

# **Solving the Parabolic Equation for Underwater Acoustic Propagation by the Split-Step Algorithm**

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package can plot their outputs. These plots include sound-speed contours, sound-speed profiles, the initial pressure field, transmission loss as a function of range, transmission loss as a function of depth, transmission-loss histograms in specified range-and-depth regions, and intensity contours as a function of range and depth. The report summarizes the theory, describes the implementation, and tells how to run the programs.

## CONTENTS

INTRODUCTION .....	1
OUTLINE OF THE THEORY AND ITS IMPLEMENTATION .....	2
PROFIL: PROGRAM TO GENERATE SOUND-SPEED PROFILES .....	5
START: PROGRAM TO GENERATE THE INITIAL PRESSURE FIELD .....	7
CSPLIT: PROGRAM TO MARCH THE PARABOLIC SOLUTION OUT IN RANGE .....	10
REFERENCES .....	13
APPENDIX A – VCUT: Program to Plot Transmission Loss Versus Depth .....	15
APPENDIX B – INTCON: Program to Plot Intensity Contours .....	17
APPENDIX C – TLHIST: Program to Plot Transmission-Loss Histograms .....	19
APPENDIX D – Sample Outputs .....	21

# SOLVING THE PARABOLIC EQUATION FOR UNDERWATER ACOUSTIC PROPAGATION BY THE SPLIT-STEP ALGORITHM

## INTRODUCTION

Computer implementations of mathematical models of underwater acoustic propagation can be used to analyze the performance of underwater acoustic surveillance systems. Such models enable more efficient use of existing systems and assist in optimal design and placement of proposed systems. They can also be used in the development of deployment strategies.

The propagation of sound in the ocean is influenced by many factors. These include processes that are deterministic and processes that can be best described statistically. Deterministic processes include refraction and diffraction; an example of processes best described statistically is scattering by either volume inhomogeneities or bathymetric irregularities. There are many theoretical formulations and computer-based algorithms for deterministic solutions of the equations governing sound-propagation in the ocean. An excellent survey of these models is contained in Ref. 1.

When the environment can be assumed to be cylindrically symmetric about the sound source, as is often the case, a two-dimensional (range-and-depth) treatment is sufficient. When the source frequency is low, say below 200 Hz, and diffraction effects should be included with the gross refraction effects, a wave-theoretic approach is better than one based on geometric optics. The parabolic-equation approximation to the Helmholtz equation for wave propagation, developed by Fock [2] and first applied to underwater acoustics by Tappert and Hardin [3,4], has proven itself applicable to a wide range of problems [4-13]. Much has been written concerning the advantages and limitations of the parabolic-equation approximation [6,14-19], the techniques which are used to solve the equation [6,20-22], and ways to improve the approximation [18,23,24].

We have developed a sequence of FORTRAN programs to model underwater acoustic propagation by solving the parabolic equation using the split-step [3,4,6] technique. The speed of this range-stepping algorithm is due to the use of a sine transform which is essentially the fast Fourier transform (FFT). Also, this approach makes it possible to take advantage of the vectorizing and pipelining capabilities of the Texas Instruments Advanced Scientific Computer (ASC) at NRL.

In our implementation we have included the option to partially correct for the errors inherent in the parabolic approximation by a technique discussed in Ref. 24. We have also provided the user with three alternatives for generating the initial pressure field required by the range-stepping algorithm: a normal-mode calculation, a functional form which is Gaussian in depth, or a user-supplied complex FORTRAN function. All of the programs in this package can produce plots of their outputs. These include sound-speed contours, sound-speed profiles, the initial pressure field, transmission loss versus range, transmission loss versus depth, intensity contours versus range and depth, and histograms of transmission loss in specified range-and-depth regions.

If the solution file is transferred (via magnetic tape) to a computer with a matrix plotter, then gray-scale plots of intensity versus range and depth can be generated. Also, if cylindrical symmetry is assumed, the effect of tilt on the performance of an array of hydrophones in the calculated pressure field can be simulated. This is done in an auxiliary program which provides either gray-scale or isometric plots of intensity as a function of arrival angle and range from the source and which provides estimates of array signal gains and 3-dB widths.

The next section briefly describes the parabolic-equation approximation, the split-step technique, and the implementation on the ASC. The three sections after the next section describe how to access and use the three main programs of the model. The inputs for each program are explained and their formats are given. The various outputs are described. Three auxiliary programs are described in Appendices A, B, and C. Appendix D contains output plots from a sample case.

All of the programs described in this report are available for use by the Navy scientific community through the Navy Laboratory Computer Network (NALCON).

## OUTLINE OF THE THEORY AND ITS IMPLEMENTATION

The Helmholtz equation for two-dimensional acoustic wave propagation in the ocean is

$$\nabla^2 \psi(r,z) + k_0^2 n^2(r,z) \psi(r,z) = 0, \quad (1)$$

where

$$k_0 = \text{reference wave number} = 2\pi F/c_0,$$

$$n(r,z) = \text{refraction index} = c_0/c(r,z),$$

and

$$\psi(r,z) = \text{time independent acoustic pressure,}$$

in which

$$F = \text{source frequency,}$$

$$c_0 = \text{reference sound speed,}$$

$$c(r,z) = \text{sound speed,}$$

and

$$z = \text{depth coordinate.}$$

Usually the reference sound speed  $c_0$  is taken to be the average sound speed over the water column at the source.

The solution of Eq. (1) is written as

$$\psi(r,z) = \frac{e^{ik_0 r}}{\sqrt{r}} \phi(r,z), \quad (2)$$

where  $\phi$  satisfies

$$\phi_{rr} + 2ik_0 \phi_r + \phi_{zz} + k_0^2 \left[ n^2(r,z) - 1 + \frac{1}{4(k_0 r)^2} \right] \phi = 0. \quad (3)$$

It is assumed that  $\phi/r^2$  is negligible in the far field and that [2]

$$|\phi_{rr}| \ll |2k_0 \phi_r|.$$

The physical assumptions of this approximation have been discussed in the literature [14-18]. Thus  $\phi$  is approximated by a function  $p$  which satisfies the parabolic equation

$$2ik_0 p_r + p_{zz} + k_0^2 [n^2(r,z) - 1] p = 0. \quad (4)$$

The solution of Eq. (4) is approximated by using the "split-step" algorithm [3,4,6], which marches in range:

$$\hat{p}(r + \Delta r, z) = e^{ik_0(n^2-1)\Delta r/2} F_z^{-1} \{ e^{-i\Delta r s^2/(2k_0)} F_z \{ \hat{p}(r, z) \} \}. \quad (5)$$

In Eq. (5),  $F_z \{ \hat{p}(r, z) \}$  is the Fourier transform of  $\hat{p}(r, z)$ ,  $s$  is the transform variable, and  $\hat{p} \approx p$ . To meet the boundary condition at the ocean surface ( $p = 0$ ), we add an image source ( $S'$ ) as indicated in Fig. 1. In the shaded regions,  $n^2$  is given an exponentially increasing imaginary part which has the effect of causing the pressure to die off as the boundaries at  $D + d$  and  $-(D + d)$  are approached. ( $D$  is the maximum water depth and usually  $d = (1/3)D$ .) This prevents energy from the periodically repeated regions containing image sources from entering the transform region. In practice, we use a discrete sine transform over the smaller region indicated in Fig. 1.

The speed of this algorithm is due to the use of a sine transform which is essentially an FFT. The subroutine [25] which performs this transform makes full use of the vectorizing and pipelining capabilities of the ASC.

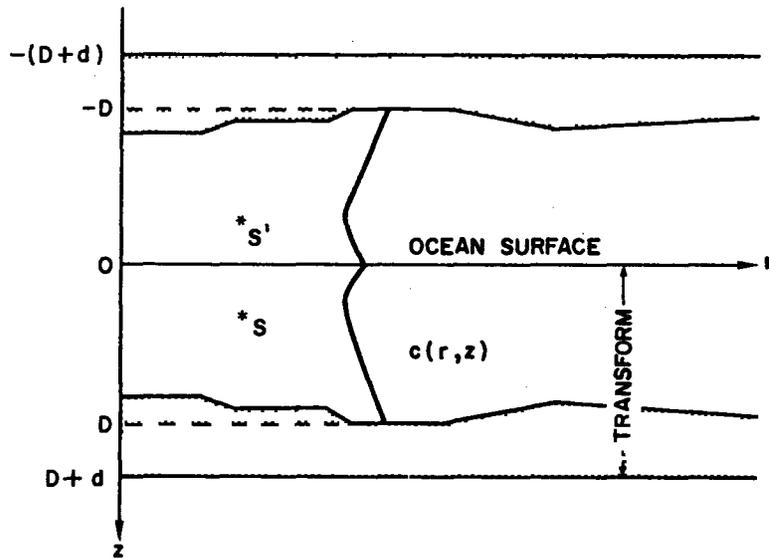


Fig. 1 — Periodically repeated transform region. In the shaded regions,  $n^2$  is given an exponentially increasing imaginary part which causes the pressure to die off as the boundaries at  $D + d$  and  $-(D + d)$  are approached.

The error which occurs from solving Eq. (4) by the split-step algorithm can be reduced by taking small range steps [6]. In our implementation we have included the option to partially correct for the "parabolic" approximations preceding Eq. (4) [24]. Analytically, we can express these corrections by

$$\psi(r,z) \approx \frac{e^{ik_0 r}}{\sqrt{r}} \left[ \hat{p}(r,z) + \frac{ir}{2k_0} \frac{\partial^2 \hat{p}}{\partial r^2} \right],$$

where the second-derivative term represents the corrections (a second-derivative term having been neglected in the parabolic equation), and dropping this term yields the usual parabolic-equation approximation.

To implement the algorithm of Eq. (5), the sound-speed field  $c(r,z)$  must be known (in order to calculate  $n(r,z)$ ), and an initial pressure field must be given (which will be marched out in range). We have separated these tasks from the range-stepping program, called CSPLIT, by writing two preliminary programs called PROFIL and START. PROFIL reads the environmental data, interpolates sound-speed profiles, writes the bathymetry and a list of sound-speed profiles on a file for use by START and CSPLIT, and draws a plot of these profiles and the ocean bottom. START creates a file containing the initial pressure field corresponding to  $\psi$  in Eq. (2). The user can choose an initial field from three alternatives: a normal-mode calculation [26,27], a functional form which is Gaussian in depth [28], or a user-supplied complex FORTRAN function which will override the Gaussian. A flow chart indicating the relationships between the basic programs is shown in Fig. 2.

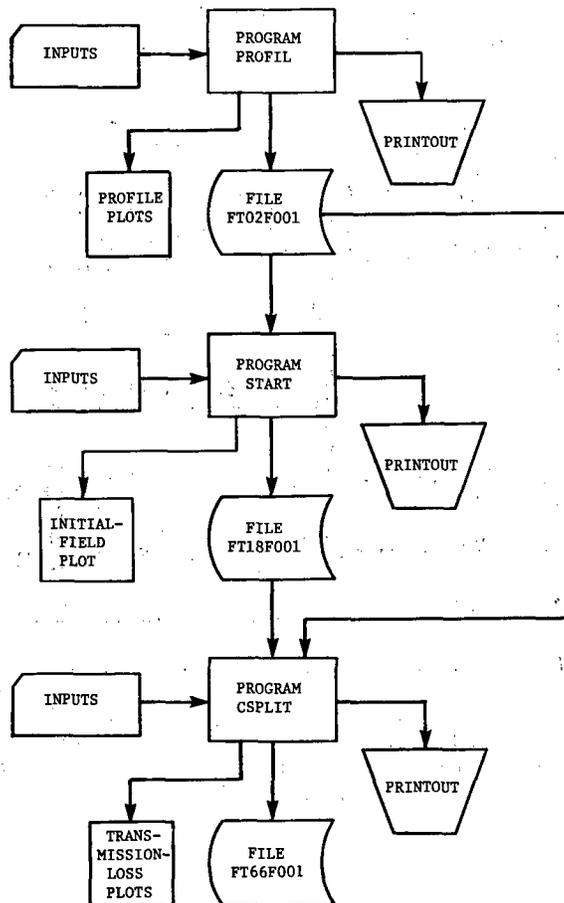


Fig. 2 — Flow chart indicating the relationships between the main programs: PROFIL, START, and CSPLIT

The direct outputs from CSPLIT are a listing and/or plot of transmission loss versus range for each input depth. If the user wishes, CSPLIT will produce a file containing the parabolic solution at every depth grid point for evenly spaced range steps. This file can be used as input to three auxiliary programs: VCUT, INTCON, and TLHIST. VCUT creates a transmission-loss-versus-depth listing and/or plot for each input range, INTCON draws intensity contours, and TLHIST draws histograms of transmission loss.

The three sections that follow describe how to access and use PROFIL, START, and CSPLIT respectively. The inputs for each program are explained and their formats are given. The various outputs are described in more detail. The auxiliary programs VCUT, INTCON, and TLHIST are described in Appendixes A, B, and C respectively. Appendix D contains output plots from a sample case.

### PROFIL: PROGRAM TO GENERATE SOUND-SPEED PROFILES

The purpose of PROFIL is to read the environment, interpolate sound-speed profiles, create a list of profiles on file FT02F001 for later use, and plot these profiles. Careful consideration should be given to the interpolation, since each profile is in effect until the range of the following profile is reached.

The following sample job deck indicates how to access and use the program:

```

/ JOB EXAMPLE$PROFIL,account number,user code,LOC=RTE7,OPT=(R),CAT=9
/ LIMIT BAND=35,SEC=400
/ LIMIT BAND=35,SEC=200
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/...
/ ASG SYS.OMOD,P/PROFIL/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ ASG CONLIB,USERCAT/D81/L60/CONLIB/OLIB,USE=SHR
/ DISSPLA VERS=8.2
/ LNK LSPACE=15000
LIBRARY CONLIB
LIBRARY DISSPOBJ
LIBRARY LIBSUBS
/ FXQT OPT=(I),CPTIME=20000
...DATA...
/ CAT YOURPATH/FT02F001,ACNM=FT02F001
/ FOSYS FT59F001,TYPE=PLOT,FORM=00
/ EOJ

```

Job category 9 (CAT = 9) will not suffice if more than 50 bands or more than 600 pseudo-seconds are required.

The inputs are as follows:

**NBOT**                      Number of bottom points:  $1 < \text{NBOT} < 101$ .

**NP**                         Value such that  $\text{ABS}(\text{NP})$  is the number of input profiles.  $\text{NP} < 0$  turns off the spherical-earth correction.

PERKINS, BAER, WRIGHT, AND ROCHE

IPLOT	Choice of plots, 0, 1, 2, or 3, with the choices being 0 for no plots, 1 for profiles only, 2 for profiles and sound-speed contours, and 3 for contours only. The plot(s) will be on file FT59F001, which may be "FOSYSed" (TYPE=PLOT) or plotted offline.
RMAX	Maximum range (km), which can be overridden by CSPLIT.
BR(I), BD(I)	Ranges and depths such that BD(I) is the depth (m) at range BR(I) (km); I = 1, NBOT, and BR(1) must be zero.
NC	Number of points on the current input profile: $2 < NC < 50$ .
NSPFYD	Number of connections specified by pairs of indices N(I) and M(I), which are described below: NSPFYD < 50.
IRGN	Number of profiles to be generated between the current input profile and the next input profile. Unless R2 > 0, these profiles will be equally spaced between the two input profiles.
R1	Starting range (km) for the current input profile.
R2	Ending range (km) for the current input profile. If R2 > 0 and IRGN > 0, then the interpolated profiles will be equally spaced between range R2 and the range of the next input profile.
D(I), C(I)	Depths and sound speeds such that C(I) is the sound speed (m/s) at depth D(I) (m); I = 1, NC and D(1) must be zero.
N(I), M(I)	Pair of indices which indicates that the N(I)th point on the current profile is to be connected to the M(I)th point on the next profile, with I = 1, NSPFYD. (Regions between input profiles are partitioned into isogradient triangular sectors, and interpolated profiles within regions are defined by the sector boundaries. The connections provide a way for the user to influence the partitioning. In many cases no connections are needed.)
KMPERI	Range scale (km/in.) for the profile plot.
MPSPI	Scale ((m/s)/in.) for plotting profiles.
RANMIN	Starting range (km) for the contour plot, required only if IPLOT = 2 or 3.
RANMAX	Ending range (km) for the contour plot, if IPLOT = 2 or 3.
DEPMIN	Starting depth (m) for the contour plot, if IPLOT = 2 or 3.
DEPMAX	Ending depth (m) for the contour plot, if IPLOT = 2 or 3.
CLMIN	Minimum sound-speed contour (m/s), if IPLOT = 2 or 3.
CLMAX	Maximum sound-speed contour (m/s), if IPLOT = 2 or 3.
VERTIN	Length (in.) of the y axis (depth), if IPLOT = 2 or 3.

- KMPI                      Horizontal (range) scale (km/in.), if IPLOT = 2 or 3.
- NCL                        Total number of contour levels (evenly spaced between CLMIN and CLMAX), if IPLOT = 2 or 3.

The formats of these inputs are as follows:

Card	Inputs	Format
1	NBOT, NP, IPLOT, RMAX	(3I5, F10.3)
2	BR(I), BD(I), I = 1, NBOT	(10F8.2)
3*	NC, NSPFYD, IRGN, R1, R2	(3I5, 2F10.3)
4*	D(I), C(I), I = 1, NC	(10F8.2)
5*†	N(I), M(I), I = 1, NSPFYD	(16I5)
6‡	KMPERI, MPSPI	(2I5)
7§	RANMIN, RANMAX, DEPMIN, DEPMAX, CLMIN, CLMAX, VERTIN, KMPI, NCL	(7F10.3, 2I5)

\*Cards 3, 4, and 5 are repeated for each profile to be inputted. The profiles are inputted in order of increasing range.

†Card 5 is omitted if 0 is inputted for NSPFYD.

‡Card 6 is omitted if IPLOT = 0 or 3.

§Card 7 is omitted if IPLOT = 0 or 1.

The outputs are the following:

- Input profiles on print file FT06F001.
- A plot of profiles on FT59F001, if IPLOT = 1 or 2. A delta or a nabla will mark the range for each profile. A nabla indicates that the profile was an input profile, and a delta indicates that the profile is an interpolated profile. Each profile is positioned with respect to its symbol so that the symbol marks a 1500-m/s reference point for plotting the profile. If a contour plot is requested, it will also be on FT59F001.
- Profiles and the bottom on FT02F001. The user should catalog FT02F001.

**START: PROGRAM TO GENERATE THE INITIAL PRESSURE FIELD**

The purpose of START is to create the initial pressure field on FT18F001. A sample job deck indicating how to access and use the program is as follows:

```

/ JOB EXAMPLE$START,account number,user code,LOC=RTE7,OPT=(R),CAT=9
/ LIMIT BAND=...,MIN=...
/ PD P,USERCAT/D81/L60/PARA
/ ASG FT02F001,YOURPATH/FT02F001,USE=SHR
/ ASG SYS.OMOD,P/START/OBJ,USE=SHR
/ ASG LIBSUBS, P/LIBSUBS/OBJ,USE=SHR
/ DISSPLA VERS=8.2
/ LNK LSPACE=15000
LIBRARY LIBSUBS
LIBRARY DISSPOBJ
/ FD FT06F001,BAND=1/15/1
/ FXQT CPTIME=...,ADDMEM=...,OPT=(I)
...DATA...
    
```

```
/ CAT YOURPATH/FT18F001,ACNM=FT18F001
/ FOSYS FT59F001,TYPE=PLOT
/ EOJ
```

Again CAT = 9 will not suffice if more than 50 bands or more than 600 pseudo-seconds are required. If ISTART = 1 or ITAPE = 0, 20 bands should suffice; otherwise the number of bands to use is  $((\text{guess for the number of modes}) * NN + 5 * NN) / 16384 + 20$ , where NN is the number of points in depth ( $NN = 2^{**}NPOW$ ).

On the ASC, CPTIME must be specified in hundredths of seconds. If the normal-mode calculation is used, the time needed may exceed the default. To do a problem with NPOW = 9, NINT = 4, and 164 modes, 45 seconds were needed. Another problem with NPOW = 11, NINT = 4, and 325 modes required 430 seconds.

The ADDMEM value to use is  $MAX(8 * NN + 4000, 8000)$ .

The file containing the profile(s) created by PROFIL must be assigned with the access name FT02F001. Other inputs are as follows:

**ISTART** An integer 0 or 1 indicating how the initial field will be calculated. If ISTART = 0, the initial field will be created by a normal-mode calculation [26,27]. If ISTART = 1, a real Gaussian [28] will be used, or the user may supply an initial field by writing a complex FORTRAN function with the name STFTN, which will return for a given depth the complex pressure at that depth. The following FORTRAN statements may be used:

```
COMPLEX FUNCTION STFTN(Z)
DOUBLE PRECISION R0,C0,F,BSS,SOURD
COMMON /OUT/ R0, C0, F,BSS,SOURD
```

```
STFTN=...
RETURN
END
```

The normal-mode start should be more accurate but can be very expensive at higher frequencies. The source must be several wavelengths away from the surface and bottom in order to use the Gaussian start.

**NPOW** Integer < 13 specifying that the number of points in depth on the initial field will be  $2^{**}NPOW$ . NPOW should be large enough so that the depth grid spacing  $\Delta z$ , which equals  $(4./3.)^{**}(\text{maximum depth along track})/2^{**}NPOW$ , is less than 1/2 wavelength.

**I PLOT** Plot choice, 0 or 1. If I PLOT = 1, a plot file for the initial field is produced.

**F** Source frequency (Hz).

**R0** Range (km) where the initial field is desired. For a normal-mode start, R0 must be at least several times the wavelength. For the Gaussian start, R0 should be set to 0.001 km.

**BSS** Speed of sound (m/s) in the bottom.

SOURD                    Depth of the source (m).  
 C0                        Reference sound speed (m/s). If C0 = 0.0, a water-column average is used.

The remaining inputs, which are not needed if ISTART = 1, are as follows:

ITAPE                    File choice 0 or 1. If ISTART = 0 and ITAPE = 1, the eigenvalues and eigenfunctions are written on FT17F001 (a QDAM file), and some unformatted information is on FT03F001 which could be used to open FT17F001. Unless there is some use for the eigenvalues and eigenfunctions, ITAPE = 0 should be used.

NINT                     Factor giving the number of points in depth used by the program in calculating the modes, which is  $NINT \cdot (2^{NPOW})$  points in depth. NINT must be a power of two, and  $NINT \cdot (2^{NPOW})$  must be no greater than 16384.  $NINT = 4$  is good in most cases.

MAXMOD                The maximum number of modes that will be used. If all modes are to be included, MAXMOD must be at least as large as the actual number of modes. If MAXMOD is less than the actual number of modes, then only the lowest modes (those with 0 to MAXMOD - 1 turning points) are included. A good approximation to the actual number of modes is  $2 \cdot F \cdot H \cdot \sqrt{1/C0^2 - 1/BSS^2}$ , where H is the water depth at the source and C0 is the average sound speed in the water column.

RHO1                    Water density (g/cm<sup>3</sup>).

RHO2                    Density (g/cm<sup>3</sup>) in the bottom.

EPSILN                 Convergence parameter, with a suggested value being EPSILN = 0.001.

The formats of the inputs are as follows:

Card	Inputs	Format
1	ISTART, NPOW, IPLOT, F, R0, BSS, SOURD, C0	(3I5, 5F10.3)
2*	ITAPE, NINT, MAXMOD, RHO1, RHO2, EPSILN	(3I5, 3F10.3)

\*Card 2 is not needed if ISTART = 1.

The outputs are the following:

- The eigenvalues and the initial pressure field on file FT06F001.
- The initial pressure field on file FT18F001. This file should be cataloged.
- Plot of the initial field on file FT59F001 (only if IPLOT = 1).
- Files FT03F001 and FT17F001 (only if ITAPE = 1). (The description of the input ITAPE explains of these files.)

**CSPLIT: PROGRAM TO MARCH THE PARABOLIC SOLUTION OUT IN RANGE**

The purpose of CSPLIT is to use the split-step algorithm in calculating the solution to the parabolic-equation approximation to the Helmholtz equation. A sample job deck indicating how to access and use the program is as follows:

```

/ JOB EXAMPLE$CSPLIT,account number,used code,LOC=RTE7,OPT=(R),CAT=9
/ LIMIT BAND=...,MIN=...
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/...
/ ASG FT02F001,YOURPATH/FT02F001,USE=SHR
/ ASG FT18F001,YOURPATH/FT18F001,USE=SHR
/ ASG SYS.OMOD,P/CSPLIT/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ DISSPLA VERS=8.2
/ LNK LSPACE=15000
LIBRARY LIBSUBS
LIBRARY DISSPOBJ
/ FD FT06F001,BAND=1/50/1
/ FD FT09F001,BAND=1/15/1,RCFM=FBA,BKSZ=3944,LREC=136
/ FXQT CPTIME=...,ADDMEN=...,OPT=(I)
...DATA...
/ CAT YOURPATH/FT66F001,ACNM=FT66F001,DTYP=TAPE
/ FOSYS FT59F001,TYPE=PLOT,FORM=00
/ FOSYS FT08F001
/ EOJ

```

Again CAT = 9 will not suffice if more than 50 bands or more than 600 pseudo-seconds are required. If ISKPR = 0, 30 bands should suffice. Otherwise the number of bands to use is  $2*NN*$  (number of range steps)/(ISKPR\*16384) + 30, where  $NN = 2**NPOW$  and ISKPR is described below.

CPTIME is approximately proportional to the number of points in depth and the number of range steps. To do a problem with 1024 points in depth and 1000 range steps, 16 seconds were needed. Also, this problem required 64 pseudo-seconds. If ICORR = 1, the run time will approximately double. The ADDMEM value is  $6*NN + 8000$ .

The file containing the profile(s) created by PROFIL must be assigned with the access name FT02F001. The file containing the initial pressure field created by START must be assigned with the access name FT18F001. Other inputs are as follows:

IBFLAG	Bottom flag, 0 or 1, with 0 implying that the bottom is flat.
IPFLAG	Profile flag, 0 or 1 with 1 being used if the sound-speed profile is to change with range. If IPFLAG = 0, the first profile on FT02F001 will be used over the entire range of the run.
IPLOT	Plot choice, 0 or 1. If IPLOT = 0, no plot files will be created.
NPOW	Integer giving the number of points in depth, which will be $2**NPOW$ . NPOW must be the same as it was for START.
NRD	Number of receiver depths: $0 < NRD < 6$ .

- ISKPR Number of range-step increments in creating file FT66F001, containing the solution. As the parabolic solution is marched out in range, the complex-valued parabolic pressure at each depth grid point is written to file FT66F001 every ISKPR range steps. If ISKPR = 0, no file is created.
- ICORR Number of steps to take between corrections. If ICORR = 0, then no corrections will be made.
- NBSS Number of ranges where the bottom sound speed is to be changed:  $0 \leq \text{NBSS} \leq 10$ . The ranges and corresponding sound speeds are the inputs RBSS(I) and XBSS(I) ( $1 \leq I \leq \text{NBSS}$ ) described below. If NBSS = 0 and CRTANG = 0.0, (as described below) then the bottom sound speed used in START will be used throughout. If NBSS > 0 is used, then probably CRTANG = 0.0 should be used.
- RBSS(I), XBSS(I) Range (km) and sound-speed (m/s) values to be inputted only if NBSS > 0. XBSS(I) is the bottom sound speed to be used from range RBSS (I) to range RBSS (I + 1) for  $1 \leq I \leq \text{NBSS} - 1$ . The bottom sound speed used in START will be used from the initial range to range RBSS(1). XBSS (NBSS) will be used from range RBSS (NBSS) to the end of the run.
- DELR Range step (km). In many cases with typical ocean sound-speed profiles and water-borne propagation, range steps as large as 5 or 10 wavelengths are permissible. When rapid sound-speed changes (such as those often seen at the water-bottom interface) are important, range steps as small as 1 or 2 wavelengths may be required [19].
- RMAXX Final range (km) for the run. If RMAXX is not positive, then the maximum range which was inputted to PROFIL will be used.
- STARTR Restart range (km). If STARTR > 0.0, then the pressure field at the range step nearest STARTR km is written on file FT19F001. This file can be used to "restart" CSPLIT by releasing FT18F001 and then renaming FT19F001 as FT18F001. If STARTR = 0.0, no restart file is created. If STARTR < 0.0, a restart file is created at the final range of the run.
- CRTANG Critical angle ( $^{\circ}$ ). If CRTANG > 0.0, then each time the sound-speed profile is changed, the bottom sound speed  $C_b$  will be changed so that  $C_b = C_w / \cos(\text{CRTANG})$ , where  $C_w$  is the water sound speed at the depth grid point immediately above the water-bottom interface at the current range. If CRTANG > 0.0 is used, then probably NBSS = 0 should be used. (If NBSS = 0, no values for RBSS(I) and XBSS(I) should be inputted.)
- RECD(I) Receiver depth (m):  $1 \leq I \leq \text{NRD}$ .
- ABCOEF Parameter effecting the attenuation in the deep bottom. This is an artificial attenuation, designed to prevent false reflections from the boundary below the bottom and is not intended to effect a realistic bottom attenuation. (The transform region is shown in Fig. 1, and the attenuation inputs DBPKM and DBPWL are described below. A value of 0.01 is recommended for ABCOEF.

- DBPKM Attenuation (dB/km) in the water. If  $DBPKM \geq 0.0$ , then the attenuation will be DBPKM dB per km. If  $DBPKM < 0.0$ , then the attenuation will be calculated as a function of the frequency.
- DBPWL Attenuation (dB/wavelength) in the bottom, where the wavelength depends on the bottom sound speed.
- SMKM Smoothing window (km) for the transmission-loss printout and plots. Gaussian weights are used with  $6\sigma = SMKM$  km.  $SMKM = 0.0$  causes no smoothing. The window will be reduced, if necessary, so that it does not extend over more than 101 range steps.
- AKMPI Scale (km/in.) for the transmission-loss plots.
- DB1, DB2 Minimum and maximum transmission-loss values for the plots.  $DB1 = DB2 = 0.0$  gives default values of  $DB1 = 40$  dB and  $DB2 = 130$  dB. If  $DB2 - DB1 > 90.0$ , then  $DB1$  will be set equal to  $DB2 - 90.0$ .

The formats for the inputs are as follows:

Card	Inputs	Format
1	IBFLAG,IPFLAG,IPLLOT,NPOW,NRD,ISKPR,ICORR,NBSS	(8I5)
2*	RBSSI,XBBS(I),I = 1,NBSS	(10F8.2)
3	DELR,RMAXX,STARTR,CRTANG	(4F8.2)
4	REDC(I),I = 1,NRD	(10F8.2)
5	ABCOEF,DBPKM,DBPWL,SMKM	(4F8.2)
6	AKMPI,DB1,DB2	(3F8.2)

\*Card 2 is omitted if  $NBSS = 0$ .

The outputs are the following:

- Transmission loss at all ranges for each receiver depth on FT06F001.
- A table summarizing profile changes and bottom-sound-speed changes on FT08F001.
- Plots of transmission loss versus range at each receiver depth on FT59F001 (only if  $IPLLOT = 1$ ).
- The pressure field at evenly spaced ranges on FT66F001 (if  $ISKPR > 0$ ). This file can be used (by VCUT, described in Appendix A) to draw transmission loss-versus-depth plots at several ranges, can be used (by INTCON, described in Appendix B) to draw intensity contours, or can be used (by TLHIST described in Appendix C) to draw histograms of transmission loss.
- A restart field on FT19F001 (if  $STARTR \neq 0$ ).

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## Appendix A

### VCUT: PROGRAM TO PLOT TRANSMISSION LOSS VERSUS DEPTH

The purpose of VCUT is to draw plots of transmission loss versus depth at several ranges using the data file created by CSPLIT. A sample job desk indicating how to access and use the program is as follows:

```

/ JOB EXAMPLE$VCUT,account number, user code,LOC=RTE7,OPT=(R),CAT=9
/ LIMIT BAND=...,MIN=1
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/...
/ ASGP FT66F001,YOURPATH/FT66F001,USE=SHR
/ ASG SYS.OMOD,P/VCUT/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ DISSPLA VERS=8.2
/ LNK LSPACE=15000
LIBRARY DISSPOBJ
LIBRARY LIBSUBS
/ WAIT
/ FXQT
...DATA...
/ FOSYS FT59F001,TYPE=PLOT,FORM=00
/ EOJ

```

Space for the data file FT66F001 which was created by CSPLIT must be provided. The default CPTIME is normally sufficient for less than ten ranges. The ADDMEM value is MAX(6\*NN + 4000,8000).

The data file created by CSPLIT must be assigned with the access name FT66F001. Other inputs are as follows:

NPOW	Integer which must be the same as for START and CSPLIT.
NRAN	Number of ranges on file FT66F001. $NRAN = IFIX((total\ number\ of\ steps\ taken\ by\ CSPLIT)/ISKPR) + 1$ .
NR	Number of ranges to be inputted: $0 < NR < 51$ . A transmission-loss-versus-depth plot will be generated for each range.
RANGE(I), DEL(I)	Range (km) and range-interval (km) values. For the plot corresponding to range RANGE(I), $I = 1, NR$ , the transmission-loss value at a given depth will be calculated from the average intensity from range RANGE(I) - DEL(I)/2.0 to range RANGE(I) + DEL(I)/2.0 at that depth. If DEL(I) = 0.0, then the plot for RANGE(I) will not incorporate a range average. RANGE(I) and DEL(I) may be modified by VCUT so that the ranges involved in the calculation are on range grid steps.

- D1 Starting depth (m) for the plots.
- D2 Final depth (m) for the plots.
- DMAX Maximum depth (m) of the water.
- AMPI Depth scale (m/in.) for the plots.
- SMM Smoothing window (m). Gaussian weights are used with  $6\sigma = \text{SMM}$  m, and  $\text{SMM}=0.0$  is used for no smoothing. Smoothing takes place after range averaging.

The formats of the inputs are as follows:

Card	Inputs	Format
1	NPOW,NRAN,NR	(3I5)
2	RANGE(I),DEL(I),I=1,NR	(8F10.3)
3	D1,D2,DMAX,AMPI,SMM	(5F10.3)

The outputs are the following:

- Depths and transmission losses for each input range on FT06F001.
- Transmission-loss-versus-depth plots for each input range on FT59F001.

## Appendix B INTCON: PROGRAM TO PLOT INTENSITY CONTOURS

The purpose of INTCON is to draw intensity contours using the data file created by CSPLIT. A sample job deck indicating how to access and use the program is as follows:

```

/ JOB EXAMPLE$INTCON,account number,user code,LOC=RTE7,OPT=(R),CAT=13
/ LIMIT BAND=...,MIN=10
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/...
/ ASGP FT02F001,YOURPATH/FT02F001,USE=SHR
/ ASGP FT66F001,YOURPATH/FT66F001,USE=SHR
/ ASG SYS.OMOD,P/INTCON/OBJ,USE=SHR
/ ASG CONLIB,USERCAT/D81/L60/CONLIB/OLIB,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ DISSPLA VERS=8.2
/ LNK LSPACE=15000
LIBRARY CONLIB
LIBRARY DISSPOBJ
LIBRARY LIBSUBS
/ WAIT
/ FXQT CPTIME=...,ADDMEM=...
...DATA...
/ FOSYS FT59F001,TYPE=PLOT,FORM=00
/ EOJ

```

As before, CAT=13 will not suffice if more than 50 bands or more than 600 pseudo-seconds are required. Space for the data file FT66F001 which was created by CSPLIT must be provided. For CPTIME, 30 seconds for each 100 range steps should suffice. The ADDMEN value is  $\text{MAX}(6 * \text{NN} + 4000, 8000)$

The file containing the profile(s) created by PROFIL must be assigned with the access name FT02F001. The data file created by CSPLIT must be assigned with the access name FT66F001. Other inputs are as follows:

NPOW	Integer which must be the same as for START and CSPLIT.
NRAN	Number of ranges on file FT66F001: $\text{NRAN} = \text{IFIX}((\text{total number of steps taken by CSPLIT}) / \text{ISKPR}) + 1$ .
NCL	Number of contour levels to be plotted: $0 < \text{NCL} < 21$ .
LMIN	Minimum contour level (m/s) to be drawn. If LMIN = 0, then the minimum contour level will be calculated.
IDELCL	Difference (m/s) between two consecutive contour levels.
XMIN	Starting range (km) for the plot.

PERKINS, BAER, WRIGHT, AND ROCHE

- XMAX Ending range (km) for the plot. If XMAX is more than 1000 range steps beyond XMIN, XMAX will be reduced by the program.
- YMIN Starting depth (m) for the plot.
- YMAX Ending depth (m) for the plot. If YMAX is more than 400 depth points deeper than YMIN, YMAX will be reduced by the program.
- AKMPI Range scale (km/in.) for the plot.
- AMPI Depth scale (m/in.) for the plot.

The formats of the inputs are as follows:

Card	Inputs	Format
1	NPOW,NRAN,NCL,LMIN,IDELCL	(5I5)
2	XMIN,XMAX,YMIN,YMAX,AKMPI,AMPI	(6F10.3)

The outputs are the following:

- Some information about the length and number of contour levels on FT06F001.
- The contour plot on FT59F001.

### Appendix C

## TLHIST: PROGRAM TO PLOT TRANSMISSION-LOSS HISTOGRAMS

The purpose of TLHIST is to draw histograms of transmission loss using the data file created by CSPLIT. A sample job deck indicating how to access and use the program is as follows:

```

/ JOB EXAMPLE$TLHIST,account number,usercode,LOC=RTE7,OPT=(R),CAT=9
/ LIMIT BAND=...,SEC=600
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/...
/ ASGP FT66F001,YOURPATH/FT66F001,USE=SHR
/ ASG SYS.OMOD,P/TLHIST/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ DISSPLA VERS=8.2
/ LNK LSPACE=20000
LIBRARY LIBSUBS
LIBRARY DISSPOBJ
/ WAIT
/ FXQT OPT=(I),CPTIME=...,ADDMEM=...
...DATA...
/ FOSYS FT59F001,TYPE=PLOT
/ EOJ

```

The data file created by CSPLIT must be assigned with the access name FT66F001. Other inputs are as follows:

NPOW	Integer which must be the same as for CSPLIT, with $2^{**}NPOW$ being the number of points in depth on file FT66F001.
NRAN	Number of ranges on file FT66F001: $NRAN = IFIX((total\ number\ of\ steps\ taken\ by\ CSPLIT)/ISKPR) + 1$ .
NHIS	Number of histograms to be plotted ( $\leq 10$ ).
NBIN	Number of bins out to one standard deviation. That is, a bin is centered at the mean, and the bin width is $\sigma/NBIN$ , where $\sigma$ is the standard deviation. A total of $4*NBIN + 1$ bins are plotted. Typically NBIN is 2 or 3. If $NBIN > 0$ , default values will be used for all seven plotting variables described below, and they are not to be input. If $NBIN = 0$ , then these plotting variables must be specified.

The following seven plot variables are not to be inputted if  $NBIN > 0$ :

XAXIS	Length (in.) of the x axis (transmission loss).
XMIN	Transmission loss (dB) value at the center of the first histogram bin.

XMAX	Transmission-loss (dB) value at the center of the last histogram bin.
BINW	The width of a histogram bin (dB).
YAXIS	Length (in.) of the y axis (number of occurrences).
YMAX	Maximum value desired on the y axis (number of occurrences). This value is automatically increased if it is not large enough.
YSTP	Tick-mark spacing on the y axis (number of occurrences).
DMAX	Maximum water depth (m).

The following four variables are required for each histogram:

D	Depth (m) at which the transmission-loss histogram is to be centered.
ND	Number of depths to be used in the calculation of transmission losses centered on depth D.
R	Range (km) at which the transmission-loss histogram is to be centered.
NR	Number of ranges to be used in the calculation of transmission losses centered on range R.

The formats of the inputs are as follows:

Card	Inputs	Format
1	NPOW,NRAN,NHIS,NBIN	(4I5)
2*	XAXIS,XMIN,XMAX,BINW,YAXIS,YMAX,YSTP	(7F10.3)
3	DMAX	(F10.3)
4†	D,ND,R,NR	(2(F10.3,I5))

\*Card 2 is inputted when NBIN = 0.

†Card 4 is repeated for each histogram (NHIS times).

The outputs are the following:

- Histogram statistics on FT06F001 including the mean, median, and one-sigma points. Any transmission losses that are too large or too small for the plot will be outputted on FT06F001.
- Histograms of transmission loss on FT59F001, including the mean and standard deviation in the heading of each plot.

## Appendix D SAMPLE OUTPUTS

Figures D1 through D7 are some plots from the program package for a sample problem. This sample problem is test case 1B from the AESD Workshop on Acoustic Propagation Modeling by Non-Ray-Tracing Techniques [4], except that the sound-speed field has been given a range dependence beginning at 70 km. This range dependence can be seen in Figs. D1 and D2, a profile plot and sound-speed contour from PROFIL. Figure D3 is a plot of the initial pressure field from START, Fig. D4 is a plot of transmission loss versus range from CSPLIT, Fig. D5 is a plot of transmission loss versus depth from VCUT, Fig. D6 is a plot of intensity contours as a function of range and depth, and Fig. D7 is a histogram of transmission-loss values.

By a transfer (via magnetic tape) of the solution file to a computer with a matrix plotter, gray-scale plots of intensity versus depth and range can be generated. Figure D8 is an example of such a plot.

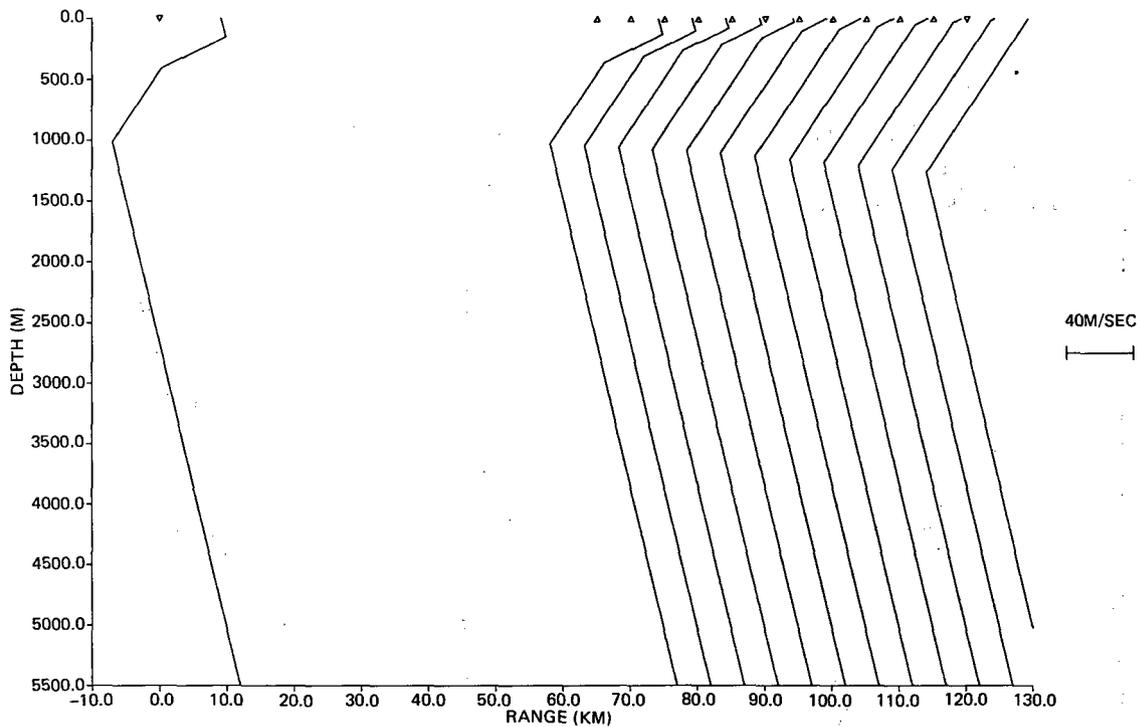


Fig. D1 — A sound-speed profile plot from PROFIL

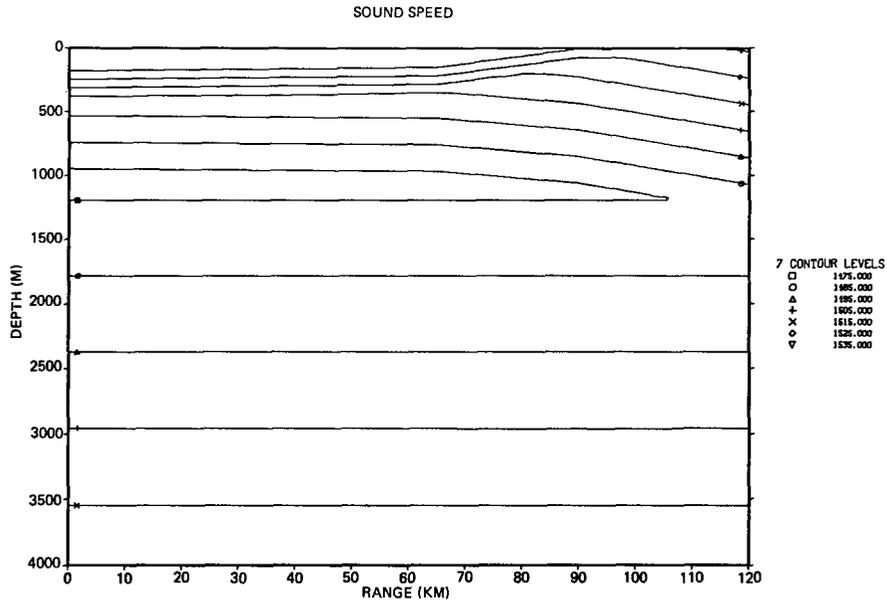


Fig. D2 — A sound-speed contour plot from PROFIL

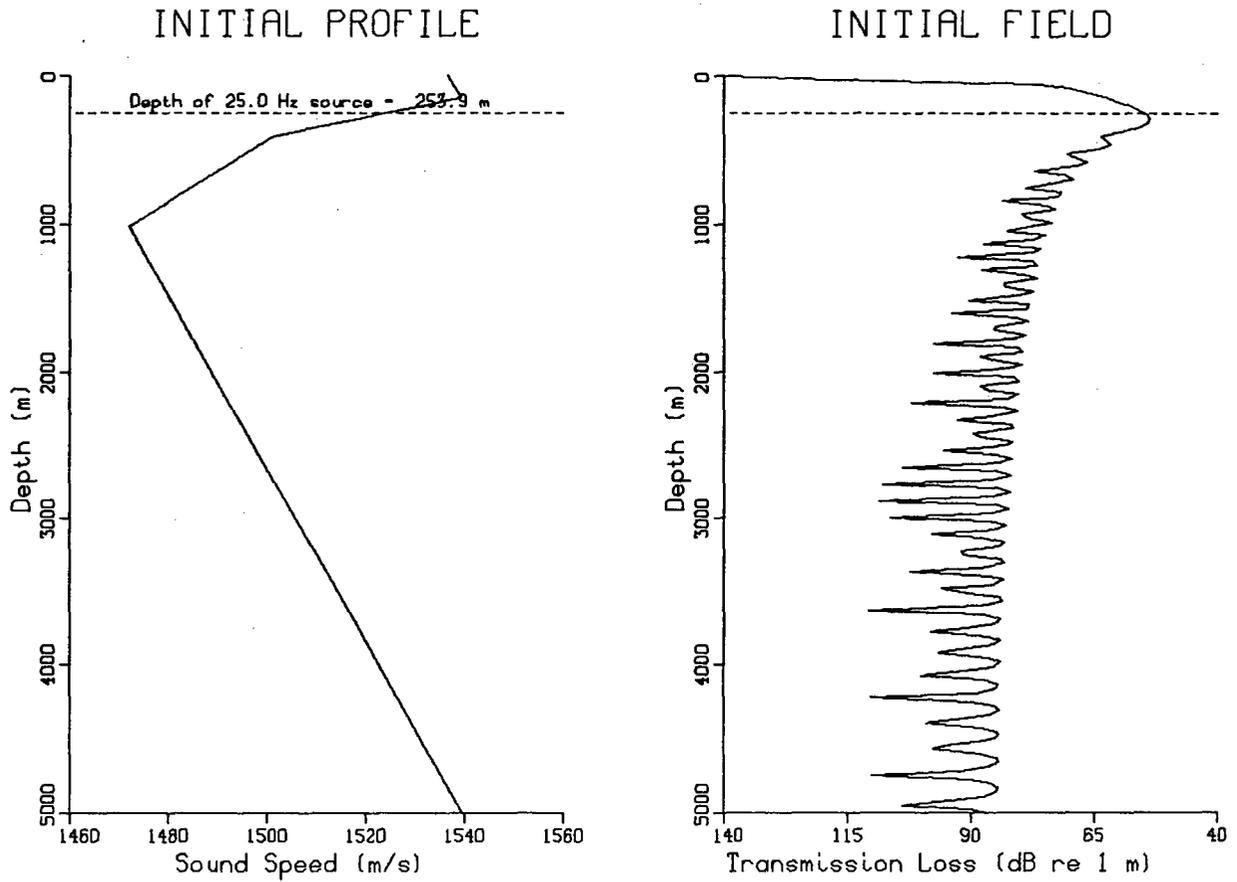


Fig. D3 — An initial pressure-field plot from START

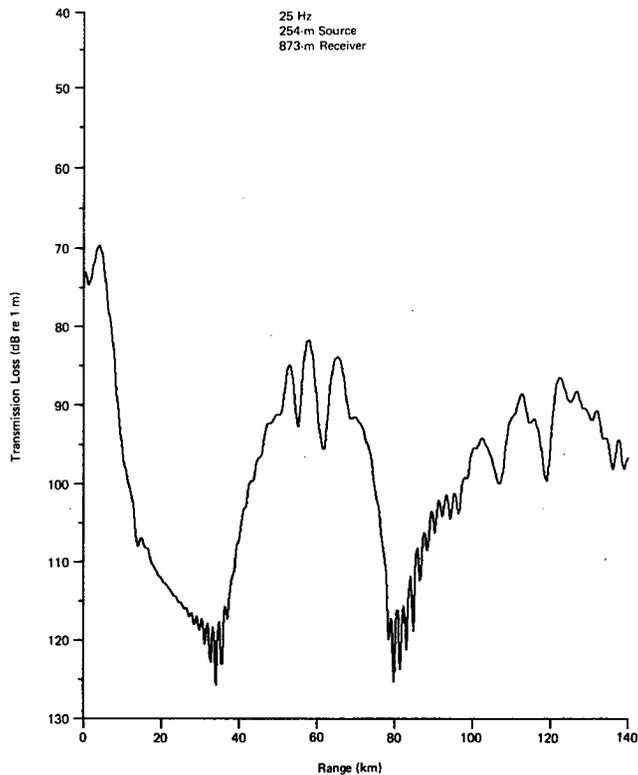


Fig. D4 — A transmission-loss plot from CSPLIT

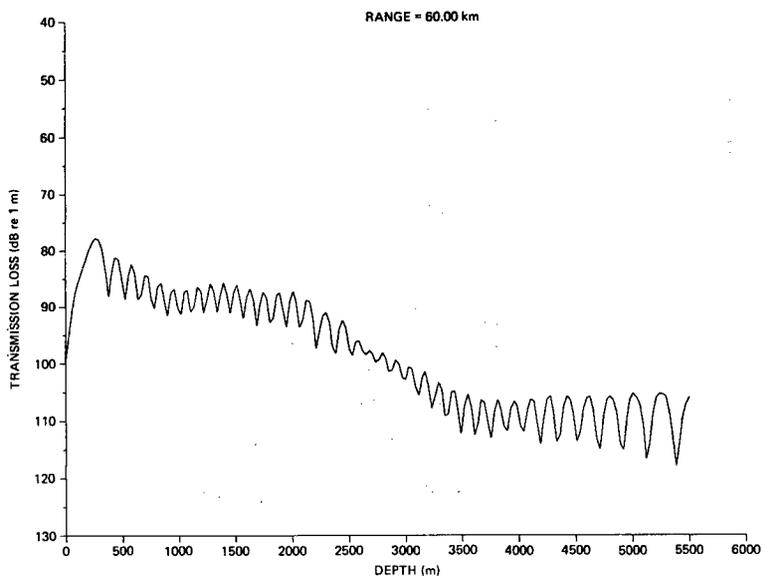


Fig. D5 — A transmission-loss plot from VCUT

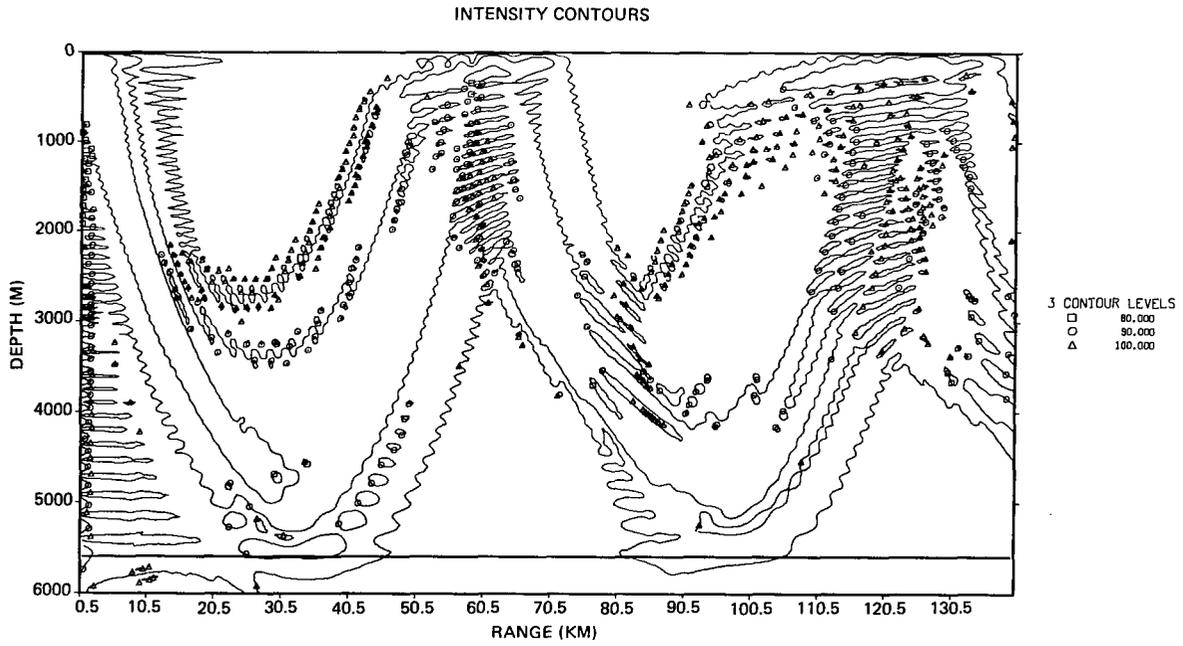


Fig. D6 — An intensity contour plot from INTCON

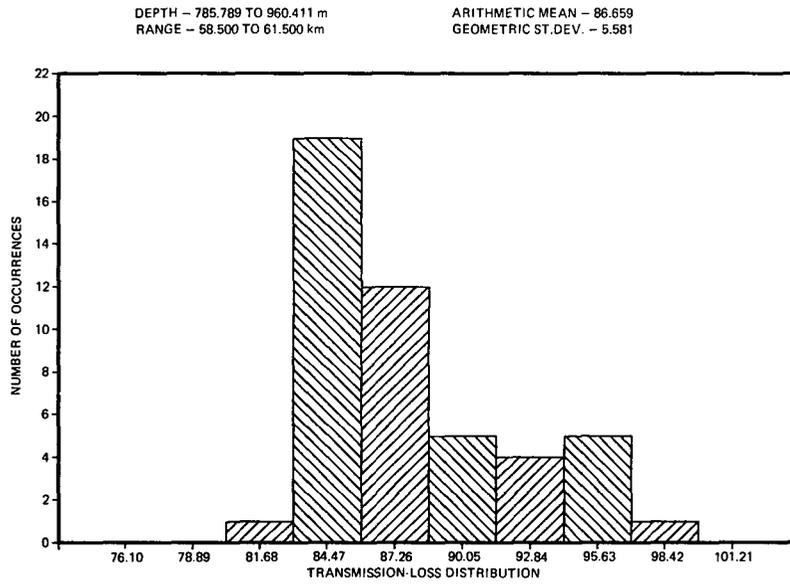


Fig. D7 — A transmission-loss histogram from TLHIST

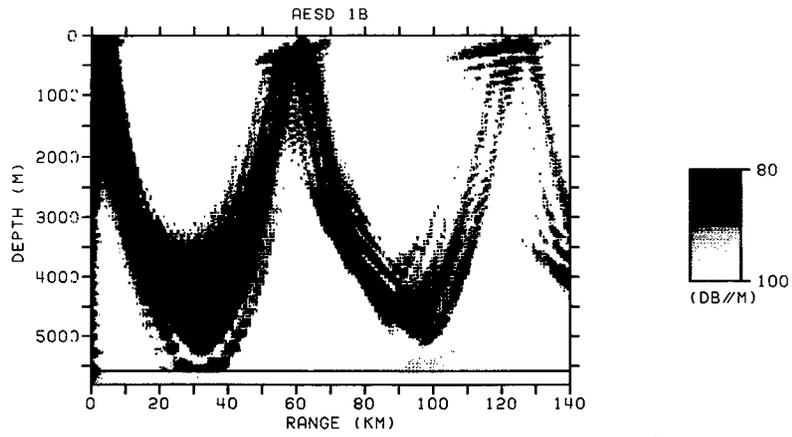


Fig. D8 — An intensity gray-scale plot generated from the solution file using a matrix plotter