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A Perturbative Treatment of Thermal Blooming in Pulsed Laser Beams

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ABSTRACT

The atmospheric propagation of pulsed, focussed, laser beams is calculated. The beam intensity is obtained by solving the equation of geometric optics and the equation of energy conservation simultaneously using a perturbative scheme. In this way the modification on propagation due to change caused by the index of refraction absorption of energy from the beam itself is included in a linear approximation. The problem of calculating the index of refraction change in a quiescent atmosphere is formulated generally with detailed expressions, valid for all times, given for the case of a paraxial Gaussian beam. Formulas for beam intensity are given as graphical results presented for the on-axis intensity of a focussed Gaussian beam.

PROBLEM STATUS

This is an interim report on a continuing problem.

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A PERTURBATIVE TREATMENT OF THERMAL BLOOMING IN PULSED LASER BEAMS

I. INTRODUCTION

The purpose of the present report is two-fold. First it presents a general method for handling problems of geometric optics in weakly inhomogeneous media, i.e. problems where the inhomogeneity may be considered to be small and treated as a perturbation on a homogeneous background. The method presented yields the intensity of the light beam being considered directly, without ray-tracing being necessary in the inhomogeneous medium once the relatively trivial case of straight line propagation in the homogeneous medium has been worked out. Focusing is thus readily included.

Second, the method is applied to the problem of thermal blooming, i.e. to the distortion of a light beam caused by energy deposition from the beam itself.[1,2] The non equilibrium case of thermal blooming of a pulsed laser beam in a quiescent atmosphere (taken to be a perfect gas)[3,4,5,6] is considered in detail, and applied to the case of a beam with Gaussian intensity profile.

Section II contains an exposition of the perturbative geometric optics method. Section III is a detailed outline of the hydrodynamics resulting from the beam-medium interaction. Section IV contains the results, while some of the calculational details are relegated to Appendices A through D.

II. GEOMETRIC OPTICS

The equation of geometrics optics is $(\nabla w)^2 = n^2$, or

$$\underset{\sim}{\nabla} w = n \underset{\sim}{\tau} \tag{1}$$

where w is the eikonal, n the index of refraction, and $\underset{\sim}{\tau}$ a unit vector tangent to the light ray paths.[7] In addition we have the conservation

of energy

$$\underset{\sim}{\nabla}\cdot(I\underset{\sim}{\tau}) = -\alpha I \tag{2}$$

where I is the beam intensity and α the absorption coefficient.

The perturbation scheme now proceeds by expanding all quantities as a series of terms of zeroth order, first order, etc. and solving the resultant equations order by order. In zeroth order we have

$$\underset{\sim}{\nabla} w_o = n_o \underset{\sim}{\tau}_o \tag{3}$$

$$\underset{\sim}{\nabla}\cdot(I_o \underset{\sim}{\tau}_o) = -\alpha I_o . \tag{4}$$

We shall suppose the solution of these equations to be known. That is, the light rays have known trajectories $r(r_o, s_o)$ where r_o is the initial point of the path and s_o is the distance measured along the path. The initial point r_o shall always be taken to be in the x-y plane. The intensity at an arbitrary point is then found in terms of the intensity in the x-y plane by solving Equation 4. This is done by using the fact that $\underset{\sim}{\tau}_o \cdot \underset{\sim}{\nabla} = \frac{d}{ds_o}$ so that Equation 4 is

$$\frac{dI_o}{ds_o} + I_o \underset{\sim}{\nabla}\cdot\underset{\sim}{\tau}_o = -\alpha I_o . \tag{5}$$

If J_o is the Jacobian of the transformation from variables x,y,z, to x_o, y_o, s_o then $\frac{dJ_o}{ds_o} = (\underset{\sim}{\nabla}\cdot\underset{\sim}{\tau}_o)J_o$,[8] so Equation 5 may be integrated to yield

$$I_o(r_o, s_o) = \frac{e^{-\alpha s_o}}{J_o}(I_o J_o)\Big|_{(s_o = 0)} . \tag{6}$$

The zero order equations are discussed in more detail in Appendix A.

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The first order equations are

$$\nabla w_1 = n_o \tau_1 + n_1 \tau_o \tag{7}$$

$$\nabla\cdot(I_o \tau_1 + I_1 \tau_o) = -\alpha I_1 . \tag{8}$$

Taking the scalar product of Equation 7 with τ_o and remembering that τ_o and τ_1 must be orthogonal in order that τ be a unit vector gives $\frac{dw_1}{ds_o} = n_1$. Integrating this equation yields

$$w_1(r_o, s_o) = w_1(r_o, o) + \int_o^{s_o} n_1 \, ds_o' . \tag{9}$$

Equation 7 now yields τ_1 in the form $\tau_1 = \frac{1}{n_o} \nabla w_1 - n_1 \tau_o$. This may be simplified by resolving the gradient operator into components along and perpendicular to the zero order trajectories. Writing

$$\nabla = \tau_o(\tau_o \cdot \nabla) - \tau_o \times (\tau_o \times \nabla) \equiv \tau_o \frac{d}{ds_o} + \nabla_p$$

then

$$\tau_1 = \frac{1}{n_o} \nabla_p w_1 . \tag{10}$$

The first order intensity equation, Equation 8, is integrated in the same way as the zeroth order equation was to yield

$$I_1 = \frac{e^{-\alpha s_o}}{J_o} (J_o I_1)(s_o = 0) - \frac{e^{-\alpha s_o}}{J_o} \int_o^{s_o} e^{\alpha s_o'} J_o \nabla_p \cdot (s_o \tau_1) ds_o' . \tag{11}$$

Equations 9, 10 and 11 yield the first order intensity for any given zero order problem, in which the first order index of refraction change is known. We shall consider only the case of paraxial beams focused a distance ℓ down the

z-axis. As discussed in Appendix A the distance s_o is then just z and $J_o = (1-\frac{z}{\ell})^2$, while effectively $\underset{\sim}{\nabla}_p = \frac{1}{1 - \frac{z}{\ell}} \underset{\sim}{\nabla}_o$ with $\underset{\sim}{\nabla}_o$ denoting differentiation with respect to the initial variables r_o. We also impose $I_1 = 0$ at $z = 0$. The equations appropriate to the case of paraxial beams are then

$$\underset{\sim}{T}_1 = \frac{1}{n_o} \frac{1}{(1 - \frac{z}{\ell})} \underset{\sim}{\nabla}_o \int_o^z n_1 \, dz' \tag{12}$$

and

$$I_1 = - \frac{e^{-\alpha z}}{(1 - \frac{z}{\ell})^2} \int_o^z e^{\alpha z'} (1 - \frac{z'}{\ell}) \underset{\sim}{\nabla}_o \cdot (I_o \underset{\sim}{T}_1) dz' \quad . \tag{13}$$

It should be remembered that the integrations are to be carried out along the rays, i.e. with r_o held fixed.

Equations 12 and 13 will be applied to the case where the first order index of refraction modification, n_1, is due to heating of the propagation medium by the beam itself. The hydrodynamical calculations necessary for the description of this index change are outlined in the next section. Before proceeding with the main development however, it is interesting to note a different application of these equations where the correct answer is known. That is, to the problem of diffraction of a paraxial, Gaussian beam propagating in a medium of constant index of refraction. To first order, that index of refraction change which will reproduce the effect of diffraction is [9]

$$n_1 = \frac{1}{2} \frac{n_o}{k^2} \frac{\nabla^2 \sqrt{I_o}}{\sqrt{I_o}} \quad . \tag{14}$$

Here k is $n_o \frac{\omega}{c}$, the wave number in the medium of constant index n_o. Taking

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the zero order intensity as

$$I_o = \frac{\mathcal{P}}{\pi a^2} \frac{e^{-r^2/[a^2(1-\frac{z}{\ell})]^2}}{(1-\frac{z}{\ell})^2} \tag{15}$$

and assuming no absorption, Equations 12 and 13 yield

$$I_1 = I_o\left(\frac{r_o^2}{a^2} - 1\right) \frac{(\frac{z}{ka^2})^2}{(1-\frac{z}{\ell})^2} \tag{16}$$

so that

$$I = I_o\left[1 + \left(\frac{r_o^2}{a^2} - 1\right) \frac{\left(\frac{z}{ka^2}\right)^2}{(1-\frac{z}{\ell})^2}\right] . \tag{17}$$

Equation 17 is just the first order expansion in powers of $\frac{1}{k^2}$ of the correct wave optics expression

$$I = \frac{\mathcal{P}}{\pi a^2 D^2} e^{-r^2/a^2D^2} \tag{18}$$

with

$$D^2 = (1-\frac{z}{\ell})^2 + \left(\frac{z}{ka^2}\right)^2 . \tag{19}$$

III. HYDRODYNAMICS

The hydrodynamic equations of a perfect gas in the presence of a beam of radiation I (r,t) which heats the medium are

$$\frac{d\rho}{dt} + \rho\, \underset{\sim}{\nabla}\cdot\underset{\sim}{v} \equiv 0 \tag{20}$$

$$\frac{\rho d\underset{\sim}{v}}{dt} = -\underset{\sim}{\nabla}\, p \tag{21}$$

$$\frac{dp}{dt} - \frac{\gamma p}{\rho}\frac{d\rho}{dt} = (\gamma-1)\,\alpha I \tag{22}$$

where ρ, p, and v are the density, pressure and velocity of the gas, γ is the ratio of specific heats, and α is the linear absorption coefficient of the medium. The time derivatives are substantial derivatives $\frac{\partial}{\partial t} + \underset{\sim}{v}\cdot\underset{\sim}{\nabla}$. Writing $\rho = \rho_o + \rho_1$, $P = P_o + P_1$, $\underset{\sim}{v} = \underset{\sim}{v}_o + \underset{\sim}{v}_1$ we linearize about the constant Po, ρo assumed to be present in the absence of the radiation. Restricting the treatment to a quiescent medium, $\underset{\sim}{v}_o$ is taken to be zero. Also, consistent with the perturbative approach, the intensity I responsible for the heating will be taken to be that of the undistorted beam. The linearized hydrodynamic equations are then

$$\frac{\partial \rho_1}{\partial t} + \rho_o \underset{\sim}{\nabla}\cdot\underset{\sim}{v}_1 = 0 \qquad (20)'$$

$$\rho_o \frac{\partial \underset{\sim}{v}_1}{\partial t} = - \underset{\sim}{\nabla} p_1 \qquad (21)'$$

$$\frac{\partial P_1}{\partial t} - \frac{c^2 \partial \rho_1}{\partial t} = (\gamma-1)\,\alpha I. \qquad (22)'$$

Here $c^2 = \frac{\gamma Po}{\rho_o}$ is the square of the speed of sound. Using Equations 20′ and 21′ we eliminate ρ_1 from Equation 22′ obtaining

$$\nabla^2 P_1 - \frac{1}{c^2}\frac{\partial^2 P_1}{\partial t^2} = - \frac{(\gamma-1)\alpha}{c^2}\frac{\partial I}{\partial t} \quad . \qquad (24)$$

This equation may be solved by using the appropriate Green's function,

$$P_1(r,t) = - (\gamma-1)\frac{\alpha}{c^2}\int G(rt,r't')\frac{\partial I}{\partial t'}(r',t')\, d^3r'dt' \qquad (25)$$

and then 23′ may be integrated directly to obtain ρ_1

$$\rho_1(r't) = - (\gamma-1)\frac{\alpha}{c^2}\int_{-\infty}^{t} I(r,t')dt' + \frac{1}{c^2}P_1(r,t) \quad . \qquad (26)$$

The boundary conditions are that all first order quantities are zero at $t = -\infty$ and the retarded Green's function is to be used. Since the velocity of light is so much larger than the velocity of sound the simplest approximation is to take the light velocity to be infinite. The beam is then regarded as an infinitely long heat source that is turned on and off at given times. Taking the z direction as the beam axis we assume beam gradients in the x-y plane to be much larger than those in the z-direction so it is appropriate to ignore all derivatives in z, in which case the problem is two dimensional, and the appropriate Green's function is just

$$\frac{-1}{2\pi} \frac{\theta(T-R/c)}{\sqrt{T^2 - R^2/c^2}} \tag{27}$$

where $R = r - r'$, $T = t - t'$ and c is the velocity of sound. All vectors are now to be taken as two dimensional and lying in the x-y plane. θ is the unit step function, $\theta(x) = 1$ for $x > o$; $\theta(x) = o$ for $x < o$. The pressure is now

$$P_1(r,t) = \frac{(\gamma-1)\alpha}{2\pi\, c^2} \int \frac{\theta(T-R/c)}{\sqrt{T^2 - R^2/c^2}} \frac{\partial I}{\partial t'}(r',t')\, d^2r'dt'. \tag{28}$$

The problem can also be treated in 3-dimensions with finite light velocity, and Appendix B has such a treatment. Essentially, Equation 28 remains valid with times t being changed to $t-z/c_L$ where c_L is the speed of light. We will continue to use Equation 28 as it stands, but remember that t means time measured relative to the time of arrival of the pulse at the z-plane being considered. Taking I as being turned on at $t = 0$ and off at $t = \tau$ with constant magnitude between,

$$I(r,t) = I(r)\theta(t)\theta(\tau-t) \ . \tag{29}$$

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Then

$$\frac{\partial I}{\partial t} = I(r)[\delta(t)-\delta(t-\tau)] \tag{30}$$

and

$$\rho_1(r,t) = -(\gamma-1)\frac{\alpha}{c^2}\int_0^t I(r)\theta(\tau-t)dt' + \frac{1}{c^2}P_1(r,t) \ . \tag{31}$$

For $t < \tau$ then,

$$P_1(r,t) = \frac{(\gamma-1)\alpha}{2\pi c^2}\int \frac{\theta(t - R/c)}{\sqrt{t^2 - \frac{R^2}{c^2}}} I(r')d^2r' \tag{32}$$

and

$$P_1(r,t) = \frac{(\gamma-1)\alpha}{c^2} I(r)t + \frac{1}{c^2}P_1(r,t) \ . \tag{33}$$

Equation 32 may be rewritten in a slightly different form for clarity of interpretation. Since the observation point $\underset{\sim}{r}$ is fixed we may transform the variable of integration in Eq. 32 to $\underset{\sim}{R} = \underset{\sim}{r}'-\underset{\sim}{r}$. Then

$$P_1(r,t) = \frac{(\gamma-1)\alpha}{2\pi c^2}\int \frac{\theta(t - R/c)}{\sqrt{t^2 - R^2/c^2}} I(r+R)d^2R \ . \tag{32$'$}$$

The pressure pulse at a point r is thus seen to be produced by the heat deposited by the beam at all points R which lie within the distance ct of the point r, i.e. which can be reached by a sound wave from point r.

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The long time behavior of the hydrodynamic variables can be readily derived from these equations. Assume that the beam intensity falls off fast enough in the x-y plane, so that there is an Rmax beyond which we can take the intensity to be essentially zero. Then for times so large that a sound wave from the most distant part of the source has had a chance to pass over the point r we have $t > \frac{Rmax}{c}$. The square root in Equation 32 may then be expanded in powers of R/ct and the dominant term yields

$$P_1 = \frac{(\gamma-1)\alpha}{2\pi c^2} \frac{\mathcal{P}}{t} \tag{34}$$

with $\mathcal{P} = \int I \, d^2 r$ being the power in the beam. Thus the pressure dies out and leaves the density with a linear behavior, $\rho_1 = (\gamma-1) \frac{\alpha}{c^2} It.$[4,5,6] The behavior for small times is more complex. However for distributions I(r) that can be expanded in a power series in r the pressure can be expanded in a power series in time. The details of the expansion are given in Appendix C. The results are

$$P_1(\underline{r},t) = (\gamma-1)\, \alpha t \sum_{n=o}^{\infty} \frac{1}{(2n+1)!} (ct)^{2n} (\nabla^2)^n I \,, \tag{35}$$

$$\rho_1(r,t) = \frac{(\gamma-1)\alpha t}{c^2} \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} (ct)^{2n} (\nabla^2)^n I \,. \tag{36}$$

Note that the n = o term of the pressure has cancelled the first term in the

density as given by Eq. 33 leaving at least a t^3 dependence.[4,5,6] It would be expected that any physically reasonable intensity distribution could be expanded as has been done here, but if a model is used in which the beam is truncated, then these expansions are only valid for points r which are at least a distance ct from the edge. Thus for short enough times, all but the outer layers of a truncated beam can be represented this way, while for ct greater than a, the radius of the beam, no point can be thus represented.

As an important example of the preceding development consider a Gaussian intensity profile,

$$I = \frac{\mathcal{P}}{\pi a^2} e^{-\frac{r^2}{a^2}} \tag{37}$$

with $\mathcal{P}$ the power in the beam and a the 1/e radius which is in general a function of z. Appendix D gives the details for this example. The result is

$$P_1(r,t) = (\gamma-1)\alpha t\, e^{-\frac{r^2}{a^2}} \frac{\mathcal{P}}{\pi a^2} \sum_{m=0}^{\infty} \frac{1}{(2n+1)!} \left(\frac{2ctr}{a^2}\right)^{2n} M(n+1,n+\tfrac{3}{2}, -(\tfrac{ct}{a})^2) \tag{38}$$

where M is the confluent hypergeometric function in the notation of Abramowitz and Stegun.[10] The density is

$$\begin{aligned} \rho_1 = & -\frac{(\gamma-1)\alpha}{c^2} \frac{\mathcal{P}}{\pi a^2} e^{-\frac{r^2}{a^2}} t \left[1 - M(1,\tfrac{3}{2},-(\tfrac{ct}{a})^2)\right] \\ & + \frac{(\gamma-1)\alpha}{c^2} \frac{\mathcal{P}}{\pi a^2} e^{-\frac{r^2}{a^2}} \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left(\frac{2ctr}{a^2}\right)^{2n} M(n+1,n+\tfrac{3}{2},-(\tfrac{ct}{a})^2) \end{aligned} \tag{39}$$

For small argument, $m(a,bx) \to 1 + \frac{a}{b} x$, so for short times, i.e. for $\frac{ct}{a}$ small, we find

$$P_1 = (\gamma-1)\alpha t \; e^{-r^2/a^2} \frac{\mathcal{P}}{\pi a^2} \tag{40}$$

$$\rho_1 = (\gamma-1)\alpha t^3 \; e^{-r^2/a^2} \frac{\mathcal{P}}{\pi a^2} \left(\frac{2}{3}\right)\left(\left(\frac{r^2}{a^2}\right) - 1\right) \tag{41}$$

in agreement with what is found using the lowest terms of Equations 35 and 36. The behavior for large times can be obtained using the properties of the hypergeometric functions for large negative argument, $m(a,b,x) \to \frac{\Gamma(b)}{\Gamma(b-a)} \left(-\frac{1}{x}\right)^a$.[10] Then, for $\left(\frac{ct}{a}\right)^2$ large we find

$$P_1 = \frac{(\gamma-1)\alpha}{\pi 2c^2} \frac{\mathcal{P}}{t} \tag{42}$$

and

$$\rho_1 = \frac{(\gamma-1)\alpha t}{c^2} \frac{\mathcal{P}}{\pi a^2} e^{-r^2/a^2} + \frac{(\gamma-1)\mathcal{P}}{2c^4 t \pi} \tag{43}$$

in agreement with the general results following from Equations 33 and 34.

The response to a uniform beam

$$I = \frac{\mathcal{P}}{\pi a^2} \theta\,(a-r)$$

may also be worked out. The result is a complex expression in terms of hypergeometric functions of one and two variables. However, the pressure and density on the z axis, r=o, are simple, and serve again to illustrate the general situation. The results follow directly from Equation 32′ and are for $ct < a$

$$P_1 = (\gamma-1)\alpha \frac{\mathcal{P}}{\pi a^2} t,$$

$$\rho_1 = 0$$

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and for $ct > a$ $$P_1 = (\gamma-1)\alpha \frac{\mathcal{P}}{\pi a^2} \left(t - \sqrt{\left(t^2 - \frac{a^2}{c^2}\right)}\right)$$

and $$\rho_1 = -(\gamma-1)\alpha \frac{\mathcal{P}}{\pi a^2 c^2} \sqrt{t^2 - \frac{a^2}{c^2}} \; .$$

Expanding the radicals in powers of $\frac{a}{ct}$ we see that the results for the long and short time regions are in agreement with the general expressions obtained previously.

The density changes given here can be related to index of refraction changes using the Lorentz-Lorenz law, which we write in the form

$$n - 1 = \frac{3}{2} N \rho$$

where N is the refractivity. If the ambient density is ρ_o and has associated with it an index of refraction n_o, then $n_o - 1 = \frac{3}{2} N \rho_o$. So if $n = n_o + n_1$ and $\rho = \rho_o + \rho_1$ we have

$$n_1 = \frac{3}{2} N \rho_1 . \tag{44}$$

Thus the first order density change defines a first order index of refraction change which is used to calculate beam intensity changes as outlined in the previous section.

IV. RESULTS

The results for the case of a paraxial, focused Gaussian beam will be presented in some detail. The zero order trajectories are

$$\underset{\sim}{r} = \left(1 - \frac{z}{\ell}\right)\underset{\sim}{r}_o \tag{45}$$

and the zero order intensity is

$$I_o = \frac{\mathcal{P}}{\pi a^2 D^2} e^{-\frac{r^2}{a^2 D^2}} \tag{46}$$

with

$$D^2 = \left(1 - \frac{z}{\ell}\right)^2 \; . \tag{47}$$

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We remark that a is now just a constant, the 1/e radius of the beam at the aperture. The z-dependence of the 1/e radius has been included in the factor D, so the notation has been altered from that used in Equations 37 through 43. Also, in all the results to be presented the exponential absorption factors have been neglected.

The general results, valid for all r and z are given first. For the short time case where

$$\rho_1 = \frac{2}{3}(\gamma-1)\alpha\left(\frac{r_o^2}{a^2} - 1\right)I_o$$

we find

$$\frac{I}{I_o} = 1 - \frac{4}{9\pi}\cdot\frac{3}{2}\frac{N(\gamma-1)\alpha t^3 \ell^2 \mathcal{P}}{a^6} e^{-\frac{r_o^2}{a^2}}\left(1-3\frac{r_o^2}{a^2}+\frac{r_o^4}{a^4}\right)\left(\frac{1-4D^3+3D^4}{D^4}\right) . \tag{48}$$

For the long time case where

$$\rho_1 = -\frac{(\gamma-1)\alpha t}{c^2} I_o$$

we find

$$\frac{I}{I_o} = 1 - \frac{2}{\pi}\cdot\frac{3}{2} N \frac{(\gamma-1)\alpha t}{c^2}\frac{\mathcal{P}}{a^4} e^{-r_o/a^2}\left(1-2\frac{r_o^2}{a^2}\right)\frac{z^2}{D^2} . \tag{49}$$

Integrated intensities may be compared as well. Consider the total intensity inside the circle $r \le \sigma D a$, i.e. the intensity originating from the area $r_o \le \sigma a$ at the aperture. We find

$$\frac{P_\sigma}{P_{o\sigma}} = 1 - \frac{4}{9\pi}\frac{3}{2} N(\gamma-1)\alpha t^3 \ell^2 \mathcal{P}\sigma^2 \frac{(1-\frac{1}{2}\sigma^2)}{1-e^{-\sigma^2}} e^{-2\sigma^2}\frac{(1-4D^3+3D^4)}{D^4} \tag{50}$$

and

$$\frac{P_\sigma}{P_{o\sigma}} = 1 - \frac{2}{\pi}\frac{3}{2} N \frac{(\gamma-1)\alpha t}{c^3}\frac{\mathcal{P}}{a^4}\frac{\sigma^2 e^{-2\sigma^2}}{1-e^{-\sigma^2}}\frac{z^2}{D^2} \tag{51}$$

for the short and long time cases respectively. In these expressions P_σ is the integral of I, $P_{o\sigma}$ that of I_o. The limit $\sigma = o$ reproduces the on-axis intensity ratios obtained by putting r = o in Equations 48 and 49. The limit $\sigma = \infty$ gives $P/P_o = 1$, which is just a statement of energy conservation. For the on-axis intensity it is also feasible to use the complete time dependent expression, Equation 39. To calculate the on-axis intensity, the density is needed to the 2nd order in r. From Equation 39 we have

$$n_1 = \frac{3}{2} N \frac{(\gamma-1)\alpha t}{c^2} \frac{\rho}{\pi a^2 D^2} e^{-r^2/a^2D^2}\left[1-M\left(1,\frac{3}{2}, -\left(\frac{ct}{aD}\right)^2\right)\right]$$

$$+ \frac{3}{2} N \frac{(\gamma-1)\alpha t}{c^2} \frac{\rho}{\pi^2 a^2 D^2} e^{-r^2/a^2D^2} \frac{1}{6}\left(\frac{2ctr}{a^2D^2}\right)^2 M\left(2,\frac{5}{2},-\left(\frac{ct}{aD}\right)^2\right) . \tag{52}$$

The two hypergeometric functions may be combined using Equation 13.4.4 of Reference 10, and defining $\nu = \frac{1}{D^2} = \frac{1}{\left(1-\frac{z}{\ell}\right)^2}$ then Equations 12 and 13 yield

$$\frac{I_1}{I_o} = -\frac{3}{2} N \frac{(\gamma-1)\alpha t}{c^2} \frac{\rho}{\pi a^4} 2\ell^2 \int_1^\nu \frac{\sqrt{\nu} - \sqrt{\nu'}}{\sqrt{\nu'}}\left[1-M\left(2,\frac{3}{2}, -\left(\frac{ct}{a}\right)^2 \nu'\right)\right] d\nu'. \tag{53}$$

The expansion of this expression for early and late times gives just the r = o limits of Equations 48 and 49. The form appropriate to an unfocussed beam is readily obtained from Equation 53 by letting $\nu \to 1 + 2\frac{z}{\ell}$ and $d\nu' \to \frac{2}{\ell} dz'$. We note also that in all cases the long time behavior is linear in time. This is not observed in the laboratory experiment of reference 11 for which the linearized hydrodynamics of this report is presumably inadequate.

Of primary interest is the intensity in the focal plane. In order to get finite results, diffraction must be included in some approximation. Unfortunately

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inclusion of diffraction to the first order, Equation 17, is no help in this regard. The procedure adopted here will be to take the distance ℓ at which the rays cross the axis to be slightly larger than the focal length f. Taking $\ell = f/(1 - \frac{f}{ka^2})$ yields trajectories

$$\underset{\sim}{r} = \left(1 - \frac{z}{f} + \frac{z}{ka^2}\right)\underset{\sim}{r}_o \quad . \tag{45}'$$

This gives the beam a spot radius of $\frac{f}{ka}$ at the focal distance, in agreement with the wave optics expression, Equation 18. The procedure adopted here should be a reasonable approximation in those cases where $\frac{z}{ka^2}$ is small. The procedure is not unique, however, and a discussion of this point is contained in reference 12 as are results obtained by different treatments of diffraction. In discussing results in the focal plane, it is convenient to make several changes of variables. We define $\nu' = \left(\frac{ka^2}{f}\right)^2 \sigma'$ so that σ' goes from zero to one, and also introduce the transit time of a sound wave across the beam at the focus

$$t_f = \frac{f}{kac} \tag{54}$$

and finally, a combination of constants having the dimension of power,

$$P_f \equiv \frac{\pi c^3}{\frac{3}{2} N (\gamma-1)\alpha k} \frac{a}{f} \quad . \tag{55}$$

The on-axis intensity ratio is

$$\frac{I}{I_o} = 1 - 2\frac{\rho}{\rho_f}\left(\frac{\ell}{f}\right)^2\left(\frac{t}{t_f}\right)\int_{\left(\frac{f}{ka^2}\right)^2}^{1} \frac{1 - \sqrt{\sigma'}}{\sqrt{\sigma'}}\left[1 - M\left(2, \tfrac{3}{2}, -\left(\frac{t}{t_f}\right)^2 \sigma'\right)\right] d\sigma' \quad . \tag{56}$$

Using the properties of the hypergeometric function given in the previous section the short time and long time limits of Equation 56 may be derived. They are

$$\frac{I}{I_o} = 1 - \frac{4}{9}\frac{\mathcal{P}}{\mathcal{P}_f}\left(\frac{t}{t_f}\right)^3 \tag{57}$$

and

$$\frac{I}{I_o} = 1 - 2\left(\frac{\mathcal{P}}{\mathcal{P}_f}\right)\frac{t}{t_f} \quad . \tag{58}$$

In the short time expression, Equation 57, higher order terms in $\frac{f}{ka^2}$ have been neglected.

Although time has been scaled according to t_f in the above equations, this is not the time unit most appropriate to the long time behavior. An examination of Equation 56 shows that the long time limit is only reached for times greater than the transit time of a sound wave across any part of the beam. The largest such transit time is $\frac{a}{c}$, the transit time across the aperture of the beam. Thus Equation 58 is only valid for times larger than $\frac{a}{c}$, which is a more appropriate scale for the long time behavior. This fact also places a restriction on the beam powers for which the present theory can be expected to be valid. Since $2\frac{\mathcal{P}}{\mathcal{P}_f}\frac{t}{t_f}$ must be less than 1, putting the time equal to $\frac{a}{c}$ yields

$$\mathcal{P} < \frac{1}{2}\mathcal{P}_f\frac{ct_f}{a} = \frac{1}{2}\frac{f}{ka^2}\mathcal{P}_f \quad .$$

Since $\frac{f}{ka^2}$ has been taken to be small, this critical power is much less than $\mathcal{P}_f$. Figure 1 shows a plot of the on-axis intensity ratio for $\frac{f}{ka^2} = 0.126$ and $\frac{\mathcal{P}}{\mathcal{P}_f} = 0.01$. The curves labelled short time and long time are plots of

Equations 57 and 58 respectively. The unlabelled curve is a graph of Equation 56 obtained by expanding the hypergeometric function in a series, integrating term by term, and numerically evaluating the resultant series. The horizontal axes show the time as scaled by t_f and by $\frac{a}{c}$. It is evident that the long time linear behavior is accurate to within a few percent for all times.

The approach to the long time limit shows up more readily when the power is not held constant, but rather the total energy in the pulse is fixed. Define

$$E_f = \mathcal{P}_f t_f = \frac{\pi c^2}{\frac{3}{2} N (\gamma-1)\alpha k^2} \quad . \tag{59}$$

Then taking the time t to be τ, the total pulse length, we have

$$\frac{\mathcal{P} t}{\mathcal{P}_f t_f} = \frac{E}{E_f}$$

where E is the total energy in the pulse. Equations 56, 57 and 58 are modified accordingly, and we have for the short and long pulse length behaviors

$$\frac{I}{I_o} = 1 - \frac{4}{9}\frac{E}{E_f}\left(\frac{\tau}{t_f}\right)^2 \tag{60}$$

and

$$\frac{I}{I_o} = 1 - 2\frac{E}{E_f} \quad . \tag{61}$$

The energy E_f is independent of the optical parameters of focal length and diameter so that Equation 61 predicts a long time limit which is dependent only on the pulse energy.

A criterion for the maximum pulse energy for which the present theory may be expected to be valid can be derived from Equation 61. Since $2\frac{E}{E_f}$ must be

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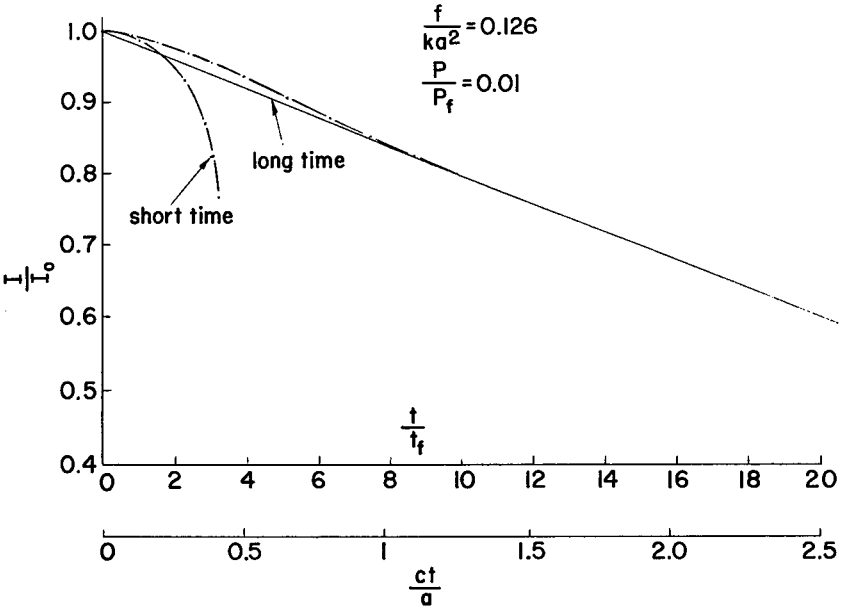

Fig. 1 - On-axis intensity ratio for fixed pulse power

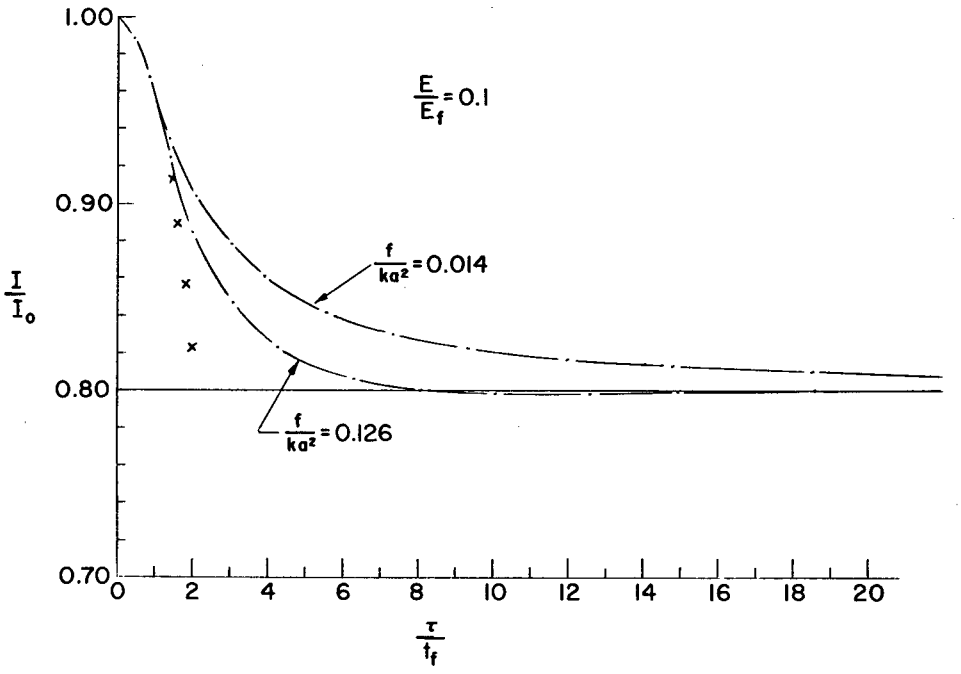

Fig. 2 - On-axis intensity ratio for fixed pulse energy

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less than 1 the criterion is simply

$$E < \frac{1}{2} E_f \ .$$

For example, for a 10.6 μm laser beam propagating in air with an absorption coefficient of 2×10^{-6} cm^{-1} the energy E_f is approximately 60 joules.

Figure 2 shows a graph of the on-axis intensity ratio for fixed pulse energy with $\frac{E}{E_f} = 0.1$ for beam parameters of $\frac{f}{ka^2} = 0.126$ and $\frac{f}{ka^2} = 0.014$. The horizontal axis is pulse length measured in units of t_f. The crosses indicate the short pulse intensity ratio as given by Equation 60. Since the pulse length is plotted in units of t_f Equation 60 is independent of $\frac{f}{ka^2}$ and the crosses denote the short pulse length behavior of both curves. Similarly, Equation 61 is the long pulse length limit of both curves and for $\frac{E}{E_f} = 0.1$ this limit is just the line $\frac{I}{I_o} = 0.8$.

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APPENDIX A

GEOMETRY OF THE UNPERTURBED RAYS

The geometric relations for the unperturbed rays are quite simple because they are not curved. The fact that the rays are focused does cause some complication, however, especially since the paraxial approximation is not made until the end. This appendix serves to define the geometry and give some of the details which ensue because of the focusing. To avoid an over abundance of subscripts, the subscript zero denoting zero-order quantities will not be written. Figure A1 defines the quantities being used. The trajectories are simply

$$\underset{\sim}{r} = \underset{\sim}{r}_o + s\,\underset{\sim}{\tau} \tag{A1}$$

with the tangent vector given in terms of the initial coordinates by

$$\underset{\sim}{\tau} = \frac{\underset{\sim}{\ell} - \underset{\sim}{r}_o}{d} \tag{A2}$$

where $\underset{\sim}{\ell} = \ell \underset{\sim}{e}_z$ and $d^2 = r_o^{\,2} + \ell^2$. In terms of the spatial coordinates,

$$\underset{\sim}{\tau} = \frac{\underset{\sim}{\ell} - \underset{\sim}{r}}{\sqrt{(\ell - r)^2}} \quad . \tag{A3}$$

In terms of coordinate components

$$\begin{aligned} x &= \left(1 - \frac{s}{d}\right) x_o \\ y &= \left(1 - \frac{s}{d}\right) y_o \\ z &= \frac{\ell}{d}\, s \end{aligned} \tag{A4}$$

from which the Jacobian is computed to be

$$J\begin{pmatrix} xyz \\ x_o y_o s \end{pmatrix} = \frac{xyz}{x_o y_o s} = \left(1 - \frac{s}{d}\right)^2 \frac{\ell}{d} \quad . \tag{A5}$$

It is instructive to calculate this also using the relation $\frac{dI}{ds} = (\underset{\sim}{\nabla} \cdot \underset{\sim}{\tau}) J$ with $\underset{\sim}{\nabla} \cdot \underset{\sim}{\tau}$ given by

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{\tau} = \frac{-2}{\sqrt{r - \ell)^2}} = \frac{-2}{d-s} \tag{A6}$$

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from Equation A3. The operator ∇_p of the text is calculated using the relations inverse to A4,

$$x_o = x/(1 - \frac{z}{\ell})$$

$$y_o = y/(1 - \frac{z}{\ell}) \tag{A7}$$

$$s = \frac{z}{\ell}\sqrt{\ell^2 + (x^2 + y^2)/(1 - \frac{z}{\ell})^2} .$$

After some manipulation we find

$$\nabla_p = \frac{1}{(1-s/d)}\left[\nabla_o + \frac{e_z}{\ell}(r_o \cdot \nabla_o) + \frac{r_o^2}{\ell d} e_z \frac{\partial}{\partial s} + \frac{r_o}{d}\frac{\partial}{\partial s}\right]. \tag{A8}$$

The paraxial approximation neglects terms of the order $\frac{r_o^2}{\ell^2}$ which results in $d = \ell$ and $s = z$.

Then

$$\nabla_p = \frac{1}{(1-\frac{z}{\ell})}\left[\nabla_o + \frac{e_z}{\ell} r_o \cdot \nabla_o + \frac{r_o}{\ell}\frac{\partial}{\partial z}\right]. \tag{A9}$$

In the text the 2nd term is neglected since it eventually occurs quadratically while the last term is neglected since it operates on the intensity which is assumed to have negligible gradients in the z-direction. However, to obtain the correct tangent vector, it is Equation A9 which should be used not merely the first term as used in the text.

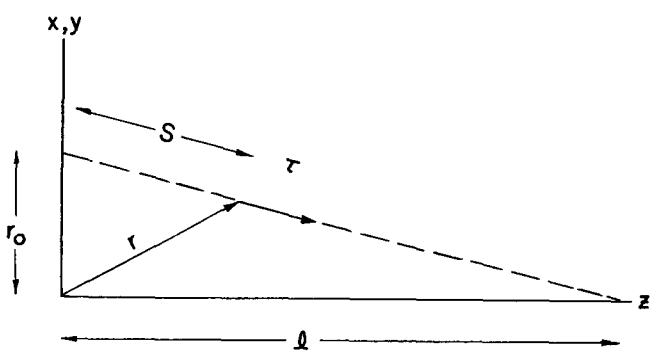

Fig. A1 - Geometry of zero order rays

APPENDIX B

PULSES WITH FINITE LIGHT VELOCITY

In this appendix we present the details of the calculations which lead from the treatment of a beam of arbitrary shape traveling with the velocity of light to a two-dimensional beam traveling with infinite velocity as used in the text. We ignore all spatial dispersion and treat the source as having a sharp front in space. Taking the pulse to be turned on at $t = 0$, off at $t = \tau$ in step function fashion, and to be steady in between we have

$$I(r,z,t) = I(r,z)[\theta(t)\theta(\tau-t)\theta(c_L t-z)\theta(z) + \theta(t-\tau)\theta(c_L t-z)\theta(z-c_L(t-\tau))\theta(t)] \ . \tag{B1}$$

This gives

$$\frac{\partial I}{\partial t} = I(r,t)[\delta(t-\frac{z}{c_L}) - \delta(t-\tau-\frac{z}{c_L})]\theta(z) \ . \tag{B2}$$

Cylindrical coordinates are used here with r denoting a vector in the x-y plane.

The retarded Green's function in 3 dimensions is

$$\frac{-1}{4\pi}\,\delta\,\frac{(t-t'-\frac{\mathcal{R}}{c})}{\mathcal{R}} \tag{B3}$$

with $\mathcal{R}$ the magnitude of the 3 dimensional vector from point r,z to point r',z'. Using Equations A2 and A3 in the expression for the pressure, Equation 5, we have

$$P_1(r,z,t)$$

$$= \frac{(\gamma-1)\alpha}{4\pi c^2}\int \delta\left(\frac{t-t'-\frac{\mathcal{R}}{c}}{\mathcal{R}}\right) I(r',z')[\delta(t'-\frac{z'}{c_L}) - \delta(t'-\tau-\frac{z'}{c_L})]\theta(z')d^2r'dz'dt' \tag{B4}$$

$$= \frac{(\gamma-1)\alpha}{4\pi c^2}\int\left[\delta\left(t-\frac{z'}{c_L}-\frac{\mathcal{R}}{c}\right) - \delta\left(t-\tau-\frac{z'}{c_L}-\frac{\mathcal{R}}{c}\right)\right]\frac{I(r',z')}{\mathcal{R}}\,\theta(z')d^2r'dz' \ . \tag{B5}$$

Only the first term of Equation A5 survives for $t < \tau$, but in any case the second term can be obtained from the first by the substitution $t \to t - \tau$. To

carry out the integration over z' define

$$g = (z'-z)\frac{c}{c_L} + \mathcal{R} . \tag{B6}$$

Then

$$z' = z + \frac{-g\frac{c}{c_L} \pm \sqrt{g^2 - \left(1 - \left(\frac{c}{c_L}\right)^2\right)R^2}}{1 - \left(\frac{c}{c_L}\right)^2} \tag{B7}$$

with the plus sign appropriate for $z' > z - \frac{\frac{c}{c_L} R}{\sqrt{1 - \left(\frac{c}{c_L}\right)^2}}$ and the minus sign for $z' < z - \frac{\frac{c}{c_L} R}{\sqrt{1 - \left(\frac{c}{c_L}\right)^2}}$. In these equations R denotes the magnitude of the 2-dimensional vector from r to r'. In the transformation to integration over g the quantity $\frac{dz'}{\mathcal{R}}$ is replaced by

$$\frac{dz'}{\mathcal{R}} = \frac{dg}{\mathcal{R}\frac{dg}{dz'}} = \pm \frac{dg}{\sqrt{g^2 - \left(1-\left(\frac{c}{c_L}\right)^2\right)R^2}} \tag{B8}$$

with the plus and minus signs chosen as above. The integration is thus divided into two regions according to the value of z' and the result is

$$P_1(r,z,t) = \frac{(\gamma-1)\alpha}{\pi c^2}\int \frac{\theta\left(t-\frac{z}{c_L} - \sqrt{1-\left(\frac{c}{c_L}\right)^2}\frac{R}{c}\right)}{\sqrt{\left(t-\frac{z}{c_L}\right)^2 - \left(1-\frac{c}{c_L}\right)^2\frac{R^2}{c_L}}}$$

$$\left[I\left(r', \frac{z - \frac{c^2}{c_L}t - c\sqrt{}}{1 - \left(\frac{c}{c_L}\right)^2}\right)\theta\left(z - \frac{c^2}{c_L}t - c\sqrt{}\right)\right. \tag{B9}$$

$$\left. + I\left(r', \frac{z - \frac{c^2}{c_2}t + c\sqrt{}}{1 - \left(\frac{c}{c_L}\right)^2}\right)\theta\left(z - \frac{c^2}{c_L}t + c\sqrt{}\right)\right] d^2r'$$

(term with $t \to t-\tau$).

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The symbol $\sqrt{}$ stands for the same square root appearing immediately after the integral sign,

$$\sqrt{} = \left[\left(t - \frac{z}{c_L}\right)^2 - \left(1 - \frac{c}{c_L}\right)^2 \frac{R^2}{c^2}\right]^{\frac{1}{2}} . \tag{B10}$$

The terms $\left(\frac{c}{c_L}\right)^2$ are of course extremely small and will be dropped in what follows. The step functions appearing with the beam intensity factors are the result of solving the half space problem, i.e. imposing $z > 0$. For $z > ct$ both of these step-functions are 1 for all R. This is due to the fact that the influence of one portion of the source on another travels with sound speed, and for $z > ct$ the edge effects due to terminating the source on the $z = 0$ plane are not felt. For small distances, such edge effects are present, and for distances $< (\frac{c}{c_L})$ ct the first terms in the bracket of equation B9 ceases to contribute at all. For distances $z > ct$ then we have

$$P_1(r,z,t) = \frac{(\gamma-1)\alpha}{4\pi c^2} \int \frac{\theta(t - \frac{z}{c_L} - \frac{R}{c})}{\sqrt{\left(t - \frac{z}{c_L}\right)^2 - \frac{R^2}{c^2}}} \tag{B11}$$

$$[I(r', z - \frac{c^2}{c_L} t - c\sqrt{}) + I(r', t - \frac{c^2}{c_L} t + c\sqrt{})]\, d^2r'$$

with

$$\sqrt{} = [(t - \frac{z}{c_L})^2 - \frac{R^2}{c^2}]^{\frac{1}{2}} . \tag{B12}$$

Finally, assuming that the intensity changes negligibly in a distance ct we can take the intensity to be independent of R and to be equal to its value

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at z

$$P_1(r,z,t) = \frac{(\gamma-1)\alpha}{2\pi c^2} \int \frac{\theta(t - \frac{z}{c_L} - \frac{R}{c})}{\sqrt{(t - \frac{z}{c_L})^2 - \frac{R^2}{c^2}}} I(r',z)\, d^2r' \qquad (B13)$$

$$- (\text{term with } t \to (t-\tau)) \; .$$

The approximation of the text consists in assuming the variation of the intensity to be slow enough so that this result is true for all $z > o$.

APPENDIX C

POWER SERIES EXPANSION OF THE PRESSURE

In this appendix we give the details of the expansion of the pressure as a power series in time. Equation 32′ of the text gives the pressure as

$$P_1(r,t) = \frac{(\gamma-1)\alpha}{2\pi c^2}\int \Theta\frac{(t-\frac{R}{c})}{\sqrt{t^2-R^2/c^2}} I(\underline{r}+\underline{R})d^2R. \qquad (32)'$$

Expanding the intensity about the point r, we have

$$I(\underset{\sim}{r}+\underset{\sim}{R}) = \sum_n \frac{(\underset{\sim}{R}\cdot\underset{\sim}{\nabla})^n}{n!} I(\underset{\sim}{r}). \qquad (C1)$$

Inserting Equation B1 into Equation 32′ and writing the integration over R in terms of polar coordinates with φ being the angle between vectors R and r, the angular integrations needed are

$$\int(\underset{\sim}{R}\cdot\underset{\sim}{\nabla})^n I(r)d\varphi. \qquad (C2)$$

writing $(\underset{\sim}{R}\cdot\underset{\sim}{\nabla})^n$ as $R^n\left(\cos\varphi\frac{\partial}{\partial x} + \sin\varphi\frac{\partial}{\partial y}\right)^n$ and expanding the gradient term by the binomial theorem leads to integrals expressible as Beta functions,[10] which are in turn expressed as products of Gamma functions. The result is

$$\frac{1}{n!}\int(\underset{\sim}{R}\cdot\underset{\sim}{\nabla})^n I d\varphi = \frac{2\pi}{2^n(\frac{n}{2})!\cdot(\frac{n}{2})!} R^n (\nabla^2)^{\frac{n}{2}} I \qquad (C3)$$

for n even. The integral is zero for n odd. The remaining integrals to be carried out in Equation 32′ are then of the form

$$\int R^n \frac{\Theta(t-\frac{R}{c})}{\sqrt{t^2-\frac{R^2}{c^2}}} RdR. \qquad (C4)$$

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Defining $R^2 = c^2t^2x$ this takes the form

$$\frac{1}{2}(ct)^n c^2 t \int_0^1 \frac{x^{n/2}\,dx}{\sqrt{1-x}}$$

which is also a Beta function[10] and is just $2^n(ct)^n c^2 t \dfrac{(\frac{n}{2})!(\frac{n}{2})!}{(n+1)!}$.

The use of Equations C1, C3, and C5 in Equation 32′ leads immediately to Equation 35 of the text.

APPENDIX D

PRESSURE AS A SERIES OF HYPERGEOMETRIC FUNCTIONS

Although Equations 35 and 36 of the text can be used to derive the density and pressure expressed in terms of hypergeometric functions, it is much easier to go back to Equations 32′,

$$P_1 = \frac{(\gamma-1)\alpha}{2\pi c^2}\int \frac{\Theta\,(t-\frac{R}{c})}{\sqrt{t^2-R^2/c^2}}\, I(\underline{r}+\underline{R})\,d^2R\ , \qquad (32)'$$

writing $I(r) = Ne^{-\frac{r^2}{a^2}}$ and defining $R^2 = c^2T^2x$ the pressure is

$$P_1(r) = \frac{(\gamma-1)\alpha t}{2}e^{-\frac{r^2}{a^2}}\int_0^1 \frac{e^{-2ctr\sqrt{x}\cos\varphi/a^2}\, e^{-c^2t^2x/a^2}}{\sqrt{1-x}}\,dx\,d\varphi\ . \qquad (D1)$$

The angle φ is the angle between the vectors r and R.

The angular integral is just the Bessel function I_o,[10] so

$$P_1 = \frac{(\gamma-1)\alpha t}{2}e^{-r^2/a^2}\int_0^1 \frac{I_o\left(\frac{2ctr}{a^2}\sqrt{x}\right)e^{-c^2t^2x/a^2}}{\sqrt{1-x}}\,dx\ . \qquad (D2)$$

If in turn, the expansion $I_o(t) = \sum_k \frac{(\frac{1}{4}t^2)^k}{k!k!}$ is used in Equation D2 the pressure is

$$P_1 = \frac{(\gamma-1)\alpha t}{2}e^{-r^2/a^2}\sum_{k=o}^{\infty}\left(\frac{ctr}{a^2}\right)^{2k}\frac{1}{k!}\,\frac{1}{k!}\int_o^1 \frac{x^k\, e^{-c^2t^2x/a^2}}{\sqrt{1-x}}\,dx\ . \qquad (D3)$$

Each integral is expressed as a confluent hypergeometric function using[10]

$$M(a,b,x) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)}\int_o^1 e^{xt}\, t^{a-1}\,(1-t)^{b-a-1}\,dt\ .$$

This leads directly to Equation 38 of the text.

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13. ABSTRACT

The atmospheric propagation of pulsed, focussed, laser beams is calculated. The beam intensity is obtained by solving the equation of geometric optics and the equation of energy conservation simultaneously using a perturbative scheme. In this way the modification on propagation due to change caused by the index of refraction absorption of energy from the beam itself is included in a linear approximation. The problem of calculating the index of refraction change in a quiescent atmosphere is formulated generally with detailed expressions, valid for all times, given for the case of a paraxial Gaussian beam. Formulas for beam intensity are given as graphical results presented for the on-axis intensity of a focussed Gaussian beam.

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