NRL Report 7071

Methods for Calculating and Analyzing Interrogation Repetition Frequency Sets Nonsynchronous for n Interrogations

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June 11, 1970

NAVAL RESEARCH LABORATORY
Washington, D.C.

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

METHODS FOR CALCULATING AND ANALYZING INTERROGATION REPETITION FREQUENCY SETS NONSYNCHRONOUS FOR n INTERROGATIONS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
An interim report on a continuing problem.

5. AUTHOR(S) *(First name, middle initial, last name)*

William K. Gardner

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 11, 1970	44	5

8a. CONTRACT OR GRANT NO. NRL Problem R03-01 b. PROJECT NO. RF-05-151-403-4150 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 7071 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Department of the Navy (Office of Naval Research), Arlington, Va. 22217

13. ABSTRACT

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DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Security Classification

9:71

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Air traffic control						
Radar beacons						
Nonsynchronous repetition Frequencies						
IFF						
Identification						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

Security Classification

UNCLASSIFIED

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PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

NRL Problem R03-01
RF-05-151-403-4150

Manuscript submitted February 13, 1970.

METHODS FOR CALCULATING AND ANALYZING INTERROGATION REPETITION FREQUENCY SETS NONSYNCHRONOUS FOR n INTERROGATIONS

INTRODUCTION

The ever-increasing densities of air traffic continually emphasize the need for effective and reliable tracking and control systems. The Air Traffic Control Radar Beacon System (ATCRBS) is in use for this purpose in every section of the United States; in Europe it is known as Secondary Surveillance Radar (SSR). Important components of the ATCRBS include interrogators with secondary radar scanning antennas at ground or control-tower locations and transponders with omnidirectional antennas installed on aircraft. The interrogators automatically request responses from the transponders at predetermined rates. The transponders accept valid requests one at a time and transmit their codes back to the receiver portion of the interrogators ("responsor"). Newer transponders also report altitude. The most modern display equipment incorporates the information received from the beacon system into the same scope display that gives range and azimuth obtained from the primary radar.

The ATCRBS performs with fairly high reliability when only a very few interrogators are operating in a given area. However, for some time, air traffic controllers have reported increasing instances of broken tracks on the monitor screens. Many of the broken tracks are believed to be the result of interference between two or more interrogator IRF's* at the time of arrival at the transponder. The term *synchronous interference* is used when interference is experienced for several interrogations in succession or at regular intervals.

When interference occurs, no reply is sent from the transponder to one or more of the interrogators involved. Both recent and current investigations reveal areas in the United States having far too many interrogator installations. The sum total of military and civil installations, each probably not initially unreasonable from the standpoint of individual requirements, appears to have reached the point of saturation in several areas. Active consideration is being given to both short-term improvement, requiring relatively minor though still costly equipment modification, and to anticipated requirements for the more distant future. With cooperation from the military and all concerned, the Federal Aviation Administration (FAA) is currently concentrating on reducing the many causes of interference within the overall system environment. This involves stricter control of frequency usage to avoid unauthorized use of assigned IRF's, a reevaluation of the operational radar power levels for each interrogator site, improved sidelobe suppression, and an investigation of ramp tester check procedures, to mention a few items.

The recently discovered case (1) of continuous strobing of an assigned IRF for several hours by an airport ramp tester well illustrates the effects of synchronous interference in the extreme. The IRF of the ramp tester was almost exactly twice that of an IRF assigned to the airport for air traffic control, with a phase angle between the two frequencies that resulted in interference on every interrogation.

*Often called PRF (pulse repetition frequency) and PRR (pulse repetition rate) by users of the system. IRF (interrogation repetition frequency) is believed to be a more precise, less ambiguous term and will be used throughout this report.

The detrimental effects of synchronous interference among assigned IRF's on beacon system performance is not so apparent at first glance. The more extreme cases of synchronous interference may be assumed to have been eliminated as the result of operational experience if not by considerations of initial choice. Nevertheless, it is worthwhile to consider methods that would result in further reduction of synchronous interference.

As a result of Foster's work (2), various approaches are possible. For example, synchronous interference could be completely eliminated with a system design philosophy based on (a) precise control of time and (b) phase synchronization of the IRF's. A low probability of occurrence of synchronous interference would result from the use of randomly spaced sequence events (interrogations) during appropriate time intervals. As a practical matter, it is difficult to control the phase relationships between two sequences of events. Randomization does not preclude serious interference. As another approach, Foster developed an algebraic procedure for choosing event repetition rates (interrogation rates) between two sequences of events (two different regularly repeated interrogation sequences) such that if interference between events of the two sequences occurs once, this type of interference will not occur again for at least the next n events (n interrogations). The idea of this approach forms the basis for this report. However, a different method for calculating the event repetition rates (interrogation rates) has been developed that is believed to be simpler to apply. Although Foster illustrated his algebraic procedure by what he calls a state diagram, it still appears to require a certain amount of background and experience for successful application. Also the effort involved appears to be considerable if attempting a search for all available IRF's within a given frequency range which would be nonsynchronous for n interrogations. Perhaps of greater importance, it was not readily apparent as to what means could be used to take advantage of the application of modern digital computers. Thus, the development of an alternative method seemed desirable. The methods presented here are based on simple arithmetic algebra, and are easily programmed for solution by digital computers.

Although the major portions of this report concern calculation methods and illustrative results, its real importance lies in potential application to air traffic control beacon systems for reduction of synchronous interference. Attention is called to the results presented in the tables, several of which should be of timely interest.

DEFINITIONS

Nonsynchronous IRF's are defined as IRF's that, if interference occurs once between the interrogations of any two IRF's, then further interference of the same type between the interrogations of these two does not occur for at least the next n interrogations. Two useful categories of the nonsynchronous relation occur, depending upon the interpretation of "the next n interrogations." The nonsynchronous IRF's are defined here as *Type 1* having *unequal coverage* when the value of n is the same for each of two IRF's, and *Type 2* having *equal coverage* when for n interrogations of the smaller nonsynchronous IRF (larger period τ_L) there are at least $[(\tau_L/\tau_s)n + 1]$ interrogations of the larger nonsynchronous IRF (smaller period τ_s).

Figure 1 illustrates the characteristics of a pair of nonsynchronous IRF's. Two sequences of interrogations, M_i and N_i, have been plotted in time. The individual, periodic interrogations are represented as instantaneous events by the higher, numbered vertical lines. Immediately following each interrogation is a length of time δ that represents the overall "busy time" of a transponder. If a transponder is already occupied with processing an interrogation associated with one IRF, and an interrogation associated with another IRF arrives at any time during the δ time interval, then the second arrival is said to be interfered with because the transponder is already busy and can neither accept nor reply to another interrogation. A comparison of the M_i and N_i sequences

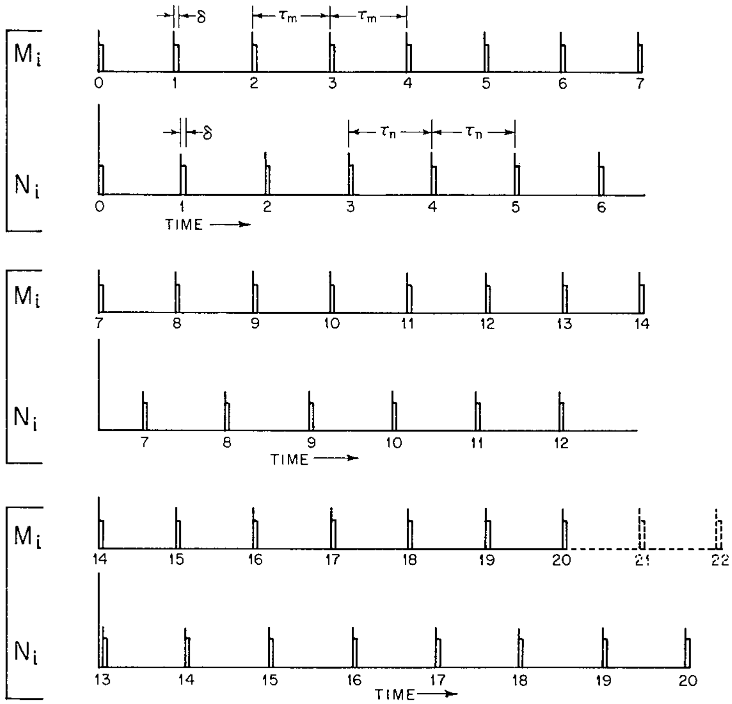

Fig. 1 - Characteristics of a pair of nonsynchronous IRF's. The two sequences of interrogations M_i and N_i plotted in time. Individual interrogations are represented by the higher, numbered vertical lines. Each interrogation is followed by a length of time δ, the busy time of a transponder. Both interrogation sequences are assumed periodic with the periods τ_m and τ_n as indicated. The solid lines, for which $M_i = N_i = 20$, illustrate Type 1 having unequal coverage. The solid plus the dotted lines, for which $M_i = 22$ and $N_i = 20$, illustrate Type 2 having (nearly) equal coverage. The total span of time occupied by the M_i sequence covers the total span of time occupied by the N_i sequence for Type 2.

shows that after the zeroth interrogations, shown in exact coincidence to serve as a phase angle reference point, the next n interrogations are interference-free for the particular choice of IRF values as represented by the periods τ_m and τ_n. The δ time intervals do not overlap. The solid-line portion of Fig. 1 represents the Type 1 category having unequal coverage. The longer total length of time occupied by 20 interrogations of sequence N_i is not "covered" by the shorter total length of time occupied by 20 interrogations of sequence M_i. The complete plot of the M_i sequence, the solid plus the dotted lines extending to 22 interrogations, illustrates the Type 2 category having "equal" (nearly equal) coverage in time of the 20 interrogations of the N_i sequence. The extended, dotted-line portion is small for the relatively low ratio of periods, $\tau_n/\tau_m = 1.08$, used for this illustration, but could be several times larger for two nonsynchronous IRF's chosen near opposite ends of the available IRF band.

Figure 1 illustrates the manner of choosing τ_m and τ_n so that the interrogations of the M_i and N_i sequences will not experience interference more than once in 20 repetitions. Actual interferences are assumed to be one-way. That is, if an interrogation of the M_i sequence arrives at a transponder less than δ μsec prior to arrival of an interrogation of the N_i sequence, then the N_i interrogation experiences interference but the M_i interrogation does not, and vice versa. It can be shown (3) that when the periods τ_m and τ_n are chosen for the Type 2 nonsynchronous IRF's as illustrated in Fig. 1 with $n = 20$, then neither sequence can experience more than one interference from the other during any sample of n successive interrogations. Furthermore, when τ_m and τ_n are chosen for the Type 1 nonsynchronous IRF's as illustrated for $n = 20$, then a burst of n interrogations with a repetition period of τ_m can experience no more than one interference from a burst of interrogations with a repetition period of τ_n and vice versa.

CORRELATION OF ANTENNA ROTATION RATE AND BEAMWIDTH WITH THE NONSYNCHRONOUS IRF VALUE

The benefits of using Type 1 nonsynchronous IRF's cannot be fully realized in a beacon system unless antenna rotation rates and beamwidths are correlated to the IRF's so that each interrogator achieves approximately n hits per scan of a target. Thus the Type 1 nonsynchronous IRF's having unequal coverage are appropriate for possible future modifications of the present ATCRBS. The Type 1 is also appropriate for systems with antennas that lock on a target and have precise on-off control of the occurrence of the IRF pulse bursts so that transponder busy time is minimized.

The Type 2 nonsynchronous IRF's, having equal coverage, are appropriate for the existing ATCRBS where correlation of IRF values with antenna rotation rates and beamwidths is not optimal because of the beacon system's dependence on the equipment characteristics of the associated primary radar. The system could be improved if acceptable nonsynchronous IRF's of Type 2 can be found that would improve the correlation with existing equipment, or if the combination of modification of antenna rotation rates, beamwidths, and acceptable values of nonsynchronous IRF's would result in improved correlation.

The ratio of antenna rotation rate to the main-beam width required to obtain just H hits per scan of the target from a given IRF may be found from Eq. (1);

$$\frac{N_{rpm}}{BW} = \frac{IRF}{6H}, \tag{1}$$

where N_{rpm} is the number of revolutions per minute, BW is the main-beam width in degrees, and IRF is the frequency in hertz of an IRF. As an approximation, BW is often taken at the half-power points of the main beam. If an accurate value of H is to be obtained from Eq. (1), other factors (4) affecting the value of BW must be considered.

Figures 2 and 3 illustrate some graphical relations obtained from Eq. (1).

If it is assumed that a beacon system design were to include exclusive use of nonsynchronous IRF's for n interrogations, the question of a suitable value for n arises. Studies of reply evaluation and use at NRL indicate that n does not need to exceed the value of 20 for military purposes. In general, nonmilitary requirements are not as severe. Therefore, it is likely that most nonmilitary functions can also be performed with $n \leq 20$.

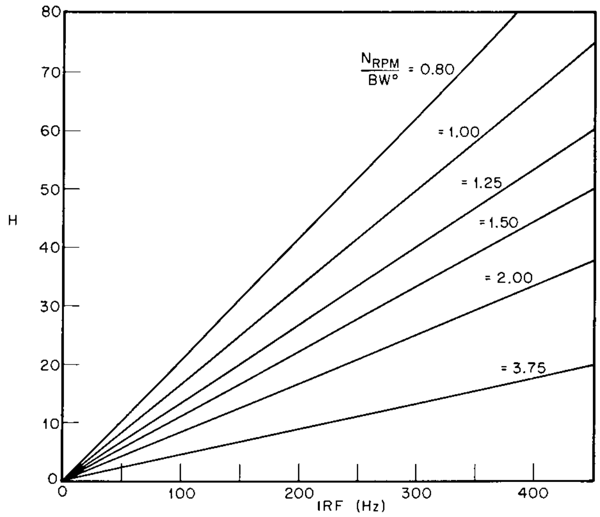

Fig. 2 - H vs IRF for several values of N_{rpm}/BW°

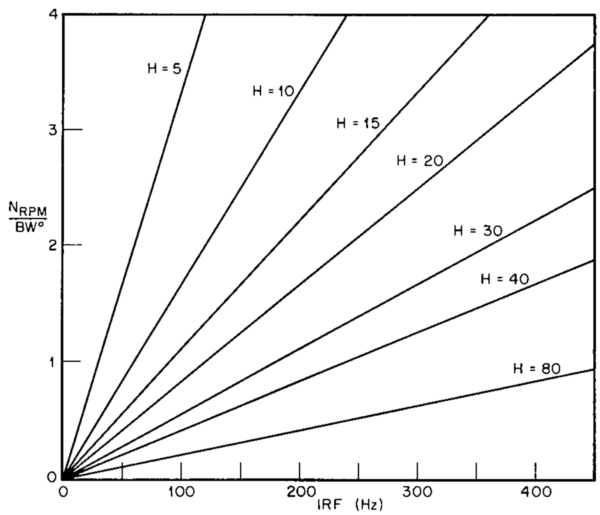

Fig. 3 - N_{rpm}/BW° vs IRF for several values of H

METHODS

To obtain better accuracy and ease of calculation, all calculation methods here are based on the use of periods in microseconds. Conversion to corresponding frequencies in hertz is done only after an integer value of the period has been calculated and found by testing to be nonsynchronous with the other members of its set.

Initial efforts* to develop an alternative method demonstrated that nonsynchronous IRF's can be found by a cut and try method. However, it was impractical to use sufficiently small increments within the available IRF frequency band to insure finding every available nonsynchronous IRF on a given search. Thus, the problem focused on the need to find a method that could locate all possible values of nonsynchronous IRF's available within a given frequency band for given values of n and δ.

Method I

The following approach was used to develop an improved method. The first nonsynchronous period is chosen equal to A, the starting value of the range of periods of interest A to Z. Although initially standing alone, A is assumed to be nonsynchronous on the expectation of later finding at least one other period to make it strictly so. The search for the set of nonsynchronous periods progresses in the direction from A to Z. Provision is included for either $A > Z$ or $A < Z$. Trial values of the periods are calculated and then tested for interference. The newest trial value is calculated in one of two ways. The first trial value following any period found by test to be nonsynchronous (including A) is always a value δ μsec beyond the newest nonsynchronous period, in the direction toward Z. In this instance, δ is the minimum possible step that can be taken in the direction toward Z without any excess use of the remaining range of periods. Whenever a trial value fails to pass the test, it will be found to interfere (or to be interfered with) at a particular multiple of its period with some other multiple of a previously accepted nonsynchronous period. The respective products of the multiples and periods will be less than δ μsec apart. Since the trial value that fails the test cannot become a member of the set in the required nonsynchronous fashion, it must be rejected. A new trial value of the period is calculated immediately after the first interference is discovered from the test.

All new trial values following a first interference point of the test are calculated in a way that will cause that particular interference to be eliminated, without any excess use of the remaining range of periods. The fact that both ways of calculating new trial values use no excess of the available remaining range of A to Z ensures that no nonsynchronous members of the particular set will be overlooked.

Many new trial periods may need to be calculated and tested before the next, if any, nonsynchronous period is found. Any trial value calculated will progress in the direction toward Z by an amount which can vary from 1 μsec minimum to $(2\delta - 1)$ maximum. Because of this range of variation, a somewhat different approach could have been used for Method I for calculating new trial values following interference. If a test were allowed to proceed to completion after the discovery of the first interference, then typically, additional interference points would be found at different multiples of the same trial period undergoing the test. If the several interference points were evaluated to find which one would result in either a large, or the largest, progression toward Z, and that conflict just eliminated, the next nonsynchronous period could be located with fewer trial

*Extensive manual calculations performed with the aid of a desk calculator by Mrs. C. J. Cooper, NRL.

value calculations and perhaps with less total testing. The same end results will be obtained whether the calculations progress from A to Z by many small steps, or fewer, mostly large, steps. This possible different approach was found at times to be useful for manual calculation. However, the advantage appeared to be quite dependent upon the relative values of n, δ, range A to Z, and whether or not A is greater than Z. Thus, for the computer implementation of Method I, it seems preferable to retain the simplicity of calculating new trial periods immediately following the first interference encountered by the test and to avoid added testing and evaluation for the other conflicts which, depending on parameter values, may offer no advantage over the first interference found.

Two test procedures for locating interference were developed for Method I. Initially, very complete tests were used. Later, shortcut test procedures were discovered and found to be sufficient. The complete tests are included, since they serve as reliable standards for checking shortened procedures and are also the basis for satisfying direct proofs that a set of IRF's claimed to be nonsynchronous for n interrogations actually is. The shortcut test procedure results in a significant saving of computer time for automatic calculation, but the advantage is considerably less for the quite tedious manual calculations where all but final confirmations can be performed by visual examination of columns of the $n\tau$ products.

The complete test for Type 1 having unequal coverage compares each multiple (1 to n) of the newest trial value of period with each multiple (1 to n) of every period previously accepted as a nonsynchronous member of the set to check that the difference between any pair of products is equal to or greater than the value of δ.

The complete test for Type 2 having equal coverage is similar to the Type 1 test, except that the additional coverage must be provided. The previously accepted periods and the newest trial periods are compared as before, but always on the basis of each integer multiple (1 to n) of the larger period τ_L, with each integer multiple $[1$ to $(\tau_L/\tau_s)n + (\delta - 1)/\tau_s + 1]$ of the smaller period τ_s.

The shortcut tests take advantage of the fact that the approximate points of interference that would be discovered by the complete tests can be predicted from the ratio of the two periods being tested. If I and J are the multiples, respectively, of two periods τ_1 and τ_2, all integers, then the case for exact coincidence is $I\tau_1 = J\tau_2$. The case for interference within the range of 1 μsec less than δ is $I\tau_1 \pm (\delta - 1) = J\tau_2$, for the J product. Thus, the values of J that represent the boundaries of possible interference for given integer values of I, δ, τ_1, and τ_2 are J = the integer of $[IR - (\delta - 1)/\tau_2]$ for the lower boundary and J = the integer of $[IR + (\delta - 1)/\tau_2 + 1]$ for the upper boundary. The "+1" insures that the upper boundary becomes the next highest integer. The term $(\delta - 1)/\tau_2$ is always a fraction less than 1, since the minimum value of δ must be less than the minimum value of τ in any calculation. Thus, for each 1- to I-integer multiple of one period, only a few (J lower to J upper) integer multiples of the other period need be tested to ensure adequate bracketing of the range of possible interference.

Two versions of the shortcut tests are required. One version is appropriate for Type 1 having unequal coverage when $A > Z$, and also for Type 2 having equal coverage when $A < Z$. The other version tests Type 1 when $A < Z$, and also Type 2 when $A > Z$.

Method I in Detail

1. Let τ equal any period (microseconds) in the range of periods A through Z. Let D equal the value of δ in microseconds. Let n equal the number of consecutive noninterfering interrogations of any pair of nonsynchronous IRF's. For Type 1, let $I = J = n$. Let $G = -1$ if $A > Z$; let $G = +1$ if $A < Z$. Let C be a "correction" applied to a trial period after the first interference to obtain a new trial period that just eliminates that interference.

2. Assume that the first nonsynchronous period of the set is $\tau_1 = A$.

3. Let the first trial value of $\tau_2 = A + GD$. (Equivalent to $A \pm \delta$, but G automatically determines the correct sign.)

4. Test for interference with one of the following tests, as is appropriate.

Complete Test for Type 1. Compare each 1 to I multiple of the newest trial value of τ with every 1 to J multiple of all the values of τ previously found by testing to be nonsynchronous (including $\tau_1 = A$). The absolute value of the difference of all products compared must be $\geq D$ if the newest trial value of τ is to pass the test and become a member of the nonsynchronous set.

Complete Test for Type 2 when $A > Z$. Compare each 1 to J multiple of all the values of τ previously found by test to be nonsynchronous (including $\tau_1 = A$) with every 1 to $[RJ + (D - 1)/\tau_s + 1]$ integer multiple of the newest trial value of τ, where $R = \tau_L/\tau_s$. The absolute value of the difference of all products compared must be $\geq D$ if the newest trial value of τ is to pass the test.

Complete Test for Type 2 when $A < Z$. Compare each 1 to I multiple of the newest trial value of τ with every 1 to $[RI + (D - 1)/\tau_s + 1]$ integer multiple of each of the previously accepted nonsynchronous periods, where $R = \tau_L/\tau_s$. The absolute value of the difference of all products compared must be $\geq D$ if the newest trial value of τ is to pass the test.

Shortcut Test for Type 1 when $A > Z$, and for Type 2 when $A < Z$. Let R equal, in sequence, the ratios of the newest trial value of τ to each of the τ's already found to be nonsynchronous. Let S_1 = the integer of $[RI - (D - 1)/\tau\,(\text{accepted})]$. Let S_3 equal the integer of $[RI + (D - 1)/\tau\,(\text{accepted}) + 1]$. Compare each 1 to I multiple of the newest trial value of τ with just the S_1 to S_3 multiples of all the values of τ previously found by test to be nonsynchronous (including $\tau_1 = A$). The absolute value of the difference of all products compared must be $\geq D$ if the newest trial value of τ is to pass the test and become a member of the nonsynchronous set.

Shortcut Test for Type 1 when $A < Z$, and for Type 2 when $A > Z$. Let R equal the ratios of each of the values of τ (in sequence) already found nonsynchronous to the newest trial value of τ. Let S_1 equal the integer of $[RJ - (D - 1)/\tau\,(\text{trial})]$. Let S_3 equal the integer of $[RJ + (D - 1)/\tau\,(\text{trial}) + 1]$. Compare each 1 to J multiple of all the values of τ previously found nonsynchronous (including $\tau_1 = A$) with just the S_1 to S_3 multiples of the newest trial value of τ. The absolute value of the difference of products compared must be $\geq D$ if the newest trial value of τ is to pass the test and become a member of the nonsynchronous set.

5. If the newest trial value of τ *passes the test*, continue with Step 6. If the newest trial value of τ *fails to pass the test*, skip to Step 9.

6. Record the value of τ and calculate and record its frequency, where IRF in hertz = $10^6/\tau$. (τ is in microseconds.)

7. Calculate the next trial value from Step 8 and test (Step 4).

8. Trial $\tau_{(k+1)} = [\text{Tested-ok } \tau_k + GD]$, where k equals the number of nonsynchronous members of the set found thus far.

9. Let C = the absolute value of $[(Y - X - GD)/I]$; if C has a fractional part, then use the next highest integer. Y is the product from the I (or J) multiple and X the

product from the J (or I) multiple that resulted in the (first) interference encountered by the test.

10. Calculate the next trial value of τ from Step 11 and test (Step 4).

11. Next trial τ = Trial $\tau_{(mrfp)}$ + GC, where mrfp means most recent failure to pass the test.

12. The calculation of the particular nonsynchronous IRF set is finished when the newest trial value of τ becomes $< Z$ if $A > Z$, or $> Z$ if $A < Z$.

Additional Techniques for Applying Method I — Although Method I was developed to insure that no nonsynchronous IRF would be overlooked in a particular set of nonsynchronous IRF's, knowledge gained from its use has suggested some rewarding cut and try techniques. Two helpful techniques have been found thus far for applying Method I when desired values of n and δ, and approximate values of the IRF's, are known in advance, but a straightforward application of Method I has failed to provide the desired IRF's. The first is termed the discard and recalculate technique and the second, the deliberate skip technique. These may also be combined or used in sequence.

The Discard and Recalculate Technique — Assume that a straightforward application of Method I has resulted in the nonsynchronous set of periods, τ_1, τ_2, τ_3, τ_4, and τ_5 in the range A through Z. Discard τ_1 and recalculate starting with τ_2. ($A = \tau_2$.) A new set (underlined subscripts) will be obtained, possibly, $\tau_{\underline{1}}$, $\tau_{\underline{2}}$, $\tau_{\underline{3}}$, $\tau_{\underline{4}}$, $\tau_{\underline{5}}$, and $\tau_{\underline{6}}$. Repeat, discarding τ_2 (= $\tau_{\underline{1}}$), and then recalculate starting with $\tau_{\underline{3}}$. Another new set (subscripts in parentheses) will be obtained, perhaps, $\tau_{(1)}$, $\tau_{(2)}$, $\tau_{(3)}$, and $\tau_{(4)}$. Each new set of nonsynchronous periods will usually offer a different combination of periods. One of the sets thus obtained may prove to be a closer match to the originally desired values.

The Deliberate Skip Technique — Assume again that a straightforward application of Method I has resulted in the nonsynchronous set of periods τ_1, τ_2, τ_3, τ_4, and τ_5, but only the value of τ_1 matches a desired value. A new set (underlined subscripts) is calculated, retaining τ_1 as the starting value. Thus, $\tau_{\underline{1}} = \tau_1$. A deliberate skip is made in selecting the first trial value of $\tau_{\underline{2}}$. The skip can be made to τ_3, τ_4, etc., or to any value of the period beyond τ_2 to Z. Assume that the result of the skip is the nonsynchronous set $\tau_{\underline{1}}$, $\tau_{\underline{2}}$, $\tau_{\underline{3}}$, and $\tau_{\underline{4}}$. If there still are insufficient matches to the desired values, the recalculation is repeated using different skip values until the best available match, for given values of δ and n, to the desired values is obtained. Only a minor modification of Method I is needed to accommodate the deliberate skip technique. In place of Step 3 (or 5c) of Method I, let the first trial value for the next nonsynchronous period of the set equal the value of period to which the skip is made.

The value to which the skip is made can be chosen in two different ways. A quick look to obtain an indication of typical, often useful, results can be obtained from educated guesses based on the previous set. The educated guesses are easy to make after just a little experience. If the estimated skip destination value does not turn out to be nonsynchronous with τ_1, Method I continues until it finds the next available nonsynchronous τ. However, the use of educated guesses may result in locating only a few of the choices for τ_2. The other way results from an abnormal application of Method I that serves to locate additional nonsynchronous values of τ_2 to which a skip could be made. The abnormal application of Method I consists of making the value of n equal to or greater than the next highest integer of n', a condition for which Method I cannot find a second interference-free τ in the normal manner. As abnormal Method I performs its search from A to Z, the interference points that occur may be observed. A record is kept of the period trial τ_2, its multiple, and the multiple of $\tau_1 = A$ whenever, for the interference points, the multiple of an appropriate τ equals or exceeds a certain limit. The record is made for Type 1 when $A > Z$ whenever the interfering multiple of trial τ_2 equals or exceeds the limit [n (normal) + 1]; for Type 1 when $A < Z$ whenever the interfering multiple of $\tau_1 = A$ equals or exceeds the limit [n (normal) + 1]; for Type 2 when $A > Z$ whenever the interfering multiple of $\tau_1 = A$ equals or exceeds the limit [n (normal) + 1], and in addition that of trial τ_2 equals or exceeds the limit of $(S_3 + 1)$; and for Type 2 when $A < Z$ whenever the interfering multiple of trial τ_2 equals or exceeds the limit [n (normal) + 1], and in addition that of $\tau_1 = A$ equals or exceeds the limit $(S_3 + 1)$. For each of the four categories, if the skip from τ_1 is made to any of the recorded trial τ_2 periods, a nonsynchronous pair will be formed with τ_1. After a choice is made from the available nonsynchronous trial τ_2 values predicted by the abnormal application, the original calculation is continued from τ_2 to Z by the normal Method I, if only the one skip from τ_1 to τ_2 is planned.

When a skip is planned from τ_2 or some τ beyond τ_2, more than one abnormal application of Method I is necessary to predict additional nonsynchronous values to which the skip can be made. For example, if a skip is planned from τ_2 to τ_3, an abnormal application of Method I will predict skip destination values that will be nonsynchronous with respect to τ_1 but not necessarily nonsynchronous with respect to τ_2, or vice versa if $A = \tau_2$ for the abnormal calculation. If two abnormal Method I calculations are made, one for $A = \tau_1$ and the other for $A = \tau_2$, then any period appearing on both records will be a nonsynchronous choice for the skip destination of τ_3. In general, as the difference between the values of n and n' becomes larger, more abnormal Method I calculations are needed to predict additional nonsynchronous skip destination values, since more nonsynchronous IRF's per set will be obtainable.

Method II

Method II is useful for determining if an existing pair of frequencies (periods) is nonsynchronous for n or more interrogations. In principle, the determination is accomplished by applying the test portion of Method I to a pair of periods for the required number of interrogations and recording some pertinent information.

Method II in Detail

1. Assume that L successive interrogations recurring with periods τ_1 and τ_2 are to be evaluated for nonsynchronous interference for n or more interrogations, where $n \leq L$. Interference is assumed for the zeroth interrogations. D is the value of δ in microseconds. Let I be the n_ith interrogation of τ_1 and J the n_jth interrogation of τ_2. Let $R = \tau_1/\tau_2$. Let $X_1 = I\tau_1$. Let $X_2 = J\tau_2$. Let $F = 10^6/\tau$, where f is the frequency in hertz and τ is the period in microseconds. Let S_1 be the integer of $[nR - (D - 1)/\tau_2]$. Let S_3 equal the integer of $[nR + (D - 1)/\tau_2 + 1]$.

2. (a) If just Type 2, or both Type 2 and Type 1, coverage is desired, assign the period values so that $\tau_1 > \tau_2$. (The information obtained for Type 2 may also be used to evaluate for Type 1.)

(b) If only the unequal coverage, Type 1, is desired, assign the period values so that $\tau_1 < \tau_2$. (However, if L is made much greater than n, sufficient information is obtained to evaluate for Type 2 also.)

(c) If L is made greater than at least 2n, the repetitive or nonrepetitive nature of the pattern of nonsynchronous or synchronous interference may be observed from either (a) or (b) above.

3. Apply the following test: Compare each 1 to L multiple (I = 1 to L) of τ_1 with the J = S_1 to S_3 multiples of τ_2. Whenever the absolute value of the difference between products, i.e., $(X_1 - X_2)$, is less than D, record the minimum necessary information, the values of I, J, and the algebraic sign of $(X_1 - X_2)$. Also record, if desired, X_1, X_2, $(X_1 - X_2)$, and the number of noninterfering interrogations between the interfering interrogations for both periods.

4. Determine the nature of any interference found as follows: If the algebraic sign of $(X_1 - X_2)$ is *negative*, then the Ith multiple of τ_1 is said to interfere with the Jth multiple of τ_2. If the algebraic sign of $(X_1 - X_2)$ is *positive*, the Jth multiple of τ_2 is said to interfere with the Ith multiple of τ_1. (In actual system operation the stated one-way direction of interference will reverse for some of the other phase-angle relations.) If the value of $(X_1 - X_2) = 0$, then acceptance of neither, either, or both interrogations recurring with τ_1 and τ_2 by a transponder in a beacon system would depend on the actual transponder circuitry.

COMPUTER PROGRAMS

The appendixes contain computer programs written in both the Basic and the Fortran IV languages. All use the shortcut version of the test procedures. Appendix A contains the Method I programs and Appendix B those for Method II. The programs were first developed and tested in Basic language using an available time sharing terminal connected by phone line to an outside commercial computer. Later the programs were converted to Fortran IV language for use on the Laboratory's CDC 3800 computer. The improved efficiency of the Fortran IV programs, due to the use wherever possible of single calculations of quantities appearing more than once, could also be incorporated into the Basic language programs. Although not required, a statement number was assigned to every Fortran IV statement to assist comparison with the Basic.

The Method I programs include two test sections, so that either Type I or Type 2 may be obtained from a single program. The input value of E is assigned a 2 when Type 1 is desired, or a 4 when Type 2 is desired. Modifications for the deliberate skip technique are listed at the end of each program. The first modification allows a skip immediately following A, the first nonsynchronous period. The second modification allows a skip at any time after the first two nonsynchronous periods have been calculated. As an alternative to removal from the program, the modifications can be circumvented, or the second modification delayed, by supplying the unmodified value when the program calls for an input. In addition, alternate internal sequences of the tests for Method I are listed. Although either sequence gives the same end results, it is conceivable that one would be preferred over the other in some special situation of analysis. The sequence arbitrarily placed in the program completes the product comparison for all the required multiples of the trial period with all the required multiples of the first period previously accepted as nonsynchronous before proceeding to the next period previously accepted as nonsynchronous. The alternate sequence completes the product comparison for the required multiples of all periods previously accepted as nonsynchronous with the first required multiple(s) of the trial period before proceeding to the next multiple(s) of the trial period.

The criteria for obtaining Type 1 or Type 2 for Method II has been discussed previously in the detailed description of Method II.

No claim is made that the computer programs are optimally coded. However, the shortcut test versions are thought to be at least reasonably efficient for the languages employed. The serial nature of the calculations makes it essential that no interference point be overlooked along the way, since all succeeding calculations will then be invalidated for the particular A to Z search of Method I. An interference point overlooked by

Method II results in an incomplete, and therefore incorrect, analysis. Thus, preference was given to a slight amount of overtesting for the shortcut test, even though it appears that the shortcut test might be shortened further, at least for most combinations of parameter values.

RESULTS

Trial use of Method I has resulted in data that give a good indication of the effects of varying various parameters. In all cases the numerical subscripts of τ and f indicate the order in which they were found, regardless of the numerical direction of search through a band of periods.

Table 1 shows a set of six Type 1 (unequal coverage) nonsynchronous frequencies obtained using Method I for $n = 20$, $\delta = 300\ \mu$sec, $A = 10{,}000\ \mu$sec, and $Z = 2500\ \mu$sec. The $n\tau$ products are listed for each 1 to n multiple of the τ's. Table 2 shows a satisfying direct proof that this set of frequencies is truly nonsynchronous as defined for Type 1. The direct proof for Type 1 consists of placing all of the $n\tau$ products in ascending or descending order. The absolute value of the difference between successive product pairs, taken in sequence, is then calculated. If every product is separated from its neighbors by at least the value of δ, then the set of frequencies (periods) has been proven to be a nonsynchronous set.

Table 1
A Set of Six Nonsynchronous IRF's Obtained by Method I
(Type 1, Unequal Coverage)*

Number of Interrogations	f_1	f_2	f_3	f_4	f_5	f_6
	100.000	103.093	106.451	110.705	114.943	169.176
n	$n\tau_1$	$n\tau_2$	$n\tau_3$	$n\tau_4$	$n\tau_5$	$n\tau_6$
1	10000	9700	9394	9033	8700	5911
2	20000	19400	18788	18066	17400	11822
3	30000	29100	28182	27099	26100	17733
4	40000	38800	37576	36132	34800	23644
5	50000	48500	46970	45165	43500	29555
6	60000	58200	56364	54198	52200	35466
7	70000	67900	65758	63231	60900	41377
8	80000	77600	75152	72264	69600	47288
9	90000	87300	84546	81297	78300	53199
10	100000	97000	93940	90330	87000	59110
11	110000	106700	103334	99363	95700	65021
12	120000	116400	112728	108396	104400	70932
13	130000	126100	122122	117429	113100	76843
14	140000	135800	131516	126462	121800	82754
15	150000	145500	140910	135495	130500	88665
16	160000	155200	150304	144528	139200	94576
17	170000	164900	159698	153561	147900	100487
18	180000	174600	169092	162594	156600	106398
19	190000	184300	178486	171627	165300	112309
20	200000	194000	187880	180660	174000	118220

*$\delta = 300\ \mu$sec, $n = 20$, $A = 10{,}000\ \mu$sec, $Z = 2500\ \mu$sec, f = frequency in Hz, τ = period in μsec.

Table 2
Differences Between the $n\tau$ Products of Table 1 When Arranged in Order*

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
6	1	5911		5	4	34800		5	7	60900	
			2789				666				2331
5	1	8700		6	6	35466		4	7	63231	
			333				666				1790
4	1	9033		4	4	36132		6	11	65021	
			361				1444				737
3	1	9394		3	4	37576		3	7	65758	
			306				1224				2142
2	1	9700		2	4	38800		2	7	67900	
			300				1200				1700
1	1	10000		1	4	40000		5	8	69600	
			1822				1377				400
6	2	11822		6	7	41377		1	7	70000	
			5578				2123				932
5	2	17400		5	5	43500		6	12	70932	
			333				1665				1332
6	3	17733		4	5	45165		4	8	72264	
			333				1805				2888
4	2	18066		3	5	46970		3	8	75152	
			722				318				1691
3	2	18788		6	8	47288		6	13	76843	
			612				1212				757
2	2	19400		2	5	48500		2	8	77600	
			600				1500				700
1	2	20000		1	5	50000		5	9	78300	
			3644				2200				1700
6	4	23644		5	6	52200		1	8	80000	
			2456				999				1297
5	3	26100		6	9	53199		4	9	81297	
			999				999				1457
4	3	27099		4	6	54198		6	14	82754	
			1083				2166				1792
3	3	28182		3	6	56364		3	9	84546	
			918				1836				2454
2	3	29100		2	6	58200		5	10	87000	
			455				910				300
6	5	29555		6	10	59110		2	9	87300	
			445				890				1365
1	3	30000		1	6	60000		6	15	88665	
			4800				900				1335

*Proof that all products are separated by at least the value of δ. (Six Type 1 nonsynchronous IRF's; δ = 300 μsec, n = 20, A = 10000 μsec, Z = 2500 μsec.)

(Table continues)

Table 2 (Continued)

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
1	9	90000		6	20	118220		2	16	155200	
			330				1780				1400
4	10	90330		1	12	120000		5	18	156600	
			3610				1800				3098
3	10	93940		5	14	121800		3	17	159698	
			636				322				302
6	16	94576		3	13	122122		1	16	160000	
			1124				3978				2594
5	11	95700		2	13	126100		4	18	162594	
			1300				362				2306
2	10	97000		4	14	126462		2	17	164900	
			2363				3538				400
4	11	99363		1	13	130000		5	19	165300	
			637				500				3792
1	10	100000		5	15	130500		3	18	169092	
			487				1016				908
6	17	100487		3	14	131516		1	17	170000	
			2847				3979				1627
3	11	103334		4	15	135495		4	19	171627	
			1066				305				2373
5	12	104400		2	14	135800		5	20	174000	
			1998				3400				600
6	18	106398		5	16	139200		2	18	174600	
			302				800				3886
2	11	106700		1	14	140000		3	19	178486	
			1696				910				1514
4	12	108396		3	15	140910		1	18	180000	
			1604				3618				660
1	11	110000		4	16	144528		4	20	180660	
			2309				972				3640
6	19	112309		2	15	145500		2	19	184300	
			419				2400				3580
3	12	112728		5	17	147900		3	20	187880	
			372				2100				2120
5	13	113100		1	15	150000		1	19	190000	
			3300				304				4000
2	12	116400		1	16	150304		2	20	194000	
			1029				3257				6000
4	13	117429		4	17	153561		1	20	200000	
			791				1639				

If the same range of periods is searched in the opposite numerical direction, a different set of nonsynchronous periods is obtained for the same values of δ and n, as may be seen by comparing Table 3 with Table 1. For Table 3, A = 2500 μsec, Z = 10,000 μsec, δ = 300 μsec, and n = 20. The direct proof for Table 3 is contained in Table 4.

Once the computer programs had been developed in the course of this investigation, the direct proofs were no longer essential, since reliable tests were built into the programs. However, it is well to bear in mind that the direct proofs are always available to double check for any purpose and might be reassuring if finalizing a system design. Numerical sorting and ordering routines are available for most digital computers, though computer cost is proportional to at least the 1.5 power of the number of items to be placed in order.

Table 3
A Different Set of Six Nonsynchronous IRF's Resulting From the Opposite Direction of Numerical Search Compared to Table 1 (Method I, Type 1, Unequal Coverage)*

Number of Interrogations	f_1	f_2	f_3	f_4	f_5	f_6
	400.000	152.439	139.159	128.041	110.375	103.263
n	$n\tau_1$	$n\tau_2$	$n\tau_3$	$n\tau_4$	$n\tau_5$	$n\tau_6$
1	2500	6560	7186	7810	9060	9684
2	5000	13120	14372	15620	18120	19368
3	7500	19680	21558	23430	27180	29052
4	10000	26240	28744	31240	36240	38736
5	12500	32800	35930	39050	45300	48420
6	15000	39360	43116	46860	54360	58104
7	17500	45920	50302	54670	63420	67788
8	20000	52480	57488	62480	72480	77472
9	22500	59040	64674	70290	81540	87156
10	25000	65600	71860	78100	90600	96840
11	27500	72160	79046	85910	99660	106524
12	30000	78720	86232	93720	108720	116208
13	32500	85280	93418	101530	117780	125892
14	35000	91840	100604	109340	126840	135576
15	37500	98400	107790	117150	135900	145260
16	40000	104960	114976	124960	144960	154944
17	42500	111520	122162	132770	154020	164628
18	45000	118080	129348	140580	163080	174312
19	47500	124640	136534	148390	172140	183996
20	50000	131200	143720	156200	181200	193680

*δ = 300 μsec, n = 20, A = 2500 μsec, Z = 10,000 μsec, f = frequency in Hz, τ = period in μsec.

Table 4
Differences Between the $n\tau$ Products of Table 3 When Arranged in Order*

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
1	1	2500		1	9	22500		1	17	42500	
			2500				930				616
1	2	5000		4	3	23430		3	6	43116	
			1560				1570				1884
2	1	6560		1	10	25000		1	18	45000	
			626				1240				300
3	1	7186		2	4	26240		5	5	45300	
			314				940				620
1	3	7500		5	3	27180		2	7	45920	
			310				320				940
4	1	7810		1	11	27500		4	6	46860	
			1250				1244				640
5	1	9060		3	4	28744		1	19	47500	
			624				308				920
6	1	9684		6	3	29052		6	5	48420	
			316				948				1580
1	4	10000		1	12	30000		1	20	50000	
			2500				1240				302
1	5	12500		4	4	31240		3	7	50302	
			620				1260				2178
2	2	13120		1	13	32500		2	8	52480	
			1252				300				1880
3	2	14372		2	5	32800		5	6	54360	
			628				2200				310
1	6	15000		1	14	35000		4	7	54670	
			620				930				2818
4	2	15620		3	5	35930		3	8	57488	
			1880				310				616
1	7	17500		5	4	36240		6	6	58104	
			620				1260				936
5	2	18120		1	15	37500		2	9	59040	
			1248				1236				3440
6	2	19368		6	4	38736		4	8	62480	
			312				314				940
2	3	19680		4	5	39050		5	7	63420	
			320				310				1254
1	8	20000		2	6	39360		3	9	64674	
			1558				640				926
3	3	21558		1	16	40000		2	10	65600	
			942				2500				2188

*Proof that all products are separated by at least the value of δ. (Six Type 1 nonsynchronous IRF's: $\delta = 300\ \mu$sec, n = 20, A = 2500 μsec and Z = 10,000 μsec.)

(Table continues)

Table 4 (Continued)

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
6	7	67788	2502	5	11	99660	944	2	20	131200	1570
4	9	70290	1570	3	14	100604	926	4	17	132770	2806
3	10	71860	300	4	13	101530	3430	6	14	135576	324
2	11	72160	320	2	16	104960	1564	5	15	135900	634
5	8	72480	4992	6	11	106524	1266	3	19	136534	4046
6	8	77472	628	3	15	107790	930	4	18	140580	3140
4	10	78100	620	5	12	108720	620	3	20	143720	1240
2	12	78720	326	4	14	109340	2180	5	16	144960	300
3	11	79046	2494	2	17	111520	3456	6	15	145260	3130
5	9	81540	3740	3	16	114976	1232	4	19	148390	5630
2	13	85280	630	6	12	116208	942	5	17	154020	924
4	11	85910	322	4	15	117150	630	6	16	154944	1256
3	12	86232	924	5	13	117780	300	4	20	156200	6880
6	9	87156	3444	2	18	118080	4082	5	18	163080	1548
5	10	90600	1240	3	17	122162	2478	6	17	164628	7512
2	14	91840	1578	2	19	124640	320	5	19	172140	2172
3	13	93418	302	4	16	124960	932	6	18	174312	6888
4	12	93720	3120	6	13	125892	948	5	20	181200	2796
6	10	96840	1560	5	14	126840	2508	6	19	183996	9684
2	15	98400	1260	3	18	129348	1852	6	20	193680	

Table 5 indicates that a lower value of δ results in a greater number of nonsynchronous IRF's. The somewhat greater A to Z range searched adds further to the number of nonsynchronous IRF's. The parameters are $\delta = 80$ μsec, $n = 20$, $A = 2222$ μsec, and $Z = 20{,}300$ μsec. The calculation is Type 1.

Nonsynchronous sets for three different values of δ are shown in Table 6, all Type 1. For each set, $n = 20$, $A = 10{,}000$ μsec, and $Z = 2500$ μsec. The start of a trend toward a greater number of nonsynchronous IRF's for lower values of δ is visible. Table 7 shows the results of further variations of δ while also varying n, all for Type 1 and $A = 4000$ μsec and $Z = 2100$ μsec. A reduction of either n or δ tends to result in the availability of a larger number of nonsynchronous IRF's per set. The irregularities and inconsistencies in the general trend seem to be typical behavior. In fact, it appears to be difficult to predict in advance of a calculation the *precise* number of resultant nonsynchronous IRF's.

Table 5
A Set of Nonsynchronous IRF's Obtained for δ = 80 μsec
(Method I, Type 1, Unequal Coverage)*

Number	τ (μsec)	Frequency (Hz)	Number	τ (μsec)	Frequency (Hz)
1	2222	450.045	18	11712	85.383
2	2302	434.405	19	11904	84.005
3	2400	416.667	20	12108	82.590
4	2578	387.898	21	12620	79.239
5	2844	351.617	22	13679	73.105
6	3383	295.596	23	13960	71.633
7	3525	283.688	24	14502	68.956
8	4177	239.406	25	14709	67.986
9	4951	201.979	26	16027	62.395
10	5247	190.585	27	16253	61.527
11	5455	183.318	28	16530	60.496
12	6485	154.202	29	17146	58.323
13	7535	132.714	30	17517	57.087
14	8685	115.141	31	18266	54.747
15	8968	111.508	32	19654	50.880
16	9337	107.101	33	20120	49.702
17	9507	105.186			

*n = 20, A = 2222 μsec, and Z = 20,300 μsec.

Table 6
Nonsynchronous IRF Sets Obtained for Three Different Values of δ
(Method I, Type 1, Unequal Coverage)*

δ	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
300	100.000	103.093	106.451	110.705	114.943	169.176	—	—
280	100.000	102.881	106.440	110.461	119.246	129.550	—	—
260	100.000	102.669	105.708	109.709	113.533	117.827	135.336	—

*n = 20, A = 10,000 μsec, and Z = 2500 μsec, or frequency range 100 to 400 Hz; f = frequency.

Table 7
Nonsynchronous IRF Sets Obtained for Various Values of n and δ
(Method I, Type 1, Unequal Coverage)*

δ (μsec)	n	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
270	14	250.000	—	—	—	—	—	—	—
	13	250.000	268.097	—	—	—	—	—	—
	12	250.000	268.097	—	—	—	—	—	—
	11	250.000	268.097	—	—	—	—	—	—
	10	250.000	268.097	294.985	460.829	—	—	—	—
	9	250.000	268.097	294.985	460.829	—	—	—	—
	8	250.000	268.097	294.985	460.829	—	—	—	—
	7	250.000	268.097	294.985	460.829	—	—	—	—
	6	250.000	268.097	289.017	341.297	—	—	—	—
	5	250.000	268.097	289.017	317.864	347.705	—	—	—
200	12	250.000	263.158	279.330	303.030	—	—	—	—
	11	250.000	263.158	279.330	303.030	—	—	—	—
	10	250.000	263.158	279.330	303.030	384.615	—	—	—
	9	250.000	263.158	277.778	294.118	360.101	—	—	—
	8	250.000	263.158	277.778	294.118	320.000	357.143	454.545	—
130	20	250.000	258.398	268.528	319.591	—	—	—	—
	19	250.000	258.398	268.528	319.591	—	—	—	—
	18	250.000	258.398	268.528	279.018	367.377	410.509	—	—
	17	250.000	258.398	268.528	279.018	367.377	410.509	—	—
	16	250.000	258.398	268.528	279.018	296.736	367.377	410.509	—
	15	250.000	258.398	268.528	279.018	296.736	364.299	—	—
	14	250.000	258.398	267.380	279.018	303.214	317.058	338.409	410.172
	13	250.000	258.398	267.380	278.784	307.787	328.084	—	—
	12	250.000	258.398	267.380	278.784	289.268	307.787	328.084	—

*A = 4000 μsec and Z = 2105 μsec, or frequency range 250 to 475 Hz; f = frequency.

An upper limit n' exists for n. Given a value of δ, the value of n for which no nonsynchronous IRF's can be found within a given frequency band is $n' = A/\delta$ when $A > Z$, or $n' = Z/\delta$ when $Z > A$. The maximum integer value of n for which at least one pair of IRF's can be anticipated is n_{max} = the integer of $(n' - 1)$. Figure 4 illustrates the relation of n' to the frequency (period) band for several different values of δ, where it is assumed that a calculation could start at various points along the period scale. Thus, as would be expected from the definitions of n', it can be seen from Fig. 4 that when $A > Z$, larger values of δ and smaller values of A both result in smaller values of n_{max}. When $A < Z$, larger values of δ and smaller values of Z both result in

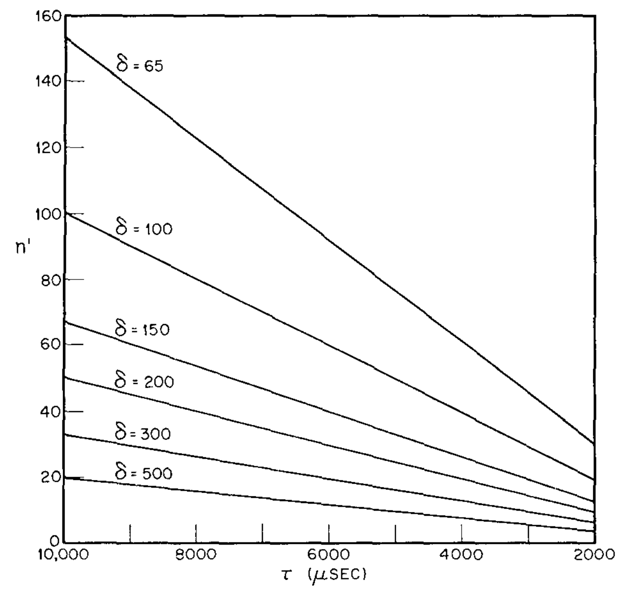

Fig. 4 - n' vs $\tau \cdot [n_{max} = (n' - 1)]$

smaller values of n_{max}. Thus, for given values of δ, any nonsynchronous frequencies found in a band which is higher in the frequency spectrum will be found nonsynchronous for fewer n_{max} interrogations than could be found in a band occupying a lower portion of the frequency spectrum.

Table 8 illustrates the greater availability of nonsynchronous IRF's of Type 1, having unequal coverage, compared to those of Type 2 having equal coverage, when $\delta = 100$ μsec, $n = 20$, $A = 2500$ μsec, and $Z = 10{,}100$ μsec. For the particular parameters, about 2.5 times as many nonsynchronous IRF's of Type 1 are obtained than those of Type 2. However, the exact ratio of Type 1 to Type 2 can be expected to vary as the calculation parameters are varied.

The direct proof for Type 2 is similar to the direct proof for Type 1 in that identified products of multiples of periods are placed in ascending or descending numerical order to prove that all neighboring products differ by at least the value of δ. However, for Type 2, the highest multiples of the periods will be different for each pair of periods. Table 9 gives the number of interrogations needed for each combination of periods of the six Type 2 nonsynchronous IRF's of Table 8. The number of interrogations in the first group of five combinations is seen to overlap the numbers of interrogations needed for the remaining combinations. Thus, an ordered list of all of the products of the first five combinations will include all products from the remaining combinations. Obviously, the Type 2 list (in this example, obtained from the first five combinations) will contain several neighboring pairs that differ by values less than δ. An evaluation (with the aid of Table 9) must be made to determine whether the interference observed between any pair occurs for multiples within or beyond the range of values required for the particular pair. Table 10 shows the direct proof for the six Type 2 nonsynchronous IRF's of Table 8. In no instance does a difference less than δ occur that is not the result of multiples that are beyond the values required for the particular pair.

Table 8
The Relative Availability of Type 1 and Type 2 Nonsynchronous IRF's for the Parameters Listed*

Type 1, Unequal Coverage			Type 2, Equal Coverage		
Number	τ (μsec)	Frequency (Hz)	Number	τ (μsec)	Frequency (Hz)
1	2500	400.000	1	2500	400.000
2	2600	384.615	2	2600	384.615
3	2700	370.370	3	2700	370.370
4	2825	353.982	4	2825	353.982
5	3557	281.136	5	4300	232.558
6	4300	232.558	6	9700	103.093
7	4406	226.963			
8	5800	172.414			
9	6194	161.447			
10	6800	147.059			
11	6900	144.928			
12	7300	136.986			
13	7400	135.135			
14	8200	121.951			
15	9584	104.341			
16	10100	99.010			

*δ = 100 μsec, n = 20, A = 2500 μsec, and Z = 10100 μsec.

Table 9
The Number of Products Required for Each Pair of Periods to Establish the Direct Proof for the Six Type 2, Nonsynchronous IRF's of Table 8

Products, $I\tau_u$		Products, $J\tau_v$
(1 to 20) τ_6	vs	(1 to 78) τ_1 (1 to 75) τ_2 (1 to 72) τ_3 (1 to 69) τ_4 (1 to 46) τ_5
(1 to 20) τ_5	vs	(1 to 35) τ_1 (1 to 34) τ_2 (1 to 32) τ_3 (1 to 31) τ_4
(1 to 20) τ_4	vs	(1 to 23) τ_1 (1 to 22) τ_2 (1 to 21) τ_3
(1 to 20) τ_3	vs	(1 to 22) τ_1 (1 to 21) τ_2
(1 to 20) τ_2	vs	(1 to 21) τ_1

Table 10
The Direct Proof for the Six Type 2 Nonsynchronous IRF's of Table 8. Proof That Any Ordered Product Differences That are Less Than δ Occur Only Beyond the Required Range of Multiples.*

x	n	$n\tau_x$	Difference
5	46	197800	
			2800
2	75	195000	
Beyond			0
1	78	195000	
Beyond			75
4	69	194925	
			525
3	72	194400	
			400
6	20	194000	
			500
5	45	193500	
			1000
1	77	192500	
			100
2	74	192400	
			300
4	68	192100	
			400
3	71	191700	
			1700
1	76	190000	
			200
2	73	189800	
			525
4	67	189275	
Beyond			75
5	44	189200	
			200
3	70	189000	
			1500
1	75	187500	
			300
2	72	187200	
			750
4	66	186450	
			150
3	69	186300	
			1300

x	n	$n\tau_x$	Difference
1	74	185000	
			100
5	43	184900	
			300
2	71	184600	
			300
6	19	184300	
			675
4	65	183625	
Peyond			25
3	68	183600	
			1100
1	73	182500	
			500
2	70	182000	
			1100
3	67	180900	
			100
4	64	180800	
			200
5	42	180600	
			600
1	72	180000	
			600
2	69	179400	
			1200
3	66	178200	
			225
4	63	177975	
			475
1	71	177500	
			700
2	68	176800	
			500
5	41	176300	
			800
3	65	175500	
			350
4	62	175150	
			150

x	n	$n\tau_x$	Difference
1	70	175000	
			400
6	18	174600	
			400
2	67	174200	
			1400
3	64	172800	
			300
1	69	172500	
			175
4	61	172325	
			325
5	40	172000	
			400
2	66	171600	
			1500
3	63	170100	
			100
1	68	170000	
			500
4	60	169500	
			500
2	65	169000	
			1300
5	39	167700	
			200
1	67	167500	
			100
3	62	167400	
			725
4	59	166675	
			275
2	64	166400	
			1400
1	66	165000	
			100
6	17	164900	
			200
3	61	164700	
			850

*δ = 100 μsec, n = 20, A = 2500 μsec, and Z = 10,100 μsec.

(Table continues)

Table 10 (Continued)

x	n	$n\tau_x$	Difference
4	58	163850	
Beyond			50
2	63	163800	
			400
5	38	163400	
			900
1	65	162500	
			500
3	60	162000	
			800
2	62	161200	
			175
4	57	161025	
			1025
1	64	160000	
			700
3	59	159300	
			200
5	37	159100	
			500
2	61	158600	
			400
4	56	158200	
			700
1	63	157500	
			900
3	58	156600	
			600
2	60	156000	
			625
4	55	155375	
			175
6	16	155200	
			200
1	62	155000	
			200
5	36	154800	
			900
3	57	153900	
			500

x	n	$n\tau_x$	Difference
2	59	153400	
			850
4	54	152550	
Beyond			50
1	61	152500	
			1300
3	56	151200	
			400
2	58	150800	
			300
5	35	150500	
			500
1	60	150000	
			275
4	53	149725	
			1225
3	55	148500	
			300
2	57	148200	
			700
1	59	147500	
			600
4	52	146900	
			700
5	34	146200	
			400
3	54	145800	
			200
2	56	145600	
			100
6	15	145500	
			500
1	58	145000	
			925
4	51	144075	
			975
3	53	143100	
			100
2	55	143000	
			500

x	n	$n\tau_x$	Difference
1	57	142500	
			600
5	33	141900	
			650
4	50	141250	
			850
3	52	140400	
Beyond			0
2	54	140400	
			400
1	56	140000	
			1575
4	49	138425	
			625
2	53	137800	
			100
3	51	137700	
			100
5	32	137600	
			100
1	55	137500	
			1700
6	14	135800	
			200
4	48	135600	
			400
2	52	135200	
			200
1	54	135000	
Beyond			0
3	50	135000	
			1700
5	31	133300	
			525
4	47	132775	
			175
2	51	132600	
			100
1	53	132500	
			200

(Table continues)

Table 10 (Continued)

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
3	49	132300		5	28	120400		1	44	110000	
			2300				400				800
1	52	130000		1	48	120000		2	42	109200	
Beyond			0				400				1200
2	50	130000		2	46	119600		3	40	108000	
Beyond			50				800				500
4	46	129950		3	44	118800		1	43	107500	
			350				150	Beyond			0
3	48	129600		4	42	118650		5	25	107500	
			600				1150				150
5	30	129000		1	47	117500		4	38	107350	
			1500				500				650
1	51	127500		2	45	117000		6	11	106700	
			100				600				100
2	49	127400		6	12	116400		2	41	106600	
			275				300				1300
4	45	127125		3	43	116100		3	39	105300	
			225	Beyond			0				300
3	47	126900		5	27	116100		1	42	105000	
			800				275				475
6	13	126100		4	41	115825		4	37	104525	
			1100				825				525
1	50	125000		1	46	115000		2	40	104000	
			200				600				800
2	48	124800		2	44	114400		5	24	103200	
			100				1000				600
5	29	124700		3	42	113400		3	38	102600	
			400				400				100
4	44	124300		4	40	113000		1	41	102500	
			100				500				800
3	46	124200		1	45	112500		4	36	101700	
			1700				700				300
1	49	122500		2	43	111800		2	39	101400	
			300	Beyond			0				1400
2	47	122200		5	26	111800		1	40	100000	
			700				1100				100
3	45	121500		3	41	110700		3	37	99900	
Beyond			25				525				1000
4	43	121475		4	39	110175		5	23	98900	
			1075				175	Beyond			25

(Table continues)

Table 10 (Continued)

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
4	35	98875		4	31	87575		1	31	77500	
Beyond			75	Beyond			75				100
2	38	98800		1	35	87500		5	18	77400	
			1300				200				1125
1	39	97500		6	9	87300		4	27	76275	
			300				900				675
3	36	97200		3	32	86400		3	28	75600	
			200				400				200
6	10	97000		5	20	86000		2	29	75400	
			800				200				400
2	37	96200		2	33	85800		1	30	75000	
			150				800				1550
4	34	96050		1	34	85000		4	26	73450	
			1050				250				350
1	38	95000		4	30	84750		5	17	73100	
			400				1050				200
5	22	94600		3	31	83700		3	27	72900	
			100				500				100
3	35	94500		2	32	83200		2	28	72800	
			900				700				300
2	36	93600		1	33	82500		1	29	72500	
			375				575				1875
4	33	93225		4	29	81925		4	25	70625	
			725				225				425
1	37	92500		5	19	81700		2	27	70200	
			700				700	Beyond			0
3	34	91800		3	30	81000		3	26	70200	
			800				400				200
2	35	91000		2	31	80600		1	28	70000	
			600				600				1200
4	32	90400		1	32	80000		5	16	68800	
			100				900				900
5	21	90300		4	28	79100		6	7	67900	
			300				800				100
1	36	90000		3	29	78300		4	24	67800	
			900				300				200
3	33	89100		2	30	78000		2	26	67600	
			700				400				100
2	34	88400		6	8	77600		1	27	67500	
			825				100	Beyond			0

(Table continues)

Table 10 (Continued)

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
3	25	67500		5	13	55900		1	18	45000	
			2500				900				800
1	26	65000		1	22	55000		2	17	44200	
Beyond			0				400				1000
2	25	65000		2	21	54600		3	16	43200	
Beyond			25				600				200
4	23	64975		3	20	54000		5	10	43000	
			175				325				500
3	24	64800		4	19	53675		1	17	42500	
			300				1175				125
5	15	64500		1	21	52500		4	15	42375	
			2000				500				775
1	25	62500		2	20	52000		2	16	41600	
			100				400				1100
2	24	62400		5	12	51600		3	15	40500	
			250				300				500
4	22	62150		3	19	51300		1	16	40000	
Beyond			50				450				450
3	23	62100		4	18	50850		4	14	39550	
			1900				850				550
5	14	60200		1	20	50000		2	15	39000	
			200				600				200
1	24	60000		2	19	49400		6	4	38800	
			200				800				100
2	23	59800		3	18	48600		5	9	38700	
			400				100				900
3	22	59400		6	5	48500		3	14	37800	
Beyond			75				475				300
4	21	59325		4	17	48025		1	15	37500	
			1125				525				775
6	6	58200		1	19	47500		4	13	36725	
			700				200				325
1	23	57500		5	11	47300		2	14	36400	
			300				500				1300
2	22	57200		2	18	46800		3	13	35100	
			500				900				100
3	21	56700		3	17	45900		1	14	35000	
			200				700				600
4	20	56500		4	16	45200		5	8	34400	
			600				200				500

(Table continues)

Table 10 (Continued)

x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference	x	n	$n\tau_x$	Difference
4	12	33900		4	8	22600		1	5	12500	
			100				100				1200
2	13	33800		1	9	22500		4	4	11300	
			1300				900				500
1	13	32500		3	8	21600		3	4	10800	
			100				100				400
3	12	32400		5	5	21500		2	4	10400	
			1200				700				400
2	12	31200		2	8	20800		1	4	10000	
			125				800				300
4	11	31075		1	8	20000		6	1	9700	
			975				225				1100
5	7	30100		4	7	19775		5	2	8600	
			100				375				125
1	12	30000		6	2	19400		4	3	8475	
			300				500				375
3	11	29700		3	7	18900		3	3	8100	
			600				700				300
6	3	29100		2	7	18200		2	3	7800	
			500				700				300
2	11	28600		1	7	17500		1	3	7500	
			350				300				1850
4	10	28250		5	4	17200		4	2	5650	
			750				250				250
1	11	27500		4	6	16950		3	2	5400	
			500				750				200
3	10	27000		3	6	16200		2	2	5200	
			1000				600				200
2	10	26000		2	6	15600		1	2	5000	
			200				600				700
5	6	25800		1	6	15000		5	1	4300	
			375				875				1475
4	9	25425		4	5	14125		4	1	2825	
			425				625				125
1	10	25000		3	5	13500		3	1	2700	
			700				500				100
3	9	24300		2	5	13000		2	1	2600	
			900				100				100
2	9	23400		5	3	12900		1	1	2500	
			800				400				

The results of an example of the discard and recalculate technique are shown in Table 11. A straightforward application of Method I for Type 2 resulted in Set (a) of nonsynchronous IRF's, where δ = 100 μsec, n = 20, A = 2500 μsec, and Z = 10,100 μsec. The first nonsynchronous IRF 2500 μsec, 400.000 Hz) was discarded. A new set was calculated starting with the second nonsynchronous IRF of the original Set (a). Thus, for the recalculation resulting in the nonsynchronous Set (b), the value of A was changed to 2600 μsec. In this particular example, Set (b) has 7 nonsynchronous IRF's compared to only 6 in Set (a). Often, however, a fewer number would be found from the first discard and recalculation. Succeeding discards and recalculations (repeating the technique) usually result in a fewer than original number of nonsynchronous IRF's. Two IRF's of Set (b) remained identical with two of Set (a), and the others are different. Thus, the discard and recalculate technique of applying Method I has made available an alternate set of IRF's which might be more appropriate in a given instance. This technique amounts to a shortening of the A to Z range originally considered. However, it provides a greater flexibility of choice compared to starting with the shortened range in the first place.

Table 11
The Results of an Example of the Discard and Recalculate Technique of Applying Method I (Type 2, Equal Coverage)*

Nonsynchronous IRF Set (a) A = 2500 μsec			Nonsynchronous IRF Set (b) A = 2600 μsec		
Number	τ (μsec)	Frequency (Hz)	Number	τ (μsec)	Frequency (Hz)
1	2500	400.000	1	2600	384.615
2	2600	384.615	2	2700	370.370
3	2700	370.370	3	2808	356.125
4	2825	353.982	4	3051	327.761
5	4300	232.558	5	5512	181.422
6	9700	103.093	6	7072	141.403
			7	10043	99.572

*δ = 100 μsec, n = 20, Z = 10,100 μsec.

Table 12 shows the results of two examples of the deliberate skip technique of applying Method I, both for Type 1. In the first example, a straightforward application of Method I gives the nonsynchronous IRF Set (a) for the conditions δ = 100 μsec, n = 20, A = 3333 μsec, and Z = 2200 μsec. The calculation for Set (b) started from the same value of A, but a skip was made to 2913 μsec for the first trial value of the second nonsynchronous IRF. In other words, it was decided to retain only the first nonsynchronous IRF from Set (a) and, after skipping, to continue the calculation in a straightforward manner to see if a set including smaller periods can be found. Set (b) resulted. Additional skips from number 1 of Set (a) to 2899 and 2860 μsec resulted in Sets (c) and (d), respectively. Thus the skips to trial τ_2 for Sets (b), (c) and (d) were actually obtained from educated guesses. Only for Set (c) did the estimated value itself prove to be nonsynchronous. An abnormal application of Method I turned up 70 nonsynchronous τ_2 skip destination values, including of course, the nonsynchronous values already listed in the four sets. Of the 70, 24 fell between 3233 and 2903 μsec, three fell between 2903 and 2842 μsec, and the remainder ranged from 2828 to 2258 μsec. τ_2 skip destination values somewhat less than 2828 μsec could be expected to result in only three or two nonsynchronous IRF's per set for the A to Z range considered. In any event, the sample sets shown are only a few of those available when skipping from τ_1 to τ_2. One of the available sets could most nearly match desired values in a given instance.

Table 12
Results of Two Examples of the Deliberate Skip Technique of Applying Method I (Type 1, Unequal Coverage)*

Example 1†

Nonsynchronous Set (a) Straightforward			Nonsynchronous Set (b) Skipped to 2913 from No. 1			Nonsynchronous Set (c) Skipped to 2899 from No. 1			Nonsynchronous Set (d) Skipped to 2860 from No. 1		
No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)
1	3333	300.030	1	3333	300.030	1	3333	300.030	1	3333	300.030
2	3233	309.310	2	2903	344.471	2	2899	344.947	2	2842	351.865
3	3131	319.387	3	2801	357.015	3	2799	357.270	3	2738	365.230
4	3010	332.226	4	2688	372.024	4	2436	410.509	4	2299	434.972
5	2899	344.947		none			none			none	

Example 2‡

Nonsynchronous Set (a) Straightforward			Nonsynchronous Set (b) Skipped to 3174 from No. 1			Nonsynchronous Set (c) Skipped to 3016 from No. 1			Nonsynchronous Set (d) Skipped to 2698 from No. 1		
No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)
1	3333	300.030	1	3333	300.030	1	3333	300.030	1	3333	300.030
2	3183	314.169	2	3174	315.060	2	3016	331.565	2	2698	370.645

Nonsynchronous Set (e) Skipped to 2539 from No. 1			Nonsynchronous Set (f) Skipped to 2272 from No. 1		
No.	τ (μsec)	Freq. (Hz)	No.	τ (μsec)	Freq. (Hz)
1	3333	300.030	1	3333	300.030
2	2539	393.856	2	2272	440.141

*$n = 20$, $A = 3333$ μsec, $Z = 2200$ μsec.
†$\delta = 100$ μsec.
‡$\delta = 150$ μsec.

For the second example, δ = 150 μsec, n = 20, A = 3333 μsec, and Z = 2200 μsec. Set (a), having only two nonsynchronous IRF's, resulted from the straightforward application of Method I. The larger value of δ is responsible for the fewer number of nonsynchronous IRF's compared to example 1. For illustration, let's assume that a beacon system installation exists that has a single interrogator whose IRF period is 3333 μsec. Assume also that one interrogator for a different beacon system must be installed in the same area. Assume further that existing equipment available for the new installation allows only a limited choice of IRF's, none of which is the 314.169 Hz obtained in Set (a). An abnormal Method I calculation located five additional nonsynchronous IRF (period) choices for use with 3333 μsec for the given parameters. Sets (b) through (f) show these choices. A quick prior attempt from educated guesses had already located Sets (d) and (e) using skips from τ_1 to 2820 and 2860 μsec, respectively. One of the five additional choices for the new installation assumed in this example may be obtainable from available equipment. The deliberate skip technique is seen to be a flexible means for adjusting the nonsynchronous IRF choices to match either IRF values already existing in a system or values desired, insofar as possible.

METHOD II RESULTS

Typical FAA beacon IRF's (PRF's) (Ref. 5) were evaluated for nonsynchronous interference. For convenience, Table 13 lists the periods with corresponding frequencies, the FAA identifying letters, and designated nominal values of the IRF's. All possible pair combinations of the 14 IRF's were analyzed for Type 2 nonsynchronous interference by Method II, for δ = 100 μsec (reasonable for newer FAA equipment) and n = 20. The nonsynchronous IRF pairs are listed below.

A & D	E & M
A & H	(F & D)
B & E	(F & E)
C & N	(*H & A*)
C & K	(*H & D*)
(*D & A*)	(I & E)
D & F	I & K
D & H	(J & D)
D & J	(K & C)
(E & B)	(K & I)
E & F	(M & E)
E & I	(N & C)

Pairs enclosed in parentheses are redundant listings to provide complete alphabetical order in left-hand column. Of the 91 possible pair combinations, 12 pairs, or 13%, were found to be nonsynchronous (when δ = 100 μsec, n = 20). Three of these, A, D, and H, were found to comprise a nonsynchronous set. Of possible interest is the fact that if the M period were modified from the listed value of 2598 μsec to the value of 2600 μsec, then the three IRF's, E, F, and M, would be found to comprise a nonsynchronous set. Also, the pair G and N would become nonsynchronous if the value of G were changed from 2565 to 2564 μsec. The pair H and L would become nonsynchronous if L were changed from the value of 2532 to 2531 μsec.

Table 13
FAA IRF Data Based on the Exact Periods Listed in Ref. 5*

Period (μsec)	Frequency (Hz)	Letter Designation	Nominal Frequency
2913	343.289	K	343
2860	349.650	A	350
2856	350.140	J	351
2820	354.610	B	355
2805	356.506	I	357
2780	359.712	C	360
2740	364.964	D	365
2700	370.370	E	370
2664	375.375	N	375
2631	380.084	H	380
2598	384.911	M	385
2565	389.864	G	390
2532	394.945	L	395
2500	400.000	F	400

*The periods are in descending numerical order.

The preceding list of nonsynchronous IRF pairs selected from all of those analyzed conforms exactly to the definition of Type 2 IRF's nonsynchronous for n cycles. The following statement applies for any phase angle relationship that could occur between these pairs: If interference does occur once between a given pair, there will be at least n interrogations before interference occurs again between that pair. The same cannot be said for the other IRF pairs of the analysis that did not merit inclusion in the list. When the calculation reveals one or more interfering interrogations between the zeroth and the (n + 1)th interrogations, it does not necessarily follow that the pattern of interfering interrogations will be the same for other phase angles. Further analysis is required to determine the possible interference patterns due to other phase angles in this case.

DISCUSSION

It appears that the ATCRBS would benefit from the exclusive use of IRF's nonsynchronous for n interrogations. To the extent that synchronous interference is reduced, some improvement could be expected even though the nonsynchronous IRF's are not ideally correlated with antenna rotation rates and main-beam widths. Ideally though, each interrogator-antenna combination should result in n hits per main-beam antenna scan to obtain the advantage of the greater availability of the Type 1 (unequal coverage) nonsynchronous IRF's.

As indicated by the results, the availability of both Type 1 and Type 2 within a given frequency band becomes greater as the values of n and δ are reduced. Unfortunately, at the present time larger values of δ are desired, especially for military systems.

In any event, the exclusive use of nonsynchronous IRF's permits a considerable reduction in the value of n. Hopefully, other means can be found to allow reduction of the value of δ, where needed, to obtain a sufficient number of nonsynchronous IRF's. Unless some other means can be found, the results presented here indicate that beacon system designs based on very large values of δ and simultaneous use of many continuously operating interrogators for a single target, are headed for serious, or even prohibitive, synchronous interference problems.

When considering a reduction of the value of n, some allowance should be made when the system has many IRF's, since the IRF's are (here) defined to be nonsynchronous for the next n interrogations on a pair basis. As a worst case, phase-angle relationships among all the IRF's of a nonsynchronous set *might* become so arranged that, during the n interrogations, one of the IRF's would experience the one interference from each in succession of the other IRF's. Fortunately, the probability of occurrence of the worst case is extremely low, but the possibility of a few IRF's interfering once in succession with one of the other IRF's is a factor that would affect the lower limit of n.

The frequencies (nonsynchronous IRF's) are listed to three decimal places so that reconversion from frequency to period will give (with rounding) the original integer value of microseconds for the period. Thus in general it is implied that the frequency stability must be at least as good to maintain the integer values of microseconds upon which the choice of nonsynchronous IRF's is made. However, there is a desirable alternative for most applications that will reduce the frequency stability requirement to only 1 part in 10^4, so that relatively inexpensive crystal control will suffice. An allowance for absorbing part of the frequency variation is included in the value of δ. That is, $\delta = \delta_1 + \delta_2$, where δ_1 is the design busy time of the transponders and δ_2 is the band in microseconds allotted to absorb a frequency variation corresponding to $\pm \delta_2/2$ μsec over and above the 1 part in 10^4 control. For example, frequency control to 1 part in 10^4 would suffice for the nonsynchronous IRF's presented here for $\delta = 100$ μsec and $n = 20$ if $\delta_1 = 80$ μsec and $\delta_2 = 20$ μsec. In the event that an insufficient number of nonsynchronous IRF's exist for $\delta = 100$ μsec and $n = 20$, further reduction of δ_1 and/or n will increase the number of nonsynchronous IRF's.

The methods presented here are applicable to any frequency band. The convenience and simplicity of using integer values of the periods is retained for other frequency bands by scaling the units of microseconds by factors of 10 prior to calculation. Furthermore, with appropriate factor-of-10 changes, the results already obtained are directly applicable to other frequency bands. In principle, however, there is no reason why the methods presented here could not be modified to originate from integer values of frequency and the calculations performed on the corresponding decimal values of the period.

Although the methods do not inherently produce optimal choices of nonsynchronous IRF's, the application techniques described provide the ability to search for optimal choices with relative ease by computer.

The methods are believed to have some general application, for example, to the determination of optimal information rates in certain types of communication systems.

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4. Operations Committee, Naval Science Dept., "Naval Operations Analysis," U.S. Naval Institute, Annapolis, Md., 1969, p. 71

5. FAA Order 6050.9, Change 1, "Radar/Beacon Pulse Repetition Frequency (PRF) Assignment Criteria and Processing Actions," 3 May 1967

Appendix A

METHOD I COMPUTER PROGRAMS

```
06 'METHOD I COMBINED TYPE 1 & TYPE 2 WITH SHORTCUT TEST.
08 'BASIC LANGUAGE PROGRAM FOR FINDING FREQUENCIES, F (HERTZ)
10 '& PERIODS, TAU (MICROSEC) NONSYNCHRONOUS FOR N INTERROGATIONS.
12 'A TO Z IS RANGE OF PERIOD BAND. D IS DELTA (MICROSEC).
14 'P SPECIFIES THE NUMBER OF ROWS(+1) OF THE STORAGE MATRIX
16 'U(P,2).  P = GENEROUS ESTIMATE, 1.5 TO 2 TIMES THE NUMBER
18 ' OF NONSYNCHRONOUS F'S EXPECTED IN THE RANGE A TO Z
20 'FOR THE VALUES OF N & D.  C IS THE CHANGE MADE TO AN
22 'UNSUCCESSFUL B FOR THE NEXT TRIAL.
24 '  TO OBTAIN TYPE 1, GIVE E THE VALUE 2.
26 '  TO OBTAIN TYPE 2, GIVE E THE VALUE 4.  ALL TYPE 1 OR 2 TAU'S
28 'PRINTED OUT WILL BE NONSYNCHRONOUS FOR THE DURATIONS COVERED.

40 INPUT P,A,Z,D,E,N
50 IF A>Z THEN 690
60 LET G=1
70 DIM U(P,2)
80 MAT U=ZER(P,2)
90 LET U(1,1)=A
100 LET U(1,2)=F=10↑6/A
110 LET U(1,0)=1
120 PRINT"NUMBER   TAU(MICROSEC)   FREQ(HZ)"
130 PRINT U(1,0),A,F
140 LET B=A+G*D
150 IF G+E=3 THEN 320
160 FOR L=1TOP
170 FOR K=1TOL
180 FOR I=1TON
190 LET Y=I*B
200 LET R=B/U(K,1)
210 LET D7=(D-1)/U(K,1)
220 LET S3=INT(I*R+D7+1)
230 LET S1=INT(I*R-D7)
240 FOR J=S1TOS3
250 LET X=J*U(K,1)
260 IF ABS(Y-X)<=D-1THEN 590
270 NEXT J
280 NEXT I
290 NEXT K
300 GOTO 480
310 NEXT L
320 FOR L=1TOP
330 FOR K=1TOL
340 LET R=U(K,1)/B
350 FOR J=1TON
360 LET X=J*U(K,1)
370 LET D7=(D-1)/B
380 LET S3=INT(J*R+D7+1)
390 LET S1=INT(J*R-D7)
400 FOR I=S1 TO S3
410 LET Y=I*B
420 IF ABS(Y-X)<=D-1THEN590
430 NEXT I
440 NEXT J
450 NEXT K
```

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```
460 GOTO 480
470 NEXT L
480 LET U(L+1,1)=B
490 LET U(L+1,2)=F=10↑6/B
500 LET U(L+1,0)=L+1
510 PRINT L+1,B,F
520 LET B=B+G*D
530 IF A<ZTHEN 670
540 IF B<ZTHEN570
550 IF G+E=3 THEN 470
560 GOTO310
570 PRINT"B="B
580 GOTO 40
590 LET C=INT(ABS((Y-X-G*D)/I)+.999)
600 LET B=B+G*C
610 IF A<ZTHEN650
620 IF B<ZTHEN570
630 IF G+E=3 THEN 330
640 GOTO 170
650 IF B>ZTHEN570
660 GOTO 630
670 IF B>Z THEN 570
680 GOTO 550
690 LET G=-1
700 GOTO 70

800 ' MODIFICATIONS FOR DELIBERATE SKIP TECHNIQUE:
810 'ADD: 132 PRINT"T"              ADD: 512 PRINT"T1"
820 '      134 INPUT T      OR           514 INPUT T1
830 'CHANGE:140 LET B=T          CHANGE:520 LET B=T1
840 'I.E.,SKIPS FROM A TO T.      I.E.,B SKIPS FROM VALUE
850 '                            AT 510 TO T1.

860 ' SOME CONVENIENT OPTIONS:  ADD: 592 PRINT B, OR 592 PRINT B,C
870 ' (INDICATES PROGRESS OF CALCULATION DURING LONG GAPS BETWEEN
880 'PRINTED OUT F'S.)   ADD:592 PRINT I;B;Y;X;U(K,1);J;C (USEFUL FOR
890 'DETAILED EXAMINATION OF THE CALCULATION). ADD: 572 MAT PRINT U
900 '(CONSOLIDATES FINAL  NONSYNCHRONOUS B'S & F'S IF 592 USED).
```

```
950 'ALTERNATE INTERNAL SEQUENCE OF TESTS:

170 FOR I=1 TO N
180 LET Y=I*B
190 FOR K=1TOL
200 LET R=B/U(K,1)
210 LET D7=(D-1)/U(K,1)
220 LET S3=INT(I*R+D7+1)
230 LET S1=INT(I*R-D7)
240 FOR J=S1 TO S3
250 LET X=J*U(K,1)
260 IF ABS(Y-X)<=D-1THEN 590
270 NEXT J
280 NEXT K
290 NEXT I

330 FOR J=1TON
340 FOR K=1TOL
350 LET X=J*U(K,1)
360 LET R=U(K,1)/B
370 LET D7=(D-1)/B
380 LET S3=INT(J*R+D7+1)
390 LET S1=INT(J*R-D7)
400 FOR I=S1TOS3
410 LET Y=I*B
420 IF ABS(Y-X)<=D-1THEN590
430 NEXT I
440 NEXT K
450 NEXT J
```

```
C     METHOD I COMBINED TYPE 1 AND TYPE 2 WITH SHORTCUT TEST.
C     FORTRAN IV LANGUAGE PROGRAM FOR FINDING PERIODS IN MICROSEC, TAU,
C     AND FREQUENCIES IN HERTZ, FREQ, NONSYNCHRONOUS FOR N INTERROGATIONS.
C     (TAU 1ST VALUE = IA, OTHERS = IB).  A TO Z IS THE RANGE OF THE
C     PERIOD BAND SEARCHED (MICROSEC.).
C     TO OBTAIN TYPE 1, GIVE IE THE VALUE 2
C     TO OBTAIN TYPE 2, GIVE IE THE VALUE 4
C     IC IS THE CHANGE MADE TO AN UNSUCCESSFUL IB FOR THE NEXT TRIAL.
C     ALL TYPE 1 OR TYPE 2 PERIODS AND FREQUENCIES PRINTED OUT WILL BE
C     NONSYNCHRONOUS FOR THE DURATIONS COVERED.

      PROGRAM NSIRFSET
C     INSERT BLANK CARD FOLLOWING DATA CARDS.
C     REVISE DIMENSION,100, AND DO LIMITS,100, 300, IF MORE THAN 100
C     NON-SYNCHRONOUS IRF'S EXPECTED.
    4 FORMAT(2I7,I4,I2,I4)
   44 FORMAT(1H0,8X*IA*8X*IZ*5X*IDELTA*3X*IE*5X*N*/
     11H0,5X,I7,3X,I7,3X,I4,5X,I2,3X,I4//)
  122 FORMAT(1X,8X*NUMBER*5X*TAU(MICROSEC)*3X*FREQ(HZ)*/)
  133 FORMAT(1X,8X,I4,9X,I7,6X,F10.3/)
  577 FORMAT(1X,6X,5HIB = I7/)
   30 DIMENSION MAT(100,2),MATZ(200),U(100)
   32 EQUIVALENCE(MAT,MATZ)
   34 DO 36 M=1,200
   36 MATZ(M)=0
   38 DO 40 N1=1,100
   40 U(N1)=0
   42 READ 4,IA,IZ,IDELTA,IE,N
   43 IF(IA.EQ.0)710,46
   46 PRINT 44,IA,IZ,IDELTA,IE,N
   48 ID1=IDELTA-1
   50 IF(IA.GT.IZ)690,60
   60 IG=1
   62 ISIGND=IG*IDELTA
   64 IGE=IG+IE
   90 MAT(1,1)=1
  100 MAT(1,2)=IA
  110 U(1)=FREQ=(1000000.)/IA
  120 PRINT 122
  130 PRINT 133,MAT(1,1),MAT(1,2),U(1)
  140 IB=IA+ISIGND
  150 IF(IGE.EQ.3)320,160
  160 DO 310 L=1,100
  170 DO 290 K=1,L
  180 DO 280 I=1,N
  190 IPROD=I*IB
  200 R=(1.0*IB)/MAT(K,2)
  210 FRAC=(1.0*ID1)/MAT(K,2)
  215 RI=R*I
```

```
220 M3=INT(RI+FRAC+1.)
230 M1=INT(RI-FRAC)
240 DO 270 J=M1,M3
250 JPROD=J*MAT(K,2)
252 IPRODIF=IPROD-JPROD
254 IABPRDF=IABS(IPRODIF)
260 IF(IABPRDF.LE.ID1)585,270
270 CONTINUE
280 CONTINUE
290 CONTINUE
300 GOTO 475
310 CONTINUE
320 DO 470 L=1,100
330 DO 450 K=1,L
340 R=(1.0*MAT(K,2))/IB
350 DO 440 J=1,N
360 JPROD=J*MAT(K,2)
370 FRAC = (1.0*ID1)/IB
375 RJ = R*J
380 M3 = INT(RJ+FRAC+1.)
390 M1 = INT(RJ-FRAC)
400 DO 430 I=M1,M3
410 IPROD=I*IB
412 IPRODIF=IPROD-JPROD
414 IABPRDF = IABS(IPRODIF)
420 IF(IABPRDF.LE.ID1)585,430
430 CONTINUE
440 CONTINUE
450 CONTINUE
460 GOTO 475
470 CONTINUE
475 L1=L+1
480 MAT(L1,1)=L1
490 MAT(L1,2)=IB
500 U(L1)=FREQ=(1000000.)/IB
510 PRINT 133,L1,IB,FREQ
520 IB=IB+ISIGND
530 IF(IA.LT.IZ)670,540
540 IF(IB.LT.IZ)570,550
550 IF(IGE.EQ.3)470,310
570 PRINT 577,IB
580 GOTO 34
585 C=ABS((1.0*(IPRODIF-ISIGND))/I)+.999
590 IC=INT(C)
600 IB=IB+IG*IC
610 IF(IA.LT.IZ)650,620
620 IF(IB.LT.IZ)570,630
630 IF(IGE.EQ.3)330,170
650 IF(IB.GT.IZ)570,630
670 IF(IB.GT.IZ)570,550
690 IG=-1
700 GOTO 62
710 END
```

```
C       MODIFICATIONS FOR
C       DELIBERATE SKIP TECHNIQUE
C       ADD, 5 FORMAT(I7)
C       ADD, 134 READ 5,IT
C       CHANGE 140, 140 IB=IT
C       SKIPS FROM IA TO IT.
C       PERMITS NEW IA TO IT TO
C       IZ CALCULATION FOR EACH PAIR
C       OF DATA CARDS.

C               OR,

C       ADD, 5 FORMAT(I7)
C       ADD, 514 READ 5,IT1
C       CHANGE 520, 520 IB=IT1
C       SKIPS FROM IB VALUE AT 510 TO
C       IT1. (SUITABLE ONLY WHEN USER
C       HAS ACCESS TO ON-LINE TYPEWRITER
C       CONTROL OF OUTPUT AND INPUT
C       OR STATEMENT REVISION).

C       OPTION FOR DETAILED EXAMINATION
C       OF THE CALCULATION.
C       ADD 594,
    594 PRINT 596,J,JPROD,IPRODIF,
        IPROD,I,IC,R,K,L
C       ADD,596
    596 FORMAT(1X,I4,3I7,2I4,F11.9,2I4)
```

Appendix B

METHOD II COMPUTER PROGRAMS

```
06   'METHOD II BASIC LANGUAGE PROGRAM FOR FINDING INTERFERENCE
08   'BETWEEN TWO INTERROGATION SEQUENCES. INTERROGATIONS RECUR
10   'WITH PERIODS T1 & T2 (MICROSEC). FOR TYPE 1 EVALUATION,
12   'INPUT VALUES OF T1<T2. FOR TYPE 2 EVALUATION, INPUT VALUES
14   'OF T1>T2. FREQUENCIES CORRESPONDING TO PERIODS ARE F1 & F2
16   '(HERTZ). D IS THE VALUE OF DELTA (MICROSEC). L IS THE MAX.
18   'NUMBER OF INTERROGATIONS TESTED. N & M PRINTED OUT ARE THE
20   'MULTIPLES OF T1 & T2 FOUND TO INTERFERE.  BETWEEN TWO
22   'SUCCESSIVE PRINTED OUT VALUES OF N OR M IS A RUN OF P OR O
24   'NON-INTERFERING INTERROGATIONS.  G1 & G2 REPRESENT THE
26   'PREVIOUS PRINTED OUT VALUES OF N OR M.  R = RATIO OF PERIODS.


40 INPUT T1,T2,D,L
50 LET R=T1/T2
60 LET F1 = 10↑6/T1
70 LET F2 = 10↑6/T2
80 PRINT"F1="F1
90 PRINT"F2="F2
100 PRINT"R="R
105 PRINT"N    P    X1     X2     M    O    INT(X1-X2)"
110 LET G1=G2=0
120 FOR N=1TOL
130 LET X1=N*T1
140 LET D7=(D-1)/T2
150 LET S3=INT(N*R+D7+1)
160 LET S1=INT(N*R-D7)
170 FOR M=S1 TO S3
180 LET X2=M*T2
190 IF ABS(X1-X2)<=D-1 THEN 230
200 NEXT M
210 NEXT N
220 IF N=L THEN 300
230 LET P=N-1-G1
240 LET O=M-1-G2
250 PRINT N;P;X1,X2,M;O, INT(X1-X2)
260 LET G1=N
270 LET G2=M
280 IF N=L THEN 300
290 GOTO 200
300 PRINT"L="L
310 GOTO 40

400 'OPTIONAL STREAMLINED VERSION; DELETE 105,110,230,240,260,270
402 'CHANGE;  250 PRINT N,INT(X1-X2),M
404 '         190 IF ABS(X1-X2)<=D-1 THEN 250

410 'OPTIONAL INPUT; ADD; 37 PRINT"D,L"; ADD 38 INPUT D,L
412 'CHANGE 40 INPUT T1,T2
```

```
C     METHOD II FORTRAN IV LANGUAGE PROGRAM FOR FINDING INTERFERENCE
C     BETWEEN TWO INTERROGATION SEQUENCES.  INTERROGATIONS RECUR WITH PERIODS
C     ITAU1 AND JTAU2 (MICROSEC.). FOR TYPE 1 EVALUATION INPUT VALUES OF
C     ITAU1 LESS THAN JTAU2.  FOR TYPE 2 EVALUATION, INPUT VALUES OF
C     ITAU1 GREATER THAN JTAU2.  FREQUENCIES IN HERTZ CORRECPONDING TO
C     PERIODS ARE FREQ1 AND FREQ2.   L IS THE MAXIMUM NUMBER OF INTER-
C     ROGATIONS TESTED.  I AND J PRINTED OUT ARE THE MULTIPLES OF ITAU1
C     AND JTAU2 FOUND TO INTERFERE.  BETWEEN TWO SUCCESSIVE PRINTED OUT
C     VALUES OF I OR J IS A RUN OF IRUN OR JRUN NONINTERFERING INTERROGATIONS.
C     IG AND JG REPRESENT THE PREVIOUS PRINTED OUT VALUES OF I OR J.
C     R = RATIO OF PERIODS.

      PROGRAM EVALIRF
C     INSERT BLANK CARD FOLLOWING ITAU1,JTAU2 DATA CARDS.
   20 FORMAT(2I4)
   40 FORMAT(2I7)
  110 FORMAT(1HO,8X*ITAU1*5X*JTAU2*4X*FREQ1*6X*FREQ2*9X*R*11X*IDELTA*/
     11HO,6X,I7,1X,I7,3X,F8.3,2X,F8.3,6X,F13.8,3X,I4)
  130 FORMAT(1HO,9X*I*6X*IRUN*9X*IPROD*10X*JPROD*8X*J*4X*JRUN*2X*
     1IPRODIF*/)
  320 FORMAT(1X,7X,I4,5X,I4,5X,I10,5X,I10,5X,I4,5X,I4,5X,I5/)
  370 FORMAT(1X,7X,4HL = I4/)
   10 READ 20, IDELTA,L
   30 READ 40,ITAU1,JTAU2
   50 IF(ITAU1.EQ.0) 400,70
   70 R=(1.0*ITAU1)/JTAU2
   80 FREQ1=(1000000.)/ITAU1
   90 FREQ2=(1000000.)/JTAU2
  100 PRINT 110,ITAU1,JTAU2,FREQ1,FREQ2,R,IDELTA
  120 PRINT 130
  140 IG=JG=0
  150 DO275  I=1,L
  160 IPROD=I*ITAU1
  170 ID1=IDELTA-1
  180 FRAC=ID1/JTAU2
  185 RI=(R*I)
  190 JS3=INT(RI+FRAC+1)
  200 JS1=INT(RI-FRAC)
  210 IF(JS1.EQ.0) 220,230
  220 JS1=1
  230 DO270 J=JS1,JS3
  240 JPROD=J*JTAU2
  250 IPRODIF=IPROD-JPROD
  260 IF(IABS(IPRODIF).LE.ID1) 290,270
  270 CONTINUE
  275 CONTINUE
  280 PRINT 370,L
  285 GO TO 30
  290 IRUN=I-1-IG
  300 JRUN=J-1-JG
  310 PRINT320,I,IRUN,IPROD,JPROD,J,JRUN,IPRODIF
  330 IG=I
  340 JG=J
  350 GO TO 270
  400 END
```

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