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Estimation of Bias Errors in Angle-of-Arrival Measurements Using Platform Motion

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An algorithm has been developed to estimate the bias errors in angle-of-arrival measurements made by electromagnetic detection devices on-board a pitching and rolling platform. The algorithm assumes that continuous exact measurements of the platform's roll and pitch conditions are available. When the roll and pitch conditions are used to transform deck-plane angular measurements of a nearly fixed target's position to a stabilized coordinate system, the resulting stabilized coordinates (azimuth and elevation) should not vary with changes in the roll and pitch conditions. If changes do occur they are a result of bias errors in the measurement system and the algorithm which has been developed uses these changes to estimate the sense and magnitude of angular bias errors.

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CONTENTS

ESTIMATION OF BIAS ERRORS IN ANGLE-OF-ARRIVAL MEASUREMENTS USING PLATFORM MOTION

INTRODUCTION

Sensors on-board naval platforms make angle-of-arrival measurements on sources of electromagnetic radiation. These measurements may be subject to bias errors. Bias errors are those errors inherent to each sensor system which may have been introduced during the construction or alignment of the sensor, or are present as a result of equipment failures. It is desirable to estimate and remove these errors from angle-of-arrival measurements, and this report describes an algorithm that has been developed for this purpose. The effectiveness of this algorithm is tested, using simulated stationary targets.

ALGORITHM DEVELOPMENT

The problem is: given the time history of measurements (azimuth and elevation) on several distributed targets, as measured by a sensor on board a pitching and rolling platform, determine the measurement bias errors. (We know the platform's pitch and roll with respect to the local stabilized coordinates.) Before describing the algorithm, the coordinate systems that are used in the development are reviewed [1].

The origin of the deck-plane coordinate system is located at the platform's center of gravity, with the z_d-axis pointing upward and normal to the deck-plane, the y_d-axis lying in the deck-plane and pointing towards the platform's bow, and the x_d-axis orthogonal to the z_d and y_d axes. The stabilized coordinate system also has its origin located at the platform's center of gravity. Its orientation is defined by a gyro compass with the z_s-axis pointing upward along the local gravity vector, the y_s-axis normal to the z_s-axis and pointing north, and the x_s-axis pointing east. The x_s and y_s axes used are rotated by the angle of the ship's heading. Although it is not usually the case, the sensor has been located at the center of gravity of the platform to simplify the algorithm. This assumption has little effect on the results and eliminates the need to translate coordinates from the sensor's phase center to the center of gravity. The sensor is assumed to measure periodically the angle-of-arrival of signals from stationary targets. The measurements are made in the deck-plane coordinate system. Azimuth is measured clockwise from the y-axis in the deck-plane, and the elevation is the angle between a radial to the target and the deck-plane with targets above the deck-plane with positive elevations.

The platform's roll and pitch are assumed to be sinusoidal functions of time (t) and are:

$$R(t) = R_M \cos\left(\frac{2\pi t}{T_R}\right) + \gamma_R \tag{1}$$

and

$$P(t) = P_M \cos\left(\frac{2\pi t}{T_P}\right) + \gamma_P \quad , \tag{2}$$

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where R_M and P_M are the maximum roll and pitch angles; T_R and T_P are the roll and pitch periods; and γ_R and γ_P are the roll and pitch phase angles. The roll and pitch periods are assumed to be independent random variables and uniformly distributed between 8 and 14 s, and 5 and 7 s. The phase angles are also assumed to be independent random variables and uniformly distributed between 0 and 2π.

If we know the roll and pitch at the instant the measurements are taken, it is then possible to develop a relationship between the elevation and azimuth in the stabilized coordinate system (e_s, a_s), and the elevation and azimuth in the deck-plane coordinate system (e_d, a_d). The equations as derived in [2] are:

$$a_s = \tan^{-1}\left[\frac{-\sin R \sin e_d + \cos R \sin a_d \cos e_d}{\cos P \cos a_d \cos e_d + W \sin P}\right], \tag{3}$$

$$e_s = \sin^{-1}\left[-\sin P \cos a_d \cos e_d + W \cos P\right], \tag{4}$$

where

$$W = \cos R \sin e_d + \sin R \sin a_d \cos e_d \ . \tag{5}$$

Assuming that the roll and pitch measurements do not contribute to errors in the transformed azimuth and elevation measurements in the stabilized coordinate system, it is possible to estimate $a_s(t)$ and $e_s(t)$ at some time, t, with the truncated Taylor series expanded about the true or mean target position ($\overline{a}_s, \overline{e}_s$); i.e.,

$$a_s = \overline{a}_s + (\partial a_s/\partial a_d)(B_a + N_a) + (\partial a_s/\partial e_d)(B_e + N_e) , \tag{6}$$

$$e_s = \overline{e}_s + (\partial e_s/\partial a_d)(B_a + N_a) + (\partial e_s/\partial e_d)(B_e + N_e) , \tag{7}$$

where B_a and B_e are the bias errors in the deck-plane azimuth and elevation measurements, and N_a and N_e represent the zero mean noise in the measurements.

Expressing Eqs. (6) and (7) in vector notation

$$\Lambda(t) = \overline{\Lambda}(t) + A(t) \cdot \left[B + N(t)\right] , \tag{8}$$

where

$$A(t) = \begin{bmatrix} \partial a_s/\partial a_d \, \partial a_s/\partial e_d \\ \\ \partial e_s/\partial a_d \, \partial e_s/\partial e_d \end{bmatrix} ; \Lambda(t) = \begin{bmatrix} a_s(t) \\ \\ e_s(t) \end{bmatrix} ; B = \begin{bmatrix} B_a \\ \\ B_e \end{bmatrix} ; \text{and } N = \begin{bmatrix} N_a \\ \\ N_e \end{bmatrix} .$$

At some later instant in time $(t + 1)$

$$\Lambda(t+1) = \overline{\Lambda}(t+1) + A(t+1) \cdot \left[B + N(t+1)\right] . \tag{9}$$

For most targets and certainly for stationary targets the true evaluation and azimuth will not change significantly over the time between measurements. For Δt equal to the time between measurements, we can safely assume that

$$\overline{\Lambda}(t) = \overline{\Lambda}(t + 1) . \tag{10}$$

However, in general $\Lambda(t) \neq \Lambda(t + 1)$ since the instantaneous R and P will change with time, and contributions to a_s and e_s from the bias errors that are fixed in the deck-plane vary with roll and pitch. To better visualize this situation, consider Fig. 1.

Figure 1 illustrates the case where the target is located directly ahead of the platform at zero elevation and there is a large azimuth bias error with no accompanying elevation bias. As seen in Fig. 1, the distance 'a' determines the magnitude of the azimuth (a_s) in the stabilized coordinate system. As the roll increases the contribution from the deck-plane bias to the stabilized azimuth coordinate (a_s) decreases; i.e., a_s as determined from Eq. (3) will vary from scan to scan because the bias errors are constant in the deck-plane coordinate system and do not change as the platform pitches and rolls. Consequently, the magnitude and sense in the bias errors will be reflected by the changes of the stabilized coordinates (a_s, e_s) as the platform pitches and rolls. To quantify this idea we subtract Eq. (8) from Eq. (9) and obtain

$$\Delta\Lambda = M \cdot B + N , \tag{11}$$

where

$$\Delta\Lambda = \Lambda(t + 1) - \Lambda(t)$$

$$M = A(t + 1) - A(t)$$

$$N = A(t + 1) \cdot N(t + 1) - A(t) \cdot N(t) .$$

Equation (11) is in the form of the observation equation for linear estimation. In this case the $\Delta\Lambda$ vector represents the measurements and the B vector of bias errors is the state vector. The state equation is:

$$B(t + 1) = I \cdot B(t) . \tag{12}$$

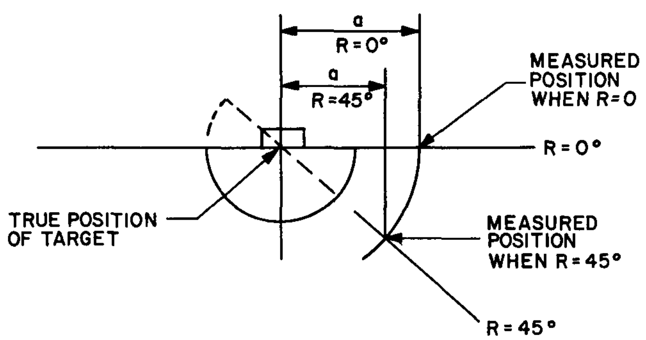

Fig. 1 — Variation in stabilized azimuth due to deck-plane azimuth bias

(The state transition matrix is the identity matrix since the bias errors are assumed to be constant over the period of interest.) Using the formulation given in Eqs. (11) and (12) it is possible to estimate the bias errors using the Kalman filter algorithm. The six steps which are required to implement the recursive Kalman filter can be found in Ref. 3 where they are applied to a similar formulation.

To complete the development of the algorithm, it is necessary to derive expressions for the elements of the A matrix. To simplify the process, Eq. (3) is rewritten as

$$a_s = \tan^{-1} [U/V] , \tag{13}$$

where

$$U = -\sin R \sin e_d + \cos R \sin a_d \cos e_d ,$$

and

$$V = \cos P \cos a_d \cos e_d + W \sin P .$$

This enables us to express the first two elements:

$$A(1,1) = \partial a_s/\partial a_d = (V^2 + U^2)^{-1} \left[V(\partial U/\partial a_d) - U(\partial V/\partial a_d)\right] , \tag{14}$$

$$A(1,2) = \partial a_s/\partial e_d = (V^2 + U^2)^{-1} \left[V(\partial U/\partial e_d) - U(\partial V/\partial e_d)\right] , \tag{15}$$

where

$$\partial U/\partial a_d = \cos R \cdot \cos a_d \cdot \cos e_d , \tag{16}$$

$$\partial V/\partial a_d = -\cos P \cdot \sin a_d \cdot \cos e_d + \sin P \cdot \sin R \cdot \cos a_d \cdot \cos e_d , \tag{17}$$

$$\partial U/\partial e_d = -\sin R \cdot \cos e_d - \cos R \cdot \sin a_d \cdot \sin e_d , \tag{18}$$

and

$$\partial V/\partial e_d = -\cos P \cdot \cos a_d \cdot \sin e_d + \sin P[\cos R \cdot \cos e_d - \sin R \cdot \sin a_d \cdot \sin e_d] . \tag{19}$$

Equation (4) is rewritten

$$e_s = \sin^{-1}(K) , \tag{20}$$

where

$$K = -\sin P \cos a_d \cos e_d + W \cos P .$$

This leads to:

$$A(2,1) = \partial e_s / \partial a_d = (1 - K^2)^{-1/2} \cdot (\partial K / \partial a_d)\,, \tag{21}$$

and

$$A(2,2) = \partial e_s / \partial e_d = (1 - K^2)^{-1/2} \cdot (\partial K / \partial e_d) \tag{22}$$

where

$$\partial K / \partial a_d = \sin P \sin a_d \cos e_d + \cos P \sin R \cos \; a_d \cos e_d \tag{23}$$

and

$$\partial K / \partial e_d = \sin P \cos a_d \sin e_d + \cos P [\cos R \cos e_d - \sin R \sin a_d \sin e_d]\;. \tag{24}$$

RESULTS

The algorithm was applied to a set of simulated data. Six stationary points in space were chosen to simulate targets which were distributed around a centrally located platform. The location of the platform and the targets together with their respective latitudes and longitudes are shown in Fig. 2. All of the targets were assigned an altitude of 10,000 m. Measurement data were generated by select-

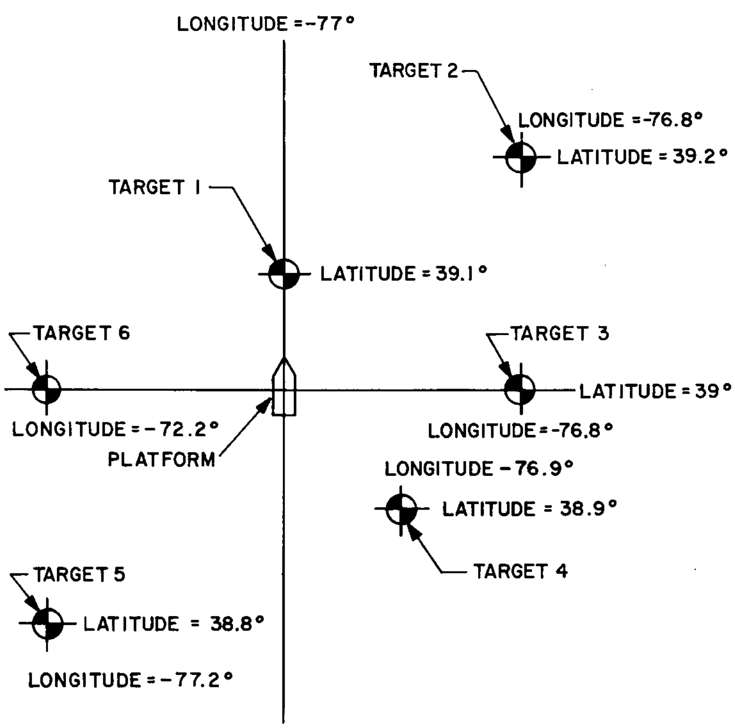

Fig. 2 — Target position

ing samples from a Gaussian distribution derived from a random number generator and by adding these samples weighted by the measurement uncertainty to the azimuth and elevation coordinates at each site. Bias errors were also added to the measurements. This resulted in a set of simulated azimuth and elevation measurements for each of the six targets. The difference in the location of each target measured periodically in time was used as the input to the Kalman filter.

Pitch and roll data were generated each time a measurement was made by assuming that the pitch period, roll period, pitch phase, and roll phase were uniformly distributed random variables. The algorithm was checked with various values of roll and pitch, bias errors, and standard deviations of the measurement's accuracy.

The first case to be considered used a relatively large roll and pitch magnitude (10° for both), 1° azimuth bias, −1° elevation bias, standard deviation of 0.5° in the azimuth measurement noise and 1° in the elevation measurement noise. In the case studied, the sensor is assumed to be a radar whose high-gain antenna rotates in azimuth. The results are displayed in Fig. 3 which plots an estimate of the bias error each time a measurement is made on an individual target. The resulting estimates were within 0.1° in elevation and 0.2° in azimuth of the actual bias errors, after 1200 observations or 200 rotations of the sensor. Figure 3 indicates that it is possible to achieve a better estimate of the elevation bias than the azimuth bias. In fact, the estimate of the elevation bias was less than 0.2° of the actual bias after 40 observations. This was expected since the algorithm depends on the difference in subsequent values of the stabilized coordinates which were created by platform motion and because of the geometry of the situation, the differences in the elevation were more pronounced.

Effects of Reducing Platform Motion

After achieving success with relatively large rolls and pitches, an attempt was made to determine the effects of limiting platform motion. The results of reducing the roll and pitch magnitudes to 5° are shown in Fig. 4. After 1200 measurements, the estimate of the elevation bias was within 0.1° of the actual bias, and the estimate of the azimuth bias was at an acceptable level of 0.3° from the actual bias error. After 500 measurements, the use of the estimated bias errors significantly improved the accuracy of angle-of-arrival estimates. Further reduction of the roll and pitch magnitudes to 1° produced unacceptable estimates of the azimuth bias as shown in Fig. 5. The estimate of the elevation bias had also deteriorated but not to the same extent. Backing up to 2° on the roll and pitch gave a significant increase in performance. The corresponding results are presented in Fig. 6. Although the estimated azimuth bias was 0.5° off the actual bias, the use of the estimated biases after 600 measurements improved the accuracy of the angle-of-arrival measurements.

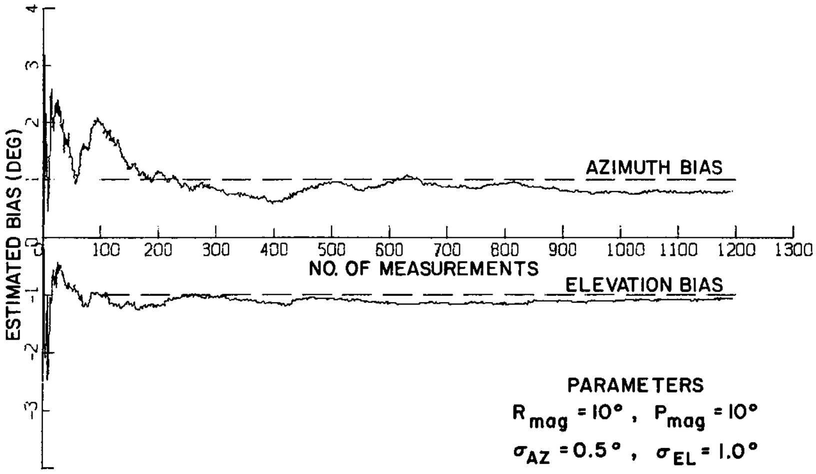

Fig. 3 — Estimate of bias errors

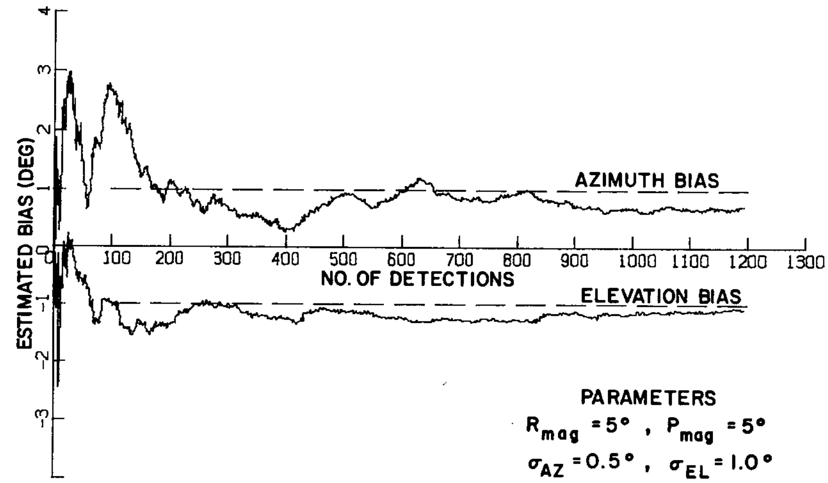

Fig. 4 — Estimate of bias errors

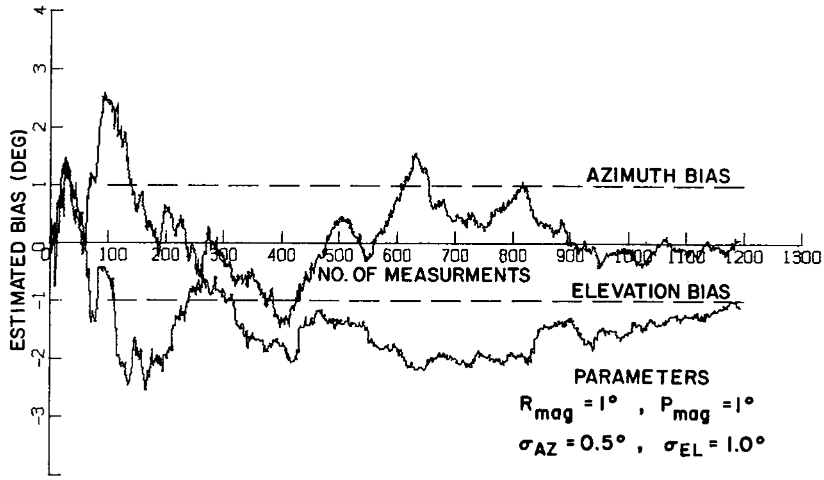

Fig. 5 — Estimate of bias errors

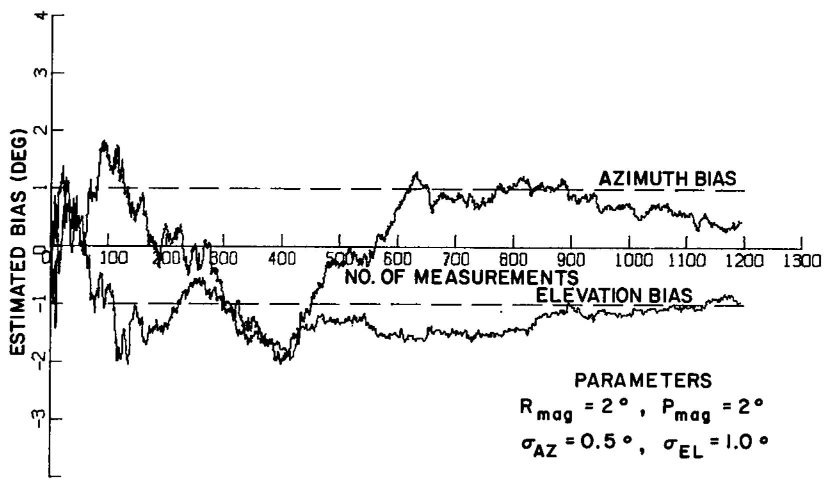

Fig. 6 — Estimate of bias errors

Performance with Larger and Smaller Bias Errors

The performance of the algorithm also was checked with larger (+3°) and smaller (0°) bias errors. To allow comparison with Fig. 5 the pitch and roll magnitudes were set at 1°. Figure 7 shows the results for the case with an azimuth bias error of 3° and an elevation bias of −3°. It appeared from Fig. 7 that the large bias errors (+3°) could be estimated more readily than smaller bias errors. An intuitive explanation for this observation is that the effects of small bias errors are lost in the noise of the measurements. This is especially true when the platform motion is reduced to a low level (1° pitch and roll). Fortunately small values of $\Delta\Lambda$ (Eq. 11) do not create singularities with the algorithm but are interpreted as an indication of zero bias. Figure 8 shows the results for the case of zero bias for 1° roll and pitch. The results are reasonably good considering the low level of platform motion. Increasing the roll and pitch to 2° significantly reduces the fluctuations in the estimate. This result is presented in Fig. 9.

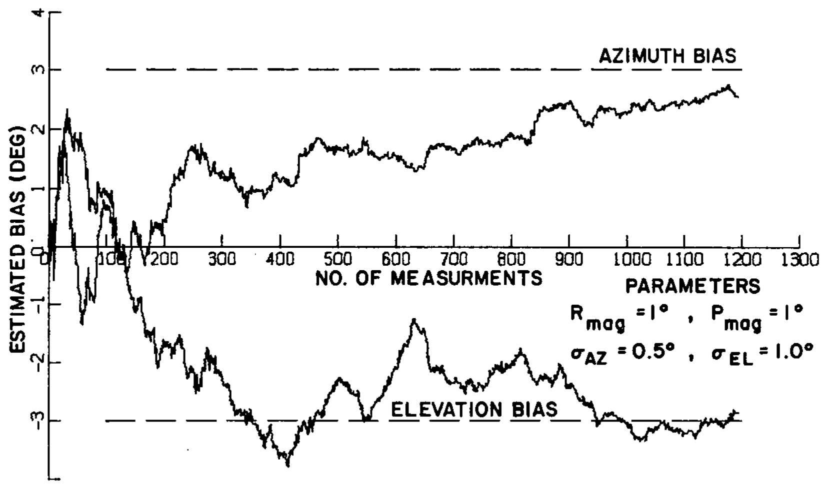

Fig. 7 — Estimate of bias errors

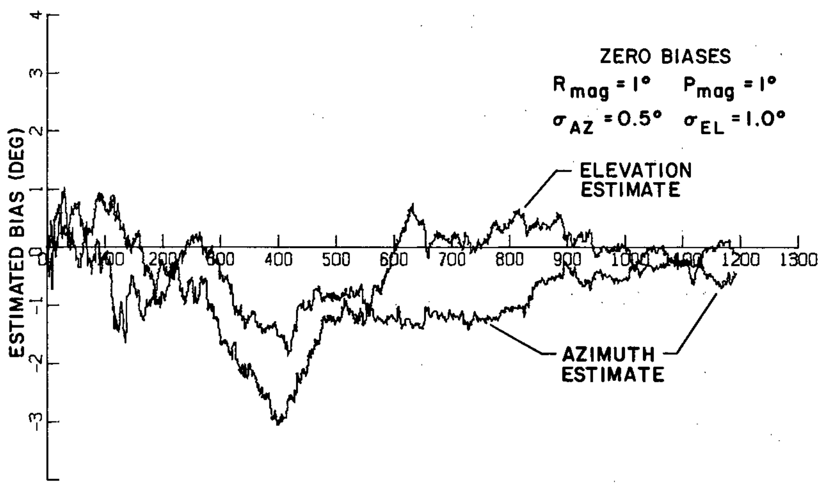

Fig. 8 — Estimate of bias errors

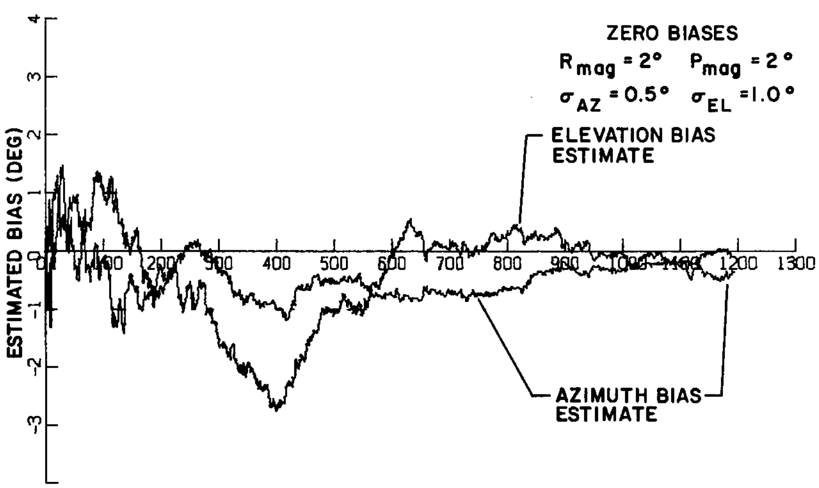

Fig. 9 — Estimate of bias errors

Effects of Target Position

An attempt was made to determine the effects of target location on the performance of the algorithm. This was accomplished by considering two targets in several different positions with platform motion restricted to 2° in roll and 1° in pitch and with 1° biases. For comparison purposes the original six targets were considered with this platform motion and the resultant bias error estimates are plotted in Fig. 10.

The first comparison was made with the results generated by locating one target on either side of the platform as shown in Fig. 11. This scenario yielded the results shown in Fig. 12, which indicates that with the given roll and pitch conditions, and target locations, one can expect a poor elevation bias estimate and deterioration in the azimuth bias. This is to be expected since roll is the dominant platform motion in this case (generally true for ships) and elevation biases cannot be detected from roll motions when the target is positioned broadside. Consequently, any measurement taken in a high-roll, low-pitch situation contributes little to the algorithm.

The next scenario placed the targets fore and aft of the platform as shown in Fig. 13. For the existing conditions this produced the results shown in Fig. 14. The improved results are attributed to the fact that the targets are in positions to take advantage of the dominant roll motion. When the targets are positioned as shown in Fig. 15, the results deteriorate (see Fig. 16). In these diagonal positions the targets are not located to take maximum advantage of either the roll or pitch motion and the behavior of the algorithm suggests that it may have trouble decoupling the roll and pitch, especially for the azimuth estimate.

From the previous results, it appears that the most desirable target location is along the axis of the dominant motion; in this case the roll axis of the platform.

Effects of Measurement Accuracy

The effects of measurement accuracy were also considered. The results produced in Fig. 10 correspond to measurements with random noise having standard deviations of 0.5° in azimuth and 1° in elevation, and a gaussian distribution. When the standard deviation in elevation is also reduced to 0.5°, a significant improvement occurs in the azimuth bias estimate. The results of this reduction are shown in Fig. 17. Further reduction of the standard deviation to 0.1° gives more pronounced improvement. These results are shown in Fig. 18.

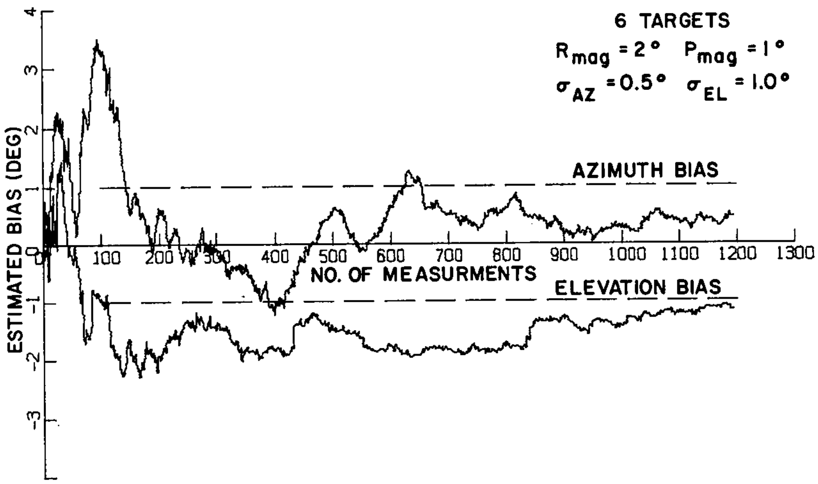

Fig. 10 — Estimate of bias errors

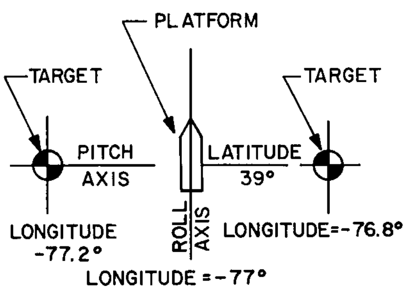

Fig. 11 — Target location

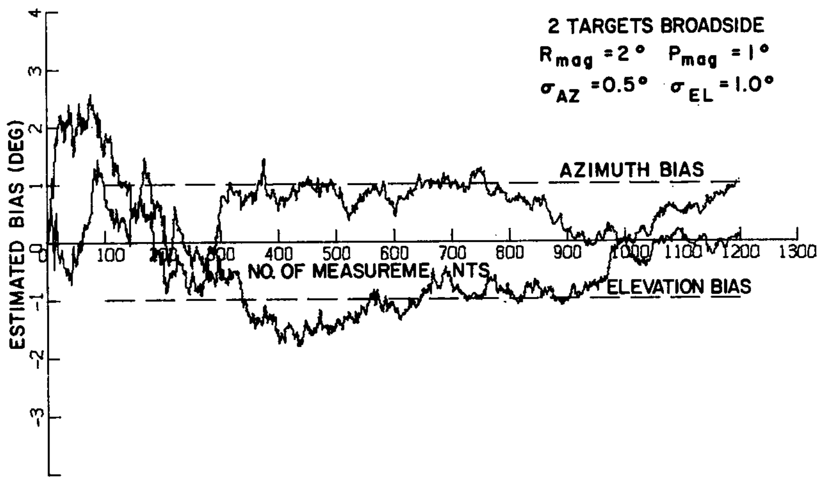

Fig. 12 — Estimate of bias errors

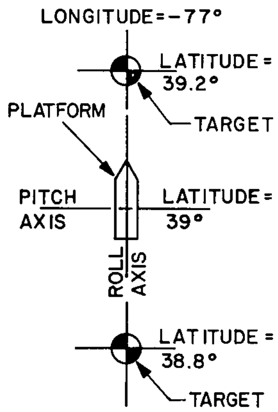

Fig. 13 — Target location

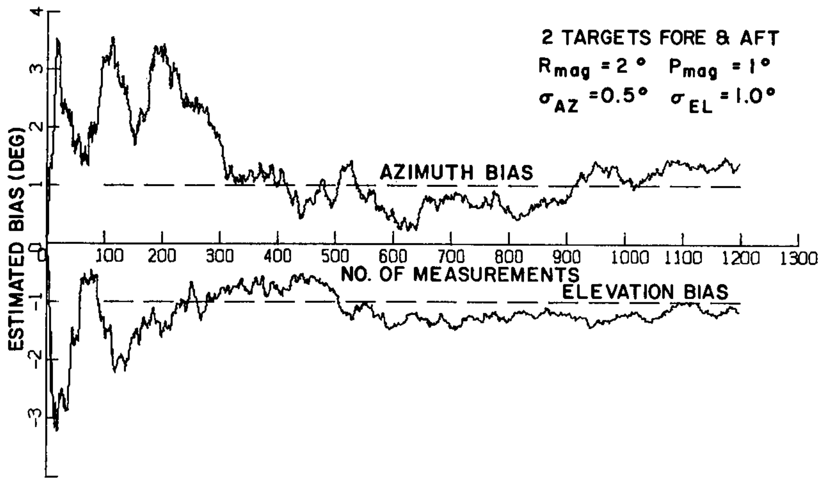

Fig. 14 — Estimate of bias errors

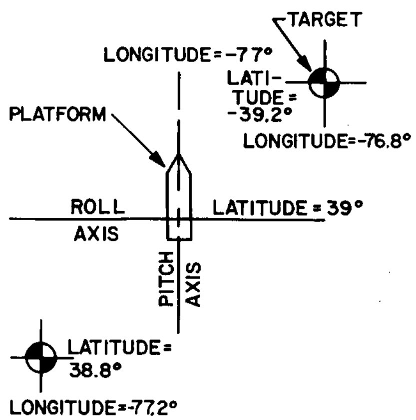

Fig. 15 — Target position

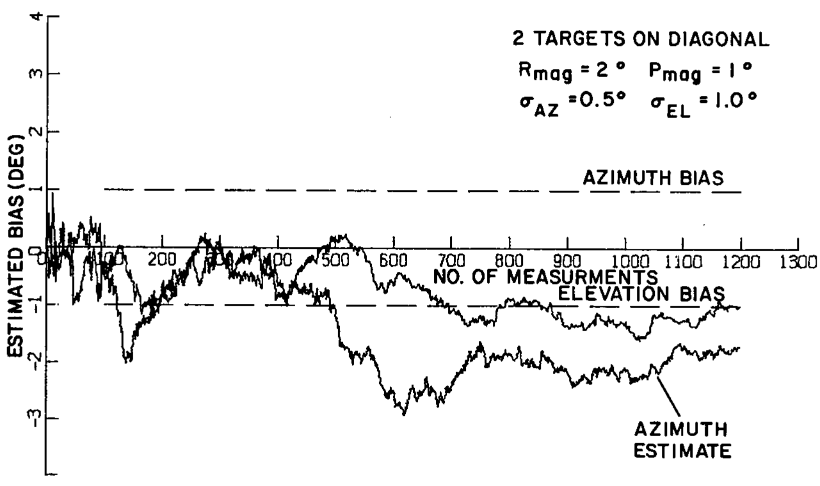

Fig. 16 — Estimate of bias errors

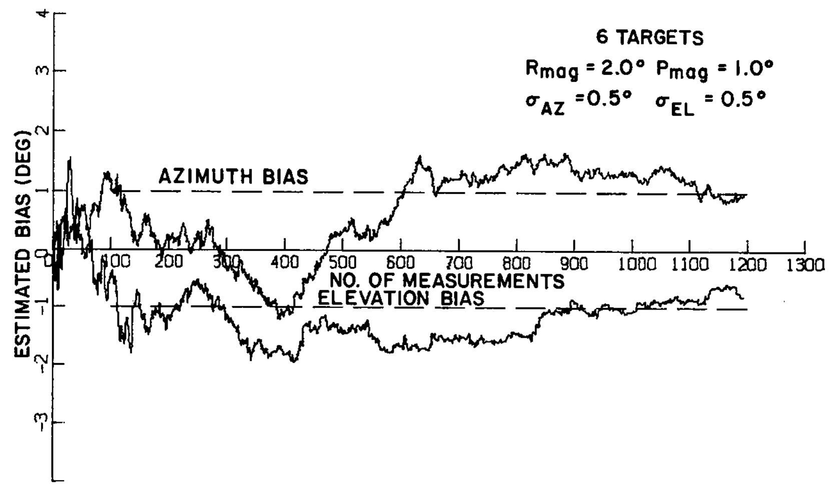

Fig. 17 — Estimate of bias errors

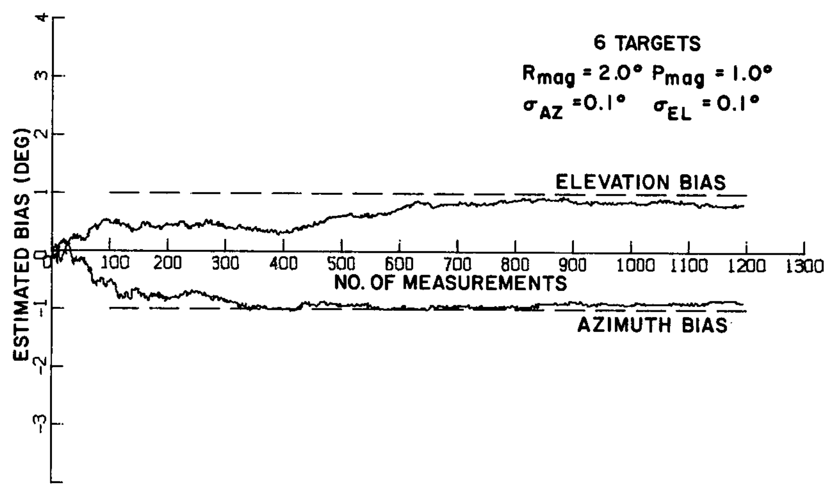

Fig. 18 — Estimate of bias errors

SUMMARY

A means of removing the bias errors in angle-of-arrival measurement equipment such as radars and direction-finding equipment located on rolling and pitching platforms was obtained. The method depends on the fact that unless there are bias errors, the angle-of-arrival measurements (on fixed or slowly moving targets or emitter,) in the stabilized coordinate system, will remain the same over short time intervals. If bias errors are present, the target or emitter angular position will change as the platform rolls and pitches. This angular deviation can be used to estimate the bias errors in the angle-of-arrival measurements made in the platform's coordinates. The relationship between the stabilized coordinates and the platform's coordinates is established by a gyro which, in the cases studied, was assumed to be perfect.

The algorithm which was developed to estimate the angular bias errors was used under various conditions of platform motion, and it was shown that useful information could be extracted from simulated measurement data even at relatively small levels of platform motion. It was also demonstrated that it is advantageous to take measurements on targets that are located along the axis of the platform's principal motion.

ACKNOWLEDGMENT

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