gaussian probability density (thermal noise, for example). (Ordinary receiver noise fits this description.) It is applicable to modulated-sine-wave systems, with proper interpretation of the meaning of S, including pulsed carrier systems.

If S is sufficiently large, a simplified approximation formula can be used. It is obtained by using the asymptotic expressions for I_0 and I_1. The result can be expressed in the following form:

$$\frac{E}{E_0} \approx \sqrt{\frac{2}{\pi}(2S+1)}\ , \ (S \gg 1). \tag{3}$$

This can be inverted to give:

$$S \approx \frac{\pi}{4}\left(\frac{E}{E_0}\right)^2 - 0.5 \tag{3a}$$

If S is sufficiently small, the following approximations can be used. They are obtained by replacing $e^{-S/2}$, $I_0(S/2)$, and $I_1(S/2)$ by their power-series expansions and dropping terms of higher than second order:

$$\frac{E}{E_0} \approx 1 + S/2 - S^2/16,\ (S \ll 1). \tag{4}$$

Solving for S gives:

$$S \approx \frac{4\ (E/E_0 - 1)}{1 + \sqrt{2 - E/E_0}}\ . \tag{4a}$$

---

*The modified Bessel functions I_0 and I_1 are related to the ordinary Bessel functions J_0 and J_1 by: $I_0(x) = J_0(ix)$, $I_1(x) = -i\ J_1(ix)$, $i = \sqrt{-1}$.

For very small values of S, the square term in Eq. (4) can be dropped, and Eq. (4a) becomes:

$$S \approx 2\ (E/E_o - 1) \tag{4b}$$

APPLICATION TO PRACTICAL MEASUREMENT

With these equations, it is possible to determine S (or as it is often written, S/N) by measuring E/E_o, using an ordinary DC voltmeter.* The only difficulty is the computational one of inverting Eq. (2) to solve for S in terms of E/E_o.

This problem can be solved by using a digital computer to produce a tabulation of values of S and resulting values of E/E_o. This has been done** in terms of decibel values ($10 \log_{10} S$) in 0.1-decibel steps, from -9.9 to +26 dB. The columns in Table 1 headed DB are $10 \log_{10} S$; the second column is S itself; the third column is $R = E/E_o$. The approximations given by Eqs. (3a) and (4a) can be used for values of S larger or smaller, respectively, than those given by Table 1.

The results are also given in graphical form by Fig. 1. This graph was made by machine plotting the digital computer results on the NRL Gerber Plotter.

The simplest application of this method is to the case of CW radio reception. In this case E_o is measured either with the transmitter turned off or with the antenna (if it is sufficiently directional) pointed away from the transmitter. Then E is measured with

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*Actually Eq. (2) is derived for the average voltage output, which would be measured by a d'Arsonval type meter. But it matters not whether the "average" is that of the peaks of the rectified RF cycles, their rms level, or some other value. The result also applies to the output current of the detector.

**The computations were programmed in Fortran for the NRL CDC-3800 by the author. A CDC CO-OP Library subroutine, designated C3-UCSD-BES, written by Gene Gilbert and George Baker of The University of California at San Diego, was used to compute the modified Bessel functions. The computational accuracy considerably exceeded the number of digits given in Table 1.

Table 1.

Digitally computed values of $R = E/E_o$ from Eq. (2), in 0.1-decibel steps of DB = 10 log S, from -9.9 to +26.0.

DB	S	R	DB	S	R	DB	S	R
-9,9	0,1023	1,0505	-5,9	0,2570	1,1246	-1,9	0,6457	1,2993
-9,8	0,1047	1,0517	-5,8	0,2630	1,1274	-1,8	0,6607	1,3058
-9,7	0,1072	1,0529	-5,7	0,2692	1,1302	-1,7	0,6761	1,3124
-9,6	0,1096	1,0541	-5,6	0,2754	1,1332	-1,6	0,6918	1,3191
-9,5	0,1122	1,0553	-5,5	0,2818	1,1362	-1,5	0,7079	1,3260
-9,4	0,1148	1,0566	-5,4	0,2884	1,1392	-1,4	0,7244	1,3330
-9,3	0,1175	1,0579	-5,3	0,2951	1,1424	-1,3	0,7413	1,3401
-9,2	0,1202	1,0592	-5,2	0,3020	1,1456	-1,2	0,7586	1,3474
-9,1	0,1230	1,0606	-5,1	0,3090	1,1488	-1,1	0,7762	1,3548
-9,0	0,1259	1,0620	-5,0	0,3162	1,1522	-1,0	0,7943	1,3624
-8,9	0,1288	1,0634	-4,9	0,3236	1,1556	-0,9	0,8128	1,3701
-8,8	0,1318	1,0649	-4,8	0,3311	1,1591	-0,8	0,8318	1,3779
-8,7	0,1349	1,0663	-4,7	0,3388	1,1626	-0,7	0,8511	1,3860
-8,6	0,1380	1,0679	-4,6	0,3467	1,1663	-0,6	0,8710	1,3941
-8,5	0,1413	1,0694	-4,5	0,3548	1,1700	-0,5	0,8913	1,4024
-8,4	0,1445	1,0710	-4,4	0,3631	1,1738	-0,4	0,9120	1,4109
-8,3	0,1479	1,0726	-4,3	0,3715	1,1776	-0,3	0,9333	1,4196
-8,2	0,1514	1,0743	-4,2	0,3802	1,1816	-0,2	0,9550	1,4284
-8,1	0,1549	1,0760	-4,1	0,3890	1,1856	-0,1	0,9772	1,4373
-8,0	0,1585	1,0777	-4,0	0,3981	1,1898	0,0	1,0000	1,4465
-7,9	0,1622	1,0795	-3,9	0,4074	1,1940	0,1	1,0233	1,4558
-7,8	0,1660	1,0813	-3,8	0,4169	1,1983	0,2	1,0471	1,4653
-7,7	0,1698	1,0832	-3,7	0,4266	1,2027	0,3	1,0715	1,4750
-7,6	0,1738	1,0851	-3,6	0,4365	1,2072	0,4	1,0965	1,4848
-7,5	0,1778	1,0870	-3,5	0,4467	1,2117	0,5	1,1220	1,4948
-7,4	0,1820	1,0890	-3,4	0,4571	1,2164	0,6	1,1482	1,5050
-7,3	0,1862	1,0910	-3,3	0,4677	1,2212	0,7	1,1749	1,5154
-7,2	0,1905	1,0931	-3,2	0,4786	1,2261	0,8	1,2023	1,5260
-7,1	0,1950	1,0952	-3,1	0,4898	1,2310	0,9	1,2303	1,5368
-7,0	0,1995	1,0974	-3,0	0,5012	1,2361	1,0	1,2589	1,5477
-6,9	0,2042	1,0996	-2,9	0,5129	1,2413	1,1	1,2882	1,5589
-6,8	0,2089	1,1018	-2,8	0,5248	1,2466	1,2	1,3183	1,5702
-6,7	0,2138	1,1041	-2,7	0,5370	1,2520	1,3	1,3490	1,5818
-6,6	0,2188	1,1065	-2,6	0,5495	1,2575	1,4	1,3804	1,5936
-6,5	0,2239	1,1089	-2,5	0,5623	1,2631	1,5	1,4125	1,6055
-6,4	0,2291	1,1114	-2,4	0,5754	1,2688	1,6	1,4454	1,6177
-6,3	0,2344	1,1139	-2,3	0,5888	1,2747	1,7	1,4791	1,6301
-6,2	0,2399	1,1165	-2,2	0,6026	1,2807	1,8	1,5136	1,6427
-6,1	0,2455	1,1191	-2,1	0,6166	1,2848	1,9	1,5488	1,6555
-6,0	0,2512	1,1218	-2,0	0,6310	1,2930	2,0	1,5849	1,6685

DB	S	R	DB	S	R	DB	S	R
2,1	1,6218	1,6818	6.1	4.0738	2,4229	10.1	10.2329	3.6989
2,2	1,6596	1,6953	6.2	4.1687	2,4474	10.2	10.4713	3.7397
2,3	1,6982	1,7090	6.3	4.2658	2,4723	10.3	10.7152	3.7809
2,4	1,7378	1,7229	6.4	4.3652	2,4975	10.4	10.9648	3.8227
2,5	1,7783	1,7371	6.5	4.4668	2,5231	10.5	11.2202	3.8649
2,6	1,8197	1,7515	6.6	4.5709	2,5490	10.6	11.4815	3.9077
2,7	1,8621	1,7661	6.7	4.6774	2,5752	10.7	11.7490	3.9510
2,8	1,9055	1,7810	6.8	4.7863	2,6018	10.8	12.0226	3.9948
2,9	1,9498	1,7961	6.9	4.8978	2,6287	10.9	12.3027	4.0391
3,0	1,9953	1,8115	7.0	5.0119	2,6560	11.0	12.5893	4.0840
3,1	2,0417	1,8271	7.1	5.1286	2,6837	11.1	12.8825	4.1294
3,2	2,0893	1,8430	7.2	5.2481	2,7117	11.2	13.1826	4.1754
3,3	2,1380	1,8591	7.3	5.3703	2,7401	11.3	13.4896	4.2219
3,4	2,1878	1,8754	7.4	5.4954	2,7688	11.4	13.8038	4.2690
3,5	2,2387	1,8921	7.5	5.6234	2,7980	11.5	14.1254	4.3166
3,6	2,2909	1,9090	7.6	5.7544	2,8274	11.6	14.4544	4.3649
3,7	2,3442	1,9261	7.7	5.8884	2,8573	11.7	14.7911	4.4137
3,8	2,3988	1,9435	7.8	6.0256	2,8876	11.8	15.1356	4.4630
3,9	2,4547	1,9612	7.9	6.1660	2,9182	11.9	15.4882	4.5130
4,0	2,5119	1,9791	8.0	6.3096	2,9493	12.0	15.8489	4.5636
4,1	2,5704	1,9973	8.1	6.4565	2,9807	12.1	16.2181	4.6148
4,2	2,6303	2,0158	8.2	6.6069	3,0125	12.2	16.5959	4.6666
4,3	2,6915	2,0346	8.3	6.7608	3,0448	12.3	16.9824	4.7190
4,4	2,7542	2,0537	8.4	6.9183	3,0774	12.4	17.3780	4.7720
4,5	2,8184	2,0730	8.5	7.0795	3,1105	12.5	17.7828	4.8257
4,6	2,8840	2,0926	8.6	7.2444	3,1439	12.6	18.1970	4.8800
4,7	2,9512	2,1125	8.7	7.4131	3,1778	12.7	18.6209	4.9350
4,8	3,0200	2,1327	8.8	7.5858	3,2121	12.8	19.0546	4.9906
4,9	3,0903	2,1532	8.9	7.7625	3,2469	12.9	19.4984	5.0469
5,0	3,1623	2,1740	9.0	7.9433	3,2821	13.0	19.9526	5.1038
5,1	3,2359	2,1950	9.1	8.1283	3,3177	13.1	20.4174	5.1615
5,2	3,3113	2,2164	9.2	8.3176	3,3537	13.2	20.8930	5.2198
5,3	3,3884	2,2381	9.3	8.5114	3,3902	13.3	21.3796	5.2788
5,4	3,4674	2,2601	9.4	8.7096	3,4272	13.4	21.8776	5.3385
5,5	3,5481	2,2824	9.5	8.9125	3,4646	13.5	22.3872	5.3989
5,6	3,6308	2,3050	9.6	9.1201	3,5025	13.6	22.9087	5.4600
5,7	3,7154	2,3280	9.7	9.3325	3,5408	13.7	23.4423	5.5219
5,8	3,8019	2,3512	9.8	9.5499	3,5796	13.8	23.9883	5.5845
5,9	3,8905	2,3748	9.9	9.7724	3,6189	13.9	24.5471	5.6478
6,0	3,9811	2,3987	10.0	10.0000	3,6587	14.0	25.1189	5.7119

DB	S	R	DB	S	R	DB	S	R
14,1	25,7040	5,7767	18.1	64,5654	9,1020	22,1	162,1810	14,3921
14,2	26,3027	5,8423	18.2	66,0693	9,2066	22,2	165,9587	14.5583
14,3	26,9153	5,9087	18.3	67,6083	9,3124	22,3	169,8244	14.7263
14,4	27,5423	5,9758	18.4	69,1831	9,4194	22,4	173,7801	14,8963
14,5	28,1838	6,0438	18,5	70,7946	9,5277	22,5	177,8279	15,0683
14,6	28,8403	6,1125	18.6	72,4436	9,6373	22,6	181,9701	15,2423
14,7	29,5121	6,1821	18,7	74,1310	9,7481	22,7	186,2087	15.4184
14,8	30,1995	6,2525	18.8	75,8578	9,8602	22,8	190,5461	15.5964
14,9	30,9030	6,3237	18.9	77,6247	9,9736	22,9	194,9845	15.7765
15,0	31,6228	6,3957	19.0	79,4325	10,0884	23,0	199,5262	15,9588
15,1	32,3594	6,4686	19.1	81,2831	10,2045	23,1	204,1738	16,1431
15,2	33,1131	6,5424	19,2	83,1764	10,3219	23,2	208,9296	16,3296
15,3	33,8844	6,6170	19,3	85,1138	10,4407	23,3	213,7962	16,5182
15,4	34,6737	6,6925	19.4	87,0964	10,5609	23,4	218,7762	16,7090
15,5	35,4813	6,7689	19,5	89,1251	10,6825	23,5	223,8721	16,9021
15,6	36,3078	6,8461	19,6	91,2011	10,8055	23,6	229,0868	17,0974
15,7	37,1535	6,9243	19,7	93,3254	10,9300	23,7	234,4229	17,2949
15,8	38,0189	7,0034	19,8	95,4993	11,0558	23,8	239,8833	17,4947
15,9	38,9045	7,0835	19,9	97,7237	11,1832	23,9	245,4709	17,6969
16,0	39,8107	7,1644	20.0	100.0000	11,3120	24,0	251,1886	17,9014
16,1	40,7380	7,2464	20.1	102,3293	11,4424	24,1	257,0396	18,1083
16,2	41,6869	7,3293	20.2	104,7129	11,5742	24,2	263,0268	18,3176
16,3	42,6580	7,4131	20.3	107,1519	11,7076	24,3	269,1535	18,5293
16,4	43,6516	7,4979	20,4	109,6478	11,8426	24,4	275,4229	18,7434
16,5	44,6684	7,5838	20,5	112,2018	11,9791	24,5	281,8383	18,9601
16,6	45,7088	7,6706	20.6	114,8154	12,1172	24,6	288,4032	19,1792
16,7	46,7735	7,7595	20,7	117,4898	12,2569	24,7	295,1209	19,4009
16,8	47,8630	7,8474	20,8	120,2264	12,3982	24,8	301,9952	19,6252
16,9	48,9779	7,9373	20,9	123,0269	12,5412	24,9	309,0295	19,8521
17,0	50,1187	8,0283	21.0	125,8925	12,6858	25,0	316,2278	20,0816
17,1	51,2861	8,1203	21,1	128,8250	12,8321	25,1	323,5937	20,3138
17,2	52,4807	8,2134	21.2	131,8257	12,9801	25,2	331,1311	20,5486
17,3	53,7032	8,3076	21,3	134,8963	13,1299	25,3	338,8442	20,7862
17,4	54,9541	8,4029	21.4	138,0384	13,2813	25,4	346,7369	21,0266
17,5	56,2341	8,4994	21,5	141,2538	13,4346	25,5	354,8134	21,2697
17,6	57,5440	8,5969	21.6	144,5440	13,5896	25,6	363,0781	21,5156
17,7	58,8844	8,6956	21,7	147,9108	13,7464	25,7	371,5352	21,7644
17,8	60,2560	8,7954	21.8	151,3561	13,9050	25,8	380,1894	22,0161
17,9	61,6595	8,8964	21,9	154,8817	14,0655	25,9	389,0451	22,2707
18,0	63,0957	8,9986	22,0	158,4893	14,2279	26,0	398,1072	22,5283

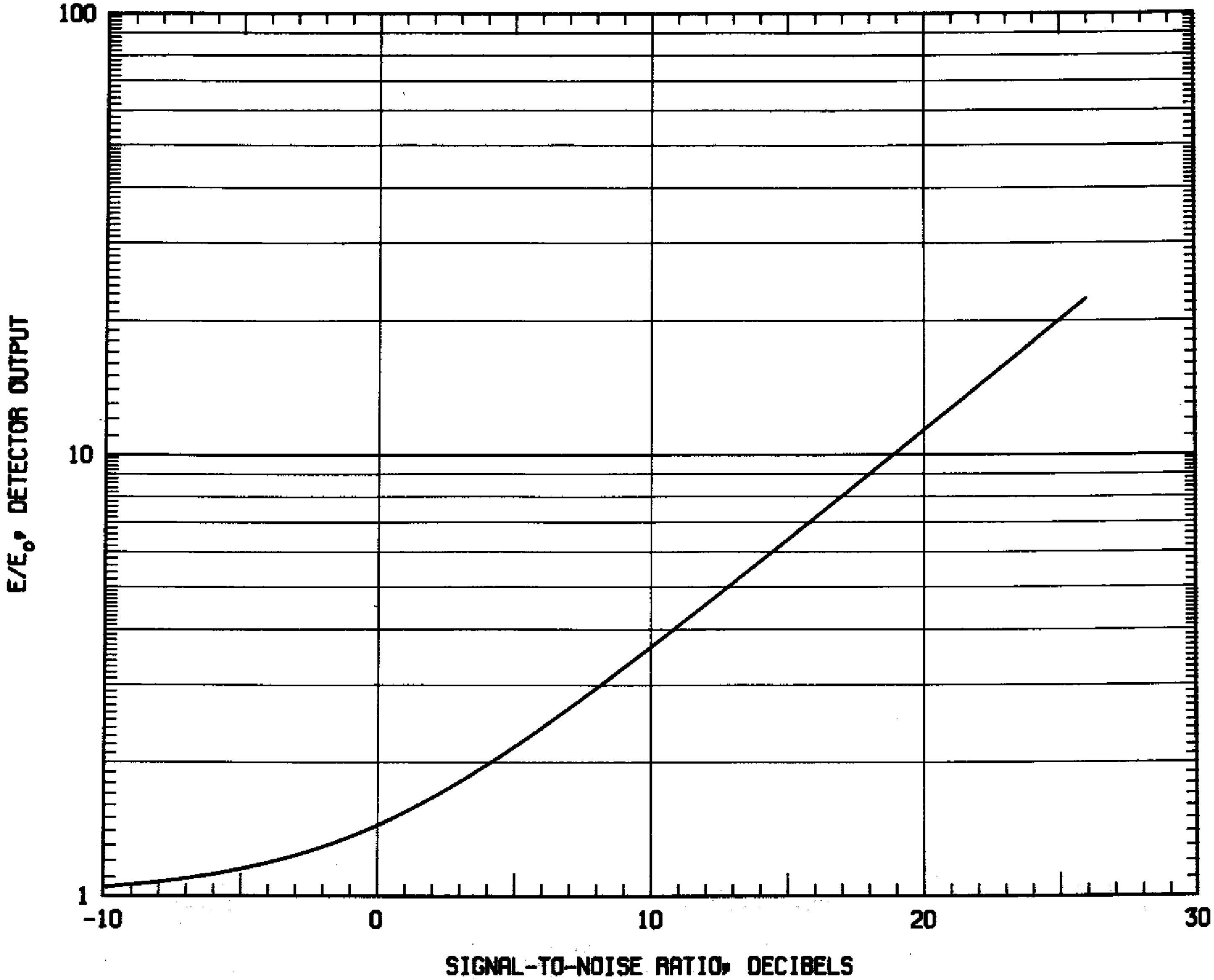

Fig. 1 - Machine plot of the ratio $R = E/E_o$ vs. $DB = 10 \log S$, as given by Table 1.

the transmitter on, or with the antenna pointed toward the transmitter. The measurement must be made with a DC instrument, in the detector output circuit prior to the insertion of any blocking capacitors. If the second method is used, care must be taken to avoid two possible sources of error: (1) sufficient signal may enter via sidelobes of the antenna pattern to give a wrong "noise only" reading; (2) the galactic noise is different in different parts of the sky, therefore care must be taken not to move the antenna pointing direction to a significantly noisier or quieter part of the sky. (Sky noise maps which have been published can be used to avoid this error.)

The method can also be readily adapted to measurements on pulsed signals (e.g., radar echoes) by measuring E during a time-gate centered on the received pulse, the duration of the gate being comparable to or shorter than the radar pulse length. E_0 is then measured with the gate shifted to a no-signal portion of the interpulse time interval. For other types of modulation, the measurement method can be varied to suit the particular modulation conditions. In all cases, the measurements must be made of quantities equivalent to the detector DC output in the CW case.

With a CW signal, the use of a DC instrument results in a smoothing of the noise fluctuations that are actually present in the detector output, because of the time constant of the meter response. Additional smoothing can be accomplished, if deemed necessary, by filtering in the detector output circuit. (A capacitor shunting the load resistor and meter is usually an adequate filter.) The noise fluctuations result in a statistical error of measurement, which is decreased by filtering (smoothing). The time constant of the usual DC meter is a quite adequate filter for the accuracy required in practical measurements when S is of the order of one or greater.

When measurements are made on radar pulse signals, the voltage reading of a single pulse does not give a smoothed result, and the statistical error may be appreciable. It can be decreased by averaging a number of successive pulses; the statistical error is inversely related to the number of pulses averaged. It is also inversely related to the signal-to-noise ratio. A quantitative discussion of this subject is given in Ref. (4), Part 2.

If the signal-to-noise ratio is not too small, it is practical to make rough estimates of the ratio E/E_0 by observing an A-scope radar display, since the A-scope vertical deflection is proportional to detector output voltage. The average values of E and E_o can be estimated fairly well by the eyeball method, if the radar pulse rate is not too low. The value of S can then be found by using Table 1 or Fig. 1. This procedure is probably practical for rough estimation of S for $S \approx 1$ or greater, corresponding to $E/E_o \approx 1.5$ or greater.

Accurate measurement of small signal-to-noise ratios by the postdetection method is difficult. However, this is also true of any other method. It is possible to measure small values of S accurately by using a long-enough averaging time. The accuracy of the actual DC measurement can be improved by noting that Eq. (4a) can be rewritten in the form:

$$S \approx \frac{4\,(\Delta E)/E_o}{1 + \sqrt{1 - (\Delta E)/E_o}}\,, \tag{4c}$$

and Eq. (4b) written in this form is:

$$S \approx 2(\Delta E)/E_o\,, \tag{4d}$$

where $\Delta E = E - E_o$. The measurement of ΔE can be accomplished with improved accuracy by "biasing out" the E_o component in the metering circuit, so that a more sensitive meter can be used. However, this must be done in such a way that the bias is not applied to the detector, but only to the metering circuit. This requires a decoupling device between the detector and the meter (e.g., a DC cathode-follower or emitter-follower).

RELATIONS FOR A NOISE-LIKE SIGNAL

In some radio systems, the received signal is simply an additional amount of noise power. This case occurs, for example, in radio astronomy. It also occurs in reception of the radar return from the ionosphere - the so-called Thomson scatter signal. Other examples could be cited.

Equation (2) does not apply in this case. The applicable equation is in fact much simpler. The output voltage is (1,3):

$$E = k\sqrt{P_n}\,, \tag{5}$$

where P_n is the noise power at the detector input and k is a constant. If E_o is the value of E when P_n is the basic system noise, then if an additional noise power ΔP_n is applied as an input "signal" and the new value of E is denoted by E_1, then

$$\frac{E_1}{E_o} = \sqrt{\frac{P_n + \Delta P_n}{P_n}} \; . \qquad (6)$$

The numerator on the right-hand side reflects the well-known fact that if two noncoherent random noise voltages are linearly added, power addition results. Equation (6) yields

$$\frac{\Delta P_n}{P_n} = \left(\frac{E_1}{E_o}\right)^2 - 1 = \left(\frac{\Delta E}{E_o}\right)^2 + \frac{2\Delta E}{E_o} \; . \qquad (7)$$

The left-hand side of this equation represents the "signal-to-noise power ratio" for the reception of a noise-like signal. In the ΔE form of this equation, the first (squared) term can be omitted when the signal-to-noise ratio is very small, and the second term can be omitted when the ratio is very large.

SQUARE-LAW DETECTOR

Although the usual radar-receiver second detector is a linear rectifier, it is of some interest to consider the results for a square-law detector, corresponding to Eqs. (1) through (7) for the linear detector. The defining relation for a square-law detector corresponding to Eq. (1) is:

$$E_o = kE_i^2 . \qquad (8)$$

Incidentally, this defines a "full-wave" square-law detector, which is usually assumed in theoretical analyses, rather than a half-wave square-law rectifier. However, use of the latter would give the same output; only the value of k would be affected.

Rice (3) gives an expression for the DC output of a square-law detector (his Eq. 4.1-14). In terms of the quantities defined in Eq. (2), the ratio of output voltage E for a sine-wave input signal with signal-to-noise ratio S to the no-signal value E_o for this detector is:

$$\frac{E}{E_o} = 1 + S. \tag{9}$$

This expression is of course much simpler than that for the linear-rectifier case, Eq. (2), and allows S to be calculated directly from measurement of E/E_o; that is:

$$S = \frac{E}{E_o} - 1. \tag{10}$$

This result holds also for the case of the noise-like signal. (Note that it is not the same as Eq. (7), in which the ratio E/E_o is squared.) This follows from the fact that the noise-only output voltage of the square-law detector is proportional to the input noise power (as shown by Rice's Eq. 4.1-14) rather than to the square root as in Eq. (5).

THEORETICAL CALCULATION OF SIGNAL-TO-NOISE RATIO

The usual reason for desiring to make a signal-to-noise ratio measurement is to compare the measured value with a theoretically calculated value. Therefore the equations for making such calculations will also be given here, for convenient reference. Their derivation is given in Ref. (5). For a radar system the applicable equation is:

$$S = \frac{P_t\ G_t\ G_r \sigma\ \lambda^2 F_t^{\ 2}\ F_r^{\ 2}}{(4\pi)^3 R_t^{\ 2}\ R_r^{\ 2}\ k\ T_s\ B_n\ L}\ , \tag{11}$$

in which the symbols have the following definitions:

S -- signal-to-noise power ratio at the output of the pre-detection portion of the receiver,

P_t -- radar transmitter power, watts,

G_t -- transmitting antenna power gain in beam maximum,

G_r -- receiving antenna power gain in beam maximum,

σ -- radar target cross section,

λ -- radar wavelength,

F_t -- transmit-path pattern-propagation factor,

F_r -- receive-path pattern-propagation factor,

R_t -- radar-transmitting-antenna-to-target distance,

R_r -- radar-target-to-receiving-antenna distance,

k -- Boltzmann's constant, 1.38×10^{-23} watt-second per degree Kelvin,

T_s -- system noise temperature, degrees Kelvin,

B_n -- receiving system noise bandwidth, hertz,

L -- system loss factor.

The definitions of these quantities are discussed in some detail in Ref. (4). The same units must be used for σ, λ, and R - e.g., meters. F_t is defined as the ratio of the wave intensity (electric or magnetic) at the radar target to that which would be observed in free space at the same range in the transmitting-antenna beam maximum. It thus takes into account the antenna pattern as well as propagation effects such as multipath interference, refractive focusing and defocusing, below-the-horizon earth shadowing, and the like. In principle it should also account for absorption, but ordinarily it is permissible and more convenient to account for absorption loss in the system loss factor L. F_r is defined analogously -- that is, it is calculated in terms of the receiving antenna as if that antenna were transmitting. For monostatic radar with the same antenna used for transmitting and receiving, $G_t = G_r$, $F_t = F_r$, and $R_t = R_r$. If the radar propagation path is "free space" and the target is in the maxima of the antenna patterns, $F_t = F_r = 1$.

The loss factor L is the total loss by atmospheric absorption, and transmission-line loss between the receiving antenna and the reference point for T_s. If the receiver passband is too narrow to pass all of the significant frequency components of the signal, L must also account for the effective loss of signal power that results. The factor L includes transmitter line loss if P_t is evaluated at the transmitter terminals, but not if P_t is the actually radiated power. If P_t is evaluated as pulse power, in the pulse-radar case, then S is the received-pulse-power-to-noise-power ratio; if P_t is average power, S is the average signal-to-noise ratio. For CW or sine-wave-modulated CW systems the average power is of course the appropriate concept.

The corresponding equation for a one-way radio system (e.g., point-to-point communication) is:

$$S = \frac{P_t \; G_t \; G_r \; \lambda^2 \; F^2}{(4\pi \; R)^2 \; k \; T_s \; B_n \; L} \; . \tag{12}$$

Here F and R are unsubscripted since there is only one propagation path. F is now defined as the ratio of the signal voltage at the output terminals of the receiving antenna to that which would be measured if the two antennas were in free space at the same separation distance, with each antenna in the beam maximum of the other. The pattern factors of both antennas are therefore contained in F.

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