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An Improved Algorithm for Adaptive Processing

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The Widrow LMS algorithm based on the method of steepest descent tends toward instability. An improved algorithm is given which is unconditionally stable and provides better cancelation performance in adaptive-array or adaptive-filter applications.

CONTENTS

AN IMPROVED ALGORITHM FOR ADAPTIVE PROCESSING

INTRODUCTION

A discrete form of a least-mean-square (LMS) algorithm based on the method of steepest descent was given by Widrow et al. [1] as a means of determining the weight vectors for minimizing interference entering an adaptive array. Their algorithm becomes unstable for fast adaptation, and this report will show that a modification to their algorithm provides unconditional stability and better performance even for slow adaptation.

DISCUSSION

The steepest-descent algorithm was given in Ref. 1 as

$$W(j+1) = W(j) - 2k_s\, E(j)\, X(j), \tag{1}$$

where $W(j+1)$ is the weight vector to be used on the $(j+1)$th input data sample, $X(j)$ is the jth input data sample, k_s is a scalar constant, and $E(j)$ is the error signal developed on the jth data sample and is given by

$$E(j) = d(j) - W^T(j)\, X(j),$$

in which $d(j)$ is the jth sample of the desired signal and W^T is the transpose of W.

In general $X(j)$ and $W(j)$ are multidimensional vector quantities. A form of this equation is given in Ref. 2 for the case of an Applebaum-Howells implementation as

$$W_i(j+1) = W_i(j)\left(1 - \frac{1}{\tau}\right) + \left(\frac{G}{\tau}\right) E(J)\, V_i^*(j), \tag{2}$$

where τ is the filter smoothing constant, G is the gain term, $V_i(j)$ is the input from the ith array element, with V_i^* being the conjugate of V_i, and

$$E(j) = P(j) - \sum_{i=1}^{N} W_i(j)\, V_i(j),$$

in which $P(j)$ is the pilot signal.

To simplify the analysis, we consider the special case of a single adaptive loop in a sidelobe-canceler configuration [3] as shown in Fig. 1. In Fig. 1 the main input is obtained from a radar antenna and the auxiliary input is obtained from an omnidirectional antenna whose gain is normally greater than the sidelobe level of the radar antenna. Without loss in generality the pilot signal is taken as the main input, since the adaptation

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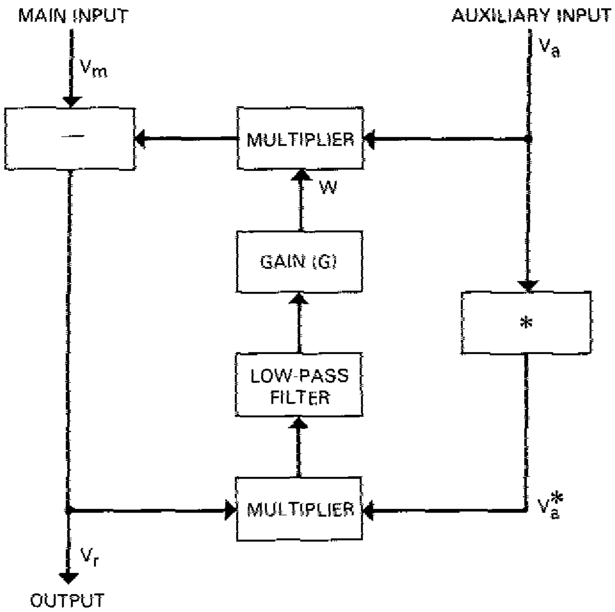

Fig. 1 — A basic sidelobe-canceler loop

criterion (LMS) is unchanged. The single sidelobe canceler corresponds to Fig. 8 in Ref. 1, with the error signal defined as

$$E(t) = V_r(t) = V_m(t) - W(t)\, V_a(t),$$

where $V_r(t)$ is the residue signal, $V_m(t)$ is the radar-channel signal, which is taken as the desired signal response in Ref. 1, $W(t)$ is a weight signal, $V_a(t)$ is the auxiliary-channel signal, and all functions correspond to complex modulation functions. The sidelobe-canceler interpretation is also discussed in Ref. 4. From (2) the adaptation algorithm for the sidelobe canceler becomes

$$W(j+1) = KW(j) + G(1-K)\, E(j)\, V_a^*(j), \tag{3a}$$

with

$$E(j) = V_m(j) - W(j)\, V_a(j), \tag{3b}$$

where

$$K = 1 - \frac{1}{\tau} = 1 - 2\pi f_{3dB},$$

in which f_{3dB} is the integrating-filter 3-dB bandwidth normalized to the sampling frequency f_s.

From (3a) it is seen that the next weight $W(j+1)$ is derived in terms of the present weight and present value of $E(j)\ V_a^*(j)$. The weight $W(j+1)$ is then used with the next auxiliary signal input to determine the residue. For fast loops, $W(j+1)$ derived from the present data is not the proper weight for the new input data. The effect is to introduce a phase shift, not present in actual loops, which causes loop instability. To avoid this instability and to provide better cancellation performance and more realistic loop simulation, a preferred algorithm is

$$W(j) = KW(j-1) + G(1-K)\ E(j)\ V_a^*(j), \tag{4a}$$

with

$$E(j) = V_m(j) - W(j)\ V_a(j). \tag{4b}$$

In this algorithm the weight applied to $V_a(j)$ is derived in terms of present input values. In effect the weight is taken prior to the delay in Fig. 7 of Ref. 1 rather than after it. Thus the weight is proper for the current data input rather than for the input data one sample interval earlier.

STABILITY CONSIDERATIONS

The steepest-descent algorithm given by (3) is

$$W(j+1) = KW(j) + G(1-K)\ [V_m(j) - W(j)\ V_a(j)]\ V_a^*(j). \tag{5}$$

For a step input with $V_a(j)$ equal to a constant and also equal to $V_m(j)$ so that the signals are perfectly correlated, (5) may be written as

$$W(j+1) = W(j)\ (K - A) + A, \tag{6}$$

where

$$A = G(1-K)\ |V_a(j)|^2.$$

Letting $W(1)$ equal 0, it is found from several iterations of (6) that the general term is

$$W(N) = \frac{A\ (1 - x^{N-1})}{1 - x}, \tag{7}$$

where

$$x = K - G(1-K)\ |V_a|^2.$$

For stability it is required that $|x| < 1$ or

$$|G(1-K)\ |V_a|^2 - K| < 1. \tag{8}$$

Substituting the value

$$K = 1 - \frac{1}{\tau} = 1 - 2\pi f_{3dB}$$

in (8) leads to

$$(1 + G) \, |V_a|^2 \, \pi f_{3dB} < 1. \tag{9}$$

Also, the weight will not ring in amplitude for A less than unity.

A stability condition derived for the sidelobe canceler version of (1) is found to be

$$|1 + 2k_s |V_a|^2| < 1,$$

which agrees with the stability condition given by equation (27) in Ref. 1, with $|V_a|^2$ in our case being equal to the unique eigenvalue.

Computer simulations were run to demonstrate the instability associated with use of (3) and are shown in Figs. 2a and 2b. In these simulations G equals 100 and $|V_a|^2$ equals 2. From (9) the stability condition for the specified values of G and $|V_a|^2$ is

$$f_{3dB} < 0.00158. \tag{10}$$

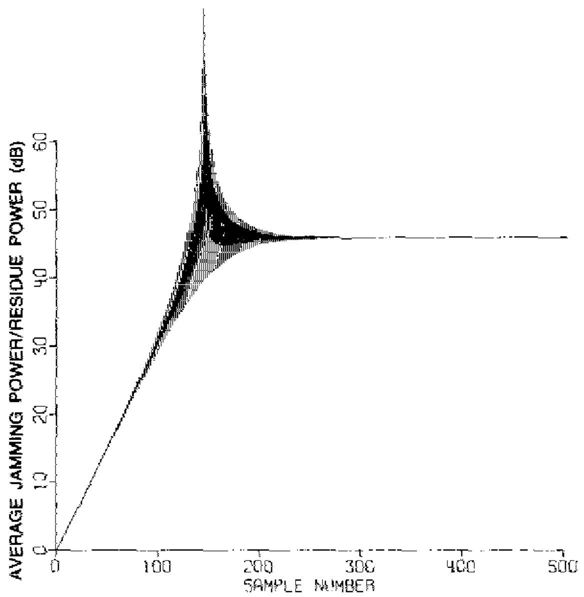

Fig. 2a — Response of the steepest-descent algorithm when f_{3dB} = 0.00155

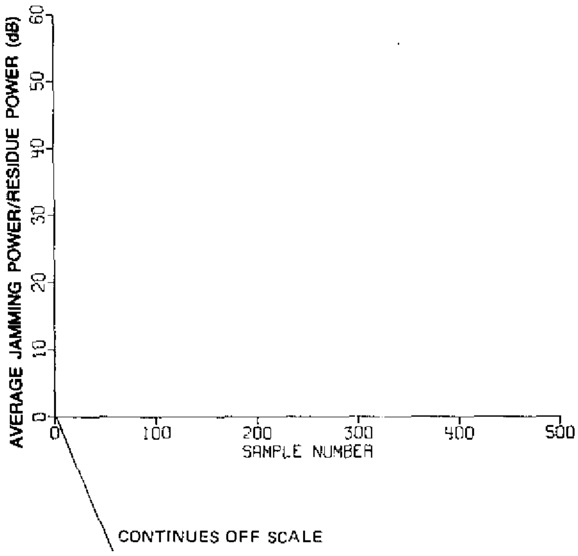

Fig. 2b — Response of the steepest-descent algorithm when f_{3dB} = 0.00162

Figure 2a shows a dampled oscillation occurring for f_{3dB} equal to 0.00155, and Fig. 2b shows instability occurring for f_{3dB} equal to 0.00162, with the weight phase alternating between 0 and 180 degrees and the weight magnitude growing unbounded.

For the improved algorithm the general weight term of (4) for a step input of constant value and V_m equal to V_a may be shown to be

$$W(N) = \frac{C\,(1 - D^N)}{1 - D}, \tag{11}$$

where

$$C = \frac{K}{1 + A}$$

and

$$D = \frac{A}{1 + A}.$$

Since D is less than unity, $W(N)$ is unconditionally stable. In Fig. 3 the response to a step input is plotted using the improved algorithm for the same value of f_{3dB} (0.00162) which caused unstable operation of the steepest-descent algorithm (Fig. 2b). There is no overshoot or ringing in Fig. 3, since the response is unconditionally stable.

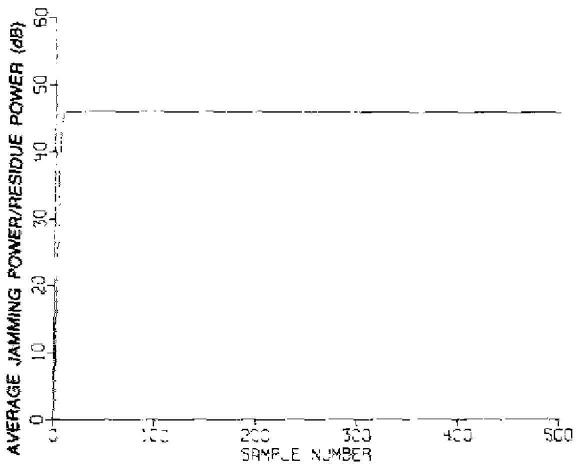

Fig. 3 — Response of the improved algorithm when f_{3dB} = 0.00162

SIMULATION OF RANDOM INPUTS

Computer simulations were run using independent samples of a Gaussian random process having a mean of 0 and a variance equal to 2. Successive samples were correlated by taking a sliding-window average of two samples and renormalizing so that the resultant power remained equal to 2. The same samples were then applied to the main and auxiliary channels of the sidelobe canceler. The steepest-descent algorithm and the improved algorithm were compared for an input step of random values which were the same for each simulation. In these simulations a constant target signal was introduced in the radar channel at sample number 250 at a clutter-to-signal level of 20 dB.

The steepest-descent and improved algorithms are shown in Figs. 4a and 4b for f_{3dB} equal to 0.00025, which corresponds to an effective loop bandwidth (B_E) to jammer bandwidth (B_J) ratio of 0.1. B_E is defined as

$$B_E = (1 + G\overline{|V_a|^2})\, f_{3dB}.$$

Comparison of Figs. 4a and 4b shows that the steepest-descent algorithm gives more points of lower cancellation (under the 40-dB line for example) than the improved algorithm gives. This is attributed to the ringing in the steepest-descent algorithm which is present even for the slower loop adaptation. Ringing will occur, as previously mentioned, when A of (7) is greater than 1. Thus use of the relation

$$A = G(1 - K)\,|V_a(j)|^2 < 1$$

and of the loop parameters G = 100 and f_{3dB} = 0.00025 leads to the requirement for ringing that

$$|V_a(j)| > 2.52.$$

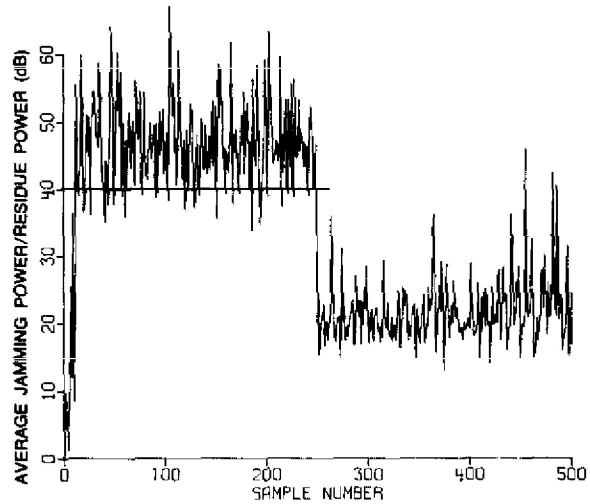

Fig. 4a — Response of the steepest-descent algorithm when f_{3dB} = 0.00025

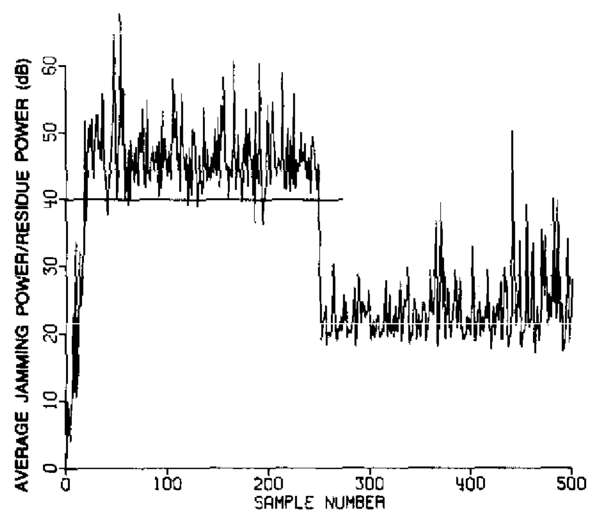

Fig. 4b — Response of the improved algorithm when f_{3dB} = 0.00025

Since $V_a(j)$ is a Gaussian random variable with a mean of 0 and a variance equal to 2, $|V_a(j)|$ is Rayleigh and the probability of $|V_a(j)|$ being greater than 2.52 is

$$P_R(|V_a(j)| > 2.52) = e^{-(2.52^2)/4} = 0.20.$$

Hence there is a 20% probability of causing ringing in this slow-loop simulation. The result of this ringing is to cause degraded cancellation of jamming signals which is due strictly to the algorithm.

In Figs. 5a and 5b the steepest-descent and improved algorithm results are shown for f_{3dB} = 0.00124, or correspondingly B_E/B_J = 0.5, and poor performance is seen to result for the steepest-descent algorithm whereas good performance is obtained with the improved algorithm. For the case of f_{3dB} = 0.0025, or B_E/B_J = 1, the steepest-descent algorithm gives unstable loop performance and (Fig. 6) the improved algorithm gives stable performance.

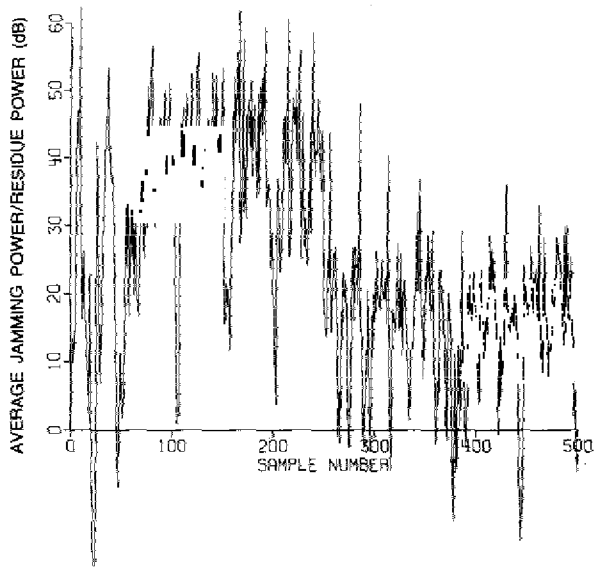

Fig. 5a — Response of the steepest-descent algorithm when f_{3dB} = 0.00124

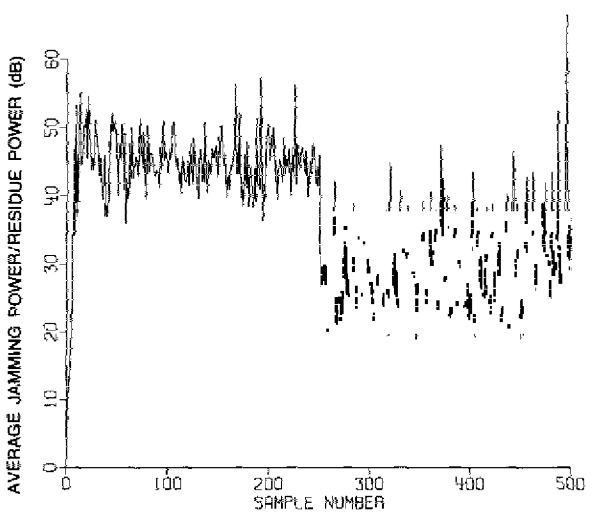

Fig. 5b — Response of the improved algorithm when f_{3dB} = 0.00124

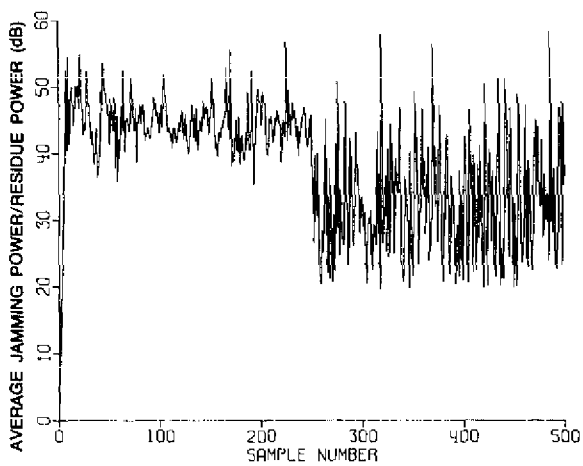

Fig. 6 — Response of the improved algorithm when f_{3dB} = 0.0025

SUMMARY

A comparison of algorithms was shown for a single loop in a sidelobe-canceler application as an illustration. The concept generalizes to any adaptive processing which minimizes the mean-square error. The generalization of (4) for M multiple loops is given for the ith weight by

$$W_i(j) = KW_i(j-1) + G(1-K)\, E(j)\, V_i^*(j), \tag{12a}$$

with

$$E(j) = V_m(j) - \sum_{n=1}^{M} W_n(j)\, V_n(j). \tag{12b}$$

A simplified version of this algorithm is given by

$$W_i(j) = KW_i(j-1) + G(1-K)\, E(j)\, V_i^*(j), \tag{13a}$$

with

$$E(j) = V_m(j) - W_i(j)\, V_i(j) - \sum_{\substack{n=1 \\ n\neq i}}^{M} W_n(j-1) V_n(j). \tag{13b}$$

In this simplified algorithm each weight is found in a closed-loop fashion, as in the single-loop case, while the other weights are frozen. The actual residue signal resulting from this algorithm is then taken to be