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Impedance Matching of a Dielectric-Loaded Waveguide Radiator in a Phased Array

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IMPEDANCE MATCHING OF A DIELECTRIC-LOADED WAVEGUIDE RADIATOR IN A PHASED ARRAY

INTRODUCTION

A multiple-frequency phased antenna is being developed by NRL (1-4), in which open-ended waveguides reduced in size and loaded with dielectric material are used. Under these conditions the radiation conductance is generally low; therefore a high-VSWR load is presented to the microwave source. The requirement for wideband operation of such an array further aggravates the matching problem.

Most matching elements commonly used can be classified as either reactive irises or impedance transformers. The reactive iris is not suitable for a wideband matching, in particular with a high-VSWR load. One can reduce the VSWR of the waveguide radiator by increasing the waveguide size or by covering the array with a dielectric sheath, but both approaches have practical limitations.

In the transformer case there is no available design method for matching a load with a certain frequency characteristic. Thus one is led to use an optimization method which consists of a direct pattern search. An example will be given of the use of such an algorithm in designing a two-section transformer for matching waveguide radiators in a phased array. This example will show that a matched condition, with an almost flat response in a 10-percent bandwidth (±5 percent of the center frequency), can be achieved with a high-VSWR load. Prior to this example the limitations of a reactive iris will be discussed.

SIMPLE IRIS MATCHING NETWORK

The simplest matching device is a one-piece reactive iris. Since the input admittance varies as a transcendental function along a transmission line, it is possible to find a location at which the real component of the input admittance is equal to the characteristic admittance of the transmission line. If a pure reactive element which has a susceptance equal to the conjugate of the susceptance of the input admittance is added at this location, then, looking from the source, the microwave structure presents a perfect match. The location and the required iris susceptance can be readily found as follows (5).

Let the complex reflection coefficient at the load point $z = 0$ (Fig. 1) be ρ; then the input admittance along the waveguide can be represented as

$$y_{in} = y_0 \frac{1 - \rho e^{-j2\beta\ell}}{1 + \rho e^{-j2\beta\ell}}, \tag{1}$$

where y_0 is the characteristic admittance of the waveguide and β is its propagation constant. In a lossless network β is always real. The quantity ℓ is the distance from the $z = 0$ point to the point where y_{in} is measured (Fig. 1).

For convenience in using the Smith chart to be presented as Fig. 2, the input admittance is normalized with respect to the characteristic admittance of the transmission line. Thus

$$y'_{in} = \frac{1 - \rho e^{-j2\beta\ell}}{1 + \rho e^{-j2\beta\ell}}. \tag{2}$$

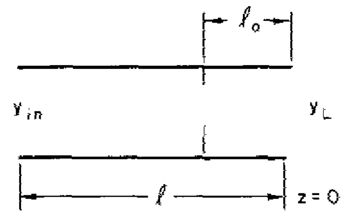

Fig. 1—Location of a matching iris in a waveguide

To avoid confusion let us adopt here a convention that $\beta\ell$ is a positive quantity. In Eq. (2) we are interested in finding an ℓ_0 such that y'_{in} has a real component of unity with an unknown imaginary part B. Therefore we may set

$$1 + jB = \frac{1 - \rho e^{-j2\beta\ell_0}}{1 + \rho e^{-j2\beta\ell_0}}. \tag{3}$$

Next we expand the complex reflection coefficient ρ into its imaginary and real parts, namely

$$\rho = \rho_r + j\rho_i, \tag{4}$$

and we represent the magnitude of ρ as ρ_m. Then

$$\rho e^{-j2\beta\ell_0} = \rho_m [\cos(\alpha - 2\beta\ell_0) + j \sin(\alpha - 2\beta\ell_0)], \tag{5}$$

where

$$\alpha = \tan^{-1} \frac{\rho_i}{\rho_r}.$$

Equation (3) can be rewritten in the form

$$\rho e^{-j2\beta\ell_0} = \frac{-jB}{2 + jB}. \tag{6}$$

Inserting Eq. (5) into Eq. (6) and separating the real and imaginary parts, we find

$$2\rho_m \cos(\alpha - 2\beta\ell_0) - \rho_m B \sin(\alpha - 2\beta\ell_0) = 0 \tag{7a}$$

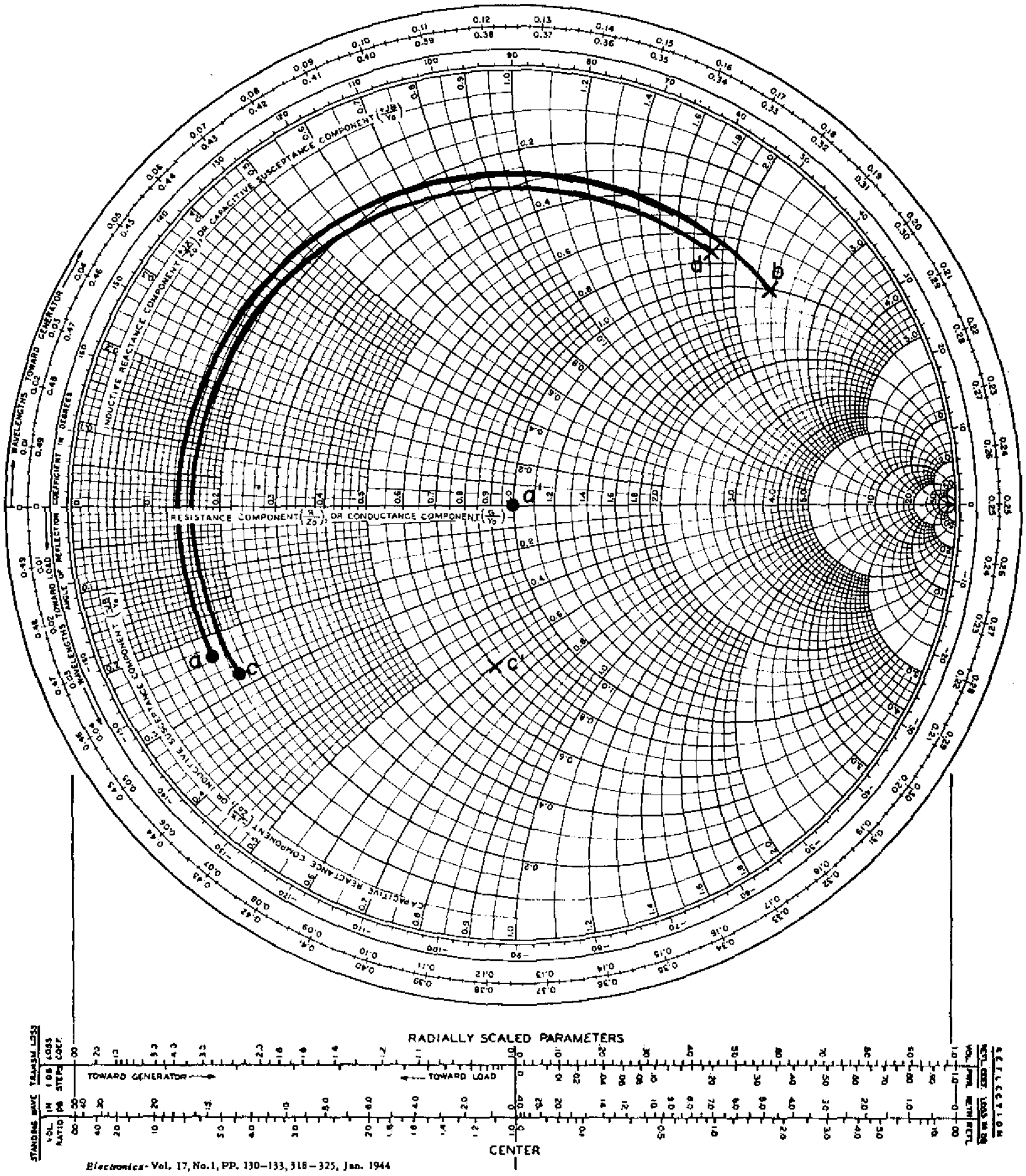

Fig. 2—Smith chart illustrating impedance matching

and

$$2\rho_m \sin(\alpha - 2\beta\ell_0) + \rho_m B \cos(\alpha - \beta\ell_0) = -B. \tag{7b}$$

This pair of transcendental equations, containing the unknowns ℓ_0 and B, can be solved to give

$$B = 2 \cot(\alpha - 2\beta\ell_0) \tag{8a}$$

and

$$\cos(\alpha - 2\beta\ell_0) = -\rho_m \tag{8b}$$

or

$$2\beta\ell_0 = n\pi + \alpha \pm \gamma_0, \tag{9a}$$

where $n = 1, 3, 5, ...,$ and

$$\gamma_0 = \cos^{-1} \rho_m .$$

If we choose

$$2\beta\ell_0 = n\pi + \alpha \pm \gamma_0, \tag{9b}$$

then B is negative, which represents an inductive susceptance; thus the matching element must be capacitive. If we choose

$$2\beta\ell_0 = n\pi + \alpha - \gamma_0, \tag{9c}$$

then the input admittance is capacitive; thus an inductive iris is required for matching. These results can be checked by use of a Smith chart. The absolute value of B can also be written

$$|B| = \frac{2\rho_m}{\sqrt{1 - \rho_m^2}}. \tag{9d}$$

Thus by use of Eqs. (9b), (9c), and (9d), one can determine the exact location and the required matching susceptance. From this known susceptance one can find the actual iris configuration from any microwave handbook (such as Ref. 7 or Ref. 8).

Actually there are an infinite number of such matching locations. For a wideband match, however one should use the least value of ℓ_0 to minimize the dispersive effect.

The disadvantage of this simple iris matching is its limitation on bandwidth, particularly with a high-VSWR load. This may be seen from a Smith chart. Suppose that a load admittance is located at point a on a Smith chart (Fig. 2). To match such a load, one traces it along a constant-ρ_m circle until it intersects the unit conductance circle, at point b. In this case a normalized susceptance of about 2.4 is required to bring the input admittance to unity. We now assume that the operating frequency is changed and that at this shifted frequency the load admittance changes slightly to point c. Due to dispersive effects β is changed at the second frequency. Thus in the same microwave structure with the same physical location of the iris, the actual input admittance at the matching point will be located at point d instead. Suppose that the iris is not sensitive to frequency change. Then, looking from the source, the admittance after matching will be located at c', which (as can be seen by comparing the radial distance from a' to c' with the scale at the lower left) represents a VSWR of 2.2, although the original difference between points a and c is very small. This dispersive effect is more pronounced if the load admittance has a high VSWR value. In this case the intersection with the unit conductance circle is in the region where the susceptance changes rapidly. Thus a slight change in position may cause a large amount of difference in both conductance and susceptance. Hence it is evident that an iris matching device is limited either to cases in which the load admittance has a high conductance component (equivalent to a low-VSWR load) or in narrow-band applications.

One may also explain this phenomenon from the point of view of a resonant circuit. The introduction of a reactive iris into the microwave structure can be viewed as forming a resonant circuit. A resonant circuit which has a low conductance or high resistance represents a high-Q circuit which, in general, is exceedingly narrow band. Thus, to increase the bandwidth, one has to increase the loss of the circuit. In this case, one must increase the radiation conductance of the radiation element.

EXAMPLE OF IRIS-MATCHED WAVEGUIDE RADIATING ELEMENTS

Figures 3a, 3b, and 3c show two cases of an example of impedance matching of an open-end waveguide in a phased array by use of an iris element. Figure 3a shows the magnitude of the reflection before and after being matched with an iris. In both of the cases presented the array structures are identical with a triangular grid of a size $d_x = 0.635\lambda$ and $d_y = 0.476\lambda$. The waveguide is loaded with dielectric material with a relative dielectric constant of 4 and in each case the waveguide has dimensions $a = 0.4356\lambda$ and $b = 0.1975\lambda$. Curve B represents the case in which the array is covered with a dielectric sheath with a relative dielectric constant of 3 and a thickness of 0.125λ. Curve A represents the case in which no dielectric sheath is used. In both cases the waveguide active admittance at boresight was computed (4,6) for a 20-percent frequency band. Subsequently a matching iris was designed

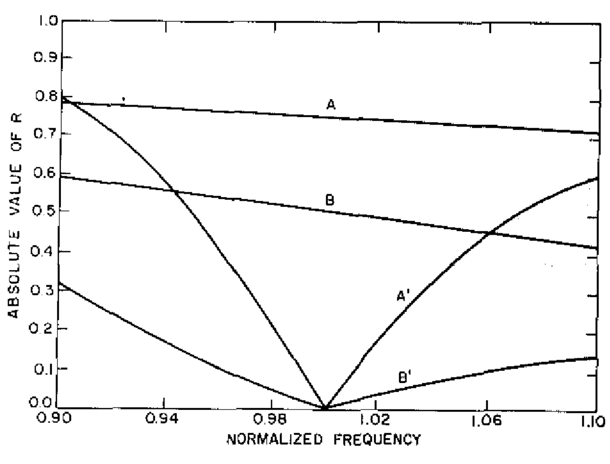

Fig. 3a—Reflection coefficient of a waveguide radiator vs frequency in two cases before (curves A and B) and after (curves A′ and B′) matching with a thin iris. Case B is the same as case A except that a dielectric sheath has been added.

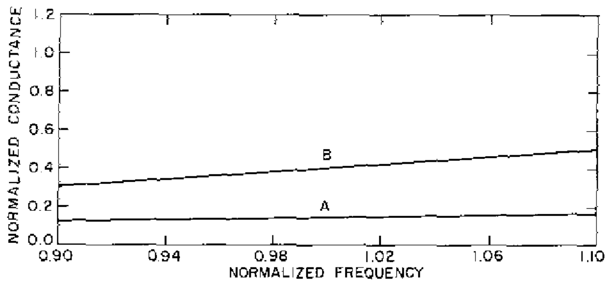

Fig. 3b—Normalized conductance for the two cases shown in Fig. 3a.

at the center frequency. The dimensions of the iris were computed by use of the formulas already developed (8,9). Then the reflection coefficient at other frequencies was computed by using the same physical dimensions of the iris and the matching location. The results are shown as curves A' and B'.

The bandwidth response of curve B' is considerably better than that of curve A'. The reason for this is probably better demonstrated in Figs. 3b and 3c. Figure 3b shows the normalized conductance for both cases. It is evident that the dielectric sheath increased the radiation conductance, which in turn influences the bandwidth response. Figure 3c shows the admittance plotted on a Smith chart both before and after matching.

In the case without a dielectric sheath (points A in Fig. 3c) the amplitude of the reflection coefficient before matching stays almost constant while the phase angle of the reflection coefficient varies.

In the presence of a dielectric sheath before matching the characteristics of the array active admittance (points B) are considerably changed; the phase angle of the reflection coefficient remains almost constant while the aplitude varies widely. This effect may actually help the case with a dielectric sheath.

From these results one may conclude that one way to improve the bandwidth is to increase the radiation conductance. In the preceding example the dielectric sheath indeed increases the radiation conductance. It acts as an impedance transformer matching the impedance at the waveguide opening to free space, thus effectively increasing the waveguide size and improving its radiation characteristics.

In Fig. 4 the effect of improvement of the radiation characteristic is further explored by actually increasing the waveguide size. The four curves represent four cases as shown in Table 1. From this table and the curves shown in Fig. 4 one sees that as the waveguide size increases, the initial reflection coefficient amplitude decreases and the response after matching has better bandwidth characteristics. In case 2 the waveguide is below cutoff when the frequency drops close to about 0.94 of the center frequency; thus it shows a high reflection when operating close to this region.

In conclusion, either covering the array with a dielectric sheath or increasing the waveguide size tends to improve the radiation conditions and thus to ease the bandwidth problem. However in some situations neither the sheath nor increased size is desirable or practical. The

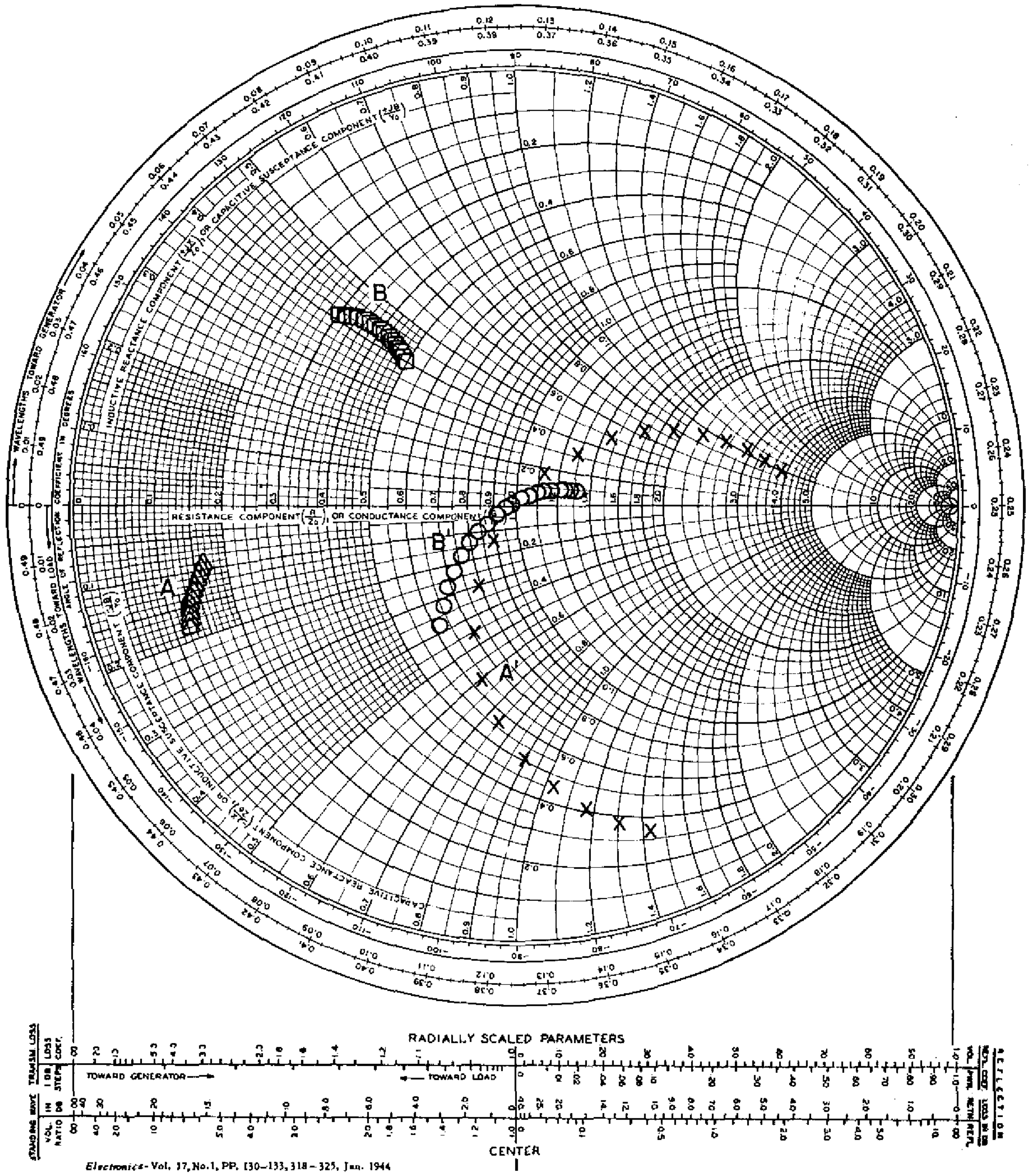

Fig. 3c—Admittance plot of the waveguide radiators in the two cases shown in Fig. 3a before and after matching with a thin iris.

use of a dielectric sheath may aggravate the deterioration of the reflection coefficient as the array is scanned (2,10). Increasing the size may sometimes not be possible; for example, in a multiple-frequency array the radiating elements are limited in size in order to fit elements of several different frequency arrays into a single aperture.

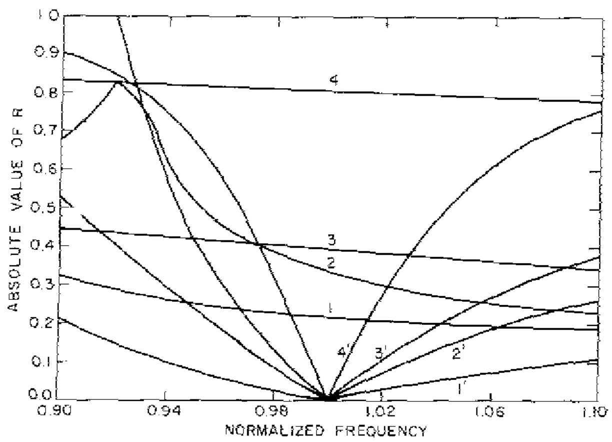

Fig. 4—Reflection coefficients of waveguide radiators before (unprimed) and after (primed) matching with a thin iris for the four cases listed in Table 1.

Table 1
The Four Cases Plotted in Fig. 4

Case	Array Dimensions (free-space wavelengths)				ϵ_r
	d_x	d_y	a	b	
1	0.635	0.476	0.6032	0.4522	1
2	0.635	0.476	0.5398	0.4046	1
3	0.635	0.476	0.4763	0.3570	2
4	0.635	0.476	0.3956	0.1575	4

Hence the use of a simple iris as a matching element is not always a satisfactory solution to the achievement of a broadband match of an array antenna.

QUARTER-WAVE TRANSFORMERS

Another commonly used matching element is a quarter-wave transformer. The fundamental principle of operation of such an element is briefly reviewed here.

The input admittance Y_{in} along a transmission line is related to the load admittance Y_2 can be represented as

$$y_{in} = y_0 \frac{y_2 \cos \beta \ell + j y_0 \sin \beta \ell}{y_0 \cos \beta \ell + j y_2 \sin \beta \ell}, \tag{10}$$

where y_0 is the characteristic admittance of the transmission line. If y_2 is real and $\beta \ell = 90$ degrees, then

$$y_{in} = \frac{y_0^2}{y_2}. \tag{11}$$

Thus two transmission lines with different characteristic admittances y_1 and y_2 can be matched by inserting between them a quarter-wave transmission line with a characteristic admittance $y_0 = \sqrt{y_1 y_2}$.

For matching a load which has a complex admittance, one must insert this transformer at a point where the admittance is real. The matching location and the required characteristic admittance can be readily determined as follows. Let the complex reflection coefficient at the load be $\rho = \rho_m e^{j\alpha}$. Then

$$y_{in} = y_0 \frac{1 - \rho_m e^{j\alpha} e^{-j2\beta\ell}}{1 + \rho_m e^{j\alpha} e^{-j2\beta\ell}}. \tag{12}$$

If one moves a distance

$$2\beta\ell_0 = \alpha + 2n\pi \tag{13a}$$

along the transmission line, then

$$\begin{aligned} y_{in} &= y_0 \frac{1 - \rho_m}{1 + \rho_m} \\ &= y_0 V \end{aligned} \tag{13b}$$

where V is the VSWR. If a distance ℓ_0' is moved, such that

$$\alpha - 2\beta\ell_0' = (2n + 1)\pi, \tag{14a}$$

then

$$y_{in} = y_0 / V. \tag{14b}$$

The required characteristic admittance of the quarter-wave transformer is then

$$y_1 = y_0 V \tag{15a}$$

for the case of Eqs. (13) and

$$y_1 = y_0 / V \tag{15b}$$

for the case of Eqs. (14).

Multiple sections of quarter-wave transformers can be cascaded into a single matching element (Fig. 5). The relations of the characteristic admittances of each section and of that of the input and output transmission lines are

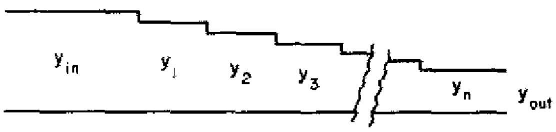

Fig. 5—A multiple-section transformer

$$y_{in} = \left(\frac{y_1}{y_2}\right)^2 \left(\frac{y_3}{y_4}\right)^2 \cdots \frac{y_n^2}{y_{out}} \tag{16a}$$

if n is odd and

$$y_{in} = \left(\frac{y_1}{y_2}\right)^2 \left(\frac{y_3}{y_4}\right)^2 \cdots \left(\frac{y_n}{y_{n-1}}\right)^2 y_{out} \tag{16b}$$

if n is even, where n is the number of transformer sections.

The fundamental principle of such transformers is based on the assumption that they are operated above cutoff. That is, the propagation constant β is real. If β is imaginary (below cutoff), the relation of Eq. (11) does not exist. The transformer section then becomes an attenuator. A wave which passes through this region, instead of undergoing only a phase change, suffers an exponential decrease in amplitude.

The quarter-wave transformer is also frequency sensitive, because in general both the propagation phase $\beta\ell$ and the characteristic admittance are functions of frequency. Thus a quarter wavelength for one frequency is not a quarter wavelength at other frequencies, and the variation of the characteristic admittance may cause further deterioration of the matching conditions.

A multiple-section transformer possesses more degrees of design freedom, but at the same time the dispersive effect is more pronounced. Numerous papers have treated this subject (11-13). However, most of these papers are concerned with the problem of matching transmission lines of different characteristics admittances (pure conductive load) for a given frequency bandwidth. No method seems available for designing a transformer with an optimum frequency response that matches to a load of complex admittance with a certain frequency characteristic. The difficulty may arise because there is no universal way to define the frequency characteristics of various load admittances. Thus it is probably impossible to develop any synthesis procedure to meet a large number of different and undefined conditions. Hence optimizing the frequency response requires a trial and error method. A description of such a method and an example of its use are given to the next two sections.

COMPUTER-AIDED DIRECT SEARCH METHOD

Any microwave network problem can generally be classified as an analysis problem or as a synthesis problem. In the analysis problem the parameters of a network are given and it is required to find how this network will behave under different conditions. In general this kind of problem is simple and straightforward. The synthesis problem is actually a design problem; that is, the network performance is given and it is required to find a set of parameters which will meet the performance requirement. This problem is in general much more difficult, and it is encountered more often in practice. Most of the time the network required to perform such a task is complicated, and the effect of variation of the parameters is interrelated. Thus it is impossible to predict the set of best parameters without actually trying the performance for all candidate sets. Usually, this design problem is solved by a trial and error method and with the help of the engineer's ingenuity and intuition. This procedure essentially turns the synthesis problem into an iterative analysis problem. A set of network parameters is chosen initially, and the network performance is calculated. The results are then analyzed, and the next move is determined based on this analysis, until finally a reasonable good design is achieved. The direct-search method presented in this section essentially follows this same procedure, except that the network is simulated on a digital computer; thus many sets of parameters can be analyzed in a very short time.

There are numerous approaches to this problem. The one used here was first developed by Hooke and Jeeves (14). Later Bandler (15) used this method to design a multiple-section transformer network for matching two waveguide sections of different characteristic impedances. He has demonstrated that his design results are superior to any hitherto known method. Bander's approach is used here because of the similarity of the NRL problem stated in the first paragraph of this report to his problem. His method is explained in detail in the following paragraphs.

Most optimization problems involve a given object function, which may be a very complicated function consisting of many parameters. Mathematically it can be represented as

$$U = U(\vec{\phi}), \tag{17}$$

where $\vec{\phi}$, which is the parameter function, is an n-dimensional vector. The optimization goal is then to find a vector $\vec{\phi}_0$ in the n-dimensional space such that either the object function U is a minimum or a maximum. To illustrate this method, we will consider optimization of a two-dimensional object function $U(\phi_1, \phi_2)$. Contours for the value of U in terms of its parameters ϕ_1 and ϕ_2 are shown in Fig. 6, in which each contour represents a constant U value. These contours are not known initially. We assume that the outer contour has the greatest U value and that each successive contour represents a lower value of U. The minimum value lies at the bottom of the valley, which is the goal of the search.

Let us assume that the search was started at the base 1 at which point U has dimensions ϕ_1^1 and ϕ_2^1 and an object function value of U_1. First we increase ϕ_1^1 to ϕ_1^2, and we find that $U_2 < U_1$. We retain the new value and also the increment δ_1^1. Next we increase ϕ_2^1 to ϕ_2^3, and we find that $U_3 < U_2$, which is a success. We retain the ϕ_2^3 and also the increment δ_2^3.

We thus find that increments δ_1^2 and δ_2^3 yield a successful result. In the hope that the pattern may be repeated again, we increase ϕ_1 and ϕ_2 from base point 3 according to the previously successful direction. This brings us to point 4. Now since $U_4 < U_3$ we retain this new value of ϕ_1^4 and ϕ_2^4 and then perform a pattern search again. After reaching point 5 in the first step of the pattern, we do not retain the second step to point 6 or an alternate step to point 7, since increasing or decreasing ϕ_2 does not improve the object function. We make another pattern move from base point 5 to point 8, using the same previous successful increments. Since $U_8 < U_5$, a new base point is thus established.

When a pattern move (as from point 8 to point 13) and subsequent moves (as from point 13 to point 14 and then to point 15 or point 16) fail, the strategy is to return to the previous base point. If the exploratory moves about the base point fail (as at ϕ^8), the pattern is destroyed, the parameter increments are reduced and the whole procedure restarted at that point (not shown in the figure). The search is terminated when the parameter increment falls below a prescribed level. At each base point the U function is always compared with a prescribed value. If it falls below that level, the search is terminated.

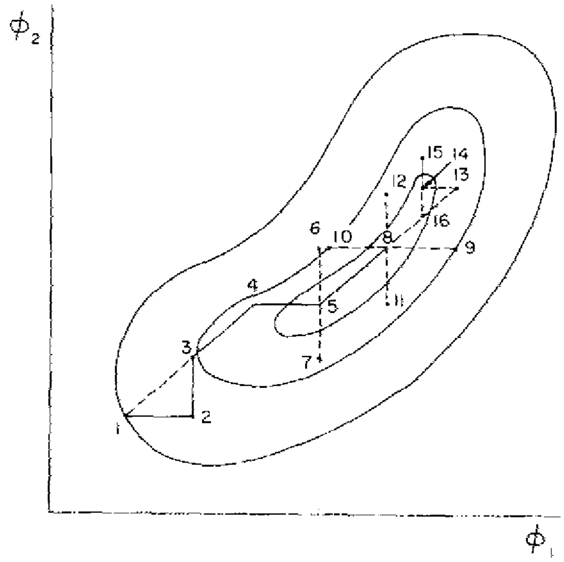

Fig. 6—Illustration of a direct-search optimization of a function $U(\phi_1,\phi_2)$. The contours are constant values of U, these contours not being initially known.

To avoid falling into a local minimum, a random move is performed when the search sequence does not yield a satisfactory result. In this random move a set of random numbers is generated, and a new base point is established according to these random numbers in a prescribed fashion. The whole search procedure is started again.

With this optimization strategy a pattern can follow along a sharp valley; hence Bandler has called it a "razor search."

A TWO-SECTION TRANSFORMER

The direct search method described in the preceding section was implemented to design a two-section transformer for matching a high-VSWR load over a 10-percent frequency band. This load is actually the active admittance of the dielectric-loaded waveguide elements of the phased array described with reference to case A in Fig. 3a. This program was developed for application to this case but is not limited to this case; hence this application serves here as an example of the direct search method. The physical dimensions of the transformer waveguides are constrained to be the same as those of the waveguide radiator. A change of the characteristic admittance is achieved by varying the dielectric constant of the filling material. Thus the discontinuity due to the variation of the waveguide dimensions can be ignored. The reason for this choice is to avoid the inconvenience of changing the waveguide dimensions. In a tightly assembled array, changing the dimensions is not practical. This kind of transformer has been discussed by Southworth (16).

The object function in this case is the VSWR at the microwave source compared with a specified desired value as follows:

$$U_n(f, \vec{\phi}) = \sum_n \left[V_n(f_n, \ell_0^n, \ell_1^n, \ell_2^n, \epsilon_1^n, \epsilon_2^n) - V_s \right]^k, \text{ if } V_n > V_s \tag{18a}$$

$$= 0, \text{ if } V_n < V_s, \tag{18b}$$

where V_n is the VSWR at nth frequency sampling point, with n being the number of sampling points across the band, V_s is the specified desired VSWR value = 1.25, k is an arbitrary constant used to control the VSWR response in the required bandwidth range, f_n is the frequency, ℓ_0 is the distance from the load to the first transformer section (Fig. 7), ℓ_1 is the length of the first transformer section, ℓ_2 is the length of the second transformer section, ϵ_1 is the relative dielectric constant of the filling material of the first section, and ϵ_2 is the relative dielectric constant of the filling material of the second section. The V function is given by

$$V_n = \frac{1 + |\rho_n|}{1 - |\rho_n|}, \tag{19}$$

where

$$\rho_n = \frac{1 - y_{in}(f_n)}{1 + y_{in}(f_n)} \tag{20}$$

in which y_{in} can be found by use of Eq. (10).

The search goal is to minimize the object function U (preferably make it zero) by varying the parameter function V. In the search procedure constraints must be applied to the parameter function so that the final optimum design is physically realizable. Thus the lengths ℓ_0, ℓ_1, and ℓ_2 must be positive, and the dielectric constants must be at least such that the dominant mode is above cutoff (is propagating). Some initial values have to be

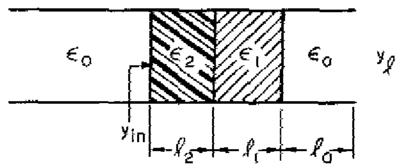

Fig. 7—A two-section impedance transformer

assigned to these parameters at the beginning of the search routine. In our particular example, they are chosen such that the waveguide has a perfect match at the midband point. The quantity V_s is assumed to be 1.25, and k is assumed to be unity.

Figure 8a shows the computed results over a 20-percent band. Curve a is the VSWR of the load before matching; it runs between 7 and 8 across a 10-percent band. Curve b shows the VSWR after matching by a two-section transformer designed for a perfect match at midband. The parameters of the transformer were then used as the initial values, and the search routine was applied with the result shown in curve c. The VSWR is reasonably constant over a 10-percent band of normalized frequencies from 0.95 to 1.05. The minimum value is 1.22, which occurs at midband. The VSWR values are slightly higher than the specified value (1.25) toward the edges of the band. At the normalized frequency $f = 0.95$ it is 1.77, and at $f = 1.05$ it is 1.52. Further search did not improve this curve.

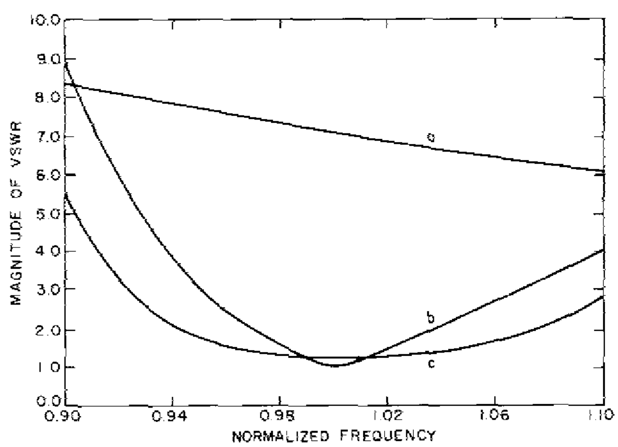

Fig. 8a—VSWR of waveguide radiators before and after matching with a two-section transformer: curve a, before matching; curve b, matched at midband; and curve c, matched by a pattern search.

Figure 8b shows the admittances on a Smith chart. Curves a, b, and c correspond to those in Fig. 8a. Curve c between $f = 0.95$ and $f = 1.05$ is good approximation to a circle about the center of the chart. The total computing time was about 30 seconds.

The dimensions and the dielectric constants of this two-section transformer for the optimum design obtained through the pattern search are

ℓ_0	ℓ_1	ϵ_1	ℓ_2	ϵ_2
0.032657λ	0.274830λ	1.683794	0.147689λ	13.423055

Such precise dimensions are difficult to realize physically. One may then ask how much dimensional error can be tolerated before the VSWR deteriorates to an unacceptable level. To answer this question, one may find the first partial derivatives of the object function

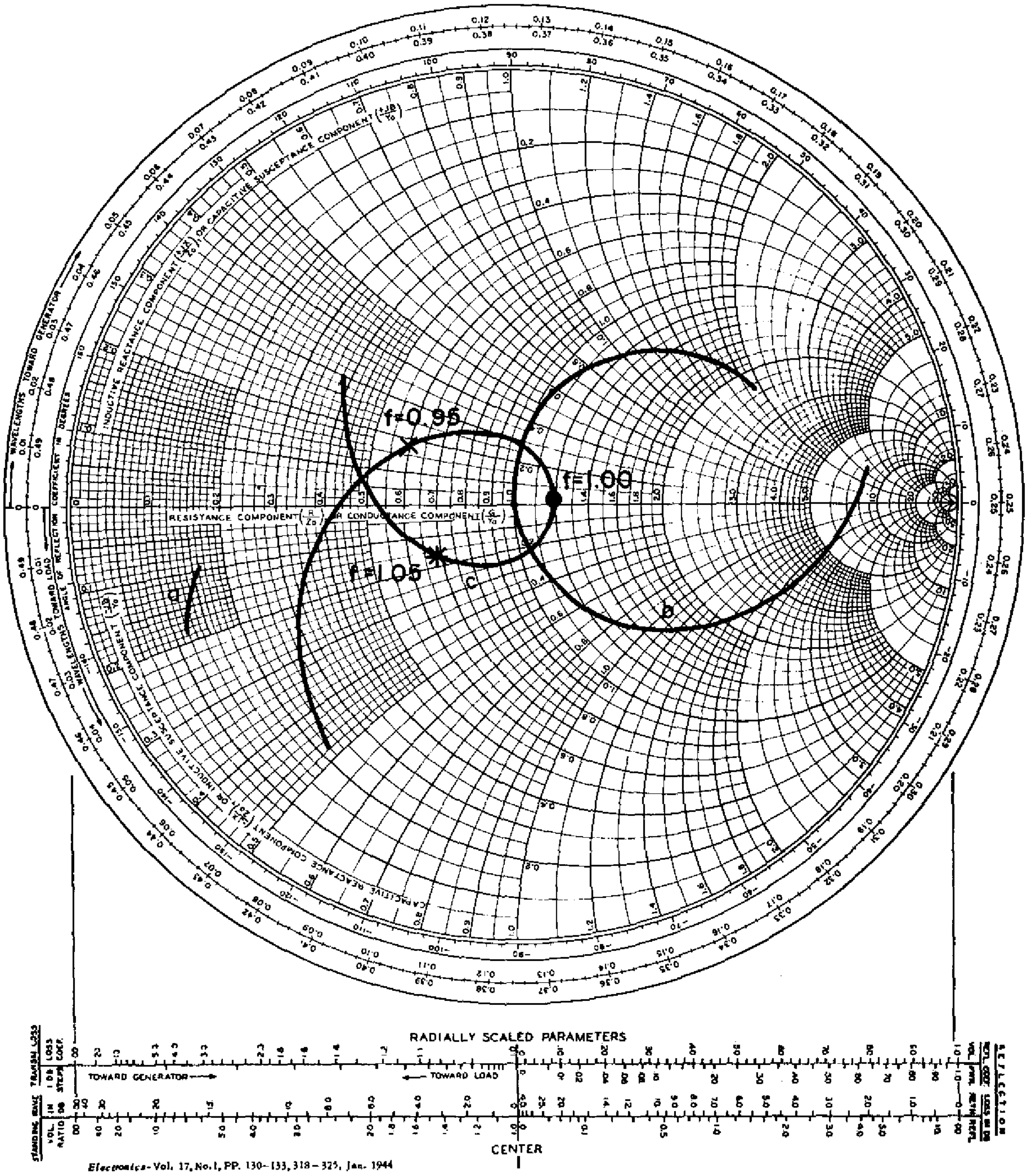

Fig. 8b—Admittance plot of waveguide radiators before and after matching with a two-section transformer: curve *a*, before matching; curve *b*, matched at midband; and curve *c*, matched by a pattern search.

$U(\vec{\phi})$ with respect to the dimension parameter function $\vec{\phi}$. If the magnitudes of these derivatives are large, it means that a small deviation of the required dimension results in a large change of the object function or, in the present case, the VSWR value. This consideration can be actually incorporated into the search routine. The search goal can be defined such that not only must the object function be minimized but also its first derivatives. The formulation of such a problem is far more complicated. Furthermore, whether such a minimum point exists, is an open question. We do not intend to pursue such an approach here. Instead we will now present a different approach, somewhat intuitive.

The two factors which affect the matching condition are the phase shift and the characteristic admittance of the transformer sections. For the dominant mode in a rectangular waveguide, they are

$$\beta \ell = 2\pi \ell' \sqrt{\epsilon_r - \left(\frac{1}{2a'}\right)^2} \tag{21a}$$

and

$$y_c = \sqrt{\frac{\epsilon_0}{\mu_0}} \sqrt{\epsilon_r - \left(\frac{1}{2a'}\right)^2}, \tag{21b}$$

where $\beta \ell$ is the phase shift along the waveguide of a length ℓ, y_c is the characteristic admittance of the TE_{10} mode, ℓ' is the waveguide length in free-space wavelengths, a' is the dimension of the waveguide in free-space wavelengths, and ϵ_r is the relative dielectric constant of the material fitting the waveguide.

These relations indicate that the phase shift is directly proportional to the ℓ' factor and that both the phase shift and the characteristic admittance are approximately proportional to the square root of ϵ_r. Furthermore the phase shift has a stronger effect than the characteristic admittance on matching. This suggests that the deviation of the VSWR response is more sensitive to the error in the ℓ' dimension than to that in the ϵ_r dimension.

Numerous computations were made and show that for the ℓ dimension an accuracy of about 0.001 wavelength is required whereas for the ϵ_r dimension an error up to more than 5 percent can be tolerated. This seems to be an acceptable level. For example, if the waveguide is operating in the range of a few gigahertz, 0.001 wavelength is equivalent to a few thousandths of an inch. This can generally be achieved both in the laboratory and in the manufacturing process. Some of the computed results are presented in Fig. 9. Curve I in Fig. 9 is the VSWR plot obtained by use of dimensions accurate to six digits, and curves II, III, and IV represent cases where the ℓ dimensions are accurate to a thousandth of a wavelength and the dielectric-constant values have errors of about 5 percent. From comparing them with curve I, it is evident that the VSWR deviations from the designed values are not excessive.

CONCLUDING REMARKS

In this report the general problem of the impedance matching of dielectric-loaded waveguide elements in a phased array has been discussed. One conclusion is that a simple iris matching element does not appear to be suitable for matching a high-VSWR load over a wide band. Increasing the radiation conductance, which means either covering the array

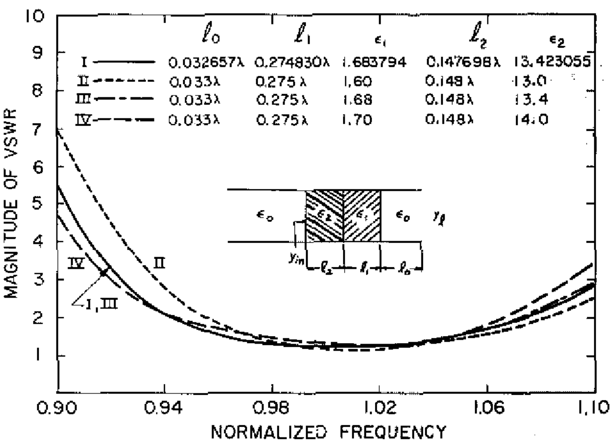

Fig. 9—Effect on the VSWR of waveguide radiators when the dimensions of the match transformer are perturbed from an accuracy of 0.00001λ to 0.001λ and the dielectric constants are perturbed about 5 percent.

with a dielectric sheath or increasing the waveguide size, tends to reduce the Q of the radiating element, thus improving the bandwidth. A sheath may however aggravate the deterioration of the reflection coefficient as the array is scanned, and increasing the waveguide size may not be practical.

A pattern-search optimization method was described which can be applied to the design of a multiple section impedance matching transformer for a loaded waveguide radiator. As an example of this technique, a two section-transformer has been designed for matching high-VSWR waveguide radiators in a phased array. It shows that a low and nearly constant VSWR can be obtained over a 10-percent band.

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