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Pulse Compression Degradation Due to Open Loop Adaptive Cancellation, Part II

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An exact expression for the perturbed sidelobe level of a compressed pulse that has been preprocessed through an adaptive canceller has been derived in Part I. This exact expression requires computer assistance to be evaluated. This report derives a "rule of thumb" expression that is a good approximation to the exact expression. Furthermore, this same approximation can be used to derive a good approximation for the canceller noise power level that is induced by having a desired signal present in the canceller weight calculation. An expression for the number of independent samples necessary to equalize the signal-induced power with the quiescent interference level is derived.

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PULSE COMPRESSION DEGRADATION DUE TO OPEN LOOP ADAPTIVE CANCELLATION, PART II

1. INTRODUCTION

An exact expression for the perturbed sidelobe level of a compressed pulse that has been preprocessed through an adaptive canceller was derived in Ref. 1. The pertinent assumptions of that analysis are:

1. the adaptive canceller was implemented by using the Sampled Matrix Inversion (SMI) algorithm [2] or its equivalent, the Gram-Schmidt canceller [3],

2. the input noises were temporally independent and Gaussian,

3. the desired signal's input vector (or code) was completely contained within the samples that were used to calculate the adaptive weights and is only present in the main channel, and

4. the adaptive weights were computed from the same data set to which they are applied (concurrent processing).

Earlier research has shown that because of finite sampling, the quiescent compressed pulse sidelobe levels are degraded by preprocessing the main channel input data stream (the uncompressed pulse) through the adaptive canceller. It was also shown that the level of degradation is independent of whether pulse compression occurs before or after the adaptive canceller under assumption three.

The exact expression [1] for pulse compression degradation requires computer assistance to evaluate this expression. This report derives a "rule of thumb" expression that is a good approximation to the exact expression.

2. BACKGROUND

Figure 1 is a functional block diagram of an adaptive canceller followed by a pulse compressor. The adaptive canceller linearly weights the auxiliary channels with weights that are calculated from a batch of sampled input data. The main channel consists of a desired signal plus noise that may or may not be correlated with the auxiliary channels. It was shown in Ref. 1 that when analyzing the pulse compression degradation it is necessary only to consider the interaction of the main channel's desired signal with the random variables in the auxiliary channels (Fig. 1). Thus for analysis purposes, the adaptive weights of x_n, $n = 1, 2, \ldots, N-1$ are only a function of the desired signal s and the samples of x_n. Furthermore, as the number of independent samples goes to infinity, the auxiliary adaptive weights go to zero [1].

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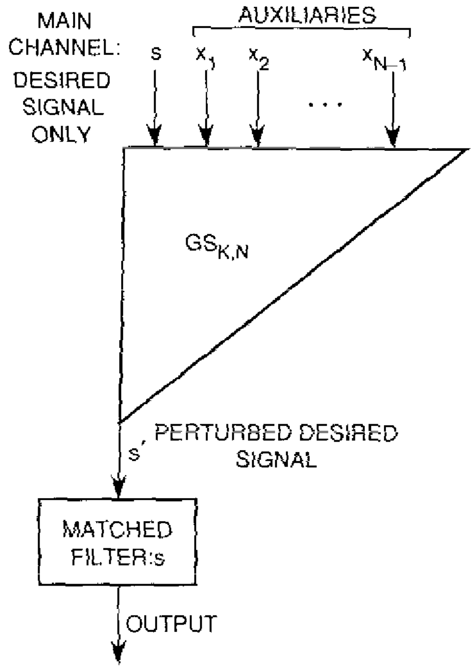

Fig. 1 — GS canceller followed by a matched filter

In Fig. 1, s represents the desired signal vector (or code) of length L, and $\mathbf{x}_n$, $n = 1, 2, \ldots, N - 1$ represents the nth auxiliary random data vector of length K. The canceller shown is the Gram-Schmidt (GS), which is numerically equivalent to the SMI algorithm [3]. We denote it by $GS_{K,N}$ where K is the number of samples per channel used to calculate the canceller weights and N is the number of input channels (main and auxiliaries).

The pulse compressor is essentially the matched filter for a given radar waveform. Most of the energy in the received radar waveform is compressed into a given single range cell and, thus, the signal level can be increased significantly for detection purposes. However, some energy does leak into the sidelobes of the compressed pulse response, resulting in low gain in range cells outside of the given range cell. If a target or piece of clutter is large enough, it can break through and be detected in these range sidelobes, falsely indicating a target detection or masking a real target. Thus it is highly desirable to maintain a low sidelobe response.

Let $\mathbf{r}$ equal the $2L - 1$ output vector of the pulse compressor. If an adaptive canceller is not being used, then it is straightforward to show that

$$\mathbf{r} = S^t \mathbf{s} \tag{1}$$

where

$$\mathbf{s} = (s_1, s_2, \ldots, s_L)^T,$$

$$S^T = \begin{bmatrix} s_L & 0 & 0 & \cdots & 0 \\ s_{L-1} & s_L & 0 & \cdots & 0 \\ s_{L-2} & s_{L-1} & s_L & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ s_1 & s_2 & s_3 & \cdots & s_L \\ 0 & s_1 & s_2 & \cdots & s_{L-1} \\ 0 & 0 & s_1 & \cdots & s_{L-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & s_1 \end{bmatrix} \tag{2}$$

and T,t denotes transpose and complex conjugate transpose, respectively. S is a $L \times (2L - 1)$ matrix called the autocorrelation function (ACF) matrix of $\mathbf{s}$. If $L < K$, we define an augmented signal vector $\mathbf{s}_{aug}$ of length K such that the first L elements are elements of $\mathbf{s}$ and the remaining elements are zero. S_{aug} is defined as the augmented $(2K - 1) \times (2K - 1)$ ACF matrix of $\mathbf{s}$ using the elements of $\mathbf{s}_{aug}$. The quantity $\mathbf{r}_{aug}$ is defined as the augmented $2K - 1$ output vector of the pulse compressor. Thus

$$\mathbf{r}_{aug} = S^t_{aug}\mathbf{s}_{aug}. \tag{3}$$

Let $\mathbf{s}'$ be the resultant output vector after $\mathbf{s}$ has been processed through the GS canceller and $\mathbf{s}'_{aug}$ be the resultant augmented GS output vector. This resultant output vector is then inputted to the matched filter of the vector $\mathbf{s}$, or equivalently, $\mathbf{s}_{aug}$. If we set $\mathbf{r}'$ equal to the response of $\mathbf{s}'_{aug}$ match filtered with $\mathbf{s}_{aug}$ then

$$\mathbf{r}' = S^t_{aug}\mathbf{s}'_{aug}. \tag{4}$$

In Ref. 1 it was shown under assumptions 1 through 4 (given in Section 1) that the average pulse compressed sidelobe level after adaptive cancellation is given by

$$SL_a(l) = \frac{K(K+1)A_{11}(K,N)}{(K-N+1)(K-N+2)}\, SL_q(l) + \frac{K(K+1)}{(K-N+1)(K-N+2)} \cdot A_{12}(K,N)\|\mathbf{s}_c(l)\|^2, \tag{5}$$

where

$SL_a(l)$ is the average pulse compressed sidelobe level after adaptive cancellation of the lth range sidelobe (sidelobes are numbered $\pm l$, $l = 1, 2, \ldots,$ these can be related directly to the elements of $\mathbf{r}'$; for example, $l = \pm 1$ are the sidelobes adjacent to the match point).

$SL_q(l)$ is the quiescent pulse compressed sidelobe level of the lth sidelobe ($K = \infty$ or equivalently no adaptive cancellation before pulse compression, these can be related directly to the elements of $\mathbf{r}$)

K is the number of independent samples per channel used to calculate the adaptive canceller weights

N is the number of channel (main and auxiliaries)

$\mathbf{s}_c(l)$ is the $K-l$th column of the augment AFC matrix S_{aug}, $l \neq K$, and

$\|\mathbf{s}_c(l)\|^2 = \mathbf{s}_c^t(l)\ \mathbf{s}_c(l)$

Note that $SL_a(l)$ and $SL_q(l)$ are normalized to the mainlobe pulse compression gain (adapted or quiescent, respectively) which is set equal to one or 0 dB.

The scalars $A_{11}(K,N)$ and $A_{12}(K,N)$ are computed as follows. Consider the two parallel adaptive cancellers shown in Fig. 2. Define

$\mathbf{u}_0, \mathbf{v}_0$ are arbitrary K-length main channel input vectors,

$\mathbf{u}_N, \mathbf{v}_N$ are K-length main channel output vector, and

$\mathbf{x}_n = (x_n(1), x_n(2), \ldots, x_n(K))^T$, $n = 1, 2, \ldots, N-1$, K-length random data vector of the nth auxiliary channel.

The elements of $\mathbf{x}_n$, $n = 1, 2, \ldots, N-1$, are assumed to have the following characteristics:

1. $x_n(k)$, $n = 1, \ldots, N-1$, $k = 1, \ldots, K$ are identically distributed circular Gaussian complex random variables (r.v.)

2. $E\{x_n(k)\} = 0$, $E\{|x_n(k)|^2\} = 1$, where $E\{\cdot\}$ denotes expectation and $|\cdot|$ denotes magnitude

3. $E\{x_{n_1}(k_1)x_{n_2}^*(k_2)\} = 0$ unless $n_1 = n_2$ and $k_1 = k_2$, where * denotes complex conjugate.

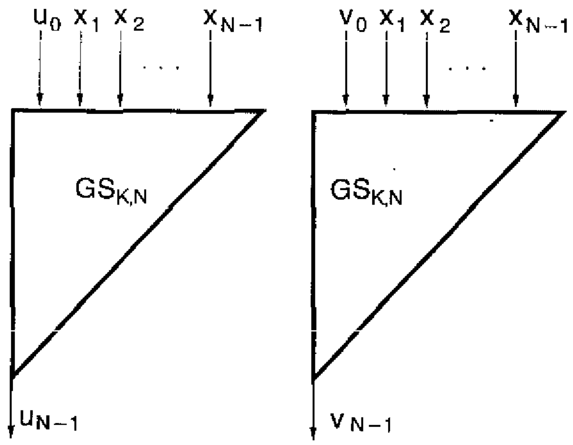

Fig. 2 — Parallel N-input GS cancellers

Define

$$a_n = 1 - \frac{2}{K - n} + \frac{1}{(K - n)(K - n + 1)}, \; n = 0, 1, \ldots, N - 2 \tag{6}$$

$$b_n = \frac{1}{(K - n)(K - n + 1)}. \tag{7}$$

It is shown in Ref. 1 that

$$\begin{bmatrix} E\{|\mathbf{u}_{N-1}^t \mathbf{v}_{N-1}|^2\} \\ E\{\|\mathbf{u}_{N-1}\|^2 \|\mathbf{v}_{N-1}\|^2\} \end{bmatrix} = \begin{bmatrix} A_{11}(K,N) & A_{12}(K,N) \\ A_{21}(K,N) & A_{22}(K,N) \end{bmatrix} \begin{bmatrix} |\mathbf{u}_0^t \mathbf{v}_0|^2 \\ \|\mathbf{u}_0\|^2 \|\mathbf{v}_0\|^2 \end{bmatrix} \tag{8}$$

where

$$\begin{bmatrix} A_{11}(K,N) & A_{12}(K,N) \\ A_{21}(K,N) & A_{22}(K,N) \end{bmatrix} = \prod_{n=0}^{N-2} \begin{bmatrix} a_n & b_n \\ b_n & a_n \end{bmatrix}. \tag{9}$$

Equations (8) and (9) resulted from solving the following coupled recursive relationships that were derived in Ref. 1:

$$E\{|\mathbf{u}_{n+1}^t \mathbf{v}_{n+1}|^2\} = E\{|\mathbf{u}_n^t \mathbf{v}_n|^2\} \left[1 - \frac{2}{K - n} + \frac{1}{(K - n)(K - n + 1)}\right]$$
$$+ E\{\|\mathbf{u}_n\|^2 \|\mathbf{v}_n\|^2\} \left[\frac{1}{(K - n)(K - n + 1)}\right] \tag{10}$$

$$E\{\|\mathbf{u}_{n+1}\|^2\|\mathbf{v}_{n+1}\|^2\} = E\{|\mathbf{u}_n^t\mathbf{v}_n|^2\}\left[\frac{1}{(K-n)(K-n+1)}\right]$$

$$+ E\{\|\mathbf{u}_n\|^2\|\mathbf{v}_n\|^2\}\left[1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right] \qquad (11)$$

where $n = 0, 1, \ldots, N-1$.

3. BOUND DERIVATION

The expression derived for SL_a given by Eq. (5), although exact, does not readily indicate how the adaptive sidelobe level varies with N and K. In this section we derive a tight upper bound on SL_a that is in terms of explicit expressions of K and N.

This bound is obtained by considering Eqs. (10) and (11). Instead of deriving a recursive relationship for $E\{\|\mathbf{u}_n\|^2\|\mathbf{v}_n\|^2\}$ in this equation, we upper-bound this expectation by using the inequality [4]

$$E\{\|\mathbf{u}_n\|^2\|\mathbf{v}_n\|^2\} \leq \sqrt{E\{\|\mathbf{u}_n\|^4\}\, E\{\|\mathbf{v}_n\|^4\}}. \qquad (12)$$

This inequality is merely another form of the Cauchy-Schwartz inequality. It allows us to upper-bound the joint moment in terms of moments of individual random variables.

It was shown in Ref. 3 that

$$E\{\|\mathbf{u}_n\|^4\} = \|\mathbf{u}_0\|^4 \,\frac{(K-n)(K-n+1)}{K(K+1)} \qquad (13)$$

and

$$E\{\|\mathbf{v}_n\|^4\} = \|\mathbf{v}_0\|^4 \,\frac{(K-n)(K-n+1)}{K(K+1)}. \qquad (14)$$

Substituting these expressions into Eq. (12) results in

$$E\{\|\mathbf{u}_n\|^2\|\mathbf{v}_n\|^2\} \leq \|\mathbf{u}_0\|^2\|\mathbf{v}_0\|^2 \,\frac{(K-n)(K-n+1)}{K(K+1)}. \qquad (15)$$

Substituting Eq. (15) into Eq. (10) results in

$$E\{|\mathbf{u}_{n+1}^t\mathbf{v}_{n+1}|^2\} \leq E\{|\mathbf{u}_n^t\mathbf{v}_n|^2\}\left\{1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right\}$$

$$+ \|\mathbf{u}_0\|^2\|\mathbf{v}_0\|^2 \,\frac{1}{K(K+1)}. \qquad (16)$$

It is apparent from Eq. (16) that $E\{|\mathbf{u}_n^t\mathbf{v}_n|^2\}$, $n = 0, 1, \ldots, N-1$ can be upper-bounded by ω_n where ω_n is found by the recursive relationship

$$\omega_{n+1} = \left[1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right]\omega_n + \frac{\|\mathbf{u}_0\|^2\|\mathbf{v}_0\|^2}{K(K+1)} \tag{17}$$

$$\text{Initial condition (IC)} \quad \omega_0 = |\mathbf{u}_0^t\mathbf{v}_0|^2.$$

We can show that ω_{N-1} has the form

$$\omega_{N-1} = \left[\prod_{n=0}^{N-2}\left(1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right)\right]|\mathbf{u}_0^t\mathbf{v}_0|^2$$

$$+ c\,\|\mathbf{u}_0\|^2\|\mathbf{v}_0\|^2, \tag{18}$$

where c is a constant to be determined. In fact, c does not depend on the initial condition, a fact that we use to find c. For $\mathbf{u}_0 = \mathbf{v}_0$, it follows from Eq. (13) that

$$\omega_{N-1} = E\{\|\mathbf{u}_{N-1}\|^4\} = \left[1 - \frac{2(N-1)}{K} + \frac{(N-1)N}{K(K+1)}\right]\|\mathbf{u}_0\|^4. \tag{19}$$

Substituting Eq. (19) into Eq. (18) and solving for c,

$$c = 1 - \frac{2(N-1)}{K} + \frac{(N-1)N}{K(K+1)} - \prod_{n=0}^{N-2}\left[1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right]. \tag{20}$$

It is shown in the Appendix that

$$\prod_{n=0}^{N-2}\left[1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right] < \left[1 - \frac{N-1}{K}\right]^2 \tag{21}$$

and

$$c < \frac{(K-N+2)(N-1)}{K(K-1)(K+1)}. \tag{22}$$

Thus inserting these inequalities into Eq. (19) results in

$$E\{|\mathbf{u}_{N-1}^t\mathbf{v}_{N-1}|^2\} < \left[1 - \frac{N-1}{K}\right]^2|\mathbf{u}_0^t\mathbf{v}_0|^2 + \frac{(K-N+2)(N-1)}{K(K-1)(K+1)}\|\mathbf{u}_0\|^2\|\mathbf{v}_0\|^2. \tag{23}$$

To find a bound on the adaptive compressed sidelobe level as was done in Ref. 1, we set $\mathbf{u}_0 = \mathbf{s}_{aug}$ and $\mathbf{v}_0 = \mathbf{s}_c$ where $\mathbf{s}_{aug}$ is the augmented K-length signal vector and $\mathbf{s}_c$ is a column of the augmented signal matrix. We note that $\|\mathbf{s}\|^2 = 1$ and that $SL_q = |\mathbf{s}_c^t\mathbf{s}|^2$. It was shown in Ref. 1 that the expected value of the match point of the compressed pulse preprocessed by a $GS_{K,N}$ canceller is

$$E\{|\mathbf{s}^{\prime t}\mathbf{s}|^2\} = \frac{(K-N+1)(K-N+2)}{K(K+1)}. \tag{24}$$

Thus if we divide both sides of Eq. (23) by the expected value of the match point we find

$$SL_a(l) < \left\{1 - \frac{N-1}{K(K-N+2)}\right\} SL_q(l) + \|\mathbf{s}_c(l)\|^2 \frac{N-1}{(K-N+1)(K-1)}. \tag{25}$$

We set

$$\tilde{Q}(K,N) = 1 - \frac{N-1}{K(K-N+2)}, \tag{26}$$

and

$$\Delta\tilde{SL}_a(K,N) = \frac{N-1}{(K-N+1)(K-1)}. \tag{27}$$

Thus

$$SL_a(l) < \tilde{Q}(K,N)SL_q(l) + \Delta\tilde{SL}_a(K,N)\|\mathbf{s}_c(l)\|^2. \tag{28}$$

Similarly, define the quiescent sidelobe level factor

$$Q(K,N) = \frac{K(K+1)A_{11}(K,N)}{(K-N+1)(K-N+2)} \tag{29}$$

and the adaptive side perturbation

$$\Delta SL_a(K,N) = \frac{K(K+1)A_{12}(K,N)}{(K-N+1)(K-N+2)} \tag{30}$$

so that Eq. (5) can be rewritten as

$$SL_a(l) = Q(K,N)SL_q(l) + \Delta SL_a(K,N)\,\|\mathbf{s}_c(l)\|^2. \tag{31}$$

4. RESULTS

We now demonstrate in graphical form that $\tilde{Q}(K,N)$ and $\Delta\tilde{SL}_a(K,N)$ are close approximations of the quiescent sidelobe level factor $Q(K,N)$ and the adaptive sidelobe perturbation $\Delta SL_a(K,N)$, respectively. Define the following ratios

$$r_Q = \frac{Q(K,N)}{\tilde{Q}(K,N)}, \tag{32}$$

and

$$r_\Delta = \frac{\Delta SL_a(K,N)}{\Delta \tilde{S}l_a(K,N)}. \tag{33}$$

We set $N_{aux} = N - 1$ and $K = MN$ where M is a positive integer and calculate r_Q and r_Δ vs N_{aux} and M. We restrict $M \geq 2$. Many cases were run ($M \leq 10$, $N \leq 100$), and the two ratios were always less than one and lower-bounded by the case when $M = 2$. Thus, we only present the curves for $M = 2$. The close approximation is verified by the plots of r_Q and r_Δ shown in Figs. 3 and 4, respectively. The worst-case approximation of $Q(K,N)$ by $\tilde{Q}(K,N)$ occurs when $N_{aux} = 1$, $M = 2$. In this case r_Q (dB) $= -1.76$ dB.

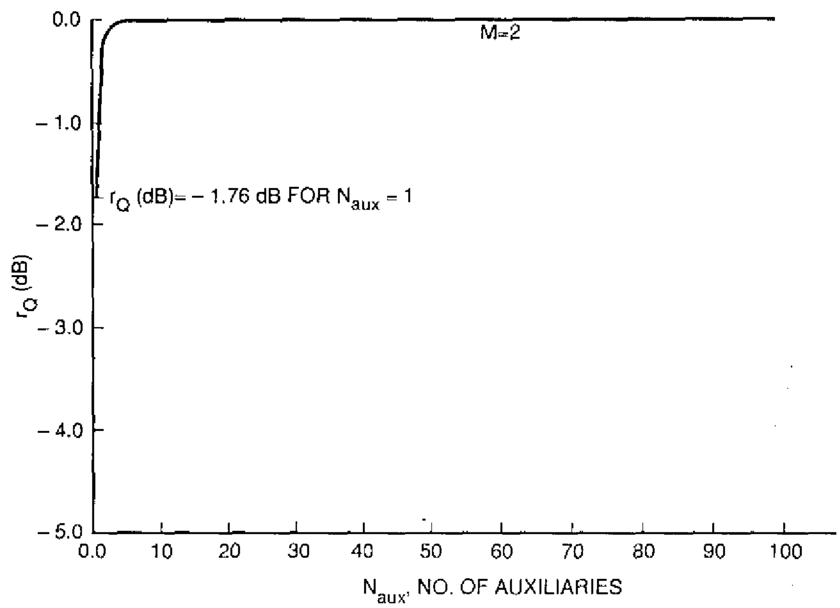

Fig. 3 — r_Q vs N_{aux}, $M = 2$

An even better approximation of $Q(K,N)$ was found by using the expression

$$\Delta \tilde{\tilde{S}}L_a(K,N) = \frac{N - 1}{(K - N + 1)K}. \tag{34}$$

Note that the difference between the expression for $\Delta\tilde{S}L_a$ given by Eqs. (34) and (27) is that the $K - 1$ is replaced by K. Define the ratio

$$r'_\Delta = \frac{\Delta SL_a(l)}{\Delta \tilde{\tilde{S}} L_a(l)}. \tag{35}$$

Figure 5 plots this ratio for $M = 2$ vs N_{aux}. Note that the worst-case approximation occurs when $N_{aux} = 2$, $M = 2$. In this case, r'_Δ (dB) $= -.51$ dB.

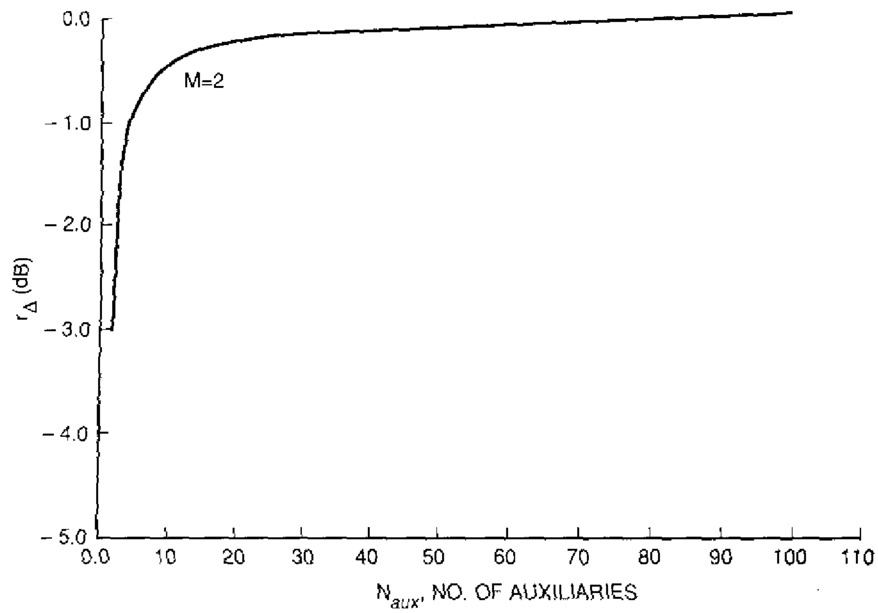

Fig. 4 — r_Δ vs N_{aux}, $M = 2$

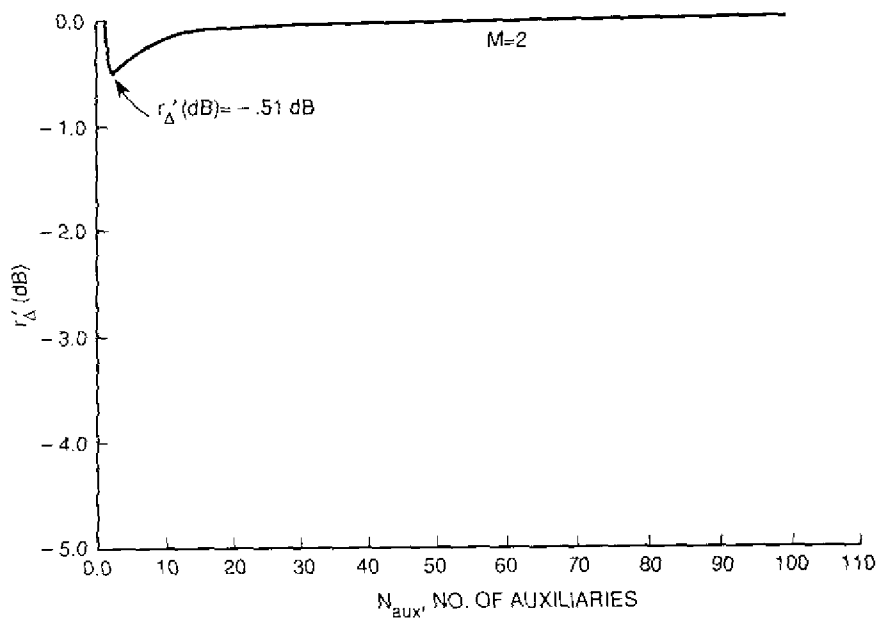

Fig. 5 — r_Δ' (dB) vs N_{aux}, $M = 2$

Because $\tilde{Q}(K, N)$ and $\Delta\tilde{SL}_a(K,N)$ are close approximations to $Q(K, N)$ and $\Delta SL_a(K,N)$ respectively, it is straightforward to show that

$$A_{11}(K,N) \doteq \left(1 - \frac{N-1}{K}\right)^2, \tag{36}$$

and

$$A_{12}(K,N) \doteq \frac{(K-N+2)(N-1)}{K(K-1)(K+1)}. \tag{37}$$

Again, if we replace $K - 1$ with K in Eq. (37), an even better approximation results:

$$A_{12}(K,N) = \frac{(K-N+2)(N-1)}{K^2(K+1)}. \tag{38}$$

Inserting the approximate expressions given by Eq. (26) for $Q(K,N)$ and Eq. (34) for $\Delta SL_a(K,N)$ into Eq. (31) results in

$$SL_a(l) \doteq \left(1 - \frac{N-1}{K(K-N+2)}\right) SL_q(l) + \frac{N-1}{(K-N+1)K} \|\mathbf{s}_c(l)\|^2. \tag{39}$$

Define $K_{3\mathrm{dB}}(l)$ to be the minimum number of independent samples such that $SL_a(l) \leq 2\overline{SL}_q$, where $\overline{SL}_q = \max\limits_{l,l \neq 0} SL_q(l)$; (i.e., the average adaptive sidelobe level at a specific range sidelobe l is at most 3 dB above the maximum quiescent sidelobe level). It is straightforward to show that

$$K_{3\mathrm{dB}}(l) \doteq \frac{N-1}{2} + \sqrt{\left(\frac{N-1}{2}\right)^2 + (N-1)\frac{\|\mathbf{s}_c(l)\|^2}{2\overline{SL}_q - SL_q(l)}} \tag{40}$$

when $SL_q(l) << 1$. The actual number of samples used to ensure that all adaptive sidelobes are below $2\overline{SL}_q$ would be

$$K_{3\mathrm{dB}} = \max_{l,l \neq 0} K_{3\mathrm{dB}}(l). \tag{41}$$

If the maximum quiescent sidelobe level occurs close to the main lobe, then $\|\mathbf{s}_c\|^2 \doteq 1$ and we find that $K_{3\mathrm{dB}}(l)$ is maximized at this maximum quiescent sidelobe level. Hence,

$$K_{3\mathrm{dB}} \doteq \frac{N-1}{2} + \sqrt{\left(\frac{N-1}{2}\right)^2 + \frac{N-1}{\overline{SL}_q}}. \tag{42}$$

Reference 1 pointed out that the pulse compression degradation analysis can be applied to quantifying the canceller degradation caused by a desired signal's presence in the samples used to calculate the adaptive weights. If the desired signal has the power σ_s^2 after pulse compression, then the average power residue caused by the signal in the $K - 1$ range bins not containing the signal can be shown to equal at most $\sigma_s^2 \Delta SL_a(K,N)$ plus possibly the signal power due to the quiescent compressed sidelobes. Let $\sigma_{\min}^2$ be the quiescent output noise power level of the canceller. Define

$$\delta(l) = \Delta SL_a(K,N) \frac{\sigma_s^2}{\sigma_{min}^2} \|s_c(l)\|^2 \tag{43}$$

and

$$\delta = \max_{l, l \neq 0} \delta(l). \tag{44}$$

One normally desires $\delta \leq 1$, otherwise the desired signal generates more range sidelobe power than the noise power residue. Because $\max_{l, l \neq 0} \|s_c\|^2 \doteq 1$ and using the good approximation for $\Delta SL_a(K,N)$ given by Eq. (34) then

$$\delta \doteq \frac{N - 1}{(K - N + 1)K} \frac{\sigma_s^2}{\sigma_{\min}^2}. \tag{45}$$

It is desirable to know the number of independent input samples K_0 such that $\delta = 1$. It can be shown that

$$K_0 = \frac{N - 1}{2} + \sqrt{\left(\frac{N - 1}{2}\right)^2 + (N - 1) \frac{\sigma_s^2}{\sigma_{\min}^2}}. \tag{46}$$

We note that $\sigma_s^2 / \sigma_{\min}^2$ equals the output signal-to-noise power ratio $(S/N)_{out}$ of the adaptive canceller. Thus Eq. (46) reduces to

$$K_0 = \frac{N - 1}{2} + \sqrt{\left(\frac{N - 1}{2}\right)^2 + (N - 1) \left(\frac{S}{N}\right)_{out}}. \tag{47}$$

5. SUMMARY

An exact expression for the perturbed sidelobe level of a compressed pulse that has been preprocessed through an adaptive canceller was derived in Ref. 1. The exact expression requires computer assistance to evaluate this expression. In this report, a "rule of thumb" expression is derived that is a good approximation to the exact expression. Furthermore, this same approximation can be used to derive a good approximation for the canceller noise power level that is induced by having a desired signal present in the canceller weight calculation. An expression for the number of independent samples necessary to equalize the signal-induced power with the quiescent interference level is also derived.

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Appendix

PROOF OF EQS. (21) AND (22)

We first prove Eq. (21). Define

$$b = \prod_{n=0}^{N-2} \left[1 - \frac{2}{K-n} + \frac{1}{(K-n)(K-n+1)}\right]$$

$$= \prod_{n=0}^{N-2} \left[1 - \frac{2}{K-n} + \frac{2}{(K-n)(K-n+1)}\right] \left[1 - \frac{1}{(K-n)(K-n-1)}\right]$$

$$= \prod_{n=0}^{N-2} \left[1 - \frac{2}{K-n} + \frac{2}{(K-n)(K-n+1)}\right] \prod_{n=0}^{N-2} \left[1 - \frac{1}{(K-n)(K-n-1)}\right] . \tag{A1}$$

Now

$$\prod_{n=0}^{N-2} \left[1 - \frac{2}{K-n} + \frac{2}{(K-n)(K-n+1)}\right] = \frac{(K-N+1)(K-N+2)}{K(K+1)}$$

$$= 1 - \frac{2(N-1)}{K} + \frac{N(N-1)}{K(K+1)}. \tag{A2}$$

Thus

$$b = \frac{(K-N+1)(K-N+2)}{K(K+1)} \prod_{n=0}^{N-2} \left[1 - \frac{1}{(K-n)(K-n-1)}\right]$$

$$< \frac{(K-N+1)(K-N+2)}{K(K+1)} \prod_{n=0}^{N-2} \left[1 - \frac{1}{(K-n)^2}\right] . \tag{A3}$$

Now

$$\prod_{n=0}^{N-2} \left[1 - \frac{1}{(K-n)^2}\right] = \frac{(K+1)(K-N+1)}{K(K-N+2)}. \tag{A4}$$

Thus

$$b < \frac{(K-N+1)^2}{K^2} = \left[1 - \frac{N-1}{K}\right]^2 . \tag{A5}$$

Next we prove Eq. (22). Set

$$a = \prod_{n=0}^{N-2} \left[1 - \frac{1}{(K - n)(K - n - 1)} \right]. \tag{A6}$$

Now

$$b = \left[1 + \frac{2(N - 1)}{K} + \frac{N(N - 1)}{K(K + 1)} \right] a \tag{A7}$$

and

$$a > \prod_{n=0}^{N-2} \left[1 - \frac{1}{(K - n - 1)^2} \right] = \frac{K(K - N)}{(K - 1)(K - N + 1)} \tag{A8}$$

or

$$1 - a < 1 - \frac{K(K - N)}{(K - 1)(K - N + 1)} = \frac{N - 1}{(K - 1)(K - N + 1)}. \tag{A9}$$

Using Eq. (A7) and Eq. (20), it can be shown that

$$c = \left\{ 1 - \frac{2(N - 1)}{K} + \frac{(N - 1)N}{K(K + 1)} \right\} (1 - a). \tag{A10}$$

Using Eq. (A9) it follows that

$$c < \left\{ 1 - \frac{2(N - 1)}{K} + \frac{(N - 1)N}{K(K + 1)} \right\} \frac{N - 1}{(K - 1)(K - N + 1)} \tag{A11}$$

or

$$c < \frac{(K - N + 2)(N - 1)}{K(K - 1)(K + 1)}. \tag{A12}$$