NRL Report 8584

Suppression of Second Time Around Radar Returns Using PRI Modulation

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May 13, 1982

NAVAL RESEARCH LABORATORY
Washington, D.C.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER NRL Report 8584	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) SUPPRESSION OF SECOND TIME AROUND RETURNS USING PRI MODULATION		5. TYPE OF REPORT & PERIOD COVERED Final report. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Karl Gerlach and G. A. Andrews		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, DC 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 53-0483-A-2
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE May 13, 1982 13. NUMBER OF PAGES 15
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Radar
PRI modulation
Second time around returns
Detection

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A technique for suppressing second time around radar returns using PRI modulation is presented and analyzed. It is shown that a modulated or staggered PRI radar system can offer considerable improvement over a nonstaggered radar system in rejecting second time around returns which cause false alarms. This improvement is a function of the number of staggered PRI, the quiescent false alarm number, the Swerling number of the false return, the transmitted signal power, the second time around noise power, and the quiescent noise power of the radar. Small changes in transmitted signal power can be traded-off with the quiescent false alarm number to significantly suppress

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20. ABSTRACT *(Continued)*

the bogus return. In addition for all other parameters being equal, if the second time around return is a Swerling case II or IV target, then there is an optimum number of staggered PRI that can be chosen to minimize the likelihood of detection of the second time around return.

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SUPPRESSION OF SECOND TIME AROUND RADAR RETURNS USING PRI MODULATION

INTRODUCTION

A second time around return (Fig. 1) is a class of radar interference that does not necessarily interfere with the detection of a desired target but rather generates false targets (undesirable detections). A low pulse repetition frequency (PRF) radar transmits a series of uniform pulses spaced T_0 seconds apart. If there is a large object (an island or mountain) located beyond the operating range, $cT_0/2$, of the radar, where c is the speed of light, then it is possible for this large object to create a substantial return (called a second time around return) at the front end of the radar receiver. In addition, the return will appear in a fixed range bin that is much closer than the actual range of the false target as seen in Fig. 1.

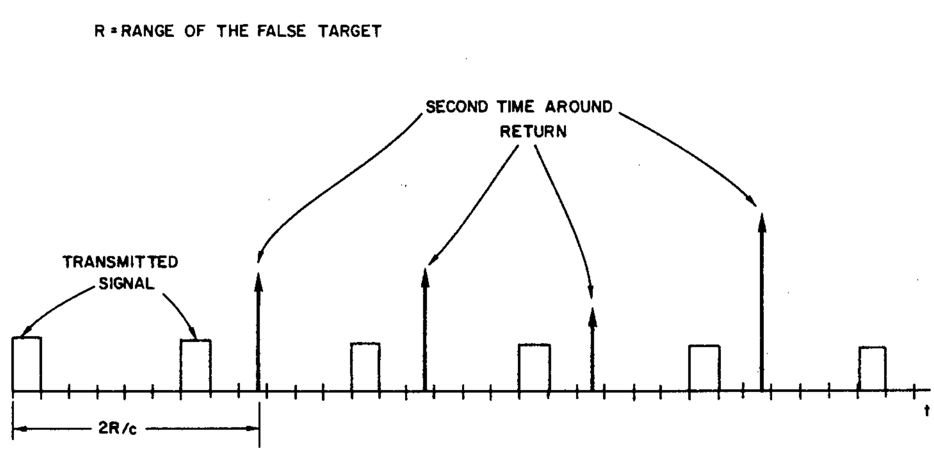

Fig. 1 — Second time around return

If we assume that we are attempting to detect slow moving targets and thus an MTI is *not* used and we process the radar returns using a CFAR threshold detector, there is a good probability that the second time around return will be detected and as a result generate a false target. Also, no amount of integration of noncoherent or coherent pulses will improve the rejection of the undesired detections.

However, we show in this report that modulating the pulse-repetition-interval (PRI) of the transmitted pulses and then integrating the returns over the respective range bins substantially improves the suppression of the second time around returns and thus reduces the false alarm rate.

Figure 2(a) illustrates the modulated PRI concept where the PRI is changed linearly. We transmit n pulses with the pulse separation reduced by ΔT seconds each time a pulse is transmitted. The smallest PRI possible is T_0, and the largest is $T_0 + (n-1)\Delta T$. We assume that $\Delta T \geqslant \tau$ where τ is the pulse width ($c\tau/2$ is the range bin size, $cT_0/2$ is the operating range of the radar, and T_0/τ is the approximate number of range bins).

Manuscript submitted January 7, 1982.

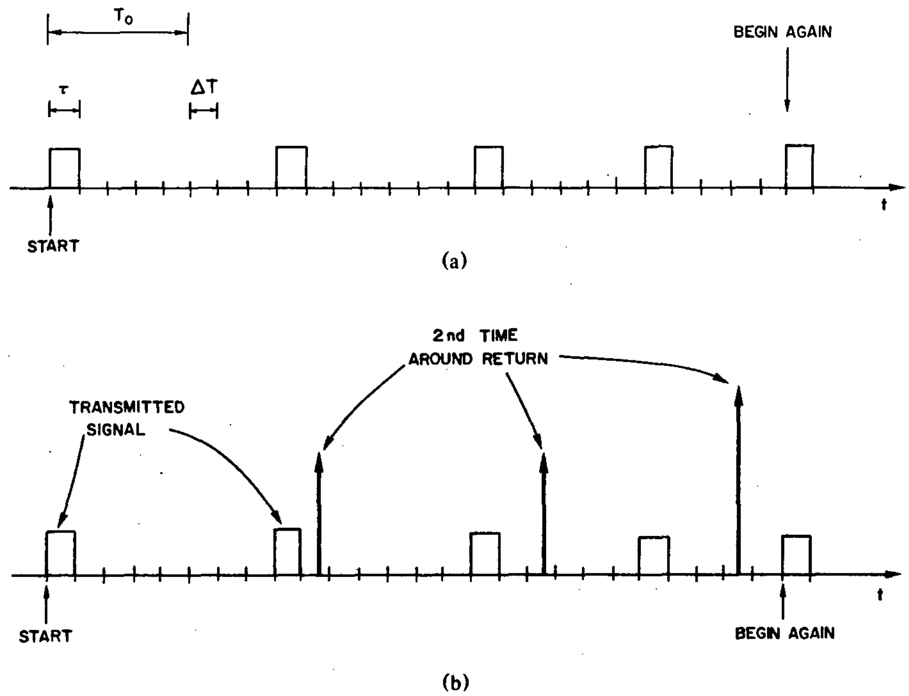

Fig. 2 (a) — Modulated PRI; $n = 4$, $\Delta T = \tau$;
(b) sliding second time around return

Figure 2(b) shows the effect of modulating the PRI on the second time around return. After the second pulse, the undesired return is in the first range bin; after the third pulse, it is in the second range bin and so on. Hence if $\Delta T \geqslant \tau$, the second time around return will slide from one range bin to another and yet never be in the same range bin twice if $n\Delta T < T_0$. If we integrate incoherently over the n pulses for a given range bin, then the effect of the second time around return in a given range bin can be diminished. This occurs because as n increases, the bias threshold also increases while the input power of the second time around return remains constant for that given range bin. Thus, the detection likelihood of that return decreases for the given range bin. However, we must remember that as n increases, the possible number of range bins that the second time around return can appear in also increases. Therefore, it will have more chances to be detected at least once in one of the $n-1$ possible range bins. The next sections present an analysis and discussion of the tradeoffs of using a modulated PRI radar to suppress second time around returns. The results of the following sections also apply if the PRI is randomly jittered so long as same PRI is not repeated over the n transmitted pulses. In addition, we assume that the range extent of the second time around return is less than the radar range bin size, $c\tau/2$.

ANALYSIS

We begin by making the following parameter definitions:

S_d = average single pulse power of the desired signal
N_q = average single pulse power of the quiescent noise
S_2 = average single pulse power of the second time return.

The queiscent noise is the receiver input noise that does not include the second time around return. The quiescent noise power and the desired false alarm rate will determine the detector threshold. We assume that second time around returns do not occur often enough to affect this threshold.

If we use a modulated PRI radar and consider a given range bin that contains only the second time around return and quiescent noise, then after envelope detection and integration the received signal voltage, r, will have the form

$$r = s_2(j) + \sum_{k=1}^{n} n_q(k) \tag{1}$$

where $s_2(j)$ is the second time around return voltage which we assume occurs on just the j th pulse and $n_q(k)$, $k = 1, 2, \ldots n$ is the quiescent noise voltages that occur on every pulse. If we consider the detection of the second time around return in noise, then from Eq. (1), we see that the integrated signal to noise power ratio for the second time around return, (S/N), can be written as

$$\left(\frac{S}{N}\right) = \frac{1}{n}\left(\frac{S_2}{N_q}\right) \tag{2}$$

assuming no integration losses. Hence we see from Eq. (2) that as n increases the integrated (S/N) decreases. The integrated (S/N) is plotted in Fig. 3 for various values of (S_2/N_q). This graph also shows the lossless integration gain or improvement of the desired signal to noise ratio as a function of n.

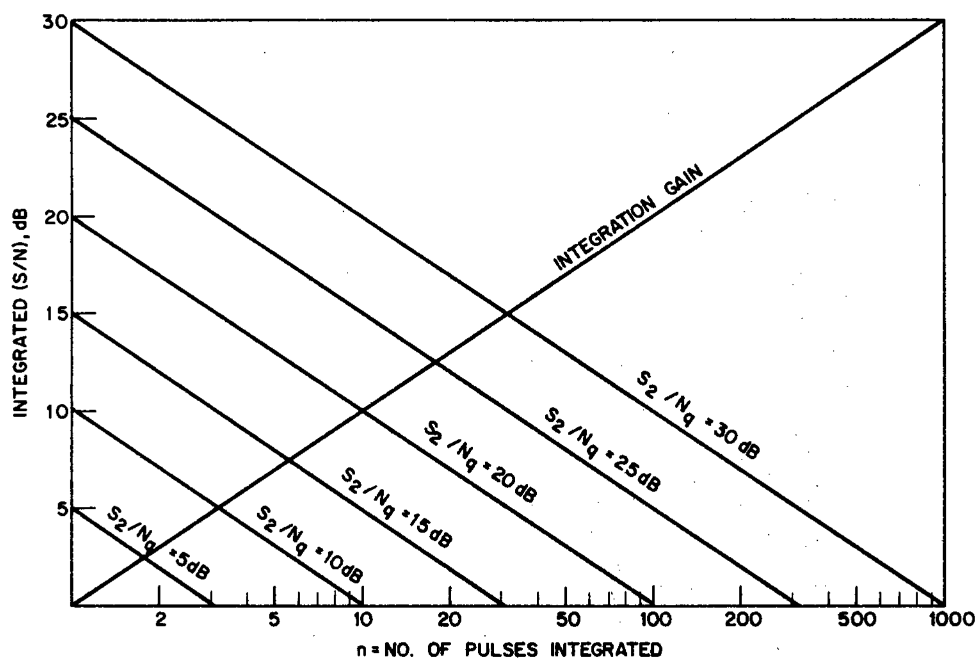

Fig. 3 — Integrated (S/N) vs no. of pulses with (S_2/N_q) as a parameter

Let $P_{fa}^{(q)}$ be the quiescent false alarm probability where $P_{fa}^{(q)}$ is the probability that a false alarm is obtained each time there is an opportunity under the condition that the second time around return is not present. The quiescent false alarm number $r_{fa}^{(q)}$ is related to the quiescent false alarm probability by the relationship [1]

$$P_{fa}^{(q)} = \frac{0.693}{r_{fa}^{(q)}}.$$

The probability, P_D, of detecting the second time around return for a given range bin is a function of (S/N), $r_{fa}^{(q)}$, and the statistical characteristics of the second time around return which we will characterize by its Swerling number, M (see the appendix for an explanation of the various Swerling cases), where $M = 0$, I, II, III, IV (0 indicates a nonfluctuating target). We must be careful when calculating this probability of detection due to the nature of the received signal, r, as seen in Eq. (1). Since the second time around return appears in this equation as a single random variable and not as a sum of random variables, the Swerling cases II and IV reduce to Swerling cases I and III respectively. To see this, we rewrite Eq. (1) as

$$r = \sum_{k=1}^{n} \left[\frac{1}{n} s_2(j) + n_q(k) \right]. \tag{3}$$

Even though $s_2(j)$ may be varying statistically from pulse to pulse, only one of these random pulses appears in a given range bin. This pulse can be modelled for the purposes of analysis as n pulses of identical amplitude, $s_2(j)/n$ in that range bin. Hence Swerling cases II and IV reduce to case I and III respectively. Therefore for each case:

$$P_D\ (\text{case} = 0) = P_D\ (S_2/nN_q,\ r_{fa}^{(q)},\ n,\ M = 0) \tag{4}$$

$$P_D\ (\text{case} = \text{I}) = P_D\ (S_2/nN_q,\ r_{fa}^{(q)},\ n,\ M = \text{I}) \tag{5}$$

$$P_D\ (\text{case} = \text{II}) = P_D\ (S_2/nN_q,\ r_{fa}^{(q)},\ n,\ M = \text{I}) \tag{6}$$

$$P_D\ (\text{case} = \text{III}) = P_D\ (S_2/nN_q,\ r_{fa}^{(q)},\ n,\ M = \text{III}) \tag{7}$$

$$P_D\ (\text{case} = \text{IV}) = P_D\ (S_2/nN_q,\ r_{fa}^{(q)},\ n,\ M = \text{III}) \tag{8}$$

The difference in performance due to the difference in Swerling cases becomes apparent if we define the performance measure, P_2, as the probability that the second time around return will be detected in at least one range bin out of a possible $n-1$ range bins. For Swerling cases 0, I, and III, P_2 is simply equal to P_D since the second time around returns do not vary from pulse to pulse and the threshold in each range bin is equal. Hence if one return exceeds this threshold, all of the returns in each range bin will exceed this threshold. However, for Swerling cases II and IV, the probabilities of detection for each range bin are independent. Thus for each Swerling case we can express P_2 as

$$P_2\ (\text{case} = 0) = P_D\ (\text{case} = 0) \tag{9}$$

$$P_2\ (\text{case} = \text{I}) = P_D\ (\text{case} = \text{I}) \tag{10}$$

$$P_2\ (\text{case} = \text{II}) = 1 - (1 - P_D\ (\text{case} = \text{II}))^{n-1} \tag{11}$$

$$P_2\ (\text{case} = \text{III}) = P_D\ (\text{case} = \text{III}) \tag{12}$$

$$P_2\ (\text{case} = \text{IV}) = 1 - (1 - P_D\ (\text{case} = \text{IV}))^{n-1} \tag{13}$$

It is possible using well-known formulas [1] and existing computer programs [2] to calculate P_D as expressed by the parameters seen in Eqs. (4) through (8). Using these results, we can calculate P_2 for each Swerling case by using Eqs. (9) through (13). We plot in Figs. 4 through 9, P_2 versus n using the quiescent false alarm number, Swerling case, and the second-time around return to quiescent noise power ratio as parameters. We also assume a square law detector is part of the radar receiver.

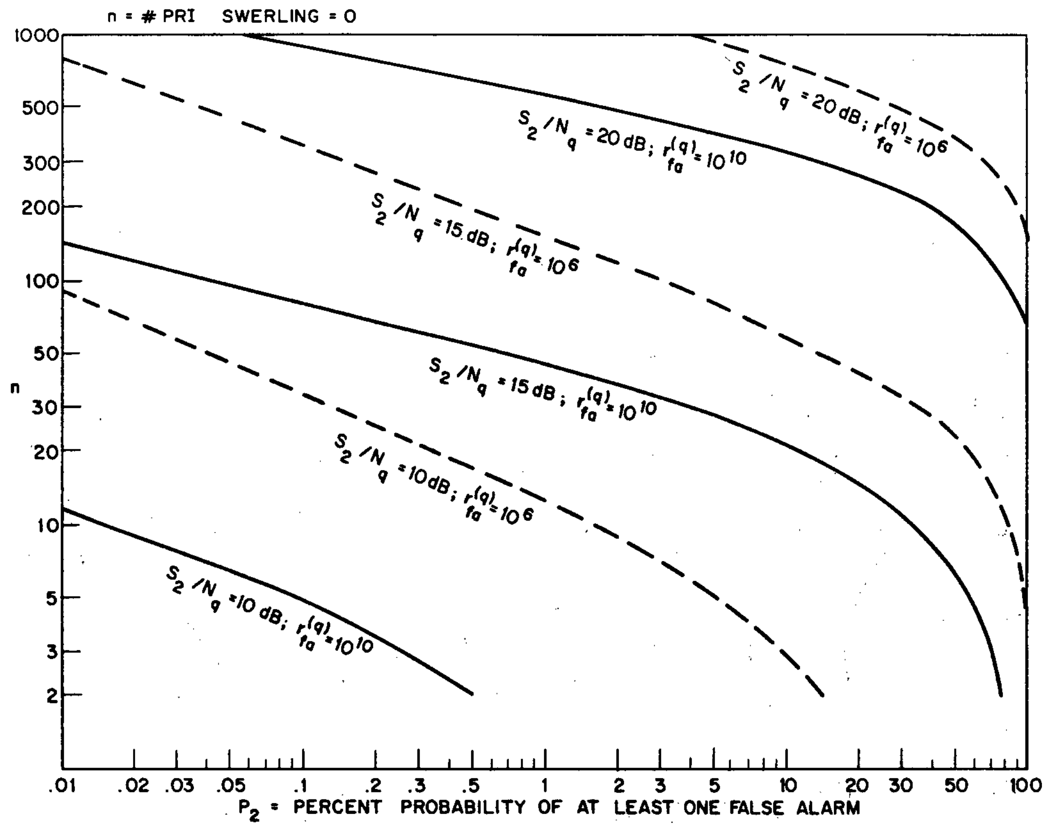

Fig. 4 — P_2 vs n; Swerling case 0

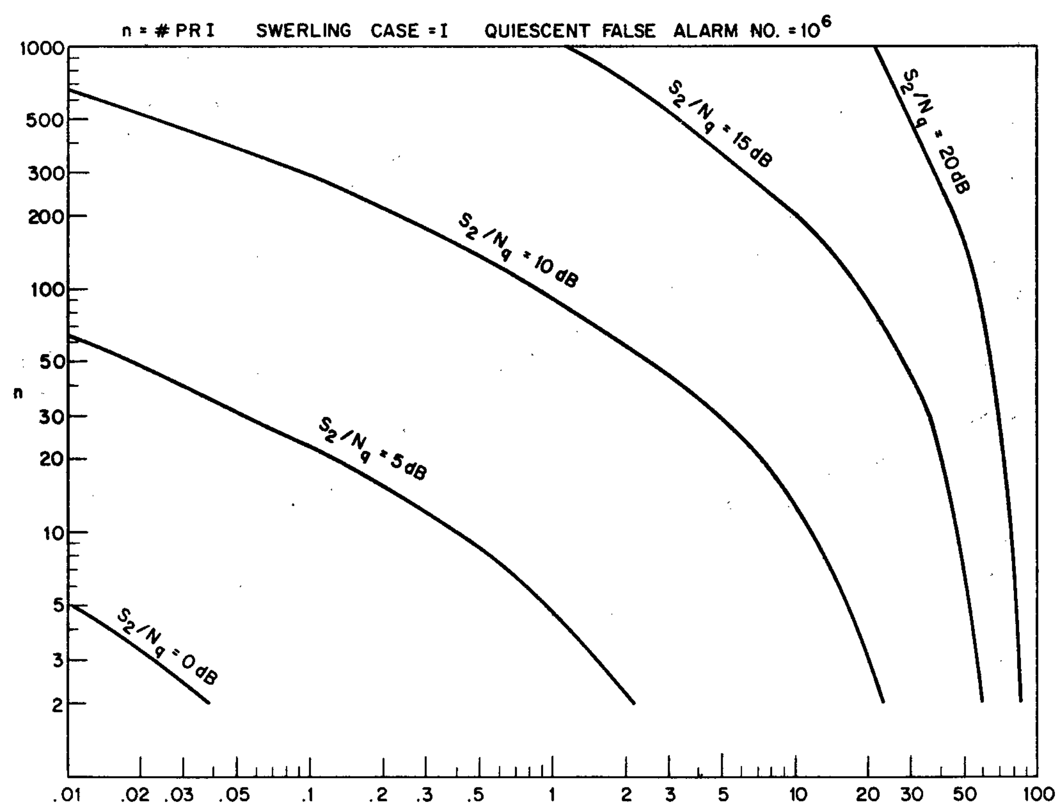

Fig. 5 — P_2 vs n; Swerling case I

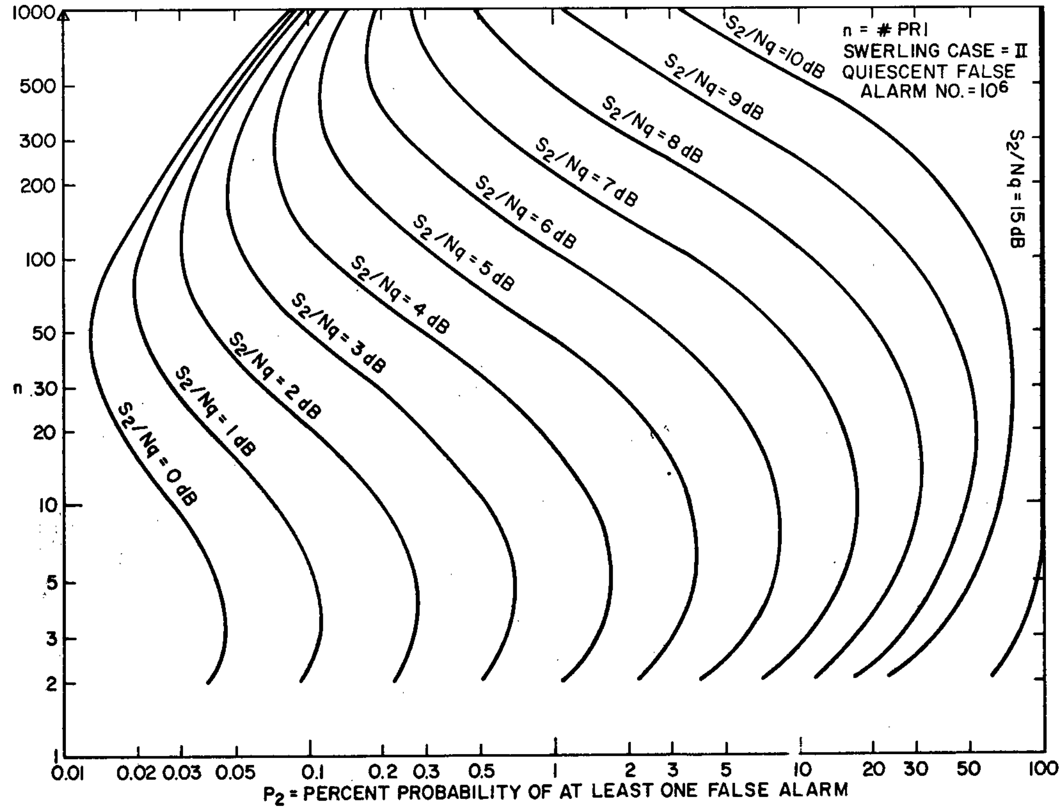

Fig. 6 — P_2 vs n; Swerling case II, $r_{fa}^{(q)} = 10^6$

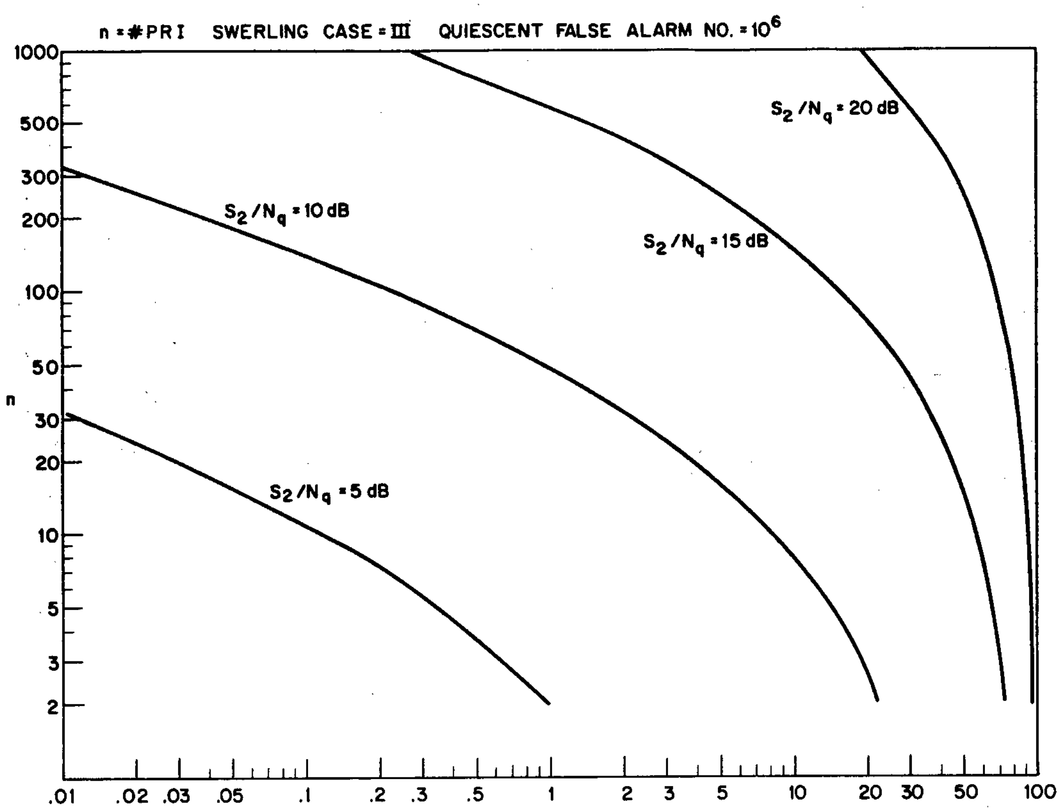

Fig. 7 — P_2 vs n; Swerling case III

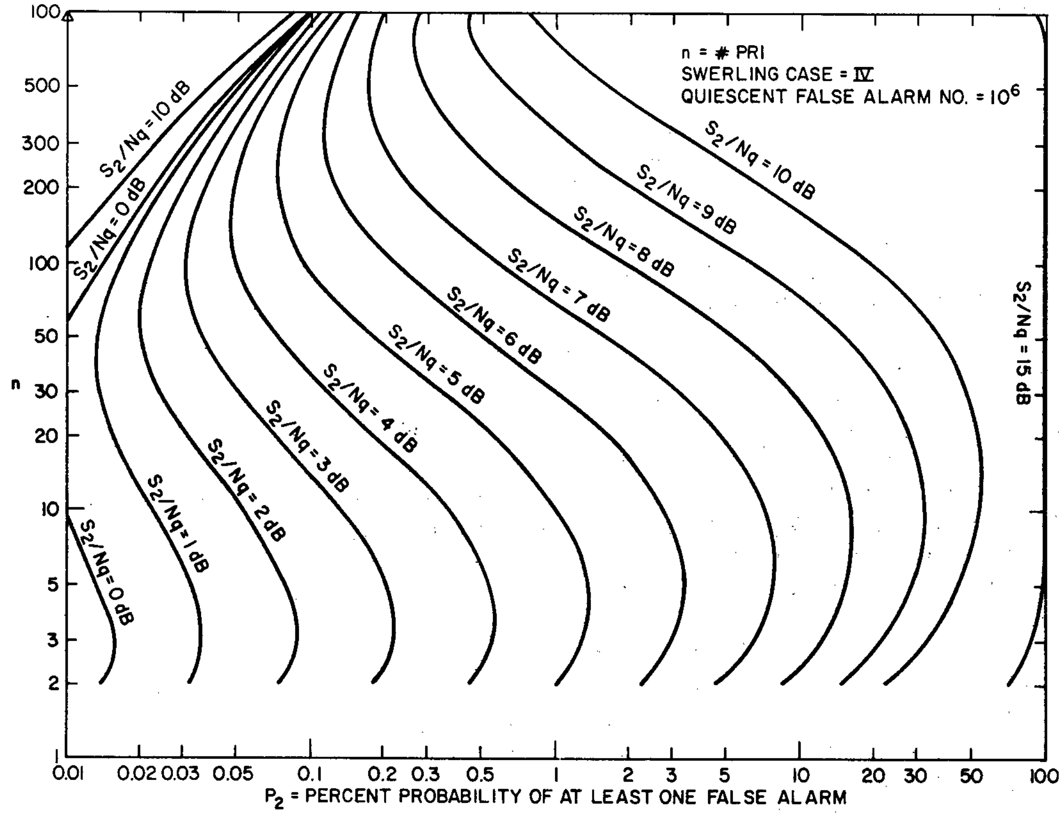

Fig. 8 — P_2 vs n; Swerling case IV

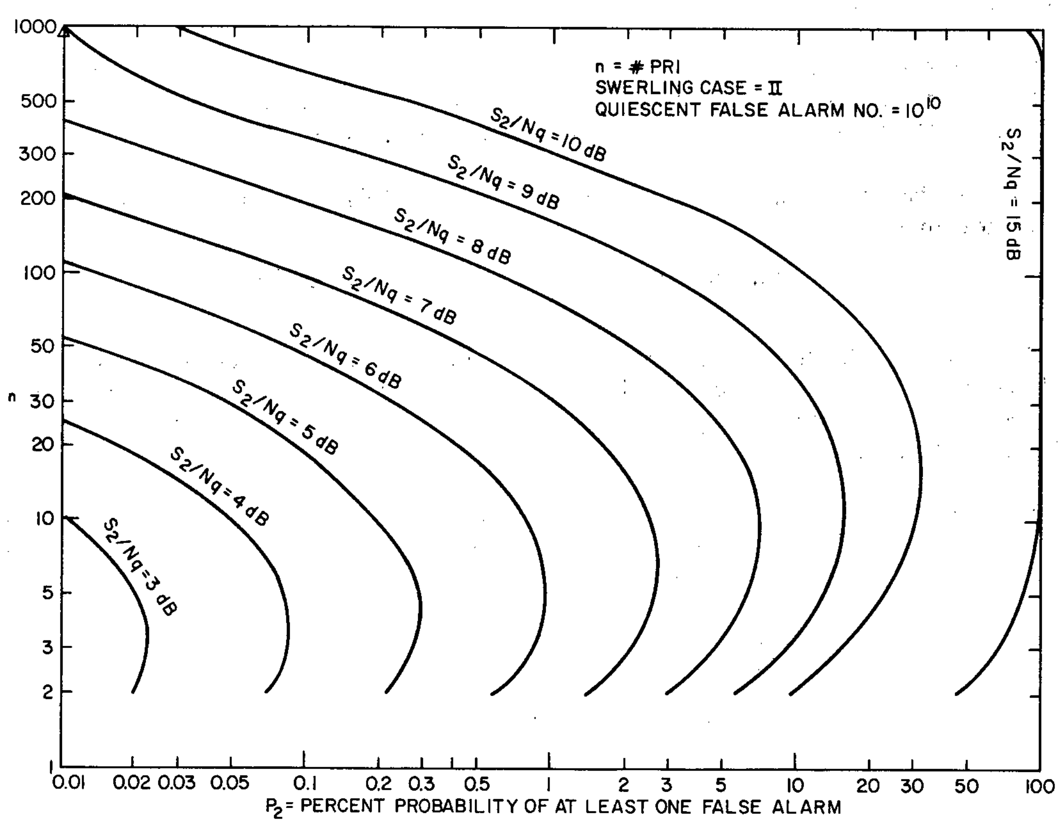

Fig. 9 — P_2 vs n; Swerling case II, $r_{fa}^{(q)} = 10^{10}$

DISCUSSION

Swerling Cases II and IV

If we keep all other parameters constant except the Swerling case, then the curves seen in Figs. 6 and 8 indicate that

$$P_2 \text{ (case = IV)} < P_2 \text{ (case = II)}. \tag{14}$$

Thus we see from Eq. (14) that if the false target can be characterized by one large reflector together with a number of small reflectors, then performance improves (P_2 becomes smaller). However, there is not a large change in performance.

The plots in Figs. 6 and 8 also obviously indicate that performance degrades as the second time around return to quiescent noise power ratio, (S_2/N_q), increases. We see from these figures that P_2 is very sensitive to (S_2/N_q). For example, if the Swerling case is II or IV and the number of PRI's, n, is approximately 50, then decreasing (S_2/N_q) by 5 dB results in a hundredfold decrease in probability of at least one false alarm. Also we see for $(S_2/N_q) > 10$ dB and the quiescent false alarm number equal to 10^6 that for most practical purposes, even a modulated PRI system does not effectively suppress the second time around return. These plots also indicate for $(S_2/N_q) < 5$ dB that the modulated PRI system can offer significant improvement if the number of PRI's is chosen properly.

Let us examine how P_2 varies with n, the number of modulated PRI's. We see that for small n, that in most cases P_2 rises to a local maximum then decreases to an absolute minimum and finally increases. In fact, it can be shown that as n approaches infinity that P_2 approaches one. Intuitively, this occurs because as $n \to \infty$, the integrated signal to noise ratio as expressed by Eq. (2) goes to zero. Thus for a given range bin, the probability of detecting the second time around return will approach the quiescent false alarm probability or $P_D \to P_{fa}^{(q)}$. Hence from Eqs. (11) or (13), we see that $P_2 \to 1$ as $n \to \infty$ and $P_D \to P_{fa}^{(q)}$. In practice, P_2 will not approach one because n is upper bounded by the number of range bins that are possible.

The local maximum exists in most cases because as n initially increases from two, there are more opportunities for the second return to be detected whereas the decrease in integrated input (S/N) as expressed by Eq. (2) does not offset this until after the local maximum.

If we asked what is the improvement of using modulated PRI over a nonmodulated PRI system, then we can show that

$$P_2 \text{ (nonmodulated, } n) \geqslant P_2 \text{ (2 pulse modulated PRI)}. \tag{15}$$

Equation (15) is true under the assumption that we are integrating more than one pulse for the nonmodulated system. The inequality becomes larger as the number of integrated pulses increases for the nonmodulated PRI system. In addition, we can show that if the number of pulses integrated is larger than the number of PRI's, then performance degrades.

We see an interesting phenomena if we compare the curves of Figs. 6 and 9. In these figures, all parameters are the same except for the quiescent false alarm number. In Fig. 6, the false alarm number is 10^6 and in Fig. 9, the false alarm number is 10^{10}. We see that by increasing the false alarm number to 10^{10} that tremendous improvement is possible. For example, if $n = 50$, $(S_2/N_q) = 5$ dB, and $r_{fa}^{(q)} = 10^6$, then $P_2 = 1\%$. However, if we raise the false alarm number to 10^{10} while holding the other parameters constant, then P_2 falls to 0.01%.

The price we pay by decreasing the quiescent false alarm probability is that the detection probability of the desired signal decreases. This is because the threshold of the detector must be raised in order to decrease the probability of a false alarm. Let us examine what occurs to the modulated PRI system performance if we increase the desired signal to quiescent noise ratio in order to maintain a constant probability of detection for desired targets.

In Fig. 10 we have plotted the required signal to noise ratio versus the number of integrated pulses necessary to obtain a probability of detection of 0.5 for a Swerling case II target using the false alarm number as a parameter. We see from the figure that in order to maintain $P_D = 0.5$ for all n, we need only to increase our transmitter power by approximately 1.5 dB when going from a false alarm number of 10^6 to 10^{10}. However by increasing the transmitter power by a given amount also increases the second time around return's power by that same amount. Hence P_2 will increase.

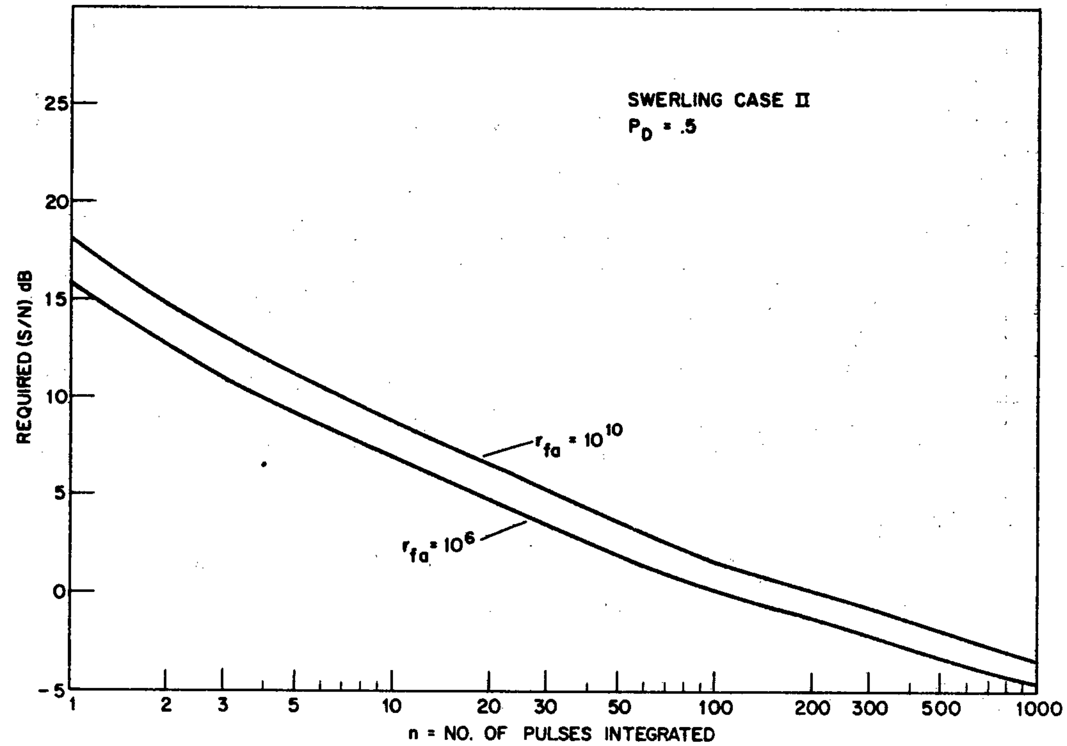

Fig. 10 — Required (S/N) vs n

For example using Fig. 6, if $n = 100$, $(S_2/N_q) = 5$ dB, $r_{fa}^{(q)} = 10^6$, and the radar returns are Swerling case II, then $P_2 = 0.35\%$. If we raise $r_{fa}^{(q)}$ to 10^{10} and also increase our transmitter power by 1.5 dB to maintain a constant probability of detection, then $(S_2/N_q) = 6.5$ dB. For this case, we can show using Fig. 9 that if $n = 100$, then $P_2 = 0.35\%$. Therefore by slightly increasing the transmitter power and increasing the detector threshold, we have decreased the probability of at least one second time around return being detected by tenfold. Hence, it would seem that modulated PRI systems work best when the quiescent false alarm number of the detector is large. Additional curves similar to those seen in Fig. 10 are found in Ref. 3 for various Swerling cases and false alarm numbers.

Plots similar to those seen in Figs 6 and 8 are possible whereby we vary the desired signal to second time around return power ratio, S_d/S_2, while holding the quiescent signal to noise ratio, S_d/N_q, a constant. These result because we can write

$$S_d/S_2 = (S_d/N_q)/(S_2/N_q). \tag{16}$$

We plot P_2 vs n with S_d/S_2 as a parameter for a Swerling case II target, $r_{fa}^{(q)} = 10^6$, and the quiescent signal to noise ratio equal to 0 dB in Fig. 11. Not unexpectantly, we see that performance degrades as S_d/S_2 decreases or equivalently as the second time around return's power increases.

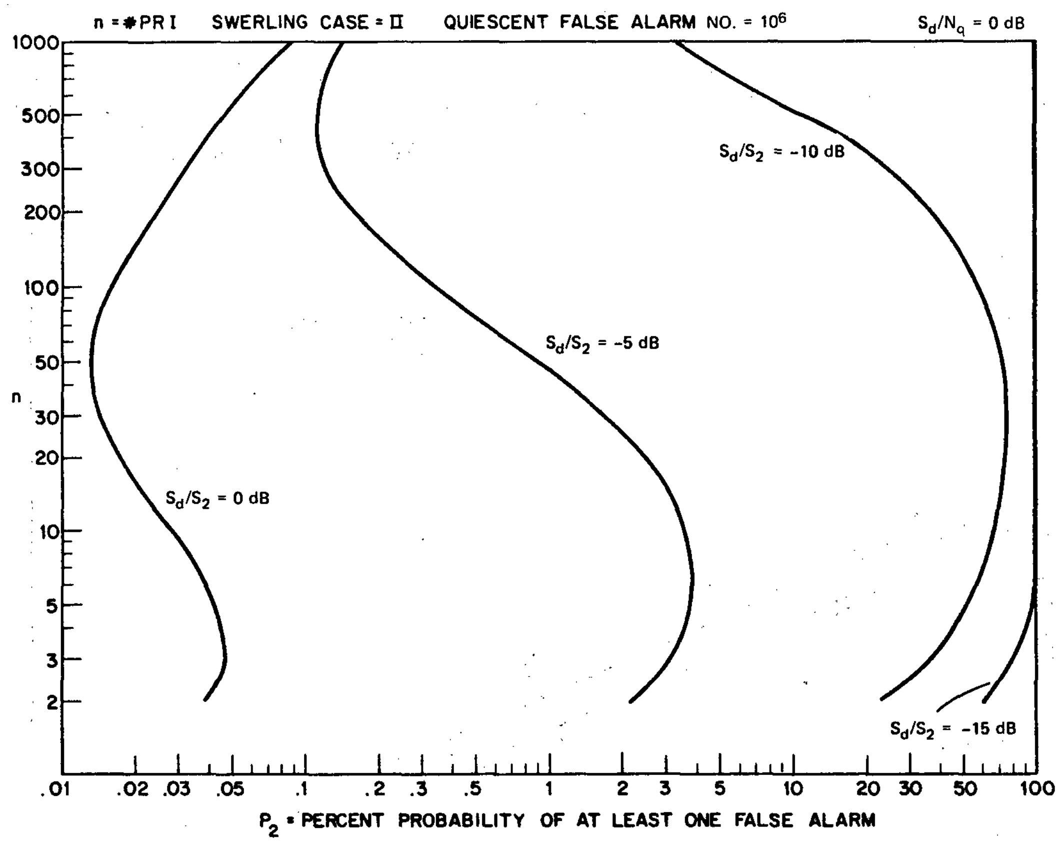

Fig. 11 — P_2 vs n with S_d/S_2 as a parameter

Swerling Cases 0, I, and III

We can order the performance of the modulated PRI radar system by Swerling number and find that

$$P_2\,(0) < P_2\,(\mathrm{III}) < P_2\,(\mathrm{I}) \tag{17}$$

where we hold all other parameters equal. Thus a nonfluctuating second time around return (case 0) is suppressed to greater extent than fluctuating returns. In addition, if the false target can be characterized by one large reflector together with a number of small reflectors, then performance improves.

Similar to Swerling cases II and IV, performance degrades as S_2/N_q increases and is very sensitive to this parameter as indicated by the curves seen in Figs. 4, 5, and 7. However, unlike cases II and IV, P_2 has no local extrema when the number of PRI's is varied. For Swerling cases 0, I, and III, P_2 is a monotonically decreasing function of n. Its maximum occurs at $n = 2$ and its minimum at $n = \infty$. In fact, it is possible to show (see the discussion on cases II and IV) that $P_2 \rightarrow P_{fa}^{(q)}$ as $n \rightarrow \infty$. Also similar to cases II and IV, significant improvement in performance is possible by raising the detection threshold (and hence the false alarm rate) as indicated by the curves seen in Fig. 4.

A DESIGN EXAMPLE

Let us determine the number of modulated PRI's necessary such that the probability of at least one false alarm due to the second time around return is less than 1%. We do this under the following conditions:

1. The second time around return is located just beyond the maximum operating range of the radar (i.e., at least as far away as a desired target at the maximum operating range).
2. The second time around return's radar cross section is three times larger than the desired target.
3. The quiescent false alarm number is 10^6.
4. The signal to quiescent noise ratio of the desired signal is 0 dB at the maximum operating range.
5. The second time around return pulses are independent from pulse to pulse and consist of many uniformly distributed scatterers (Swerling case II).

From conditions 1, 2 and 4, we can show that $(S_2/N_q)_{dB} = 4.8$ dB.

To find n, the required number of modulated PRI, we use Fig. 6 and the above given parameters. From this figure, we see that n is approximately 45. Note that we can reduce the number of modulated PRI significantly by raising the quiescent false alarm number and increasing our transmitter power slightly in order to maintain a constant probability of detection for the desired target (see Fig. 10). Hence we see that the processing complexity can be reduced by using more transmitter power. Also note that we placed the second time around return at the best possible range for its detection. In most situations the bogus return will be located much farther away than the maximum operating range of the radar so that its cross section can increase considerably while still maintaining P_2 less than 1%.

CONCLUSIONS

We have shown that a staggered PRI radar system can offer considerable improvement over a nonstaggered radar system in rejecting second time around returns which cause false alarms. This improvement is a function of the number of staggered PRI, the quiescent false alarm number, the Swerling number of the false return, the transmitted signal power, the second time around noise power, and the quiescent noise power of the radar. Small changes in transmitted signal power can be traded-off with the quiescent false alarm number to significantly suppress the bogus return. In addition for all other parameters being equal, if the second time around return is a Swerling case II or IV target, then there is an optimum number of staggered PRI that can be chosen to minimize the likelihood of detection of the second time around return.

REFERENCES

1. J.V. DiFranco and W.L. Rubin. *Radar Detection,* Artech House Inc., Dedham, Mass., 1980
2. D.J. Kaplan, A. Grindlay, and L. Davis: "A Surveillance Radar System Evaluation Model (SRSEM) Handbook," NRL Report 8037, Jan. 1977.
3. L.V. Blake, "Prediction of Radar Range," Chap. 2 of *Radar Handbook,* M.I. Skolnik, ed., McGraw Hill, New York, 1970.

Appendix
SWERLING CASES

Swerling [A1] employed four different fluctuation models of radar cross section in calculating the probability of detection of targets modeled in this way. The four fluctuation models are as follows:

Case 1. The echo pulses received from a target on any one scan are of constant amplitude throughout the entire scan but are independent (uncorrelated) from scan to scan. This assumption ignores the effect of the antenna beam shape on the echo amplitude. The probability-density function for the cross section σ is given by the density function

$$p(\sigma) = \frac{1}{\sigma_{av}} \exp\left(-\frac{\sigma}{\sigma_{av}}\right) \quad \sigma \geqslant 0 \tag{A1}$$

where σ_{av} is the average cross section over all target fluctuations.

Case 2. The probability-density function for the target cross section is also given by Eq. (A1), but the fluctuations are more rapid than in case 1 and are taken to be independent from pulse to pulse instead of from scan to scan.

Case 3. In this case, the fluctuation is assumed to be independent from scan to scan as in case 1, but the probability-density function is given by

$$p(\sigma) = \frac{4\sigma}{\sigma_{av}^2} \exp\left(-\frac{2\sigma}{\sigma_{av}}\right). \tag{A2}$$

Case 4. The fluctuation is pulse to pulse according to Eq. (A2).

We refer to the nonfluctuating radar cross section as Case 0.

REFERENCES

A1. P. Swerling, "Probability of Detection for Fluctuating Targets," *IRE Trans.* **IT-6**, 269-308, April 1960.