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The Relationship of Drag Reduction to the Transient Laminar Shear Flow Properties of Polymer Solutions

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CONTENTS

Abstract	ii
Problem Status	ii
Authorization	ii
INTRODUCTION	1
TRANSIENT LAMINAR SHEAR FLOWS	1
TURBULENT-FLOW PREDICTIONS	7
DISCUSSION	9
CONCLUSIONS	10
ACKNOWLEDGMENTS	11
NOTATION	11
REFERENCES	13

ABSTRACT

The laminar flow of a polymer solution near a flat plate of infinite extent was investigated theoretically for three cases: (a) the plate is impulsively started and moves in its own plane with a constant velocity; (b) the plate executes linear harmonic oscillations in its own plane; (c) motion of the plate in this plane is a random function of time. By combining the results with a simple model for turbulent flow near a wall, a number of experimentally observed characteristics of the drag reduction phenomenon are predicted. The merits of this transient shear explanation of drag reduction were compared to those of previously offered hypotheses. Some justification was found for expecting the transient shear flow mechanism to be dominant at large flow rates.

PROBLEM STATUS

This is an interim report; work is continuing on other phases of the problem.

AUTHORIZATION

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THE RELATIONSHIP OF DRAG REDUCTION TO THE TRANSIENT LAMINAR SHEAR FLOW PROPERTIES OF POLYMER SOLUTIONS

INTRODUCTION

The ability of soluble, high-molecular-weight polymer molecules to reduce drag in turbulent flow has been known for more than two decades. The explanation for this important phenomenon is still not known with certainty, however. Some investigators (1) have asserted that the interaction of an individual polymer molecule or aggregate with a turbulent eddy is of primary importance, so that drag reduction may not be viewed as a continuum phenomenon. Others have adopted the continuum point of view and related measured or theoretically predicted solution flow properties to characteristics of the turbulent boundary layer. For example, Metzner (2) and others have measured large elongational viscosities for dilute polymer solutions and have proposed a mechanism for drag reduction on this basis. More recently a continuum explanation related to the behavior of polymer solutions in transient laminar shear flows has been suggested by Hansen (3) and independently by Meek and Baer (4). In the present work new theoretical results on the behavior of these solutions in such flows are presented. These results are combined with a simple mathematical model of the turbulent boundary layer to predict the drag reduction behavior in turbulent pipe flow. Finally, the relative merits of the transient shear, high elongational viscosity, and individual molecule explanations for drag reduction are considered.

TRANSIENT LAMINAR SHEAR FLOWS

In each of the three, nonsteady, laminar shear flows of interest, a polymer solution is supposed bounded on one side by a flat plate of infinite extent. The solution is initially at rest in the first case, and at time $t = 0$ the plate is instantaneously accelerated and moves in its own plane with a constant velocity U . That is to say,

$$\begin{aligned} v_x &= 0 & \text{for } y > 0, & \quad t = 0; \\ v_x &= U & \text{for } y = 0, & \quad t \geq 0; \\ v_x &\rightarrow 0 & \text{for } y \rightarrow \infty, & \end{aligned} \tag{1}$$

where x denotes* distance parallel to the plate velocity, y the distance perpendicular to the plate, and v_x the liquid velocity in the x direction (Fig. 1). In the second case

$$v_x = U \cos \omega t \quad \text{for } y = 0$$

and (2)

$$v_x \rightarrow 0 \quad \text{for } y \rightarrow \infty,$$

and the flow after the decay of transients created in starting the plate from rest is treated. The third case has the boundary conditions

*The last section of this report lists the definition of symbols.

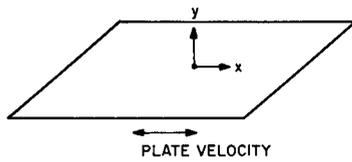


Fig. 1 - The xy coordinate system

$$v_x = F(t) \quad \text{for } y = 0$$

and

$$v_x \rightarrow 0 \quad \text{for } y \rightarrow \infty,$$

(3)

where $F(t)$ denotes a random function of time.

The fluid motion is in all cases governed by Newton's Second Law and the constitutive equation for the polymer solution. The former takes the form

$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial \tau}{\partial y} \quad (4)$$

by virtue of the invariance of v_x , the only nonzero velocity component, with x . In Eq. (4), τ denotes the shear stress in the x and y planes, and ρ denotes the liquid density. The constitutive equation assumed to characterize the polymer solution is the convected Maxwell model (5), which in these circumstances is

$$\frac{\partial v_x}{\partial y} = \frac{\tau}{\mu} + \frac{\lambda}{\mu} \frac{\partial \tau}{\partial t} \quad (5)$$

The solution viscosity μ and relaxation time λ are assumed independent of shear rate. Recent rheological studies (6,7) suggest that these assumptions together with Eq. (5) give a qualitatively correct description of the stress-strain rate relationship in the polymer solution.

Equations (4) and (5) for the impulsive start of the plate can be solved by the evaluation of certain contour integrals (8). For the oscillating plate, a separation-of-variables technique may be employed. (The random-motion case may be treated as a superposition of oscillatory solutions.) These procedures need not be carried out, however, because the equations have the same form as the generalized telegraphy equations for the case of zero leakage conductance. Solutions for arbitrary leakage conductance and boundary conditions analogous to Eqs. (1) and (2) have been given in the transmission-line literature (8,9). Thus τ and v_x are obtained directly from these solutions by substituting $-\tau$ for current, v_x for potential, $1/\mu$, λ/μ , and ρ for resistance, inductance, and capacitance per unit length respectively, and zero for leakage conductance.

The following expressions for the fluid velocity and wall shear stress are obtained for the impulsive-start case from the transmission line analogy:

$$\frac{v_x}{U} = e^{-t/2\lambda} [I_0(z_1) + 2y_1(z_2, z_1) + 2y_2(z_2, z_1)] \delta(t - \xi); \quad (6)$$

$$\frac{\tau_0}{\rho U^2} \sqrt{\frac{\lambda U^2}{\nu}} = I_0\left(\frac{t}{2\lambda}\right) e^{-t/2\lambda} \quad (7)$$

Here $I_n(z_1)$ denotes the modified Bessel function of order n with argument z_1 , and $\delta(t - \xi)$ denotes the unit step function. Also

$$z_1 = \frac{\sqrt{t^2 - \xi^2}}{2\lambda};$$

$$z_2 = \frac{(t - \xi)}{2\lambda};$$

$$\xi = y \sqrt{\frac{\rho\lambda}{\mu}};$$

$$y_n(z_2, z_1) = \sum_{m=0}^{\infty} \left(\frac{z_2}{z_1}\right)^{n+2m} I_{n+2m}(z_1) \dots$$

The velocity field given by Eq. (6) is shown in Fig. 2 for four finite values of $t/2\lambda$ and for a Newtonian fluid, for which $\lambda = 0$ or $t/\lambda = \infty$. It is evident from this plot that a transverse disturbance in a fluid for which $\lambda \neq 0$ propagates with a finite velocity, whereas the propagation velocity is infinitely large in a Newtonian fluid. The large change in fluid velocity at the propagating wavefront for $\lambda \neq 0$ probably in reality occurs over a small range of y rather than discontinuously, due to nonlinear effects which have been ignored in Eq. (5).

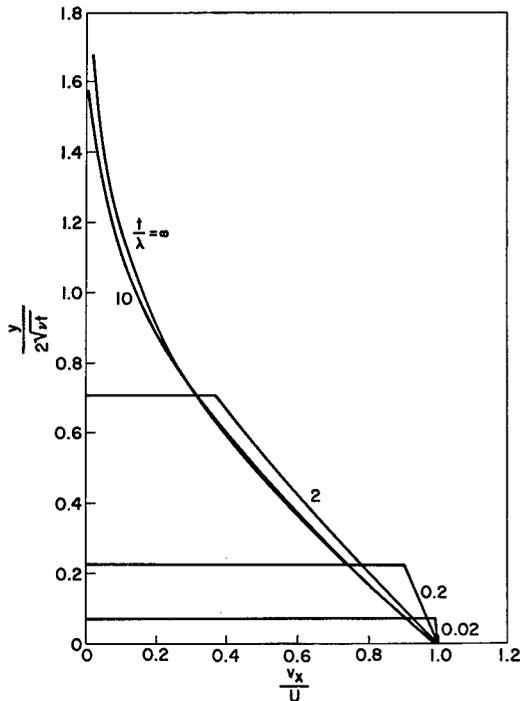


Fig. 2 - The velocity profile in the fluid for five values of t/λ

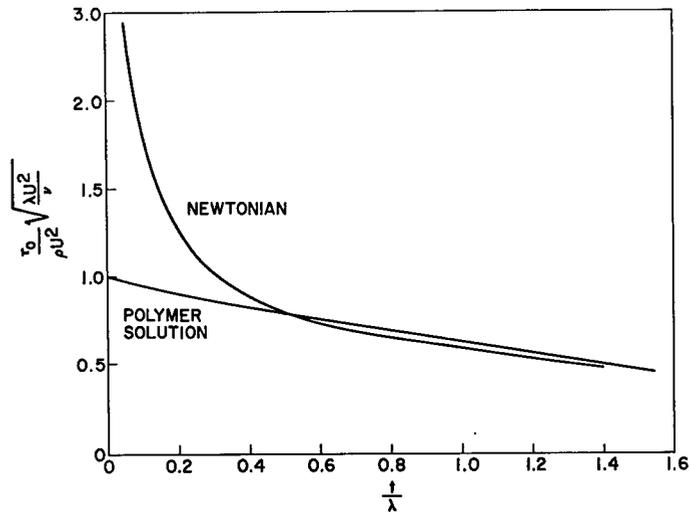


Fig. 3 - The instantaneous value of the wall shear stress on an impulsively started plate

The variation in wall shear stress with $t/2\lambda$ given by Eq. (7) is shown in Fig. 3. It is qualitatively similar to experimental results (10) for $t/2\lambda$ greater than about 0.3, as well as to the wall shear stress predicted in this range of $t/2\lambda$ using a three-constant constitutive equation (10). Neither theoretical nor experimental results have been given previously for smaller values of this parameter. The corresponding result for a Newtonian fluid is also shown. It is obtained (11) from the equation

$$\frac{\tau_0}{\rho U^2} = \sqrt{\frac{\nu}{\pi t U^2}} = \sqrt{\frac{\nu}{2\pi\lambda U^2} \frac{2\lambda}{t}} \quad (8)$$

Of importance in the viscous-sublayer model introduced subsequently is the time average of τ_0 , defined as

$$\bar{\tau}_0 = \frac{1}{T} \int_0^T \tau_0 dt \quad (9)$$

The integral on the right-hand side of this equation cannot be evaluated in closed form when τ_0 is given by Eq. (7). The integration has therefore been carried out numerically using Simpson's Rule. The result is shown in Fig. 4 in the form $(\bar{\tau}_0/\rho U^2) \sqrt{\lambda U^2/\nu}$ as a function of $T/2\lambda$. Also shown is $(\bar{\tau}_0/\rho U^2) \sqrt{\lambda U^2/\nu}$ for a Newtonian liquid, obtained by integrating Eq. (8). The (normalized) time-averaged wall shear stress is seen to be less for the polymer solution than for the Newtonian liquid when $T/2\lambda$ is small compared to unity and is seen to asymptotically approach the Newtonian value as $T/2\lambda$ becomes large.

In the oscillating plate case v_x and τ_0 from the solution of the relevant transmission-line problem (9) are given by

$$v_x = Ue^{-My} \cos(\omega t - Ny) \quad (10)$$

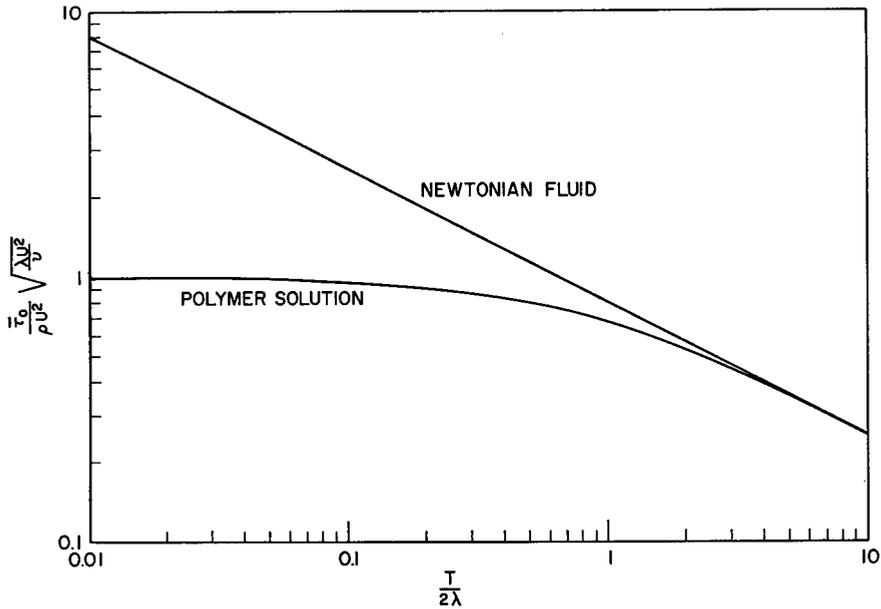


Fig. 4 - The time-average value of the wall shear stress on the impulsively started plate

and

$$\tau_0 = \frac{\mu U}{\sqrt{1 + \omega^2 \lambda^2}} (M \sin \omega t - N \cos \omega t) , \tag{11}$$

where

$$M^2 = \frac{\omega \rho}{2\mu} \left(-\omega \lambda + \sqrt{1 + \omega^2 \lambda^2} \right)$$

and

$$N^2 = \frac{\omega \rho}{2\mu} \left(\omega \lambda + \sqrt{1 + \omega^2 \lambda^2} \right) .$$

The time average of the wall shear stress is not a useful indicator of the effect of the polymer additive on drag in this case, since the sign of τ_0 varies as the plate oscillates. The mean-square value $\overline{\tau_0^2}$, defined by the following expression, is significant:

$$\overline{\tau_0^2} = \frac{\omega}{2\pi} \int_{\omega t=K}^{\omega t=K+2\pi} \tau_0^2 dt . \tag{12}$$

Here K is an arbitrary constant. Substitution of Eq. (11) into Eq. (12) gives

$$\overline{\tau_0^2} = \frac{\rho \mu U^2 \omega}{2 \sqrt{1 + \omega^2 \lambda^2}} . \tag{13}$$

The corresponding calculation for a Newtonian liquid gives

$$\frac{\overline{\tau_0^2}}{\tau_0^2} = \frac{\rho \mu U^2 \omega}{2} . \quad (14)$$

(This result may also be obtained from Eq. (13) by remembering that the relaxation time of a Newtonian liquid is assumed zero.) Consequently, in the root-mean-square sense, drag is lower for the polymer solution than for the Newtonian liquid of the same density and viscosity when $\omega\lambda$ is significant compared to unity.

Finally, in the case where v_x at $y = 0$ is a random function of time, this function may be represented as the superposition of simple harmonic velocities of different amplitudes, frequencies, and phase (12). The elemental contribution ΔF_ω to $F(t)$ of the component with frequency ω may be written in complex notation as

$$\Delta F_\omega = \frac{1}{2} \left(\Delta U_\omega e^{j\omega t} + \Delta U_\omega^* e^{-j\omega t} \right) . \quad (15)$$

Here ΔU_ω denotes the complex magnitude of this velocity component, ΔU_ω^* denotes its complex conjugate, and $j = \sqrt{-1}$. The mean-square value of $F(t)$ is given (12) as

$$\overline{[F(t)]^2} = \sum_\omega \frac{(\Delta U_\omega) (\Delta U_\omega^*)}{2} \quad (16)$$

if $F(t)$ is the sum of velocity components of discrete frequencies. For the more general case involving a continuous frequency spectrum, the distribution function $S(\omega)$ is introduced and defined as

$$S(\omega) = \lim_{\Delta\omega \rightarrow 0} \frac{(\Delta U_\omega) (\Delta U_\omega^*)}{\Delta\omega} . \quad (17)$$

Substituting Eq. (17) in Eq. (16) and replacing the summation by an integral over all frequencies gives

$$\overline{[F(t)]^2} = \frac{1}{2} \int_0^\infty S(\omega) d\omega . \quad (18)$$

Equation (11), written in complex form, gives the following relationship among $\Delta\tau_0$, the contribution to the wall shear stress associated with the frequency ω , ΔU_ω , and ΔU_ω^* :

$$\Delta\tau_0 = \frac{\rho \omega}{2 \sqrt{1 + \omega^2 \lambda^2}} \left[(Mj + N) \Delta U_\omega e^{j\omega t} + (-Mj + N) \Delta U_\omega^* e^{-j\omega t} \right] . \quad (19)$$

This equation is of the same form as Eq. (15). Consequently, the mean-square values of τ_0 are obtained from Eq. (16), (17), and (18) by replacing the coefficients of the exponential terms in Eq. (15) by those in Eq. (19). The results, for $F(t)$ associated with discrete and continuous frequency spectra, are respectively

$$\bar{\tau}_0^2 = \sum_{\omega} \frac{\rho \mu \omega}{\sqrt{1 + \omega^2 \lambda^2}} \frac{(\Delta U_{\omega}) (\Delta U_{\omega}^*)}{2} \quad (20)$$

and

$$\bar{\tau}_0^2 = \frac{1}{2} \int_0^{\infty} \frac{\rho \mu \omega}{\sqrt{1 + \omega^2 \lambda^2}} S(\omega) d\omega . \quad (21)$$

The corresponding equations for a Newtonian fluid are the same except that $\omega \lambda$ in the denominator is zero. Since all terms in the summation in Eq. (20) are positive, as is the integrand in Eq. (21) for all ω , the conclusion follows that $\bar{\tau}_0^2$ is less for a polymer solution than for a Newtonian fluid for any $F(t)$. This effect will be pronounced when $F(t)$ is composed primarily of high-frequency components, i.e., when $\omega \lambda \gg 1$ where $S(\omega)$ or $(\Delta U_{\omega}) (\Delta U_{\omega}^*)$ is large.

TURBULENT-FLOW PREDICTIONS

Drag reduction in turbulent flow is due primarily to the effect of the polymer additive on the fluid motion very near the wall. The presence of the additive outside the viscous sublayer and buffer zones in turbulent pipe flow, for example, have been shown by experiment (13) to have little or no effect on the relationship between flow rate and wall shear stress. In what follows, therefore, the turbulent flow near the wall of a liquid described by Eq. (5) is examined theoretically.

The conceptual model adopted for turbulent flow in the wall region is that first proposed for a Newtonian liquid by Einstein and Li (11) and recently refined by Meek and Baer (14). Localized regions of periodic growth and decay of the sublayer are postulated. The decay is assumed to occur in a negligible amount of time compared to the growth, which is governed by viscous processes. The flow near the wall in the growth stage is represented by Stokes' first problem, i.e., by the impulsive start of a flat plate described by Eq. (1). That is to say, the time-averaged wall shear stress in the turbulent flow is set equal to $\bar{\tau}_0$ from Eqs. (8) and (9), T being identified with the period of growth of the periodic region and U being identified with U_{δ} , the liquid velocity at the outer edge of this region. After terms are rearranged and the friction velocity u_* (defined as $\sqrt{\bar{\tau}_0/\rho}$) is introduced, the following expression is obtained:

$$\left(\frac{U_{\delta}}{u_*} \right)^2 = \frac{\pi}{4} \frac{u_*^2 T}{\nu} . \quad (22)$$

Recent experimental studies (14) indicate that this simple model gives qualitatively correct predictions for turbulent flow in a pipe. Quantitative agreement has been obtained by one investigator (15) when U_{δ} was replaced by V in Eq. (22), V denoting the average flow velocity in the pipe.

For the turbulent flow of the polymer solution, the relationship among $\bar{\tau}_0$, T , and U , obtained by substituting Eq. (7) into Eq. (9), may be used and U again identified with U_{δ} . The result is

$$\left(\frac{U_{\delta}}{u_*} \right)^2 = \frac{\pi}{4} \frac{u_*^2 T}{\nu} \frac{1}{D^2} , \quad (23)$$

where D denotes the ratio of τ_0 for the polymer solution to that for the Newtonian fluid at the same value of T . Rearrangement of Eq. (23) gives

$$\frac{\left(\frac{U_\delta}{u_*}\right)^2}{\left(\frac{\pi u_*^2 \lambda}{2\nu}\right)} = \frac{\left(\frac{T}{2\lambda}\right)}{D^2}. \quad (24)$$

The quotient on the left-hand side of this equation is plotted as a function of $T/2\lambda$ in Fig. 5 together with the corresponding result for a Newtonian fluid.

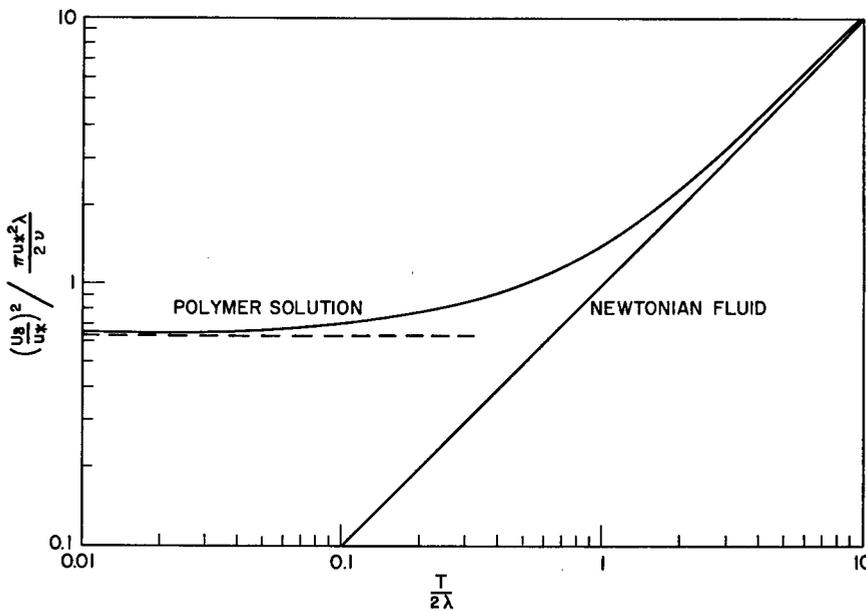


Fig. 5 - Turbulent-flow predictions for a Newtonian fluid and a polymer solution

Several features of this plot are significant. First, for sufficiently large $T/2\lambda$ the behavior of the polymer solution in the wall region is essentially Newtonian. This result is consistent with the experimentally observed flow behavior of polymer solutions at low wall shear stresses (large T) or low polymer concentrations and molecular weights (small λ) or both.

Second, the deviation from Newtonian behavior evident in Fig. 5 corresponds to a reduction in turbulent-flow drag. For small $T/2\lambda$ the nondimensional quantity $(U_\delta/u_*)^2 / (\pi u_*^2 \lambda / 2\nu)$ approaches an asymptote given by

$$\left(\frac{U_\delta}{u_*}\right)^2 = \frac{u_*^2 \lambda}{\nu}. \quad (25)$$

(This result follows directly from Fig. 4 or may be obtained by evaluating the limit of $(T/2\lambda)/D^2$ as $T/2\lambda$ approaches zero.) Thus as the wall shear stress increases at small $T/2\lambda$, so also does the ratio U_δ/u_* . (U_δ is of the order of magnitude of the mean flow velocity in the pipe.) In contrast the ratio of U_δ to u_* in the turbulent pipe flow of a Newtonian fluid has a constant value to a good approximation (11). That the slight deviation from Newtonian behavior for $T/2\lambda$ large compared to unity corresponds to a slight reduction in drag has been shown by Meek and Baer (4).

A consequence of Eq. (25) is that the quotient U_δ/u_* at a given u_* has a maximum value for a specific polymer additive. For very dilute solutions λ increases with concentration and molecular weight from the negligibly small value of the solvent alone (16, 17). When the concentration is large, λ is independent of concentration and molecular weight (18). Also ν increases slowly with concentration. Thus the ratio λ/ν is negligibly small for the solvent, increases with concentration in very dilute solutions, and decreases with increasing concentration in concentrated solutions. At some intermediate concentration, therefore, λ/ν attains a maximum value. (Presently available data do not resolve the question of whether or not the maximum value of λ/ν will depend on molecular weight.) Substitution of this maximum value into Eq. (25) gives the maximum drag-reduction condition.

Experiments by Virk (19) confirm that such a condition exists. The ratio of V to u_* at a given u_* ceased to increase with concentration for sufficiently concentrated solutions. The maximum value of V/u_* was also independent of molecular weight. Virk's "maximum drag-reduction asymptote" was characterized by a variation in V with $u_*^{1.38}$, whereas Eq. (25) predicts that U_δ will be proportional to u_*^2 . Since U_δ and V are of the same order of magnitude, these two results are qualitatively similar. The functional relationship between the two velocities is not known precisely, however, so it is not known if Virk's result and Eq. (25) are in quantitative agreement.

Third, another important characteristic of Fig. 5 is that it shows a continuously increasing effect of the polymer additive with decreasing $T/2\lambda$ rather than the initiation of the phenomenon at some critical wall shear stress. The existence of a well-defined "onset" condition is anticipated from the experimental work of Virk (19) and others (17, 20). This apparent discrepancy between the theory and experiments may be due to a weakness in the flow model chosen for the wall region. Alternatively, the prediction of the theory may be correct, but the deviation from Newtonian behavior may be detectable by presently used experimental methods only above some fairly-well-defined wall shear stress.

DISCUSSION

A number of important questions must be raised about the explanation for turbulent flow drag reduction given above. First, its basis is the solution of the impulsively started plate problem, where there are no shear stresses in the liquid for $t < 0$. In reality the lifetime of such shear-free regions is very small, or they may be absent completely at large flow rates in turbulent flow (21). It must be asked whether the drag will still be reduced in these circumstances in which there is not time for the polymer molecules to relax to their static configurations between successive applications for shear gradients.* The answer is provided by the treatment given previously of the laminar shear flow caused by a plate with random motion resulting in Eqs. (20) and (21). The results of this treatment predict a significant reduction in the mean-square wall shear stress whenever the distribution function $S(\omega)$ is large for $\omega\lambda$ of order unity or larger. No particular

*Private communication with A. B. Metzner.

initial condition on the velocity or shear stress in the polymer solution is specified in obtaining this result. Therefore, regardless of the form which $S(\omega)$ in this laminar flow problem should have to best approximate the transient shear character of the turbulent boundary layer, a decrease in turbulent-flow drag is expected when the product of λ and a characteristic frequency of the wall region is large.

A related question is how important the transient shear effects are in causing drag reduction, compared to the elongational effects discussed by previous investigators (2).^{*} A part of the answer may be that for a polymer solution to have a large elongational viscosity, it must be in an elongational flow of sufficient duration to allow alignment of the polymer molecules along the streamlines. It has not yet been established that this condition is satisfied in the wall region of turbulent flows where drag reduction is observed.[†] Also, the lifetime of a given elongational region near the wall decreases with increasing flow rate (21), so that the drag reduction due to elongational effects may decrease with increasing flow rate. In contrast the deviation from Newtonian behavior expected from the transient-shear analysis and in general from that observed in experiments increases with increasing flow rate (with decreasing $\tau/2\lambda$). Thus, although the relative importance of the transient-shear and elongational-viscosity mechanisms is not known with certainty at present, there is some justification for expecting the former to be dominant at large flow rates.

A third question is whether a continuum treatment of drag reduction for the very dilute polymer solutions of greatest interest is permissible. A tentative answer may be offered on the basis of a comparison of the average distance (L) between polymer molecules and the thickness (y) of the periodic region. The former has been computed as a function of polymer concentration by Patterson (20). The latter is of the order of $70\mu/\sqrt{\tau_0\rho}$ (14). In experiments conducted by Hansen and Little (17, 22, 23) using an aqueous solution of polyethylene oxide WSR-205, τ_0 was of the order of 100 dynes/cm², and μ and ρ were about the same as for water. The value of y in these circumstances was about 0.07 cm. At a concentration of 0.1 ppm by weight, L for this system was approximately 2×10^{-4} cm. An increase to 100 ppm decreased L by one order of magnitude. Thus the corresponding range of L/y was about 3×10^{-3} to 3×10^{-4} . Because this ratio is orders of magnitude less than unity, the conclusion follows that a continuum treatment of the problem is appropriate in these circumstances (which are typical of those in which much of the drag-reduction data appearing in the literature were obtained). This conclusion must be regarded as tentative, however, because aggregation may be important for drag reduction in very dilute solutions (23); it was ignored in the calculation of L (20).

CONCLUSIONS

The drag exerted on an impulsively started flat plate by a polymer solution of infinite extent is less than that by a Newtonian fluid of the same density and viscosity. The same is true if the plate executes harmonic or random motion in its own plane. Combination of the impulsive-start solution with a simple flow model for the wall region leads to the conclusion that the dissolved polymer additive causes drag reduction in turbulent flow. As is observed in experiments, the predicted reduction in drag is negligibly small at low flow rates, low polymer concentrations, and low molecular weights. A maximum drag reduction condition for a given polymer additive is also predicted.

^{*}Private communication with W. S. Ament.

[†]Private communication with A. B. Metzner.

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NOTATION

- D = ratio of $\bar{\tau}_0$ from Eq. (7) and (9) to that from Eqs. (8) and (9) for a given value of $T/2\lambda$.
- F(t) = a random function of time; ΔF_ω = the contribution to F(t) associated with the frequency ω .
- j = $\sqrt{-1}$.
- M, N = functions of $\omega\rho/\mu$ and $\omega\lambda$.
- S(ω) = the distribution function in terms of which F(t) is specified.
- T = the period over which the wall shear stress is averaged and also the period associated with the flow near the wall in turbulent flow.
- t = time.
- U = plate velocity; ΔU_ω = the contribution to U of the harmonic velocity with frequency ω ; ΔU_ω^* = the complex conjugate of ΔU_ω .
- U_δ = the fluid velocity at the outer edge of the periodic wall region in turbulent flow.
- u_* = friction velocity.
- V = the average fluid velocity in a pipe.
- v_x = the x component of fluid velocity.
- x = distance parallel to the plate velocity.
- y = distance perpendicular to the plate.
- z_1 = $\sqrt{(t^2 - \xi^2)}/2\lambda$.
- z_2 = $(t - \xi)/2\lambda$.
- $(\quad)^2$ = the mean-square value of ().
- λ = Maxwell relaxation time of a polymer solution.
- ρ = density.
- ν = kinematic viscosity.
- μ = viscosity.

τ = shear stress in the x and y planes.

τ_0 = instantaneous value of the wall shear stress; $\Delta\tau_0$ = contribution to τ_0 associated with the harmonic velocity component with frequency ω .

$\bar{\tau}_0$ = time-average value of the wall shear stress.

ω = circular frequency.

ξ = $y \sqrt{\rho\lambda/\mu}$.

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13. ABSTRACT

The laminar flow of a polymer solution near a flat plate of infinite extent was investigated theoretically for three cases: (a) the plate is impulsively started and moves in its own plane with a constant velocity; (b) the plate executes linear harmonic oscillations in its own plane; (c) motion of the plate in this plane is a random function of time. By combining the results with a simple model for turbulent flow near a wall, a number of experimentally observed characteristics of the drag reduction phenomenon are predicted. The merits of this transient shear explanation of drag reduction were compared to those of previously offered hypotheses. Some justification was found for expecting the transient shear flow mechanism to be dominant at large flow rates.

14. KEY WORDS	LINK A		LINK B		LINK C	
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Drag reduction Polymer solution Transient shear flows Laminar flow Turbulent flow Newtonian liquid High elongational viscosity Continuum phenomenon Constitutive equation Convective Maxwell model Impulsive-start case Viscous-sublayer model Maxwell relaxation time Friction velocity Wall shear stress						