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Design Theory for a Constant-Beamwidth Transducer

W. JAMES TROTT

Transducer Branch
Acoustics Division

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Design theory is presented for constructing an underwater acoustic transducer (piezoelectric) having a frequency-independent directivity index. The transmitting current response is shown to be constant above a low-frequency limit. Bandwidths of several decades can be achieved for application to surveillance tracking, communication, bottom mapping, mine detection, or target identification. Theoretical design data are presented graphically.

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CONTENTS

DESIGN THEORY FOR A CONSTANT-BEAMWIDTH TRANSDUCER

INTRODUCTION

If a transducer array consists of elements mounted on a spherical surface, shaded with real coefficients proportional to the Legendre functions $P_n(\cos\theta)$, there exists a frequency range for which the transducer has a constant directivity index. If the elements are piezoelectric, the transmitting current response is constant over the frequency band. The transfer impedance, the ratio of the pressure $p(r, \theta, \phi)$ at a radial distance r divided by the velocity $v(r_0, \theta, \phi)$ of the source at the radius of the sphere r_0, is independent of direction:

$$\frac{p(r, \theta, \phi)}{v(r_0, \theta, \phi)} = \rho c\left(\frac{r_0}{r}\right), \tag{1}$$

where ρc is the wave impedance. The nodal lines $P_n(\cos\theta) = 0$, at which the velocity amplitude on the radiating sphere is equal to zero, determine the conical surface in space for which the pressure amplitude in space vanishes. Since the transfer impedance is real, there can be no transfer of energy across this conical surface. The shaded array can therefore be terminated at θ_n, the first null of $P_n(\cos\theta)$. This will yield a transducer array having a constant transmitting current response, constant beamwidth, and essentially no minor lobes. A transducer of this type should have numerous sonar applications requiring a broad spectral content with constant waveform within the radiating beam.

By reciprocity, a transducer array with constant beamwidth (piezoelectric elements) will have a free-field voltage sensitivity that is inversely proportional to frequency over the same frequency band. A transducer array shaded to $P_1(\cos\theta) = \cos\theta$ over half of a spherical surface will have a $\cos\theta$ pattern above the frequency at which the diameter of the sphere is a half wavelength. Below this frequency the free-field voltage sensitivity is constant, and the radiation pattern becomes omnidirectional. If the array is shaded to $P_1(\cos\theta) = \cos\theta$ over the entire sphere, then the $\cos\theta$ pattern exists below this frequency, and the sensitivity is proportional to frequency. Thus a sphere with a diameter of a half wavelength at 10 kHz would have a maximum sensitivity at this frequency and a sensitivity 17 dB lower at both 1 and 100 kHz if the two hemispheres are shaded to $\cos\theta$ and separately connected. Three mutually orthogonal cos patterns could be used for three-dimensional surveillance tracking.

This report presents design equations and design graphs for determining the transducer characteristics fitting a sonar application. The electronic instrumentation required for shading, beam steering, filtering, and crossover networks is not discussed.

Manuscript submitted September 18, 1975.

DESIGN THEORY

Stenzel [1] derives the equation for the radiated pressure from a rigid sphere having a velocity amplitude proportional to the first-order Legendre function:

$$v = v_0 P_1(\cos\theta) e^{i\omega t} = v_0 \cos\theta \; e^{i\omega t}. \tag{2}$$

The pressure amplitude at a radial distance r from a sphere of radius r_0 is

$$p = \rho c v_0 k^3 r_0^3 (4 + k^4 r_0^4)^{-1/2} (k^2 r^2)^{-1} (1 + k^2 r^2)^{1/2} \cos\theta, \tag{3}$$

where k is the wave number. We treat the far-field condition $kr \gg 1$, which may exist at the surface of the sphere when $kr_0 \gg 1$. Thus

$$p = \rho c v_0 k^3 r_0^3 (4 + k^4 r_0^4)^{-1/2} (kr)^{-1} \cos\theta. \tag{4}$$

To relate v_0 to the driving current I, the sphere can be assumed to be a piezoceramic shell that has been sawed, 80% through the thickness (diced), to yield a mosaic of elements equivalent to long bars polarized parallel to the length of the bars, the length being the thickness l of the shell. Reference 2 shows that

$$\sum I = \sum \psi v = \sum \frac{dA}{l} \frac{d_{33}}{S_{33}^E} v, \tag{5}$$

where dA is differential area, d_{33} is the piezoelectric constant, and S_{33}^E is the strain coefficient for a constant electric field. The sum can be expressed as an integral

$$I = \iint \psi v = 2\int_0^{2\pi} \int_0^{\pi/2} \frac{d_{33} v_0}{l S_{33}^E} \cos\theta \; (r_0^2 \sin\theta \; d\theta d\phi). \tag{6}$$

Integrating and rearranging,

$$v_0 = \frac{l S_{33}^E}{2\pi r_0^2 d_{33}} I. \tag{7}$$

Substituting for v_0 in Eq. (4) from Eq. (7), the far-field radiated sound pressure per ampere is

$$\frac{p}{I} = \frac{\rho c l S_{33}^E k^3 r_0^3 \cos\theta}{2\pi r_0^2 d_{33} kr (4 + k^4 r_0^4)^{1/2}}. \tag{8}$$

At low frequencies the transmitting current response S at $r = 1$ m and $\theta = 0$ is

$$S = \frac{\rho c l S_{33}^E}{4\pi d_{33}} k^2 r_0, \; kr_0 \ll 1. \tag{9}$$

At high frequencies

$$S = \frac{\rho c l S_{33}^E}{2\pi d_{33}} \frac{1}{r_0}, \; kr_0 \gg 1. \tag{10}$$

The free-field voltage sensitivity M is the product of the transmitting current response andtthe reciprocity parameter J:

$$M = SJ = S\frac{2r\lambda}{\rho c},$$

where ρc is the wave impedance, density ρ times the speed of sound c, and λ is the wavelength of sound. Therefore

$$M = \frac{2lS_{33}^E}{d_{33}} \frac{kr_0}{(4 + k^4 r_0^4)^{1/2}} . \tag{11}$$

At low frequencies, Eq. (11) reduces to

$$M = \frac{lS_{33}^E}{d_{33}} kr_0, \; kr_0 \ll 1, \tag{12}$$

and the free-field voltage sensitivity is proportional to frequency. At high frequencies, Eq. (11) becomes

$$M = \frac{2lS_{33}^E}{d_{33}} \left(\frac{1}{kr_0}\right), \; kr_0 \gg 1, \tag{13}$$

in which the free-field voltage sensitivity is inversely proportional to frequency.

Taking the derivative of M with respect to k and setting it to zero shows that the peak sensitivity is at $kr_0 = \sqrt{2}$ or a sphere diameter $/\lambda = 0.45$. The peak sensitivity therefore is

$$M_{\max} = \frac{lS_{33}^E}{d_{33}} \tag{14}$$

at $kr_0 = \sqrt{2}$. For Navy Type II soft lead zirconate titanate,

$$S_{33}^E = (5.3 \times 10^{10})^{-1} \;\; m^2/N,$$

and

$$d_{33} = 374 \times 10^{-12} \quad m/V.$$

Assuming l = 0.01 m (the thickness resonance is approximately 150 kHz),

$$M_{max} = 5.04 \times 10^{-4} \quad V/Pa,$$

and the free-field voltage sensitivity level is then

$$20 \log M_{max} = -185.9 \text{ dB re } 1 \; V/\mu Pa. \tag{15}$$

If M_{max} is at 10 kHz, then at both 1 and 100 kHz

$$20 \log M = -202.9 \text{ dB re } 1 \; V/\mu Pa. \tag{16}$$

A plot of the free-field voltage sensitivity level is shown if Fig. 1 as a function of kr_0. It shows a 17-dB variation in sensitivity level over a 100-to-1 frequency range with a constant cos θ directivity.

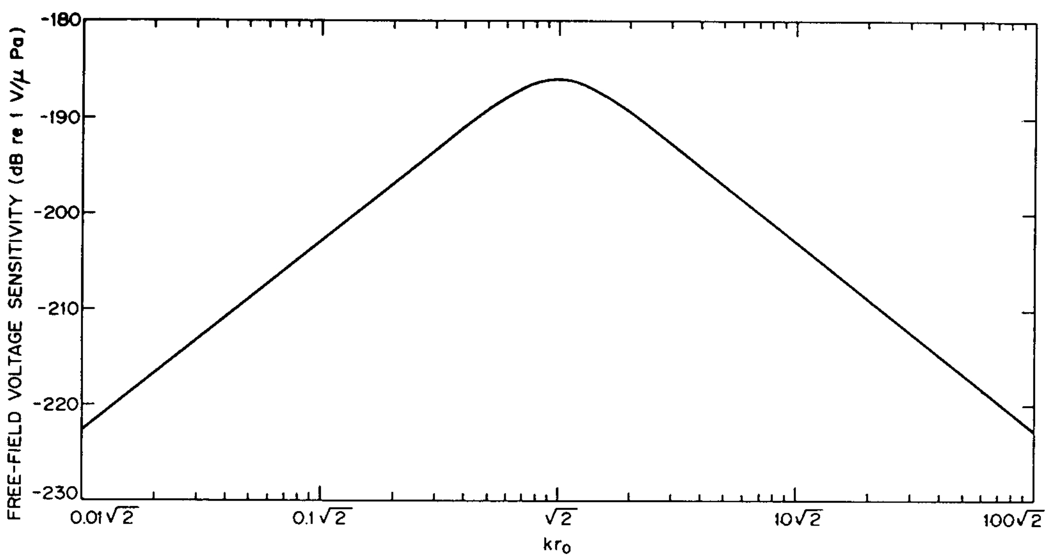

Fig. 1 — Free-field voltage sensitivity for a rigid sphere shaded to $P_1(\cos\theta)$ for $0 \leqslant \theta \leqslant \pi$

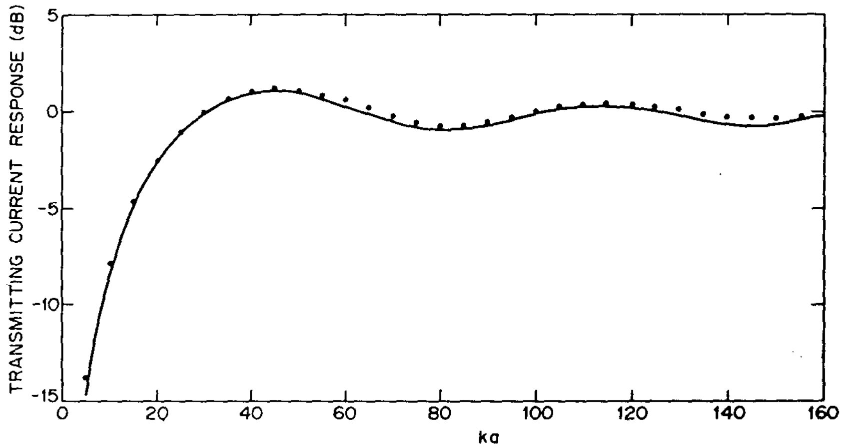

Fig. 2 — Transmitting current response; relative level for P_5 (cos θ) shading $0 \leqslant \theta \leqslant \theta_5$. Curve represents a rigid sphere. Points represent an acoustically transparent sphere.

The preceding derivation is based on a velocity function on a rigid sphere. A structure that is rigid over a frequency range of 100 to 1 is hard to find. As previously stated, the shading function can be terminated at θ_n, $P_n(\cos \theta_n) = 0$. Rogers [3] expanded the velocity distribution from $0 \leqslant \theta \leqslant \theta_n$ into a series of spherical harmonics and computed the radiation patterns and the relative transmitting current response for a piezoelectric transducer. Typical beam patterns for P_5, P_7, and P_{10} were computed and showed radiation more than 30 dB below the axis pressure outside the main beam. Above a low-frequency limit, both the beamwidth of the far-field pattern and the transmitting current response are nearly constant. We have made calculations for the case of an acoustically transparent array of elements on a spherical surface since the Rogers paper. The same properties of constant beamwidth and transmitting current response were observed for this case. Figure 2 shows the transmitting current response as a function of ka for the rigid sphere (curve) and the transparent sphere (points) for $P_5(\cos \theta)$ shading terminated at its first nodal line. This means that above the frequency of maximum M on Fig. 1, a spherical surface array of transparent elements shaded proportional to cos θ and covering one hemisphere ($0 \leqslant \theta \leqslant \pi/2$) will have the sensitivity shown. Instead of being proportional to frequency below the maximum M, the sensitivity will be constant, and the directivity pattern will approach omnidirectionality at low frequencies. Thus in the example calculated for maximum M at 10 kHz, an array of 488 elements spaced 10° on a sphere of 3.4-cm radius could be used to cover the frequency range to 200 kHz. Above 10 kHz each hemisphere array would be a separate connection. Below 10 kHz each hemisphere would yield a constant sensitivity, and the radiation would approach omnidirectionality. Combining the hemispheres for full cos θ shading would yield the proportional-to-frequency sensitivity function of Fig. 1 and the cos θ pattern. The sensitivity of the hemisphere array would be 6 dB above the level shown in Fig. 1, agreeing with the curves of Fig. 3. The sensitivity of an array of

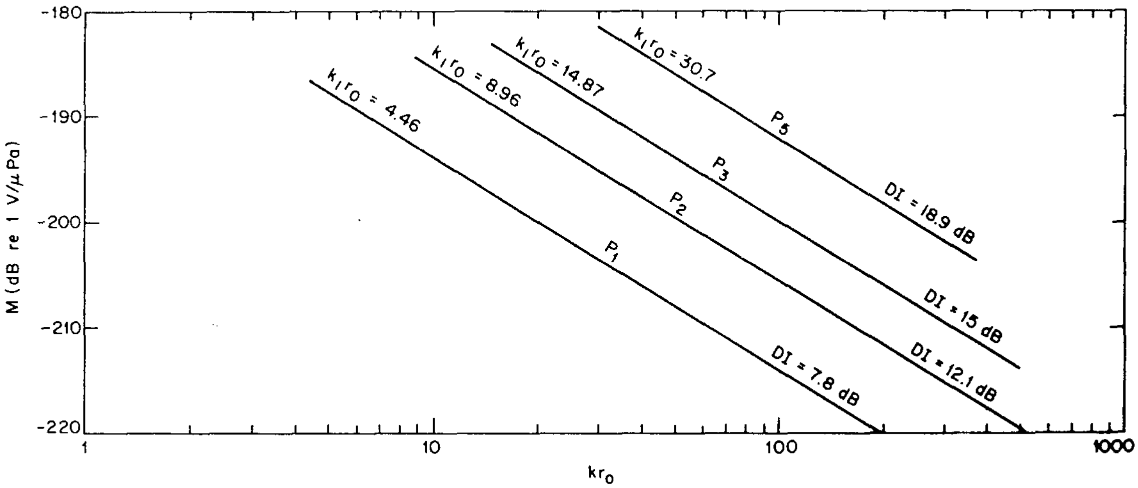

Fig. 3 — Free-field voltage sensitivity level for a spherical shell 1 cm thick of Navy Type II piezoceramic (Table 2; limits from Table 3)

capped cylinders would be lower than the example shown, the impedance would be lower, and construction costs would be less.

Returning to Eq. (4) for $kr_0 \gg 1$, the equation can be expressed more generally in terms of $P_n(\cos\theta)$:

$$p_n = \rho c v_0 \left(\frac{r_0}{r}\right) P_n\ (\cos\theta). \tag{17}$$

$$I = \int \psi v = \int_0^{\theta_n} \psi v_0 P_n\ (\cos\theta)\ \sin\theta\ d\theta,$$

or

$$I = \frac{v_0 d_{33}}{l S_{33}^E}\, 2\pi r_0^2 \int_0^{\theta_n} P_n(\cos\theta)\ \sin\theta\ d\theta,$$

or

$$v_0 = \frac{I l S_{33}^E}{2\pi r_0^2 d_{33}} \left[\int_0^{\theta_n} P_n\ (\cos\theta)\ \sin\theta\ d\theta\right]^{-1}.$$

Therefore

$$\frac{p_n}{I} = \frac{\rho c l S_{33}^E P_n\ (\cos\theta)}{2\pi d_{33} r_0 r} \left[\int_0^{\theta_n} P_n\ (\cos\theta)\ \sin\theta\ d\theta\right]^{-1},$$

and the transmitting current response S is p_n at $r = 1$ m for $I = 1A$ and $\theta = 0$ or

$$S = \frac{\rho c l S_{33}^E}{2\pi d_{33} r_0} \left[\int_0^{\theta_n} P_n (\cos\theta) \sin\theta \, d\theta \right]^{-1} . \tag{18}$$

By reciprocity, $M = JS_0$, and

$$M = \frac{2lS_{33}^E}{kr_0 d_{33}} \left[\int_0^{\theta_n} P_n (\cos\theta) \sin\theta \, d\theta \right]^{-1} . \tag{19}$$

Thus the transmitting current response is shown to be constant above a low-frequency limit ($kr_0 \gg 1$), and the free-field voltage sensitivity is inversely proportional to frequency or wave number k.

From Ref. 3 the low-frequency limit for a constant beamwidth and a constant transmitting current response is

$$f_1 = \frac{8n + 27}{10\pi\, \theta_n} \frac{c}{r_0}, \tag{20}$$

or

$$k_1 r_0 = \frac{8n + 27}{5\theta_n} .$$

Figures 3 and 4 show examples of free-field voltage sensitivity and transmitting current response with the lower limit of kr_0 for several orders of Legendre functions. From Ref. 3,

$$\theta_n = \frac{4.8}{2n + 1} . \tag{21}$$

The directivity index for the constant beamwidth is

$$DI = 10 \log \frac{(2n + 1)^2}{1.56}, \; n > 1. \tag{22}$$

This can be calculated from the shading function:

$$R_\theta = 2 \left[\int_0^{\theta_n} P_n^2 (\cos\theta) \sin\theta \, d\theta \right]^{-1} , \tag{23}$$

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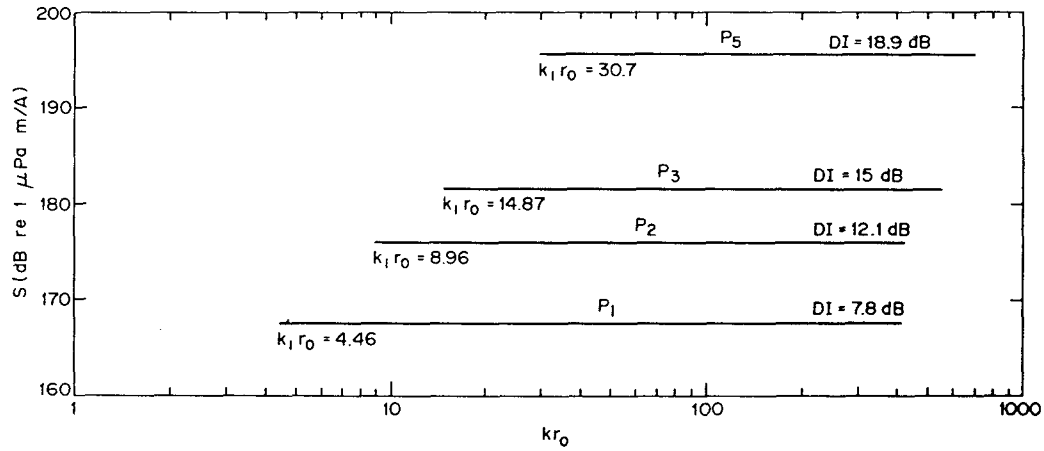

Fig. 4 — Transmitting current response level for a spherical shell 1 cm thick of Navy Type II piezoceramic (Table 1; limits from Table 3)

where $DI = 10 \log R_\theta$. Tables 1 through 3 list the results of the calculations for the transmitting current response level, free-field voltage sensitivity level, low-frequency limit, and directivity index.

The number of elements required to produce the constant beamwidth transducer can be calculated on the basis of 0.5λ spacing for a conservative design or 0.8λ spacing as an absolute upper bound. The area of the spherical surface is

$$\text{area} = 2\pi r_0^2 \, (1 - \cos \theta_n). \tag{24}$$

For half-wavelength spacing, each element represents $(c_0/2f_2)^2$, where f_2 is the upper frequency limit. From Eqs. (20) and (21)

$$r_0 = \frac{(8n + 27)\,(2n + 1)c_0}{48\pi f_1}.$$

Table 1
Transmitting Current Response

P_n	20 log S (dB re 1 μPa m/A)
P_1	167.6 - 20 log r_0
P_2	175.9 - 20 log r_0
P_3	181.6 - 20 log r_0
P_5	195.4 - 20 log r_0

Table 2
Free-Field Voltage Sensitivity

P_n	20 log M (dB re 1 $V/\mu Pa$)
P_1	-173 - 20 log kr_0
P_2	-165.6 - 20 log kr_0
P_3	-159.9 - 20 log kr_0
P_5	-152 - 20 log kr_0

Table 3
Low-Frequency Limit and
Directivity Index

P_n	$k_1 r_0$	DI (dB)
P_1	4.46	7.8
P_2	8.96	12.1
P_3	14.87	15.0
P_5	30.7	18.9

The number of elements N depends on the ratio of the upper to lower frequency limits:

$$N = \frac{[(8n + 27)(2n + 1)]^2}{288\pi} \left(\frac{f_2}{f_1}\right)^2 (1 - \cos \theta_n). \tag{25}$$

The diameter of the transducer array is the diameter of the spherical cap or $2r_0 \sin \theta_n$. Array parameters are shown in Table 4 for Legendre shading functions of order 3, 5, 7, and 10. The number of elements range from 126 elements for a one-octave frequency band and 50° beamwidth to 2288 elements for a two-octave frequency band and 14° beamwidth.

Table 4
Array Parameters for Four Legendre Shading Functions
and One- and Two-Octave Frequency Bands

P_n	θ_n	Total beamwidth -6 dB	DI	N	
				1 octave	2 octaves
3	39°	50°	15.0	126	502
5	25°	31°	18.9	225	900
7	18°	23°	21.6	335	1342
10	13°	14°	24.5	572	2288

For receiving-array applications the elements can be capped piezoceramic cylinders connected through MicroCoax® cabling to electronic circuitry. In the example cited (Fig. 1) the 488 connections could fit in a tube having a diameter of less than 1 inch. Conceivably elements could be shared in three or more beams. Bearing would be accurately determined from the ratio of signals on two beams, the receiver signal frequency content and phase being unaffected by direction of reception within the beam. Beam steering from a cosine and a sine pattern could be obtained through a resolver circuit.

CONCLUSIONS

Design theory has been derived and theoretical data presented graphically and in tables for developing a constant-beamwidth transducer with constant transmitting current response over a wide frequency range. One example suggests that a surveillance tracking transducer having a 400-to-1 frequency range could be built from 488 elements. Other applications as a source may include communication, bottom mapping, mine detection, and target identification. These are all areas where the requirements are broad frequency bands and a signal spectral content independent of bearing within the beam.

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