NRL Report 7124

On the Photo-Induced Beta Decay of Protons and Neutrons in Stellar Interiors

JOHN N. HAYES

Theory Branch
Nuclear Physics Division

January 6, 1971

NAVAL RESEARCH LABORATORY
Washington, D.C.

CONTENTS

ABSTRACT

In this first phase of a study on photon-induced beta decay, the reaction rate for $\gamma + p \longrightarrow n + e^{+} + \nu_e$ is determined for conditions approximating those in stellar interiors. It is shown that the lifetime of the proton in such conditions drops from infinity to only microseconds as the temperature varies from 0 to 10^{12}°K. Hence the stability of the proton against this reaction is not seriously altered and will not affect nucleosynthesis (which depends on the availability of protons). However, the reactions $\gamma + n \longrightarrow p + e^{-} + \bar{\nu}_e$ and $e^{-} + p \longrightarrow n + \nu_e$ are not restrained by threshold energy requirements. If the neutron's lifetime is drastically reduced by this latter reaction to the point that it cannot live long enough to react with nuclei, then nucleosynthesis in stars will be curtailed at very high temperatures. Attempts in this latter case to proceed with calculations along the same line as the $\gamma + p$ calculation run immediately into a divergence problem, as is shown here. The correct handling of this reaction will be the subject of another report when completed.

PROBLEM STATUS

This is a final report on one phase of a continuing problem.

AUTHORIZATION

NRL Problem H01-06
Project RR 002-06-41-5003

Manuscript submitted May 21, 1970.

ON THE PHOTO-INDUCED BETA DECAY OF PROTONS AND NEUTRONS IN STELLAR INTERIORS

INTRODUCTION

The theory of stellar structure and stellar evolution that has been developed in recent years calls upon the data of nuclear physics very extensively (1); in particular, the luminosity of a star depends very directly upon the rate of energy production in the interior of a star through nuclear reactions. The kinds of nuclear reactions that act as the source of energy depend upon the star and on its stage of evolution. But in virtually all stages of evolution, proton reactions play a significant role, and in the late stages of evolution neutron reactions are important. In all models of stars, the pertinent nuclear reactions are studied under the assumption that the proton and the neutron are stable particles, i.e., they maintain their identities for times much longer than the times required for the pertinent nuclear reactions to occur in the stellar milieu. It is thus very important to determine the validity of this assumption.

There are, in fact, some reactions which will occur in the stellar medium which cast some doubt on this assumption. For protons, there are two reactions which render the proton unstable, namely

$$\gamma + p \longrightarrow n + e^{+} + \nu_e$$

and

$$e^{-} + p \longrightarrow n + \nu_e \ ,$$

while for the neutron the reactions

$$\gamma + n \longrightarrow p + e^{-} + \bar{\nu}_e$$

and

$$e^{+} + n \longrightarrow p + \bar{\nu}_e$$

render the neutron unstable. Now at ordinary room temperatures and densities, these reactions are completely negligible. Even at these temperatures and densities representative of the core of stars on all portions of the main sequence we expect them to be negligible. For example, for the γ, p reaction a threshold of $(m_n + m_e - m_p)\, c^2$ is required for the reaction to proceed — an energy which is of the order of 1.80 MeV. If the photon at the peak of the Planck spectrum has this energy, the temperature must be of the order of $2 \times 10^{10}\,{}^{\circ}$K, which is a temperature not reached in stars on the main sequence (2). For the e, p reaction, temperatures of $1 \times 10^{10}\,{}^{\circ}$K are required for this reaction to be significant in depopulating the protons in a star. For the neutron reactions, there is not a significant neutron population in the main sequence stars to render these reactions important.

However, in certain of the red giant stages, and in phases subsequent to the red giant stage, such high temperatures are approached.* It is true that chemical evolution of the star has depleted the hottest regions of their original hydrogen content, which would suggest that the above-cited proton reactions would be unimportant once again; however, this need not be the case. First, one must take into account the fact that thermonuclear reactions which take place in this evolved matter will cause protons and/or neutrons to be ejected as final-state products, and these in turn will react with the ambient nuclei. Whether or not they react with these nuclei as protons and/or neutrons, or vice versa, depends upon the lifetime of these particles against the reactions cited above, as compared to the lifetimes of protons and neutrons against the specifically nuclear reactions. Second, there is good evidence that convection currents may carry large quantities of virtually pure hydrogen into the deep interior even in the red giant stage (3); such convective currents could become a prolific source of neutrons if the photo-induced beta decay of the proton has a lifetime constant which is short compared with the convection time scales and with the diffusion times in the interior of the star.

Finally, in the collapse of the interior of a star evolving to the supernova stage, extremely high temperatures are reached with a prolific production of neutrons. In some theories of nucleosynthesis, too many neutrons are produced; it would appear that the γ, n reaction cited above, which has never been included in such theories, will place limits on neutron production and consequently affect the distribution of the elements produced in such an explosion. For these reasons, we feel that a study of these reactions is warranted.

In this report, we will calculate the lifetime of the proton against photo-induced beta decay as a function of the ambient temperature, the source of the photons being the black-body radiation in the ambient medium. The result is found to be independent of all stellar parameters except the temperature. While the e, p reaction has a lower threshold, and might therefore be expected to be dominant over the γ, p reaction, we note that the lifetime of the proton against the e, p reaction depends directly upon the density of electrons, while this is absent in the problem at hand. Further, the number of photons per unit volume increases with the temperature; hence, for a given density there will be a temperature at which this reaction will be more important than the e, p reaction. Bahcall has treated the electron-proton reaction (4).

In the next section, the necessary field theoretic preliminaries that set the method of calculation are discussed; in the section following that, the integrations required to determine the lifetime are developed. Finally, the last section is devoted to a discussion of results and related problems.

FIELD THEORY PRELIMINARIES

To determine the scattering amplitude for the reaction $\gamma + p \longrightarrow n + e^+ + \nu_e$, we shall use standard field theoretic methods (5) and the usual perturbation expansion of the S-matrix which is good for the cases of electromagnetic interactions and for the weak interactions. The electromagnetic field interacts with all charged particles and is therefore coupled to both the electron field and the proton field. The four fermions p, n, e^+, and ν_e interact directly with one another through the weak interaction, which we take to be in the form of the V-A theory of Marshak and Sudarshan (6), and of Feynman and Gellmann (7). The fact that the axial vector coupling constant for neutron decay is not identical in magnitude with the vector coupling constant is ignored here. To treat this case exactly complicates the calculation without being particularly illuminating, and our result will not deviate significantly from that in which $|C_A/C_V| = 1.18$.

*See, for instance, Ref. 1.

The perturbation Hamiltonian is thus taken to be

$$H(t) = H_{e\gamma}(t) + H_{p\gamma}(t) + H_w(t) \tag{1}$$

where

$$\left.\begin{aligned} H_{e\gamma}(t) &= -ie \int_V d^3x : \bar{\psi}_e(x)\, \gamma^\lambda \psi_e(x)\, A_\lambda(x) : \\ H_{p\gamma}(t) &= +ie \int_V d^3x : \bar{\psi}_p(x)\, \gamma^\lambda \psi_p(x)\, A_\lambda(x) : \\ H_w(t) &= \frac{g}{\sqrt{2}} \int_V d^3x : \bar{\psi}_n(x)(1-\gamma^5)\gamma^\mu \psi_p(x)\, \bar{\psi}_\nu(x)(1-\gamma^5)\gamma^\mu \psi_e(x) : \\ &\qquad + \text{adjoint} \end{aligned}\right\} \tag{2}$$

with $e = -4.803 \times 10^{-10}$ esu and $g = 1.418 \times 10^{-49}$ erg-cm^3.

The field operators will be expanded in plane waves as follows:

$$\left.\begin{aligned} \psi_f(x) &= \sum_{\vec{q},s} \sqrt{\frac{m_f c^2}{\hbar\omega_q V}} \left\{ U_s(\mathbf{q})\, b_s(\mathbf{q})\, e^{iq\cdot x} + V_s(\mathbf{q})\, d_s^+(\mathbf{q})\, e^{-iq\cdot x} \right\} \\ \bar{\psi}_f(x) &= \sum_{\vec{q},s} \sqrt{\frac{m_f c^2}{\hbar\omega_q V}} \left\{ \bar{U}_s(\mathbf{q})\, b_s^+(\mathbf{q})\, e^{-iq\cdot x} + \bar{V}_s(\mathbf{q})\, d_s(\mathbf{q})\, e^{iq\cdot x} \right\} \\ A_\mu(x) &= \sum_{\vec{k},m} \sqrt{\frac{2\pi\hbar^2 c^2}{\hbar\omega_k V}} \left\{ a_m(\mathbf{k})\, \epsilon_\mu^m(\mathbf{k})\, e^{ik\cdot x} + a_m^+(\mathbf{k})\, \epsilon_\mu^{m*}(\mathbf{k})\, e^{-ik\cdot x} \right\} \end{aligned}\right\} \tag{3}$$

where the subscript f stands for any of the four fermions p, n, e^+, and ν_e. Here, we give the neutrino a finite mass at the outset of the calculation, taking $m_{\nu_e} \to 0$ after the calculation is concluded. Clearly, our plane waves are normalized to unity in a box of volume V.

The initial state will be characterized by the wave numbers and polarization numbers $\mathbf{k}, m$ and $\mathbf{p}, u$ of the photon and proton, respectively. The wave and spin numbers of the final state particles are designated by $\mathbf{n}, r$ for the neutron, $\mathbf{e}, s$ for the positron, and ν, t for the neutrino. To the lowest order in the coupling constants, the S matrix elements for this process are given by

$$\langle f|S|i\rangle = \frac{eg}{\sqrt{2}\hbar} \sqrt{\frac{2\pi}{\hbar\omega_k V}} \sqrt{\frac{m_e c^2}{\hbar\omega_e V}} \sqrt{\frac{m_\nu c^2}{\hbar\omega_\nu V}} \sqrt{\frac{M_n c^2}{\hbar\omega_n V}} \sqrt{\frac{M_p c^2}{\hbar\omega_p V}}\, \epsilon_\lambda^m(\mathbf{k})\, \mathfrak{M}_\lambda V T \delta_{p+k,\,n+e+\nu} \tag{4}$$

where

$$\mathfrak{M}_\lambda = \mathfrak{M}_\lambda^e + \mathfrak{M}_\lambda^p \; ,$$

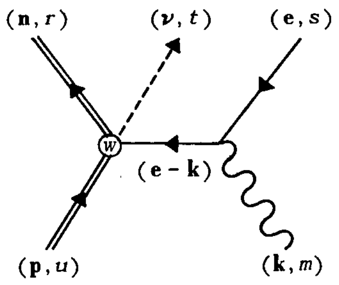
$$\mathfrak{M}_\lambda^e = +\bar{U}_r(\mathbf{n})(1-\gamma^5)\gamma^\mu U_u(\mathbf{p})\,\bar{U}_t(\nu)(1-\gamma^5)\gamma^\mu \left\langle\frac{-i\gamma\cdot(e-k)-\mu_e}{(e-k)^2+\mu_e^2}\right\rangle \gamma^\lambda V_s(\mathbf{e}) \; ,$$

$$\mathfrak{M}_\lambda^p = -\bar{U}_r(\mathbf{n})(1-\gamma^5)\gamma^\mu \left\langle\frac{i\gamma\cdot(p+k)-\mu_p}{(p+k)^2+\mu_p^2}\right\rangle \gamma^\lambda U_u(\mathbf{p})\,\bar{U}_t(\nu)(1-\gamma^5)\gamma^\mu V_s(\mathbf{e}) \; , \qquad (5)$$

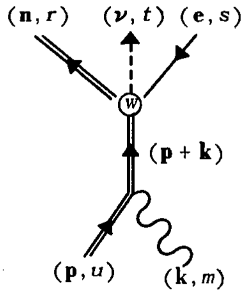
and

$$\mu_f = m_f c/\hbar \; .$$

The terms $\mathfrak{M}_\lambda^e$ and $\mathfrak{M}_\lambda^p$ correspond to the Feynman diagrams shown below.



$\mathfrak{M}_\lambda^e$



$\mathfrak{M}_\lambda^p$

THE LIFETIME OF THE PROTON IN A BATH OF BLACKBODY RADIATION

Collision Probability and Transition Rate ω

The quantity $\langle f|S|i\rangle$ specified in the previous section is the scattering amplitude for the process $\gamma + p \longrightarrow n + e^+ + \nu$ described above for the initial and final states. The quantity $|\langle f|S|i\rangle|^2$ represents the probability that given one photon per unit volume, and one proton in this same volume V, a collision will occur between the two within a time t, leading to this particular specified final state. The total probability that this particular initial state will give rise to a collision in time t leading to any allowed final state is obtained by summing up the probabilities for all final states:

$$dP_i(t) = \sum_{\substack{\text{final}\\ \text{states } f}} |\langle f|S|i\rangle|^2$$

$$= \sum_{r,s,t} \int \frac{d^3nV}{(2\pi)^3} \int \frac{d^3eV}{(2\pi)^3} \int \frac{d^3\nu V}{(2\pi)^3} \, |\langle f|S|i\rangle|^2 \; .$$

Now if $N_\gamma(\mathbf{k},m)$ is the number of photons of polarization m per unit volume in wave number space, then

$$dP_p(t) = \sum_m VN_\gamma(\mathbf{k},m) \frac{d^3k}{(2\pi)^3} dP_i(t)$$

$$= \sum_{m,r,s,t} \frac{V^4}{(2\pi)^{12}} N_\gamma(\mathbf{k},m)\, d^3k \int d^3n \int d^3e \int d^3\nu |\langle f|S|i\rangle|^2 .$$

(For a Planck distribution, $N_\gamma(\mathbf{k},m)$ is really independent of m, of course.) Summing over all possible photon momenta gives the probability that the proton $(\mathbf{p},u)$ will transform, together with some photon, into some final state of the desired configuration in time t:

$$P_p(t) = \sum_{m,r,s,t} \frac{V^4}{(2\pi)^{12}} \int d^3kN_\gamma(\mathbf{k},m) \int d^3n \int d^3e \int d^3\nu |\langle f|S|i\rangle|^2 .$$

If the probability is $N(\mathbf{p},u)\,d^3p$ that the proton is in a state $(\mathbf{p},u)$ to $(\mathbf{p}+d\mathbf{p},u)$, then by averaging over initial proton states we get

$$P(t) = \frac{1}{2} \sum_{\substack{rstu\\m}} \frac{V^4}{(2\pi)^{12}} \int d^3kN_\gamma(\mathbf{k},m) \int d^3pN(\mathbf{p},u) \int d^3n \int d^3e \int d^3\nu |\langle f|S|i\rangle|^2 .$$

For an equilibrium photon and proton distribution, N_γ and N are independent of m and u, respectively, leaving

$$P(t) = \frac{1}{2} \sum_{\substack{rstu\\m}} \frac{V^4}{(2\pi)^{12}} \int d^3kN_\gamma(\mathbf{k}) \int d^3pN(\mathbf{p}) \int d^3n \int d^3e \int d^3\nu |\langle f|S|i\rangle|^2$$

with

$$\int N(\mathbf{p})\, d^3p = 1$$

and

$$\int N_\gamma(\mathbf{k})\, d^3k = \frac{1}{2} N_\gamma(T)$$

where $N_\gamma(T)$ is the total number of photons at a temperature T in a cubic centimeter.

As is well known, the quantity

$$\omega = \lim_{\substack{t\to\infty\\V\to\infty}} \frac{P(t)}{t}$$

is finite, independent of t and V, and is the well-known transition rate whose reciprocal is τ, the lifetime which we seek.

$$\omega = \int d^3k N_\gamma(\mathbf{k}) \int d^3p N(\mathbf{p}) \lim_{V,\, t\to\infty} \frac{1}{t} \cdot \frac{1}{2} \sum_{\substack{rstu\\m}} \frac{V^4}{(2\pi)^{12}} \int d^3n \int d^3e \int d^3\nu \,|\langle f|S|i\rangle|^2 .$$

Utilizing Eq. (5) of the previous section, we may write the transition rate in the form

$$\omega = \int \frac{d^3k}{k_0} N_\gamma(\mathbf{k}) \int \frac{d^3p}{p_0} N(\mathbf{p})\, G(p,k) \tag{6}$$

where

$$G(p,k) = \frac{\pi}{2} \frac{e^2 g^2}{\hbar^3 c^2} \frac{\mu_p\mu_n\mu_e\mu_\nu}{(2\pi)^8} \int \frac{d^3n}{n_0} \int \frac{d^3e}{e_0} \int \frac{d^3\nu}{\nu_0} \sum_{\substack{rstu\\m}} |e^m \cdot \mathfrak{M}|^2\, \delta(p+k-n-e-\nu) .$$

Here $q_0 = \omega_q/c$, the μ's are reciprocal Compton wavelengths, and we have used

$$\lim_{t\to\infty} Vt\delta_{p+k,\,n+e+\nu} = \frac{(2\pi)^4}{c}\, \delta(p+k-n-e-\nu) .$$

As a kind of check, one can easily show that the dimensions of ω are indeed $\sec^{-1}$. Using the customary properties of the Dirac delta function, one may readily cast G into the form

$$\begin{aligned} G(p,k) = {} & \frac{\pi}{2(2\pi)^8} \frac{\alpha g^2}{\hbar^2 c} \mu_p\mu_n\mu_e\mu_\nu \int d^4n \int d^4e \int d^4\nu \\ & \times \theta(n_0)\delta(n^2+\mu_n^2)\,\theta(e_0)\,\delta(e^2+\mu_e^2)\,\theta(\nu_0)\,\delta(\nu^2) \\ & \times \sum_{\substack{rstu\\m}} |e_\lambda^m(\mathbf{k}) \cdot \mathfrak{M}_\lambda|^2 \delta(p+k-n-e-\nu) . \end{aligned} \tag{7}$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant.

The above expression is an invariant function of p and k; hence it can only be a function of p^2, k^2, and $p\cdot k$. But $p^2 = \mu_p^2$ and $k^2 = 0$, so G may be regarded as a function of $p\cdot k$ only. Equivalently it may be regarded as a function of $(p+k)^2$ only. Putting $(p+k) = q$, we can designate $G(p,k)$ now by $G(q^2)$, abusing our notation only slightly.

Without actually evaluating $G(q^2)$, its Lorentz-invariant character enables us to reduce the expression for ω somewhat. We have

$$\omega = \int \frac{d^3k}{k_0} N_\gamma(\mathbf{k}) \int \frac{d^3p}{p_0} N(\mathbf{p})\, G(q^2) .$$

Now

$$N(\mathbf{p}) = \frac{\lambda_0^{3/2}}{(2\pi\mu_p)^{3/2}} \exp\left[(m_p c^2/kT) - \lambda_0 p_0\right] , \tag{8}$$

where

$$\lambda_0 = \frac{\hbar c}{kT}$$

and

$$p_0 = \sqrt{\mathbf{p}^2 + \mu_p^2}$$

is the Boltzmann distribution for the protons. Although this form is not exact, it is nevertheless indistinguishable from the correct one for the temperatures inserted in all astrophysical problems. The photon distribution is that of Planck:

$$N_\gamma(\mathbf{k}) = [\exp(+\lambda_0 k_0) - 1]^{-1}$$

where λ_0 is the same as in Eq. (8) and $k_0 = \omega_k/c$. Then

$$\omega = \frac{\lambda_0^{3/2}}{(2\pi\mu_p)^{3/2}} \exp\left(\frac{m_p c^2}{kT}\right) \int \frac{d^3k}{k_0} \int \frac{d^3p}{p_0} \frac{\exp[-\lambda_0(p_0 + k_0)]}{1 - \exp(-\lambda_0 k_0)} G(q^2)$$

$$= \frac{4\lambda_0^{3/2}}{(2\pi\mu_p)^{3/2}} \exp\left(\frac{m_p c^2}{kT}\right) \int d^4k\theta(k_0)\,\delta(k^2) \int d^4p\theta(p_0)\,\delta(p^2 + \mu_p^2)$$

$$\times \frac{\exp(-\lambda_0 q_0)}{1 - \exp(-\lambda_0 k_0)} G(q^2) \; .$$

More convenient choices of variables of integration are $q = p + k$ and k; thus

$$\omega = \frac{4\lambda_0^{3/2}}{(2\pi\mu_p)^{3/2}} \exp\left(\frac{m_p c^2}{kT}\right) \int d^4q G(q^2) \exp(-\lambda_0 q_0) \int d^4k \frac{\theta(k)\,\delta(k^2)\,\theta(q_0 - k_0)\,\delta[(q-k)^2 + \mu_p^2]}{1 - \exp(-\lambda_0 k_0)}$$

where the last factor is an integral which may be evaluated exactly. Indeed,

$$\int d^4k \frac{\theta(k_0)\,\theta(q_0 - k_0)\,\delta(k^2)\,\delta\left((q-k)^2 + \mu_p^2\right)}{1 - \exp(-\lambda_0 k_0)} =$$

$$-\frac{\pi}{2} \frac{\theta(q_0)}{\lambda_0|\mathbf{q}|} \ln \left[\frac{\exp\left[\frac{-\lambda_0}{2}\left(\frac{\mu_p^2 + q^2}{q_0 - |\mathbf{q}|}\right)\right] - 1}{\exp\left[\frac{-\lambda_0}{2}\left(\frac{\mu_p^2 + q^2}{q_0 + |\mathbf{q}|}\right)\right] - 1} \right].$$

Then the transition rate becomes

$$\omega = -\frac{2\pi}{\lambda_0}\left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2} \exp\left(\frac{m_p c^2}{kT}\right) \int \frac{d^4q}{|\mathbf{q}|} \theta(q_0)\, G(q^2) \exp(-\lambda_0 q_0) \ln \left[\frac{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2 + q^2}{q_0 - |\mathbf{q}|}\right)\right] - 1}{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2 + q^2}{q_0 + |\mathbf{q}|}\right)\right] - 1} \right].$$

Because the integrand is independent of the direction of the vector $\mathbf{q}$, the angular integration may be done immediately. Putting

$$d^4q\,|\mathbf{q}|^{-1} = dq_0 |\mathbf{q}|\, d\,|\mathbf{q}|\, d\Omega_q = \frac{1}{2}\, dq_0\, dx\, d\Omega$$

where $x = \mathbf{q}^2$, we get

$$\omega = -\frac{4\pi^2}{\lambda_0} \exp\left(\frac{m_p c^2}{kT}\right)\left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2} \int dq_0 \int dx\, \theta(q_0)\, G(x - q_0^2) \exp(-\lambda_0 q_0)\, ln \left[\frac{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2 + x - q_0^2}{q_0 - \sqrt{x}}\right)\right] - 1}{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2 + x - q_0^2}{q_0 + \sqrt{x}}\right)\right] - 1}\right].$$

In this expression we have placed no limits on the integrations over x and q_0. There are natural limitations on the domains, however, that will appear when the function G is actually evaluated, but it is easy to see what these domains are without detailed evaluation of $G(q^2)$. First, we note from the definition of G that it is a Lorentz-invariant function; hence we may choose an inertial frame that makes its evaluation easiest, express the result in an invariant fashion, and thereby have the integral for all inertial frames. Second, we note that G expressed as an integral over neutron, electron, and neutrino variables has a delta function in the integrand. Now this restricts the variables $\mathbf{n}$, $\mathbf{e}$, $\boldsymbol{\nu}$ and their corresponding energies to those values such that

$$\mathbf{q} = \mathbf{n} + \mathbf{e} + \boldsymbol{\nu}$$

and

$$q_0 = \sqrt{\mathbf{n}^2 + \mu_n^2} + \sqrt{\mathbf{e}^2 + \mu_e^2} + \sqrt{\boldsymbol{\nu}^2}\ .$$

In the inertial frame that coincides with the center of momentum of the initial state, we have $\mathbf{q} = 0$. Thus for the momenta $\mathbf{n} = \mathbf{e} = \boldsymbol{\nu}$, we have

$$q_0 = \mu_n + \mu_e\ .$$

For any $q_0 < (\mu_n + \mu_e)$, the δ function must necessarily vanish, and then $G(q^2)$ will vanish. Hence

$$G(-q_0^2) = 0\,, \quad \text{if} \quad q_0 < \mu_n + \mu_e\,,$$

or

$$G(-q_0^2) = \theta(q_0)\,\theta(q_0 - \mu)\,G(-q_0^2)\,,$$

$$= \theta(q_0)\,\theta(+q_0^2 - \mu^2)\,G(-q_0^2)$$

where

$$\mu_n + \mu_e \equiv \mu\ .$$

Casting this result in a covariant form, we get

$$G(q^2) = \theta(q_0)\,\theta(-q^2 - \mu^2)\,G(+q^2)\ .$$

Now in terms of our new variable x,

$$G(q^2) = \theta(q_0)\,\theta(q_0^2 - \mu^2 - x)\,G(x - q_0^2)$$

$$= \theta(q_0)\,\theta(q_0^2 - \mu^2)\,\theta(q_0^2 - \mu^2 - x)\,G(x - q_0^2)\ .$$

This shows that $G(q^2) \neq 0$ only if $q_0 \geq \mu$, while $0 \leq x \leq q_0^2 - \mu^2$; these are the limits on the q_0 and x integrations. Thus

$$\omega = -\frac{4\pi^2}{\lambda_0} \exp \frac{m_p c^2}{kT} \left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2}$$

$$\times \int_\mu^\infty dq_0 \int_0^{q_0^2-\mu^2} dx\, G\left(x - q_0^2\right) \exp\left(-\lambda_0 q_0\right) \ln \left[\frac{\exp\left[-\dfrac{\lambda_0}{2}\left(\dfrac{\mu_p^2 + x - q_0^2}{q_0 - \sqrt{x}}\right)\right] - 1}{\exp\left[-\dfrac{\lambda_0}{2}\left(\dfrac{\mu_p^2 + x - q_0^2}{q_0 + \sqrt{x}}\right)\right] - 1}\right] .$$

This expression may still be reduced to a single integral. First interchange the order of integration so that

$$\omega = -\frac{4\pi^2}{\lambda_0} \exp \left(\frac{m_p c^2}{kT}\right) \left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2}$$

$$\times \int_0^\infty dx \int_{\sqrt{x+\mu^2}}^\infty dq_0\, G\left(x - q_0^2\right) \exp\left(-\lambda_0 q_0\right) \ln \left[\frac{\exp\left[-\dfrac{\lambda_0}{2}\left(\dfrac{\mu_p^2 + x - q_0^2}{q_0 - \sqrt{x}}\right)\right] - 1}{\exp\left[-\dfrac{\lambda_0}{2}\left(\dfrac{\mu_p^2 + x - q_0^2}{q_0 + \sqrt{x}}\right)\right] - 1}\right] .$$

Make the change of variable from q_0 to w where

$$w = q_0^2 - x \geq p^2 ;$$

thus

$$\omega = -\frac{2\pi^2}{\lambda_0} \exp \left(\frac{m_p c^2}{kT}\right) \left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2}$$

$$\times \int_0^\infty dx \int_{\mu^2}^\infty \frac{dw}{\sqrt{w + x}}\, G(-w) \exp\left(-\lambda_0 \sqrt{w + x}\right) \ln \left[\frac{\exp\left[-\dfrac{\lambda_0}{2}\left(\dfrac{\mu_p^2 - w}{\sqrt{w+x} + \sqrt{x}}\right)\right] - 1}{\exp\left[-\dfrac{\lambda_0}{2}\left(\dfrac{\mu_p^2 - w}{\sqrt{w+x} + \sqrt{x}}\right)\right] - 1}\right]$$

$$= -\frac{2\pi^2}{\lambda_0}\exp\left(\frac{m_p c^2}{kT}\right)\left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2}\int_{\mu^2}^{\infty} dw\, G(-w)$$

$$\times\int_0^\infty dx\,\frac{\exp(-\lambda_0\sqrt{w+x})}{\sqrt{w+x}}\, ln\left[\frac{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2 - w}{\sqrt{w+x}-\sqrt{x}}\right)\right] - 1}{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2 - w}{\sqrt{w+x}+\sqrt{x}}\right)\right] - 1}\right].$$

Next put $w = \mu^2\xi$; then

$$w = -\frac{2\pi^2\mu^2}{\lambda_0}\left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2}\exp\left(\frac{m_p c^2}{kT}\right)\int_1^\infty d\xi\, G(-\mu^2\xi)$$

$$\times\int_0^\infty dx\,\frac{\exp(-\lambda_0\sqrt{\mu^2\xi+x})}{\sqrt{\mu^2\xi+x}}\, ln\left[\frac{\exp\left[-\frac{\lambda_p}{2}\left(\frac{\mu_p^2-\mu^2\xi}{\sqrt{\mu^2\xi+x}-\sqrt{x}}\right)\right] - 1}{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p - \mu^2\xi}{\sqrt{\mu^2\xi+x}+\sqrt{x}}\right)\right] - 1}\right].$$

The further transformation $x = \mu^2\xi y$ is useful; since $\xi \neq 0$, y ranges from 0 to infinity. Further, $dx = \mu^2\xi\, dy$, so

$$\int_0^\infty dx\,\frac{\exp(-\lambda_0\sqrt{\mu^2\xi+x})}{\sqrt{\mu^2\xi+x}}\, ln\left[\frac{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu_p^2-\mu^2\xi}{\sqrt{\mu_p^2\xi\pm x}-\sqrt{x}}\right)\right] - 1}{\exp\left[-\frac{\lambda_0}{2}\left(\frac{\mu^2+\mu^2\xi}{\sqrt{\mu^2\xi+x}+\sqrt{x}}\right)\right] - 1}\right]$$

$$= \mu\sqrt{\xi}\int_0^\infty dy\,\frac{\exp(-\lambda_0\mu\sqrt{\xi}\sqrt{1+y})}{\sqrt{1+y}}\, ln\left[\frac{\exp\left[-\frac{\lambda_0}{2\mu\sqrt{\xi}}\left(\frac{\mu_p^2-\mu^2\xi}{\sqrt{1+y}-\sqrt{y}}\right)\right] - 1}{\exp\left[-\frac{\lambda_0}{2\mu\sqrt{\xi}}\left(\frac{\mu_p^2-\mu^2\xi}{\sqrt{1+y}+\sqrt{y}}\right)\right] + 1}\right].$$

These tedious transformations will be drawing to an end soon, but first we put $1 + y = t^2$; then the right-hand side of our last equation becomes

$$2\mu\sqrt{\xi}\int_1^\infty dt\,\exp(-\lambda_0\mu\sqrt{\xi}\,t)\, ln\left[\frac{\exp\left[\frac{\lambda_0\mu(\xi-\xi_0)}{2\sqrt{\xi}}\left(\frac{1}{t-\sqrt{t^2-1}}\right)\right] - 1}{\exp\left[\frac{\lambda_0\mu(\xi-\xi_0)}{2\sqrt{\xi}}\,(t-\sqrt{t^2-1})\right] - 1}\right]$$

where $\mu_p^2/\mu^2 \equiv \xi_0$ and we have used the identity $(t - \sqrt{t^2 - 1})(t + \sqrt{t^2 - 1}) = 1$. We next integrate by parts. The above expression becomes

$$\mu \frac{(\xi - \xi_0)}{\sqrt{\xi}} \int_1^\infty \frac{dt}{\sqrt{t^2 - 1}} \left[\frac{\exp\left[\left(-\lambda_0 \mu \sqrt{\xi}\ t\right) + \left(\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\right)\left(\frac{1}{t - \sqrt{t^2 - 1}}\right)\right]}{\exp\left[\left(\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\right)\left(\frac{1}{t - \sqrt{t^2 - 1}}\right)\right] - 1} \right] (t - \sqrt{t^2 - 1})^{-1}$$

$$+ \frac{\mu(\xi - \xi_0)}{\sqrt{\xi}} \int_1^\infty \frac{dt}{\sqrt{t^2 - 1}} \left[\frac{\exp\left[-\lambda_0 \mu \sqrt{\xi}\ t + \left(\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\right)(t - \sqrt{t^2 - 1})\right]}{\exp\left[\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\ (t - \sqrt{t^2 - 1})\right] - 1} \right] (t - \sqrt{t^2 - 1})\,.$$

Let $t - \sqrt{t^2 - 1} = Z$; then the above expression becomes

$$\frac{\mu(\xi - \xi_0)}{\sqrt{\xi}} \left[\int_0^1 \frac{dZ}{Z^2} \frac{\exp\left[-\frac{\lambda_0 \mu \sqrt{\xi}}{2}\left(Z + \frac{1}{Z}\right)\right]}{1 - \exp\left[-\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}\ Z}\right]} + \int_0^1 dZ \frac{\exp\left[-\frac{\lambda_0 \mu \sqrt{\xi}}{2}\left(Z + \frac{1}{Z}\right)\right]}{1 - \exp\left[\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\right] Z} \right].$$

In the first integral put $Z = 1/\zeta$; then the above expression becomes

$$\frac{\mu(\xi - \xi_0)}{\sqrt{\xi}} \left[\int_1^\infty d\zeta \frac{\exp\left[-\frac{\lambda_0 \mu \sqrt{\xi}}{2}\left(\zeta + \frac{1}{\zeta}\right)\right]}{1 - \exp\left[-\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\ \zeta\right]} + \int_0^\infty dZ \frac{\exp\left[-\frac{\lambda_0 \mu \sqrt{\xi}}{2}\left(Z + \frac{1}{Z}\right)\right]}{1 - \exp\left[-\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\ Z\right]} \right]$$

$$= \frac{\mu(\xi - \xi_0)}{\sqrt{\xi}} \int_0^\infty dZ \frac{\exp\left[-\frac{\lambda_0 \mu \sqrt{\xi}}{2}\left(Z + \frac{1}{Z}\right)\right]}{1 - \exp\left[-\frac{\lambda_0 \mu (\xi - \xi_0)}{2\sqrt{\xi}}\ Z\right]}\,.$$

Finally, put $(\lambda_0 \mu (\xi - \xi_0)/2\sqrt{\xi})\ Z = x$; then our last expression becomes

$$\frac{2}{\lambda_0} \int_0^\infty dx \frac{\exp\left[-\frac{\xi}{\xi - \xi_0}\left(x + \frac{\lambda_0^2 \mu^2 (\xi - \xi_0)^2}{4\xi x}\right)\right]}{1 - \exp(-x)}\,,$$

and our transition rate becomes

$$\omega = \frac{4\pi^2\mu^2}{\lambda_0^2}\left(\frac{\lambda_0}{2\pi\mu_p}\right)^{3/2} \exp\left(\frac{m_p c^2}{kT}\right)\int_1^\infty d\xi\, G(-\mu^2\xi)$$

$$\times \int_0^\infty dx \frac{\exp\left[-\dfrac{\xi}{\xi-\xi_0}\left(x + \dfrac{\lambda_0^2\mu^2(\xi-\xi_0)^2}{4\xi x}\right)\right]}{1-\exp(-x)} . \tag{9}$$

It can be shown that this last factor that we have been transforming, the integral over x, can, for temperatures below 10^{12}°K, be approximated by the modified Bessel function $K_1(\lambda_0\mu\sqrt{\xi})$, which in turn may be approximated by its asymptotic values

$$\sim(\lambda_0\mu\sqrt{\xi})^{-1} \exp(-\lambda_0\mu\sqrt{\xi}) ;$$

this reduces ω to a single integral over ξ. We shall see, however, that $G(\mu^2\xi)$ is sufficiently complicated such that a good analytical evaluation of the integral becomes possible only in a temperature region that is far too low to be interesting. Hence we shall resort to a numerical computation of the integral over ξ, and therefore we shall do the same for the x integral above. We note that the above expression for ω is exact, within the framework of first-order perturbation theory.

Matrix Element Evaluation

Because the final state of the reaction $\gamma + p \rightarrow n + e^+ + \nu_e$ is a three-body state, the evaluation of the matrix elements is quite tedious and difficult. In this subsection, only the highlights of the calculation are presented, the details being relegated to an appendix or, if they are consequences of standard field theoretic techniques, being left to the reader to establish.

The sequence of expressions to be evaluated is readily discerned by reference to Eq. (7) which defines $G(+q^2)$. The quantities $\mathfrak{M}_\lambda$ were given by Eq. (5). Because the theory must be gauge invariant,

$$\sum_{rstu}\sum_{m=1}^{2} \epsilon_\lambda^m(\mathbf{k})\,\epsilon_{\lambda'}^{m*}(\mathbf{k})\mathfrak{M}_\lambda\mathfrak{M}_{\lambda'}^* = \sum_{rstu}\sum_{m=1}^{4} \epsilon_\lambda^m(\mathbf{k})\,\epsilon_{\lambda'}^{m*}(\mathbf{k})\mathfrak{M}_\lambda\mathfrak{M}_{\lambda'}^*$$

$$= \sum_{rstu} \delta_{\lambda\lambda'}(-1)^{\delta_{\lambda 4}}\mathfrak{M}_\lambda\mathfrak{M}_{\lambda'}^* = \sum_{rstu} \mathfrak{M}_\lambda\mathfrak{M}_\lambda^*(-1)^{\delta_{\lambda 4}} \tag{10}$$

where a repeated Greek index is understood to be summed. The first step in the evaluation of G is therefore the determination of

$$\sum_{rstu} \mathfrak{M}_\lambda\mathfrak{M}_\lambda^*(-1)^{\delta_{\lambda 4}} .$$

This can be done from the standard trace technique of field theory; in the appendix, we show

$$\sum_{rstu} \mathfrak{M}_\lambda \mathfrak{M}_\lambda^* (-1)^{\delta_{\lambda 4}} = \frac{2^4}{\mu_p\mu_n\mu_e\mu_\nu} \frac{n \cdot \nu}{(p \cdot k)^2 (e \cdot k)^2} \Big\{ [\mu_e^2 (p \cdot k)^2 + \mu_p^2 (e \cdot k)^2] q \cdot (k - e)$$

$$- e \cdot k p \cdot k [\langle q \cdot (k - e)\rangle^2 + \langle e \cdot (q - k)\rangle^2] \Big\} . \qquad (11)$$

After converting three-dimensional integrals to four-dimensional integrals by introducing the appropriate Dirac delta functions, G may be cast in the form

$$G(q^2) = \frac{1}{2^2 \pi^7} \frac{\alpha g^2}{\hbar^2 c} \int d^4 e\, \theta(e_0)\, \delta(e^2 + \mu_e^2)$$

$$\times \left\{ \frac{[\mu_e^2 (p \cdot k)^2 + \mu_p^2 (e \cdot k)^2] q \cdot (k - e) - e \cdot k p \cdot k [\langle q \cdot (k - e)\rangle^2 + \langle e \cdot (q - k)\rangle^2]}{(p \cdot k)^2 (e \cdot k)^2} \right\}$$

$$\times \int d^4 n \int d^4 \nu\, \theta(n_0)\, \theta(\nu_0)\, \delta(n^2 + \mu_n^2)\, \delta(\nu^2)\, \delta(q - e - n - \nu)\, n \cdot \nu .$$

The neutron and neutrino integrations may be done exactly:

$$\int d^4 n \int d^4 \nu\, \theta(n_0)\, \theta(\nu_0)\, \delta(n^2 + \mu_n^2)\, \delta(\nu^2)\, \delta(q - e - n - \nu)\, n \cdot \nu$$

$$= \frac{\pi}{4} \frac{\left[(q - e)^2 + \mu_n^2\right]^2}{(q - e)^2} \theta(q_0 - \mu_0)\, \theta\left(-(q - e)^2 - \mu_n^2\right) .$$

The expression for G is thus reduced to just an integral over the electron variables:

$$G(q^2) = \frac{1}{2^4 \pi^6} \frac{\alpha g^2}{h^2 c} \int d^4 e\, \theta(e_0)\, \delta(e^2 + \mu_e^2)\, \theta(q_0 - \mu_0)\, \theta\; -(q - e)^2 - \mu_n^2$$

$$\times \frac{\left[(q - e)^2 + \mu_n^2\right]^2}{(q - e)^2} \left\{ \frac{-e \cdot k}{p \cdot k} + \frac{\mu_p^2 q \cdot (k - e) + 2 p \cdot k e \cdot q}{(p \cdot k)^2} \right.$$

$$\left. + \frac{1}{e \cdot k} \left(- \frac{\langle q \cdot (k - e)\rangle^2 + \langle q \cdot e\rangle^2}{p \cdot k} \right) + \frac{\mu_e^2 q \cdot (k - e)}{(e \cdot k)^2} \right\} .$$

Recall that G is an invariant function of the invariant q^2; it will be easier to do the remaining integrations in the center-of-momentum reference frame. In this case, $q^2 = -q_0^2$. To get the value of G in any other reference from its value for the center-of-momentum value of q we need only replace q_0 by $\sqrt{-q^2}$ everywhere; in fact, however, we shall only require its value in the center of momentum. Hence we set $\mathbf{q} = 0$; then the separation of the various terms into factors independent of the direction of $\mathbf{e}$ and those dependent on the direction of $\mathbf{e}$ is evident; the direction of $\mathbf{e}$ appears only in those factors involving $\mathbf{e} \cdot \mathbf{k}$. Such a separation facilitates the evaluation of the integrals over the angular coordinates of e. The step functions are independent of the angular coordinates and therefore pose no restrictions on them, but they do pose restrictions on the limits of integration of the variable e_0; clearly $\mu_e \le e_0 \le [q_0^2 + \mu_e^2 - (\mu_n^2/2q_0)]$. The Dirac delta function $\delta(e^2 + \mu_e^2)$ will enable the integration over the variables $|\mathbf{e}|$ to be done immediately.

The integral is thus reduced to a three-dimensional integral. In the center-of-momentum frame, $\mathbf{q} = 0$, and

$$q \cdot k = p \cdot k = \frac{1}{2}\left(\mu_p^2 - q_0^2\right),$$

$$k_0 = q \cdot k / q_0 = \left(q_0^2 - \mu_p^2\right)/2q_0 .$$

With $\eta = \cos(\mathbf{k}, \mathbf{e})$, we also have

$$e \cdot k = + \frac{q \cdot k}{q_0}\left(e_0 - |\mathbf{e}|\eta\right).$$

From these results, we may cast the terms of the integrand of the right-hand side of Eq. (10) into the following forms:

$$-\frac{e \cdot k}{p \cdot k} = -\frac{e_0 - |\mathbf{e}|\eta}{q_0},$$

$$\frac{\mu_p^2 q \cdot (k - e) + 2p \cdot k e \cdot q}{(p \cdot k)^2} = \frac{2\mu_p^2}{q_0^2 - \mu_p^2} - \frac{4q_0^3 e_0}{\left(q_0^2 - \mu_p^2\right)^2} \equiv -g_{II},$$

$$-\frac{\langle q \cdot (k - e)\rangle^2 + \langle q \cdot e\rangle^2}{e \cdot k p \cdot k} = -\frac{q_0}{e_0 - |\mathbf{e}|\eta} - \frac{q_0}{e_0 - |\mathbf{e}|\eta}\,\frac{8q_0^2 e_0^2}{\left(q_0^2 - \mu_p^2\right)^2},$$

$$-\frac{q_0}{e_0 - |\mathbf{e}|\eta} \cdot \frac{4q_0 e_0}{q_0^2 - \mu_p^2} = -\frac{q_0}{e_0 - |\mathbf{e}|\eta}\left(1 + \frac{4q_0 e_0}{q_0^2 - \mu_p^2} + \frac{8q_0^2 e_0^2}{\left(q_0^2 - \mu_p^2\right)^2}\right),$$

and

$$\equiv -\frac{q_0}{e_0 - |\mathbf{e}|\eta}\, g_{III}$$

$$\frac{\mu_e^2 q \cdot (k - e)}{(e \cdot k)^2} = -\frac{q_0^2}{\left(e_0 - |\mathbf{e}|\eta\right)^2}\left(\frac{2\mu_e^2}{q_0^2 - \mu_p^2} - \frac{4\mu_e^2 q_0 e_0}{\left(q_0^2 - \mu_p^2\right)^2}\right)$$

$$\equiv -\frac{q_0^2}{\left(e_0 - |\mathbf{e}|\eta\right)^2}\, g_{IV} .$$

We continue with the evaluation of $G(-\mu^2\xi)$ by noting that

$$\int d^4 e\,\theta(e_0)\,\theta(q_0 - \mu)\,\theta\left(-(q - e)^2 - \mu_n^2\right)\delta(e^2 + \mu_e^2)\,F(e_0, |\mathbf{e}|, \Omega_{\hat{e}}; q_0)$$

$$= \frac{1}{2}\int_{\mu_e}^{(q_0^2 + \mu_e^2 - \mu_n^2)/2q_0} de_0 \sqrt{e_0^2 - \mu_e^2}\int d\Omega_{\hat{e}} F\left(e_0, \sqrt{e_0^2 - \mu_e^2}, \Omega_{\hat{e}}; q_0\right)$$

and that

$$\int d\Omega_{\hat{e}} \frac{e_0 - |e|\eta}{q_0} = \frac{4\pi e_0}{q_0},$$

$$\int d\Omega_e \cdot 1 = 4\pi,$$

$$\int d\Omega_{\hat{e}} \frac{1}{e_0 - \eta\sqrt{e_0^2 - \mu_e^2}} = \frac{2\pi}{\sqrt{e_0^2 - \mu_e^2}}\, ln\left(\frac{e_0 + \sqrt{e_0^2 - \mu_e^2}}{e_0 - \sqrt{e_0^2 - \mu_0^2}}\right),$$

$$\int d\Omega_{\hat{e}} \frac{1}{(e_0 - \eta\sqrt{e_0^2 - \mu_e^2})^2} = \frac{4\pi}{\mu_e^2};$$

then

$$G(-q_0^2) = \frac{2\pi}{2^5\pi^6}\frac{\alpha g^2}{\hbar^2 c}\int_{\mu_e}^{(q_0^2+\mu_e^2-\mu_n^2)/2q_0} de_0 \sqrt{e_0^2 - \mu_e^2}\; \frac{\left[(q-e)^2 + \mu_n^2\right]^2}{-(q-e)^2}$$

$$\times\left(\frac{2e_0}{q_0} + 2g_{II} + \frac{q_0}{\sqrt{e_0^2 - \mu_0^2}}\, ln\left(\frac{e_0 + \sqrt{e_0^2 - \mu_e^2}}{e_0 - \sqrt{e_0^2 - \mu_e^2}}\right) g_{III} + \frac{2q_0^2}{\mu_e^2}\, g_{IV}\right).$$

In this last expression, replace e_0 by $\mu_e x$ and q_0 by $\mu\sqrt{\xi}$; then, $G(-\mu^2\xi)$ becomes

$$G(-\mu^2\xi) = \frac{1}{2^3\pi^5}\frac{\alpha g^2}{h^2 c}\mu\mu_e^3 g(\xi)$$

where

$$g(\xi) = \sqrt{\xi}\int_1^{(\mu^2\xi+\mu_e^2-\mu_n^2)/2\mu\mu_e\sqrt{\xi}} dx\, \sqrt{x^2 - 1}\; \frac{\left(\dfrac{\mu^2\xi + \mu_e^2 - \mu_n^2}{2\mu\mu_e\sqrt{\xi}} - x\right)^2}{\dfrac{\mu^2\xi + \mu_e^2}{2\mu\mu_e\sqrt{\xi}} - x}$$

$$\times\left\{\frac{2\mu_e}{\mu}\frac{x}{\sqrt{\xi}} + \frac{4(\xi+\xi_0)}{\xi - \xi_0} - \frac{16\mu_e}{\mu}\frac{x\xi^{3/2}}{(\xi-\xi_0)^2}\right.$$

$$\left. + \frac{\mu}{\mu_e}\frac{\sqrt{\xi}}{\sqrt{x^2 - 1}}\left(1 + \frac{4\mu_e}{\mu}\frac{x\sqrt{\xi}}{\xi - \xi_0} + \frac{8\mu_e^2}{\mu^2}\frac{x^2\xi}{(\xi-\xi_0)^2}\right) ln\left(\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}\right)\right\}.$$

If it were not for the presence of the factor $[(\mu^2\xi + \mu_e^2)/(2\mu\mu_e\sqrt{\xi}) - x]^{-1}$, this integration could be performed analytically. This factor varies in value from $\mu\sqrt{\xi}/2\mu_e$ at $x = 1$ to $\mu/2\mu_e\sqrt{\xi}$ at $x = (\mu^2\xi + \mu_e^2 - \mu_n^2)/(2\mu\mu_e\sqrt{\xi})$. For ξ very nearly unity, this factor remains nearly a constant. For low enough temperatures, $g(\xi)$ is multiplied by

$$\exp\left[-\frac{[(m_n + m_e)\, c^2 \sqrt{\xi}\,]}{kT}\right]$$

so that values of g for ξ significantly different from unity are reduced over those near unity by $\exp[-(m_n + m_e)/kT]\; c^2(\sqrt{\xi} - 1)$, a very small number. It would appear that we could thus approximate $g(\xi)$ by evaluating the factor $[(\mu^2\xi + \mu_e^2)/(2\mu\mu_e\sqrt{\xi}) - x]^{-1}$ at some point near $x = 1$. But caution is necessary since such an approximation scheme will not give results valid for temperatures that might be interesting in cosmological theories and also because $g(1)$ itself is zero. Further, the remaining integral, though it may be evaluated exactly, does not yield a function that renders the ξ integration tractable at all. To get any results, in this fashion, that are at all convincing appears to require restricting the temperature to values below $10^9\,^\circ$K, whereas we know that a large value for ω is likely only at $10^9\,^\circ$K or higher. For these reasons, we leave $g(\xi)$ in the last form and resort to a numerical computation of the integrals to give ω as a function of temperature. The results of the computation are compiled in Table 1.

DISCUSSION AND SUMMARY

It is important to note that the transition rate and lifetime as tabulated is in need of a correction that is temperature dependent. The correction has not been made in this report, but even so, the trend of the results is clearly delineated by the given values of ω and τ. The necessary correction to the given results stems from the fact that at high enough temperatures, the radiation field creates its own positive-negative electron pairs, and these are always created in the lowest allowed energy states. The probability that a given positron state will be occupied will be between zero and one-half; hence the availability of final states to the emitted positron is limited. Indeed, the positron phase space factor must be modified by subtracting the number of occupied positron states. Hence if $\rho(\epsilon)$ represents the density of positron states at an energy ϵ, as used in this report, the correction to the density amounts to subtracting $\rho(\epsilon)\cdot(e^{\epsilon/kT} + 1)^{-1}$, or by replacing $\rho(\epsilon)$ by $\rho(\epsilon)\, e^{\epsilon/kT}(e^{\epsilon/kT} + 1)^{-1}$; ϵ, of course, includes the rest-mass energy. For temperatures less than $6\times10^9\,^\circ$K, this correction is very small ($\leq 5\%$); for temperatures *greatly* in excess of $6\times10^9\,^\circ$K, the correction amounts to multiplying ω by one-half, or to doubling the lifetimes τ. Hence, the orders of magnitude of the results are not affected, and the qualitative features of the results are preserved.

J. N. Bahcall (4) has given the lifetime of the proton against the reaction $e^- + p \rightarrow n + \nu_e$ as

$$\tau_{cp} \approx \frac{1175 \pm 30}{K}\ \text{sec}$$

where, for nondegenerate matter, K is given by

$$K \approx 2\beta^{-5}\, e^{\nu} e^{-x}(x^2 + 6x + 12)\ ,$$

with

$$\beta = m_e c^2/kT$$

and

$$x \approx 2.54\beta\ .$$

Table 1
Numerical Computations Give the Following Values for ω as a Function of T

T_9	ω (sec^{-1})	$1/\omega = \tau$ (sec)
0.100	6.34×10^{-96}	1.58×10^{95}
1.000	4.17×10^{-16}	2.40×10^{15}
2.000	1.47×10^{-11}	6.80×10^{10}
3.00	2.47×10^{-9}	4.05×10^{8}
4.00	4.92×10^{-8}	2.03×10^{7}
5.00	3.83×10^{-7}	2.61×10^{6}
6.00	1.79×10^{-6}	5.57×10^{5}
7.00	6.10×10^{-6}	1.64×10^{5}
8.00	1.68×10^{-5}	5.93×10^{4}
9.00	3.98×10^{-5}	2.51×10^{4}
10.00	8.38×10^{-5}	1.19×10^{4}
11.00	1.61×10^{-4}	6.18×10^{3}
12.00	2.91×10^{-4}	3.43×10^{3}
13.00	4.96×10^{-4}	2.02×10^{3}
14.00	8.05×10^{-4}	1.24×10^{3}
15.00	12.58×10^{-4}	7.95×10^{2}
...	...	...
100	69.7	1.43×10^{-2}
1000	1.39×10^{7}	7.20×10^{-8}

Nondegeneracy is assured if $\rho/T_9^{3/2} < 1.52\times10^6$; we shall restrict ourselves to a range of ρ and T where the nondegeneracy criteria are met in order to get a qualitative comparison of this reaction to the $\gamma + p$ reaction. Under these conditions, τ_{ep} reduces to

$$\rho\tau_{ep} \approx 8.63\times10^9 + \frac{6.54}{T_9}\,(12.90 + 5.14\,T_9 + .341\,T_9^2)^{-1}\ (\text{gm-sec/cm}^3)\ .$$

The following short table of values of $\rho\tau_{ep}$ versus T_9 indicates how $\rho\tau_{ep}$ behaves

T_9	$\rho\tau_{ep}$ (gm-sec/cm^3)
1	1.63×10^{15}
2	1.22×10^{12}
4	9.44×10^{9}
6	1.56×10^{9}
8	7.53×10^{8}
10	3.95×10^{8}
100	2.20×10^{6}
1000	2.50×10^{2}

Figure 1 shows a plot of $\tau(\gamma + p)$, as well as of τ_{ep} for $\rho = 10^3$ and $\rho = 10^4$. The expected crossover does indeed occur, clearly at lower temperatures for lower densities. At temperatures of 10^{12}°K, the stability, or lack thereof, of the proton is determined almost solely by the γ, p reaction, except for extreme densities.

The instability of the proton cannot be an important factor for a star unless the temperatures where protons abound are of the order of 6×10^9°K or higher. Hence, the process has no effect on main sequence stars at all, as was shown in the introduction. Such high temperatures are reached in stellar interiors only in those stars that are incipient supernova; Chiu (8) has shown that the relaxation time of such a star for cooling due to neutrino emission is of the order of 10^7 sec or less for temperatures greater than 10^9°K. While τ and τ_ν have the same order of magnitude, τ_ν is not the proper number with which to compare the proton lifetime; that number should be the collision time of the proton, which is many orders of magnitude smaller at these temperatures and densities than is τ_p. Hence we conclude that the reactions $\gamma + p \rightarrow n + e^+ + \nu_e$ and $e^- + p \rightarrow n + \nu_e$ are not of importance to problems of stellar interiors and nucleosynthesis in stellar interiors. We mentioned in our introduction the possibility the hydrogen could be convected into stellar interiors and that under such an eventuality, these reactions might be important; however, in presupernova stars, the hot cores are not convective (8). Therefore, once again the proton may be regarded as stable.

The reactions cited here may be of relevance in certain cosmological theories of the early stages of the universe, where temperatures of the order of 10^{12}°K are postulated.

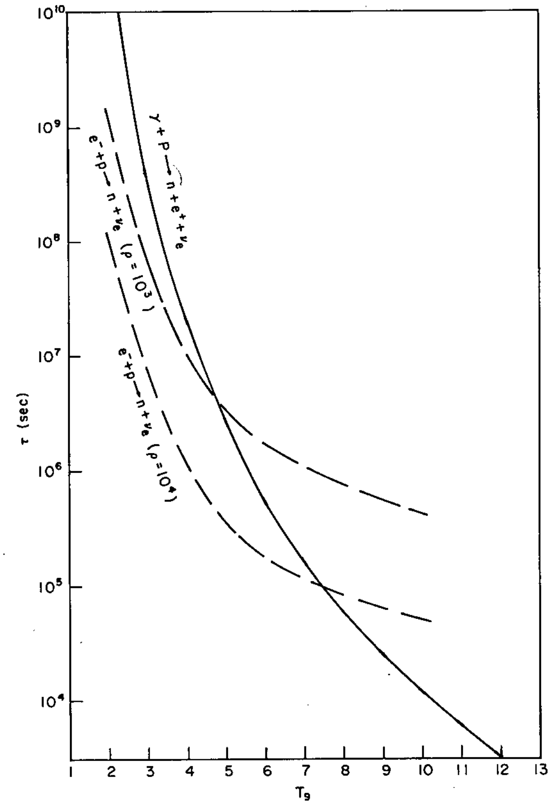


Fig. 1 - Graph of proton lifetime vs ambient temperature T for the γ, p and e^-, p reactions

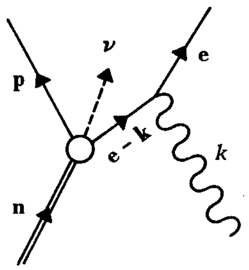
Even though we concluded that the photoinduced β-decay of the proton is of no importance for astrophysics, this study raises some rather interesting questions that are being studied further. We have seen that in spite of the rather large threshold for the $\gamma + p \rightarrow n + e^+ + \nu_e$ reaction and the smallness of the coupling constant, τ_p possesses such a steep temperature dependence as to shorten the lifetime of the proton to the order of 1 day at $6\times10^9\,^\circ$K. Now the neutron is supposed, in many theories of stellar nucleosynthesis, to be quite abundantly produced in stellar interiors and is needed to synthesize the elements of high atomic numbers. The neutron in these theories is taken to be stable; in fact of course, it is not, having a natural lifetime of the order of 10^4 sec; but since this is a very long time as compared with the lifetime against all nuclear reactions in the stellar material, the neutron stability is a good assumption. However, when we include reactions such as

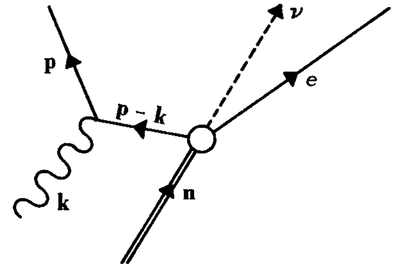
$$\gamma + n \rightarrow p + e^- + \bar{\nu}_e$$

and

$$e^+ + n \rightarrow p + \bar{\nu}_e$$

we may once again question the validity of the assumption of neutron stability. Disregarding the second reaction, which requires positrons to be present, photons are always naturally present in great abundance; we emphasize that now there is no threshold to overcome. Hence the natural β-activity of the neutron should be enhanced by the blackbody bath surrounding it. Now if the lifetime of the proton is cut from infinity at $0\,^\circ$K to 10^5 sec at $6\times10^9\,^\circ$K, when a threshold barrier must be overcome, to what degree will the neutron lifetime be cut at the same temperature? To calculate this, the natural procedure is to begin with the same interaction Hamiltonian as used in the second section above. The pertinent Feynman diagrams are given below.





The calculation proceeds along the same lines as the present one did. One important difference present however is that due to the lack of a minimum threshold energy; hence photons of zero momentum may be absorbed. Determining the matrix element and its square is tedious but straightforward. The transition rate is then the integral of the Planck distribution for the photons with the square of the matrix element, the integration extending from $\omega = 0$ to $\omega = \infty$ where ω is the circular frequency of the photons.

From the Feynman rules for writing the matrix elements from the diagrams and the fact that the final state momenta $\mathbf{p} + \mathbf{e}$ lie on the mass shells, the frequency dependence of the matrix elements is $\omega^{-3/2}$, so the square of the matrix elements is ω^{-3}. Hence our integral over $d\omega$ is of a function whose behavior near $\omega \sim 0$ is given by

$$\frac{\omega^2}{\exp\left(\frac{h\omega}{kT}\right) - 1} \cdot \frac{1}{\omega^3} \approx \frac{kT\omega^2}{\omega^4} = \frac{1}{\omega^2} .$$

Therefore the integral will diverge because of the singularity at $\omega = 0$. This renders the transition rate infinite and the lifetime of the neutron zero *for any temperature* $T \neq 0$. The singularity does not appear to be renormalizable. It owes its origin in part to the frequency dependence of the Planck distribution and in part to the use of perturbation theory. If the Planck spectrum is replaced by a laser beam whose frequency is allowed to go to zero, the matrix element still possesses this anomalous behavior. The source of the trouble appears to lie in the use of perturbation theory for the electromagnetic interaction. Fried and Eberly (9) have studied Compton scattering by a laser beam, by means of perturbation theory in the external potential, and have shown that in any finite order, as $\omega \to 0$, the cross section becomes infinite; however, if the scattering is calculated to all orders and summed exactly, the singularity at $\omega = 0$ disappears and, for low energy densities, one recovers the Thomson cross section. The success of this technique lies in the fact that the laser beam possesses a unique frequency; this allows the infinite sum to be evaluated (as a continued fraction). It would appear that the Planck spectrum affords us no such advantages. The problem is still under investigation, but no conclusions can be drawn at the present time.

ACKNOWLEDGMENT

My very deep thanks go to Arlo D. Anderson for performing the necessary numerical computations to determine the lifetime.

REFERENCES

1. Clayton, D.D., "Principles of Stellar Evolution and Nucleosynthesis," New York: McGraw-Hill, 1968

2. Schwarzschild, K., "Structure and Evolution of Stars," New York:Dover, 1958

3. Wallerstein, G., and Greenstein, J.L., in "Stellar Evolution," Stein and Cameron, eds., New York:Plenum, 1960. Also see article by V.L. Peterson and M.H. Wrubel in same volume.

4. Bahcall, J.N., J. Appl. Phys., 139:332 (1964)

5. Kållen, G., "Elementary Particle Physics," Cambridge, Mass.:Addison-Wesley, 1964

6. Marshak and Sudarshan, Phys. Rev., 109:1860 (1958)

7. Feynman, R.P., and Gell-Mann, M., Phys. Rev., 109:193 (1958)

8. Chiu, H.Y., in "Stellar Evolution," R.F. Stein and A.G.W. Cameron, eds., New York: Plenum, 1966

9. Fried, Z., and Eberly, J., Phys. Rev., 136(B):871 (1964)

UNCLASSIFIED

Appendix

COMPUTATIONAL DETAILS

In this section we give some of the computational details of the evaluation of Eq. (11). From Eq. (5) we have

$$\left.\begin{aligned} \mathfrak{M}_\lambda^{e*} &= (-1)^{\delta_{\lambda 4}+1}\bar{U}_u(\mathbf{p})\gamma^\mu(1+\gamma^5)U_r(\mathbf{n})\nu_s(\mathbf{e})\gamma^\lambda \frac{\langle -i\gamma\cdot(e-k)-\mu_e\rangle}{(e-k)^2+\mu_e^2}\gamma^\mu(1+\gamma^5)U_t(\boldsymbol{\nu}) \\ &\text{and} \\ \mathfrak{M}_\lambda^{p*} &= (-1)^{\delta_{\lambda 4}}U_u(\mathbf{p})\gamma^\lambda \frac{\langle i\gamma\cdot(p+k)-\mu_p\rangle}{(p+k)^2+\mu_p^2}\gamma^\mu(1+\gamma^5)U_r(\mathbf{n})\bar{\nu}_s(\mathbf{e})\gamma^u(1+\gamma^5)U_t(\boldsymbol{\nu}) \ . \end{aligned}\right\} \quad \text{(A1)}$$

We shall need the following relations:

$$\left.\begin{aligned} \sum_{\ell=1,2} u_\ell(\eta)\,\bar{u}_\ell(\eta) &= \frac{\mu - i\gamma\cdot\eta}{2\mu} \\ -\sum_{\ell=1,2} \nu_\ell(\eta)\,\bar{\nu}_\ell(\mathbf{x}) &= \frac{\mu + i\gamma\cdot\eta}{2\mu} \ . \end{aligned}\right\} \quad \text{(A2)}$$

Then, setting $\mu_\nu = 0$ in the numerators, we obtain

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^{e}\mathfrak{M}_\lambda^{e*}(-1)^{\delta_{\lambda 4}} = \frac{-2^{-4}}{\mu_p\mu_n\mu_e\mu_\nu}$$

$$\times \frac{1}{(e\cdot k)^2}\, Tr\left[(1-\gamma^5)\gamma^\mu\langle\mu_p - i\gamma\cdot p\rangle\gamma^\rho(1+\gamma^5)\langle\mu_n - i\gamma\cdot n\rangle\right]$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu\langle i\gamma\cdot(e-k)+\mu_e\rangle\langle 2\mu_e - i\gamma\cdot e\rangle\langle i\gamma\cdot(e-k)+\mu_e\rangle\gamma^\rho i\gamma\cdot\nu\right] \quad \text{(A3)}$$

and

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^{p}\mathfrak{M}_\lambda^{p*}(-1)^{\delta_{\lambda 4}} = \frac{-2^{-4}}{\mu_p\mu_n\mu_e\mu_\nu}\cdot\frac{1}{(p\cdot k)^2}$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu\langle i\gamma\cdot(p+k)-\mu_p\rangle\langle 2\mu_p + i\gamma\cdot p\rangle\langle i\gamma\cdot(p+k)-\mu_p\rangle\gamma^\rho\cdot(1+\gamma^5)\langle\mu_n - i\gamma\cdot n\rangle\right]$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu\langle\mu_e + i\gamma\cdot e\rangle\gamma^\rho i\gamma\cdot\nu\right] \ . \quad \text{(A4)}$$

Also,

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^e \mathfrak{M}_\lambda^{p*} (-1)^{\delta_{\lambda 4}} = \frac{-2^{-6}}{(p \cdot k)(e \cdot k)\,\mu_p \mu_n \mu_e \mu_\nu}$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu \langle \mu_p - (\gamma \cdot p)\gamma^\lambda \langle i\gamma \cdot (p+k) - \mu_p \rangle \gamma^\rho (1+\gamma^5)\langle \mu_n - i\gamma \cdot n\rangle\right]$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu \langle -i\gamma \cdot (e-k) - \mu_e\rangle \gamma^\lambda \langle \mu_e + i\gamma \cdot e\rangle \gamma^\rho (1+\gamma^5)\, i\gamma \cdot \nu\right] \;. \tag{A5}$$

The evaluation of the traces is a straightforward but tedious task. We merely state some needed intermediate results and the traces themselves:

$$\langle i\gamma \cdot (e-k) + \mu_e\rangle \langle 2\mu_e - i\gamma \cdot e\rangle \langle i\gamma \cdot (e-k) + \mu_e\rangle = 2\mu_e(\mu_e^2 + e \cdot k)$$

$$+2\mu_e^2\, i\gamma \cdot e - 2(\mu_e^2 - e \cdot k)\, i\gamma \cdot k \;.$$

$$\langle i\gamma \cdot (p+k) - \mu_p\rangle \langle 2\mu_p + i\gamma \cdot p\rangle \langle i\gamma \cdot (p+k) - \mu_p\rangle = 2\mu_p(\mu_p^2 - k \cdot p) \tag{A6}$$

$$-2\mu_p^2\, i\gamma \cdot p - 2(\mu_p^2 + k \cdot p)\, i\gamma \cdot k \;.$$

Then

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^e \mathfrak{M}_\lambda^{e*} (-1)^{\delta_{\lambda 4}} = \frac{-1}{2^4 \mu_p \mu_n \mu_e \mu_\nu}$$

$$\times\, \frac{1}{(e \cdot k)^2}\, Tr\left[(1-\gamma^5)\gamma^\mu \langle \mu_p - i\gamma \cdot p\rangle \gamma^\rho (1+\gamma^5)\langle \mu_n - i\gamma \cdot n\rangle\right]$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu \langle 2\mu_e(\mu_e^2 + e \cdot k) + 2\mu_e^2\, i\gamma \cdot e - 2(\mu_e^2 - e \cdot k)\, i\gamma \cdot k\rangle \gamma^\rho i\gamma \cdot \nu\right] \;,$$

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^p \mathfrak{M}_\lambda^{p*} (-1)^{\delta_{\lambda 4}} = \frac{-1}{2^4 \mu_p \mu_n \mu_e \mu_\nu}$$

$$\times\, \frac{1}{(p \cdot k)^2}\, Tr\left[(1-\gamma^5)\gamma^\mu \langle \mu_e + i\gamma \cdot e\rangle q^\rho i\gamma \cdot \nu\right]$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu \langle 2\mu_p(\mu_p^2 - p \cdot k) - 2\mu_p^2\, i\gamma \cdot p - 2(\mu_p^2 + p \cdot k)\rangle \gamma^\rho (1+\gamma^5)\langle \mu_n - i\gamma \cdot n\rangle\right] \;,$$

and

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^e \mathfrak{M}_\lambda^{p*} (-1)^{\delta_{\lambda 4}} = \frac{+1}{2^6 \mu_p \mu_n \mu_e \mu_\nu (p \cdot k)(e \cdot k)}$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu \langle \mu_p - i\gamma \cdot p\rangle \gamma^\lambda \langle i\gamma \cdot (p+k) - \mu_p\rangle \gamma^\rho (1+\gamma^5)\langle \mu_n - i\gamma \cdot n\rangle\right]$$

$$\times\, Tr\left[(1-\gamma^5)\gamma^\mu \langle i\gamma \cdot (e-k) + \mu_e\rangle \gamma^\lambda \langle \mu_e + i\gamma \cdot e\rangle \gamma^\rho (1+\gamma^5)\, i\gamma \cdot \nu\right] \;.$$

Now we evaluate the traces, some details of which will be given. First let

$$Tr\,(1-\gamma^5)\,\gamma^\mu\gamma^\alpha\gamma^\lambda\gamma^\beta\gamma^\rho\gamma^\gamma \equiv T^{\mu\alpha\lambda\beta\rho\gamma}$$

and

$$Tr(1-\gamma^5)\,\gamma^\mu\gamma^\lambda\gamma^\rho\gamma^\sigma \equiv R^{\mu\lambda\rho\sigma}\ .$$

All the above traces may be expressed in terms of these tensors. Thus,

$$Tr[(1-\gamma^5)\,\gamma^\mu\langle\mu_p - i\gamma\cdot p\rangle\gamma^\rho(1+\gamma^5)\langle\mu_n - i\gamma\cdot n\rangle] = -2p^\lambda n^\sigma R^{\mu\lambda\rho\sigma}\ ,$$

$$Tr[(1-\gamma^5)\,\gamma^\mu\langle 2\mu_e(\mu_e^2 + e\cdot k) + 2\mu_e^2\, i\gamma\cdot e - 2\,(\mu_e^2 - e\cdot k)\ i\gamma\cdot k\rangle\gamma^\rho i\gamma\cdot\nu] =$$

$$[-2\mu_e^2 e^\lambda\nu^\sigma + 2\,(\mu_e^2 - e\cdot k)\,k^\lambda\nu^\sigma]\,R^{\mu\lambda\rho\sigma}\ ,$$

$$Tr\,[(1-\gamma^5)\,\gamma^\mu\langle 2\mu_p^2(\mu_p^2 - k\cdot p) - 2\mu_p^2\, i\gamma\cdot p - 2\,(\mu_p^2 + k\cdot p)\ i\gamma\cdot k\rangle\gamma^\rho(1+\gamma^5)\langle\mu_n - i\gamma\cdot n\rangle] =$$

$$2\,[-2\mu_p^2 p^\lambda n^\sigma - 2\,(\mu_p^2 + p\cdot k)\,k^\lambda n^\sigma]\,R^{\mu\lambda\rho\sigma}\ ,$$

$$Tr\,[(1-\gamma^5)\,\gamma^\mu\langle\mu_p - i\gamma\cdot p\rangle\gamma^\lambda\langle i\gamma(p+k) - \mu_p\rangle\gamma^\rho(1+\gamma^5)\langle\mu_n - i\gamma\cdot n\rangle] =$$

$$-2\,ip^\alpha(p+k)^\beta n^\gamma T^{\mu\alpha\lambda\beta\rho\gamma} + 2\,i\mu_p^2 n^\sigma R^{\mu\lambda\rho\sigma}\ ,$$

and

$$Tr\,[(1-\gamma^5)\,\gamma^\mu\langle i\gamma\cdot(e-k) + \mu_e\rangle\,\gamma^\lambda\langle\mu_e + i\gamma\cdot e\rangle\gamma^\rho(1+\gamma^5)\ i\gamma\cdot\nu] =$$

$$-2\,i\,(e-k)^\alpha e^\beta\nu^\gamma T^{\mu\alpha\lambda\beta\rho\gamma} + 2\,i\mu_e^2\nu^\sigma R^{\mu\lambda\rho\sigma}$$

Therefore,

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^e\mathfrak{M}_\lambda^{e*}(-1)^{\delta_{\lambda 4}} = \frac{-1}{2^2\mu_p\mu_n\mu_e\mu_\nu}$$

$$\times\ \frac{1}{(e\cdot k)^2}\ p^\lambda n^\sigma[\mu_e^2 e^{\lambda'}\nu^{\sigma'} - (\mu_e^2 - e\cdot k)\,k^{\lambda'}\nu^{\sigma'}]\,R^{\mu\lambda\rho\sigma}R^{\mu\lambda'\rho\sigma'}\ ,$$

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^p\mathfrak{M}_\lambda^{p*}(-1)^{\delta_{\lambda 4}} = \frac{-1}{2^2\mu_p\mu_n\mu_e\mu_\nu}$$

$$\times\ \frac{1}{(p\cdot k)^2}\ e^\lambda\nu^\sigma[\mu_p^2 p^{\lambda'} n^{\sigma'} + (\mu_p^2 + p\cdot k)\,k^{\lambda'}n^{\sigma'}]\,R^{\mu\lambda\rho\sigma}R^{\mu\lambda'\rho\sigma'}\ ,$$

and

$$2\ \mathrm{Re} \sum_{r,s,t,u} \mathfrak{M}_\lambda^{e} \mathfrak{M}_\lambda^{p*} (-1)^{\delta_{\lambda 4}} = \frac{-1}{2^3 \mu_p \mu_n \mu_e \mu_\nu}$$

$$\times \frac{1}{(p\cdot k)(e\cdot k)} \Big(p^\alpha (p+k)^\beta n^\gamma (e-k)^{\alpha'} e^{\beta'} \nu^{\gamma'} T^{\mu\alpha\lambda\beta\rho\gamma} T^{\mu\alpha'\lambda\beta'\rho\gamma'}$$

$$-\mu_p{}^2 n^\sigma (e-k)^{\alpha'} e^{\beta'} \nu^{\gamma'} R^{\mu\lambda\rho\sigma} T^{\mu\alpha'\lambda\beta'\rho\gamma'}$$

$$-\mu_e{}^2 \nu^{\sigma'} p^\alpha (p+k)^\beta n^\gamma T^{\mu\alpha\lambda\beta\rho\gamma} R^{\mu\lambda\rho\sigma'} + \mu_p{}^2 \mu_e{}^2 n^\sigma \nu^{\sigma'} R^{\mu\lambda\rho\sigma} R^{\mu\lambda\rho\sigma'} \Big)$$

The following list of identities are needed:

$$Tr[\gamma^\mu \gamma^\lambda \gamma^\rho \gamma^\sigma] = 4(\delta^{\mu\lambda}\delta^{\rho\sigma} - \delta^{\mu\rho}\delta^{\lambda\sigma} + \delta^{\mu\sigma}\delta^{\lambda\rho}) ,$$

$$Tr[\gamma^5 \gamma^\mu \gamma^\lambda \gamma^\rho \gamma^\sigma] = 4\epsilon^{\mu\lambda\rho\sigma} ,$$

$$\gamma^\alpha \gamma^\beta \gamma^\gamma = \gamma^\alpha \delta^{\beta\gamma} - \gamma^\beta \delta^{\gamma\alpha} + \gamma^\gamma \delta^{\alpha\beta} + \epsilon^{\alpha\beta\gamma\sigma} \gamma^5 \gamma^\sigma ,$$

$$\epsilon^{\mu\lambda\rho\sigma} \epsilon^{\mu\alpha\beta\gamma} = 1!\,[\delta^{\lambda\alpha}(\delta^{\rho\beta}\delta^{\sigma\gamma} - \delta^{\rho\gamma}\delta^{\sigma\beta})$$

$$+\delta^{\lambda\beta}(\delta^{\rho\gamma}\delta^{\sigma\alpha} - \delta^{\rho\alpha}\delta^{\sigma\gamma}) + \delta^{\lambda\gamma}(\delta^{\rho\alpha}\delta^{\sigma\beta} - \delta^{\rho\beta}\delta^{\sigma\alpha})] ,$$

$$\epsilon^{\mu\lambda\rho\sigma} \epsilon^{\mu\lambda\beta\gamma} = 2!\,(\delta^{\rho\beta}\delta^{\sigma\gamma} - \delta^{\rho\gamma}\delta^{\sigma\beta}) ,$$

$$\epsilon^{\mu\lambda\rho\sigma} \epsilon^{\mu\lambda\rho\gamma} = 3!\,\delta^{\sigma\gamma} ,$$

$$\epsilon^{\mu\lambda\rho\sigma} \epsilon^{\mu\lambda\rho\sigma} = 4! ,$$

and

$$T^{\mu\alpha\lambda\beta\rho\gamma} = \delta^{\mu\alpha} R^{\lambda\beta\rho\gamma} - \delta^{\mu\lambda} R^{\alpha\beta\rho\gamma} + \delta^{\alpha\lambda} R^{\mu\beta\rho\gamma} - \epsilon^{\mu\alpha\lambda\sigma} R^{\sigma\beta\rho\gamma} .$$

From these relations one can prove that

$$R^{\mu\lambda\rho\sigma} R^{\mu\lambda\rho\sigma'} = +2^8 \delta^{\sigma\sigma'}$$

$$T^{\mu\alpha\lambda\beta\rho\gamma} R^{\mu\lambda\rho\sigma'} = -2^7 \delta^{\alpha\beta} \delta^{\gamma\sigma'}$$

$$T^{\mu\alpha\lambda\beta\rho\gamma} T^{\mu\alpha'\lambda\beta'\rho\gamma'} = 2^8 \delta^{\alpha\alpha'} \delta^{\beta\beta'} \delta^{\gamma\gamma'}$$

$$R^{\mu\lambda\rho\sigma} R^{\mu\lambda'\rho\sigma'} = +2^6 \delta^{\lambda\lambda'} \delta^{\sigma\sigma'} .$$

Then

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^e \mathfrak{M}_\lambda^{e*}(-1)^{\delta_{\lambda 4}} = \frac{+2^4}{\mu_e \mu_n \mu_e \mu_\nu} \cdot \frac{n \cdot \nu}{(e \cdot k)^2} \left[(\mu_e^2 - e \cdot k)\, p \cdot k - \mu_e^2 e \cdot p\right] ,$$

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda^p \mathfrak{M}_\lambda^{p*}(-1)^{\delta_{\lambda 4}} = \frac{+2^4}{\mu_p \mu_n \mu_e \mu_\nu} \frac{(-n \cdot \nu)}{(p \cdot k)^2} \left[(\mu_p^2 + p \cdot k)\, e \cdot k + \mu_p^2 p \cdot e\right] ,$$

and

$$2 \operatorname{Re} \sum_{r,s,t,u} \mathfrak{M}_\lambda^e \mathfrak{M}_\lambda^{p*}(-1)^{\delta_{\lambda 4}} = \frac{+2^4}{\mu_p \mu_n \mu_e \mu_\nu} \cdot \frac{(-n \cdot \nu)}{p \cdot k e \cdot k} \left[2p \cdot (e - k)\, e \cdot (p + k) - \mu_p^2 e \cdot k + \mu_e^2 p \cdot k\right] .$$

From these results we obtain

$$\sum_{r,s,t,u} \mathfrak{M}_\lambda \mathfrak{M}_\lambda^*(-1)^{\delta_{\lambda 4}} = \frac{2^4}{\mu_p \mu_n \mu_e \mu_\nu} \frac{n \cdot \nu}{(p \cdot k)^2 (e \cdot k)^2}$$

$$\times \Big\{ \left[\mu_e^2 (p \cdot k)^2 + \mu_p^2 (e \cdot k)^2\right] q \cdot (k - e)$$

$$- e \cdot k p \cdot k \left[(q \cdot (k - e)^{\,2} + \; e \cdot (q - k))^2\right] \Big\}$$

where $q = p + k$.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

ON THE PHOTO-INDUCED BETA DECAY OF PROTONS AND NEUTRONS IN STELLAR INTERIORS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

A final report on one phase of a continuing problem.

5. AUTHOR(S) (First name, middle initial, last name)

John N. Hayes

6. REPORT DATE January 6, 1971	7a. TOTAL NO. OF PAGES 30	7b. NO. OF REFS 9

8a. CONTRACT OR GRANT NO. NRL Problem H01-06 b. PROJECT NO. RR 002-06-41-5003 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 7124 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Department of the Navy (Office of Naval Research), Washington, D.C. 20360

13. ABSTRACT

In this first phase of a study on photon-induced beta decay, the reaction rate for $\gamma + p \rightarrow n + e^+ + \nu_e$ is determined for conditions approximating those in stellar interiors. It is shown that the lifetime of the proton in such conditions drops from infinity to only microseconds as the temperature varies from 0 to 10^{12}°K. Hence the stability of the proton against this reaction is not seriously altered and will not affect nucleosynthesis (which depends on the availability of protons). However, the reactions $\gamma + n \rightarrow p + e^- + \bar{\nu}_e$ and $e^- + p \rightarrow n + \nu_e$ are not restrained by threshold energy requirements. If the neutron's lifetime is drastically reduced by this latter reaction to the point that it cannot live long enough to react with nuclei, then nucleosynthesis in stars will be curtailed at very high temperatures. Attempts in this latter case to proceed with calculations along the same line as the $\gamma + p$ calculation run immediately into a divergence problem, as is shown here. The correct handling of this reaction will be the subject of another report when completed.

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Photoelectrons						
Beta decay						
Neutron reactions						
Proton reactions						
Half life						
Stellar evolution						
Stellar physics						