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The Conditional Probability Distribution of Crystal Structure Invariants

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Abstract: Improved joint probability distributions of several crystal structure factors have recently been obtained. These lead to a number of associated conditional probability distributions. In particular, the conditional probability distribution of $\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3})$, given

$$A = \frac{2}{N^{1/2}} |E_{\vec{h}_1} E_{\vec{h}_2} E_{\vec{h}_3}|,$$

where $\vec{h}_1 + \vec{h}_2 + \vec{h}_3 = 0$, has been derived and is here tabulated.

I. BACKGROUND

For a crystal consisting of N identical point-atoms per unit cell the normalized structure factor $E_{\vec{h}}$ is a complex number defined by

$$E_{\vec{h}} = |E_{\vec{h}}| e^{i\varphi_{\vec{h}}} = \frac{1}{N^{1/2}} \sum_{j=1}^N e^{2\pi i \vec{h} \cdot \vec{r}_j}, \quad (1-1)$$

where \vec{r}_j is the position vector of the j th atom, and \vec{h} is an arbitrary vector with integer components in reciprocal space. Once the values of a sufficient number of normalized structure factors are known, the crystal structure may be determined by means of the E -map synthesis

$$\rho(\vec{r}) = \left\langle |E_{\vec{h}}| e^{i\varphi_{\vec{h}}} e^{-2\pi i \vec{h} \cdot \vec{r}} \right\rangle_{\vec{h}}, \quad (1-2)$$

the maxima of which coincide with the atomic positions. This seemingly straightforward method of crystal structure determination is complicated by the fact that only the normalized structure factor amplitudes $|E_{\vec{h}}|$ are determined experimentally, whereas (1-2) is equally dependent on the phases $\varphi_{\vec{h}}$. The so-called phase problem is essentially the problem of determining the phases of the normalized structure factors when only the magnitudes are known.

From (1-1) it is evident that, because of the position vector \vec{r}_j , the normalized structure factors, and consequently their associated phases, depend not only on the crystal structure but also on the choice of origin. This further complicates the problem of direct phase determination, since it requires that some provision be made for specifying the origin. Fortunately the linear combinations

$$\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3}, \quad (1-3)$$

where

$$\vec{h}_1 + \vec{h}_2 + \vec{h}_3 = 0, \quad (1-4)$$

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depend only on the crystal structure and are independent of the position of the origin. For this reason the linear combinations (1-3) are called structure invariants.

Only the magnitudes of the linear combinations (1-3) or, equivalently, the values of the structure invariants

$$\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3}) \quad (1-5)$$

are determined by the magnitudes $|E_{\vec{k}}|$. Early efforts by Karle and Hauptman [1,2] led to formulas for computing the structure invariants (1-5) directly from the known values of the normalized structure factor amplitudes. These formulas, however, were found to be invalid if the structure contained a significant number of chance or induced interactions. While an improved formula by Hauptman [3] overcame this obstacle in the case that the interactions were exact, it was discovered that in practice interactions are often not exact but only approximate [4], so that even the improved formula produced cosine values as large as 4 or 5. Since the average

$$\langle (|E_{\vec{k}}|^2 - 1)^3 \rangle_{\vec{k}}$$

is known to increase as a result of the presence of these interactions, for both the exact and the approximate interactions, and since this average occurs as a factor in the scaling constant in the improved formula by Hauptman [3], it was evident that the scaling constant should be replaced by a continuous scaling curve. This curve, K , was determined in such a way as to force the empirical conditional distribution of $\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3})$ for the given crystal to agree as closely as possible with the known theoretical conditional probability distribution of $\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3})$ [4].

Numerous successful applications have been made using the modified formula

$$\begin{aligned} & |E_{\vec{h}_1} E_{\vec{h}_2} E_{\vec{h}_3}| \cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3}) \\ & \approx K \left\langle (|E_{\vec{k}}|^{1/2} - \overline{|E|^{1/2}}) \ (|E_{\vec{h}_1 + \vec{k}}|^{1/2} - \overline{|E|^{1/2}}) \ (|E_{-\vec{h}_3 + \vec{k}}|^{1/2} - \overline{|E|^{1/2}}) \right\rangle_{\vec{k}} \\ & + \frac{\sigma_3}{4 \sigma_2^{3/2}} \left[\frac{3}{2} (|E_{\vec{h}_1} E_{\vec{h}_2}|^2 + |E_{\vec{h}_2} E_{\vec{h}_3}|^2 + |E_{\vec{h}_3} E_{\vec{h}_1}|^2) + |E_{\vec{h}_1}|^2 + |E_{\vec{h}_2}|^2 + |E_{\vec{h}_3}|^2 - \frac{7}{2} \right], \end{aligned} \quad (1-6)$$

where \vec{k} ranges over all vectors with integer components in reciprocal space, and

$$\overline{|E|^{1/2}} = \left\langle |E_{\vec{k}}|^{1/2} \right\rangle_{\vec{k}}$$

As attempts are made to solve more complex crystal structures, the need to compute cosines with even greater accuracy is expected to increase. A recent effort by Hauptman [5] produced a formula which has already achieved some success where the previous formulas failed:

$$\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3}) \approx M(D - KS), \quad (1-7)$$

where

$$D = \frac{1}{m_1 + m_2 + m_3} (m_1 D_1 + m_2 D_2 + m_3 D_3) \quad (1-8)$$

and

$$D_i = \left\langle (|E_{-\vec{h}_i + \vec{k}}|^2 - 1) \mid |E_{\vec{k}}| > t, |E_{\vec{h}_j + \vec{k}}| > t \right\rangle_{\vec{k}}, \quad (1-9)$$

where $i = 1, 2, 3$ when $j = 2, 3, 1$ respectively, m_i is the number of contributors to the average D_i , and t is a fixed real number greater than one.

Similarly,

$$S = \frac{1}{n_1 + n_2 + n_3} (n_1 S_1 + n_2 S_2 + n_3 S_3), \quad (1-10)$$

where n_i is the number of contributors to the average S_i , and

$$S_i = \left\langle (|E_{-\vec{h}_i + \vec{k}}|^2 - 1) \mid |E_{\vec{k}}| > t \right\rangle_{\vec{k}} + \left\langle (|E_{-\vec{h}_i + \vec{k}}|^2 - 1) \mid |E_{\vec{h}_j + \vec{k}}| > t \right\rangle_{\vec{k}}, \quad (1-11)$$

where $i = 1, 2, 3$ when $j = 2, 3, 1$ respectively. However when \vec{k} is allowed to range over all vectors in reciprocal space for which $|E_{\vec{h}_j + \vec{k}}| > t$, it is evident that in (1-11) every contributor to the second average coincides with a contributor to the first average for some i . Thus, if (1-11) is replaced by the simpler

$$S_i = 2 \left\langle (|E_{-\vec{h}_i + \vec{k}}|^2 - 1) \mid |E_{\vec{k}}| > t \right\rangle_{\vec{k}}, \quad (1-12)$$

where $i = 1, 2, 3$, S is still correctly given by (1-10).

The invariants (1-7) can be arranged in decreasing order of $|E_{\vec{h}_1} E_{\vec{h}_2} E_{\vec{h}_3}|$ and partitioned in such a way that in each group the values of $|E_{\vec{h}_1} E_{\vec{h}_2} E_{\vec{h}_3}|$ are essentially constant. Then K may be determined for each group so that the resulting fraction of negative cosines agrees with that given by the theoretical distribution. The parameter M may next be determined in such a way as to force the average value of $\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3})$ to coincide with the theoretical expected value. Finally, the group may be arranged in decreasing order of $M(D - KS)$ and scaled so that the empirical distribution of cosines conforms to the known theoretical distribution. As an additional benefit, (1-7) requires less computing time than (1-6), because the averages are taken over restricted ranges.

Once the values of a suitable set of structure invariants (1-5) have thus been determined, the individual phases may then be found by a least-squares procedure which includes selecting the origin and the enantiomorph [4]. Finally the E -map synthesis (1-2) yields the crystal structure.

II. THE CONDITIONAL DISTRIBUTION AND CERTAIN OF ITS PARAMETERS

Let $Y = \varphi_{\vec{h}} + \varphi_{\vec{k}} + \varphi_{-\vec{h}-\vec{k}}$ and let $X = \cos Y$, where \vec{h} is a fixed vector and \vec{k} ranges uniformly throughout reciprocal space. Now let

$$A = \frac{2}{N^{1/2}} |E_{\vec{h}} E_{\vec{k}} E_{\vec{h} + \vec{k}}|,$$

where N is the number of atoms in the unit cell. Then the conditional probability that $X > x$, given A , is defined by the density function

$$\left. \begin{array}{ll} \frac{\exp Ax}{\pi I_0(A) \sqrt{1-x^2}}, & \text{if } |x| < 1, \\ 0, & \text{if } |x| > 1, \end{array} \right\} \quad (2-1)$$

where I_0 is the Bessel function of imaginary argument.

The following conditional expected values and variances, which may be derived from (2-1), have important applications:

$$E(\cos Y) = \frac{I_1(A)}{I_0(A)}, \quad (2-2)$$

$$E(\cos^2 Y) = \frac{1}{2} + \frac{I_2(A)}{2I_0(A)} = 1 - \frac{I_1(A)}{AI_0(A)}, \quad (2-3)$$

$$E(\sin^2 Y) = \frac{1}{2} - \frac{I_2(A)}{2I_0(A)} = \frac{I_1(A)}{AI_0(A)}, \quad (2-4)$$

$$\text{Var}(\cos Y) = 1 - \frac{I_1(A)}{AI_0(A)} - \frac{I_1^2(A)}{I_0^2(A)}, \quad (2-5)$$

and

$$\text{Var}(\sin Y) = \frac{I_1(A)}{AI_0(A)}. \quad (2-6)$$

The cumulative distribution function,

$$\int_x^1 \frac{e^{Ax}}{\pi I_0(A) \sqrt{1-x^2}} dx, \quad (2-7)$$

is displayed in Appendix A, and functions (2-2) through (2-6) are listed in Appendix B for A values ranging from 0.0 to 15.0 in steps of 0.1.

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APPENDIX A

THE CONDITIONAL PROBABILITY THAT $\cos(\varphi_{\vec{h}} + \varphi_{\vec{k}} + \varphi_{-\vec{h}-\vec{k}}) > x$,

$$\text{GIVEN } A = \frac{2}{N^{1/2}} |E_{\vec{h}} E_{\vec{k}} E_{\vec{h}+\vec{k}}|$$

X	A											
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.0199	0.0220	0.0242	0.0264	0.0287	0.0311	0.0335	0.0359	0.0383	0.0407	0.0431	
0.995	0.0316	0.0349	0.0383	0.0418	0.0455	0.0492	0.0530	0.0568	0.0606	0.0644	0.0682	
0.990	0.0448	0.0494	0.0543	0.0592	0.0644	0.0696	0.0749	0.0803	0.0856	0.0910	0.0963	
0.985	0.0550	0.0606	0.0665	0.0726	0.0788	0.0852	0.0917	0.0983	0.1048	0.1114	0.1179	
0.980	0.0635	0.0700	0.0768	0.0838	0.0910	0.0984	0.1059	0.1134	0.1209	0.1285	0.1359	
0.975	0.0711	0.0784	0.0859	0.0937	0.1018	0.1100	0.1183	0.1267	0.1351	0.1435	0.1518	
0.970	0.0779	0.0859	0.0941	0.1027	0.1115	0.1204	0.1295	0.1387	0.1479	0.1570	0.1661	
0.960	0.0901	0.0992	0.1088	0.1186	0.1287	0.1390	0.1494	0.1599	0.1704	0.1809	0.1913	
0.950	0.1089	0.1110	0.1216	0.1326	0.1438	0.1553	0.1669	0.1785	0.1902	0.2018	0.2134	
0.940	0.1106	0.1217	0.1333	0.1452	0.1575	0.1700	0.1826	0.1953	0.2080	0.2206	0.2332	
0.930	0.1196	0.1316	0.1440	0.1569	0.1700	0.1834	0.1970	0.2106	0.2243	0.2378	0.2513	
0.920	0.1280	0.1407	0.1540	0.1677	0.1817	0.1960	0.2104	0.2249	0.2393	0.2537	0.2680	
0.910	0.1358	0.1494	0.1634	0.1778	0.1926	0.2077	0.2229	0.2382	0.2534	0.2685	0.2835	
0.900	0.1433	0.1575	0.1723	0.1874	0.2030	0.2188	0.2347	0.2507	0.2666	0.2825	0.2981	
0.880	0.1573	0.1728	0.1888	0.2053	0.2222	0.2393	0.2565	0.2738	0.2911	0.3081	0.3250	
0.860	0.1702	0.1868	0.2040	0.2217	0.2397	0.2580	0.2765	0.2949	0.3133	0.3315	0.3494	
0.840	0.1823	0.2000	0.2182	0.2370	0.2561	0.2754	0.2949	0.3144	0.3337	0.3529	0.3717	
0.820	0.1937	0.2123	0.2316	0.2513	0.2714	0.2917	0.3121	0.3325	0.3528	0.3728	0.3924	
0.800	0.2046	0.2241	0.2442	0.2648	0.2858	0.3070	0.3283	0.3495	0.3706	0.3913	0.4117	
0.780	0.2150	0.2353	0.2562	0.2777	0.2995	0.3215	0.3436	0.3655	0.3873	0.4087	0.4298	
0.760	0.2250	0.2460	0.2678	0.2900	0.3126	0.3353	0.3581	0.3807	0.4031	0.4252	0.4468	
0.740	0.2346	0.2564	0.2788	0.3018	0.3250	0.3485	0.3719	0.3952	0.4182	0.4408	0.4629	
0.720	0.2439	0.2664	0.2895	0.3131	0.3370	0.3611	0.3851	0.4090	0.4325	0.4556	0.4782	
0.700	0.2530	0.2761	0.2998	0.3241	0.3486	0.3732	0.3978	0.4222	0.4462	0.4697	0.4927	
0.680	0.2617	0.2855	0.3098	0.3346	0.3597	0.3849	0.4100	0.4348	0.4593	0.4832	0.5065	
0.660	0.2703	0.2946	0.3195	0.3449	0.3705	0.3962	0.4217	0.4470	0.4718	0.4961	0.5198	
0.640	0.2787	0.3035	0.3290	0.3549	0.3809	0.4071	0.4330	0.4587	0.4839	0.5085	0.5325	
0.620	0.2869	0.3123	0.3382	0.3645	0.3911	0.4176	0.4440	0.4700	0.4956	0.5205	0.5446	
0.600	0.2949	0.3208	0.3472	0.3740	0.4009	0.4279	0.4546	0.4810	0.5068	0.5319	0.5563	
0.580	0.3028	0.3291	0.3560	0.3832	0.4105	0.4378	0.4649	0.4915	0.5176	0.5430	0.5675	
0.560	0.3106	0.3373	0.3646	0.3922	0.4199	0.4475	0.4749	0.5018	0.5281	0.5537	0.5784	
0.540	0.3182	0.3453	0.3730	0.4010	0.4290	0.4570	0.4846	0.5117	0.5382	0.5640	0.5888	
0.520	0.3257	0.3532	0.3813	0.4096	0.4379	0.4662	0.4940	0.5214	0.5481	0.5739	0.5989	
0.500	0.3331	0.3610	0.3894	0.4180	0.4467	0.4751	0.5033	0.5308	0.5576	0.5836	0.6086	
0.480	0.3404	0.3687	0.3973	0.4263	0.4552	0.4839	0.5122	0.5399	0.5669	0.5930	0.6180	
0.460	0.3476	0.3762	0.4052	0.4344	0.4636	0.4925	0.5210	0.5488	0.5759	0.6020	0.6271	
0.440	0.3547	0.3836	0.4129	0.4424	0.4718	0.5009	0.5295	0.5575	0.5847	0.6108	0.6360	
0.420	0.3618	0.3910	0.4205	0.4502	0.4798	0.5091	0.5379	0.5660	0.5932	0.6194	0.6445	
0.400	0.3688	0.3982	0.4280	0.4579	0.4877	0.5172	0.5461	0.5742	0.6015	0.6277	0.6528	
0.380	0.3757	0.4054	0.4354	0.4655	0.4955	0.5251	0.5541	0.5823	0.6096	0.6358	0.6609	
0.360	0.3825	0.4125	0.4427	0.4730	0.5031	0.5328	0.5619	0.5902	0.6175	0.6437	0.6687	
0.340	0.3893	0.4195	0.4499	0.4804	0.5106	0.5404	0.5696	0.5979	0.6252	0.6514	0.6764	
0.320	0.3961	0.4265	0.4571	0.4877	0.5180	0.5479	0.5771	0.6054	0.6327	0.6589	0.6838	
0.300	0.4028	0.4334	0.4641	0.4949	0.5253	0.5553	0.5845	0.6128	0.6401	0.6662	0.6910	
0.280	0.4094	0.4402	0.4711	0.5020	0.5325	0.5625	0.5917	0.6200	0.6473	0.6733	0.6980	
0.260	0.4160	0.4470	0.4780	0.5090	0.5396	0.5696	0.5989	0.6271	0.6543	0.6802	0.7048	
0.240	0.4226	0.4537	0.4849	0.5159	0.5466	0.5766	0.6059	0.6341	0.6612	0.6870	0.7115	
0.220	0.4292	0.4604	0.4917	0.5228	0.5535	0.5835	0.6127	0.6409	0.6679	0.6937	0.7180	
0.200	0.4357	0.4670	0.4984	0.5295	0.5603	0.5903	0.6195	0.6476	0.6745	0.7001	0.7244	
0.180	0.4422	0.4736	0.5050	0.5363	0.5670	0.5970	0.6262	0.6542	0.6810	0.7065	0.7306	
0.160	0.4486	0.4801	0.5117	0.5429	0.5736	0.6036	0.6327	0.6607	0.6873	0.7127	0.7366	
0.140	0.4551	0.4867	0.5182	0.5495	0.5802	0.6102	0.6392	0.6670	0.6936	0.7188	0.7425	
0.120	0.4615	0.4932	0.5248	0.5560	0.5867	0.6166	0.6455	0.6732	0.6997	0.7247	0.7483	
0.100	0.4679	0.4996	0.5312	0.5625	0.5932	0.6230	0.6518	0.6794	0.7057	0.7305	0.7539	

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x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.0431	0.0455	0.0479	0.0502	0.0525	0.0547	0.0569	0.0590	0.0611	0.0632	0.0652
0.995	0.0682	0.0719	0.0757	0.0793	0.0829	0.0864	0.0899	0.0932	0.0965	0.0997	0.1029
0.990	0.0963	0.1016	0.1068	0.1120	0.1170	0.1220	0.1268	0.1315	0.1362	0.1407	0.1451
0.985	0.1179	0.1243	0.1307	0.1369	0.1431	0.1491	0.1550	0.1607	0.1663	0.1718	0.1771
0.980	0.1359	0.1433	0.1507	0.1578	0.1649	0.1718	0.1786	0.1851	0.1916	0.1978	0.2040
0.975	0.1518	0.1600	0.1682	0.1762	0.1840	0.1917	0.1992	0.2065	0.2136	0.2206	0.2274
0.970	0.1661	0.1751	0.1839	0.1927	0.2012	0.2095	0.2177	0.2257	0.2334	0.2410	0.2484
0.960	0.1913	0.2016	0.2117	0.2217	0.2314	0.2410	0.2503	0.2594	0.2682	0.2768	0.2852
0.950	0.2134	0.2248	0.2360	0.2470	0.2578	0.2683	0.2786	0.2886	0.2983	0.3078	0.3170
0.940	0.2332	0.2456	0.2577	0.2697	0.2813	0.2927	0.3039	0.3147	0.3252	0.3354	0.3453
0.930	0.2513	0.2645	0.2775	0.2903	0.3028	0.3149	0.3268	0.3383	0.3495	0.3603	0.3709
0.920	0.2680	0.2820	0.2958	0.3093	0.3225	0.3353	0.3478	0.3600	0.3717	0.3832	0.3942
0.910	0.2835	0.2983	0.3128	0.3269	0.3407	0.3542	0.3673	0.3800	0.3923	0.4043	0.4158
0.900	0.2981	0.3135	0.3287	0.3434	0.3579	0.3719	0.3855	0.3987	0.4115	0.4239	0.4359
0.880	0.3250	0.3416	0.3578	0.3737	0.3891	0.4041	0.4187	0.4328	0.4464	0.4596	0.4723
0.860	0.3494	0.3670	0.3842	0.4010	0.4173	0.4331	0.4484	0.4632	0.4775	0.4913	0.5046
0.840	0.3717	0.3902	0.4082	0.4258	0.4429	0.4594	0.4753	0.4907	0.5056	0.5199	0.5336
0.820	0.3924	0.4117	0.4304	0.4487	0.4664	0.4835	0.5000	0.5158	0.5311	0.5458	0.5600
0.800	0.4117	0.4316	0.4510	0.4699	0.4881	0.5057	0.5226	0.5389	0.5546	0.5696	0.5840
0.780	0.4298	0.4503	0.4703	0.4896	0.5083	0.5263	0.5436	0.5603	0.5762	0.5915	0.6062
0.760	0.4468	0.4679	0.4883	0.5081	0.5272	0.5456	0.5632	0.5801	0.5963	0.6118	0.6266
0.740	0.4629	0.4844	0.5053	0.5255	0.5449	0.5636	0.5815	0.5986	0.6150	0.6306	0.6455
0.720	0.4782	0.5001	0.5213	0.5418	0.5616	0.5805	0.5986	0.6159	0.6324	0.6481	0.6631
0.700	0.4927	0.5150	0.5366	0.5573	0.5773	0.5964	0.6147	0.6321	0.6487	0.6645	0.6795
0.680	0.5065	0.5292	0.5510	0.5720	0.5922	0.6114	0.6298	0.6474	0.6640	0.6799	0.6949
0.660	0.5198	0.5427	0.5648	0.5860	0.6063	0.6257	0.6442	0.6617	0.6784	0.6943	0.7092
0.640	0.5325	0.5556	0.5779	0.5992	0.6197	0.6392	0.6577	0.6753	0.6920	0.7078	0.7227
0.620	0.5446	0.5680	0.5904	0.6119	0.6324	0.6520	0.6705	0.6881	0.7048	0.7205	0.7354
0.600	0.5563	0.5798	0.6024	0.6240	0.6446	0.6642	0.6827	0.7003	0.7169	0.7326	0.7474
0.580	0.5675	0.5912	0.6139	0.6355	0.6562	0.6758	0.6943	0.7118	0.7284	0.7440	0.7586
0.560	0.5784	0.6021	0.6249	0.6466	0.6672	0.6868	0.7053	0.7228	0.7392	0.7547	0.7692
0.540	0.5888	0.6127	0.6355	0.6572	0.6778	0.6974	0.7158	0.7332	0.7496	0.7649	0.7793
0.520	0.5989	0.6228	0.6456	0.6674	0.6880	0.7075	0.7259	0.7431	0.7594	0.7746	0.7888
0.500	0.6086	0.6326	0.6554	0.6772	0.6977	0.7171	0.7354	0.7526	0.7687	0.7837	0.7978
0.480	0.6180	0.6420	0.6649	0.6866	0.7071	0.7264	0.7446	0.7616	0.7775	0.7924	0.8063
0.460	0.6271	0.6511	0.6740	0.6956	0.7160	0.7353	0.7533	0.7702	0.7860	0.8007	0.8144
0.440	0.6360	0.6600	0.6827	0.7043	0.7247	0.7438	0.7617	0.7784	0.7940	0.8086	0.8221
0.420	0.6445	0.6685	0.6912	0.7127	0.7329	0.7519	0.7697	0.7863	0.8017	0.8161	0.8294
0.400	0.6528	0.6768	0.6994	0.7208	0.7409	0.7598	0.7774	0.7938	0.8091	0.8233	0.8364
0.380	0.6609	0.6848	0.7073	0.7286	0.7486	0.7673	0.7848	0.8010	0.8161	0.8301	0.8430
0.360	0.6687	0.6925	0.7150	0.7362	0.7560	0.7746	0.7918	0.8079	0.8228	0.8366	0.8493
0.340	0.6764	0.7001	0.7224	0.7435	0.7631	0.7815	0.7986	0.8145	0.8292	0.8428	0.8553
0.320	0.6838	0.7074	0.7296	0.7505	0.7700	0.7862	0.8052	0.8208	0.8353	0.8487	0.8610
0.300	0.6910	0.7145	0.7366	0.7573	0.7767	0.7947	0.8115	0.8269	0.8412	0.8544	0.8665
0.280	0.6980	0.7214	0.7433	0.7639	0.7831	0.8010	0.8175	0.8328	0.8469	0.8598	0.8717
0.260	0.7048	0.7281	0.7499	0.7703	0.7893	0.8070	0.8233	0.8384	0.8523	0.8650	0.8767
0.240	0.7115	0.7346	0.7563	0.7765	0.7953	0.8128	0.8289	0.8438	0.8574	0.8700	0.8815
0.220	0.7180	0.7410	0.7625	0.7825	0.8012	0.8184	0.8343	0.8490	0.8624	0.8747	0.8860
0.200	0.7244	0.7471	0.7685	0.7883	0.8068	0.8238	0.8395	0.8540	0.8672	0.8793	0.8903
0.180	0.7306	0.7532	0.7743	0.7940	0.8122	0.8291	0.8446	0.8588	0.8718	0.8837	0.8945
0.160	0.7366	0.7590	0.7800	0.7995	0.8175	0.8341	0.8494	0.8634	0.8762	0.8879	0.8985
0.140	0.7425	0.7648	0.7855	0.8048	0.8226	0.8390	0.8541	0.8679	0.8804	0.8919	0.9023
0.120	0.7483	0.7704	0.7909	0.8100	0.8276	0.8438	0.8586	0.8722	0.8845	0.8958	0.9060
0.100	0.7539	0.7758	0.7962	0.8150	0.8324	0.8483	0.8630	0.8763	0.8884	0.8995	0.9095

UNCLASSIFIED

X	A										
	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.0652	0.0671	0.0690	0.0709	0.0727	0.0744	0.0762	0.0778	0.0795	0.0811	0.0827
0.995	0.1029	0.1059	0.1089	0.1118	0.1147	0.1174	0.1201	0.1228	0.1254	0.1279	0.1304
0.990	0.1451	0.1493	0.1535	0.1576	0.1616	0.1654	0.1692	0.1729	0.1766	0.1801	0.1836
0.985	0.1771	0.1823	0.1874	0.1924	0.1972	0.2019	0.2065	0.2110	0.2153	0.2196	0.2238
0.980	0.2040	0.2099	0.2157	0.2214	0.2269	0.2323	0.2375	0.2426	0.2476	0.2525	0.2573
0.975	0.2274	0.2340	0.2404	0.2467	0.2528	0.2587	0.2645	0.2702	0.2757	0.2811	0.2863
0.970	0.2484	0.2555	0.2625	0.2693	0.2759	0.2824	0.2886	0.2948	0.3007	0.3066	0.3123
0.960	0.2852	0.2933	0.3012	0.3089	0.3164	0.3237	0.3307	0.3377	0.3444	0.3510	0.3574
0.950	0.3170	0.3259	0.3346	0.3431	0.3513	0.3592	0.3670	0.3745	0.3819	0.3890	0.3960
0.940	0.3453	0.3549	0.3643	0.3733	0.3821	0.3907	0.3990	0.4071	0.4149	0.4226	0.4300
0.930	0.3709	0.3811	0.3910	0.4006	0.4099	0.4190	0.4277	0.4363	0.4445	0.4526	0.4604
0.920	0.3942	0.4050	0.4154	0.4255	0.4352	0.4447	0.4539	0.4628	0.4714	0.4798	0.4880
0.910	0.4158	0.4270	0.4379	0.4483	0.4585	0.4683	0.4779	0.4871	0.4960	0.5047	0.5132
0.900	0.4359	0.4475	0.4587	0.4696	0.4800	0.4902	0.5000	0.5095	0.5187	0.5277	0.5363
0.880	0.4723	0.4845	0.4964	0.5078	0.5189	0.5295	0.5398	0.5498	0.5594	0.5687	0.5777
0.860	0.5046	0.5174	0.5297	0.5416	0.5531	0.5641	0.5747	0.5850	0.5949	0.6045	0.6137
0.840	0.5336	0.5468	0.5596	0.5718	0.5836	0.5949	0.6058	0.6163	0.6264	0.6361	0.6455
0.820	0.5600	0.5735	0.5865	0.5990	0.6110	0.6225	0.6336	0.6442	0.6544	0.6642	0.6737
0.800	0.5840	0.5979	0.6111	0.6238	0.6359	0.6476	0.6587	0.6694	0.6797	0.6895	0.6990
0.780	0.6062	0.6202	0.6336	0.6464	0.6586	0.6703	0.6815	0.6922	0.7025	0.7123	0.7217
0.760	0.6266	0.6407	0.6542	0.6671	0.6794	0.6911	0.7023	0.7130	0.7232	0.7330	0.7424
0.740	0.6455	0.6597	0.6733	0.6862	0.6985	0.7102	0.7214	0.7320	0.7422	0.7518	0.7611
0.720	0.6631	0.6774	0.6909	0.7038	0.7161	0.7278	0.7389	0.7494	0.7595	0.7690	0.7781
0.700	0.6795	0.6938	0.7073	0.7202	0.7324	0.7440	0.7550	0.7654	0.7753	0.7848	0.7937
0.680	0.6949	0.7091	0.7226	0.7354	0.7475	0.7590	0.7699	0.7802	0.7900	0.7992	0.8080
0.660	0.7092	0.7234	0.7369	0.7496	0.7616	0.7729	0.7837	0.7938	0.8034	0.8125	0.8211
0.640	0.7227	0.7368	0.7502	0.7628	0.7746	0.7859	0.7964	0.8064	0.8159	0.8248	0.8332
0.620	0.7354	0.7494	0.7627	0.7751	0.7868	0.7979	0.8083	0.8181	0.8274	0.8361	0.8443
0.600	0.7474	0.7613	0.7743	0.7867	0.7982	0.8091	0.8194	0.8290	0.8380	0.8466	0.8546
0.580	0.7586	0.7724	0.7853	0.7975	0.8089	0.8196	0.8296	0.8391	0.8479	0.8563	0.8641
0.560	0.7692	0.7829	0.7957	0.8076	0.8189	0.8294	0.8392	0.8485	0.8571	0.8652	0.8728
0.540	0.7793	0.7928	0.8054	0.8172	0.8282	0.8386	0.8482	0.8572	0.8657	0.8736	0.8810
0.520	0.7888	0.8021	0.8146	0.8262	0.8370	0.8471	0.8566	0.8654	0.8737	0.8813	0.8885
0.500	0.7978	0.8109	0.8232	0.8346	0.8453	0.8552	0.8644	0.8731	0.8811	0.8886	0.8955
0.480	0.8063	0.8193	0.8314	0.8426	0.8530	0.8628	0.8718	0.8802	0.8880	0.8953	0.9020
0.460	0.8144	0.8272	0.8391	0.8501	0.8604	0.8699	0.8787	0.8869	0.8945	0.9015	0.9081
0.440	0.8221	0.8347	0.8464	0.8572	0.8672	0.8765	0.8852	0.8931	0.9005	0.9074	0.9137
0.420	0.8294	0.8418	0.8533	0.8639	0.8737	0.8828	0.8912	0.8990	0.9062	0.9128	0.9190
0.400	0.8364	0.8486	0.8598	0.8702	0.8799	0.8887	0.8969	0.9045	0.9115	0.9179	0.9239
0.380	0.8430	0.8550	0.8660	0.8762	0.8856	0.8943	0.9023	0.9097	0.9164	0.9227	0.9284
0.360	0.8493	0.8611	0.8719	0.8819	0.8911	0.8996	0.9073	0.9145	0.9211	0.9271	0.9327
0.340	0.8553	0.8669	0.8775	0.8873	0.8963	0.9045	0.9121	0.9190	0.9254	0.9313	0.9367
0.320	0.8610	0.8724	0.8828	0.8924	0.9011	0.9092	0.9166	0.9233	0.9295	0.9352	0.9404
0.300	0.8665	0.8776	0.8878	0.8972	0.9058	0.9136	0.9208	0.9274	0.9334	0.9389	0.9439
0.280	0.8717	0.8826	0.8926	0.9018	0.9101	0.9178	0.9248	0.9311	0.9370	0.9423	0.9471
0.260	0.8767	0.8874	0.8972	0.9061	0.9143	0.9217	0.9285	0.9347	0.9404	0.9455	0.9502
0.240	0.8815	0.8919	0.9015	0.9103	0.9182	0.9255	0.9321	0.9381	0.9435	0.9485	0.9530
0.220	0.8860	0.8963	0.9056	0.9142	0.9219	0.9290	0.9354	0.9412	0.9465	0.9513	0.9557
0.200	0.8903	0.9004	0.9096	0.9179	0.9255	0.9323	0.9386	0.9442	0.9494	0.9540	0.9582
0.180	0.8945	0.9044	0.9133	0.9215	0.9288	0.9355	0.9416	0.9471	0.9520	0.9565	0.9606
0.160	0.8985	0.9081	0.9169	0.9248	0.9320	0.9385	0.9444	0.9497	0.9545	0.9588	0.9628
0.140	0.9023	0.9117	0.9203	0.9280	0.9350	0.9414	0.9471	0.9522	0.9569	0.9611	0.9648
0.120	0.9060	0.9152	0.9236	0.9311	0.9379	0.9441	0.9496	0.9546	0.9591	0.9631	0.9668
0.100	0.9095	0.9185	0.9267	0.9340	0.9407	0.9466	0.9520	0.9568	0.9612	0.9651	0.9686

UNCLASSIFIED

X	A										
	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.0827	0.0843	0.0858	0.0873	0.0887	0.0902	0.0916	0.0930	0.0943	0.0957	0.0970
0.995	0.1304	0.1329	0.1352	0.1376	0.1399	0.1421	0.1443	0.1465	0.1486	0.1507	0.1528
0.990	0.1836	0.1870	0.1903	0.1936	0.1968	0.1999	0.2030	0.2060	0.2090	0.2119	0.2148
0.985	0.2238	0.2279	0.2319	0.2359	0.2397	0.2435	0.2472	0.2509	0.2544	0.2580	0.2614
0.980	0.2573	0.2620	0.2665	0.2710	0.2754	0.2797	0.2839	0.2880	0.2921	0.2961	0.3000
0.975	0.2864	0.2913	0.2966	0.3015	0.3063	0.3110	0.3157	0.3202	0.3247	0.3291	0.3334
0.970	0.3123	0.3179	0.3233	0.3286	0.3338	0.3389	0.3439	0.3488	0.3534	0.3584	0.3630
0.960	0.3574	0.3637	0.3698	0.3757	0.3816	0.3873	0.3929	0.3983	0.4037	0.4090	0.4141
0.950	0.3961	0.4029	0.4095	0.4160	0.4223	0.4285	0.4345	0.4405	0.4463	0.4520	0.4575
0.940	0.4301	0.4373	0.4444	0.4513	0.4580	0.4646	0.4710	0.4773	0.4834	0.4894	0.4953
0.930	0.4605	0.4681	0.4755	0.4828	0.4898	0.4967	0.5034	0.5100	0.5164	0.5227	0.5288
0.920	0.4880	0.4960	0.5037	0.5112	0.5185	0.5254	0.5326	0.5394	0.5460	0.5525	0.5588
0.910	0.5132	0.5214	0.5293	0.5371	0.5446	0.5519	0.5591	0.5661	0.5728	0.5795	0.5859
0.900	0.5364	0.5448	0.5529	0.5608	0.5686	0.5760	0.5833	0.5904	0.5974	0.6041	0.6107
0.880	0.5777	0.5865	0.5949	0.6031	0.6110	0.6188	0.6262	0.6335	0.6406	0.6475	0.6542
0.860	0.6138	0.6227	0.6313	0.6397	0.6477	0.6554	0.6632	0.6705	0.6777	0.6846	0.6913
0.840	0.6455	0.6545	0.6633	0.6717	0.6798	0.6877	0.6953	0.7027	0.7094	0.7167	0.7234
0.820	0.6737	0.6828	0.6916	0.7000	0.7081	0.7160	0.7235	0.7309	0.7379	0.7448	0.7514
0.800	0.6990	0.7081	0.7168	0.7252	0.7333	0.7411	0.7485	0.7558	0.7627	0.7694	0.7759
0.780	0.7218	0.7308	0.7395	0.7478	0.7558	0.7634	0.7708	0.7779	0.7847	0.7913	0.7976
0.760	0.7424	0.7513	0.7599	0.7681	0.7759	0.7835	0.7907	0.7976	0.8043	0.8107	0.8168
0.740	0.7611	0.7699	0.7784	0.7864	0.7941	0.8015	0.8085	0.8153	0.8218	0.8280	0.8339
0.720	0.7782	0.7869	0.7951	0.8030	0.8105	0.8177	0.8246	0.8312	0.8378	0.8435	0.8492
0.700	0.7938	0.8023	0.8104	0.8181	0.8254	0.8324	0.8391	0.8455	0.8514	0.8574	0.8629
0.680	0.8081	0.8164	0.8243	0.8318	0.8390	0.8458	0.8523	0.8584	0.8643	0.8699	0.8752
0.660	0.8212	0.8293	0.8370	0.8444	0.8513	0.8579	0.8642	0.8701	0.8758	0.8812	0.8863
0.640	0.8332	0.8412	0.8487	0.8558	0.8626	0.8690	0.8750	0.8808	0.8862	0.8914	0.8963
0.620	0.8444	0.8521	0.8594	0.8663	0.8729	0.8791	0.8849	0.8904	0.8956	0.9006	0.9053
0.600	0.8546	0.8622	0.8693	0.8760	0.8823	0.8883	0.8939	0.8992	0.9042	0.9090	0.9135
0.580	0.8641	0.8715	0.8783	0.8848	0.8909	0.8967	0.9021	0.9072	0.9120	0.9166	0.9208
0.560	0.8729	0.8800	0.8867	0.8930	0.8989	0.9044	0.9096	0.9145	0.9191	0.9235	0.9276
0.540	0.8810	0.8879	0.8944	0.9005	0.9062	0.9115	0.9165	0.9212	0.9256	0.9298	0.9336
0.520	0.8886	0.8953	0.9015	0.9074	0.9129	0.9180	0.9228	0.9273	0.9315	0.9355	0.9392
0.500	0.8956	0.9021	0.9081	0.9138	0.9191	0.9240	0.9286	0.9329	0.9369	0.9407	0.9442
0.480	0.9021	0.9084	0.9142	0.9197	0.9248	0.9295	0.9339	0.9380	0.9410	0.9455	0.9488
0.460	0.9081	0.9142	0.9199	0.9251	0.9300	0.9346	0.9388	0.9427	0.9464	0.9498	0.9530
0.440	0.9138	0.9197	0.9251	0.9302	0.9349	0.9392	0.9433	0.9470	0.9505	0.9538	0.9568
0.420	0.9190	0.9247	0.9300	0.9348	0.9394	0.9435	0.9474	0.9510	0.9543	0.9574	0.9603
0.400	0.9239	0.9294	0.9345	0.9392	0.9435	0.9475	0.9512	0.9546	0.9578	0.9608	0.9635
0.380	0.9285	0.9338	0.9387	0.9432	0.9473	0.9512	0.9547	0.9580	0.9610	0.9638	0.9664
0.360	0.9327	0.9379	0.9426	0.9469	0.9509	0.9546	0.9580	0.9611	0.9640	0.9666	0.9691
0.340	0.9367	0.9417	0.9462	0.9504	0.9542	0.9577	0.9609	0.9639	0.9667	0.9692	0.9715
0.320	0.9404	0.9452	0.9495	0.9536	0.9572	0.9604	0.9637	0.9665	0.9697	0.9715	0.9737
0.300	0.9439	0.9485	0.9527	0.9566	0.9601	0.9633	0.9662	0.9689	0.9714	0.9737	0.9758
0.280	0.9472	0.9516	0.9556	0.9593	0.9627	0.9658	0.9686	0.9712	0.9735	0.9757	0.9777
0.260	0.9502	0.9545	0.9584	0.9619	0.9651	0.9681	0.9708	0.9732	0.9755	0.9775	0.9794
0.240	0.9531	0.9572	0.9609	0.9643	0.9674	0.9702	0.9728	0.9751	0.9773	0.9792	0.9810
0.220	0.9558	0.9597	0.9633	0.9666	0.9695	0.9722	0.9747	0.9769	0.9789	0.9807	0.9824
0.200	0.9583	0.9621	0.9655	0.9686	0.9715	0.9740	0.9764	0.9785	0.9804	0.9822	0.9837
0.180	0.9606	0.9643	0.9676	0.9706	0.9733	0.9758	0.9780	0.9800	0.9818	0.9835	0.9850
0.160	0.9628	0.9663	0.9695	0.9724	0.9750	0.9773	0.9795	0.9814	0.9831	0.9847	0.9861
0.140	0.9649	0.9683	0.9713	0.9741	0.9768	0.9788	0.9808	0.9827	0.9843	0.9858	0.9871
0.120	0.9668	0.9701	0.9730	0.9757	0.9781	0.9802	0.9821	0.9839	0.9854	0.9868	0.9881
0.100	0.9686	0.9718	0.9746	0.9771	0.9794	0.9815	0.9833	0.9850	0.9864	0.9878	0.9890

UNCLASSIFIED

X	A											
	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.0970	0.0983	0.0996	0.1009	0.1021	0.1034	0.1046	0.1058	0.1070	0.1082	0.1093	
0.995	0.1528	0.1549	0.1569	0.1589	0.1608	0.1628	0.1647	0.1666	0.1684	0.1702	0.1721	
0.990	0.2148	0.2176	0.2204	0.2232	0.2259	0.2286	0.2312	0.2338	0.2364	0.2389	0.2414	
0.985	0.2614	0.2648	0.2682	0.2715	0.2748	0.2780	0.2812	0.2843	0.2874	0.2904	0.2934	
0.980	0.3000	0.3039	0.3077	0.3114	0.3151	0.3188	0.3223	0.3259	0.3294	0.3328	0.3362	
0.975	0.3334	0.3376	0.3418	0.3459	0.3500	0.3539	0.3579	0.3617	0.3655	0.3693	0.3730	
0.970	0.3630	0.3676	0.3721	0.3765	0.3808	0.3851	0.3893	0.3934	0.3973	0.4015	0.4055	
0.960	0.4141	0.4192	0.4242	0.4291	0.4339	0.4386	0.4433	0.4479	0.4524	0.4568	0.4612	
0.950	0.4575	0.4630	0.4684	0.4736	0.4788	0.4839	0.4888	0.4937	0.4986	0.5033	0.5080	
0.940	0.4953	0.5011	0.5067	0.5123	0.5177	0.5230	0.5283	0.5334	0.5385	0.5434	0.5483	
0.930	0.5288	0.5348	0.5407	0.5464	0.5520	0.5576	0.5630	0.5683	0.5735	0.5786	0.5837	
0.920	0.5588	0.5650	0.5710	0.5769	0.5827	0.5884	0.5939	0.5994	0.6047	0.6099	0.6151	
0.910	0.5859	0.5922	0.5984	0.6045	0.6103	0.6161	0.6218	0.6273	0.6327	0.6380	0.6432	
0.900	0.6107	0.6171	0.6233	0.6295	0.6354	0.6413	0.6470	0.6525	0.6580	0.6633	0.6686	
0.880	0.6542	0.6607	0.6671	0.6733	0.6793	0.6852	0.6909	0.6965	0.7020	0.7073	0.7125	
0.860	0.6913	0.6979	0.7042	0.7104	0.7164	0.7223	0.7280	0.7335	0.7389	0.7442	0.7493	
0.840	0.7234	0.7299	0.7362	0.7423	0.7482	0.7540	0.7596	0.7650	0.7703	0.7755	0.7805	
0.820	0.7514	0.7578	0.7639	0.7699	0.7757	0.7814	0.7868	0.7921	0.7973	0.8022	0.8071	
0.800	0.7759	0.7822	0.7882	0.7941	0.7997	0.8052	0.8105	0.8156	0.8205	0.8253	0.8300	
0.780	0.7976	0.8037	0.8096	0.8152	0.8207	0.8260	0.8311	0.8360	0.8408	0.8454	0.8498	
0.760	0.8168	0.8227	0.8284	0.8339	0.8392	0.8442	0.8491	0.8539	0.8584	0.8628	0.8670	
0.740	0.8339	0.8396	0.8451	0.8504	0.8555	0.8603	0.8650	0.8695	0.8739	0.8780	0.8821	
0.720	0.8492	0.8547	0.8600	0.8651	0.8699	0.8746	0.8791	0.8834	0.8875	0.8914	0.8952	
0.700	0.8629	0.8682	0.8733	0.8781	0.8828	0.8872	0.8913	0.8956	0.8995	0.9032	0.9068	
0.680	0.8752	0.8803	0.8852	0.8898	0.8942	0.8985	0.9025	0.9064	0.9103	0.9136	0.9170	
0.660	0.8863	0.8912	0.8958	0.9002	0.9044	0.9085	0.9123	0.9160	0.9194	0.9228	0.9260	
0.640	0.8963	0.9009	0.9054	0.9096	0.9136	0.9174	0.9210	0.9245	0.9278	0.9309	0.9339	
0.620	0.9053	0.9097	0.9140	0.9180	0.9218	0.9254	0.9288	0.9321	0.9352	0.9381	0.9409	
0.600	0.9135	0.9177	0.9217	0.9255	0.9291	0.9325	0.9358	0.9389	0.9418	0.9445	0.9472	
0.580	0.9208	0.9249	0.9287	0.9323	0.9357	0.9390	0.9420	0.9449	0.9476	0.9502	0.9527	
0.560	0.9276	0.9314	0.9350	0.9385	0.9417	0.9447	0.9476	0.9503	0.9529	0.9553	0.9576	
0.540	0.9336	0.9373	0.9407	0.9440	0.9470	0.9490	0.9526	0.9552	0.9576	0.9598	0.9620	
0.520	0.9392	0.9427	0.9459	0.9490	0.9519	0.9544	0.9571	0.9595	0.9618	0.9639	0.9659	
0.500	0.9442	0.9475	0.9506	0.9535	0.9562	0.9588	0.9612	0.9634	0.9655	0.9675	0.9694	
0.480	0.9488	0.9519	0.9549	0.9576	0.9602	0.9626	0.9648	0.9669	0.9689	0.9707	0.9725	
0.460	0.9530	0.9560	0.9587	0.9613	0.9637	0.9660	0.9681	0.9701	0.9719	0.9736	0.9753	
0.440	0.9568	0.9596	0.9622	0.9647	0.9670	0.9691	0.9711	0.9729	0.9746	0.9762	0.9777	
0.420	0.9603	0.9630	0.9654	0.9677	0.9699	0.9719	0.9737	0.9755	0.9771	0.9786	0.9800	
0.400	0.9635	0.9660	0.9683	0.9705	0.9725	0.9744	0.9761	0.9778	0.9793	0.9807	0.9820	
0.380	0.9664	0.9688	0.9710	0.9730	0.9749	0.9767	0.9783	0.9798	0.9812	0.9825	0.9837	
0.360	0.9691	0.9713	0.9734	0.9753	0.9771	0.9788	0.9803	0.9817	0.9830	0.9842	0.9853	
0.340	0.9715	0.9736	0.9756	0.9774	0.9791	0.9806	0.9821	0.9834	0.9846	0.9857	0.9868	
0.320	0.9737	0.9758	0.9776	0.9793	0.9809	0.9824	0.9837	0.9849	0.9861	0.9871	0.9881	
0.300	0.9758	0.9777	0.9794	0.9811	0.9825	0.9839	0.9851	0.9863	0.9874	0.9883	0.9892	
0.280	0.9777	0.9795	0.9811	0.9826	0.9840	0.9853	0.9865	0.9875	0.9885	0.9894	0.9903	
0.260	0.9794	0.9811	0.9826	0.9841	0.9854	0.9866	0.9877	0.9887	0.9896	0.9904	0.9912	
0.240	0.9810	0.9826	0.9840	0.9854	0.9866	0.9877	0.9888	0.9897	0.9906	0.9913	0.9921	
0.220	0.9824	0.9839	0.9853	0.9866	0.9877	0.9888	0.9897	0.9906	0.9914	0.9921	0.9928	
0.200	0.9837	0.9852	0.9865	0.9877	0.9888	0.9897	0.9906	0.9915	0.9922	0.9929	0.9935	
0.180	0.9850	0.9863	0.9876	0.9887	0.9897	0.9905	0.9915	0.9922	0.9929	0.9935	0.9941	
0.160	0.9861	0.9874	0.9885	0.9896	0.9906	0.9914	0.9922	0.9929	0.9936	0.9941	0.9947	
0.140	0.9871	0.9884	0.9894	0.9904	0.9913	0.9921	0.9929	0.9935	0.9941	0.9947	0.9952	
0.120	0.9881	0.9892	0.9903	0.9912	0.9920	0.9928	0.9935	0.9941	0.9947	0.9952	0.9956	
0.100	0.9890	0.9901	0.9910	0.9919	0.9927	0.9934	0.9940	0.9946	0.9951	0.9956	0.9960	

UNCLASSIFIED

X	A											
	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.1093	0.1105	0.1116	0.1128	0.1139	0.1150	0.1161	0.1172	0.1182	0.1193	0.1203	
0.995	0.1721	0.1739	0.1756	0.1774	0.1791	0.1808	0.1825	0.1842	0.1859	0.1875	0.1892	
0.990	0.2414	0.2439	0.2464	0.2488	0.2512	0.2536	0.2559	0.2582	0.2605	0.2628	0.2650	
0.985	0.2934	0.2964	0.2993	0.3022	0.3051	0.3079	0.3107	0.3134	0.3162	0.3189	0.3215	
0.980	0.3362	0.3395	0.3428	0.3461	0.3493	0.3525	0.3556	0.3587	0.3618	0.3648	0.3678	
0.975	0.3730	0.3766	0.3802	0.3838	0.3873	0.3908	0.3942	0.3976	0.4009	0.4042	0.4075	
0.970	0.4055	0.4094	0.4132	0.4170	0.4208	0.4245	0.4281	0.4317	0.4353	0.4388	0.4423	
0.960	0.4612	0.4655	0.4697	0.4739	0.4780	0.4820	0.4860	0.4900	0.4939	0.4977	0.5015	
0.950	0.5080	0.5125	0.5170	0.5215	0.5259	0.5302	0.5344	0.5386	0.5427	0.5468	0.5508	
0.940	0.5483	0.5531	0.5578	0.5624	0.5670	0.5715	0.5759	0.5802	0.5845	0.5887	0.5928	
0.930	0.5837	0.5886	0.5934	0.5982	0.6029	0.6073	0.6120	0.6164	0.6208	0.6251	0.6293	
0.920	0.6151	0.6201	0.6250	0.6299	0.6346	0.6393	0.6439	0.6484	0.6528	0.6572	0.6614	
0.910	0.6432	0.6483	0.6533	0.6581	0.6629	0.6676	0.6723	0.6768	0.6812	0.6856	0.6899	
0.900	0.6686	0.6737	0.6787	0.6836	0.6884	0.6931	0.6977	0.7022	0.7067	0.7110	0.7153	
0.880	0.7125	0.7176	0.7226	0.7275	0.7322	0.7369	0.7414	0.7459	0.7503	0.7545	0.7587	
0.860	0.7493	0.7543	0.7592	0.7640	0.7686	0.7732	0.7776	0.7819	0.7861	0.7903	0.7943	
0.840	0.7805	0.7853	0.7901	0.7947	0.7992	0.8036	0.8078	0.8120	0.8160	0.8200	0.8238	
0.820	0.8071	0.8118	0.8163	0.8208	0.8251	0.8293	0.8333	0.8373	0.8412	0.8449	0.8486	
0.800	0.8300	0.8345	0.8389	0.8431	0.8472	0.8512	0.8550	0.8588	0.8624	0.8660	0.8694	
0.780	0.8498	0.8541	0.8583	0.8623	0.8662	0.8700	0.8736	0.8771	0.8806	0.8839	0.8871	
0.760	0.8670	0.8711	0.8751	0.8789	0.8826	0.8861	0.8899	0.8929	0.8961	0.8992	0.9022	
0.740	0.8821	0.8859	0.8897	0.8933	0.8967	0.9001	0.9033	0.9064	0.9094	0.9123	0.9151	
0.720	0.8952	0.8989	0.9024	0.9058	0.9090	0.9122	0.9152	0.9181	0.9209	0.9236	0.9262	
0.700	0.9068	0.9102	0.9136	0.9167	0.9198	0.9227	0.9255	0.9282	0.9308	0.9333	0.9357	
0.680	0.9170	0.9202	0.9233	0.9263	0.9291	0.9319	0.9345	0.9370	0.9394	0.9417	0.9439	
0.660	0.9260	0.9290	0.9319	0.9347	0.9373	0.9399	0.9423	0.9446	0.9469	0.9490	0.9510	
0.640	0.9339	0.9367	0.9395	0.9421	0.9445	0.9469	0.9491	0.9513	0.9533	0.9553	0.9572	
0.620	0.9409	0.9436	0.9461	0.9485	0.9508	0.9530	0.9551	0.9571	0.9590	0.9608	0.9625	
0.600	0.9472	0.9496	0.9520	0.9543	0.9564	0.9584	0.9604	0.9622	0.9639	0.9656	0.9672	
0.580	0.9527	0.9550	0.9572	0.9593	0.9613	0.9632	0.9650	0.9666	0.9683	0.9698	0.9712	
0.560	0.9576	0.9598	0.9618	0.9638	0.9656	0.9673	0.9690	0.9706	0.9720	0.9734	0.9748	
0.540	0.9620	0.9640	0.9659	0.9677	0.9694	0.9710	0.9725	0.9740	0.9753	0.9766	0.9778	
0.520	0.9659	0.9678	0.9695	0.9712	0.9728	0.9743	0.9757	0.9770	0.9782	0.9794	0.9805	
0.500	0.9694	0.9711	0.9728	0.9743	0.9758	0.9771	0.9784	0.9796	0.9808	0.9819	0.9829	
0.480	0.9725	0.9741	0.9756	0.9771	0.9784	0.9797	0.9809	0.9820	0.9830	0.9840	0.9849	
0.460	0.9753	0.9768	0.9782	0.9795	0.9808	0.9819	0.9830	0.9840	0.9850	0.9859	0.9867	
0.440	0.9777	0.9791	0.9805	0.9817	0.9828	0.9839	0.9849	0.9858	0.9867	0.9875	0.9883	
0.420	0.9800	0.9813	0.9825	0.9836	0.9847	0.9857	0.9866	0.9874	0.9882	0.9890	0.9897	
0.400	0.9820	0.9832	0.9843	0.9853	0.9863	0.9872	0.9881	0.9889	0.9896	0.9903	0.9909	
0.380	0.9837	0.9849	0.9859	0.9869	0.9878	0.9886	0.9894	0.9901	0.9908	0.9914	0.9920	
0.360	0.9853	0.9864	0.9873	0.9882	0.9891	0.9898	0.9905	0.9912	0.9918	0.9924	0.9929	
0.340	0.9868	0.9877	0.9886	0.9895	0.9902	0.9909	0.9916	0.9922	0.9927	0.9933	0.9937	
0.320	0.9881	0.9890	0.9898	0.9905	0.9912	0.9919	0.9925	0.9930	0.9936	0.9940	0.9945	
0.300	0.9892	0.9901	0.9908	0.9915	0.9922	0.9928	0.9933	0.9938	0.9943	0.9947	0.9951	
0.280	0.9903	0.9910	0.9917	0.9924	0.9930	0.9935	0.9940	0.9945	0.9949	0.9953	0.9957	
0.260	0.9912	0.9919	0.9926	0.9932	0.9937	0.9942	0.9947	0.9951	0.9955	0.9958	0.9962	
0.240	0.9921	0.9927	0.9933	0.9939	0.9944	0.9948	0.9952	0.9956	0.9960	0.9963	0.9966	
0.220	0.9928	0.9934	0.9940	0.9945	0.9949	0.9954	0.9957	0.9961	0.9964	0.9967	0.9970	
0.200	0.9935	0.9941	0.9946	0.9950	0.9955	0.9958	0.9962	0.9965	0.9968	0.9971	0.9973	
0.180	0.9941	0.9946	0.9951	0.9955	0.9959	0.9963	0.9966	0.9969	0.9972	0.9974	0.9976	
0.160	0.9947	0.9951	0.9956	0.9960	0.9963	0.9967	0.9970	0.9972	0.9975	0.9977	0.9979	
0.140	0.9952	0.9956	0.9960	0.9964	0.9967	0.9970	0.9973	0.9975	0.9978	0.9980	0.9981	
0.120	0.9956	0.9960	0.9964	0.9967	0.9970	0.9973	0.9976	0.9978	0.9980	0.9982	0.9983	
0.100	0.9960	0.9964	0.9967	0.9971	0.9973	0.9976	0.9978	0.9980	0.9982	0.9984	0.9985	

UNCLASSIFIED

X	A											
	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.1203	0.1214	0.1224	0.1234	0.1244	0.1255	0.1265	0.1274	0.1284	0.1294	0.1304	
0.995	0.1892	0.1908	0.1924	0.1940	0.1956	0.1971	0.1987	0.2002	0.2017	0.2032	0.2047	
0.990	0.2650	0.2672	0.2694	0.2716	0.2738	0.2759	0.2781	0.2802	0.2822	0.2843	0.2864	
0.985	0.3215	0.3242	0.3268	0.3294	0.3320	0.3345	0.3370	0.3395	0.3420	0.3444	0.3469	
0.980	0.3678	0.3708	0.3738	0.3767	0.3795	0.3824	0.3852	0.3880	0.3907	0.3935	0.3962	
0.975	0.4075	0.4107	0.4139	0.4170	0.4202	0.4232	0.4263	0.4293	0.4323	0.4353	0.4382	
0.970	0.4423	0.4457	0.4491	0.4525	0.4558	0.4591	0.4623	0.4655	0.4687	0.4718	0.4749	
0.960	0.5015	0.5053	0.5090	0.5126	0.5162	0.5198	0.5233	0.5268	0.5302	0.5336	0.5369	
0.950	0.5508	0.5547	0.5586	0.5625	0.5663	0.5700	0.5737	0.5773	0.5809	0.5845	0.5880	
0.940	0.5928	0.5969	0.6009	0.6049	0.6088	0.6127	0.6165	0.6202	0.6239	0.6276	0.6311	
0.930	0.6293	0.6335	0.6376	0.6416	0.6456	0.6495	0.6534	0.6572	0.6610	0.6646	0.6682	
0.920	0.6614	0.6656	0.6698	0.6738	0.6778	0.6818	0.6856	0.6894	0.6932	0.6969	0.7005	
0.910	0.6899	0.6941	0.6982	0.7023	0.7063	0.7102	0.7141	0.7178	0.7216	0.7252	0.7288	
0.900	0.7153	0.7195	0.7236	0.7276	0.7316	0.7355	0.7393	0.7430	0.7467	0.7503	0.7539	
0.880	0.7587	0.7628	0.7668	0.7707	0.7746	0.7783	0.7820	0.7856	0.7892	0.7927	0.7961	
0.860	0.7943	0.7982	0.8021	0.8058	0.8095	0.8131	0.8166	0.8201	0.8234	0.8267	0.8299	
0.840	0.8238	0.8276	0.8312	0.8348	0.8383	0.8417	0.8450	0.8482	0.8514	0.8545	0.8575	
0.820	0.8486	0.8521	0.8556	0.8589	0.8622	0.8654	0.8685	0.8715	0.8745	0.8773	0.8801	
0.800	0.8694	0.8727	0.8760	0.8791	0.8822	0.8852	0.8881	0.8909	0.8936	0.8963	0.8988	
0.780	0.8871	0.8902	0.8933	0.8962	0.8990	0.9018	0.9045	0.9071	0.9096	0.9120	0.9144	
0.760	0.9022	0.9051	0.9079	0.9106	0.9133	0.9158	0.9183	0.9207	0.9230	0.9252	0.9274	
0.740	0.9151	0.9178	0.9204	0.9229	0.9253	0.9277	0.9299	0.9321	0.9343	0.9363	0.9383	
0.720	0.9262	0.9287	0.9311	0.9334	0.9356	0.9378	0.9399	0.9419	0.9438	0.9457	0.9474	
0.700	0.9357	0.9380	0.9402	0.9423	0.9444	0.9464	0.9483	0.9501	0.9519	0.9534	0.9552	
0.680	0.9439	0.9460	0.9481	0.9500	0.9519	0.9537	0.9554	0.9571	0.9587	0.9602	0.9617	
0.660	0.9510	0.9530	0.9549	0.9566	0.9584	0.9600	0.9616	0.9631	0.9645	0.9659	0.9673	
0.640	0.9572	0.9590	0.9607	0.9623	0.9639	0.9654	0.9668	0.9682	0.9695	0.9708	0.9720	
0.620	0.9625	0.9642	0.9657	0.9672	0.9687	0.9700	0.9713	0.9726	0.9738	0.9749	0.9760	
0.600	0.9672	0.9687	0.9701	0.9715	0.9728	0.9740	0.9752	0.9763	0.9774	0.9784	0.9794	
0.580	0.9712	0.9726	0.9739	0.9752	0.9763	0.9775	0.9785	0.9795	0.9805	0.9814	0.9823	
0.560	0.9748	0.9760	0.9772	0.9783	0.9794	0.9804	0.9814	0.9823	0.9832	0.9840	0.9848	
0.540	0.9778	0.9790	0.9801	0.9811	0.9821	0.9830	0.9839	0.9847	0.9855	0.9862	0.9869	
0.520	0.9805	0.9816	0.9826	0.9835	0.9844	0.9852	0.9860	0.9867	0.9874	0.9881	0.9887	
0.500	0.9829	0.9838	0.9847	0.9856	0.9864	0.9871	0.9878	0.9885	0.9891	0.9897	0.9903	
0.480	0.9849	0.9858	0.9866	0.9874	0.9881	0.9888	0.9894	0.9900	0.9906	0.9911	0.9916	
0.460	0.9867	0.9875	0.9883	0.9890	0.9896	0.9902	0.9908	0.9913	0.9919	0.9923	0.9928	
0.440	0.9883	0.9890	0.9897	0.9903	0.9909	0.9915	0.9920	0.9925	0.9929	0.9934	0.9938	
0.420	0.9897	0.9903	0.9910	0.9915	0.9921	0.9924	0.9930	0.9935	0.9939	0.9943	0.9946	
0.400	0.9909	0.9915	0.9921	0.9926	0.9931	0.9935	0.9939	0.9943	0.9947	0.9950	0.9954	
0.380	0.9920	0.9925	0.9930	0.9935	0.9939	0.9943	0.9947	0.9951	0.9954	0.9957	0.9960	
0.360	0.9929	0.9934	0.9939	0.9943	0.9947	0.9950	0.9954	0.9957	0.9960	0.9963	0.9965	
0.340	0.9937	0.9942	0.9946	0.9950	0.9953	0.9957	0.9960	0.9963	0.9965	0.9968	0.9970	
0.320	0.9945	0.9949	0.9952	0.9956	0.9959	0.9962	0.9965	0.9967	0.9970	0.9972	0.9974	
0.300	0.9951	0.9955	0.9958	0.9961	0.9964	0.9967	0.9969	0.9972	0.9974	0.9976	0.9977	
0.280	0.9957	0.9960	0.9963	0.9966	0.9969	0.9971	0.9973	0.9975	0.9977	0.9979	0.9980	
0.260	0.9962	0.9965	0.9967	0.9970	0.9972	0.9975	0.9977	0.9978	0.9980	0.9982	0.9983	
0.240	0.9966	0.9969	0.9971	0.9974	0.9976	0.9978	0.9979	0.9981	0.9983	0.9984	0.9985	
0.220	0.9970	0.9972	0.9975	0.9977	0.9979	0.9980	0.9982	0.9984	0.9985	0.9986	0.9987	
0.200	0.9973	0.9976	0.9978	0.9980	0.9981	0.9983	0.9984	0.9986	0.9987	0.9988	0.9989	
0.180	0.9976	0.9978	0.9980	0.9982	0.9984	0.9985	0.9986	0.9987	0.9988	0.9989	0.9990	
0.160	0.9979	0.9981	0.9983	0.9984	0.9986	0.9987	0.9988	0.9989	0.9990	0.9991	0.9992	
0.140	0.9981	0.9983	0.9985	0.9986	0.9987	0.9988	0.9989	0.9990	0.9991	0.9992	0.9993	
0.120	0.9983	0.9985	0.9986	0.9988	0.9989	0.9990	0.9991	0.9992	0.9992	0.9993	0.9994	
0.100	0.9985	0.9987	0.9988	0.9989	0.9990	0.9991	0.9992	0.9993	0.9993	0.9994	0.9994	

UNCLASSIFIED

X	A										
	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.998	0.1304	0.1313	0.1323	0.1332	0.1342	0.1351	0.1360	0.1369	0.1378	0.1387	0.1396
0.995	0.2047	0.2062	0.2077	0.2092	0.2106	0.2121	0.2135	0.2149	0.2163	0.2177	0.2191
0.990	0.2864	0.2884	0.2904	0.2924	0.2944	0.2963	0.2983	0.3002	0.3021	0.3040	0.3059
0.985	0.3469	0.3493	0.3517	0.3540	0.3564	0.3587	0.3610	0.3633	0.3655	0.3678	0.3700
0.980	0.3962	0.3989	0.4015	0.4042	0.4068	0.4094	0.4119	0.4145	0.4170	0.4195	0.4219
0.975	0.4382	0.4411	0.4440	0.4468	0.4496	0.4524	0.4552	0.4579	0.4606	0.4633	0.4660
0.970	0.4749	0.4780	0.4810	0.4840	0.4870	0.4900	0.4929	0.4958	0.4986	0.5015	0.5043
0.960	0.5369	0.5402	0.5435	0.5468	0.5500	0.5531	0.5563	0.5594	0.5624	0.5655	0.5685
0.950	0.5880	0.5915	0.5949	0.5983	0.6016	0.6049	0.6081	0.6114	0.6145	0.6177	0.6208
0.940	0.6311	0.6347	0.6382	0.6416	0.6450	0.6484	0.6517	0.6550	0.6582	0.6614	0.6646
0.930	0.6682	0.6718	0.6753	0.6788	0.6822	0.6856	0.6890	0.6922	0.6955	0.6987	0.7018
0.920	0.7005	0.7041	0.7076	0.7111	0.7145	0.7178	0.7212	0.7244	0.7276	0.7308	0.7339
0.910	0.7288	0.7324	0.7359	0.7393	0.7427	0.7460	0.7493	0.7525	0.7556	0.7588	0.7618
0.900	0.7539	0.7574	0.7608	0.7642	0.7675	0.7708	0.7740	0.7771	0.7802	0.7832	0.7862
0.880	0.7961	0.7994	0.8027	0.8059	0.8090	0.8121	0.8152	0.8181	0.8211	0.8239	0.8267
0.860	0.8299	0.8331	0.8362	0.8392	0.8422	0.8451	0.8479	0.8507	0.8534	0.8560	0.8587
0.840	0.8575	0.8605	0.8633	0.8661	0.8689	0.8716	0.8742	0.8767	0.8793	0.8817	0.8841
0.820	0.8801	0.8829	0.8855	0.8881	0.8906	0.8931	0.8955	0.8979	0.9002	0.9024	0.9046
0.800	0.8988	0.9014	0.9038	0.9062	0.9085	0.9107	0.9129	0.9151	0.9172	0.9192	0.9211
0.780	0.9144	0.9167	0.9189	0.9211	0.9232	0.9253	0.9273	0.9292	0.9311	0.9329	0.9347
0.760	0.9274	0.9295	0.9315	0.9335	0.9354	0.9373	0.9391	0.9408	0.9425	0.9441	0.9457
0.740	0.9383	0.9402	0.9420	0.9438	0.9456	0.9472	0.9489	0.9504	0.9519	0.9534	0.9548
0.720	0.9474	0.9492	0.9509	0.9525	0.9540	0.9555	0.9570	0.9584	0.9598	0.9615	0.9623
0.700	0.9552	0.9568	0.9583	0.9597	0.9611	0.9625	0.9638	0.9650	0.9662	0.9674	0.9685
0.680	0.9617	0.9631	0.9645	0.9658	0.9671	0.9683	0.9694	0.9706	0.9716	0.9727	0.9737
0.660	0.9673	0.9685	0.9698	0.9709	0.9721	0.9732	0.9742	0.9752	0.9761	0.9771	0.9780
0.640	0.9720	0.9731	0.9742	0.9753	0.9763	0.9773	0.9782	0.9791	0.9799	0.9807	0.9815
0.620	0.9760	0.9770	0.9780	0.9789	0.9798	0.9807	0.9815	0.9823	0.9831	0.9838	0.9845
0.600	0.9794	0.9803	0.9812	0.9820	0.9828	0.9836	0.9843	0.9850	0.9857	0.9863	0.9869
0.580	0.9823	0.9831	0.9839	0.9847	0.9854	0.9861	0.9867	0.9873	0.9879	0.9885	0.9890
0.560	0.9848	0.9855	0.9862	0.9869	0.9875	0.9882	0.9887	0.9893	0.9898	0.9903	0.9908
0.540	0.9869	0.9876	0.9882	0.9888	0.9894	0.9899	0.9904	0.9909	0.9914	0.9918	0.9922
0.520	0.9887	0.9893	0.9899	0.9904	0.9909	0.9914	0.9919	0.9923	0.9927	0.9931	0.9934
0.500	0.9903	0.9908	0.9913	0.9918	0.9922	0.9927	0.9931	0.9934	0.9938	0.9941	0.9945
0.480	0.9916	0.9921	0.9926	0.9930	0.9934	0.9937	0.9941	0.9944	0.9947	0.9950	0.9953
0.460	0.9928	0.9932	0.9936	0.9940	0.9943	0.9947	0.9950	0.9953	0.9955	0.9958	0.9960
0.440	0.9938	0.9942	0.9945	0.9948	0.9951	0.9954	0.9957	0.9960	0.9962	0.9964	0.9967
0.420	0.9946	0.9950	0.9953	0.9956	0.9958	0.9961	0.9963	0.9966	0.9968	0.9970	0.9972
0.400	0.9954	0.9957	0.9959	0.9962	0.9964	0.9967	0.9969	0.9971	0.9973	0.9974	0.9976
0.380	0.9960	0.9962	0.9965	0.9967	0.9969	0.9971	0.9973	0.9975	0.9977	0.9978	0.9980
0.360	0.9965	0.9968	0.9970	0.9972	0.9974	0.9976	0.9977	0.9979	0.9980	0.9982	0.9983
0.340	0.9970	0.9972	0.9974	0.9976	0.9978	0.9979	0.9981	0.9982	0.9983	0.9984	0.9985
0.320	0.9974	0.9976	0.9978	0.9979	0.9981	0.9982	0.9983	0.9985	0.9986	0.9987	0.9988
0.300	0.9977	0.9979	0.9981	0.9982	0.9983	0.9985	0.9986	0.9987	0.9988	0.9989	0.9989
0.280	0.9980	0.9982	0.9983	0.9985	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991
0.260	0.9983	0.9984	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9992
0.240	0.9985	0.9986	0.9988	0.9989	0.9989	0.9990	0.9991	0.9992	0.9992	0.9993	0.9994
0.220	0.9987	0.9988	0.9989	0.9990	0.9991	0.9992	0.9992	0.9993	0.9994	0.9994	0.9995
0.200	0.9989	0.9990	0.9991	0.9991	0.9992	0.9993	0.9993	0.9994	0.9994	0.9995	0.9995
0.180	0.9990	0.9991	0.9992	0.9993	0.9993	0.9994	0.9994	0.9995	0.9995	0.9996	0.9996
0.160	0.9992	0.9992	0.9993	0.9994	0.9994	0.9995	0.9995	0.9996	0.9996	0.9996	0.9997
0.140	0.9993	0.9993	0.9994	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997
0.120	0.9994	0.9994	0.9995	0.9995	0.9996	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998
0.100	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998

UNCLASSIFIED

	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	8.0
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.985	0.3700	0.3722	0.3744	0.3766	0.3787	0.3809	0.3831	0.3851	0.3873	0.3893	0.3913
0.990	0.3059	0.3078	0.3109	0.3135	0.3155	0.3175	0.3195	0.3215	0.3235	0.3253	0.3273
0.995	0.2192	0.2203	0.2218	0.2232	0.2245	0.2259	0.2272	0.2285	0.2299	0.2312	0.2325
0.998	0.1396	0.1403	0.1414	0.1423	0.1432	0.1446	0.1458	0.1466	0.1473	0.1483	0.1493
0.999	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.940	0.6646	0.6677	0.6708	0.6738	0.6768	0.6798	0.6827	0.6856	0.6884	0.6913	0.6940
0.945	0.4219	0.4244	0.4264	0.4282	0.4316	0.4346	0.4364	0.4384	0.4404	0.4423	0.4436
0.950	0.4660	0.4686	0.4722	0.4738	0.4744	0.4759	0.4774	0.4789	0.4804	0.4819	0.4825
0.955	0.5043	0.5070	0.5098	0.5125	0.5142	0.5160	0.5179	0.5195	0.5212	0.5228	0.5240
0.960	0.5685	0.5714	0.5744	0.5773	0.5794	0.5820	0.5841	0.5861	0.5881	0.5901	0.5920
0.965	0.6208	0.6239	0.6269	0.6299	0.6329	0.6358	0.6387	0.6416	0.6444	0.6472	0.6500
0.970	0.7618	0.7648	0.7678	0.7707	0.7736	0.7765	0.7793	0.7820	0.7847	0.7874	0.7903
0.975	0.7018	0.7049	0.7080	0.7120	0.7170	0.7210	0.7258	0.7296	0.7334	0.7372	0.7412
0.980	0.8267	0.8293	0.8322	0.8349	0.8375	0.8403	0.8426	0.8451	0.8479	0.8500	0.8522
0.984	0.8841	0.8864	0.8887	0.8907	0.8931	0.8953	0.8974	0.8994	0.9014	0.9034	0.9053
0.988	0.8587	0.8612	0.8637	0.8662	0.8686	0.8709	0.8732	0.8755	0.8777	0.8799	0.8820
0.992	0.9041	0.9067	0.9087	0.9108	0.9128	0.9148	0.9168	0.9188	0.9208	0.9229	0.9250
0.996	0.9457	0.9473	0.9488	0.9502	0.9516	0.9530	0.9543	0.9556	0.9568	0.9581	0.9592
0.999	0.9347	0.9364	0.9381	0.9393	0.9413	0.9428	0.9443	0.9458	0.9472	0.9485	0.9499
0.9995	0.9450	0.9465	0.9478	0.9491	0.9505	0.9518	0.9531	0.9545	0.9558	0.9569	0.9581
0.9999	0.9945	0.9954	0.9964	0.9971	0.9978	0.9984	0.9989	0.9993	0.9996	0.9998	0.9999
0.99995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
0.99999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

A

X

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APPENDIX B

THE CONDITIONAL EXPECTED VALUES, GIVEN A , OF

$$\cos(\varphi_{\vec{n}} + \varphi_{\vec{k}} + \varphi_{-\vec{n}-\vec{k}}), \cos^2(\varphi_{\vec{n}} + \varphi_{\vec{k}} + \varphi_{-\vec{n}-\vec{k}})$$

$$\sin^2(\varphi_{\vec{n}} + \varphi_{\vec{k}} + \varphi_{-\vec{n}-\vec{k}}),$$

AND VARIANCES OF

$$\cos(\varphi_{\vec{n}} + \varphi_{\vec{k}} + \varphi_{-\vec{n}-\vec{k}}), \text{ AND } \sin(\varphi_{\vec{n}} + \varphi_{\vec{k}} + \varphi_{-\vec{n}-\vec{k}})$$

A	E(cos Y)	E(cos ² Y)	E(sin ² Y)	Var(cos Y)	Var(sin Y)
0,00	0.000000	0.500000	0.500000	0.500000	0.500000
0,10	0.049938	0.500624	0.499376	0.498130	0.499376
0,20	0.099503	0.502483	0.497517	0.492583	0.497517
0,30	0.148337	0.505542	0.494458	0.483538	0.494458
0,40	0.196104	0.509740	0.490260	0.471284	0.490260
0,50	0.242500	0.515001	0.484999	0.456195	0.484999
0,60	0.287263	0.521229	0.478771	0.438709	0.478771
0,70	0.330177	0.528318	0.471682	0.419301	0.471682
0,80	0.371075	0.536156	0.463844	0.398459	0.463844
0,90	0.409837	0.544625	0.455373	0.376659	0.455375
1,00	0.446390	0.553610	0.446390	0.354346	0.446390
1,10	0.480703	0.562997	0.437003	0.331922	0.437003
1,20	0.512782	0.572681	0.427319	0.309734	0.427319
1,30	0.542668	0.582563	0.417437	0.288075	0.417437
1,40	0.570423	0.592555	0.407445	0.267173	0.407445
1,50	0.596133	0.602578	0.397422	0.247203	0.397422
1,60	0.619899	0.612563	0.387437	0.228289	0.387437
1,70	0.641829	0.622454	0.377546	0.210509	0.377546
1,80	0.662040	0.632200	0.367800	0.193904	0.367800
1,90	0.680649	0.641764	0.358236	0.178481	0.358236
2,00	0.697775	0.651113	0.348887	0.164223	0.348887
2,10	0.713531	0.660223	0.339777	0.151096	0.339777
2,20	0.728030	0.669077	0.330923	0.139050	0.330923
2,30	0.741375	0.677663	0.322337	0.128025	0.322337
2,40	0.753667	0.685972	0.314028	0.117958	0.314028
2,50	0.764997	0.694001	0.305999	0.108781	0.305999
2,60	0.775451	0.701750	0.298250	0.100426	0.298250
2,70	0.785107	0.709220	0.290780	0.092826	0.290780
2,80	0.794039	0.716415	0.283585	0.085917	0.283585
2,90	0.802312	0.723341	0.276659	0.079637	0.276659
3,00	0.809985	0.730005	0.269995	0.073929	0.269995
3,10	0.817115	0.736415	0.263585	0.068738	0.263585
3,20	0.823749	0.742579	0.257421	0.064017	0.257421
3,30	0.829932	0.748505	0.251495	0.059718	0.251495
3,40	0.835705	0.754204	0.245796	0.055802	0.245796
3,50	0.841104	0.759685	0.240315	0.052229	0.240315
3,60	0.846161	0.764955	0.235045	0.048967	0.235045
3,70	0.850906	0.770025	0.229975	0.045984	0.229975
3,80	0.855366	0.774904	0.225096	0.043252	0.225096
3,90	0.859564	0.779599	0.220401	0.040748	0.220401
4,00	0.863523	0.784119	0.215881	0.038448	0.215881
4,10	0.867260	0.788473	0.211527	0.036333	0.211527
4,20	0.870795	0.792668	0.207332	0.034384	0.207332
4,30	0.874142	0.796711	0.203289	0.032587	0.203289
4,40	0.877317	0.800610	0.199390	0.030925	0.199390
4,50	0.880331	0.804371	0.195629	0.029388	0.195629
4,60	0.883198	0.808000	0.192000	0.027962	0.192000
4,70	0.885927	0.811505	0.188495	0.026638	0.188495
4,80	0.888529	0.814890	0.185110	0.025407	0.185110
4,90	0.891011	0.818161	0.181839	0.024260	0.181839
5,00	0.893383	0.821323	0.178677	0.023190	0.178677

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A	E(cos Y)	E(cos ² Y)	E(sin ² Y)	Var(cos Y)	Var(sin Y)
5,10	0.895652	0.824382	0.175618	0.022190	0.175618
5,20	0.897823	0.827342	0.172658	0.021255	0.172658
5,30	0.899905	0.830207	0.169793	0.020379	0.169793
5,40	0.901901	0.832981	0.167019	0.019556	0.167019
5,50	0.903817	0.835670	0.164330	0.018784	0.164330
5,60	0.905659	0.838275	0.161725	0.018057	0.161725
5,70	0.907430	0.840802	0.159198	0.017372	0.159198
5,80	0.909135	0.843253	0.156747	0.016727	0.156747
5,90	0.910777	0.845631	0.154369	0.016117	0.154369
6,00	0.912359	0.847940	0.152060	0.015541	0.152060
6,10	0.913886	0.850183	0.149817	0.014995	0.149817
6,20	0.915359	0.852361	0.147639	0.014479	0.147639
6,30	0.916782	0.854479	0.145521	0.013989	0.145521
6,40	0.918158	0.856538	0.143462	0.013524	0.143462
6,50	0.919488	0.858540	0.141460	0.013082	0.141460
6,60	0.920775	0.860489	0.139511	0.012662	0.139511
6,70	0.922021	0.862385	0.137615	0.012262	0.137615
6,80	0.923228	0.864231	0.135769	0.011881	0.135769
6,90	0.924398	0.866029	0.133971	0.011518	0.133971
7,00	0.925532	0.867781	0.132219	0.011171	0.132219
7,10	0.926633	0.869488	0.130512	0.010840	0.130512
7,20	0.927701	0.871153	0.128847	0.010524	0.128847
7,30	0.928738	0.872776	0.127224	0.010222	0.127224
7,40	0.929745	0.874359	0.125641	0.009932	0.125641
7,50	0.930725	0.875903	0.124097	0.009655	0.124097
7,60	0.931677	0.877411	0.122589	0.009389	0.122589
7,70	0.932603	0.878883	0.121117	0.009134	0.121117
7,80	0.933504	0.880320	0.119680	0.008890	0.119680
7,90	0.934381	0.881724	0.118276	0.008655	0.118276
8,00	0.935235	0.883096	0.116904	0.008430	0.116904
8,10	0.936068	0.884436	0.115564	0.008214	0.115564
8,20	0.936878	0.885747	0.114253	0.008005	0.114253
8,30	0.937669	0.887028	0.112972	0.007805	0.112972
8,40	0.938440	0.888281	0.111719	0.007612	0.111719
8,50	0.939192	0.889507	0.110493	0.007426	0.110493
8,60	0.939925	0.890706	0.109294	0.007247	0.109294
8,70	0.940641	0.891880	0.108120	0.007075	0.108120
8,80	0.941340	0.893030	0.106970	0.006908	0.106970
8,90	0.942023	0.894155	0.105845	0.006747	0.105845
9,00	0.942690	0.895257	0.104743	0.006592	0.104743
9,10	0.943342	0.896336	0.103664	0.006443	0.103664
9,20	0.943979	0.897394	0.102606	0.006298	0.102606
9,30	0.944601	0.898430	0.101570	0.006158	0.101570
9,40	0.945210	0.899446	0.100554	0.006023	0.100554
9,50	0.945806	0.900441	0.099559	0.005892	0.099559
9,60	0.946389	0.901418	0.098582	0.005766	0.098582
9,70	0.946959	0.902375	0.097625	0.005643	0.097625
9,80	0.947518	0.903315	0.096685	0.005525	0.096685
9,90	0.948064	0.904236	0.095764	0.005410	0.095764
10,00	0.948600	0.905140	0.094860	0.005298	0.094860

A	E(cos Y)	E(cos ² Y)	E(sin ² Y)	Var(cos Y)	Var(sin Y)
10.10	0.949124	0.906027	0.093973	0.005190	0.093973
10.20	0.949638	0.906898	0.093102	0.005084	0.093102
10.30	0.950142	0.907753	0.092247	0.004984	0.092247
10.40	0.950635	0.908593	0.091407	0.004886	0.091407
10.50	0.951119	0.909417	0.090583	0.004790	0.090583
10.60	0.951593	0.910227	0.089773	0.004698	0.089773
10.70	0.952058	0.911023	0.088977	0.004607	0.088977
10.80	0.952515	0.911804	0.088196	0.004520	0.088196
10.90	0.952962	0.912572	0.087428	0.004435	0.087428
11.00	0.953402	0.913327	0.086673	0.004352	0.086673
11.10	0.953833	0.914069	0.085931	0.004272	0.085931
11.20	0.954256	0.914799	0.085201	0.004194	0.085201
11.30	0.954672	0.915516	0.084484	0.004118	0.084484
11.40	0.955080	0.916221	0.083779	0.004044	0.083779
11.50	0.955481	0.916915	0.083085	0.003972	0.083085
11.60	0.955874	0.917597	0.082403	0.003902	0.082403
11.70	0.956261	0.918268	0.081732	0.003833	0.081732
11.80	0.956641	0.918929	0.081071	0.003767	0.081071
11.90	0.957014	0.919579	0.080421	0.003702	0.080421
12.00	0.957381	0.920218	0.079782	0.003639	0.079782
12.10	0.957742	0.920848	0.079152	0.003578	0.079152
12.20	0.958097	0.921467	0.078533	0.003518	0.078533
12.30	0.958446	0.922078	0.077922	0.003459	0.077922
12.40	0.958789	0.922678	0.077322	0.003402	0.077322
12.50	0.959126	0.923270	0.076730	0.003347	0.076730
12.60	0.959458	0.923853	0.076147	0.003292	0.076147
12.70	0.959785	0.924426	0.075574	0.003239	0.075574
12.80	0.960106	0.924992	0.075008	0.003188	0.075008
12.90	0.960422	0.925549	0.074451	0.003137	0.074451
13.00	0.960734	0.926097	0.073903	0.003088	0.073903
13.10	0.961040	0.926638	0.073362	0.003040	0.073362
13.20	0.961342	0.927171	0.072829	0.002993	0.072829
13.30	0.961639	0.927696	0.072304	0.002947	0.072304
13.40	0.961931	0.928214	0.071786	0.002902	0.071786
13.50	0.962219	0.928724	0.071276	0.002859	0.071276
13.60	0.962503	0.929228	0.070772	0.002816	0.070772
13.70	0.962782	0.929724	0.070276	0.002774	0.070276
13.80	0.963058	0.930213	0.069787	0.002733	0.069787
13.90	0.963329	0.930696	0.069304	0.002693	0.069304
14.00	0.963596	0.931172	0.068828	0.002654	0.068828
14.10	0.963860	0.931641	0.068359	0.002615	0.068359
14.20	0.964119	0.932104	0.067896	0.002578	0.067896
14.30	0.964375	0.932561	0.067439	0.002541	0.067439
14.40	0.964628	0.933012	0.066988	0.002505	0.066988
14.50	0.964877	0.933457	0.066543	0.002470	0.066543
14.60	0.965122	0.933896	0.066104	0.002436	0.066104
14.70	0.965364	0.934329	0.065671	0.002402	0.065671
14.80	0.965602	0.934757	0.065243	0.002369	0.065243
14.90	0.965837	0.935179	0.064821	0.002337	0.064821
15.00	0.966070	0.935595	0.064405	0.002305	0.064405

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13. ABSTRACT

New methods for solving complex noncentrosymmetric crystal structures have been developed at NRL. A vital part of the method involves evaluating the structure invariants $\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3})$, where $\vec{h}_1 + \vec{h}_2 + \vec{h}_3 = 0$, directly from the normalized structure factor amplitudes. The theoretical conditional probability distribution of $\cos(\varphi_{\vec{h}_1} + \varphi_{\vec{h}_2} + \varphi_{\vec{h}_3})$, given

$$A = \frac{2}{N^{1/2}} |E_{\vec{h}_1} E_{\vec{h}_2} E_{\vec{h}_3}|,$$

and the theoretical expected values of the structure invariants are essential to this computation. This report tabulates the conditional distribution and the expectations.

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