NRL Report 8625

# Effects of Bandwidth Limitation on Polyphase Coded Pulse Compression Systems

B. L. LEWIS, F. F. KRETSCHMER, JR., AND F. C. LIN

*Target Characteristics Branch*
*Radar Division*

September 20, 1982

NAVAL RESEARCH LABORATORY
Washington, D.C.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

| | |
|---|---|
| 1. REPORT NUMBER: NRL Report 8625 | 2. GOVT ACCESSION NO. |
| 3. RECIPIENT'S CATALOG NUMBER | |
| 4. TITLE (and Subtitle): EFFECTS OF BANDWIDTH LIMITATION ON POLYPHASE CODED PULSE COMPRESSION SYSTEMS | 5. TYPE OF REPORT & PERIOD COVERED: An interim report on one phase of the problem. |
| | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s): B. L. Lewis, F. F. Kretschmer, Jr., and F. C. Lin | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Research Laboratory, Washington, DC 20375 | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 62712N, SF12-131-691, 53-0605-0-2 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS: Naval Sea Systems Command, Washington, DC 20362 | 12. REPORT DATE: September 20, 1982 |
| | 13. NUMBER OF PAGES: 16 |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) | 15. SECURITY CLASS. (of this report): UNCLASSIFIED |
| | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Digital signal processing
Pulse compression
Radar

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The effects of both pre- and postcompression bandwidth limitations on frequency-derived polyphase-coded pulse compressors are illustrated. Such band limitation is found to decrease the peak to maximum range-time-sidelobe ratios of the Frank and P3 coded compressor outputs but to significantly increase the ratio using the P1, P2, and P4 codes. The band limited P1, P2, and P4 coded systems were found to produce peak to maximum sidelobe ratios close to those of so-called perfect codes such as the Barker codes without restrictions on pulse-compression ratios.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

## CONTENTS

# EFFECTS OF BANDWIDTH LIMITATION ON POLYPHASE CODED PULSE COMPRESSION SYSTEMS

## INTRODUCTION

Bandwidth limitations are found in all well-designed radar systems. Such bandwidth limitations are the results of attempts to maximize the signal to thermal noise ratio in the receiver.

Radar receiver bandwidth limitations are very detrimental to some phase coded pulse compression systems but actually improve the performance of compressors that employ recently developed polyphase codes [1]. The purpose of this report is to document the effects of both pre- and postcompression bandwidth limitations on the performance of digitally implemented polyphase coded pulse compressors.

It is assumed that the radar transmits the phase codes as a train of contiguous constant-amplitude pulses with discrete phase changes from pulse to pulse in the train.

## POLYPHASE PULSE COMPRESSION CODES TO BE CONSIDERED

The codes to be considered in detail are limited to what have been called frequency derived polyphase codes [1-6], i.e., the Frank, P1, P2, P3, and P4 codes. These codes are the phase weights or their conjugates taken in succession that would be used in a digital filter to separate the resolvable frequency components of analog frequency modulation waveforms sampled at a rate equal to the bandwidth over which the frequency is varied (herein called the Nyquist rate). In this case, resolvable frequencies are separated by integer multiples of the reciprocal of the duration of the signal being processed in the digital filter.

For a pulse compression ratio $\rho = N^2$, where $N$ is the number of resolvable frequencies, the Frank code is defined by

$$\phi_{i,j} = (2\pi/N)(i-1)(j-1) \tag{1}$$

where $i = 1,2,3, \ldots N$ and $j = 1,2,3, \ldots N$. The index $i$ designates the $i$th steering weight of the $j$th frequency filter. In forming the code, $i$ ranges from 1 to $N$ for each value of $j$. For example, with $\rho = 16$ and $N = 4$, the Frank code would consist of 16 code elements $\phi_{1,1}, \phi_{2,1}, \phi_{3,1}, \phi_{4,1}, \phi_{1,2}, \phi_{2,2}, \ldots, \phi_{4,4}$ where $\phi_{2,2} = (2\pi/N)(2-1)(2-1) = \pi/2$. Note that the Frank code would be obtained by inphase "I" and quadrature "Q" detecting a step-approximation-to-a-linear-modulation-waveform (SALFMW) with a coherent local oscillator of frequency equal to the first frequency step of the SALFMW and sampling at the Nyquist rate starting at the leading edge of the waveform.

The P1 code is similar to the Frank code in being derived from a SALFMW. However, the local oscillator used in the I, Q detectors in deriving the P1 code would have a frequency equal to the average frequency of the SALFMW. Because of this difference, the Frank code can be thought of as the result of a single sideband detection while the P1 code would be the result of a double sideband detection. The phase of the $i$th element of the $j$th frequency of the P1 code is defined by

$$\phi_{i,j} = -(\pi/N)[N-(2j-1)][(j-1)N+(i-1)]. \tag{2}$$

Manuscript submitted May 25, 1982.

It should be noted that, for $N$ odd, the DC term is in the middle of the P1 code instead of at the beginning as in the Frank code. The P1 code has frequency symmetry about its center while the Frank code is unsymmetrical.

The P2 code differs from both the Frank and P1 codes by being derived from a Butler matrix such as used in phased array antennas. It is palindromic in that it has conjugate symmetry across each frequency or beam and even symmetry about the center of the code. The $i$th code element of the $j$th beam or frequency of the P2 code is defined by

$$\phi_{i,j} = \{(\pi/2)[(N-1)/N] - (\pi/N)(i-1)\}\{N+1-2j\}. \tag{3}$$

The P3 code is derived from an I, Q detected and sampled linear-frequency-modulation-waveform (LFMW) where the local oscillator frequency is the lowest frequency of the LFMW. The $i$th code element is defined by

$$\phi_i = \pi(i-1)^2/N^2 = \pi(i-1)^2/\rho. \tag{4}$$

The P4 code is obtained by moving the local oscillator to the center frequency of the LFMW and sampling at the Nyquist rate starting at the leading edge of the LFMW. The P4 code is defined by code element phases of

$$\phi_i = [\pi(i-1)^2/\rho] - \pi(i-1). \tag{5}$$

The P4 code differs from the P3 code by having the largest code element to code element phase changes on the ends of the code instead of the middle as in the P3 code. In this way, the P4 code differs from the P3 code in the same way as the P1 code differs from the Frank code.

Note that the P1 code can be made completely symmetrical (palindromic) by subtracting $\phi_{N,j}$ from each code element in the lower sideband frequencies when $N$ is odd. This makes the autocorrelation function real rather than complex. Similarly, the P3 and P4 codes can be made palindromic by taking the first sample of the LFMW 1/2 period of a code element after the leading edge of the waveform while still sampling at the Nyquist rate, i.e., I and Q sampling rates equal to the waveform bandwidth.

## PHASE CODED WAVEFORM SPECTRA

The spectrum of a Frank coded waveform with 100 code elements is illustrated in Fig. 1 out to the second nulls in the spectrum. The envelope is (sin $X$)/$X$, but there is unsymmetrical fine structure. The abscissa is normalized to a frequency equal to the reciprocal of the duration of a code element.

Figure 2 represents the spectrum of a P1 coded waveform. It also has a (sin $X$)/$X$ envelope, but its fine structure is symmetrical, unlike that of the Frank code. This difference is attributed to the differences of the time order of the frequencies represented by the code groups in the two waveforms.

The spectra of the P2 and P4 codes are very similar to that of the P1 code, and that of the P3 code resembles that of the Frank code.

Note that if the I, Q detected LFMW that was used to derive the P3 and P4 codes is sampled faster than at the Nyquist rate, the spectrum of the resultant phase code on a waveform changes dramatically from that of the P3 and P4 codes. Figures 3 and 4 show the spectra obtained by sampling at 2 and 5 times faster than the Nyquist rate. These spectra are nearly rectangular instead of (sin $X$)/$X$. This explains the difference in the peak range-time-sidelobes of the Frank and P codes compared to an analog LFMW compressor since a (sin $X$)/$X$ spectrum has a time function with zero sidelobes while a rectangular spectrum produces a (sin $X$)/$X$ time function. The phase codes derived by sampling at the Nyquist rate have peak sidelobe levels more than the pulse compression ratio below the match point

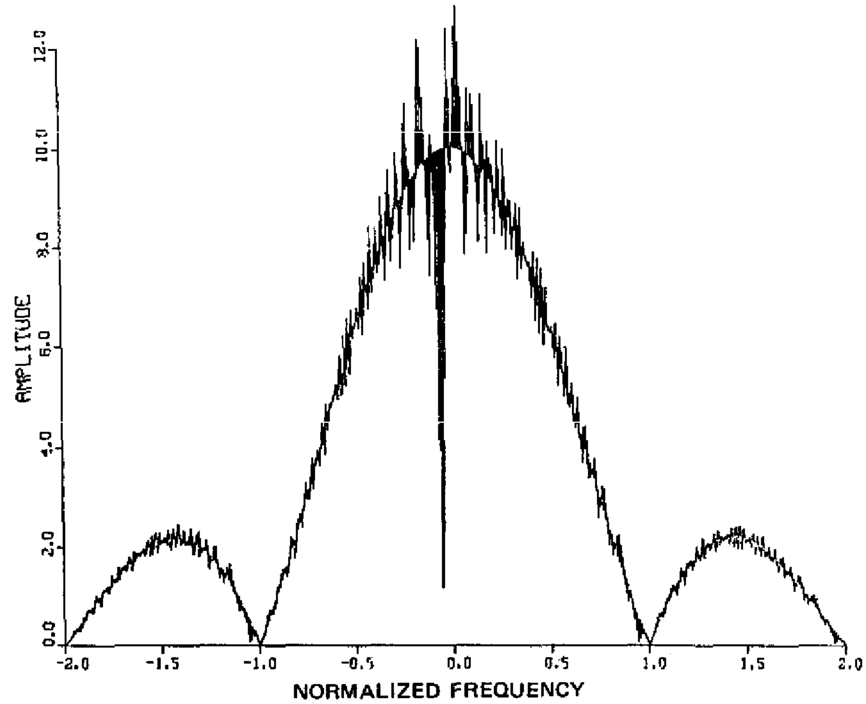


Fig. 1 — Spectrum of a Frank coded waveform

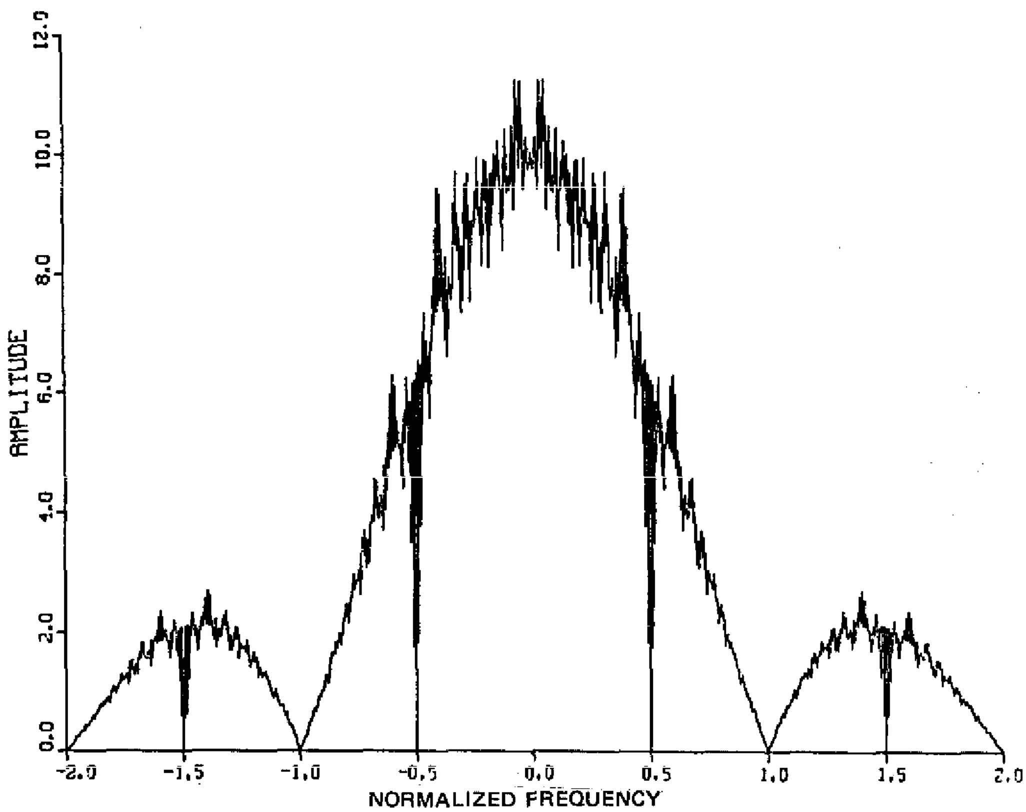


Fig. 2 — Spectrum of a P1 coded waveform

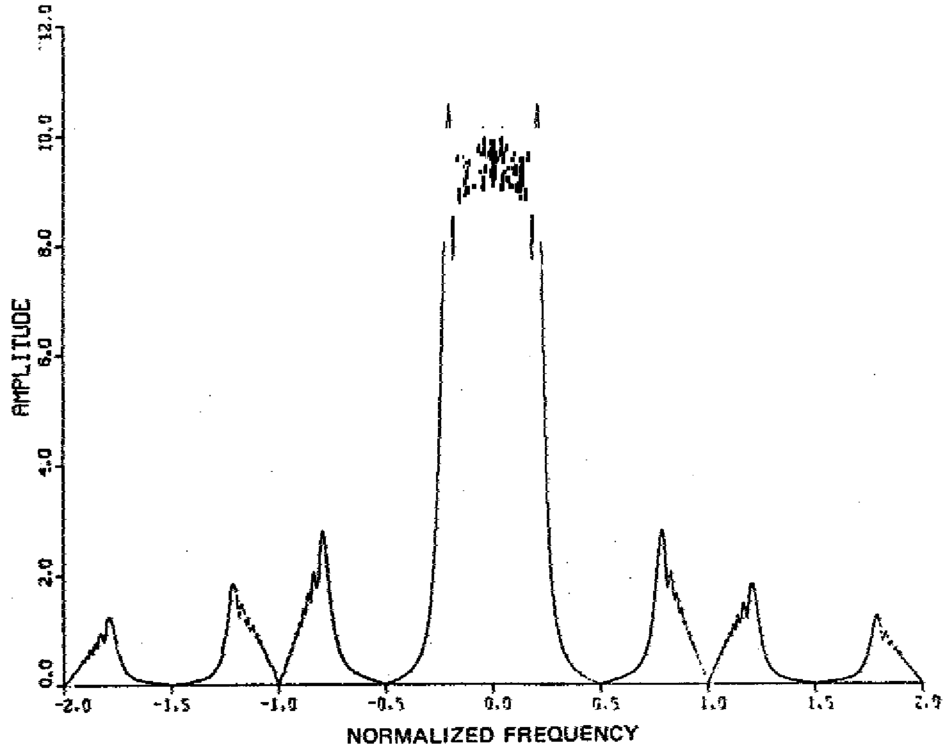


Fig. 3 — Spectrum of a linear frequency modulated waveform baseband sampled at twice the Nyquist rate

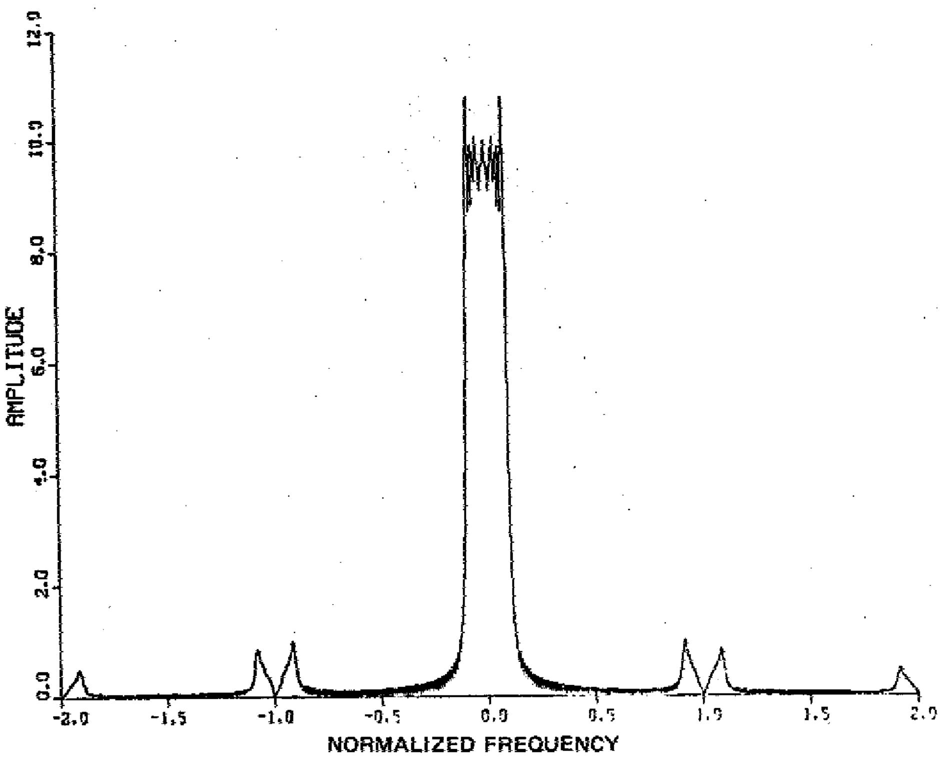


Fig. 4 — Spectrum of a linear frequency modulated waveform baseband sampled at 5 times the Nyquist rate

while the analog waveform has a (sin $X$)/$X$ autocorrelation function independent of the pulse compression ratio. The finite sidelobes in the time functions of the Nyquist rate phase coded compressor outputs are due to the fine structure in the phase coded waveform spectra.

## PRECOMPRESSION BANDLIMITING EFFECTS

The effects of precompression band-limitations on the response of polyphase coded pulse-compressors were evaluated by using two different techniques. In one technique, a fourth order Butterworth filter was placed ahead of the compressor and autocorrelation functions were determined for several different code leading edge arrival times with respect to a sampling pulse. In a second technique, the received code was sampled at 5 times the Nyquist rate and sliding window 5 sample averages (Fig. 5) were taken digitally to reduce the signal bandwidth. The resultant signal was then sampled at the Nyquist rate and compressed in a compressor matched to the unbandlimited code.

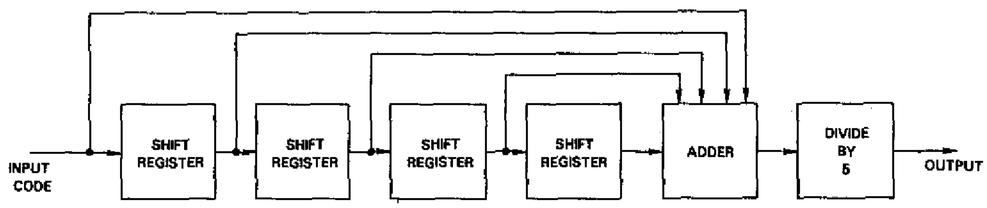


Fig. 5 — Sliding window 5 sample averager

Figures 6(a) and 7(a) show the real part or I values of a 144-element complex number Frank and modified P1 code [1] respectively. The modified P1 code was a rearranged Frank code representing 12 frequencies. The code group using the $\pi$ phase increments from code element to code element was placed on the left-hand side of the uncompressed waveform and was followed by the frequency groups having progressively smaller negative phase increments from code element to code element. This modification was employed to permit a 12-point fast Fourier transform (FFT) circuit to be used to generate and compress both codes. In this case, each code element of the waveform generated for the test was 0.5 $\mu$s in duration.

Figures 6(b) and 7(b) show the effect of a 2 MHz bandwidth fourth order Butterworth filter on the I parts of the codes. Figures 6(c), 6(d), 7(c), and 7(d) show the corresponding imaginary parts or Q values of the complex numbers specifying the code elements of the two codes without and with filtering, respectively,

Figures 6(e) and 7(e) show the autocorrelation functions (compressor outputs) of the two codes sampled 0.1 sampling period prior to the leading edge of the waveform. Note that the peak range-time-sidelobes of both compressor outputs are $\pi^2\rho = 144\ \pi^2$ below the peak response.

Figures 6(f), 6(g), 7(f), and 7(g) show the effect of the 2 MHz filter on the compressor outputs using the two different codes sampled 0.3 and 0.5 sampling periods ahead of the leading edge of the waveforms. In both cases, the average peak gain loss was 1.5 dB. However, the filter reduced the P1 code peak range-time-sidelobes by an average of 3 dB without changing the Frank code peak sidelobe levels. This occurred because the filter effectively amplitude reduced the edges of the P1 code waveform more than the center while it reduced the amplitude of the center of the Frank code waveform more than its edges. In this way, the filter decreased the average Frank code peak response to peak sidelobe ratio by 1.5 dB but decreased the P1 average ratio by 1.5 dB.

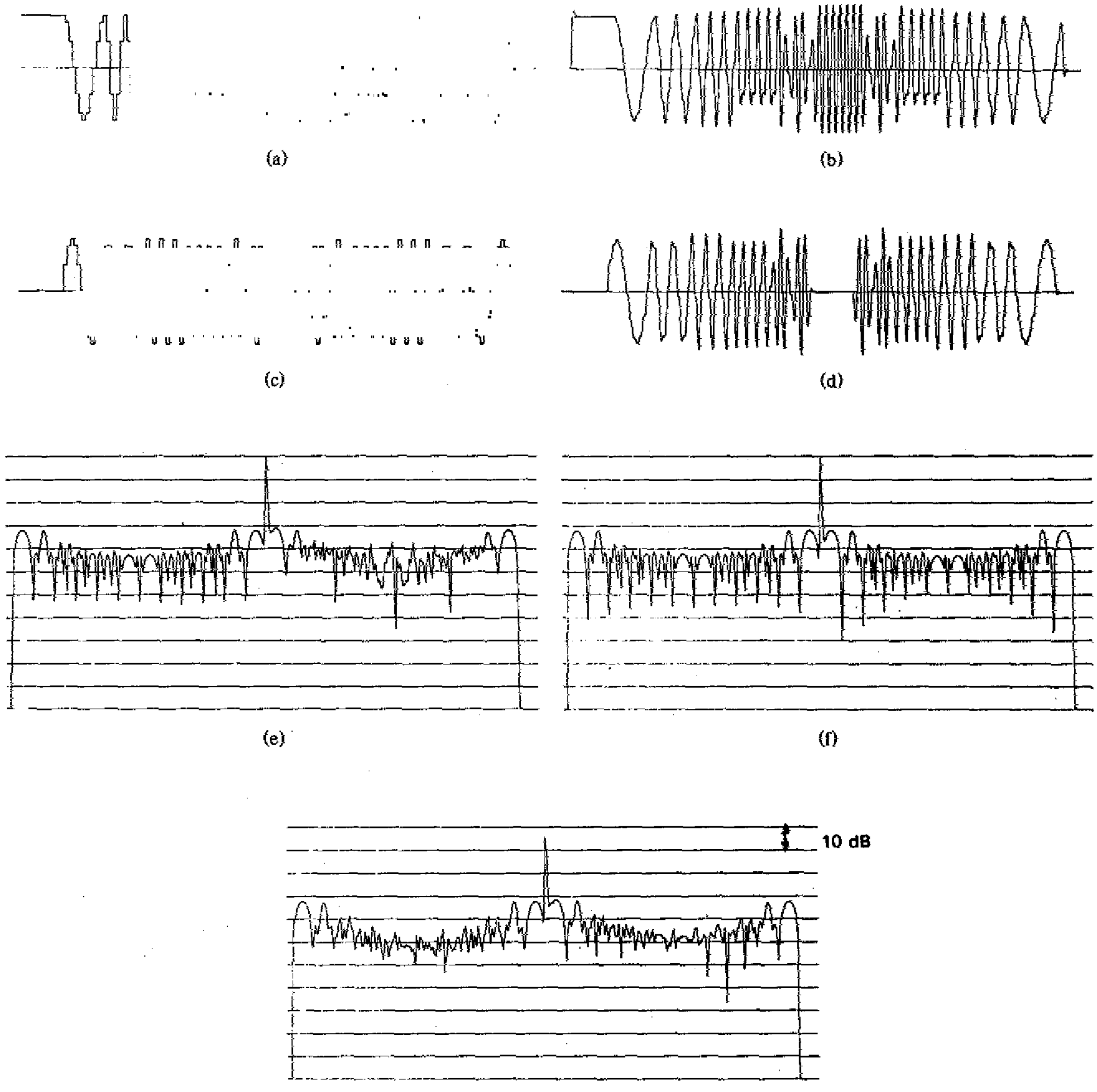


Fig. 6 — Frank code, $\rho = 144$. (a) Unfiltered I; (b) filtered I, 2 MHz; (c) unfiltered Q; (d) filtered Q, 2 MHz; (e) autocorrelation function sampled 0.1 sampling period early; (f) sampled 0.3 sampling period early; (g) sampled 0.5 sampling period early.

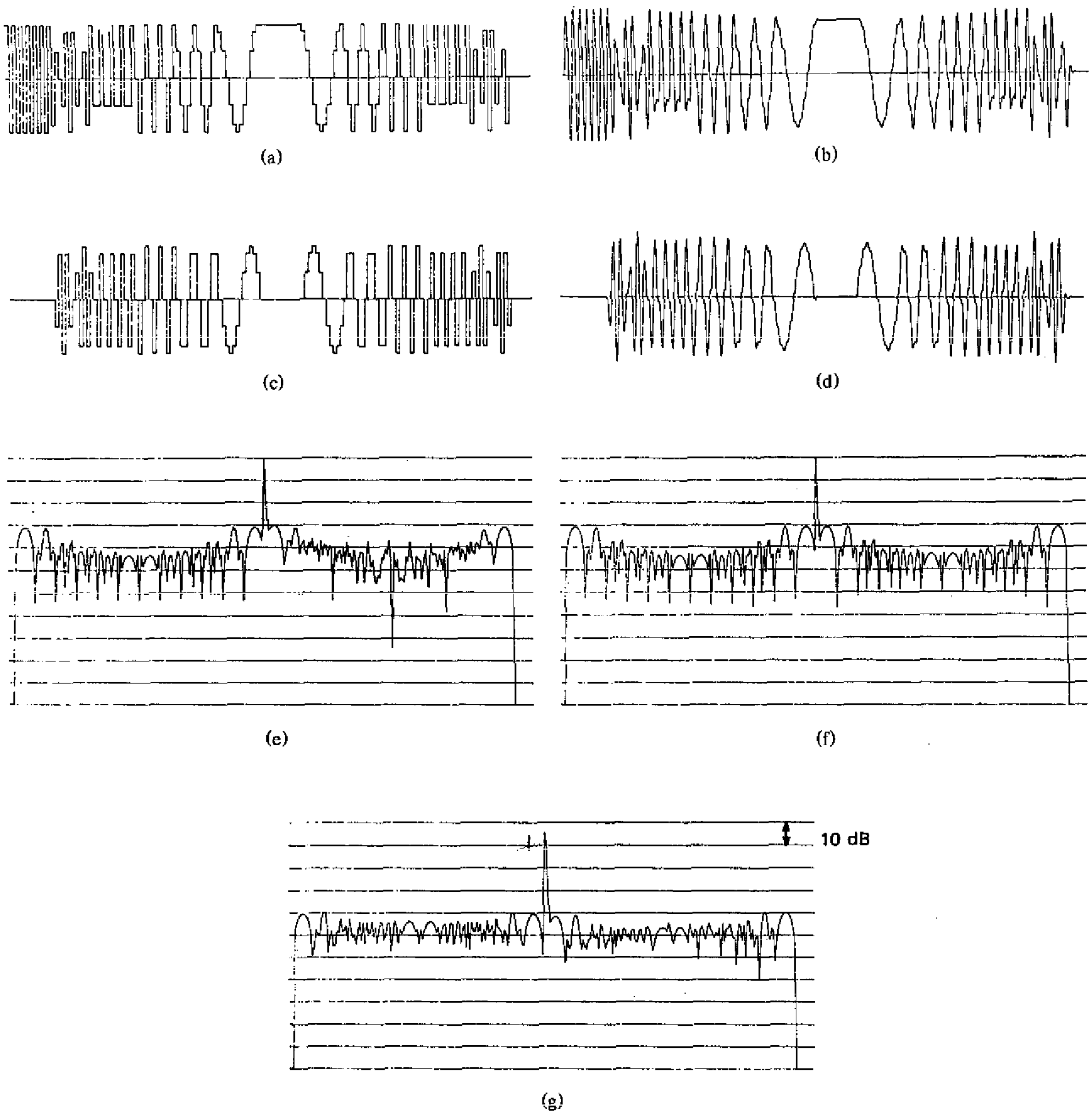


Fig. 7 — P1 code, $\rho$ = 144. (a) Unfiltered I; (b) filtered I, 2 MHz; (c) unfiltered Q; (d) filtered Q, 2 MHz; (e) autocorrelation function sampled 0.1 sampling period early; (f) sampled 0.3 sampling period early; (g) sampled 0.5 sampling period early.

Figures 8 and 9 show the effect of reducing the filter bandwidth to 1 MHz without changing the input signal bandwidth of 2 MHz. Note that the filter drops both peaks by the same amounts without significantly altering the Frank code peak sidelobes. However, the P1 code peak sidelobes are reduced.

The P3 code yielded results similar to those shown in Figs. 6 and 8 for the Frank code, and the P2 and P4 codes yielded results similar to those shown in Figs. 7 and 9 for the P1 codes.

Figure 10 illustrates the effect (averaged over the possible arrival times) of oversampling the polyphase codes by 5 to 1, averaging with a 5 sample sliding window and pulse compressing with a compressor matched to the unfiltered phase code. Again, each code had 100 code elements. Note that the peak sidelobes of the Frank and P3 codes were unaffected while the peak sidelobes of the P1, P2, and P4 codes were reduced.

## POSTCOMPRESSION BANDLIMITING EFFECTS

Postcompression bandlimiting effects were investigated by using a two-sample sliding window adder on the output of a digital pulse compressor with pulse compression ratios of 100 and 400.

Figure 11 illustrates the effect of a two-sample sliding window adder after pulse compression on the various polyphase codes with a pulse compression ratio $\rho$ of 100. Figure 12 is similar data for a pulse compression ratio $\rho$ of 400. Note that the peak response is not changed because the output of the adder was not divided by 2 to normalize the data. Note also that the Frank and P3 code peak sidelobes increase by 6 dB due to the adder so the adder would not have changed them if the data had been normalized by dividing by 2 after addition. The most important thing to note, however, is that the peak sidelobes of the P1, P2, and P4 codes are actually reduced by a significant amount, about 5 dB for $\rho = 100$ and 11 dB for $\rho = 400$. Thus, if the data had been normalized, the peak response would have dropped by 6 dB and the peak sidelobes by 11 dB and 17 dB for $\rho = 100$ and $\rho = 400$ respectively with the P1, P2, and P4 codes.

It is interesting to note that with $\rho = 100$ or $\rho = 400$ and the sliding window two sample adder, the peak sidelobes of the P1, P2, and P4 codes are only about 5 dB above those that would have been obtained with a so-called perfect code like the Barker code [5] if one could be found with these high pulse compression ratios. The Barker codes have peak sidelobes down by the square of the pulse compression ratio.

## CONCLUSIONS

The symmetrical P1, P2, and P4 code peak to range-time-sidelobe ratios are increased by either pre- or postcompression bandlimiting. Such bandlimiting, however, reduces this ratio using the Frank or P3 code.

Bandlimiting the P1, P2, or P4 code results in peak to maximum range time sidelobes within 6 dB of those of a so-called perfect code like the Barker codes. However, the P1, P2, and P4 codes can provide any desired pulse compression ratio in contrast to the Barker codes that are limited to pulse compression ratios of 13.

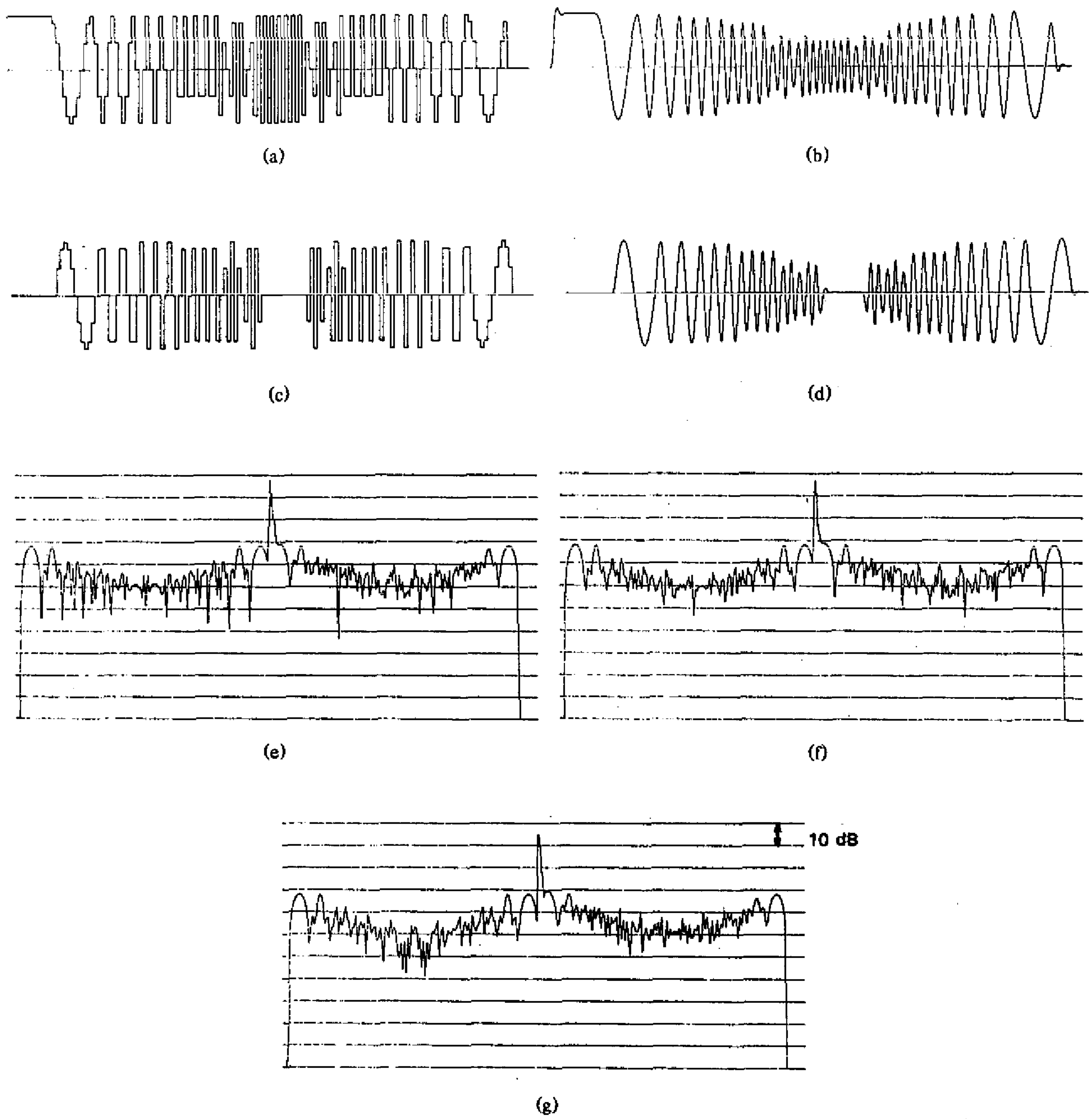


Fig. 8 — Frank code, $\rho$ = 144. (a) Unfiltered I; (b) filtered I, 1 MHz; (c) unfiltered Q; (d) filtered Q, 1 MHz; (e) autocorrelation function sampled 0.1 sampling period early; (f) sampled 0.3 sampling period early; (g) sampled 0.5 sampling period early.

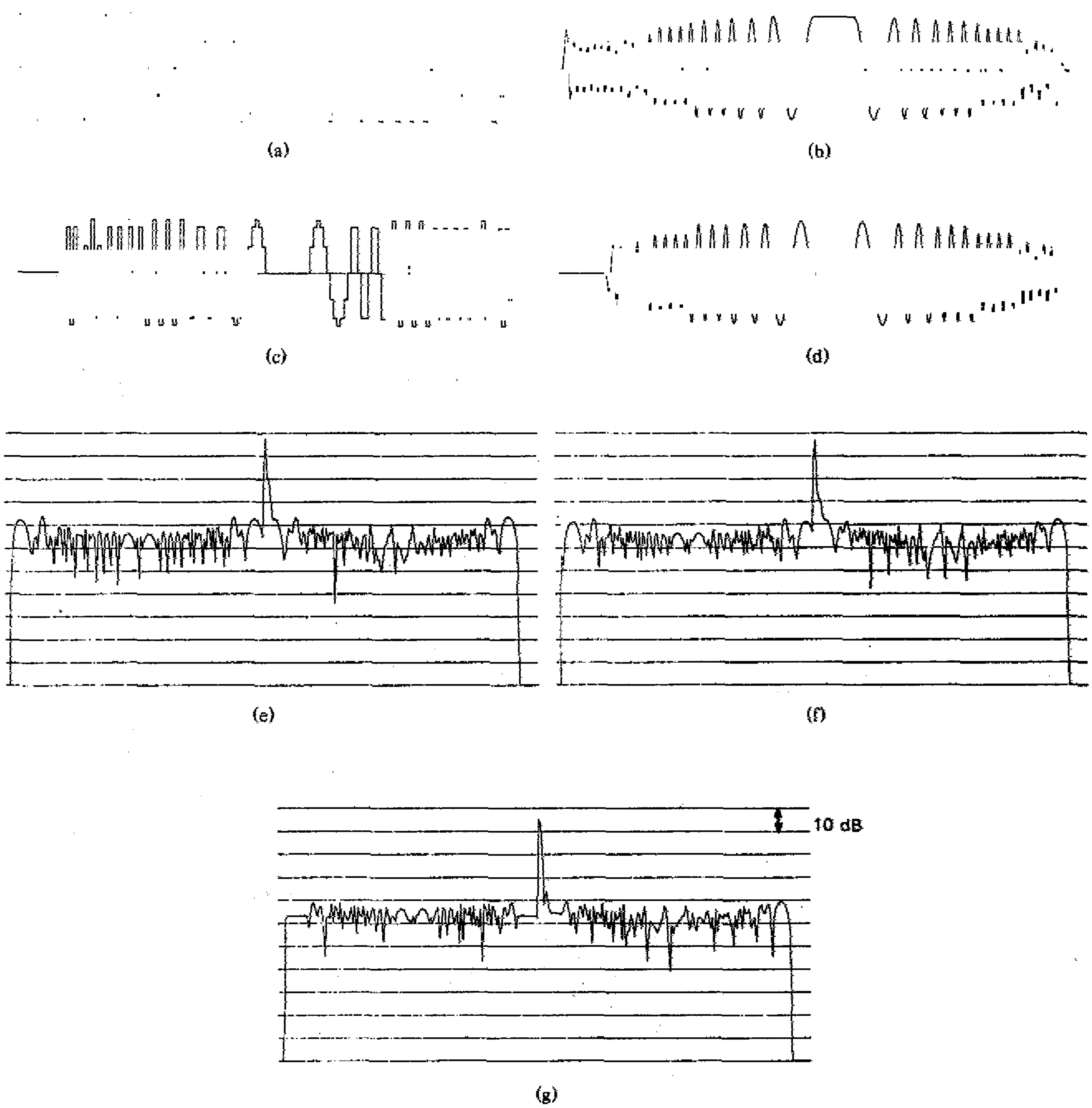


Fig. 9 — P1 code, $\rho = 144$. (a) Unfiltered I; (b) filtered I, 1 MHz; (c) unfiltered Q; (d) filtered Q, 1 MHz; (e) autocorrelation function sampled 0.1 sampling period early; (f) sampled 0.3 sampling period early; (g) sampled 0.5 sampling period early.

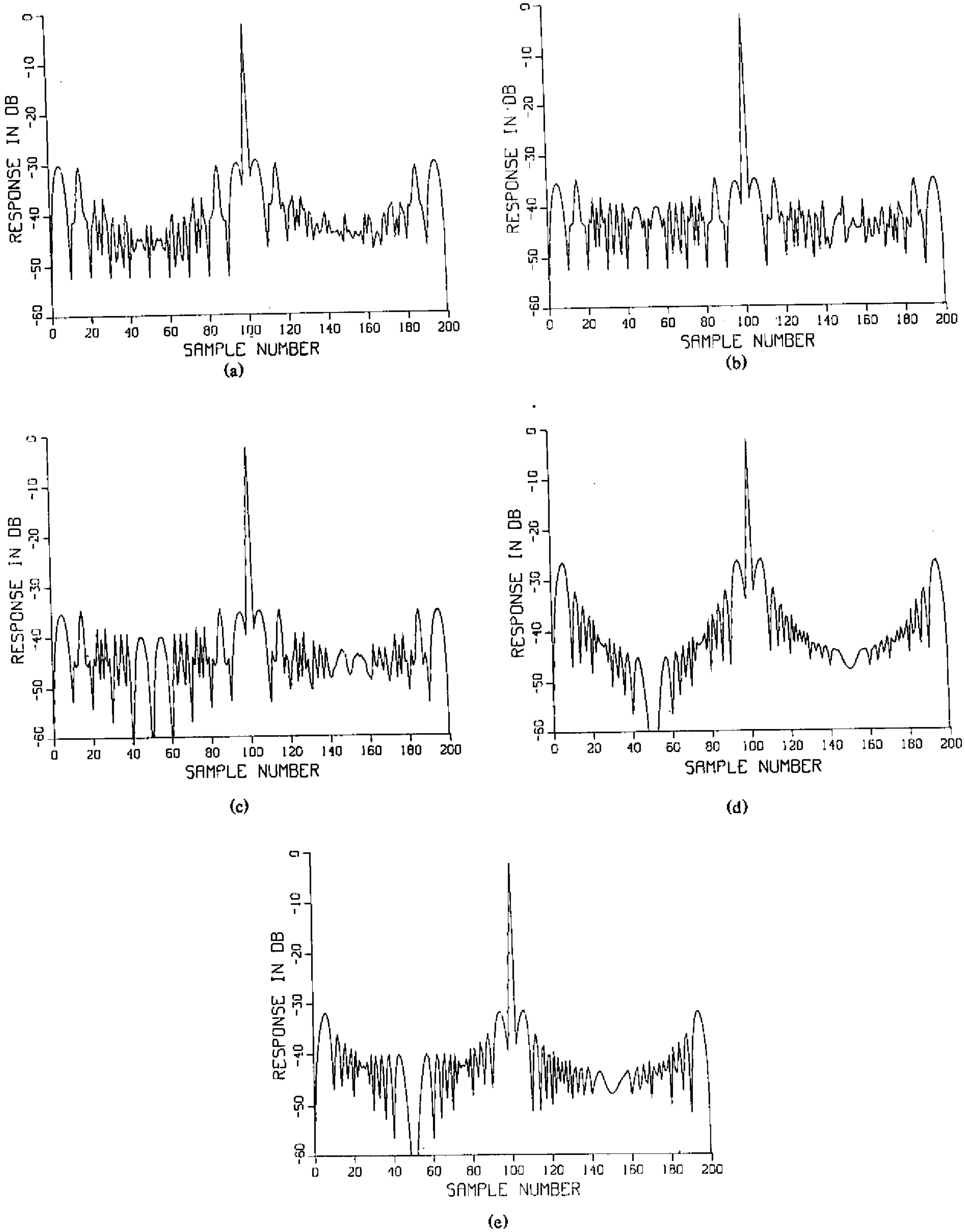


Fig. 10 — Average response over arrival times of various code compressors to waveforms over sampled by 5, averaged by 5 and resampled at the Nyquist rate for the unbandlimited waveform with $\rho$ = 100. (a) Frank code; (b) P1 code; (c) P2 code; (d) P3 code; (e) P4 code.

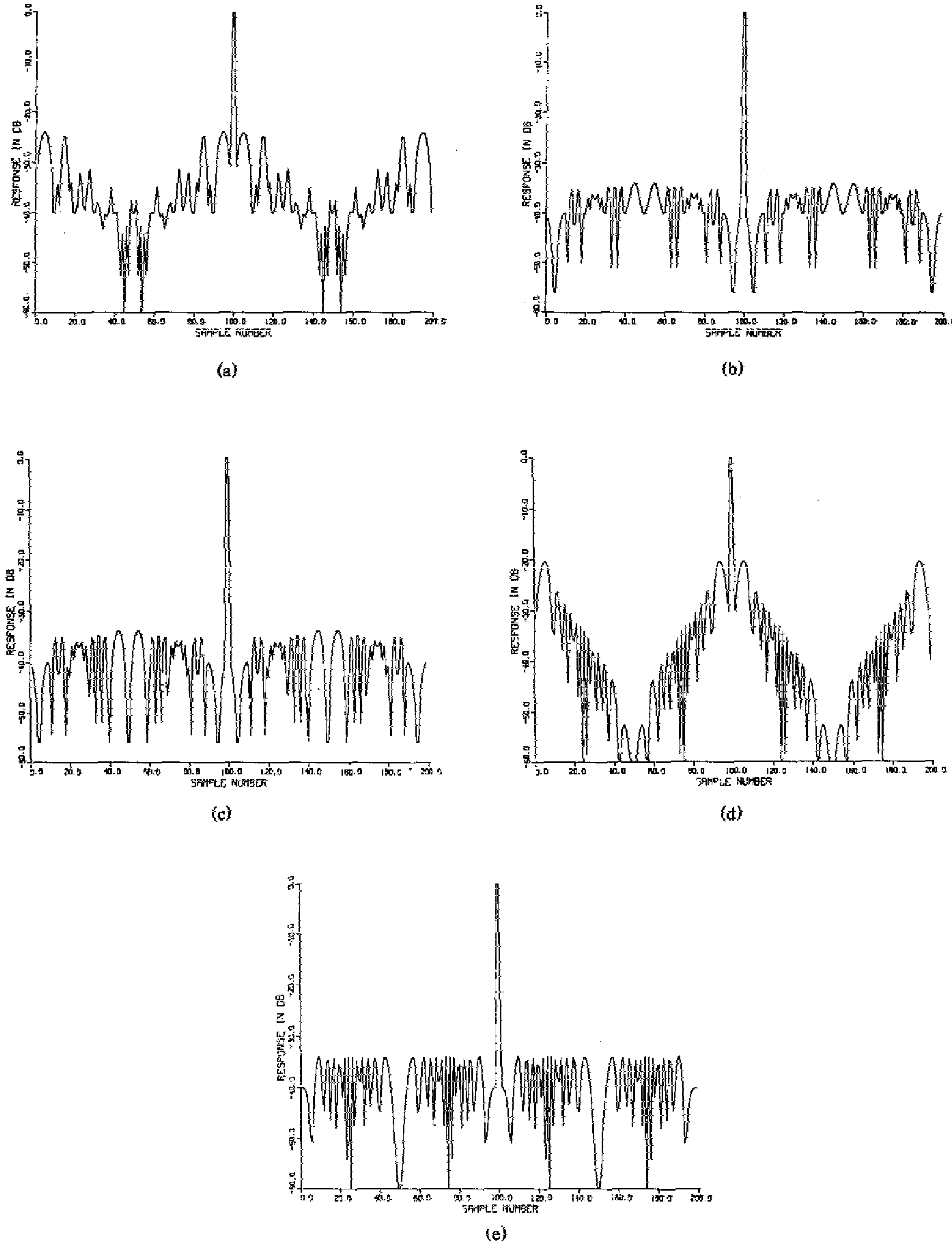


Fig. 11 — Compressed pulse waveforms of polyphase codes sampled at the Nyquist rate with $\rho = 100$ and with sliding window 2 sample adder at output. (a) Frank code; (b) P1 code; (c) P2 code; (d) P3 code; (e) P4 code.

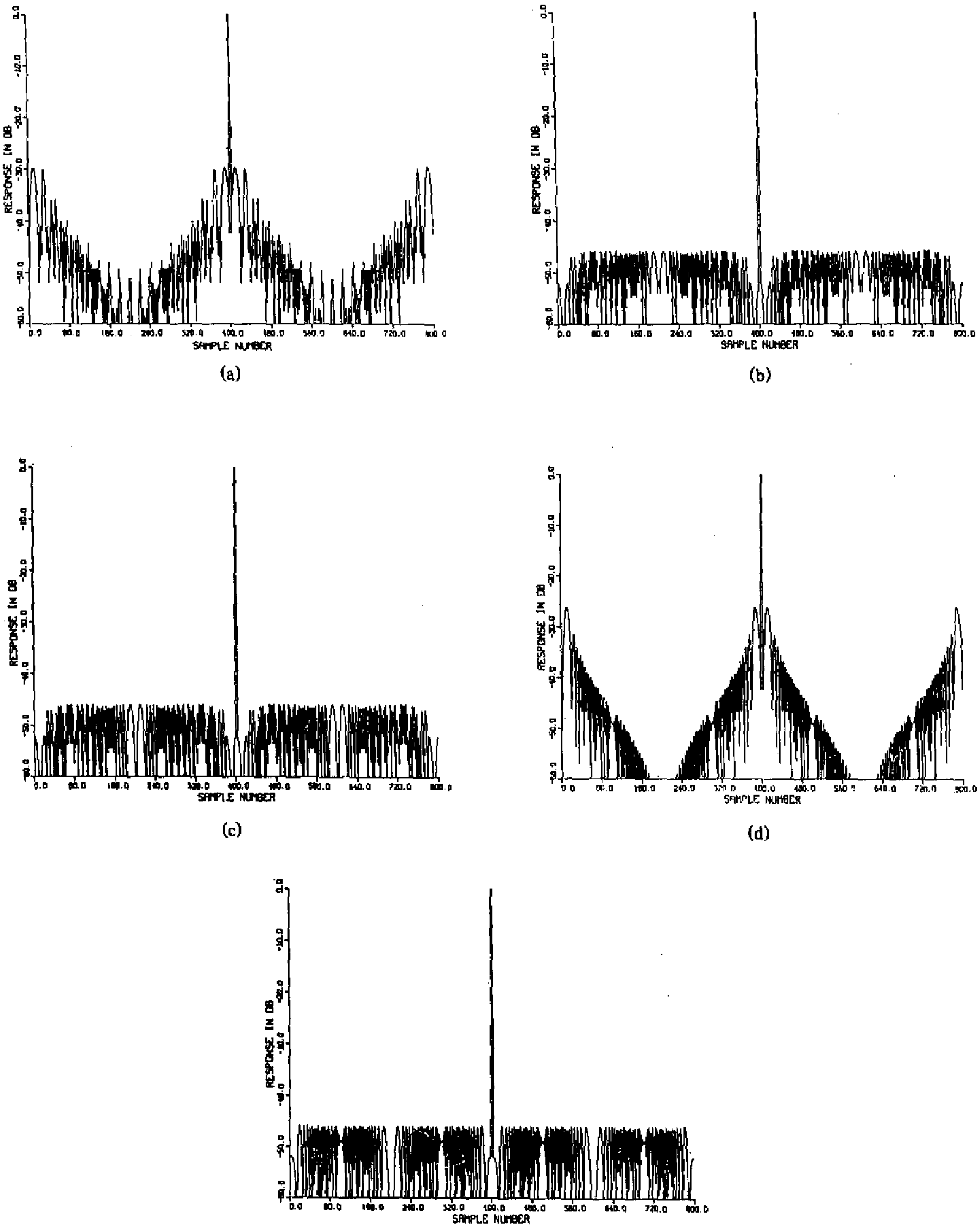


Fig. 12 — Compressed pulse waveforms of polyphase codes sampled at the Nyquist rate with $\rho = 400$ and with sliding window 2 sample adder at output. (a) Frank code; (b) P1 code; (c) P2 code; (d) P3 code; (e) P4 code.

## REFERENCES


1. B.L. Lewis and F.F. Kretschmer, Jr., "A New Class of Polyphase Pulse Compression Codes and Techniques," IEEE Trans. Aerospace and Electronic Systems **AES-17**(3), 364-372, May 1981.

2. B.L. Lewis and F.F. Kretschmer, Jr., "Linear Frequency Modulation Derived Polyphase Pulse-Compression Codes and an Efficient Digital Implementation," NRL Report 8541, November 1981.

3. F.F. Kretschmer, Jr., and B.L. Lewis, "Polyphase Pulse Compression Waveforms," NRL Report 8540, January 1982.

4. R.L. Frank, "Polyphase Codes With Good Nonperiodic Correlation Properties," IEEE Trans. Information Theory **IT-9**, 43-45, January 1963.

5. F.E. Nathanson, *Radar Design Principles: Signal Processing and the Environment,* McGraw-Hill, 1969.

6. C.E. Cook and M. Bernfeld, *Radar Signals: An Introduction to Theory and Application*, Academic Press, 1967.