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Atmospheric Propagation with Thermal Blooming

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ABSTRACT

Thermal blooming of a laser beam is described by a model equation for the trajectory of a selected profile ray. The model equation is applied to an examination of optical focusing for increasing transmitted power density. It is shown that the model equation is asumptotic in the far field to the geometric far field solution based on Snell's law for a stratified medium.

PROBLEM STATUS

An interim report on a continuing NRL Problem

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ATMOSPHERIC PROPAGATION WITH THERMAL BLOOMING

INTRODUCTION

Thermal defocusing of high-energy laser beams in liquids is a well known phenomenon (1-11) that has been used to measure weak absorptions (2). The analogous phenomenon in the atmosphere has been investigated in a simulated experiment (12).

As an intense laser beam propagates through the atmosphere, part of its energy will be absorbed by constituents of the air. This absorption will cause local heating with decrease in air density and refractive index in the beam. The bending of light rays toward regions of higher refractive index will cause the beam to spread. This phenomenon is referred to as thermal blooming or thermal defocusing, and it is the purpose of this report to develop simple equations descriptive of this effect under certain assumed conditions.

It is assumed that there is no wind and that convection and conduction can be neglected. True conduction is negligible, and conduction by microturbulence will be considered negligible for truly static air. Gravitational convection will be present but may be neglected for short time regimes because severe thermal blooming requires but a very small density differential.

As the atmosphere is rarely static, one might wonder what utility there is to a description based on such an assumption. The point is that static air represents the worst situation from the standpoint of thermal blooming and yet is the one situation that allows a simple closed form description of the development of thermal blooming. When wind is taken into account, a completely different and somewhat more approximate approach is needed. The figures for static air serve as references for the improvement due to wind, which will be considered in a later report.

It is assumed that the beam power density and the refractive index are radial Gaussian functions with the same spread parameter at a particular range and time. The assumption that the beam distribution remains Gaussian implies that there is no ray crossing, whereas the far field solution shows that ray crossing must be complete, that is, that the beam is turned inside out. It is important therefore to select as the profile ray one that is compatible with the far field solution. One can then hope that ray crossing will tend to balance out at intermediate ranges and that a solution that is asymptotically accurate at $z = 0$ and $z = \infty$ will be reasonably so at intermediate ranges.

Incidentally it is not necessary to use a Gaussian function for this method of solution to apply. It is, however, necessary to assume that the functional form of the radial distribution remains the same as the beam spreads. Alternate convenient distribution functions are

$$\begin{cases} P = 1 + \cos \dfrac{r^2}{b^2} , \quad 0 \quad r < \sqrt{\pi}\, b , \\[2ex] P = 0 , \quad r > \sqrt{\pi}\, b , \end{cases} \tag{1}$$

and

$$\begin{cases} P = 1 + \cos \dfrac{r^2}{b^2}, & 0 \le r \le \sqrt{\dfrac{\pi}{2}}\, b , \\ P = 0 , & r > \sqrt{\dfrac{\pi}{2}}\, b . \end{cases} \tag{2}$$

It is assumed that the total power in the beam is independent of range. With an absorption coefficient of 8×10^{-7} cm^{-1}, high-power beams will have done most of their bending in the first few kilometers before an appreciable fraction of the total beam power has been lost. For example at 2 kilometers the overall path transmission is

$$e^{-\alpha z} = e^{-0.16} = 0.85 . \tag{3}$$

THE MODEL EQUATION

Normalization of the Distribution

Consider a spreading beam as shown in Fig. 1. Let $a(z, t)$ be the characteristic beam radius with value a_0 at $z = 0$. The ray $r = a$ will generate a flared tube which will be considered the beam profile.

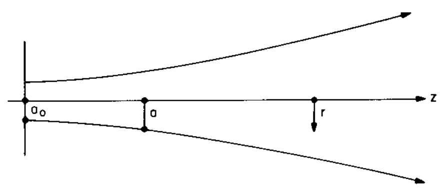

Fig. 1 - Beam geometry

Assuming that the radial distribution of power is Gaussian and remains Gaussian, the power flux can be written

$$P = N e^{-r^2/a^2} , \tag{4}$$

where N is a normalization constant. The total beam power W is then

$$W = 2\pi N \int_0^\infty e^{-r^2/a^2} \, r \, dr = \pi a^2 N . \tag{5}$$

Therefore

$$P = \frac{W}{\pi a^2} e^{-r^2/a^2} . \tag{6}$$

The Eikonal Equation

The change in refractive index Δn due to a change in density $\Delta\rho$ is

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$$\Delta n = (n_0 - 1)\frac{\Delta\rho}{\rho} \tag{7}$$

$$= (n_0 - 1)\frac{\alpha t P}{C_p \theta \rho} \tag{8}$$

$$= (n_0 - 1)\frac{\alpha t}{C_p \theta \rho}\frac{W}{\pi a^2} e^{-r^2/a^2} , \tag{9}$$

where α is the absorption coefficient, C_p is the specific heat at constant pressure, ρ is the density, θ is the ambient temperature, and t is the time.

It will be noted that in Eq. (8)

$$\int_0^t P \, dt$$

has been replaced by Pt. The nature of this approximation may be judged from Fig. 2, which shows a plot of P for $r = a$ as the envelope of a set of Gaussian distributions. For example consider some down-beam point with radial coordinate r, where $a < r$ at $t = 0$ and $a = r$ as the profile ray bends and passes through the point r. It is when $r = a$ that the profile ray solution applies. For definiteness let $r = 15$ cm and let $a = 10$ cm at $t = 0$. From Fig. 2, $P = 1.07$ at $t = 0$ and $P = 1.63$ when $a = r$. For a linear decrease of P with time,

$$\int_0^t P \, dt = P_{av} t = 1.35 \ t ,$$

whereas $P_t t = 1.63 \ t$. Thus the approximation increases the predicted severity of blooming.

With the beam propagating along the z axis the deviation of the beam toward a higher refractive index is expressed in the equation (from Ref. 13) for the radius of curvature R,

$$\frac{1}{R} = \frac{1}{n}\frac{dn}{dr} \approx \frac{dn}{dr} , \tag{10}$$

since $n \approx 1$. The index of refraction n is

$$n = n_0 - \Delta n ,$$

so

$$n = n_0 - (n_0 - 1)\frac{\alpha t}{C_p \theta \rho}\frac{W}{\pi a^2} e^{-r^2/a^2} . \tag{11}$$

Then the radial index gradient is

$$\frac{dn}{dr} = (n_0 - 1)\frac{\alpha t}{C_p \theta \rho}\frac{W}{\pi a^2}\frac{2r}{a^2} e^{-r^2/a^2} . \tag{12}$$

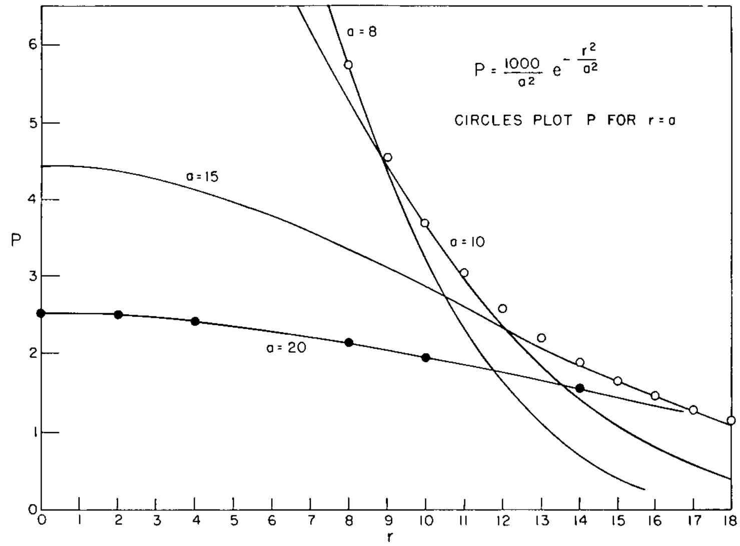

Fig. 2 - Profile power density as an envelope of a set of Gaussian distributions

Combining Eqs. (10) and (12) gives

$$\frac{1}{R} = \frac{2(n_0 - 1)\,\alpha W t r}{\pi C_p \theta \rho a^4}\, e^{-r^2/a^2} , \tag{13}$$

and, at $r = a$,

$$\frac{1}{R} = \frac{2(n_0 - 1)\,\alpha W t}{\pi e C_p \theta \rho a^3} . \tag{14}$$

But

$$\frac{1}{R} \approx \frac{d^2 a}{dz^2} , \tag{15}$$

and Eqs. (14) and (15) combined give the Eikonal equation for the ray $r = a$:

$$\frac{d^2 a}{dz^2} = \frac{2(n_0 - 1)\,\alpha W t}{\pi e C_p \theta \rho a^3} \tag{16}$$

or

$$\frac{d^2 a}{dz^2} = \frac{k^2}{a^3} , \tag{17}$$

where

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$$k^2 = \frac{2(n_0 - 1)\,\alpha W t}{\pi e C_p \theta \rho} .$$

Solution of the Eikonal

If

$$p = \frac{da}{dz} \quad \text{and} \quad \frac{d^2 a}{dz^2} = p\,\frac{dp}{da} , \tag{18}$$

Eq. (17) becomes

$$p\,\frac{dp}{da} = \frac{k^2}{a^3} . \tag{19}$$

Integration with $p = 0$ at $a = a_0$ gives

$$p^2 = k^2 \left(\frac{1}{a_0^2} - \frac{1}{a^2}\right) \tag{20}$$

and

$$p = \frac{da}{dz} = \frac{k}{aa_0} \sqrt{a^2 - a_0^2} . \tag{21}$$

Integration with $a = a_0$ at $z = 0$ gives

$$a^2 = \frac{k^2}{a_0^2} z^2 + a_0^2 , \tag{22}$$

which may be recognized as a hyperbola starting from $r = a_0$ at $z = 0$ with $dr/dz = 0$ at $z = 0$ and asymptotic to the cones

$$r^2 = \frac{k^2}{a_0^2} z^2 . \tag{23}$$

Numerical Applications

Assigning the appropriate numerical values to the atmospheric parameters in Eq. (16) gives

$$\frac{k^2}{a_0^2} = \frac{2(n_0 - 1)\,\alpha W t}{\pi e C_p \theta \rho a_0^2} = \frac{W t \times 10^{-7}}{513\ a_0^2} , \tag{24}$$

where

$$(n_0 - 1) = 3 \times 10^{-4},$$

$$\alpha = 8 \times 10^{-7}\ \text{cm}^{-1},$$

$$C_p = 0.96\ \text{joule-g}^{-1},$$

$$\theta = 300\,°\text{K},$$

$$\rho = 10^{-3}\ \text{g-cm}^{-3},$$

$$W = \text{watts},$$

$$t = \text{seconds}.$$

From Eq. (22)

$$a^2 = \frac{Wt\ z^2 \times 10^{-7}}{513\ a_0^2} + a_0^2\ . \tag{25}$$

For $Wt = 5130$ and $a_0 = 10$ cm Eq. (25) gives

$$a^2 = z^2 \times 10^{-8} + 100\ , \tag{26}$$

and for $Wt = 51{,}300$ and $a_0 = 10$ cm Eq. (25) gives

$$a^2 = z^2 \times 10^{-7} + 100\ . \tag{27}$$

If P_m is the power flux at the center of the beam at $z = 0$, then

$$P_m t = \frac{Wt}{\pi a_0^2}\ , \tag{28}$$

and for the preceding cases

$$P_m t = \frac{5130}{100\ \pi} = 16.3\ \text{joule-cm}^{-2} \tag{29}$$

and

$$P_m t = \frac{51300}{100\ \pi} = 163\ \text{joule-cm}^{-2}\ . \tag{30}$$

The beam profiles are plotted in Fig. 3 from Eqs. (26) and (27).

The effect of initial diameter on a constant-energy beam is shown in Fig. 4.

In Fig. 5, the energy density at the beam center is kept constant as the initial beam diameter is varied. In the far field it is seen that the direction and consequently the diameter is set by the energy density of the beam at $z = 0$.

In the far-field approximation when

$$a^2 \approx \frac{Wt\ z^2 \times 10^{-7}}{513\ a_0^2} \tag{31}$$

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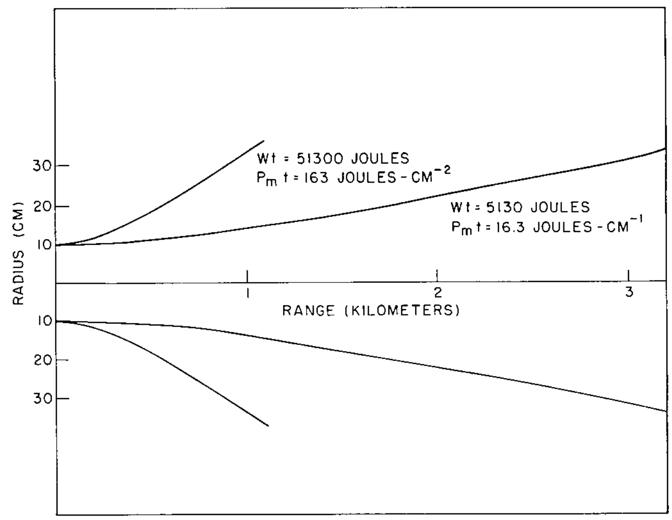

Fig. 3 - Effect of the beam energy on blooming

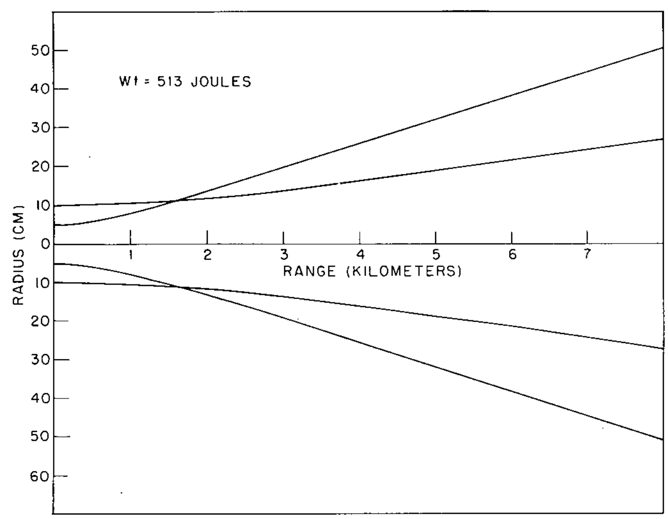

Fig. 4 - Effect of the initial diameter on a constant-energy beam (Wt = 513 joules)

and thermal blooming is dominant, it is seen that delivered energy is independent of beam energy Wt and is determined by the solid angle of the source aperture as seen from the receiver. Thus

$$\frac{Wt}{\pi a^2} \approx \frac{513\ a_0^2}{\pi z^2 \times 10^{-7}}$$

$$\approx 163 \times 10^{-7}\ \omega\ , \tag{32}$$

where $\omega = a_0^2/z^2$ is the solid angle subtended by the source aperture at the receiver.

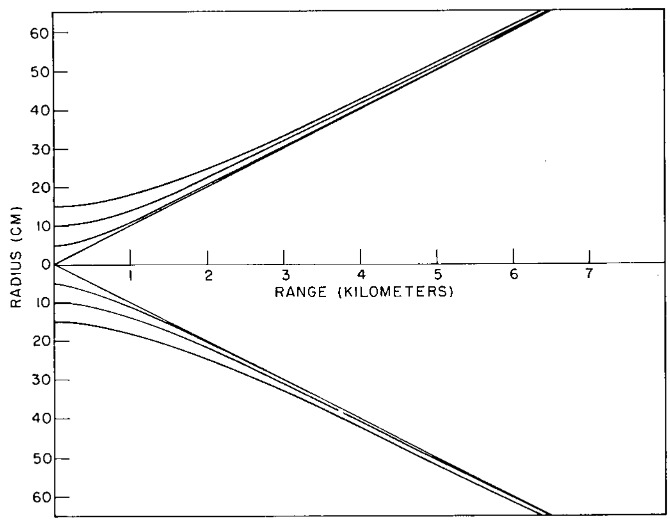

Fig. 5 - Profiles for constant energy density at the center of the initial beam ($P_m t$ = 16.3 joule-cm^{-2})

EFFECT OF OPTICAL FOCUSING

It is natural to investigate to what extent optical focusing may be useful in increasing the power density at a distant receiver.

As before, the energy deposited in the atmosphere is assumed to have a Gaussian radial distribution. Let the slope of the profile ray be m. Then Eq. (19) integrates to

$$\frac{p^2}{2} = -\frac{k^2}{2a^2} + A . \tag{33}$$

Substitution of the boundary conditions $p = m$ at $a = a_0$ gives

$$A = \frac{m^2}{2} + \frac{k^2}{2a_0^2} . \tag{34}$$

Thus

$$p = \frac{da}{dz} = \sqrt{\frac{L^2}{a_0^2} - \frac{k^2}{a^2}} , \tag{35}$$

where $L^2 = k^2 + m^2 a_0^2$. This can be written

$$\frac{1}{a_0} \int dz = \frac{1}{2L^2} \int \frac{2L^2 a \, da}{\sqrt{L^2 a^2 - k^2 a_0^2}} \tag{36}$$

to give

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$$\frac{1}{L^2}\sqrt{L^2 a^2 - k^2 a_0^2} = \frac{z}{a_0} + B \ . \tag{37}$$

With $z = 0$ at $a = a_0$, Eq. (37) gives

$$B = \frac{a_0}{L^2}\sqrt{L^2 - k^2} \ , \tag{38}$$

which, when substituted back into Eq. (37) leads to

$$a^2 = \frac{k^2}{a_0^2} z^2 + (mz + a_0)^2 \ . \tag{39}$$

For the atmosphere, Eq. (24) gives

$$\frac{k^2}{a_0^2} = \frac{Wt \times 10^{-7}}{513 \ a_0^2} \ ,$$

and Eq. (39) may be written

$$a^2 = \frac{Wt \times 10^{-7}}{513 \ a_0^2} z^2 + (mz + a_0)^2 \ . \tag{40}$$

It is immediately obvious from Eq. (39) or Eq. (40) that a is a minimum at a particular range if the beam is focused for that range. Thus a is a minimum at range z when

$$m = -\frac{a_0}{z} \ . \tag{41}$$

As a practical matter the improvement achievable by optical focusing may be trivial. This will be the situation if the spread due to thermal blooming (or other mechanisms) is large compared to the original aperture. In Eq. (39), a is little affected by a_0 if $kz/a >> a_0$. This will be illustrated by a numerical example. For Wt = 1850 joules and a_0 = 10 cm, Eq. (40) becomes

$$a^2 = 36 \times 10^{10} z^2 + (10 + mz)^2 \ . \tag{42}$$

This equation is plotted in Fig. 6 for the three slopes $m = 0$, $m = -2.5 \times 10^{-5}$, and $m = -5 \times 10^{-4}$ corresponding to no focusing, a 4-km focal point, and a 0.2-km focal point.

THE FAR FIELD OF A THERMALLY DEFOCUSSED BEAM

The far field refers to the final angular distribution of beam energy relative to the beam axis. In this analysis, diffraction effects are neglected, and the initial beam is assumed to be a plane wave with a Gaussian radial power distribution at the aperture. The assumption of a Gaussian distribution is not necessary but is consistent with the earlier model. (The appendix treats a non-Gaussian, finite distribution.) Because of the cylindrical symmetry, each ray travels in a plane passing through the beam axis, and the refractive index gradient direction lies in this plane. Under these conditions Snell's law for a stratified medium may be applied if the down-beam flaring of the heated region is neglected. Snell's law says that if the direction of the refractive index gradient is

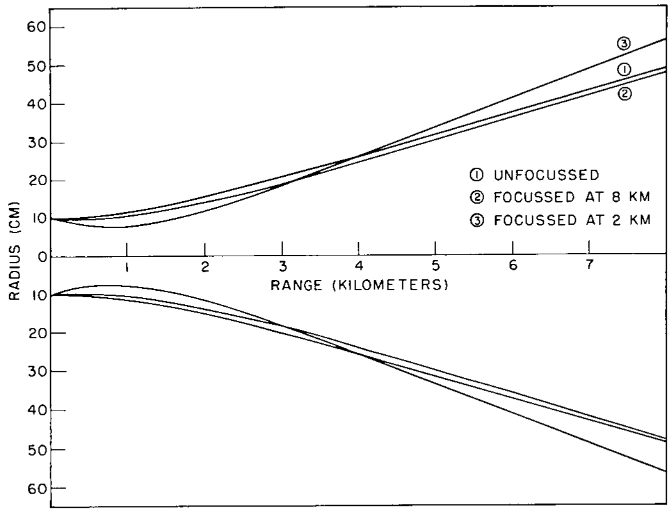

Fig. 6 - Effect of optical focusing on the beam profile

constant for a ray passing through a nonhomogeneous medium, then $n \sin \phi$ is constant, where ϕ is the angle between ray direction and gradient direction and n is the index of refraction of the medium. Let ψ be the angle between a ray and the beam axis in the far field, and let the ray start parallel to the beam axis from a aperture point where the index is n. Applying Snell's law to find the far-field angle in terms of the near-field index gives

$$\cos \psi = \frac{n}{n_0} . \tag{43}$$

From Eq. (11), for small ψ

$$1 - \frac{\psi^2}{2} = \frac{1}{n_0} \left[n_0 - (n_0 - 1) \frac{\alpha t}{C_p \theta \rho} \frac{W}{\pi a_0^2} e^{-r^2/a_0^2} \right] \tag{44}$$

and, with $n_0 \approx 1$,

$$\psi = \left[2(n_0 - 1) \frac{\alpha t}{C_p \theta \rho} \frac{W}{\pi a_0^2} \right]^{1/2} e^{-r^2/2a_0^2} . \tag{45}$$

Also

$$\frac{d\psi}{dr} = - \frac{r}{a_0^2} \psi . \tag{46}$$

Now if $S(\psi)$ is the far-field radiant intensity,

$$S(\psi)\, 2\pi\psi (-d\psi) = P(r)\, 2\pi r \, dr \tag{47}$$

or

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$$\psi S(\psi) \frac{d\psi}{dr} = -P(r)\, r \,, \tag{48}$$

and from Eq. (46)

$$\psi S(\psi) \left(- \frac{r}{a_0^2} \psi \right) = -P(r)\, r$$

or

$$S(\psi) = \frac{a_0^2}{\psi^2} P(r) \,. \tag{49}$$

Substituting for P and ψ from Eqs. (6) and (45) gives

$$S(\psi) = \frac{C_p \theta \rho a_0^2}{2(n_0 - 1)\alpha t} \tag{50}$$

with

$$0 \le \psi \le \left[2(n_0 - 1) \frac{\alpha t}{C_p \theta \rho} \frac{W}{\pi a_0^2} \right]^{1/2}$$

and $S(\psi) = 0$ elsewhere. The upper bound for ψ follows from Eq. (45) with $r = 0$. The rays from the point with greatest index change show the maximum angular deviation.

The far field is described as starting from a point of infinite intensity and spreading as a disk with radius proportional to $t^{1/2}$. The intensity within the disk is spatially uniform but varies with time as t^{-1}. In the far field the beam is radially inverted, or turned inside out. The edge of the beam is formed by the rays from the center of the initial beam, and ray crossing is complete.

When focusing is used, ray crossing may be induced in the near field, and in many situations the far-field equations will allow quick estimates of what to expect. An example is given below of the application of the far-field equations to the situation where the receiving area is smaller than the initial beam. Focusing is assumed.

When the received beam is smaller than the receiving area, the total beam power may be considered to be delivered. Let the receiving area be a disk of angular diameter 2ϕ. From Eq. (45) the beam will expand to the receiving area in time

$$t_1 = \frac{C_p \theta \rho \pi a_0^2 \phi^2}{2(n_0 - 1)\alpha W} \,, \tag{51}$$

and the total energy delivered at some subsequent time t will be

$$E = Wt_1 + \pi\phi^2 \int_{t_1}^{t} \frac{C_p \theta \rho a_0^2}{2(n_0 - 1)\alpha} \frac{dt}{t}$$

$$= Wt_1 + Wt_1 \log_e \frac{t}{t_1}, \quad t_1 < t$$

$$= Wt, \quad 0 \le t \le t_1$$

$$= Wt_1 \left(1 + \log_e \frac{t}{t_1}\right), \quad t_1 < t . \tag{52}$$

Letting

$a_0 = 100$ cm,

d = receiving diameter = 100 cm,

R = range = 10^6 cm, and

$t_1 = 1$ sec,

one has

$$\phi = 5 \times 10^{-5} ,$$

$$W = 1.5 \times 10^4 \text{ watts} ,$$

and

$$\begin{cases} E = 1.5\,\pi \times 10^4\, t , & 0 \le t \le 1 , \\ E = 1.5\,\pi \times 10^4 (1 + \log_e t) , & 1 < t . \end{cases} \tag{53}$$

Equations (53) are plotted in Fig. 7. Energy on target increases linearly until the beam area equals the target area at time t_1. After t_1 the effect of thermal blooming slows energy deposition to a logarithmic increase with time.

ASYMPTOTIC BEHAVIOR OF THE MODEL EQUATION

The derivation of Eq. (39) insures that in the near field, i.e., as $z \to 0$, the predicted slopes and curvatures will be correct. It is easily shown that this description is asymptotically valid in the far field, i.e., as $z \to \infty$.

Applying Snell's law for a stratified medium gives

$$n \cos \beta_1 = n_0 \cos \beta_2 , \tag{54}$$

where n is the index at $z = 0$ and $r = a_0$, $\beta_1 = m$ is the initial slope of the profile ray, and β_2 is the far-field slope of the profile ray.

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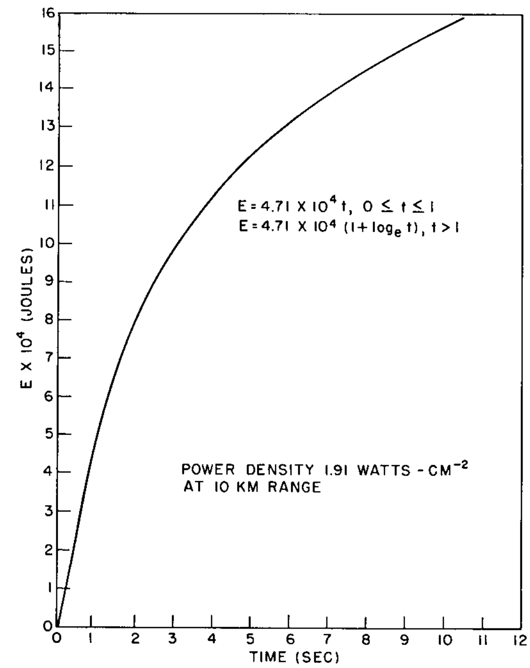

Fig. 7 - Received energy determined from far-field equations (power density of 1.91 watts-cm^{-2} at a 10-km range)

Expanding gives

$$n - \frac{nm^2}{2} = n_0 - \frac{n_0\beta_2^2}{2} . \tag{55}$$

But from Eq. (11) for $r = a = a_0$

$$n_0 - n = \frac{(n_0 - 1)\,\alpha t W}{C_p \theta \rho \pi a_0^2 e} = \frac{k^2}{2\,a_0^2} ; \tag{56}$$

so

$$\frac{n_0\beta_2^2}{2} = \frac{k^2}{2\,a_0^2} + \frac{nm^2}{2} \tag{57}$$

and, since $n_0 \approx n \approx 1$,

$$\beta_2 = \sqrt{\frac{k^2}{a_0^2} + m^2} . \tag{58}$$

This is the asymptotic form of Eq. (39) as $z \to \infty$, i.e.,

$$\beta_2 = \lim_{3\to\infty} \frac{a}{z}$$

$$= \lim \left[\frac{k^2}{a_0^2} + \frac{(mz + a_0)^2}{z^2} \right]^{1/2}$$

$$= \sqrt{\frac{k^2}{a_0^2} + m^2} \ .$$

CONCLUSIONS AND SUMMARY

A model equation (Eq. (39) has been derived to describe the phenomenon of thermal blooming in static air. The equation describes the path of a profile ray with the correct slope and curvature in the near field and the proper direction in the far field. In the intermediate field the solution to the ray path is approximate.

Aside from the approximations made in the derviation of the model equation, which should not invalidate qualitative or rough quantitative predictions, one important environmental assumption is made. This is the assumption of no wind and static air, which represents the worst situation from the standpoint of thermal blooming. The figures obtained for static air will be extreme numbers that can serve as references for the improvement due to wind. The static air solution is a transient solution involving time. When wind is present, it will be more convenient to look for steady state solutions. This will be done in a future report.

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Appendix

FAR FIELD WITH FINITE APERTURE

The far-field pattern refers to the directional intensity distribution of the aperture considered as a point source, and, since diffraction is being neglected, it is here generated solely by the geometrical optics of the heated atmosphere. The detailed distribution in the far field is expected to be a function of the radial power distribution at the aperture. The far field for a radial Gaussian distribution has been found to be a disk of uniform spatial intensity but starting as a point and spreading in time. It will be of interest to examine the far field for another functional distribution of power density and in particular for a distribution that sharply defines a finite aperture. Let

$$\begin{cases} P = P_0 \left(1 - \dfrac{r^2}{r_0^2}\right) , & r \le r_0 \\ P = 0 , & r > r_0 \end{cases} \tag{A1}$$

where r_0 is the radius of a finite aperture.

In place of Eq. (45) we will now have

$$\psi^2 = 2 (n_0 - 1) \frac{\alpha t}{C_p \theta \rho} P_0 \left(1 - \frac{r^2}{r_0^2}\right) \tag{A2}$$

or, with

$$K = \frac{2 (n_0 - 1) \alpha t}{C_p \theta \rho} ,$$

$$\psi^2 = K P_0 \left(1 - \frac{r^2}{r_0^2}\right) = KP , \tag{A3}$$

which, when differentiated with respect to r, gives

$$\psi \frac{d\psi}{dr} = - \frac{K P_0 r}{r_0^2} . \tag{A4}$$

Equation (48) still applies, i.e.,

$$S(\psi) \psi \frac{d\psi}{dr} = -P(r) \, dr ,$$

and making use of Eq. (A4) gives

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$$S(\psi)\left(-\frac{KP_0 r}{r_0^2}\right) = -\frac{\psi^2}{K} r \tag{A5}$$

or

$$S(\psi) = \frac{r_0^2}{K^2 P_0}\psi^2 , \tag{A6}$$

where from Eq. (A2)

$$\psi_{max} = (KP_0)^{1/2} .$$

To compare Eqs. (A5) and (A6) with Eq. (50) let a_0 be related to r_0 so that beam power W and maximum power density P_0 are the same for the two distributions. For the finite aperture,

$$W = 2\pi \int_0^{r_0} Pr\, dr$$

$$= 2\pi P_0 \int_0^{r_0} \left(r - \frac{r^2}{r_0^2}\right) dr \tag{A7}$$

or

$$W = \frac{\pi P_0 r_0^2}{2} .$$

For the Gaussian aperture it has been shown that

$$W = \pi P_0 a_0^2 , \tag{A8}$$

and the condition for matching W's and P_0's is that

$$a_0^2 = \frac{r_0^2}{2} . \tag{A9}$$

Rewriting Eq. (50) in terms of K and r_0 gives for the Gaussian distribution

$$\begin{cases} S(\psi) = \dfrac{a_0^2}{K} = \dfrac{r_0^2}{2K} , & 0 \le \psi \le \sqrt{KP_0} \\ S(\psi) = 0 , & \psi > \sqrt{KP_0} \end{cases} \tag{A10}$$

and for the finite aperture

$$\begin{cases} S(\psi) = \dfrac{r_0^2}{K^2 P_0}\,\psi^2 , & 0 \le \psi \le \sqrt{KP_0} , \\[2ex] S(\psi) = 0 & , \quad \psi > \sqrt{KP_0} , \end{cases} \tag{A11}$$

where in each case

$$K = \frac{2\,(n_0 - 1)\,\alpha t}{C_p\,\theta\rho} .$$

At the same instant of time, it is seen that the far-field disks have the same diameters as expected but that the intensity within the disk is not uniform for the finite aperture. It varies as ψ^2 from $S = 0$ for $\psi = 0$ to $S = r_0^2/K$ for $\psi = \sqrt{KP_0}$. The disk edge intensity is thus twice the uniform value of intensity found for the Gaussian distribution.

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13. ABSTRACT

Thermal blooming of a laser beam is described by a model equation for the trajectory of a selected profile ray. The model equation is applied to an examination of optical focusing for increasing transmitted power density. It is shown that the model equation is asymptotic in the far field to the geometric far field solution based on Snell's law for a stratified medium.

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