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SINGLE TUBE HARMONIC GENERATOR DESIGN

by

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CONTENTS

ABSTRACT

This report takes the expressions for the Fourier coefficients as known to the analysis art for several waveforms and expresses them in a form more suitable for harmonic generator design. Three waveforms, two of them obtainable by simple grid circuit distortion of a sine wave drive, are considered and their relative worth determined for use in one-tube harmonic generators. In addition, some of the effects of grid drive amplitude and grid bias are discussed relative to the three wave-forms, isosceles triangle, symmetrical trapezoid, and the fractional sine wave.

PROBLEM STATUS

This is an interim report on this problem: work is continuing.

AUTHORIZATION

This problem was initiated at the request of the Bureau of Aeronautics and has the Bureau number of TED NRL 31A262.

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INTRODUCTION

The problem of frequency multiplication is considered from the standpoint of simplicity of the multiplier circuit. With this restriction it has been found that certain design factors reveal themselves to be worthy of more emphasis than has been given them in prior art. Former authors have been primarily interested in the value of the Fourier series coefficients, C_n, as a function of harmonic number n. In designing multipliers, particularly those of simplified circuit types, maximum performance for a specified harmonic in the output is of primary importance.

The following theoretical treatment deals with the fractional sine wave, the isosceles triangle wave, and the symmetrical trapezoid wave. The isosceles triangle waveform is considered for comparison purposes only. The other two are waveforms which serve as classification types for those waveforms easily obtainable from sine wave drive in grid circuits.

WAVEFORM DESIGN CONSIDERATIONS

The design of harmonic generators in general requires a consideration of:

1. Waveform distortion possibilities.
2. Circuits to obtain these waveforms.
3. Amplitude changes of the fundamental and its harmonics produced by the distortion.
4. The subsequent gain required to get the desired harmonic component up to the prescribed signal level.
5. Means for obtaining the specified gain.

The present paper is restricted to theoretical discussion of design factors which arise when one attempts to obtain in a circuit containing one tube, an output signal (at a specified harmonic of the input signal frequency) whose amplitude is equal to the amplitude of the input signal.

For the simplified circuit case, it is important to set forth the Fourier coefficient relations for the three waveforms and to emphasize some important facts. The subject matter of Figures 1, 2 and 3 has appeared in many articles and textbooks of prior literature.* These figures are reproduced in this paper for ready reference to the several waveform parameters hereinafter used.

* Reference Data for Radio Engineers, FTRL, p. 167

The three waveforms previously mentioned are obtainable in voltage form from the Fourier Series expansion

$$e = \sum_{n=o}^{\infty} C_n \cos nx, \qquad n = 0, 1, 2, \ldots .$$

Where e is the voltage and C_n the Fourier coefficients. The expressions corresponding to the three waveforms for the coefficients, C_n, are:

$$C_n = 2\,\frac{Af}{T}\left[\frac{\sin\frac{n\pi f}{T}}{\frac{n\pi f}{T}}\right]^2 \tag{1}$$

for the isosceles triangle wave in Figure 1,

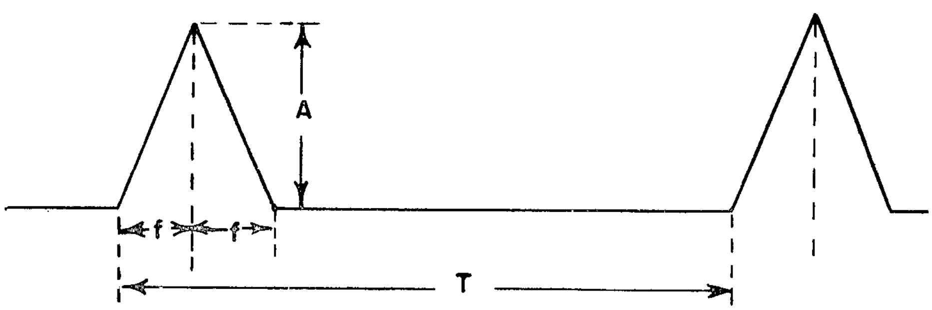

Fig. 1 Isosceles Triangle Wave

$$C_n = \frac{A}{n}\,\frac{d}{T}\,\frac{1}{1-\cos\frac{\pi d}{T}}\left[\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)\frac{\pi d}{T}} - \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)\frac{\pi d}{T}}\right] \tag{2}$$

for the fractional sine wave in Figure 2,

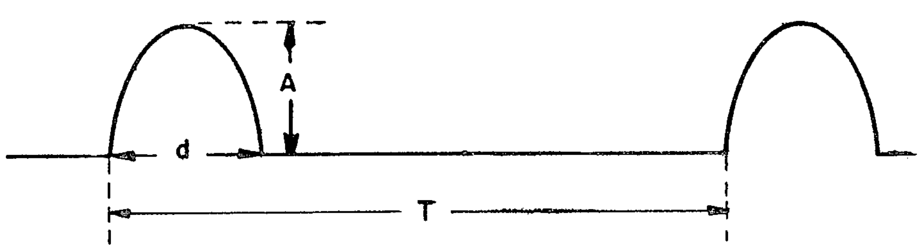

Fig. 2 Fractional Sine Wave

and

$$C_n = 2A\frac{f+d}{T}\left[\frac{\sin\frac{n\pi f}{T}}{\frac{n\pi f}{T}}\right]\left[\frac{\sin\frac{n\pi(f+d)}{T}}{\frac{n\pi(f+d)}{T}}\right] \tag{3}$$

for the symmetrical trapezoid waveform,

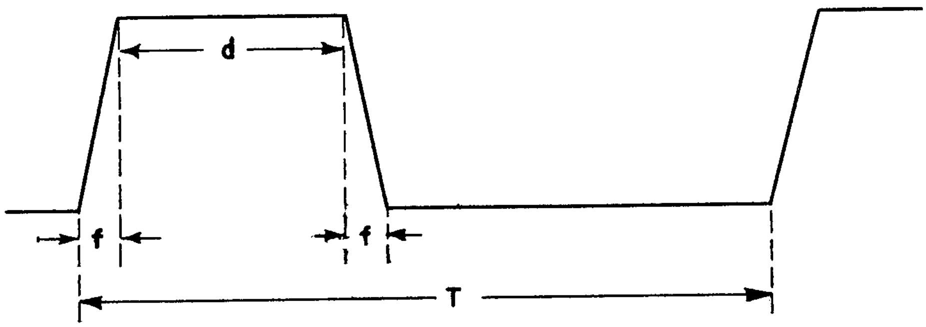

Fig. 3 Isosceles Trapezoid Wave

As previously mentioned, the coefficient (C_n/A) has been of interest to numerous authors, and graphs of this ratio, or related functions, have been published.†,‡,§ Figures 4, 5, and 6 are particular functions which relate to the present treatments. Figures 4 is a plot for the isosceles triangle wave of (nC_n/A) as a function of angle, Figure 5 is a plot for the fractional sine wave of (C_n/A) as a function of angle, and Figure 6 is a plot of (nC_n/A) as a function of angle for the symmetrical trapezoid wave. Each of the waveforms and the related important functions will be treated in turn.

Isosceles Triangle Wave

The expression for C_n of this particular waveform is given by equation (1). This expression shows C_n as a function of n and f only. Rearranging equation (1) the ratio (nC_n/A) can be expressed by

$$\frac{nC_n}{A} = \frac{2}{\pi}\left[\frac{\sin\frac{n\pi f}{T}}{\frac{n\pi f}{T}}\right]\sin\frac{n\pi f}{T} = \frac{2}{\pi}\,\frac{T}{n\pi f}\left[\sin\frac{n\pi f}{T}\right]^2 \tag{4}$$

Equation (4) is a form which can reveal a particular design feature. Figure 4, which is a plot of (nC_n/A) as a function of (nf/T), is a universal curve suitable for use with any desired harmonic number. Differentiating equation (4) to get maximum (nC_n/A), the following expressions and conclusions are evident.

† Stansel, F. R., BSTJ, Vol. 20, 1941, p. 332

‡ Terman, F. E., Radio Engineer's Handbook, p. 212

§ Lattin, W. J., Vol. 33, Number 1, November 1945, Proc. IRE

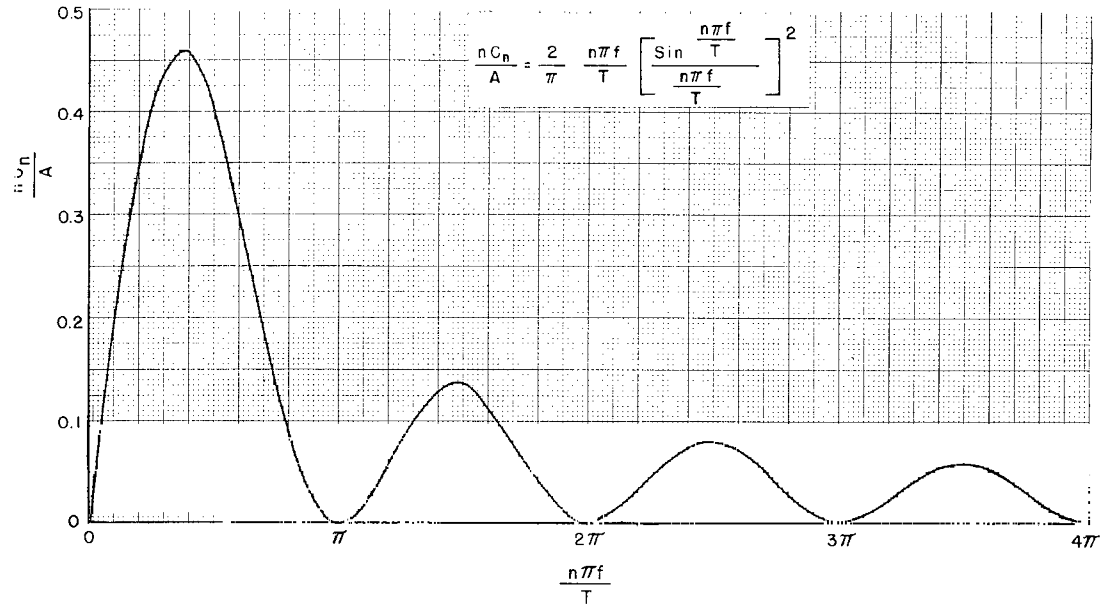

Fig. 4 Design Graph for Isosceles Triangle Wave Distortion

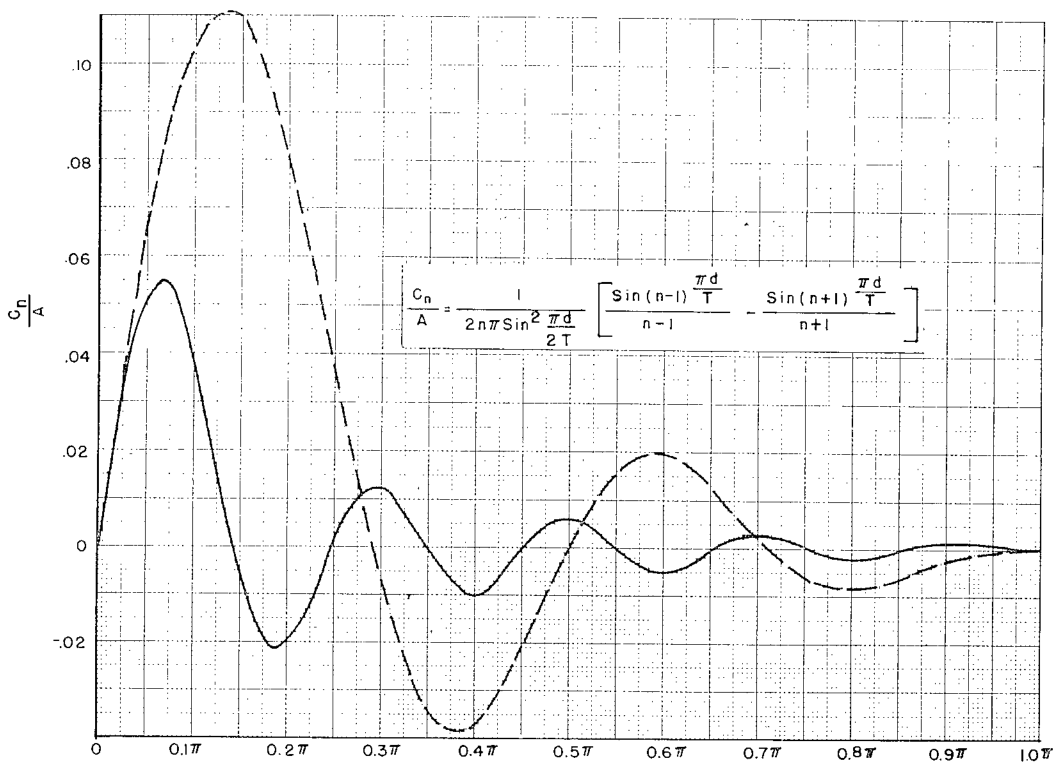

Fig. 5 Design Graph for Fractional Sine Wave Distortion

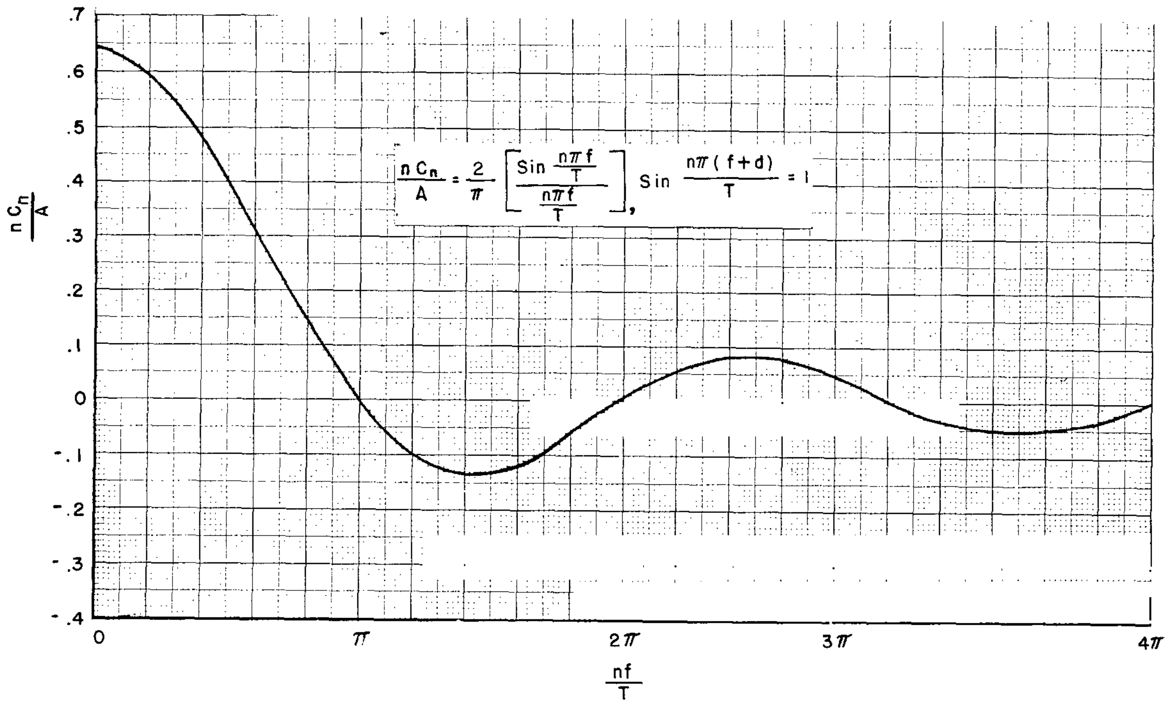

Fig. 6 Design Graph for Isosceles Trapezoid Distortion

Let $\frac{n\pi f}{T} = \theta,$

then $\frac{nC_n}{A} = \frac{2}{\pi}\,\frac{\sin^2\theta}{\theta}\,,$

and $\frac{d}{d\theta}\left(\frac{nC_n}{A}\right) = \frac{2}{\pi}\sin\theta\left[\frac{2\,\theta\cos\theta - \sin\theta}{\theta^2}\right] = \theta$

$$2\,\theta\cos\theta = \sin\theta, \ \tan\theta = 2\theta.$$

The smallest value of θ satisfying this equation is $\theta = 67°$. It is evident that decreasing θ below 67° is not desirable.

At $\theta = 0, \ \frac{nC_n}{A} = \frac{2}{\pi}\,\theta = 0,$

and $\frac{d}{d\theta}\left(\frac{nC_n}{A}\right) = \frac{2}{\pi}\theta\left[\frac{2\,\theta - \theta}{\theta^2}\right] = \frac{2}{\pi}$

At $\frac{n\pi f}{T} = 67°, \ \frac{nC_n}{A} = 0.46$

It is also to be noted that for optimum θ, $\frac{C_n}{A} \propto \frac{1}{n}$. In this case $\frac{C_n}{A} = \frac{0.46}{n}$.

Fractional Sine Wave

The fractional sine wave, equation (2), does not lend itself to such compact graphical expression. However, in Figure 5 two members of a family of curves relating C_n/A to d/T for two values (n = 5, n = 10) of the parameter n are given. Here the maximum with the largest value of C_n/A, as determined from Figure 5, comes at about 125°/n. This is very close to the value secured for the isosceles triangle wave when one remembers that 2f is comparable to d. A form of equation (2) expressing the ratio C_n/A which was used for plotting purposes is:

$$\frac{C_n}{A} = \frac{1}{n\pi} \frac{1}{2\sin^2\frac{\pi d}{2T}} \left[\frac{\sin(n-1)\frac{\pi d}{T}}{n-1} - \frac{\sin(n+1)\frac{\pi d}{T}}{n+1} \right] \qquad (5)$$

Symmetrical Trapezoid Wave

The trapezoid wave coefficients involve the use of two shaping parameters f and d making completely adequate graphical presentation difficult. However, if equation (3) is expressed in the form

$$\frac{nC_n}{A} = \frac{2}{\pi} \left[\frac{\sin\frac{n\pi f}{T}}{\frac{n\pi f}{T}} \right] \left[\sin \frac{n\pi(f+d)}{T} \right] \qquad (6)$$

and it is assumed that it will always be possible to make $\sin\ n\pi(f+d)/T = 1$ then a universal curve, Figure 6, can be drawn for the remaining factors by a plot of

$$\frac{nC_n}{A} = \frac{2}{\pi} \left[\frac{\sin\frac{n\pi f}{T}}{\frac{n\pi f}{T}} \right] \qquad (7)$$

Figure 6 gives nC_n/A as a function of nf/T. No matter what value d has, the graph will have the same shape. It's overall size will be proportional to $\sin n\pi(f+d)/T$. It is also important to notice that nC_n/A increases very slowly when nf/T goes from $\pi/2$ to zero. It thus proves to be poor economy to decrease nf/T below an angle of about 60°.

SIGNAL DISTORTION METHODS

It was previously pointed out that simplicity of circuit was a paramount consideration in the work which initiated this analysis. In such circuits, it is not practical to consider the use of a triangular wave, because the shaping process requires the use of additional tubes. For instance, if a frequency multiplication of 10 is required, it can be readily obtained by using two tubes in the usual manner, one doubling and one quintupling. The latter would be just as satisfactory as using a tube for shaping and another in the fashion herein disclosed. In line with the latter, it can be said that if considerations of proper waveform in the distorting circuit do not lead to a circuit that requires only one tube and a few other components, the advantages of the subject analysis are of secondary importance.

With the foregoing limitations one can consider the problem of distorting a sine wave.

Simple methods for distorting a sine wave are limited in number. Large grid bias can be used to obtain a fractional sine wave. A trapezoid wave, or a closely allied wave form, can be secured by a combination of grid bias and clipping due to grid current flow. In choosing between these two types, another consideration appears; of importance in this choice is the ratio of pulse amplitude $A_1 + A_2$ to driving Voltage E, Figures 7 and 8.

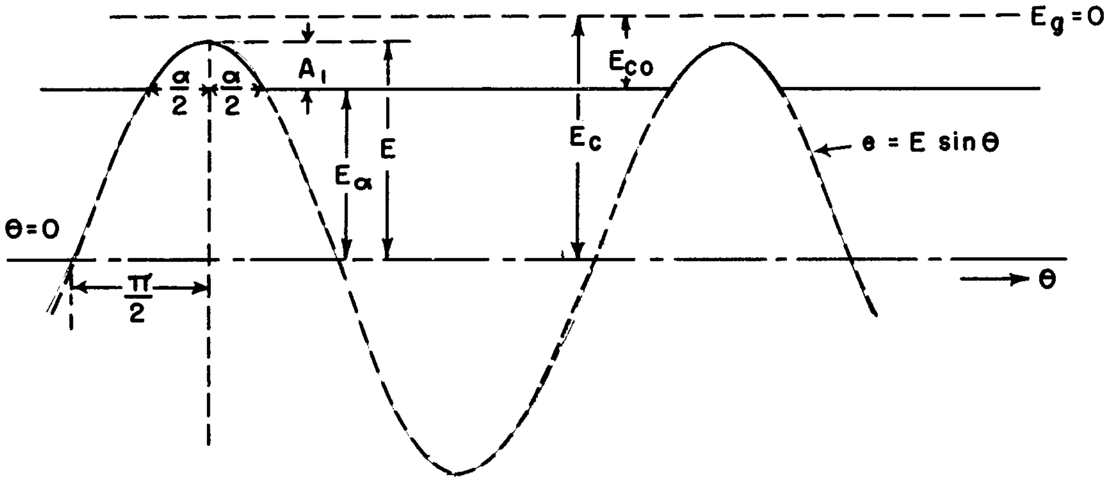

Fig. 7 Definition of Symbols for Fractional Sine Wave

In Figure 7, α is the angle during which the grid potential permits the tube to conduct, where E_{co} is the grid voltage required for cut off, E_c is the grid bias, and E_g is the grid to cathode voltage.

Thus

$$A_1 = E - E_\alpha = E \sin\frac{\pi}{2} - E \sin\left(\frac{\pi}{2} + \alpha\right)$$

or

$$A_1 = E\left[\sin\frac{\pi}{2} - \sin\left(\frac{\pi}{2} + \alpha\right)\right] = E(1 - \cos\alpha),$$

since

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \sin\frac{\pi}{2}\cos\alpha + \cos\frac{\pi}{2}\sin\alpha = \cos\alpha \quad ,$$

$$A_1 = E\left[1 - \left(1 - \frac{\alpha^2}{2!} + \ldots\right)\right]$$

$$\frac{A_1}{E} = \frac{\alpha^2}{2!} \tag{8}$$

Contrast this latter with the situation prevailing for trapezoid shaping, where the flat top can be obtained by grid current flow and the shaping at the base by high grid bias. Figure 8 presents the details.

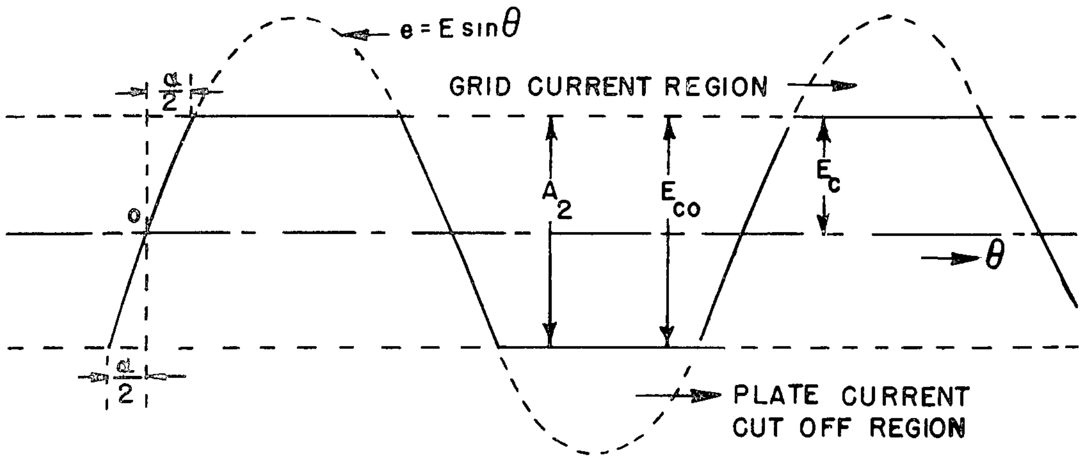

Fig. 8 Definition of Symbols for Isosceles Trapezoid Wave

$$A_2 = 2E \sin \frac{\alpha}{2}$$

If α is small, i.e. n large,

$$A_2 = \alpha E, \frac{A_2}{E} = \alpha \qquad (9)$$

The values of α in the two cases are nearly equal, so from equations (8) and (9)

$$\frac{A_1}{A_2} \approx \frac{\frac{\alpha^2}{2}}{\alpha} = \frac{\alpha}{2} \qquad (10)$$

For n = 10 the value of α should be 12.5° or .218 radians and $A_1/A_2 \approx .11$, $A_2/A_1 \approx 9$.

The functions plotted (Figures 5 and 6) show that the angle, α, required for optimum output at a specified harmonic, n, is approximately given for large n, by

$$\alpha = \frac{125^\circ}{n} = \frac{2.2 \text{ radians}}{n}$$

As an attempt is made to use higher values of n, it becomes increasingly important to try to utilize the rapid rise in the driving voltage when θ is close to $J\pi$ rather than the comparatively slow rise near $K\ \pi/2$. (J and K are intergers.) For smaller values of the harmonic index, n, the advantage ratio A_2/A_1 gained by using trapezoid shaping, is lost.

It is possible to make a good estimate of the voltage loss suffered by both waveforms in the shaping process. Both fractional sine wave shaping and trapezoid shaping result in values of C_n/A given approximately (by reference to Figures 5 and 6) for large n, by

$$\frac{C_n}{A} = \frac{0.5}{n}$$

For fractional sine wave shaping

$$A_1 = \frac{E\alpha^2}{2} = \frac{E(2.2)^2}{2n^2}$$

$$\frac{C_n}{E} = \frac{(0.5)(2.2)^2}{2n^3} \approx \frac{1}{n^3}$$

and for trapezoid shaping

$$A_2 = E\alpha = \frac{E2.2}{n}$$

therefore,

$$\frac{C_n}{E} = \frac{(0.5)(2.2)}{n^2} \approx \frac{1}{n^2}$$

In applications where the desired frequency multiple is 10 or more, trapezoid shaping is required. In fact, even for this type of shaping a single tube frequency multiplier cannot be expected to multiply frequency by much more than ten in most frequency ranges, without loss of voltage amplitude. This is due to the fact that stable gains of 100 at the output frequency are not easy to obtain in many of the frequency ranges where multiplication is attempted. If some regeneration is permitted, factors larger than $n = 10$ can easily be secured, but unless great care is exercised, instability is invited by such design.

It is obvious that α must be properly chosen to accentuate a desired harmonic. For either type of shaping, α is determined by the drive amplitude E, the grid bias E_c, and the cut off bias E_{co}.

CONCLUSIONS

(1) If the value of C_n/A for a specified harmonic only is considered, and if the wave shape of each type is adjusted so that maximum output for the specified harmonic is obtained, then the value of C_n/A is not particularly sensitive to the type of wave shape chosen.

(2) In distorting circuits using active components, such as vacuum tubes, it is often more important to obtain a desired voltage output rather than a particular power output so that power supplied to the plate circuit of the tube may be converted to sizeable quantities of power at the desired frequency. Here trapezoid shaping is most useful.

(3) Below a value $n = 10$, there is no particular choice to be made between trapezoidal and fractional sine wave shaping except that each one must be adjusted properly.

(4) Above $n = 10$, trapezoidal shaping is required, unless an input-output voltage ratio less than one is permissible. Regeneration may be used with care to increase the output-input voltage ratio.

(5) In some cases good efficiency of conversion on a power basis is required. W. R. Ferris,** of Radio Division I, NRL, has presented considerations to be applied where the

** W. R. Ferris, "Harmonic Generation," meeting of URSI and IRE, Washington, D.C., May 1947.

available power from the driving source is barely adequate, and where the distorting component is a passive device such as a rectifier crystal. Fractional sine wave shaping is best under such circumstances because of the low average power obtained in the pulse.

(6) The foregoing analysis, limited to a one tube multiplier multiplying by 10 or more should be subjected to verification on a similitude basis.

(7) In the case of fractional sine wave shaping, the correct conduction angle for a specified output frequency can be chosen by proper selection of tube type, grid drive, and grid bias.

(8) For symmetrical trapezoid shaping, the optimum choice of rise time, fall time, and flat top duration can be selected by correct choice of tube type, grid drive and grid bias.

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