

# Generalized List Detection for Coded MFSK/FH Signaling on Fading and Jamming Channels

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# GENERALIZED LIST DETECTION FOR CODED MFSK/FH SIGNALING ON FADING AND JAMMING CHANNELS

## INTRODUCTION

Modern digital communication systems are frequently required to operate in severe interference environments. These disturbances include a variety of fading and jamming phenomena arising from natural and man-made sources. Usually, when the interference type is well understood, adequate performance can be achieved by using the right combination of modulation, error control coding, and spread spectrum techniques. This was demonstrated in 1975 by Viterbi and Jacobs [1], who showed that coding can lead to considerable improvement in the performance of multiple frequency shift keying (MFSK) frequency hopped (FH) systems on Rayleigh fading and worst-case partial band Gaussian interference channels.

In the presence of intelligent jamming there are special difficulties which must be addressed. Of particular concern is the fact that pure soft-decision decoding\* does not work effectively in an intelligent jamming environment unless the communication receiver has side-information concerning the state of the jamming signal† [2,3]. This is true because a pure soft-decision receiver without side-information is vulnerable to a jamming strategy where high jamming power can be concentrated on a single element of a coded transmission sequence and lead to a large number of decoding errors. Whereas pure soft-decision decoding usually outperforms hard-decision decoding in nonintelligent interference environments, the opposite is true against intelligent jamming when the receiver does not possess jamming state information.

One approach to alleviating the problem of soft-decision decoding in the absence of side-information is to quantize the demodulator outputs with a finite number of threshold levels. The difficulty with threshold quantization detection is that optimum threshold setting requires perfect automatic gain control (AGC), which is difficult to maintain in jamming or fading.

An alternative approach called list-metric detection is presented in this report. With this technique, demodulator outputs are ranked in magnitude from the highest to the lowest, and decoder metrics are assigned to these outputs according to their position in the ranking rather than their magnitude. The process of rank-ordering is equivalent to partitioning the  $M$ -dimensional observation space of the  $M$  demodulator outputs into several regions corresponding to the different ordered lists. In this sense, listing is another form of soft-decision quantization because it creates a discrete memoryless channel (DMC) with more outputs than inputs as seen by the encoder/decoder. Note that this form of quantization does not depend upon thresholds which are difficult to maintain during jamming.‡

Previous work [5-7] has considered list-of- $L$  detection, which is a special case of list-metric detection. It has been shown [5,6] that list-of- $L$  detection with optimum metrics is inferior to threshold quantization detection for additive white Gaussian (AWGN) channels. The results of these previous list-detection studies are generalized in this report by introducing greater flexibility in the choice of metrics. Also, analysis of the optimum metric results is extended to include fading and jamming channels.

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\*Pure soft-decision decoding refers to the situation where the actual analog demodulator outputs are provided to the decoder.

†The assumption that the receiver has jamming state side-information is made in [1].

‡Viterbi [4] has proposed a related approach using a simple-to-implement ratio-threshold technique. This has been shown to be particularly effective against tone jamming.

The second section of this report is devoted to a background discussion of the methodology for evaluation of communication system performance with an arbitrary choice of metrics. A general expression of the performance figure of merit, the cutoff rate parameter  $R_0$ , is derived for list-metric detection in the third section of this report. In the fourth section, the optimal metric choice for list-metric detection is presented. Performance is evaluated in the fifth and sixth sections of this report for two severe communication channels, a highly idealized worst-case tone jamming channel and the Rayleigh fading channel. Conclusions are drawn and a recommendation is made for future investigation in the last section.

## METHODOLOGY FOR ANALYZING PERFORMANCE OF CODED COMMUNICATION SYSTEMS

In this section we present a methodology [2] for analyzing a general communication system such as that shown in Fig. 1. Included in this system are a channel encoder, signal modulator, and frequency hopper, along with their inverse operations. Also shown is a processor which acts as an interface between the demodulator and decoder.

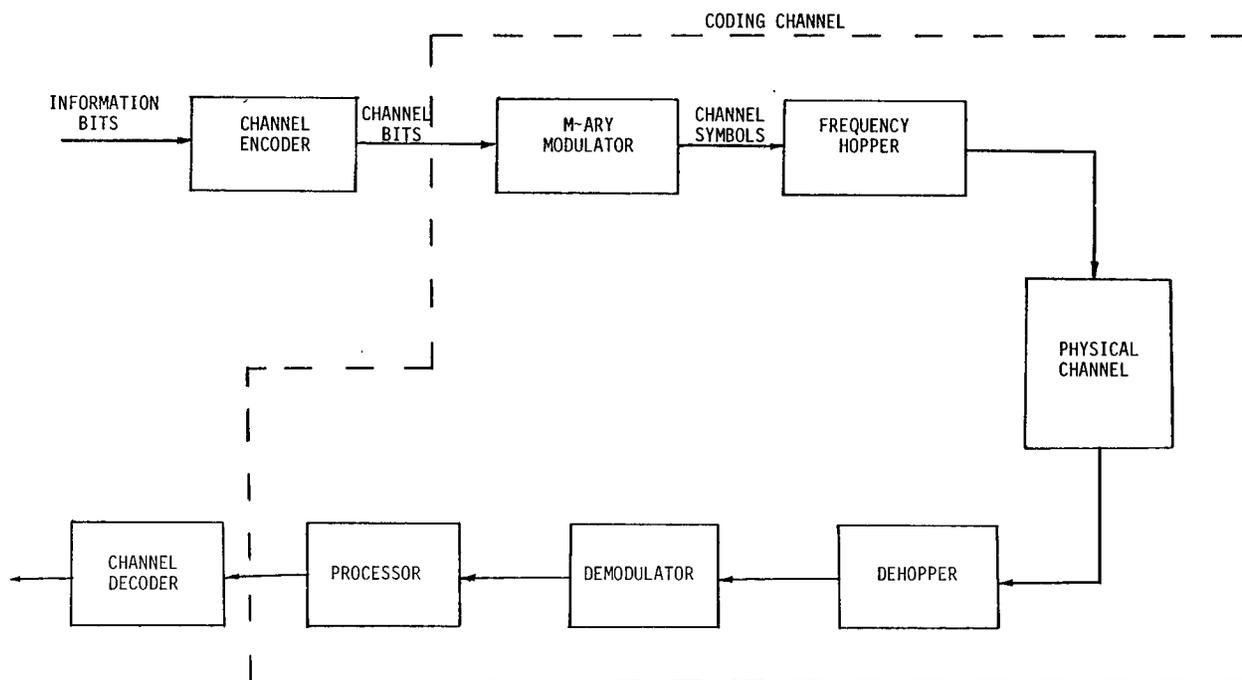


Fig. 1 — A general communication system

The ultimate quantity of interest is the probability of bit error, and to derive a bound on this quantity, it is useful to decouple the coding aspects of the communication system from the remaining parts. The coding channel, that is, all but the channel encoder/decoder, can be characterized by the cutoff rate parameter  $R_0$  which represents the practically achievable data rate [8]. For any specific code and for most modulations of interest, it is possible to bound the decoded bit error probability by a function purely of  $R_0$ ; that is,

$$P_b \leq B(R_0). \quad (1)$$

The cutoff rate parameter  $R_0$  depends solely on  $E_c/N_0$ , the ratio of channel bit energy to one-sided noise power spectral density, that is, we may write

$$R_0 = f \left( \frac{E_c}{N_0} \right). \quad (2)$$

Furthermore, we have

$$\frac{E_c}{N_0} = R \frac{E_b}{N_0}, \quad (3)$$

where  $E_b$  is the energy per information bit and  $R$  is the code rate (channel bits per information bit). Thus, from Eqs. (1-3),  $P_b$  can be bounded by a function of  $E_b/N_0$ , for the coded system.

Since the function  $B(R_0)$  is unique for each code, and the parameter  $R_0$  is independent of the code, it is possible to decouple the coder/decoder from the coding channel. Various modulation systems can be compared by using the cutoff rate parameter  $R_0$  and specific codes can be evaluated separately.

The coding channel is shown in Fig. 2. For convenience it is separated into two parts: a memoryless\* channel which includes all of the modulation/demodulation functions and a processor which provides metrics to the decoder. For the memoryless channel, a modulator input  $x$  is transformed into a demodulator output  $y$  according to a forward conditional probability density function  $p(y|x)$ . (For MFSK the output  $y$  is actually an  $M$ -dimensional vector  $y$  whose components are the  $M$  matched filter outputs  $y_1, y_2, \dots, y_M$ . The vector notation is deleted here to avoid confusion with output sequence vectors.) For each demodulator output  $y$ , the processor generates a set of  $M$  metrics  $m(y, x)$ , one for each of the input hypotheses  $x$ . Throughout this report we assume that decisions are made according to the highest metric.

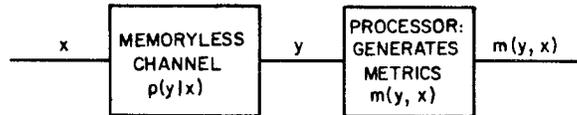


Fig. 2 — Coding channel

Consider two input sequences (vectors) of length  $N$ ,  $\mathbf{x}$  and  $\mathbf{x}'$ , and the pairwise error probability of the receiver choosing  $\mathbf{x}'$  when  $\mathbf{x}$  is transmitted assuming that these are the only two possible transmitted sequences. This probability is denoted by  $\Pr\{\mathbf{x} \rightarrow \mathbf{x}'\}$ . Using the Chernoff bound [8] with free parameter  $\lambda \geq 0$ , we obtain

$$\begin{aligned} \Pr\{\mathbf{x} \rightarrow \mathbf{x}'\} &= \Pr \left\{ \sum_{n=1}^N m(y_n, x_n) \leq \sum_{n=1}^N m(y_n, x'_n) \mid \mathbf{x} \right\} \\ &= \Pr \left\{ \sum_{n=1}^N [m(y_n, x'_n) - m(y_n, x_n)] \geq 0 \mid \mathbf{x} \right\} \\ &\leq E \left\{ e^{\lambda \sum_{n=1}^N [m(y_n, x'_n) - m(y_n, x_n)]} \mid \mathbf{x} \right\}, \end{aligned} \quad (4)$$

where  $E$  is the expectation operator, and the subscript  $n$  refers to position in the sequence.

\*The memoryless property can be achieved either by interleaving or by frequency hopping. Successive channel inputs are treated in a statistically independent manner.

Since the channel is memoryless, we write Eq. (4) as

$$\Pr\{\mathbf{x} \rightarrow \mathbf{x}'\} \leq \prod_{n=1}^N E\{e^{[m(y_n, x'_n) - m(y_n, x_n)]} | x_n\}. \quad (5)$$

Defining

$$D(x, x'; \lambda) = E\{e^{\lambda[m(y, x') - m(y, x)]} | x\}, \quad (6)$$

we see that Eq. (5) becomes

$$\Pr\{\mathbf{x} \rightarrow \mathbf{x}'\} \leq \prod_{n=1}^N D(x_n, x'_n; \lambda). \quad (7)$$

The cutoff parameter  $R_0$  is defined [9] by

$$R_0 = \max_{\lambda \geq 0} \max_{p(\cdot)} R_0[p(\cdot); \lambda], \quad (8)$$

where  $p(\cdot)$  is the input probability density function and  $R_0[p(\cdot); \lambda]$  is derived from the relationship

$$\begin{aligned} 2^{-R_0[p(\cdot); \lambda]} &= E\{D(x, x'; \lambda)\} \\ &= \sum_x \sum_{x'} p(x)p(x')D(x, x'; \lambda). \end{aligned} \quad (9)$$

For orthogonal MFSK signaling the coding channel is symmetrical and the maximization over  $p(\cdot)$  occurs with equally likely input signals, i.e.,  $p(\cdot) = 1/M$  for all  $M$  inputs. Furthermore, because of the symmetry of the orthogonal MFSK signal set, we recognize that

$$D(x, x'; \lambda) = \begin{cases} 1 & , \quad x = x'. \\ D(\lambda) & , \quad x \neq x'. \end{cases} \quad (10)$$

With this simplification we may write the right-hand side of Eq. (9) as

$$\sum_x \sum_{x'} p(x)p(x')D(x, x'; \lambda) = \frac{1 + (M-1)D(\lambda)}{M}, \quad (11)$$

and the cutoff rate parameter becomes

$$R_0 = \log_2 M - \log_2 [1 + (M-1)D], \quad (12)$$

where

$$D = \min_{\lambda \geq 0} D(\lambda). \quad (13)$$

In Eq. (12) we see that for orthogonal signaling there is a simple one-to-one correspondence between  $D$  and  $R_0$ .

In the next section we shall use these results to evaluate  $R_0$  for the list-metric coding channel.

## LIST-METRIC DETECTION AND ITS $R_0$ EXPRESSION

Consider a coded MFSK/FH receiver as shown in Fig. 3. The demodulator consists of a bank of  $M$ -matched filter-envelope detectors whose outputs are  $y_1, y_2, \dots, y_M$ . These  $M$  numbers are presented to a nonlinear processor which produces a metric vector  $\mathbf{N} = [N_1, N_2, \dots, N_M]$ .

A list metric is obtained by ranking the matched-filter outputs and by giving the  $l$ th largest output a metric value  $N_l$ . The list metric can be written as  $m(y, x) = N_{l_x}$ .

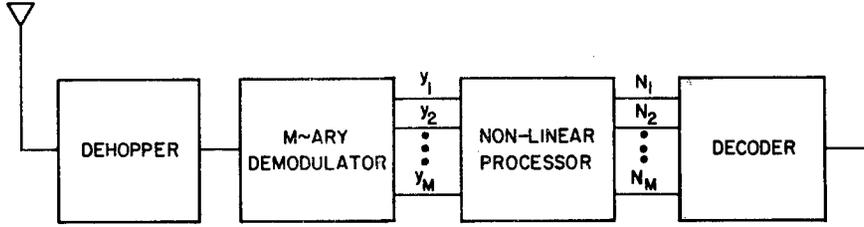


Fig. 3 — General receiver configuration

From Eqs. (6) and (10) we may write

$$\begin{aligned}
 D(\lambda) &= E\{e^{\lambda[N_{l_x} - N_{l_x'}]} | x\} \\
 &= \sum_{k=1}^M E\{e^{\lambda[N_{l_x} - N_{l_x'}]} | x, l_x = k\} \Pr\{l_x = k | x\} \\
 &= \sum_{k=1}^M \left[ \frac{1}{M-1} \sum_{\substack{i=1 \\ i \neq k}}^M e^{\lambda[N_i - N_k]} \right] \Pr\{l_x = k | x\}.
 \end{aligned} \tag{14}$$

It follows that

$$D = \min_{\lambda \geq 0} \frac{1}{M-1} \sum_{k=1}^M \left[ \sum_{i=1}^M e^{\lambda[N_i - N_k]} - 1 \right] q_k, \tag{15}$$

where  $q_k = \Pr\{l_x = k | x\}$  is the probability that the sent signal appears in  $k^{\text{th}}$  position on the ordered list.

The channel is usually characterized by the forward conditional probability density functions  $p_0(\alpha)$  and  $p_1(\alpha)$  for each matched-filter output  $\alpha$ , where  $p_1(\alpha)$  is the output density when the input is signal plus noise, and  $p_0(\alpha)$  is the output density when the input is noise only. Then, for  $M$ -ary orthogonal signaling the position probabilities  $q_k$  are given by

$$q_k = \binom{M-1}{k-1} \int_0^\infty \left[ \int_0^\beta p_0(\alpha) d\alpha \right]^{k-1} \left[ 1 - \int_0^\beta p_0(\alpha) d\alpha \right]^{m-k} p_1(\beta) d\beta, \tag{16}$$

which satisfy the conditions

$$q_k \geq 0, \quad k = 1, 2, \dots, M$$

and

$$\sum_{k=1}^M q_k = 1.$$

Thus the vector  $\mathbf{q} = [q_1, q_2, \dots, q_M]$  is an equivalent characterization of the channel.

In Eq. (15), we may define the array sum  $S(\lambda, \mathbf{N})$  by

$$S(\lambda, \mathbf{N}) = \sum_{k=1}^M \sum_{i=1}^M q_k e^{\lambda(N_i - N_k)}. \tag{17}$$

The minimum of this expression with respect to  $\lambda \geq 0$  we designate by  $S$ , that is,

$$S = \min_{\lambda \geq 0} S(\lambda, \mathbf{N}). \tag{18}$$

From this it follows that

$$D = \frac{1}{M-1} [-1 + S]. \quad (19)$$

Then from Eq. (12) we see that

$$\begin{aligned} R_0 &= \log_2 M - \log_2 S \\ &= \log_2 M - \log_2 \left[ \min_{\lambda \geq 0} \sum_{k=1}^M \sum_{i=1}^M q_k e^{\lambda(N_i - N_k)} \right]. \end{aligned} \quad (20)$$

This is the general expression for  $R_0$  for list-metric detection, and it is applicable for any choice of list-metric vector  $\mathbf{N}$ . In the next section we derive an expression for the particular vector  $\mathbf{N}$  which maximizes  $R_0$  for list-metric detection.

### OPTIMAL CHOICE OF $\mathbf{N}$ FOR LIST METRIC DETECTION

The choice of metric vector  $\mathbf{N}$  which maximizes  $R_0$  in Eq. (20) can be found by minimizing the right hand side of Eq. (17) with respect to  $\lambda$  and  $\mathbf{N}$  [10]. That is, we determine the quantity

$$\hat{S} = \min_{\lambda \geq 0} \min_{\mathbf{N}} S(\lambda, \mathbf{N}). \quad (21)$$

To perform this minimization we first differentiate  $S(\lambda, \mathbf{N})$  with respect to  $\lambda$  and equate to zero,

$$\frac{\partial S(\lambda, \mathbf{N})}{\partial \lambda} = \sum_{k=1}^M \sum_{i=1}^M q_k (N_i - N_k) e^{\lambda(N_i - N_k)} = 0. \quad (22)$$

This double summation has all zeros on the diagonal and the remaining portion can be written in two parts, one for each side of the diagonal.

$$\begin{aligned} \frac{\partial S(\lambda, \mathbf{N})}{\partial \lambda} &= \sum_{k=2}^M \sum_{i=1}^{k-1} q_k (N_i - N_k) e^{\lambda(N_i - N_k)} \\ &+ \sum_{i=2}^M \sum_{k=1}^{i-1} q_k (N_i - N_k) e^{\lambda(N_i - N_k)} = 0. \end{aligned} \quad (23)$$

From the symmetry of the terms in Eq. (23), we may write

$$\frac{\partial S(\lambda, \mathbf{N})}{\partial \lambda} = \sum_{k=2}^M \sum_{i=1}^{k-1} (N_i - N_k) [q_k e^{\lambda(N_i - N_k)} - q_i e^{\lambda(N_i - N_k)}] = 0. \quad (24)$$

Next, we minimize  $S(\lambda, \mathbf{N})$  with respect to  $\mathbf{N}$ . This is accomplished by differentiating with respect to  $N_j$  for  $j = 1, 2, \dots, M$  and equating all derivatives to zero.

$$\frac{\partial S(\lambda, \mathbf{N})}{\partial N_j} = \frac{\partial}{\partial N_j} \left[ \sum_{i=1}^M e^{\lambda N_i} \right] \sum_{k=1}^M q_k e^{-\lambda N_k} + \frac{\partial}{\partial N_j} \left[ \sum_{k=1}^M q_k e^{-\lambda N_k} \right] \sum_{i=1}^M e^{\lambda N_i} = 0, \quad (25)$$

or

$$\frac{\partial S(\lambda, \mathbf{N})}{\partial N_j} = \lambda \sum_{k=1}^M q_k e^{\lambda(N_j - N_k)} - \lambda \sum_{i=1}^M q_j e^{-\lambda(N_i - N_j)} = 0, \quad j = 1, 2, \dots, M. \quad (26)$$

In Eq. (26) the summations are carried out independently so we may arbitrarily replace the index  $i$  by  $k$  in the second sum.

$$\frac{\partial S(\lambda, \mathbf{N})}{\partial N_j} = \lambda \sum_{k=1}^M q_k e^{\lambda(N_j - N_k)} - \lambda \sum_{k=1}^M q_j e^{-\lambda(N_k - N_j)} = 0, \quad j = 1, 2, \dots, M. \quad (27)$$

Also, in Eq. (27), we may use the subscript  $i$  in place of  $j$ .

$$\frac{\partial S(\lambda, \mathbf{N})}{\partial N_i} = \lambda \sum_{k=1}^M [q_k e^{\lambda(N_i - N_k)} - q_i e^{-\lambda(N_i - N_k)}] = 0, \quad i = 1, 2, \dots, M. \quad (28)$$

In Eqs. (24) and (28), we have the identical expression in brackets. Thus for  $\lambda$  and  $N$  to maximize  $R_0$ , it is sufficient that

$$\lambda(N_i - N_k) = \ln \sqrt{\frac{q_i}{q_k}} \text{ for all pairs } 1 \leq i, k, \leq M. \quad (29)$$

A particular solution of Eq. (29) gives the optimal choice

$$\lambda = \frac{1}{2}, \quad (30)$$

and

$$N_i = \ln q_i. \quad (31)$$

The results of Eq. (29) may be used in the optimal  $R_0$  expression

$$R_0 = \log_2 M - \log_2 \left[ \min_{\lambda \geq 0} \min_N \sum_{k=1}^M \sum_{i=1}^M q_k e^{\lambda(N_i - N_k)} \right] \quad (32)$$

to yield

$$\begin{aligned} R_0 &= \log_2 M - \log_2 \left[ \sum_{k=1}^M \sum_{i=1}^M q_k \sqrt{\frac{q_i}{q_k}} \right] \\ &= \log_2 M - \log_2 \left[ \sum_{k=1}^M \sum_{i=1}^M \sqrt{q_k q_i} \right] \\ &= \log_2 M - 2 \log_2 \sum_{i=1}^M \sqrt{q_i}. \end{aligned} \quad (33)$$

In some circumstances it is desirable to use a list of demodulator outputs which has been shortened to a length  $L$  (for  $L \leq M$ ). In doing so, the lowest  $M-L$  outputs are treated indistinguishably as a single off-list group. This can be accounted for by using a position probability vector

$$\mathbf{q} = [q_1, q_2, \dots, q_L, q_0, q_0, \dots, q_0] \quad (34)$$

where

$$q_0 = \frac{\sum_{l=L+1}^M q_l}{M - L}.$$

It follows from Eq. (31) that the list-metric vector becomes

$$\mathbf{N} = [\ln q_1, \ln q_2, \dots, \ln q_L, \ln q_0, \dots, \ln q_0]. \quad (35)$$

\*The logarithms used in this metric may be taken to any base and have an arbitrary common bias as in the case of the Fano metric used in sequential decoding [9].

The optimal  $R_0$  corresponding to Eq. (33) for the shortened list-of- $L$  is seen to be

$$\begin{aligned}
 R_0 &= \log_2 M - 2 \log_2 \left[ \sum_{i=1}^L \sqrt{q_i} + \sum_{i=L+1}^M q_0 \right] \\
 &= \log_2 M - 2 \log_2 \left[ \sum_{i=1}^L \sqrt{q_i} + \sqrt{(M-L) \sum_{i=L+1}^M q_i} \right].
 \end{aligned} \tag{36}$$

This result is the same as that for list-of- $L$  detection as presented in [5-7].

To obtain the optimal  $R_0$  of Eq. (36) for list-of- $L$  detection, the list-metric vector  $\mathbf{N}$  given by Eq. (35) must be provided to the decoder. This metric information is based upon perfect knowledge of the channel statistics (that is, perfect knowledge of the first  $L$  components of  $\mathbf{q}$ ). Consequently, the degree to which the optimal list-metric detection performance can be realized depends upon how well these channel parameters can be measured or estimated by the receiver. In the next two sections of this report, we assume that this perfect channel knowledge is available in determining the optimal  $R_0$  for both tone jamming and Rayleigh fading channels.

## PARTIAL BAND TONE-JAMMING CHANNELS

The performance of uncoded orthogonal MFSK/FH signaling against worst-case partial band tone jamming was found by Houston [11]. In this section we consider the performance of list-metric detection on this tone-jamming channel.

The MFSK/FH signaling format is shown in Fig. 4. A total hopping bandwidth  $W$  is divided into  $b$  subbands with each tone symbol being transmitted on a different frequency hop. Within a hopping subband, one of  $M$  tones carrying  $k = \log_2 M$  bits is sent with signal power  $S$ . Candidate tones are orthogonally spaced with a frequency spacing  $\Delta = \frac{1}{T_s} = R_s$ , where  $T_s$  is the symbol duration and  $R_s$  is the symbol rate.

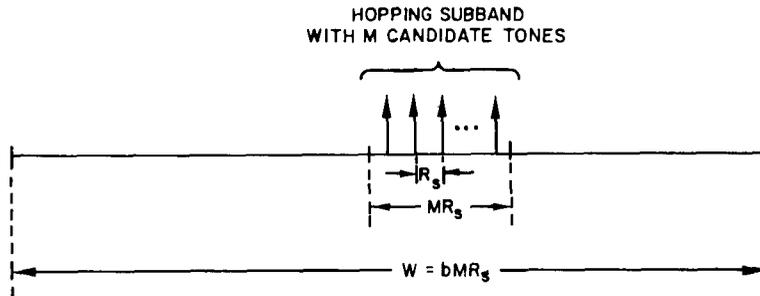


Fig. 4 — MFSK/FH signaling format

In the worst-case jamming strategy, the jammer places tones in as many hopping subbands as possible, with a maximum of one tone per subband. The jamming tone power is taken to slightly exceed the signal power, but for purposes of analytical convenience they are taken to be equal. In this tone jamming strategy, it is assumed (perhaps unrealistically) that the jammer has perfect knowledge of the communicator's signal power level and frequency slots.

For a total jamming power  $J$ , the jammer will attempt to force incorrect symbol decisions in  $n$  subbands by placing in them tones with power  $J/n = S$ . A jamming tone will successfully cause a symbol error if it hits one of the  $M - 1$  nontransmitted tone slots in the transmission subband. Thus, the probability of symbol error is

$$P_s = \frac{n}{b} \frac{M-1}{M}, \quad (37)$$

where  $n/b \leq 1$  is the fraction of the subbands which are jammed. For orthogonal signals, it follows that the probability of bit error is

$$P_b = \frac{M/2}{M-1} P_s = \frac{1}{2} \frac{n}{b}, \quad \frac{n}{b} \leq 1. \quad (38)$$

Since the number of the subbands jammed is

$$n = \frac{J}{S}, \quad n \leq b, \quad (39)$$

and the total number of subbands is

$$b = \frac{W}{MR_s}, \quad (40)$$

where  $MR_s$  is the bandwidth of one subband, it follows that Eq. (38) may be written as

$$P_b = \begin{cases} \frac{1}{2} \frac{J}{S} \frac{MR_s}{W}, & \frac{SW}{JR_s} \geq M \\ \frac{1}{2} & \frac{SW}{JR_s} \leq M. \end{cases} \quad (41)$$

However, since  $S/R_s$  is the symbol energy and  $J/W$  is the equivalent noise power spectral density  $N_0$ , (W/Hz) we may write

$$P_b = \begin{cases} \frac{1}{2} \frac{M}{E_s/N_0}, & \frac{E_s}{N_0} \geq M \\ \frac{1}{2} & \frac{E_s}{N_0} \leq M, \end{cases} \quad (42)$$

or

$$P_b = \begin{cases} \frac{1}{2} \frac{M}{k} \frac{1}{E_b/N_0}, & \frac{E_b}{N_0} \geq \frac{M}{k} \\ \frac{1}{2} & \frac{E_b}{N_0} \leq \frac{M}{k}. \end{cases} \quad (43)$$

Furthermore, the optimal jamming fraction may be determined as

$$\frac{q}{b} = 2P_b = \begin{cases} \frac{M}{k} \frac{1}{E_b/N_0}, & \frac{E_b}{N_0} \geq \frac{M}{k} \\ 1 & \frac{E_b}{N_0} \leq \frac{M}{k}. \end{cases} \quad (44)$$

Over the usual range of interest ( $E_b/N_0 \geq M/k$ ), the dependence of  $P_b$  on  $E_b/N_0$  is inverse linear, with the probability of bit error increasing by a factor  $M/\log_2 M$  as  $M$  increases.

With this background, we now consider the performance of a coded MFSK/FH system with list-metric detection. For the worst-case tone-jamming channel with a maximum of one jamming tone per subband, the position probability vector  $\mathbf{q}$  may be written as

$$\mathbf{q} = [q_1, q_2, 0, \dots, 0] \quad (45)$$

where  $q_2$  is the probability of symbol error, given by

$$q_2 = \begin{cases} \frac{M-1}{E_s/N_0}, & \frac{E_s}{N_0} \geq M \\ \frac{M-1}{M}, & \frac{E_s}{N_0} \leq M, \end{cases} \quad (46)$$

and  $q_1$  is the probability of correct symbol decision ( $1 - q_2$ ) given by

$$q_1 = \begin{cases} 1 - \frac{M-1}{E_s/N_0}, & \frac{E_s}{N_0} \geq M \\ \frac{1}{M}, & \frac{E_s}{N_0} \leq M. \end{cases} \quad (47)$$

The optimal list-metric for this channel is  $\mathbf{N} = \ln \mathbf{q}$ .

From Eq. (36), the cutoff rate parameter for list-of-2 detection on a worst-case tone-jamming channel is

$$R_0 = \log_2 M - 2 \log_2 [\sqrt{q_1} + \sqrt{q_2}]. \quad (48)$$

The list-of-2 curves for  $R_0$  vs  $E_s/N_0$  are plotted in Fig. 5. These curves exhibit an unusual behavior. For large  $E_s/N_0$ ,  $R_0$  approaches  $\log_2 M$ . As  $E_s/N_0$  decreases,  $R_0$  decreases to a minimum value of  $\log_2 M - 1$  when  $q_1 = q_2 = 1/2$ . (In a sense, the missing bit per channel use corresponds to the total uncertainty which exists as to whether the sent signal appears first or second in the ranking.) The condition  $q_1 = q_2 = 1/2$  occurs when  $\frac{M/2}{M-1}$  of the subbands contain a jamming tone. In each subband which has a jamming tone, there is probability  $(M-1)/M$  that the jamming tone will hit a slot other than that occupied by the sent signal, thereby causing a symbol error.

As  $E_s/N_0$  further decreases,  $R_0$  increases because  $q_2 > q_1$  and the metrics  $\ln q_1$  and  $\ln q_2$  become reverse weighted. That is, the second ranked output will receive a higher metric than the first if there is a higher probability that the sent signal will appear second on the list. The improvement in  $R_0$  continues as  $q_2/q_1$  increases until the full band is jammed with one tone per subband. When this occurs the ranking probabilities are

$$q_1 = \frac{1}{M} \quad (49)$$

and

$$q_2 = \frac{M-1}{M}. \quad (50)$$

The limiting cutoff rate parameter becomes

$$R_0 = \frac{M}{1 + \frac{2\sqrt{M-1}}{M}} \quad (51)$$

for all values of  $E_s/N_0 \leq M$ . Here, the jamming cannot be made more effective (as long as there is only one jamming tone per subband) because there is always a probability  $1/M$  that the jamming tone will coincide with the sent signal.

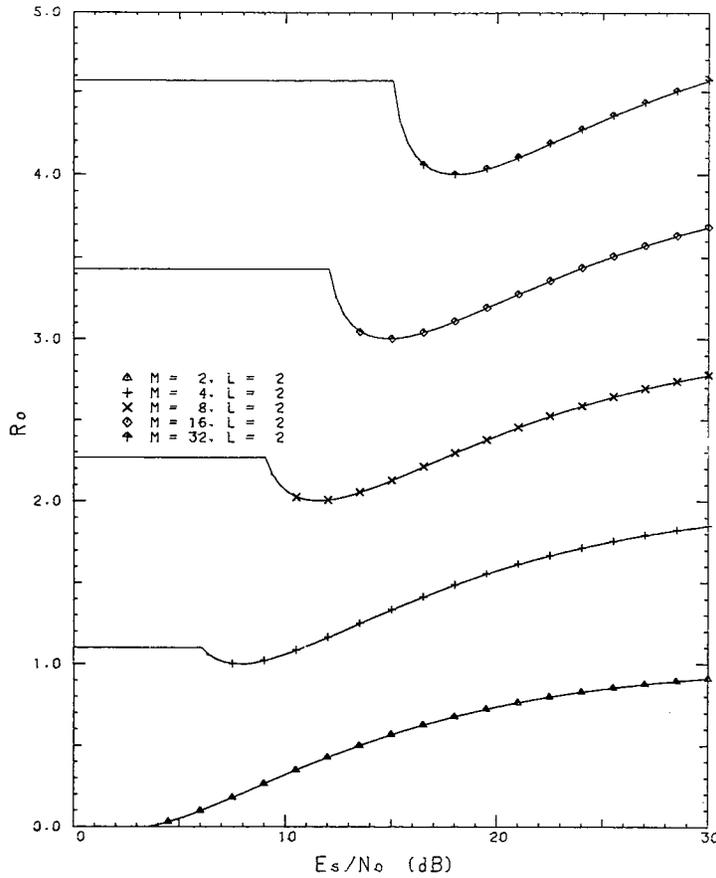


Fig. 5 -  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel

Figure 6 shows the list-of-1 (hard-decision) results for  $M = 2, 4, \dots, 32$ . These were determined from the  $R_0$  expression of Eq. (46) with  $L = 1$ ,

$$R_0 = \log_2 M - 2 \log_2 [\sqrt{q_1} + \sqrt{(M-1)q_2}]. \quad (52)$$

For comparison, the list-of-1 and the list-of-2 results are plotted together for  $M = 2, 4, \dots, 32$  in Figs. 7 through 11. These show the improvement of list-of-2 detection over hard-decision detection. The poor performance of hard-decision detection at low signal-to-noise ratios is due to the fact that the hard-decision metric vector

$$N = \left[ \ln q_1, \ln \frac{q_2}{M-1}, \ln \frac{q_2}{M-1}, \dots, \ln \frac{q_2}{M-1} \right] \quad (53)$$

does not contain information that the sent signal has high probability of appearing second on the list of demodulator outputs.

In a companion set of curves  $E_b/N_0$  vs code rate  $R$  is plotted in Figs. 12 through 18. Here the code rate is taken to be equal to the cutoff rate parameter (normalized to channel bits instead of channel symbols). Since there are  $\log_2 M$  channel bits-per-channel symbol, the code rate at cutoff is defined as

$$R = \frac{R_0}{\log_2 M}, \quad (54)$$

and it follows that

$$\frac{E_b}{N_0} = \frac{1}{R \log_2 M} \frac{E_s}{N_0} = \frac{1}{R_0} \frac{E_s}{N_0}. \quad (55)$$

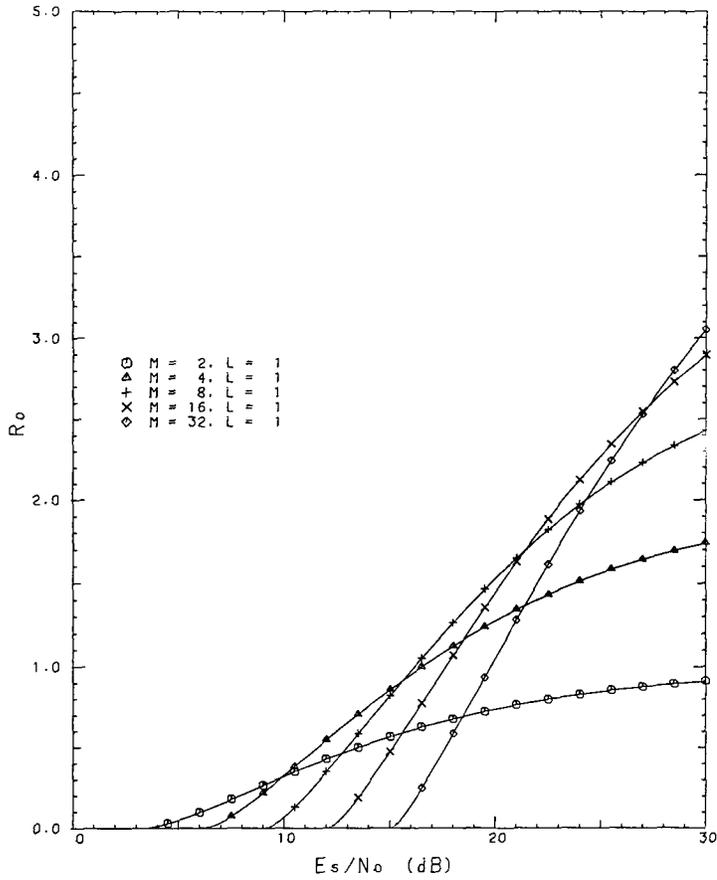
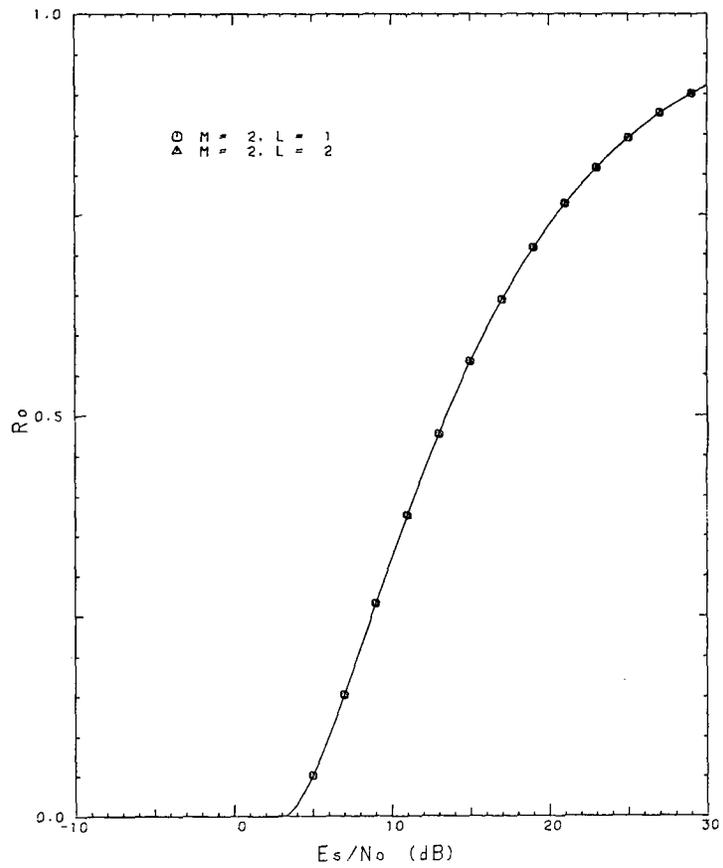


Fig. 6 —  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel

Fig. 7 —  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel



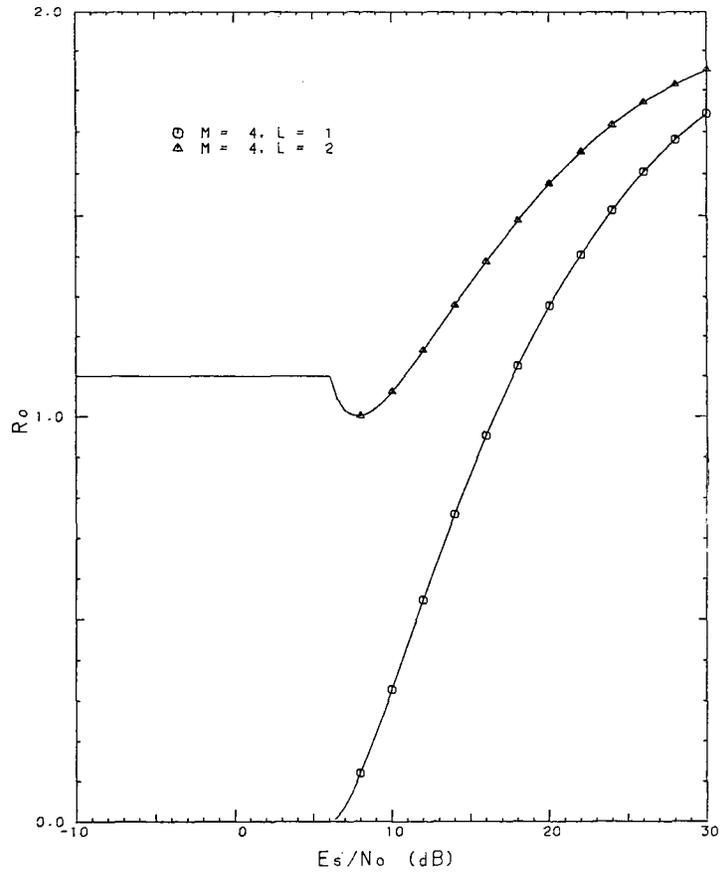


Fig. 8 —  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel

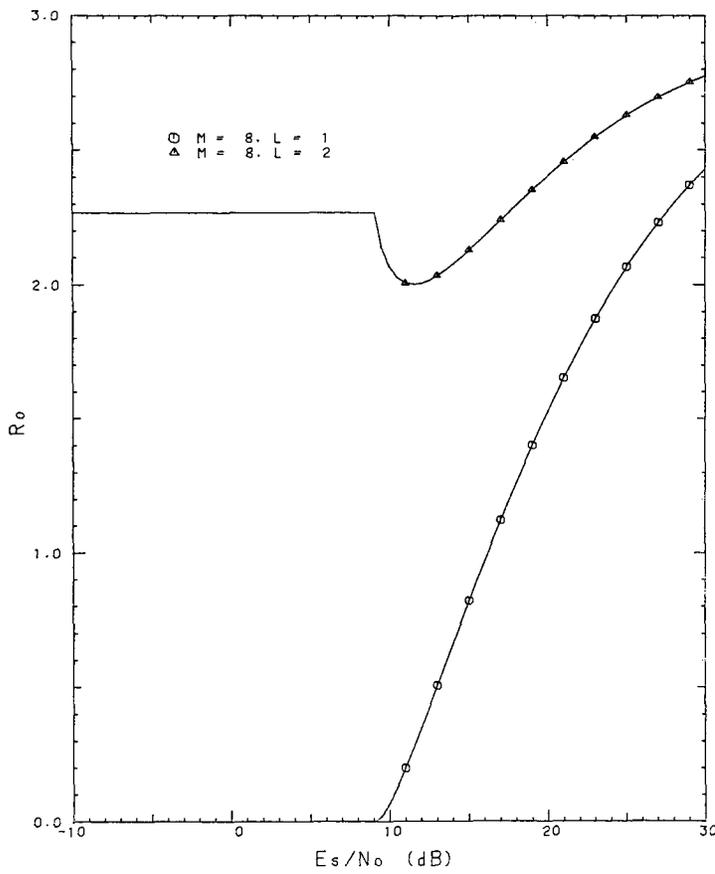


Fig. 9 —  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel

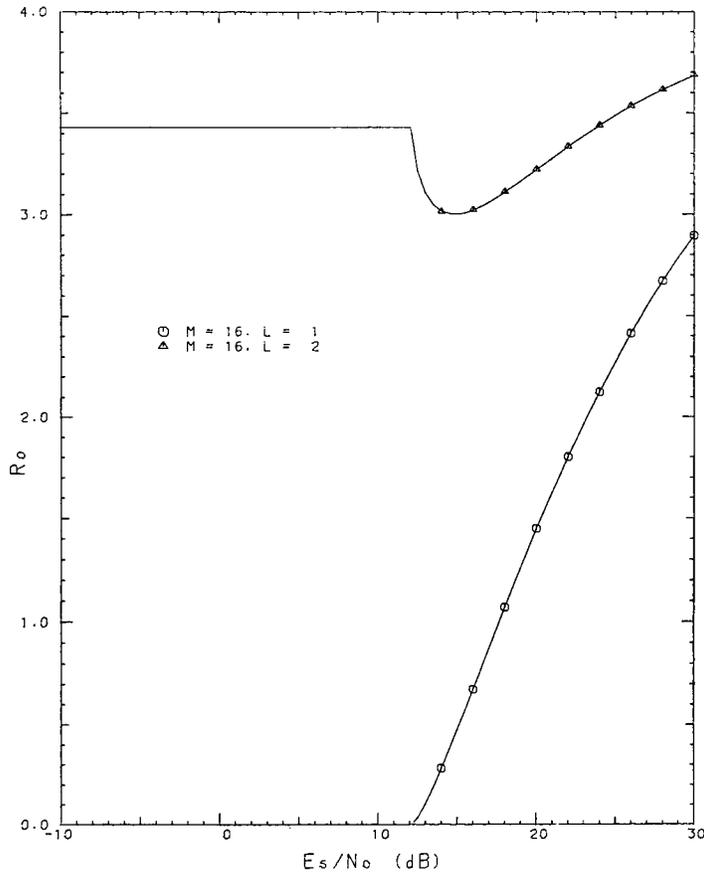


Fig. 10 —  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel

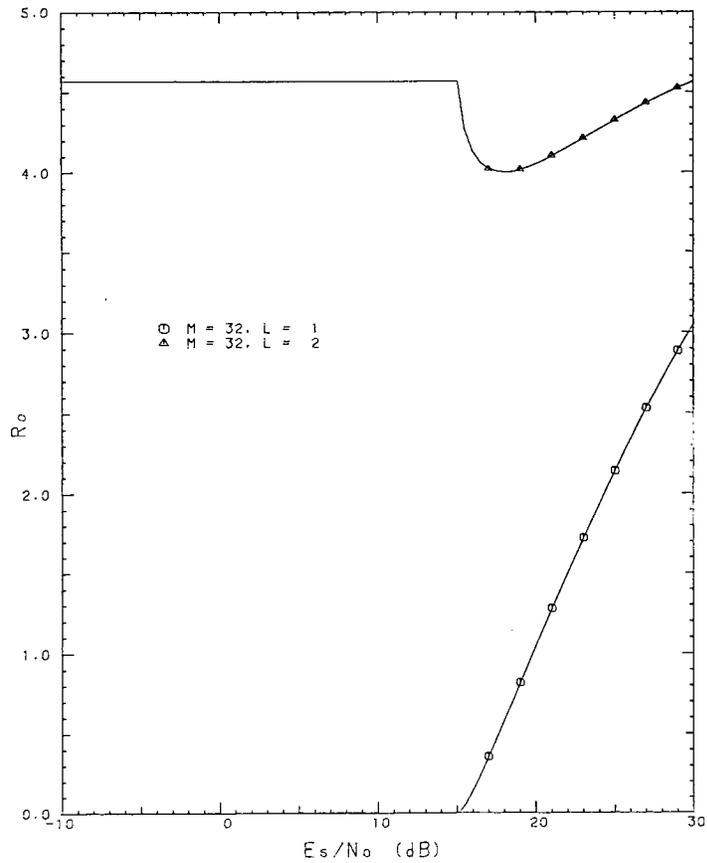


Fig. 11 —  $R_0$  vs  $E_s/N_0$  for optimal tone-jamming channel

Fig. 12 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

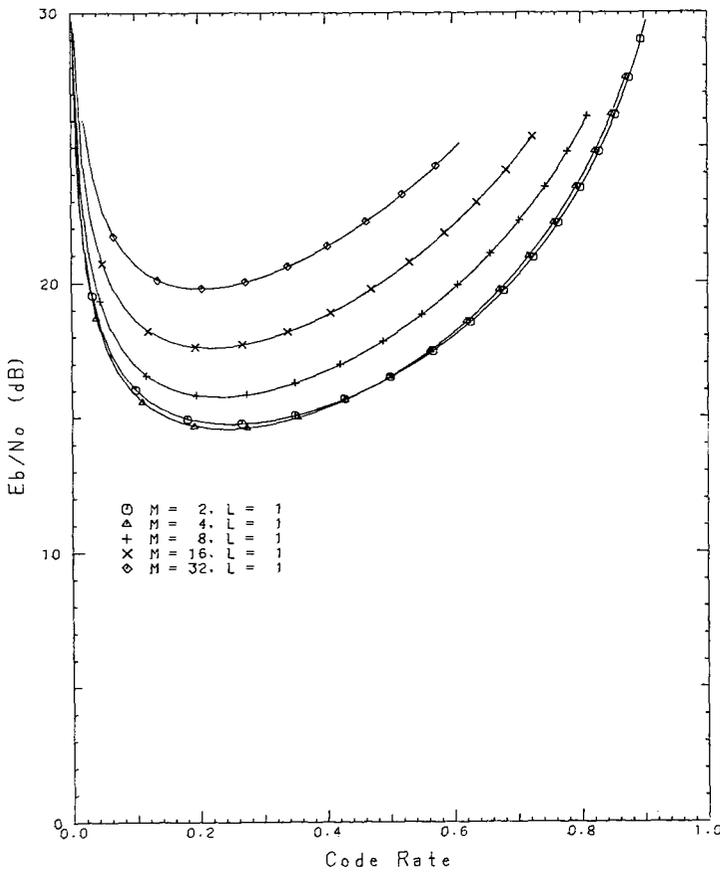
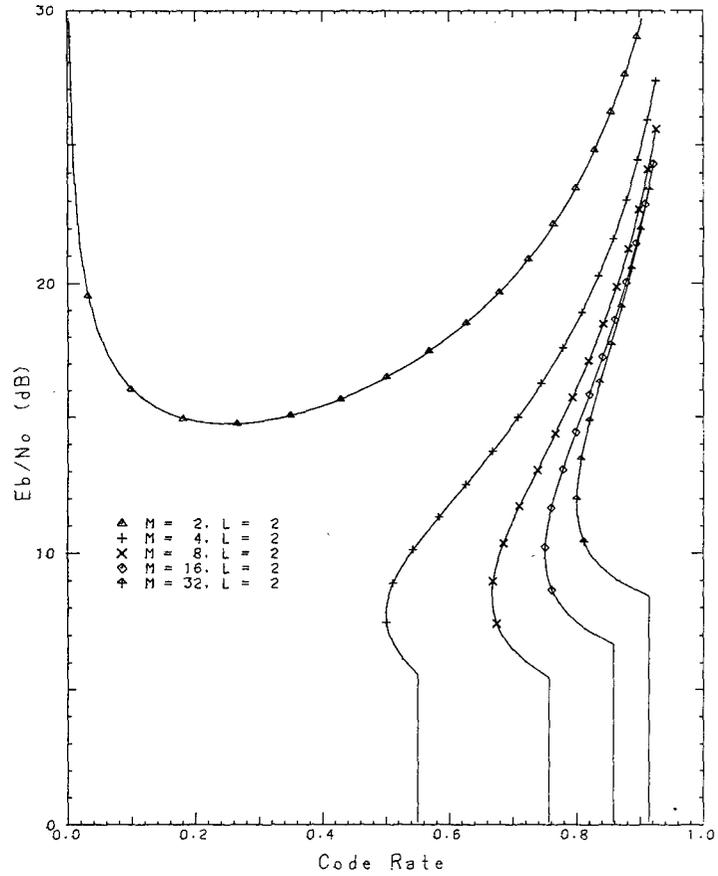


Fig. 13 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

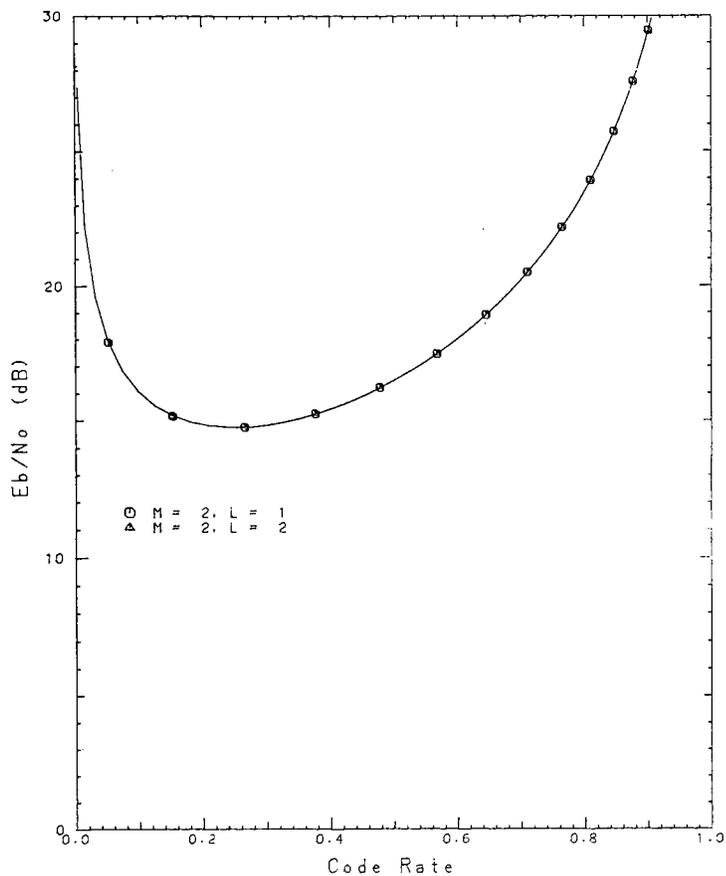


Fig. 14 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

Fig. 15 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

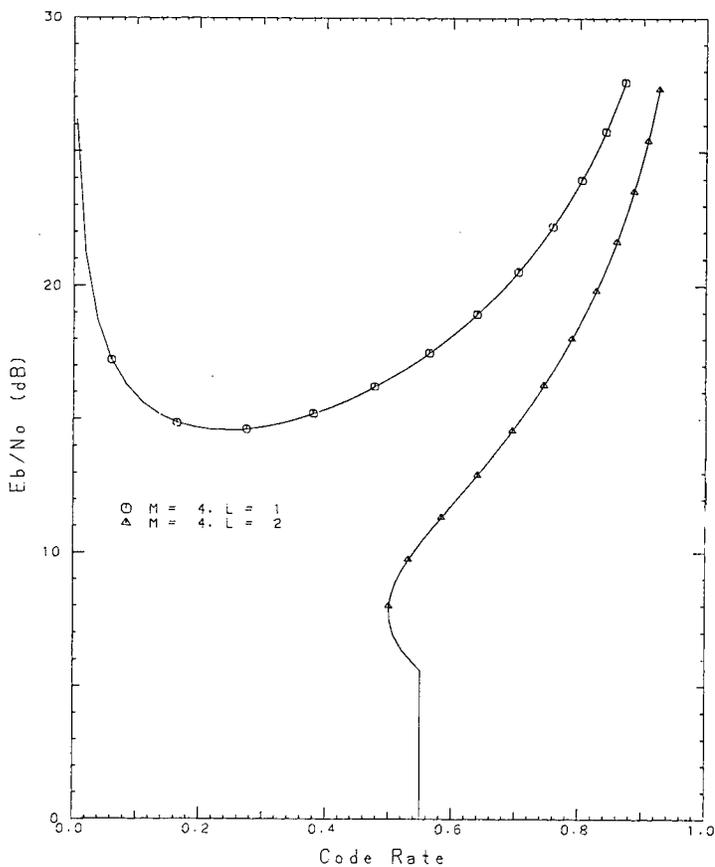


Fig. 16 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

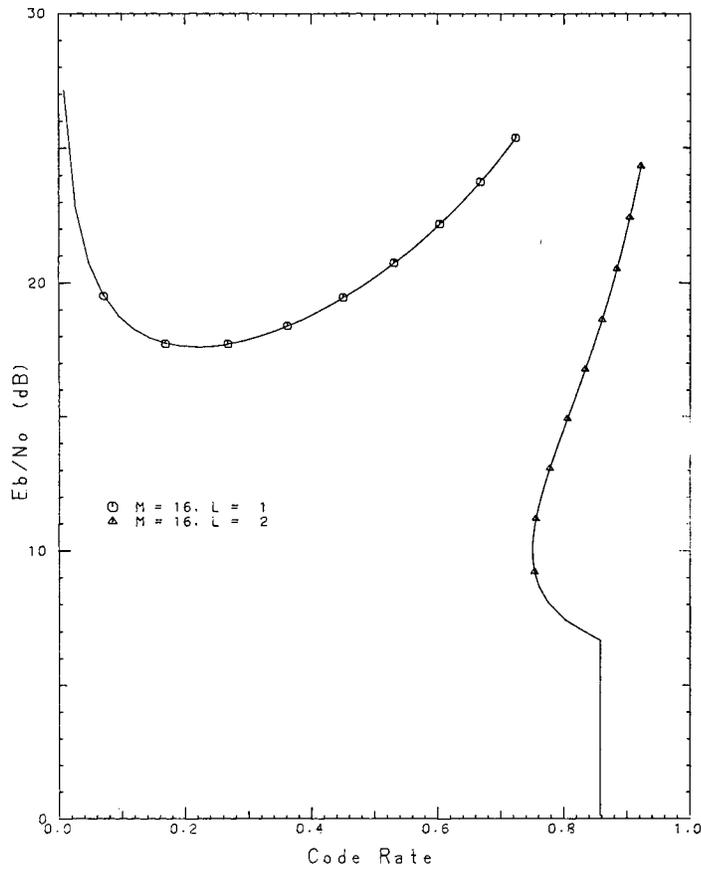
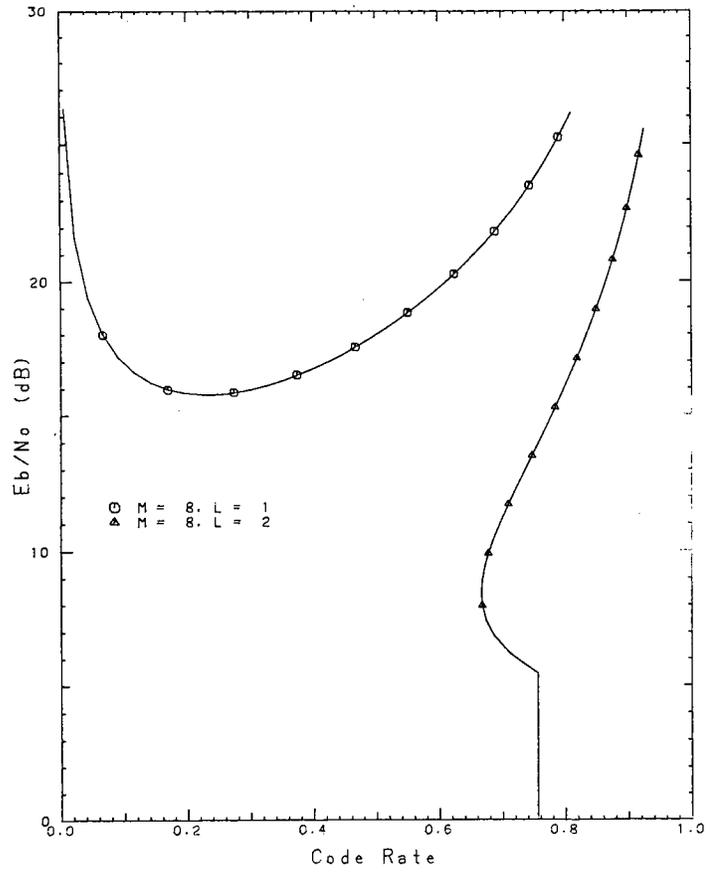


Fig. 17 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

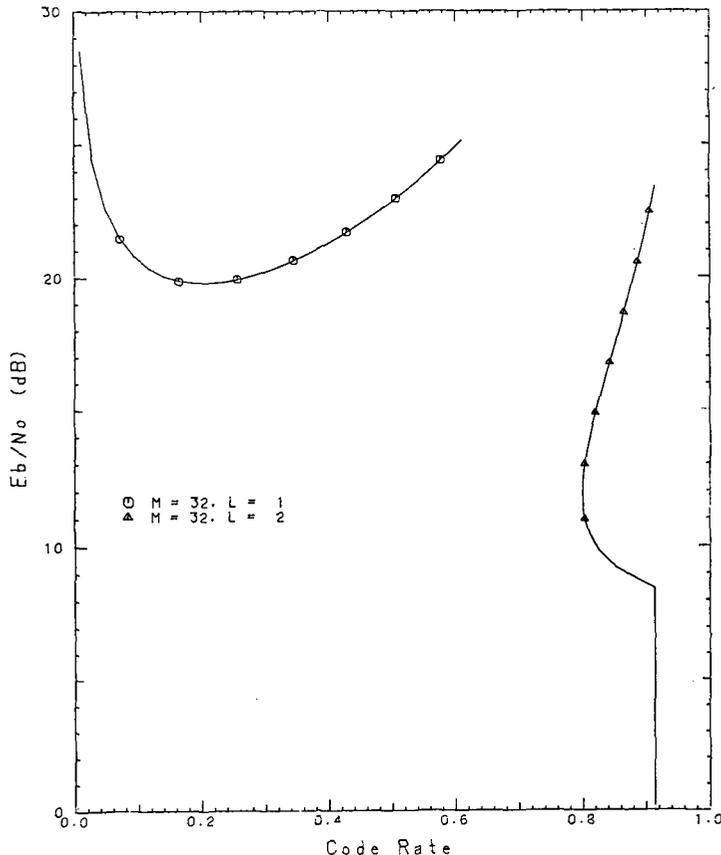


Fig. 18 -  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for optimal tone-jamming channel

The  $L = 1$  (hard decision) curves in Fig. 13 exhibit the usual concave characteristic for non-coherent systems, with the minimum  $E_b/N_0$  occurring at low code rates. At very low code rates,  $E_b/N_0$  increases due to combining losses which occur in coded noncoherent systems. In Figs. 14 through 18, we again see the improvement of list-of-2 detection over hard decision detection.

Note that the channel model assumed here is idealized and somewhat unrealistic because the jammer is constrained to a maximum of one jamming tone per subband. Clearly, if more jamming tones are used, the performance of list-of-2 detection would deteriorate and a higher list-of- $L$  detection scheme would be required. The purpose of the present study is merely to show the potential improvement available from using list-metric detection in a jamming environment. More complicated jamming models and larger list sizes are considered in a separate study on list metric detection [10].

### RAYLEIGH FADING CHANNELS

In this section we consider the performance of coded MFSK signaling with list metric detection on a Rayleigh fading channel. For this channel the received signal is

$$r(t) = r \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_i t + \theta) + n(t), \quad 0 \leq t \leq T_s, \quad (56)$$

where  $n(t)$  is an AWGN process with two-sided power spectral density  $N_0/2$ ,  $\theta$  is a uniformly distributed random phase variable, and  $r$  is a Rayleigh amplitude random variable with normalized probability density function

$$p(r) = 2re^{-r^2}, \quad r \geq 0. \quad (57)$$

With this normalization,  $r^2 E_s$  is the received energy random variable with average value  $\bar{E}_s = E_s$ . The amplitude and phase random variables are assumed to be constant over the duration of the symbol, but independent from one symbol to the next because of the memoryless property of the channel.

The optimal (maximum likelihood) receiver for  $M$  ary orthogonal signals on a noncoherent Rayleigh fading channel consists of a bank of  $M$  envelope detectors, one matched to each possible signal. The outputs of these envelope detectors are  $M$  independent Rayleigh random variables [9]. For the  $M - 1$  envelope detectors with no signal present (noise only) the output density is

$$p_N(\alpha) = \frac{2\alpha}{N_0} e^{-\frac{\alpha^2}{N_0}}, \quad \alpha \geq 0, \quad (58)$$

and for the envelope detector with signal plus noise present the output density is

$$p_{s+N}(\alpha) = \frac{2\alpha}{N_0 + E_s} e^{-\frac{\alpha^2}{N_0 + E_s}}, \quad \alpha \geq 0. \quad (59)$$

The probability that the sent signal appears  $l^{\text{th}}$  on the ordered list of demodulator outputs is given by this expression comes about because the cumulative probability distribution of

$$q_l = \binom{M-1}{l-1} \int_0^\infty \left[ 1 - e^{-\frac{\alpha^2}{N_0}} \right]^{M-l} \left[ e^{-\frac{\alpha^2}{N_0}} \right]^{l-1} \left[ \frac{2\alpha}{N_0 + E_s} e^{-\frac{\alpha^2}{N_0 + E_s}} \right] d\alpha, \quad l = 1, 2, \dots, M. \quad (60)$$

This is the probability that a Rayleigh random variable with variance parameter  $(N_0 + E_s)/2$  will be less than  $l - 1$  Rayleigh random variables with variance parameter  $N_0/2$  but greater than  $M - l$  others. The simplicity of this expression comes about because the cumulative probability distribution of a Rayleigh random variable is given by

$$\int_0^a \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} dr = 1 - e^{-\frac{a^2}{2\sigma^2}}. \quad (61)$$

To evaluate  $q_l$  in Eq. (60), we make the substitutions

$$u^2 = \frac{\alpha^2}{N_0},$$

and

$$1 + \frac{E_s}{N_0} = \frac{1}{b}.$$

It follows that

$$q_l = 2b \binom{M-1}{l-1} \int_0^\infty u [1 - e^{-u^2}]^{M-l} [e^{-u^2}]^{l-1} e^{-bu^2} du, \quad l = 1, 2, \dots, M. \quad (62)$$

Using the binomial expansion we have

$$[1 - e^{-u^2}]^{M-l} = \sum_{n=0}^{M-l} \binom{M-l}{n} (-1)^n e^{-nu^2}, \quad (63)$$

and Eq. (62) becomes

$$\begin{aligned} q_l &= 2b \binom{M-1}{l-1} \int_0^\infty u e^{-(l-1)u^2} e^{-bu^2} \sum_{n=0}^{M-l} \binom{M-l}{n} (-1)^n e^{-nu^2} du \\ &= 2b \binom{M-1}{l-1} \sum_{n=0}^{M-l} \binom{M-l}{n} (-1)^n \int_0^\infty u e^{-(b+n+l-1)u^2} du, \quad l = 1, 2, \dots, M. \end{aligned} \quad (64)$$

Since

$$\int_0^{\infty} u e^{-(n+l+b-1)u^2} du = \frac{1}{2(n+l+b-1)} \quad (65)$$

we may simplify Eq. (64) to become

$$q_l = b \binom{M-1}{l-1} \sum_{n=0}^{M-l} \binom{M-l}{n} (-1)^n \frac{1}{n+l+b-1}, \quad l = 1, 2, \dots, M \quad (66)$$

where

$$b = \frac{1}{1 + \frac{E_s}{N_0}}$$

From Eq. (66) we are able to express the ranking probability vector  $q$  as a function of  $E_s/N_0$ . With this it is possible to determine the cutoff rate parameter  $R_0$  as a function of  $E_s/N_0$ . This can be done for list-of- $L$  detection using the expression

$$R_0 = \log_2 M - 2 \log_2 \left[ \sum_{l=1}^L \sqrt{q_l} + \sqrt{(M-L) \left( 1 - \sum_{l=1}^L q_l \right)} \right]. \quad (67)$$

The plots of  $R_0$  vs  $E_s/N_0$  are given in Figs. 19 through 23. For convenience all of the  $L = 1$  (hard decision) curves are collected in Fig. 24.

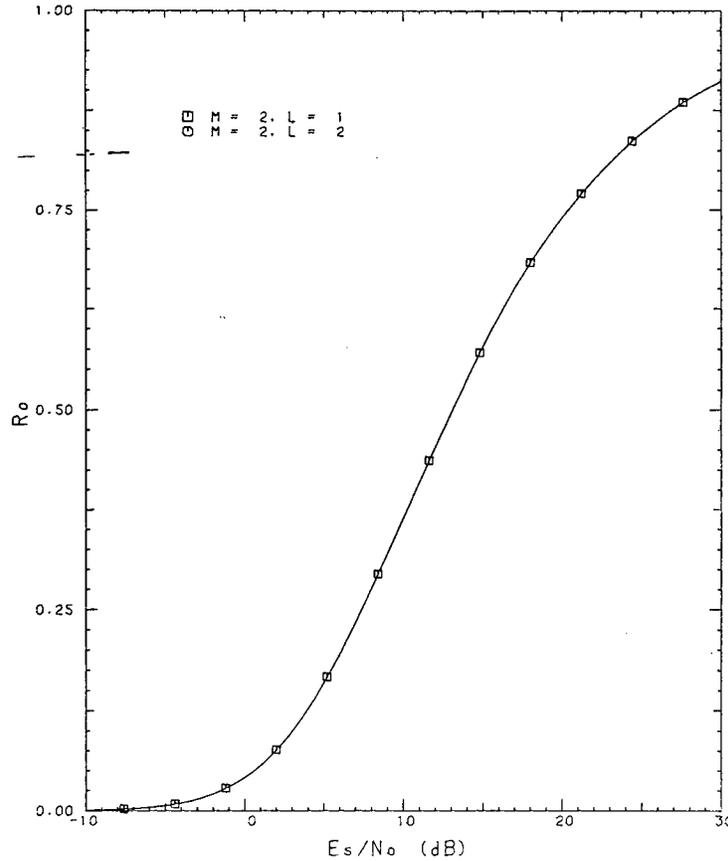


Fig. 19 -  $R_0$  vs  $E_s/N_0$  for Rayleigh fading channels

Fig. 20 —  $R_0$  vs  $E_s/N_0$  for Rayleigh fading channels

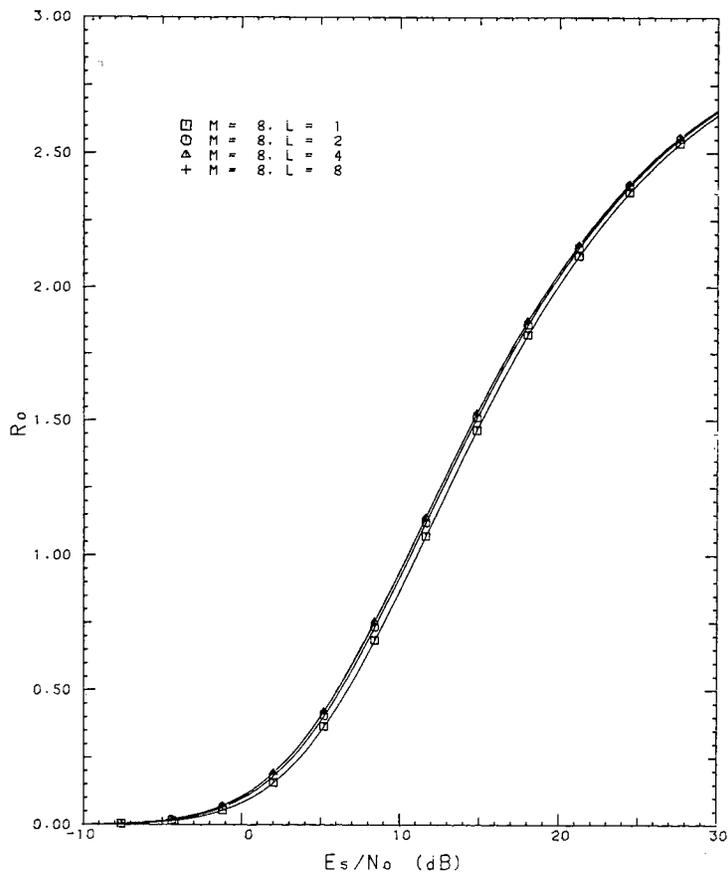
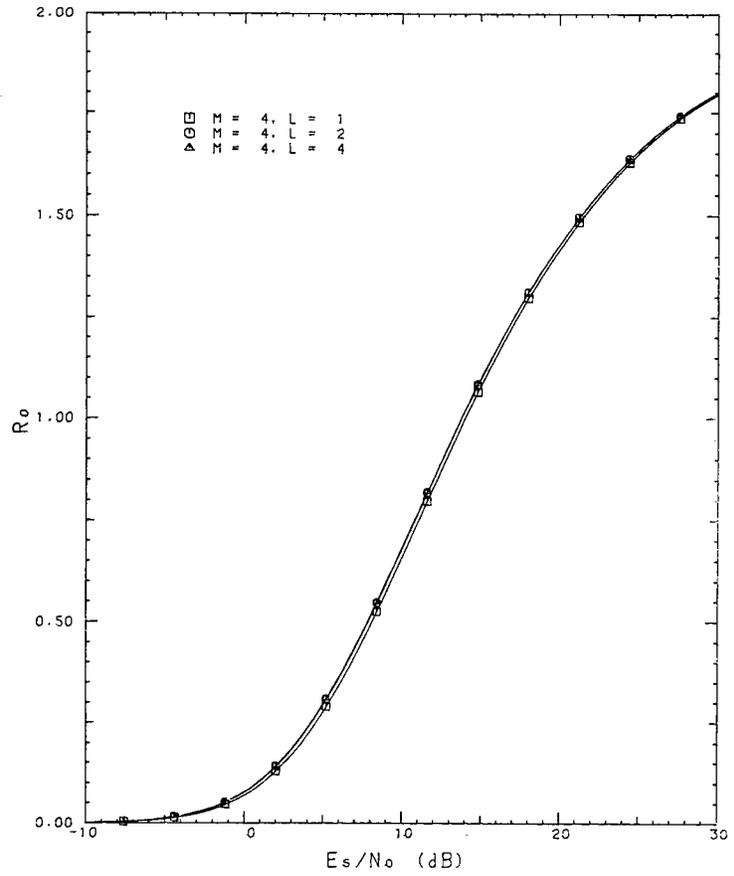


Fig. 21 —  $R_0$  vs  $E_s/N_0$  for Rayleigh fading channels

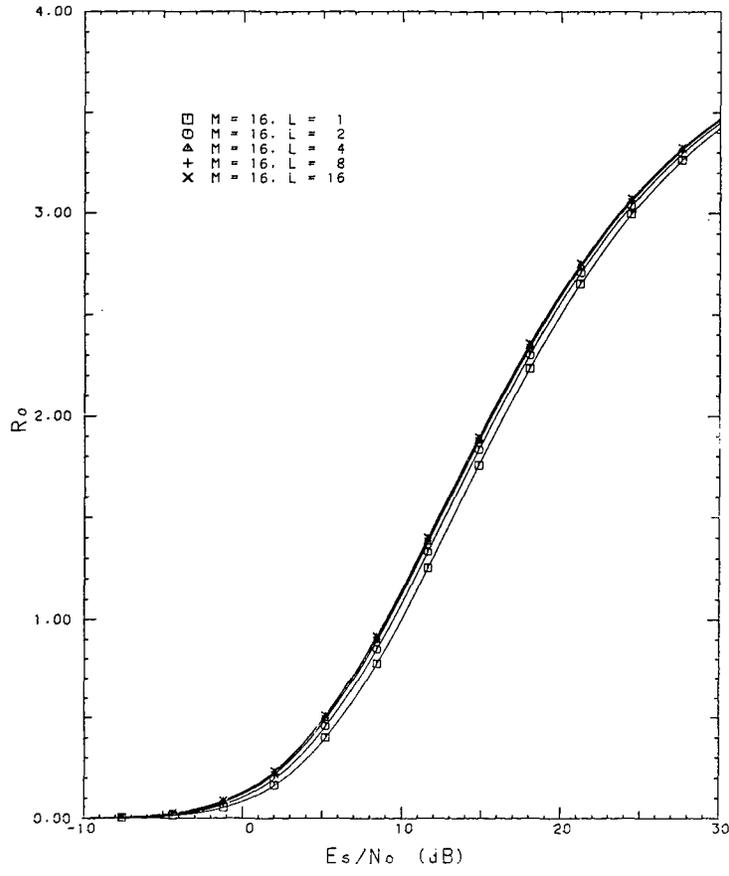
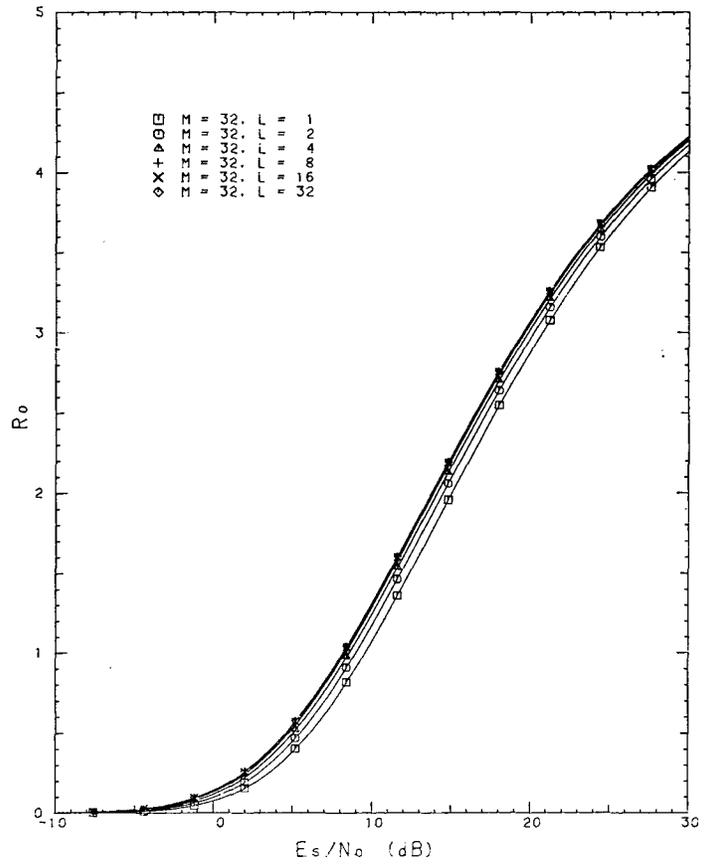


Fig. 22 —  $R_0$  vs  $E_s/N_0$  for Rayleigh fading channels

Fig. 23 —  $R_0$  vs  $E_s/N_0$  for Rayleigh fading channels



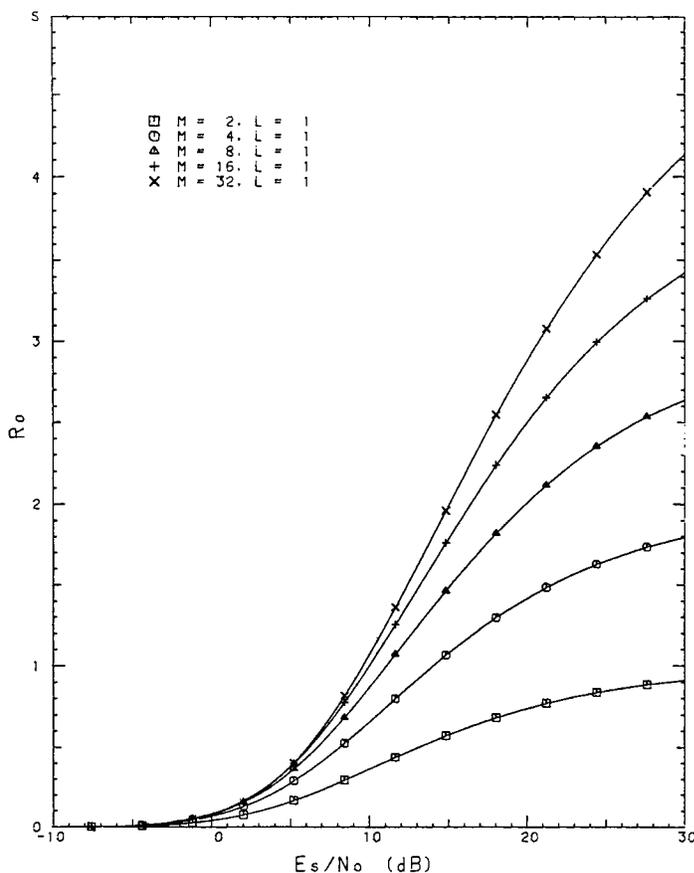


Fig. 24 —  $R_0$  vs  $E_s/N_0$  for Rayleigh fading channels

In Figs. 25 through 29 are plotted the minimum required  $E_b/N_0$  vs code rate  $R$  (information bits per channel bit), assuming that the encoder is operating at the cutoff rate. For this condition the code rate and  $E_b/N_0$  are given respectively by Eqs. (54) and (55). Again, the  $L = 1$  curves are gathered in Fig. 30. In Figs. 25 through 29 we see that it is possible to operate in the vicinity of  $E_b/N_0 = 10$  dB provided that the code rate is kept in the low-rate range, that is, approximately  $R = 1/4$  or less. From hard-decision detection to larger list sizes, there is a modest improvement (approximately 1 dB). This improvement mainly exists for small list sizes ( $L = 2$  and 4), and further improvement is negligible for  $L > 4$ . Thus, for Rayleigh fading channels, there is some improvement in using list-of- $L$  detection over hard-decision detection, but large list sizes offer little advantage.

### CONCLUSIONS AND RECOMMENDATIONS

This report contains the results of an introductory study on list-metric detection. This investigation was motivated by the need for more effective detection schemes to operate in fading and jamming environments. With list-metric detection, it is possible to achieve a performance improvement over hard-decision detection while avoiding the difficulties of pure soft-decision detection (without jammer state information) on intelligent interference channels.

List-metric detection is a general technique and includes list-of- $L$  detection as a special case. In this report, list-of- $L$  detection results are derived for MFSK/FH signaling on a Rayleigh fading channel and on an idealized worst-case partial band tone-jamming channel. Results show that the improvement (over hard-decision detection) for Rayleigh fading is modest but for the tone-jamming channel the advantage is considerable.

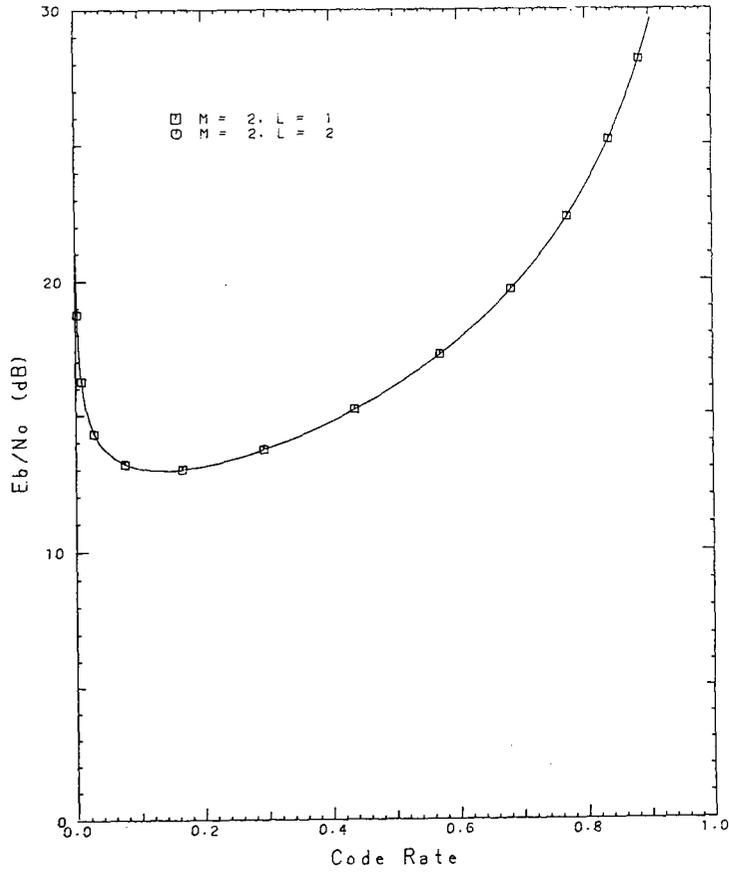


Fig. 25 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for Rayleigh fading channels

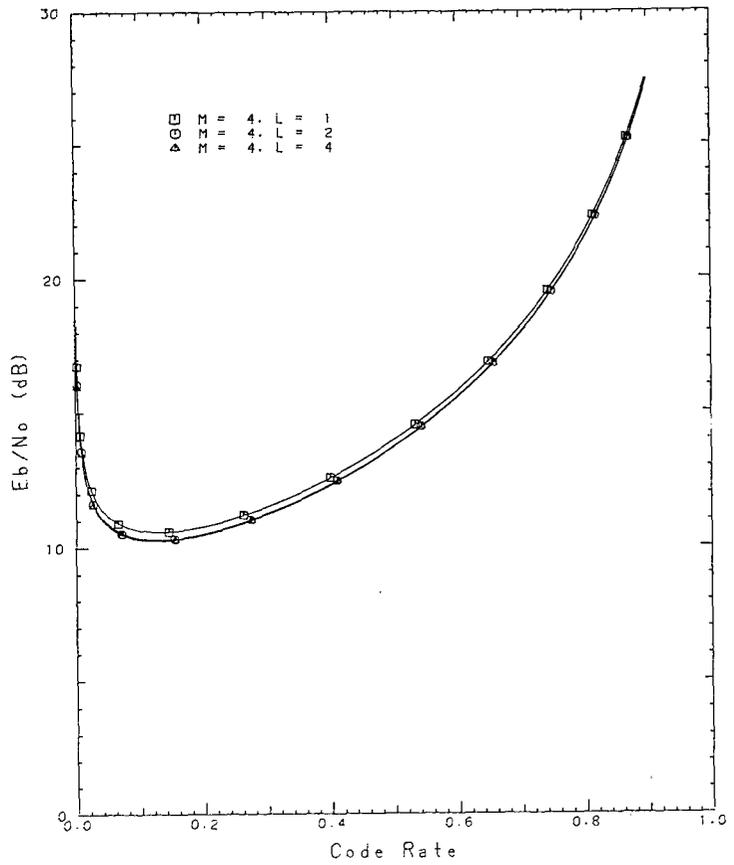


Fig. 26 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for Rayleigh fading channels

Fig. 27 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for Rayleigh fading channels

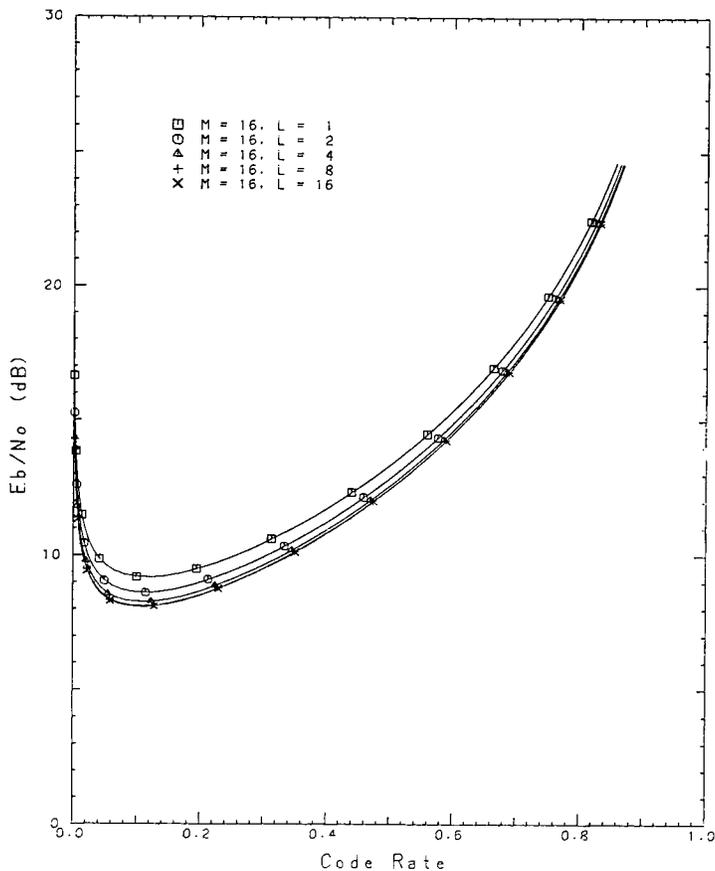
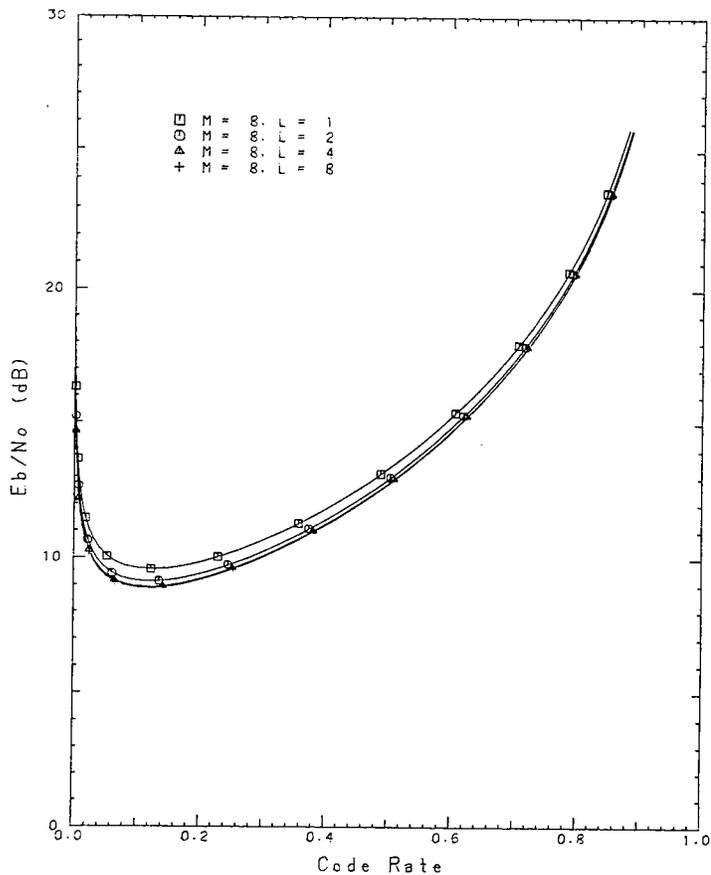


Fig. 28 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for Rayleigh fading channels

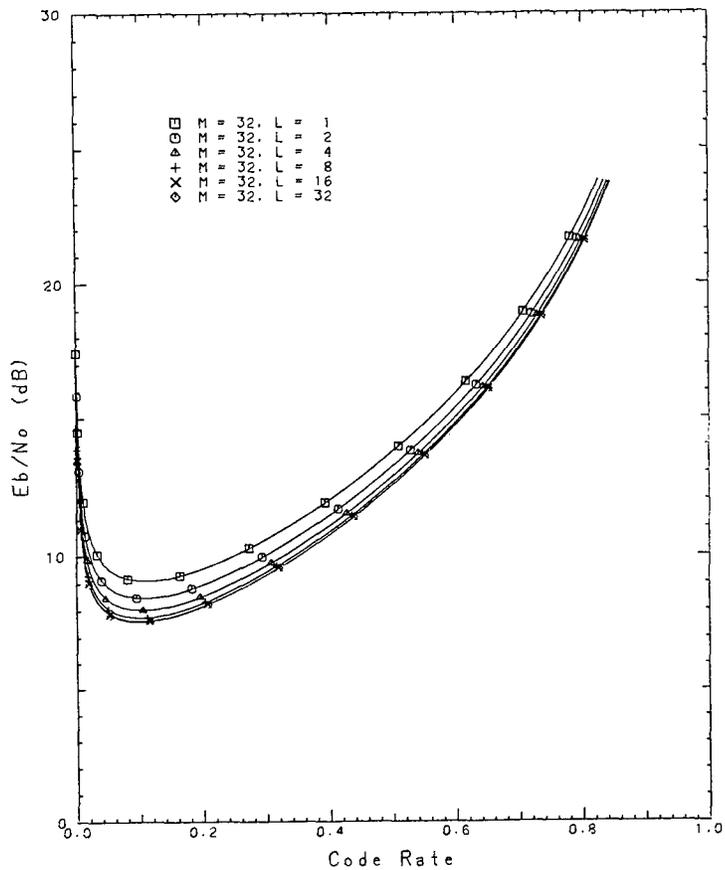
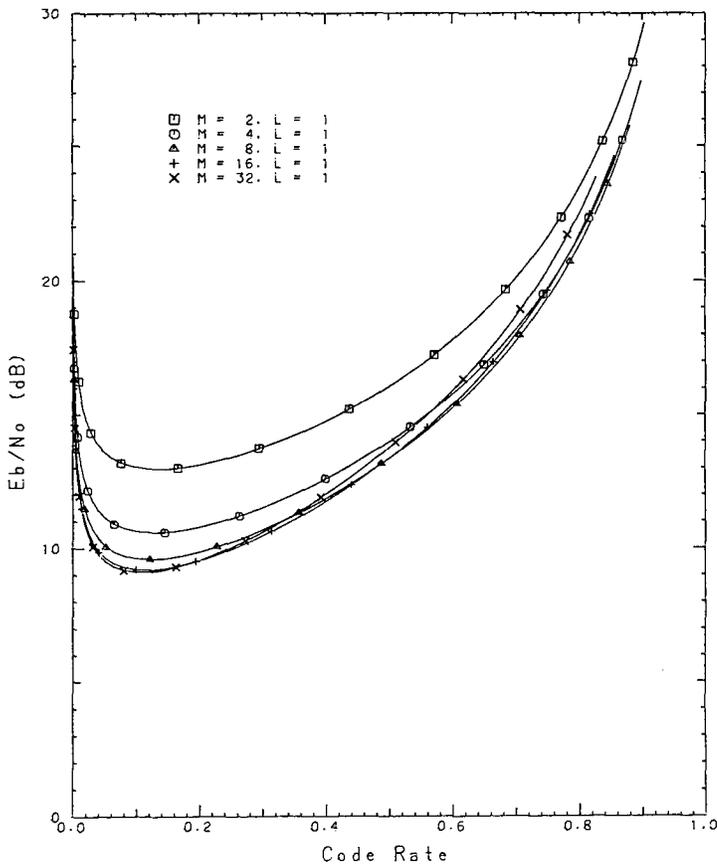


Fig. 29 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for Rayleigh fading channels

Fig. 30 —  $E_b/N_0$  vs code rate at  $R = R_0/\log_2 M$  for Rayleigh fading channels



In a companion study on list-metric detection [10], several aspects of the present investigation are extended and amplified. These extensions include:

- Analysis of list-metric detection on partial band Gaussian noise channels and partial band tone jamming channels with multiple tones per subband.
- Comparison of list-metric detection with and without jammer state information.
- Consideration of performance for suboptimal fixed assignment list metrics.

Refer to Ref. 10 for a detailed analysis of these issues. In general, their conclusions are that list-metric detection is effective (compared to soft-decision energy-metric detection) when no jammer state information is available and performs within 2 dB of soft-decision energy-metric detection when jammer state information is available. Furthermore, fixed metrics can be effective only over limited ranges of signal-to-noise ratio.

A key issue concerning the use of list-metric detection is the ability of the receiver to generate optimum (log-likelihood) list metrics. To do this, the receiver must estimate (at least imperfectly) the probability of the sent signal appearing at each position on the ordered list of outputs. In general, this can be a formidable problem for fading or jamming channels but, if accomplished, it can lead to an appreciable performance improvement *r* fixed assignment metrics.

Future studies in this research area should be concentrated on the problem of channel estimation for the purpose of adaptively generating optimal-list metrics.

#### ACKNOWLEDGMENT

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