Naval Research Laboratory
Washington, DC 20375-5000

NRL Report 9130

Coherent Detection of Radar Targets in K-Distributed, Correlated Clutter

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Radar Division

August 5, 1988

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S): NRL Report 9130	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION: Naval Research Laboratory; 6b. OFFICE SYMBOL (If applicable): Code 5340	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code): Washington, DC 20375-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING / SPONSORING ORGANIZATION: Office of Naval Research; 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code): Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. 61153N; PROJECT NO. 021-05-43; TASK NO.; WORK UNIT ACCESSION NO. DN480-006

11. TITLE (Include Security Classification)
Coherent Detection of Radar Targets in K-Distributed, Correlated Clutter

12. PERSONAL AUTHOR(S)
Sangston, K. J.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Interim	FROM ______ TO ______	1988 August 5	20

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Detection theory; Radar; Non-Gaussian; Radar signal processing; K-Distribution

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A processor is obtained for detecting a radar target in correlated, nonhomogeneous Gaussian (K-distributed) clutter. When this processor and a matched filter are excited with nonhomogeneous Gaussian data, the performance of the new processor exceeds that of the matched filter. The new processor was obtained by approximating the Neyman-Pearson test for this problem.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: K. J. Sangston	22b. TELEPHONE (Include Area Code): (202) 767-3472; 22c. OFFICE SYMBOL: 5340.1S

DD Form 1473, JUN 86 — Previous editions are obsolete. — SECURITY CLASSIFICATION OF THIS PAGE

CONTENTS

COHERENT DETECTION OF RADAR TARGETS IN K-DISTRIBUTED, CORRELATED CLUTTER

INTRODUCTION

The coherent detection of radar targets in a background of correlated non-Gaussian clutter is a difficult problem that has recently begun to motivate active research. As a problem in detection theory, this problem is well understood. The optimum detection procedure is given by the Neyman-Pearson likelihood ratio. However, to formulate the likelihood ratio, a representation, such as a closed-form description, of the multivariate probability density function (pdf) is required. The modeling of this multivariate pdf usually is accomplished by extending knowledge about the univariate pdf and the covariance matrix associated with the process. In the absence of knowledge of the complete structure of the multivariate pdf, this extension is necessarily arbitrary and is usually motivated by the researcher's desire for mathematical tractability.

One such extension of the problem of detecting radar targets in non-Gaussian clutter has been described by Cantrell [1] and by Farina et al. [2]. The approach taken in these studies was based on the transformation noise model described by Martinez et al. [3]. This model maps a given multivariate pdf, usually Gaussian, into a non-Gaussian multivariate pdf in such a way that the resulting pdf has the required marginal pdf's and the required covariance matrix. Cantrell [4] also applied this same technique to the radar problem by starting with a bivariate complex Cauchy pdf instead of a Gaussian pdf. In each of these cases, the result was the specification, based on the likelihood ratio, of a detection scheme that yields improved detection performance over the performance obtained by processing the same data with a detector designed for Gaussian noise. Although this type of comparison is not a measure of the optimality of the non-Gaussian detector, it does reveal the benefit to be gained by correctly identifying the clutter process as non-Gaussian rather than assuming that it is Gaussian.

Other methods for constructing a multivariate pdf with specified marginals and a specified covariance matrix are available, and one of these alternate methods is used in this report. The approach followed here is motivated by physical arguments about the nature of electromagentic scattering from the surface of the sea [5] and is applicable to the problem of detecting radar targets in sea clutter. The clutter process is modeled as a nonhomogeneous Gaussian process where the clutter power level is a random variable. This model results in the so-called spherically invariant random process (SIRP), which has been described in the literature [6-11]. The problem of target detection in a SIRP has been the subject of little investigation. Yao [8] examined the case in which the detection threshold is set equal to 1. Goldman [12] examined the conditions under which correlation receivers are optimal. Picinbono and Vezzosi [13,14] examined the asymptotic detection of a weak signal in the presence of a real SIRP and found that the asymptotic detector is independent of the pdf of the unknown power level. Spooner [15] examined the performance of the optimum detector for the problem in which the form of the pdf of the power level is known and the data are comprised of independent samples. None of these results is directly applicable to this study, although Picinbono and Vezzosi did derive the form of the likelihood ratio for the problem considered here. This report examines the problem of radar target detection with a small number of samples.

Manuscript approved March 4, 1988.

In this report, the multivariate pdf that describes the non-Gaussian clutter process is obtained by averaging a multivariate complex Gaussian pdf with respect to a random variable that multiplies the normalized correlation matrix, i.e., the variance or (in the vocabulary of sea clutter researchers) the power level. The likelihood ratio is then obtained in closed form. The resulting test depends on the unknown signal amplitude and unknown initial phase. An approximation to this test that is independent of the signal amplitude and initial phase is then formulated, and the detection performace of this approximate test is evaluated for some specific examples. This detection performance is compared to the performance of the optimum test in which the signal amplitude and phase are known and to the performance of the Gaussian matched-filter detector against the same clutter data.

MULTIVARIATE PROBABILITY DENSITY FUNCTION

A univariate pdf is assumed that describes the amplitude behavior of the complex clutter process as well as the normalized correlation matrix of the clutter process. The extension of this information to the multivariate pdf proceeds in two steps. First, since the clutter process is modeled as a complex process, some assumption about the statistics of the phase of the process must be made. For radar clutter modeling, the phase of the clutter is generally assumed to be uniformly distributed. With this assumption and the assumption that the amplitude and phase of a given sample are independent, the univariate complex pdf can be immediately obtained. This pdf must then be extended to the multivariate domain.

The assumption of nonhomogeneous Gaussian clutter automatically satisfies the above-stated assumptions. By nonhomogeneous Gaussian clutter, we mean that on any given detection decision, the observation samples are drawn from a population whose statistics are described by the following Gaussian pdf:

$$f(x \mid \tau) = \frac{1}{(2\pi)^m \mid \Lambda \mid \tau^m} \exp -\frac{1}{2} \left\{ \frac{\bar{x}^t \Lambda^{-1} x}{\tau} \right\}, \tag{1}$$

where

Λ is a positive definite normalized correlation matrix,

t indicates transpose,

the bar indicates conjugate,

m is the dimension of the problem (i.e., number of pulses),

x is the received signal vector of length m, and

τ is the unknown power level that varies from one oservation to the next according to a pdf $f(\tau)$.

The resulting multivariate pdf that describes the non-Gaussian clutter process is obtained as

$$f(x) = \int_0^\infty f(x \mid \tau) f(\tau) d\tau$$

$$= \frac{1}{(2\pi)^m \mid \Lambda \mid} \int_0^\infty \frac{\left\{ \exp -\frac{1}{2} \frac{\bar{x}^t \Lambda^{-1} x}{\tau} \right\}}{\tau^m} f(\tau) d\tau \tag{2}$$

The choice of $f(\tau)$ influences the statistical behavior of this process. To ensure that this statistical behavior matches the desired univariate amplitude pdf $g(|x|)$, $f(\tau)$ is chosen as the solution to the integral equation

$$g(|x|) = \int_0^\infty g(|x| \mid \tau) f(\tau) d\tau$$

$$= \int_0^\infty \frac{|x|}{\tau} \exp\left\{\frac{-|x|^2}{2\tau}\right\} f(\tau) d\tau, \qquad (3)$$

where $g(|x| \mid \tau)$ is a Rayleigh pdf conditioned on the parameter τ. This integral equation is in general very difficult to solve. However, if $g(|x|)$ is chosen to be the so-called K distribution [16], which has been shown to provide a good fit to sea clutter amplitude statistics [17, 18], the solution to this equation can be found. It is given by the well-known gamma pdf,

$$f(\tau) = \frac{\left[\frac{\nu}{\eta}\right]^\nu}{\Gamma(\nu)} \tau^{\nu-1} \exp\left\{-\frac{\nu}{\eta}\tau\right\}, \qquad (4)$$

where $\Gamma(\cdot)$ is a gamma function, η fixes the mean of the distribution, and ν controls the skewness. Figure 1 shows curves that describe the behavior of the gamma density for different values of ν.

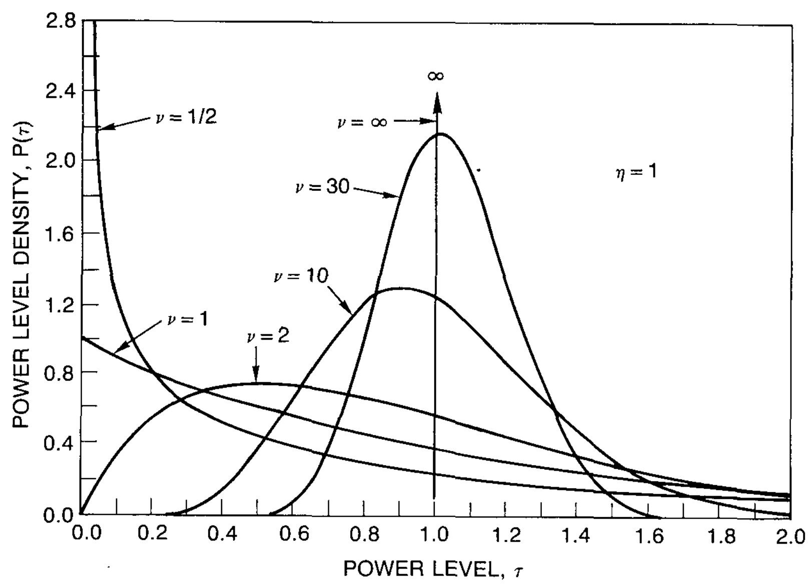


Fig. 1 — Power level probability density function

With the choice of the gamma pdf for the variation of the power level from one observation to the next, the univariate amplitude pdf is given by the K-distribution

$$g(|x|) = \frac{\frac{2\nu}{\eta}|x|}{\Gamma(\nu)} \left[\frac{|x|\sqrt{\frac{2\nu}{\eta}}}{2}\right]^{\nu-1} K_{\nu-1}\left[|x|\sqrt{\frac{2\nu}{\eta}}\right]. \qquad (5)$$

The multivariate complex pdf of the clutter process is obtained by solving the integral in Eq. (2) [19]. This is given by

$$f(x) = \frac{2\left(\frac{\nu}{\eta}\right)^{m}}{\Gamma(\nu)(2\pi)^{m}\,|\Lambda|}\left(\frac{\sqrt{\frac{2\nu}{\eta}\,\bar{x}^{t}\Lambda^{-1}x}}{2}\right)^{\nu-m} K_{\nu-m}\left(\sqrt{\frac{2\nu}{\eta}\,\bar{x}^{t}\Lambda^{-1}x}\right), \tag{6}$$

where $K_\nu(\cdot)$ is a modified Bessel function of the second kind of order ν. Figure 2 shows curves that describe the univariate amplitude pdf. The covariance matrix of the complex pdf is easily shown to be equal to $\eta\Lambda$.

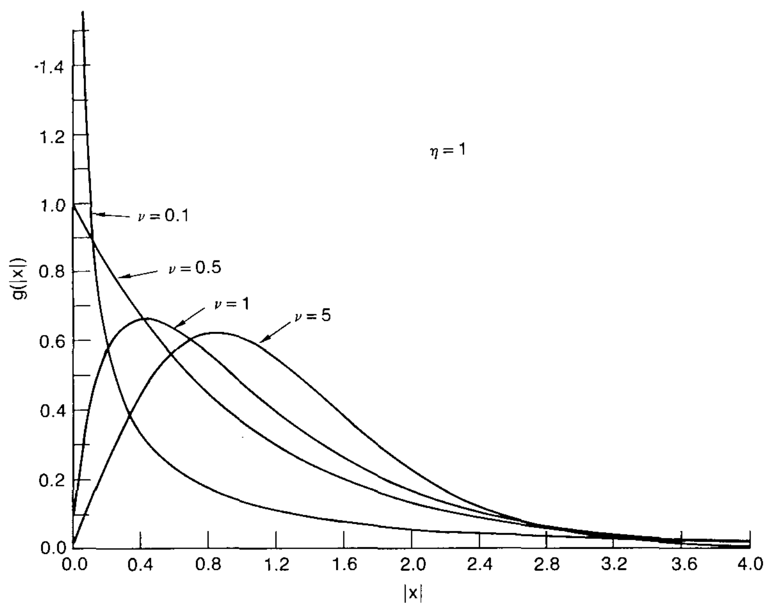


Fig. 2 — K-amplitude probability density function

NEYMAN-PEARSON TEST

The radar detection problem is defined as a problem in binary hypothesis testing [20], and the optimum detection procedure is given by the well-known Neyman-Pearson test. The problem is formulated as follows:

> Select a complex vector sample y from one of two possible statistical populations
>
> $$H_0:\ y = n$$
> $$H_1:\ y = n + s$$
>
> where n is a complex vector of length m of noise samples and s is a complex vector of length m of signal samples that is independent of n. The desired test must decide in an optimal manner if the observed sample y was caused by noise alone or by signal plus noise. A choice in favor of H_1 is said to be a detection (either true or false).

The Neyman-Pearson test is optimal in the sense that for a fixed probability of false alarm, it maximizes the probability of detection. The test is given in the form of a likelihood ratio,

$$\lambda = \frac{f(y \mid H_1)}{f(y \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} T, \tag{7}$$

where $f(y \mid H_0)$ represents the pdf of y, given hypothesis H_0, and $f(y \mid H_1)$ represents the pdf of y, given hypothesis H_1. For the specific problem studied here, the likelihood ratio is given by

$$\lambda_{NP} = \frac{\left[\sqrt{\frac{2\nu}{\eta} \overline{(y - s)}^t \Lambda^{-1} (y - s)}\right]^{\nu - m}}{\left[\sqrt{\frac{2\nu}{\eta} \bar{y}^t \Lambda^{-1} y}\right]^{\nu - m}} \cdot \frac{K_{\nu - m}\left[\sqrt{\frac{2\nu}{\eta} \overline{(y - s)}^t \Lambda^{-1} (y - s)}\right]}{K_{\nu - m}\left[\sqrt{\frac{2\nu}{\eta} \bar{y}^t \Lambda^{-1} y}\right]}. \tag{8}$$

Here the signal may be written as

$$s = \alpha e^{j\phi_s} \hat{s}, \tag{9}$$

where α is the signal amplitude, ϕ_s is the initial signal phase, and $\hat{s}$ is a complex steering vector that represents the pulse to pulse phase shift of the signal caused by the Doppler shift. This steering vector is assumed to be known; in practice, a bank of filters is built to determine the Doppler shift of the target. However, the signal amplitude and initial phase are are assumed to be unknown. As a result, the likelihood ratio as given above is not suitable as a detection procedure. The given test must be

(1) accompanied by an estimator of the unknown quantities,

(2) approximated to remove the dependency on the unknowns, or

(3) averaged over an assumed statistical distribution of the unknown quantity.

In this problem, step (3) is mathematically intractable, and a combination of steps (1) and (2) is used.

ESTIMATION OF UNKNOWN SIGNAL AMPLITUDE

The estimate of the unknown signal amplitude is based on the maximum likelihood (ML) estimator of the signal amplitude under the assumption that the signal phase is known. This ML estimate is discussed in Appendix A. The estimate is given by

$$\tilde{\alpha} = \frac{\left| s^t \Lambda^{-1} y \right|}{s^t \Lambda^{-1} s}. \tag{10}$$

Figure 3 is a histogram of the typical accuracy of this estimate against simulated data. Note that the estimate is essentially an implementation of the matched filter. It is of interest to examine the behavior of this estimate under the hypothesis of noise only. Appendix B shows that the probability of the estimate being greater than or equal to a given value (provided that the hypothesis of noise

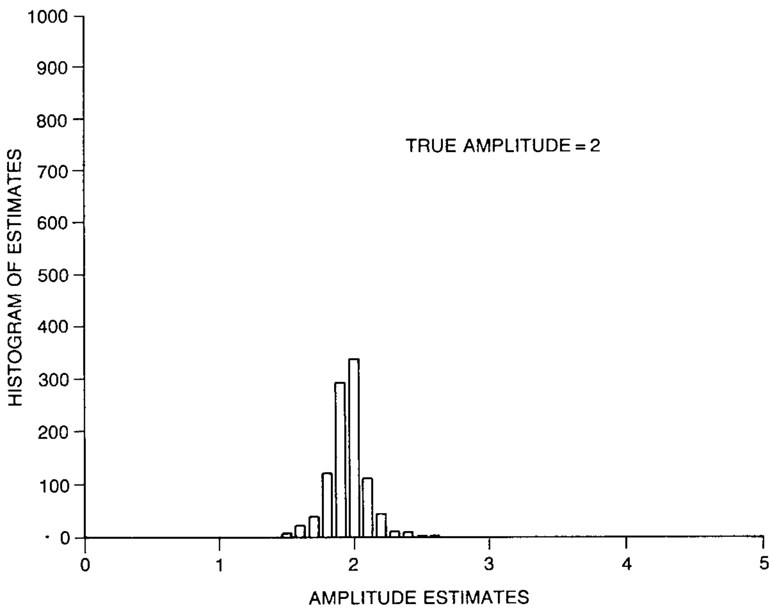


Fig. 3 — Amplitude estimates; $\nu = 1$, $\eta = 1$, $\rho = 0.98$, two pulses

alone is true) may be solved for in closed form. As a result, we may establish a threshold, say T_α, such that the probability of the amplitude estimate is greater than or equal to this threshold when the hypothesis of noise alone is true and is equal to a desired probability of false alarm. If the amplitude estimate exceeds this threshold, we can declare a detection without further processing.

APPROXIMATION OF NEYMAN-PEARSON TEST TO REMOVE DEPENDENCY ON THE SIGNAL PHASE

To remove the dependency of the likelihood ratio on the initial signal phase, the quadratic form

$$(\overline{y - s})^t \Lambda^{-1} (y - s)$$

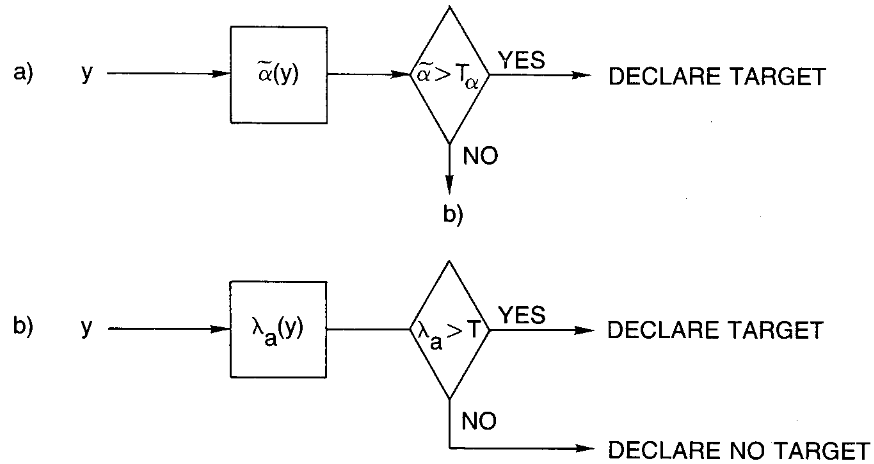
is replaced by

$$|\bar{y}^t \Lambda^{-1} y + \bar{\hat{s}}^t \Lambda^{-1} s - 2\alpha \, |\bar{\hat{s}}^t \Lambda^{-1} y| \, | \, .$$

If the quadratic form given above is expanded, the crux of this approximation is seen to be the approximation of $\mathrm{Re}(s^t \Lambda^{-1} y)$ by $|s^t \Lambda^{-1} y|$. This is the implementation of the matched filter when the signal phase is uniformly distributed.

EVALUATION OF DETECTION PERFORMANCE

At this point, the detection procedure may be evaluated. To facilitate the numerical work, the log-likelihood ratio is evaluated in each case. The log-likelihood ratio yields exactly the same detection performance as the likelihood ratio. It does however require different detection threshold settings. Figure 4 is schematic of the detection procedure. As shown, the estimate of the signal amplitude is first formulated. If this estimate exceeds a threshold, a detection is declared and processing stops. If the estimate does not exceed the threshold, the approximate log-likelihood ratio test is formulated and a detection decision based on it is made.



$$\tilde{\alpha} = \frac{|\bar{\hat{s}}^t \Lambda^{-1} y|}{|\bar{\hat{s}}^t \Lambda^{-1} \hat{s}|}$$

$$\lambda_a = \ln \left[\frac{\left(\sqrt{\frac{2\nu}{\eta} \left(\bar{y}^t \Lambda^{-1} y - \frac{|\bar{\hat{s}}^t \Lambda^{-1} y|^2}{\bar{\hat{s}}^t \Lambda^{-1} \hat{s}} \right)} \right)^{\nu - m}}{\left(\sqrt{\frac{2\nu}{\eta} \bar{y}^t \Lambda^{-1} y} \right)^{\nu - m}} \; \frac{K_{\nu - m}\left(\sqrt{\frac{2\nu}{\eta} \left(\bar{y}^t \Lambda^{-1} y - \frac{|\bar{\hat{s}}^t \Lambda^{-1} y|^2}{\bar{\hat{s}}^t \Lambda^{-1} \hat{s}} \right)} \right)}{K_{\nu - m}\left(\sqrt{\frac{2\nu}{\eta} \bar{y}^t \Lambda^{-1} y} \right)} \right]$$

Fig. 4 — Detection procedure for approximate detector

For the examples evaluated, $P_{fa} = 10^{-7}$, $m =$ two pulses, ρ (the correlation between the two pulses) $= 0.98$, $\eta = 1$, $\nu = 0.1, 1, 5$, and the steering vector is given as

$$\hat{s}^t = (1 + j0 \quad 0 + j1),$$

which represents a 90° phase rotation between pulses. Finally, the signal-to-clutter ratio is computed as

$$s/c = \alpha^2 / 2\eta .$$

As stated above, the threshold at the output of the amplitude estimator may be obtained in closed form (Appendix B). Figure 5 shows curves describing this result for the three examples computed here.

A closed-form evaluation of the detection threshold at the output of the log-likelihood ratio test did not prove possible, but the thresholds were determined by using importance-sampling techniques [21,22]. Figure 6 shows curves describing these results for the three examples computed. To ensure that the probability of false alarm (P_{fa}) at the output of the approximate detector is essentially equal to the desired value (10^{-7}), the thresholds were chosen so that the P_{fa} caused by the amplitude estimation procedure alone and the P_{fa} caused by the evaluation of the log-likelihood function alone were each equal to 0.5×10^{-7}. This setting ensures the P_{fa} at the output of the approximate detector is $\leq 10^{-7}$. Table 1 shows the actual threshold settings used to compute the various curves.

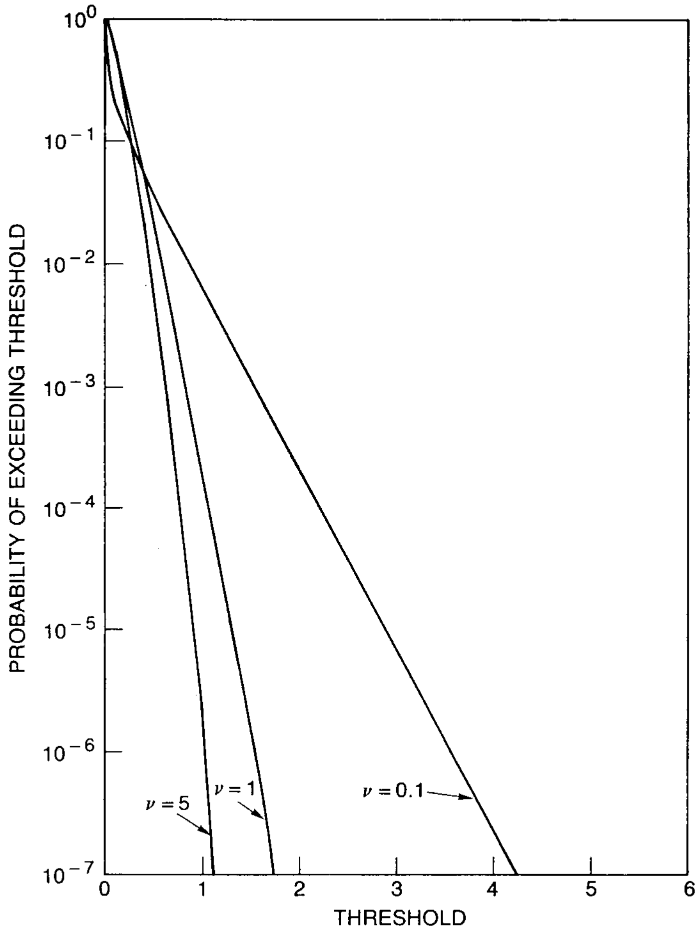


Fig. 5 — Threshold settings for T_α

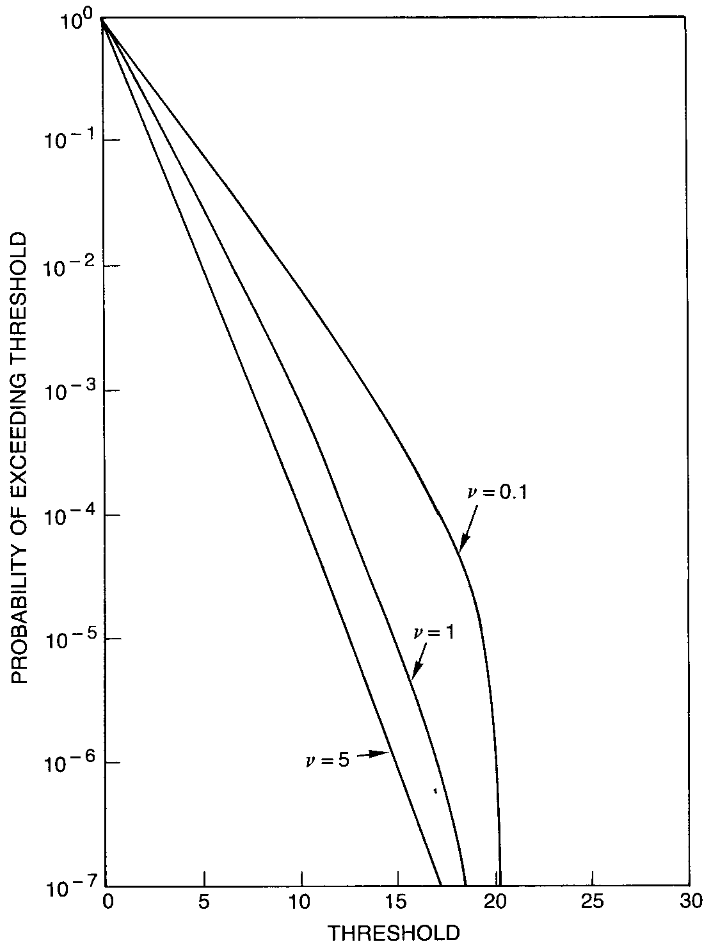


Fig. 6 — Detection thresholds at output of approximate log-likelihood ratio test

Table 1 — Threshold Settings Used to Calculate Various Curves

Approximate Test, λ_0

ν	T_α	T
0.1	4.5	21.0
1	1.85	19.8
5	1.16	18.1

Gaussian Matched Filter, λ_{mf}

ν	T
0.1	217
1	89.4
5	57.1

Optimum Neyman-Pearson Test, λ_{NP}

s/c	T
6.53	12.9
4.95	12.5
0.51	12.0

$\nu = 0.1$

s/c	T
2.10	12.5
0.51	14.0
−0.73	14.1
−2.18	14.1
−3.01	14.0

$\nu = 1$

s/c	T
−9.03	9.1
−4.95	13.5
−3.01	13.8
−1.42	12.3

$\nu = 5$

To evaluate the detection performance, the Monte Carlo technique was used. The data were generated as follows:

(1) a random number was generated from the distribution describing the power level variation:

(2) a pair of complex Gaussian samples with the desired correlation and the power level from step (1) was generated:

(3) a signal of a given amplitude, with random initial phase and the desired phase rotation from pulse to pulse was added to the clutter data; for clutter only, the amplitude was set to 0.

This sequence results in two observations and thus one detection decision. Figures 7 through 9 show performance curves based on 1000 repetitions. The performance curves of the optimum Neyman-Pearson test and the performance curves of the matched filter are also shown. The performance curves for the optimum test assume that the signal amplitude and the initial phase are known. The Monte Carlo technique was used to compute the performance of the test at several points; the other values of the curves were interpolated based on these points. As shown in Fig. 9, the performance of the optimum test for low values of signal-to-clutter ratio was extrapolated. This extrapolation was necessary since the setting of the detection thresholds in this regime proved to be prohibitively difficult. As a result, this particular curve is not intended to be an exact evaluation of the test's performance but only a heuristic indication of this performance. The performance curves for the

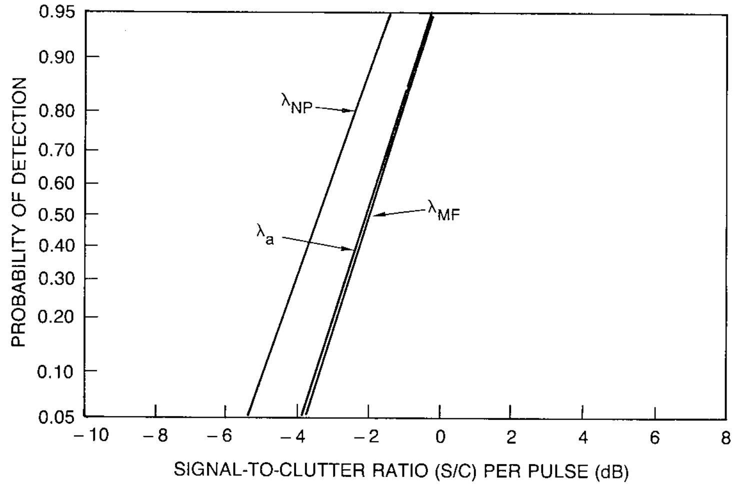


Fig. 7 — Detection performance, $\nu = 5$

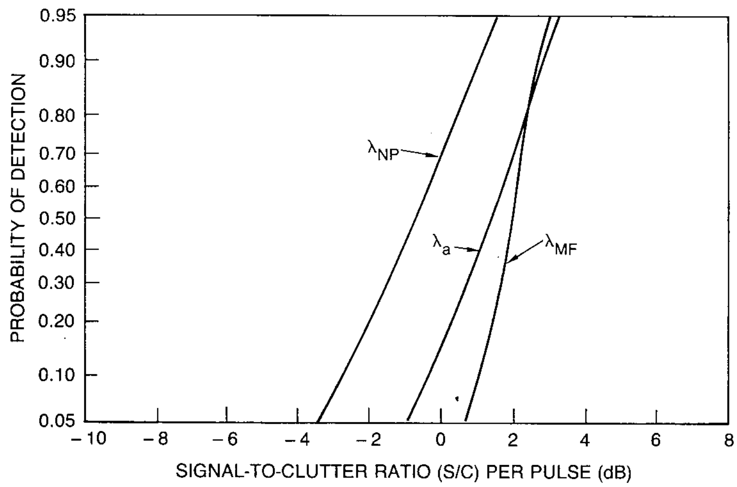


Fig. 8 — Detection performance, $\nu = 1$

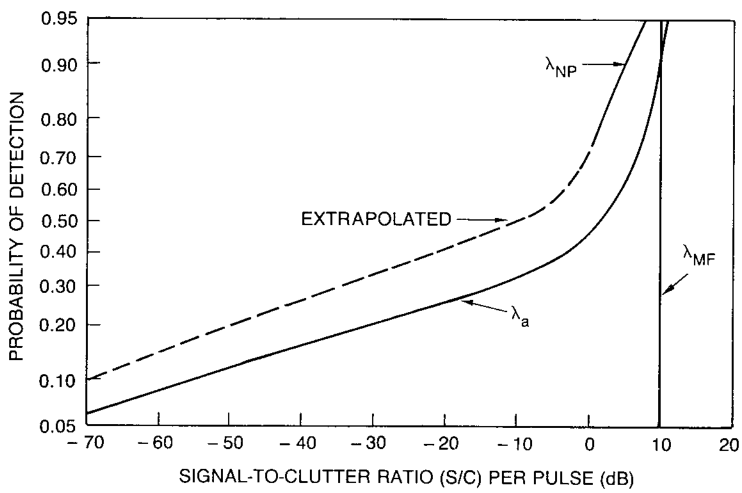


Fig. 9 — Detection performance, $\nu = 0.1$

matched-filter case were also obtained by Monte Carlo simulation. However, the detection thresholds for this case were obtained in closed form, as indicated in Appendix B.

The curves indicate that the optimum test can detect small signals in spiky clutter (the clutter is said to get spikier as $\nu \rightarrow 0$) much better than the matched filter can, although the performances approach each other as the signal strength increases. The approximate test yields performance that lies between these two cases. Also, performance of the approximate test and of the matched filter are seen to approach each other as $\nu \rightarrow \infty$. This occurs because the K distribution approaches the Rayleigh as $\nu \rightarrow \infty$.

SUMMARY

A procedure for modeling a complex non-Gaussian clutter process as a nonhomogeneous complex Gaussian process is described. The multivariate pdf associated with this clutter process is constructed so that it has a desired marginal amplitude pdf and a desired covariance matrix. The general procedure requires the solution of an integral equation. For this study, the K distribution is used for a marginal amplitude pdf, because this pdf matches sea clutter statistics and the solution to the integral equation for this problem is known.

An approximation to the optimum Neyman-Pearson test is implemented and evaluated. This approximation is independent of the signal amplitude and initial phase. The resulting detection performance of this approximate test is less than that of the optimal test but is better than the performance of the matched filter when applied against the same clutter data.

Because of the arbitrariness of constructing multivariate pdf's from limited knowledge of the statistics of the clutter process, the performance of the detector cannot be truly evaluated until it is driven by actual clutter data. However, since the clutter modeling was based on a physical model of the scattering of electromagnetic waves from the surface of the sea, this detector is expected to perform well against sea clutter data.

ACKNOWLEDGMENTS

The author thanks Dr. Ben Cantrell of NRL for suggesting this problem and for patiently monitoring the progress of this work during the months that it took to do the research.

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Appendix A
AMPLITUDE ESTIMATION

Under the signal hypothesis, the pdf of the observed vector y may be written as

$$f(y) = \frac{1}{(2\pi)^m |\Lambda|} \int_0^\infty \frac{\exp\left\{-\frac{1}{2}\frac{\overline{(y-s)}^t \Lambda^{-1}(y-s)}{\tau}\right\}}{\tau^m} f(\tau) d\tau, \tag{A1}$$

where

$$s = \alpha e^{j\phi}\hat{s} .$$

The maximum-likelihood estimate of the signal strength α occurs at $\frac{\partial f(y)}{\partial \alpha} = 0$. Assuming that the partial differentiation may be taken inside the integral, we have

$$\frac{\partial f(y)}{\partial \alpha} = \frac{1}{(2\pi)^m |\Lambda|} \int_0^\infty \frac{f(\tau)}{\tau^m} \frac{\partial}{\partial \alpha} \left[\exp\left\{-\frac{1}{2}\frac{\overline{(y-s)}^t \Lambda^{-1}(y-s)}{\tau}\right\}\right] d\tau \tag{A2}$$

Expanding the quadratic form and performing the derivative shows that the above expression equals 0 for arbitrary $f(\tau)$ if

$$\mathrm{Re}(\bar{\hat{s}}^t \Lambda^{-1} y) - \alpha \bar{\hat{s}}^t \Lambda^{-1} s = 0. \tag{A3}$$

Solving this equation yields

$$\tilde{\alpha} = \mathrm{Re}(\bar{\hat{s}} \Lambda^{-1} y)/\bar{\hat{s}} \Lambda^{-1} \hat{s} . \tag{A4}$$

for the estimate of α.

Implementation of this estimator, however, requires knowledge of the signal initial phase. To account for our lack of knowledge, we implement

$$\tilde{\alpha} = |\bar{\hat{s}}^t \Lambda^{-1} y| / \bar{\hat{s}}^t \Lambda^{-1} \hat{s} . \tag{A5}$$

Appendix B

THRESHOLD SETTING FOR THE MATCHED FILTER

In this report, the form of the matched filter considered is

$$\lambda_{mf} = | \bar{\hat{s}}^t \Lambda^{-1} y | / a \,, \tag{B1}$$

where

$a = 1$ for the detection curves

$a = \bar{\hat{s}}^t \Lambda^{-1} s$ for the setting of T_α.

Conditioned on the value of τ and under the noise hypothesis, the pdf of λ_{mf} is easily shown to be

$$f(\lambda_{mf}) = \frac{\lambda_{mf}}{\beta\tau} \exp\left\{ -\frac{\lambda_{mf}^2}{2\beta\tau} \right\}, \tag{B2}$$

where

$$\beta = \frac{\bar{\hat{s}}^t \Lambda^{-1} \hat{s}}{a^2} ;$$

Still conditioned on the value of τ, we have for the probability of a false alarm

$$P_{fa/\tau} = \exp\left\{ -\frac{T^2}{2\beta\tau} \right\} \tag{B3}$$

The unconditional probability of a false alarm is then

$$\begin{aligned} P_{fa} &= \int_0^\infty \exp\left\{ -\frac{T^2}{2\beta\tau} \right\} f(\tau) d\tau \\ &= \frac{\left[\frac{\nu}{\eta} \right]^\nu}{\Gamma(\nu)} \int_0^\infty \tau^{\nu-1} \exp\left\{ -\left[\frac{\nu}{\eta}\tau - \frac{T^2}{2\beta\tau} \right] \right\} d\tau \\ &= \frac{2}{\Gamma(\nu)} \left[\frac{\sqrt{\frac{2\nu}{\eta\beta}}\, T}{2} \right]^\nu K_\nu \left[\sqrt{\frac{2\nu}{\eta\beta}}\, T \right] . \end{aligned}$$

The thresholds T_α were set by letting $\beta = 1/\bar{\hat{s}}^t \Lambda^{-1} s$ (i.e., $a = \bar{\hat{s}}^t \Lambda^{-1} s$), and the thresholds at the output of the matched filter detector were set by letting $\beta = \bar{\hat{s}}^t \Lambda^{-1} \hat{s}$ (i.e., $a = 1$). These results were checked and verified by Monte Carlo simulation.