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13. ABSTRACT

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ABSTRACT

A generalized method, reported in NRL Report 7300, for determining the fixed-base natural frequencies of an in situ or laboratory mechanical structure or substructure was tested by experiments. The test structure was composed of a simple steel beam on three supports which were mounted on flexible members of a trusslike frame. The fixed-base natural frequencies of the lowest two modes of the test beam were determined by the aforementioned semianalytical method. Theoretical calculations and a standard resonance test were also performed for comparison. Results confirmed the applicability and usefulness of the developed method, as predicted by the theoretical analysis.

PROBLEM STATUS

This is an interim report; work is continuing on other phases of the problem.

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DETERMINATION OF FIXED-BASE NATURAL FREQUENCIES OF A COMPOSITE STRUCTURE OR SUBSTRUCTURES: EXPERIMENT AND APPLICATION

INTRODUCTION

As a result of recent developments, the application of matrix analysis to the solution of dynamic problems of large composite structures has been widespread. The division of a composite structure into mutually coupled substructures with constraints is one of the essential steps in this analysis. These constrained substructures are, in turn, treated separately to produce results that are then synthesized to obtain the dynamic properties of the total composite structure. Methods of structure synthesis are categorized according to the applied internal boundary conditions among the divided substructures: the displacement method primarily emphasizes the matching of internal boundary displacements, and the force method basically considers the equilibrium of interaction redundant forces. Among the developed methods reported to date, a displacement method treated by Hurty (1) is particularly interesting in certain engineering applications. In his method, a composite structure is divided into fixed-base (or fixed-constraint) substructures. The dynamic problem of the total composite structure is then reduced to the solution of two separate problems:

1. The dynamic problem of the fixed-base substructures

2. The mechanical coupling between the adjacent substructures.

This division is advantageous in the treatment of composite structures consisting of functional substructures. Normally, the spatial configuration of a functional substructure is well determined, and the imposed fixed-base constraint isolates the particular substructure from its surroundings. Consequently, the obtained dynamic properties of this substructure are independent of space and time, and such data can be of permanent value in engineering practice. Parallel but independent to this treatment, the determination of fixed-base natural frequencies has been studied both theoretically and experimentally (2-5) at the U.S. Naval Research Laboratory (NRL).

To design a structure which undergoes dynamic loading, a knowledge of the natural frequencies of free vibration of the structure is required. Two commonly used approaches for determining the fixed-base natural frequencies of a structure are

1. Analytical approach. This approach requires modeling the structure under study. In general, the modeling problem is very complicated and virtually relies on individual experience and intuitive judgment.

2. Experimental approach. This approach is essentially a structure resonance method. By sweeping a range of exciting frequencies, the resonance mode of the structure

is actually excited by the applied oscillatory force at the given fixed-base supporting condition. The fixed-base natural frequencies of the structure are identified by the resonance frequencies. This method fails when the supporting base becomes flexible.

This acquisition of information concerned with the fixed-base natural frequencies is made difficult by the induced uncertainties and the limitations of these two methods.

These limitations are overcome by a recently developed semianalytical (6) method of determining the fixed-base natural frequencies of composite structures. The structure dynamic response measurements at the constraints of the substructure, and the invariance properties of the total structure, are used to determine the substructure's true fixed-base natural frequencies. The mathematical analysis is based on a "conceptual lumped-mass model" to derive expressions for special functions in terms of measurable physical quantities. These quantities are the appropriate dynamic responses of the structure and the applied forces. Such special functions contain the information of the fixed-base natural frequencies of the divided substructures and the natural frequencies of the total structure. By proper deduction from those special functions, the fixed-base natural frequencies of the substructure considered can be concluded.

A conceptual lumped-mass model is a lumped-mass model obtained without actually modeling the real structure. The advantage of this concept is that it bypasses the complicated modeling problem and allows us to approach a continuous model with ease. Furthermore, in doing this, pointwise measurements in experiment become rigorously justified. The results derived from this method are actual physical quantities of the substructures or the total structure considered as a whole.

The experimental verification of this semianalytical method not only provides a definite way to obtain the fixed-base natural frequencies desired, but also proves the applicability of the lumped-mass model itself, in general. Such proof consolidates the matrix analysis of composite structures on its basic theoretical ground.

EXPERIMENTS

Semianalytical Method

The success of this method in computer simulations (6) was followed by laboratory experiments conducted at the random vibration laboratory, NRL. Since the shake test itself is well-known in structure dynamic response studies, only a few important aspects essential for the conclusions of meaningful results are presented in this report. It is understood that the natural frequencies of a structure are linear dynamic properties of the structure. The corresponding mathematical model used in the theoretical analysis is linear time-invariant and deterministic. Therefore, the key governing rule of the experimental or test measurements is to confine the operation within the range of linearity. In practical terms, the noise level must be kept low.

Review of Theoretical Results—Physical quantities, as well as theoretically derived special functions pertaining to the experimental performance and practical application of this method, are summarized below:

Mobility elements

$$m_{ij} = \frac{\overline{q}_i}{\overline{f}_j} \cos \phi_{ij} = -\frac{\overline{\ddot{q}}_i}{\overline{f}_j} \cos \phi_{ij} = \frac{\overline{\dot{q}}_i}{\overline{f}_j} \sin \phi_{ij} \tag{1}$$

Mobility functions

$$\Omega(m_{ij}; \omega) = |[m_{ij}]| = \frac{|[Z^e]|\,|[Z^b]|}{|[Z]|} \tag{2}$$

$$i,j = 1, 2, \ldots, t.$$

t = total number of supports

$$\Omega'(m_{ij}; \omega) = |[m_{ij}]| = \frac{|[Z^{e'}]|\,|[Z^b]|}{|[Z]|} \tag{3}$$

$$i,j = 1, 2, \ldots, t{+}1$$

$(t+1)$th point on equipment substructure.

Resonance function

$$\psi(m_{ij}; \omega) = \frac{\Omega'(m_{ij}; \omega)}{\Omega(m_{ij}; \omega)} = \left\| \frac{|[Z^{e'}]|}{|[Z^e]|} \right\| \tag{4}$$

where

$$\Omega(m_{ij}; \omega) = \left| \begin{bmatrix} m_{11} & m_{1t} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ m_{t1} & m_{tt} \end{bmatrix} \right| \tag{5}$$

m_{ij} = The mobility element at the location associated with the ith generalized coordinate due to the shake force at the location associated with the jth generalized coordinate

$\overline{q}_i, \overline{\dot{q}}_i, \overline{\ddot{q}}_i$ = The amplitude of the displacement, velocity, and acceleration responses respectively, associated with the ith generalized coordinate

ω = The exciting frequency

ϕ_{ij} = The relative phase difference between the ith response and the jth force.

$\overline{f_j}$ = The force amplitude. Its application point is associated with the jth generalized coordinate.

[ ] = Square matrix

| | = Determinant

|| || = Absolute value

$[Z^e]$ = Impedance matrix of the equipment substructure (a different equipment substructure is referred to when a prime is marked on the right upper corner.)

$[Z^b]$ = Impedance matrix of the base substructure

$[Z]$ = Impedance matrix of the total structure

The measurable quantities in Eq. (1) are obtained by a series of shake tests. These measurable quantities are related to the impedance matrixes through the theoretically derived Eqs. (1) through (5). The dynamic properties contained in the impedance matrixes of the substructure as well as the total structure, which hardly can be obtained otherwise, are then revealed through limited number of measurements by simple shake test.

Experimental Procedure—It is noticed in the theoretical results that the relative phases of the responses with respect to the applied force are necessary in the mobility and resonance calculation, even in an undamped linear time-invariant case. In practice, the measured responses always involve frequency components other than the exciting frequency, in spite of the fact that the exciting frequency can be a fairly well controlled value. The main sources of these undesirable frequency components are of electrical and mechanical origin. Contributions due to electrical distortion are often mendable. The mechanically induced distortion is mainly due to the nonlinear dynamic behavior of the total structure, which is an inherent property of the structure under test and cannot be removed without altering the structure itself. However, such nonlinear responses are avoidable if a specific experimental procedure is established so that the test is conducted within the dynamic range where the linear time-invariant mathematical model is representative for the structure under investigation. Within this range, the mobility elements of a given structure are independent of space and time, but are functions of the exciting frequency alone. By utilizing this invariance property of the mobility elements as a guideline, the determination of fixed-base natural frequencies of a substructure becomes possible.

For the very reasons described above, preparatory work is needed before taking useful data. This work is described below.

1. Force amplitude determination. Either too small or too large a force amplitude will cause an increasing noise to signal ratio, consequently reducing the reliability of the mobility element calculation. Although the mechanisms are different for these two cases, the former is believed due to harmonic instability (7, 8), and the latter is more likely

due to the inelastic property of the structure. Their effect on this particular method is the same. To avoid this difficulty, an appropriate magnitude of the force amplitude has to be first selected. This is done by varying the force amplitude applied on the structure, while response measurements, taken at a given constant exciting frequency, are recorded by oscillograph. The mobility elements corresponding to each force amplitude setting are calculated and plotted against force amplitudes. A typical plot is shown in Fig. 1. Within the range designated by R_f in the plot, the mobility element is force-amplitude independent for a given exciting frequency ω. Responses due to forces within the dynamic range are essentially governed by the linear time-invariant analytical model.

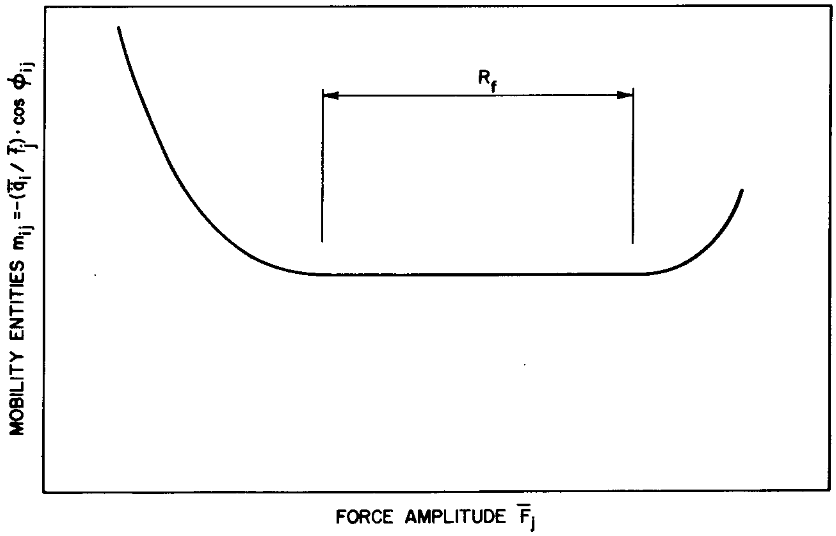


Fig. 1—Force amplitude determination in semianalytical method

2. Narrowband filter application. One is aware that nonlinear responses of varying degree are always present in real structures. Only structures with small nonlinear dynamic behavior are considered here because predominantly nonlinear systems certainly cannot be described by a linear analytical model. Direct measurement of relative phase angle by a zero crossing type of phase meter or lissajous type of measurement on the oscilloscope is very difficult at times. This difficulty is overcome, for slightly damped cases, by using a narrowband filter and comparing the phase difference between the two fundamental components. The amplitudes of the filtered fundamental components are then used for mobility element calculation.

After the preparatory work is done, the remaining task of the experiment or test is rather routine and will not be discussed here. The necessary data are collected and then used to calculate the value of the mobility functions and the resonance function corresponding to each exciting frequency. This is a straightforward routine on any digital computer, and no special program is needed.

It is important to point out that the resonance function is not defined when

$$|[Z^b]| = 0 \qquad \text{and} \qquad |[Z]| = 0,$$

although the analytical form in Eq. (4) shows that the resonance function is independent of the impedance determinants $|[Z^b]|$ and $|[Z]|$. The reason is that we are not measuring the impedance elements, but the mobility elements—indirectly through force and response measurements, according to Eq. (1). It is then evident from Eqs. (2), (3), and (4) that, when $|[Z^b]| = 0$ and $|[Z]| = 0$, the resonance function becomes 0/0 and ∞/∞, respectively. Such induced ambiguities can be eliminated by data analysis and present no real problem. The technique for doing this task is illustrated in the schematic diagram given in Fig. 2.

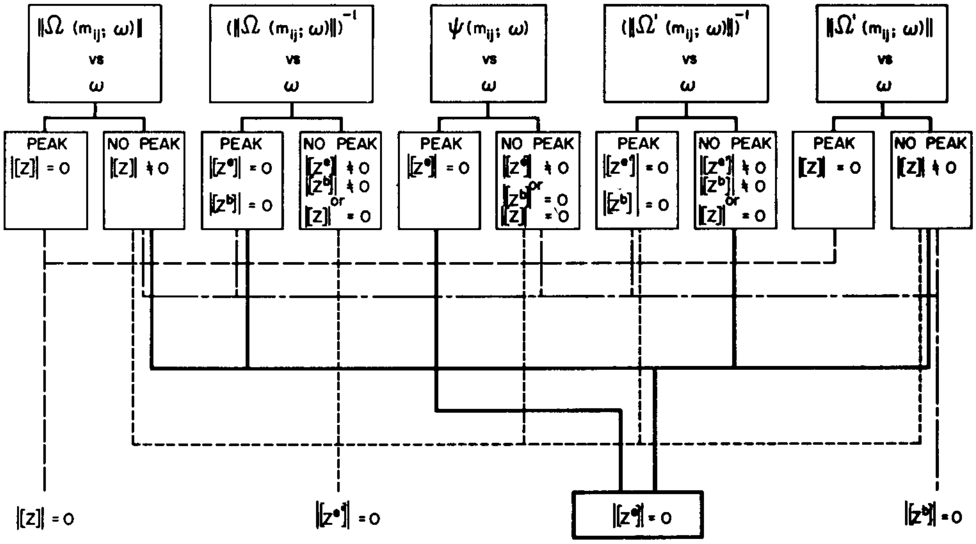


Fig. 2—Data analysis procedure

Description of the Test Structure—The mechanical system used in a laboratory experiment to test the validity and applicability of the semianalytical method reported in the NRL Report 7300 is shown in Fig. 3. The structure is essentially composed of three substructures:

- Equipment—The equipment substructure consists of a 1-in. X 2-in. X 4-ft steel beam with three simple supports at the quarter points along its length.

- Support—The support substructure consists of all the common points or connecting parts of the equipment and the base.

- Base—The base is the remaining part of the total structure other than the equipment and the support. The base consists of a metallic trusslike frame constructed partially of steel frame and partially of aluminum channels.

Experimental Results—According to the analytical results summarized in Eqs. (1) through (5), there is only a limited number of response measurements on the supports, and one additional location on the equipment substructure is needed in the mobility and resonance function calculation. Therefore, three piezoelectric-type accelerometers are mounted at the supports of the beam and one at the tip of the beam as shown in Fig. 4. A piezoelectric-type force gage is installed between the magnetic shaker and the shake point on the structure. The fixed-base natural frequencies of the lowest two modes of the

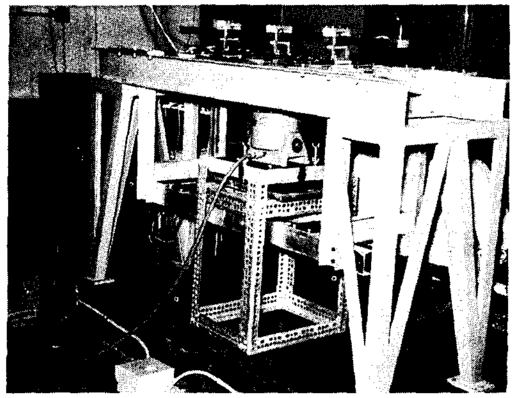


Fig. 3—Test structure for semianalytical method

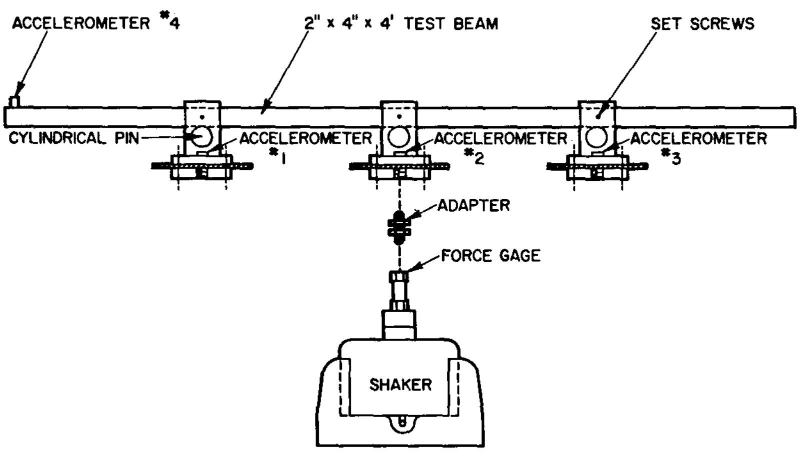


Fig. 4—Details of the test beam and its support system

continuous-test beam are obtained by use of the semianalytical method described. In order to demonstrate another important feature of this method (the location of the force application points is irrelevant as long as they are applied within the limit of the support and base substructures), two sets of applied forces are used:

1. Shaker at the support points. Figure 5 shows a resonance peak at $f = 125$ Hz. Figure 6 shows two resonance peaks, at $f = 125$ Hz and $f = 140$ Hz. It is found that in Fig. 7, there is no natural frequency of the total structure observed inside the frequency range 100 to 200 Hz. Consequently, it implies that $|[Z^b]| \neq 0$ at the fixed-base natural frequencies of the equipment substructure. If $|[Z^b]| = 0$ at the fixed-base natural frequencies of the equipment substructure, this physically means inner resonance and the determinant $|[Z]|$ would have vanished at those frequencies. The remaining possibility of the missing resonance peak at $f = 140$ Hz in Fig. 5 is that the determinant $|[Z^{e'}]|$ equals zero at $f = 140$ Hz. Figure 8 does show a peak at $f = 140$ Hz which confirms the reason for the

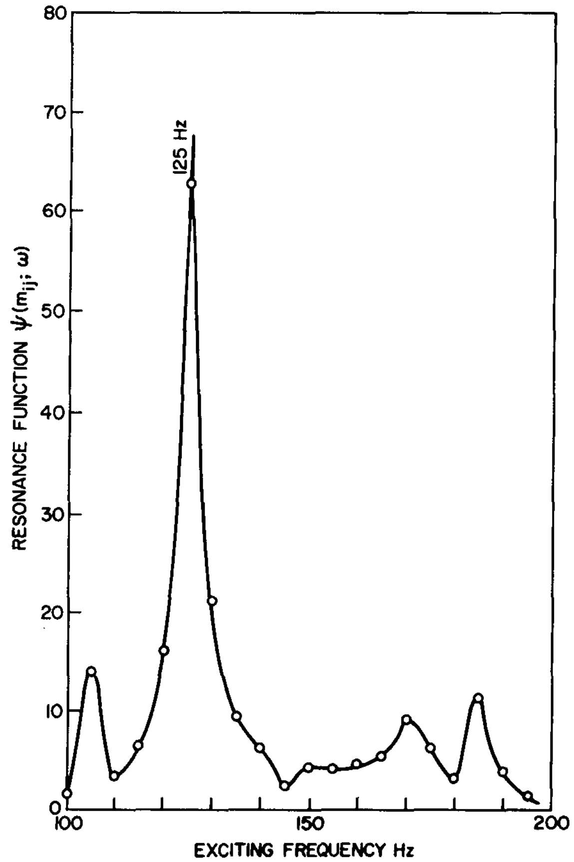


Fig. 5—Resonance function $\psi(m_{ij}; \omega)$ vs exciting frequency for shaker at the supports

missing peak in the resonance plot. It can be concluded that the experimental result of this case, with the shaker at the supports, is that the measured fixed-base natural frequencies of the lowest two modes of the continuous-test beam are

$$f_1 = 125 \text{ Hz} \qquad \text{and} \qquad f_2 = 140 \text{ Hz}.$$

2. Shaker at points on the base substructure. In this case a situation occurred similar to the previous case. A missing resonance peak at $f = 125$ Hz is noticed in the resonance function plot in Fig. 9. Figures 10 and 11 combined with Fig. 12, by the same reasoning, show the existence of the fixed-base natural frequency of the test beam at $f = 125$ Hz. In addition, there is an apparent frequency shift at the resonance peak in Fig. 9, $f = 135$ Hz, compared with the result of the previous case, $f = 140$ Hz. Figure 12 shows a peak at $f = 140$ Hz where the determinant $|[Z]|$ vanishes. Therefore the frequency shift is due to the resonance mode of the total structure. In engineering applications, such a frequency shift can be clarified and removed by altering the base substructure. The experimental result of this case is that the natural frequencies are

$$f_1 = 125 \text{ Hz} \qquad \text{and} \qquad f_2 = 135 \text{ Hz}.$$

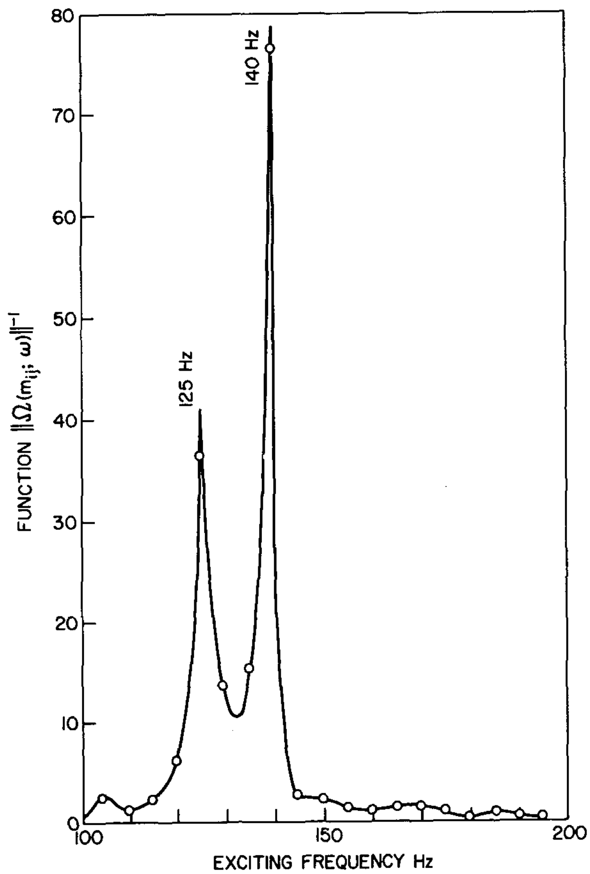


Fig. 6—Absolute value of the inverse mobility function $||\Omega(m_{ij};\ \omega)||^{-1}$ vs exciting frequency for shaker at the supports

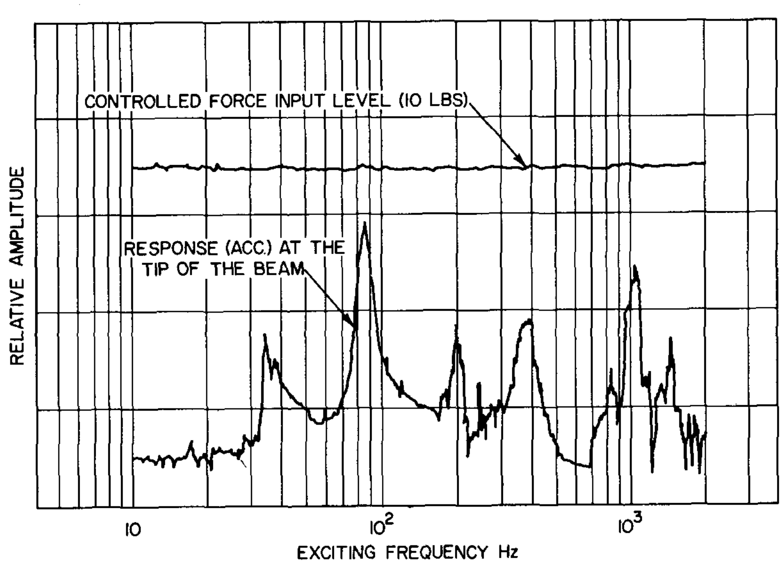


Fig. 7—Standard resonance test for determining natural frequencies of the total structure shown in Fig. 3a

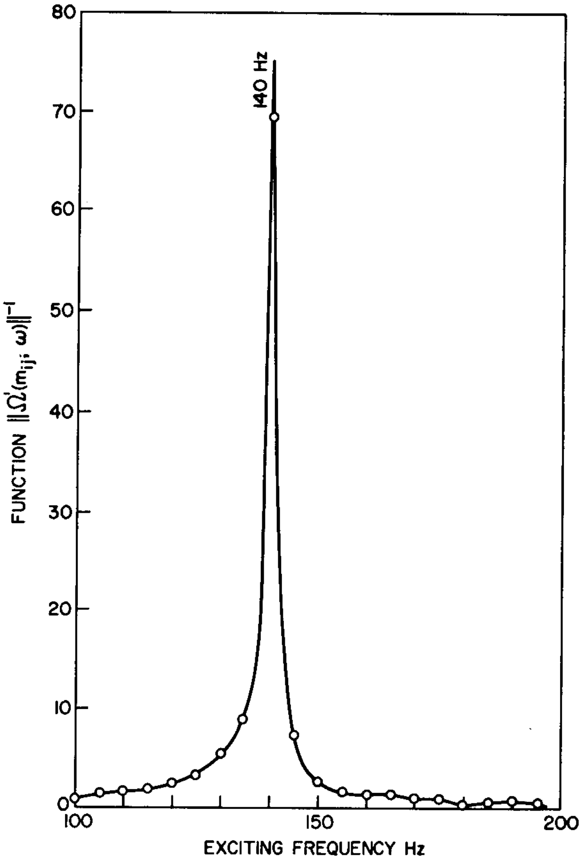


Fig. 8—Absolute value of the inverse mobility function $||\Omega'(m_{ij};\ \omega)||^{-1}$ vs exciting frequency for shaker at the supports and at the tip of the beam

Standard Resonance Test Method

For the purpose of comparing results, the same continuous-test beam is tested under the boundary condition of fixed constraints. There is a basic difference between the semianalytical method and the standard resonance test method. The former is based on the resonance condition without actually exciting the resonance modes, and the latter requires the actual excitation of the resonance modes as a necessity. Therefore, the applicability of the standard resonance test is limited by the rigidity of the shake table upon which the test beam is mounted.

Description of the Test Structure—The test structure is shown in Fig. 13. It is also composed of three substructures. The equipment and support are the same as those described in the semianalytical method. The base substructure consists of the mounting table of the shaker, a 2-in. × 18-in. × 18-in. aluminum plate.

Experimental Procedure—The standard resonance test method is performed by sweeping through a prescribed exciting frequency range and monitoring the response at an appropriate location on the equipment. The shaking force is controlled at a definite amplitude level by servo devices. The same number and location of the accelerometers used in the semianalytical method are used. The accelerometer at the center support is used to control the shake level. The two auxiliary accelerometers at the side supports are monitored at the control shake level to obtain information from which the recorded variation in relative

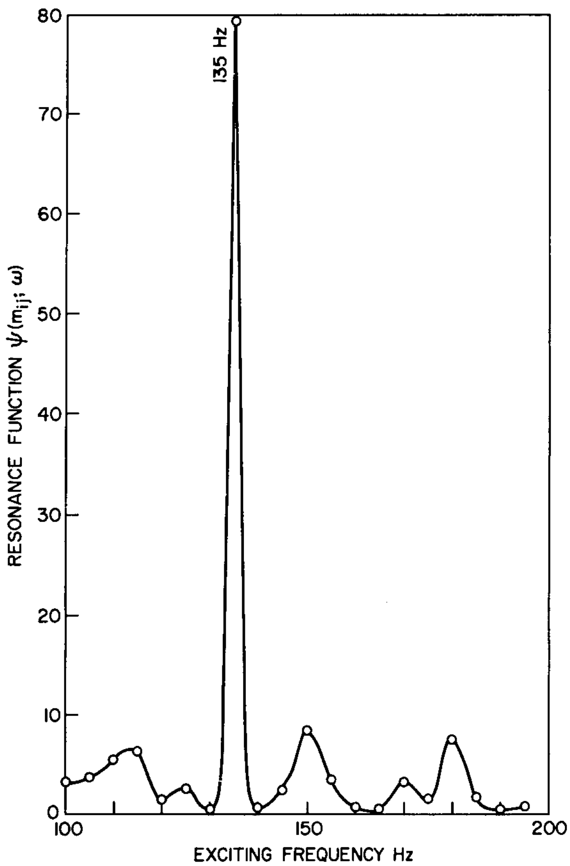


Fig. 9—Resonance function $\psi(m_{ij}; \omega)$ vs exciting frequency for shaker at points on the base substructure

amplitudes throughout the sweep frequency range enable us to determine the applicability of the shake table for this particular test. Within the frequency range, where the shake table responds to the excitation as a rigid-body motion, the fixed-base natural frequencies of the equipment substructure are identified by resonance peaks on the monitored response signal of the accelerometer at the tip of the beam.

Experimental Result—The result of the standard resonance test shows that the broadened resonance peak occurred around 120 Hz in Fig. 14. Judging from the shape of the broadening, we believe that the resolution of this test is not high enough to separate the two supposedly existing resonance peaks at the peak-to-peak frequency difference δf of about 10 Hz. It is concluded that there is a resonance peak at the frequency of 120 Hz. The possible existence is inferred of another close-by resonance frequency $f \cong 140$ Hz corresponding to the shoulder, occurring on the limb of the peak on the higher frequency side, i.e.,

$$f_1 = 120 \text{ Hz} \qquad \text{and} \qquad f_2 = 140 \text{ Hz (inconclusive).}$$

THEORETICAL CALCULATION

The theoretical calculation is based on Bernoulli-Euler beam theory. The frequency formula is

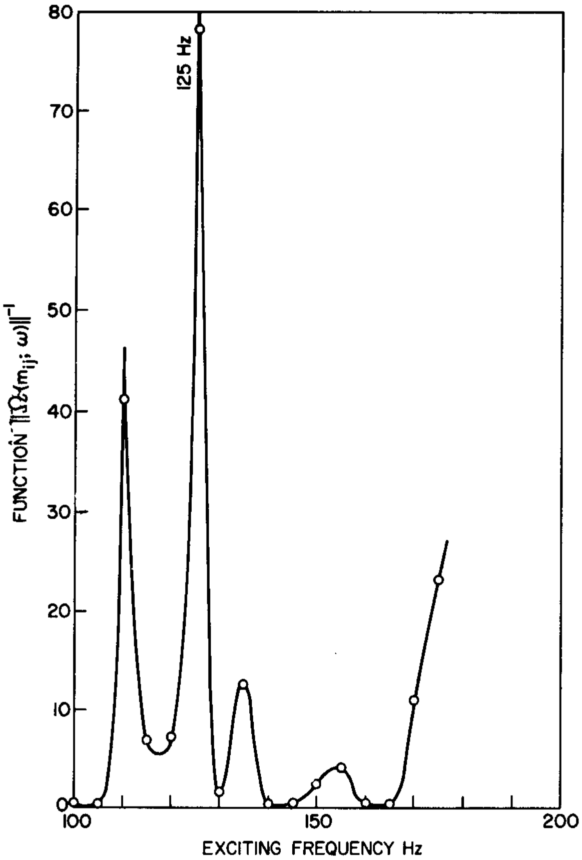


Fig. 10—Absolute value of the inverse mobility function $\|\Omega(m_{ij};\ \omega)\|^{-1}$ vs exciting frequency for shaker at points on the base substructure

$$\omega^2 = \frac{(k\ell)^4 EIg}{A\ell^4},$$

where

k = The eigenvalue corresponding to the given boundary condition

ℓ = The length of the spans

A = The cross-sectional area of the beam

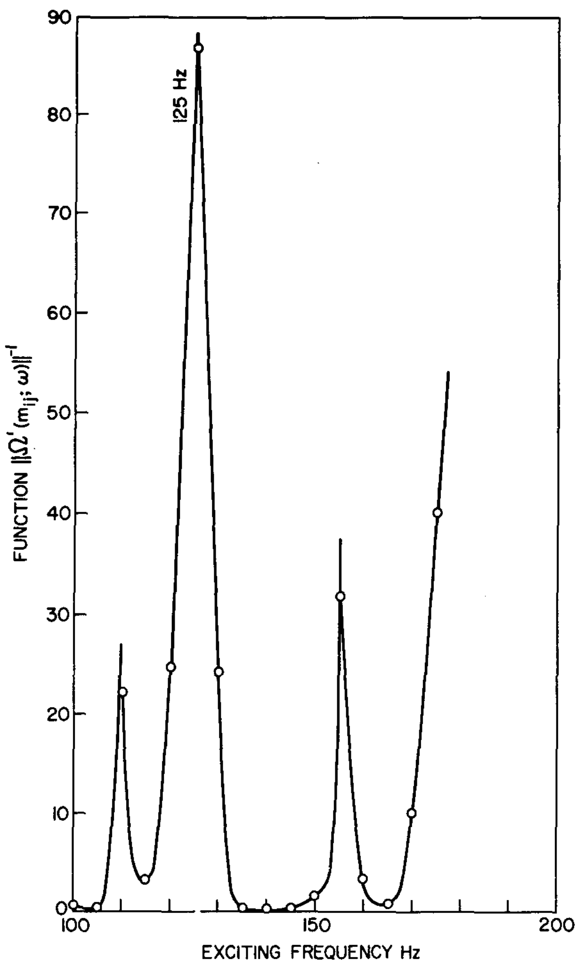
E = The Young's modulus of the beam

I = The second moment with respect to the horizontal neutral axis of the beam cross section

g = The gravitational constant

γ = The specific weight of the beam material.

Fig. 11—Absolute value of the inverse mobility function $||\Omega'(m_{ij}; \omega)||^{-1}$ vs exciting frequency for shaker at points on the base substructure and at the tip of the beam



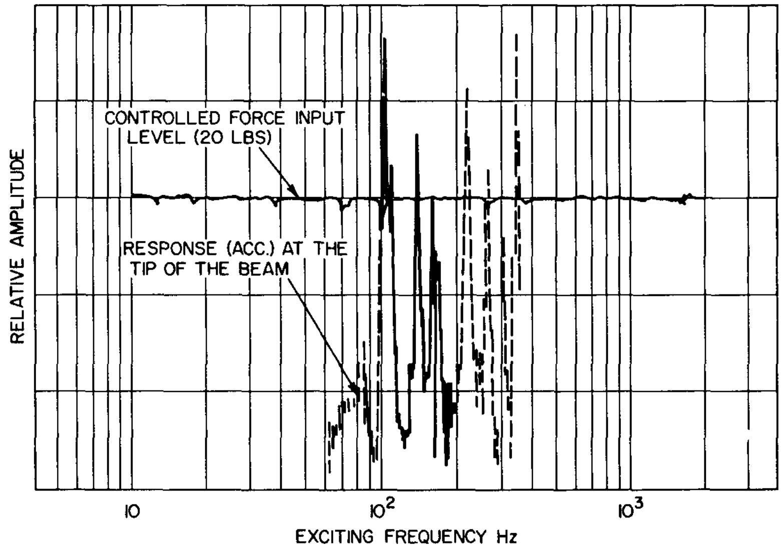


Fig. 12—Resonance test for determining natural frequencies of the total structure shown in Fig. 3b

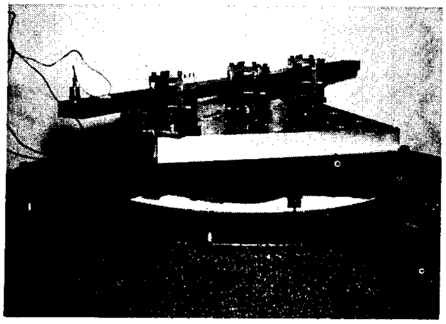


Fig. 13—Configuration of the test beam mounting

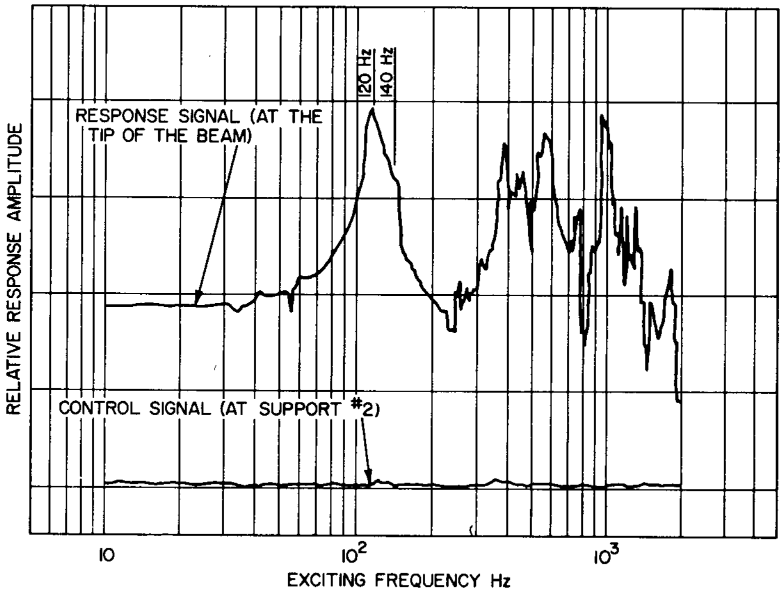


Fig. 14—Standard resonance test result of the continuous-test beam

The physical constants used in this calculation are

$\ell = 12$ in.

$A = 2$ sq in.

$E = 30 \times 10^6$ psi

$I = 1/6$ in.^{4}

$g = 386.4$ in./sec^2

$\gamma = 0.28$ lb/cu in.

The calculated natural frequencies of the lowest two modes are

$$f_1 = 138 \text{ Hz} \qquad \text{and} \qquad f_2 = 148 \text{ Hz}.$$

DISCUSSION AND CONCLUSION

The experimental and theoretical results are summarized in Table 1 for immediate comparison.

Table 1
Comparison of Results

Method	Natural Frequencies (Hz)	
	f_1	f_2
Semianalytical method		
Shaker at supports	125	140
Shaker at points on the base substructure	125	135
Standard resonance test method	120	140 (inconclusive)
Theoretical calculation Bernoulli-Euler beam theory	138	148

One observes that the discrepancies between the experimental results are smaller than those between either experimental result and the calculated theoretical value. There is a discrepancy of less than 5 percent between the results of the two experimental methods. Figures 6 and 14 show that, in this case, the semianalytical method gives a better resolution of the fixed-base natural frequencies than the standard resonance test method. The discrepancies between the experimental results and the theoretical results calculated from the Bernoulli-Euler beam theory amount to 10 to 15 percent. An examination of the possible causes of the theoretically predicted higher natural frequencies reveals that neither shear and rotary inertia, nor viscous damping effects alone, can be counted on as the sole mechanism to explain such a large discrepancy. Further work to improve the theoretically calculated values by considering more complicated models is not to be attempted. The consistent experimental results enable us to conclude the following:

1. The semianalytical method gives accurate structural dynamics information.

2. The semianalytical method provides a simple means to determine fixed-base natural frequencies of a composite structure in situ. Such measurements can not be done by any existing standard test method.

3. The semianalytical method overcomes the induced uncertainties and the limitations of the analytical and conventional experimental approaches.

REFERENCES

1. Hurty, W.C., "Dynamic Analysis of Structural Systems Using Component Modes," AIAA J. 3 (No. 4):678-685 (Apr. 1965)
2. O'Hara, G.J., "Mechanical Impedance and Mobility Concepts," J. Acoust. Soc. Amer. Vo. 41 (No. 5):1180-1184 (1967)
3. Petak, L.P., and O'Hara, G.J., "Determination of Fixed Base Natural Frequencies of Dual Foundation Shipboard Equipments by Shake Tests," NRL Report 6451, Aug. 23, 1966
4. Petak, L.P., and Kaplan, R.E., "Resonance Testing in The Determination of Fixed Base Natural Frequencies of Shipboard Equipment," NRL Report 6176, Dec. 15, 1964
5. Remmers, G.M., "Experimental Technique for Determining Fixed-Base, Natural Frequencies of Structures on Single Nonrigid Attachment Points," Shock & Vib. Bull. 38, Part 2:261-270 (Aug. 1968)
6. Ni, Chen-chou, and Skop, R., "Determination of Fixed-Base Natural Frequency of Multiple Foundation Mechanical Systems by Shake Test," NRL Report 7300, Nov. 17, 1971
7. Tseng, W.T., and Dugundji, J., "Non-linear Vibrations of a Beam under Harmonic Excitation," Trans. ASME 92, Ser. E; J. Appl. Mech. 37 (No. 2):292-297 (June 1970)
8. Mettler, E., "Schwingungs- und Stabilitätsprobleme bei mechanischen Systemen mit harmonischer Erregung," Z. angew. Math. Mech. 45:475-484 (1965)