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Photoelastic Analysis of Shrinkage Stresses and Its Application to the Micromechanics of Composites

VINCENT J. PARKS

Mechanics of Materials Branch
Ocean Technology Division

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A review of progress over the past 20 years in the development of photoelastic methods to determine the stresses and strains caused by restrained shrinkage in multimaterial bodies and in particular in composites demonstrates that the photoelastic method has been developed into a viable method for both two-dimensional and three-dimensional analysis of stresses caused by restrained shrinkage. The method has been used to solve a number of important problems and presumably will be used to solve more. Three-dimensional photoelasticity is generally applied to the microanalysis of shrinkage stresses in composite materials subjected to slow curing or

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) slow thermal changes. Two-dimensional photoelastic analysis of composites is seen to be hampered by an inherent condition called pinching, which restricts the analysis to regions away from the interfaces of the composite material.

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PHOTOELASTIC ANALYSIS OF SHRINKAGE STRESSES AND ITS APPLICATION TO THE MICROMECHANICS OF COMPOSITES

INTRODUCTION

A number of early photoelastic studies of shrinkage stresses were conducted to analyze solid-propellant rocket motors. A solid-propellant rocket grain is a soft rubberlike combustible material cast in and bonded to a rigid rocket shell or case. The shrinkage after casting the grain in the shell, or the shrinkage due to thermal change, produces strains and stresses in the grain. The photoelastic methods developed to study stresses in rocket grains can be applied to shrinkage problems of all kinds, such as the peeling of a printed circuit from a printed circuit board due to thermal change, the stresses in rubber tires when the rubber is cast on plastic or metal plies, or tension cracks in prestressed concrete. Currently the shrinkage problem which seems to be of greatest interest to photoelasticians is the microanalysis of stresses due to shrinkage in composite materials such as fiberglass.

With regard to the rocket-motor problem, the obvious load on a rocket grain is the internal pressure due to burning. Within certain limits the pressure loading and the shrinkage loading* on propellant grains produce similar stress distributions. Two-dimensional photoelasticity was used to analyze pressure-loaded models to determine the stress concentration factors for both pressure and shrinkage loading [1].

As rocket-grain design progressed beyond these limits of similarity, it became necessary to model the shrinkage loading directly. Two-dimensional models loaded by curing shrinkage [2], direct thermal shrinkage [3], and mechanical displacement [4] were used to analyze the shrinkage load.

Further progress in rocket-grain design made it necessary to develop three-dimensional models [5]. Since most two-dimensional experimental analysis was done appropriately enough with the photoelastic method, three-dimensional photoelasticity seemed the first choice for three-dimensional studies. The advantages were obvious, but there were serious difficulties to overcome. Unless one resorts to the sandwich method or the scattered-light method (this method was subsequently used successfully [6] in three-dimensional rocket-grain studies), it is necessary to *lock* the stress into the model. Since the loading is a curing or thermal shrinkage, the locking-in process becomes intimately associated with the loading process.

The object of this report is to summarize the development of the photoelastic method for analysis of shrinkage stresses. These methods were first applied in many cases to rocket-grain problems but have wider application and are currently being used in general in the studies of composites.

*When material of a body shrinks due to thermal change or curing of the material, and is restrained from free shrinkage, stresses and strains are set up. The restrained shrinkage is the causative factor of these stresses and strains. In this report this shrinkage will be referred to as a load, or loading, on the body.

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SPECIAL REQUIREMENTS

Zero-Gradient Requirement

An important shrinkage load on a solid-propellant rocket grain is caused by a slowly changing temperature or a slowly curing propellant material. At any instant the temperature (or stage of curing) of the propellant is the same at every point. This is equivalent to saying there is zero thermal gradient (or zero gradient of curing) in the propellant. If the shell also shrinks, the effective, load-causing shrinkage is represented by the difference in shrinkage coefficients of the propellant material and the shell material. This definition (or restriction) of the problem allows the application of the three-dimensional stress-freezing method of photoelasticity to be applied. To fully justify the application, a linear-behavior limitation during cooling is required.

Linear-Behavior Requirement

Locking-in or freezing of a photoelastic pattern is sometimes thought to occur at a certain *critical temperature*, the locking-in occurring as the model is cooled below this temperature. If this were the case, the photoelastic model could be heated above the critical temperature, restrained, and slowly cooled to the critical temperature, at which point some given amount of shrinkage would have occurred. The restraint of this shrinkage would have set up a stress and strain field. By slowly cooling past the critical temperature, this stress and strain field would be locked in.

This description is oversimplified, since it is known that when a photoelastic model is mechanically loaded and the load removed below the critical temperature, the photoelastic response tends to decay. Loading, or increasing the load, below the critical temperature produces an increase in photoelastic response. It is found that for epoxy materals there is a range of some 50° F through which the material must pass before the response is fully locked in or frozen. This range is often called the transition zone between the rubbery and glassy states of the material. The bounds of this transition zone are called the upper critical temperature and the lower critical temperature [7]. These concepts are illustrated in Fig. 1.

The simplified description of the locking-in process is adequate to describe the locking-in of stresses due to mechanical load, since mechanical loading in the transition zone is typically held constant. The upper critical temperature is called the critical temperature, and the lower critical temperature is not specified.

For shrinkage load the transition zone is important and in most shrinkage problems is of vital importance. The total shrinkage depends to a large degree on the maximum temperature. Failure may occur if the shrinkage is excessive. To avoid failure in photoelastic analysis of bodies subjected to shrinkage, the maximum temperature is often not allowed to go beyond the upper critical temperature. Thus in many photoelastic analyses of shrinkage all the significant loading and all the locking-in occur in the transition zone.

the cast surface. It can occur whether the casting is restrained from shrinkage or free to shrink and is essentially unrelated to bonding or other forms of restraint.

The rind effect has always been a nuisance to photoelasticians, and typically the cast surface with the rind effect is cut away after casting. Fifteen years ago according to Hetenyi [8] "a certain rind effect [was] seemingly unavoidable in cast surfaces." Since that time a number of investigators have cast rind-free surfaces. Various recommendations have been made to obtain rind-free surfaces. These include choice of material (primarily endothermic hardeners), mold manufacture (generally rubber molds are suggested, although some still recommend metal), and surface protection from the atmosphere. Invariably however slow heating and cooling are recommended, and it is felt that this is probably the primary factor in rind-free casting.

The rind effect aptly illustrates the need to restrict the analysis of shrinkage to zero thermal gradient and to linear behavior. The thermal gradient associated with the rind effect acts in from the surface as shown in Fig. 2 and has several manifestations which are also illustrated in Fig. 2. It creates a gradient of strain, gradients of material properties (specifically Young's modulus (E) and the photoelastic fringe value (f_σ)) and a spacial variation of the stage of locking-in of the photoelastic response. Obviously these effects are coupled with each other and with curing in a complicated way, so that it can no longer be said that the photoelastic fringes are proportional to stress and strain. And in the end the locked-in photoelastic fringes do not correspond to stress or strain. Even in the relatively simple one-dimensional analysis along a normal to a flat cast surface it is difficult to apply the usual analysis to predict stress and strain from fringes produced by the rind effect. Analysis of rind stress in a typical body with corners, discontinuities, and curved surfaces would be much more difficult.

An additional difficulty is indicated by the fact that the rind effect cannot be eliminated by annealing the material. This suggests the material itself is permanently altered by curing of this sort and further indicates that the requirement of linear behavior is violated.

A further point is that similar effects occur in fully cured photoelastic material that is cooled too quickly from the upper critical temperature. Occasionally frozen photoelastic patterns with thermal gradients are produced on purpose. Early investigators often simply quenched a hot model to demonstrate frozen stress fringes. However, no quantitative analysis of stresses or strain can be recalled.

Because of these difficulties it is felt that where there is a shrinkage gradient in a body, photoelastic analysis of shrinkage stresses with the frozen-stress method is still not possible, and that analysis must be restricted to problems of zero thermal gradient and to model materials with linear viscoelastic response.

Material-Property-Proportionality Requirement

In addition to the zero-gradient and linear-behavior requirements the third, and last, special requirement to insure a valid photoelastic solution to the shrinkage problem is that the material properties remain proportional. Specifically it is required that the proportions

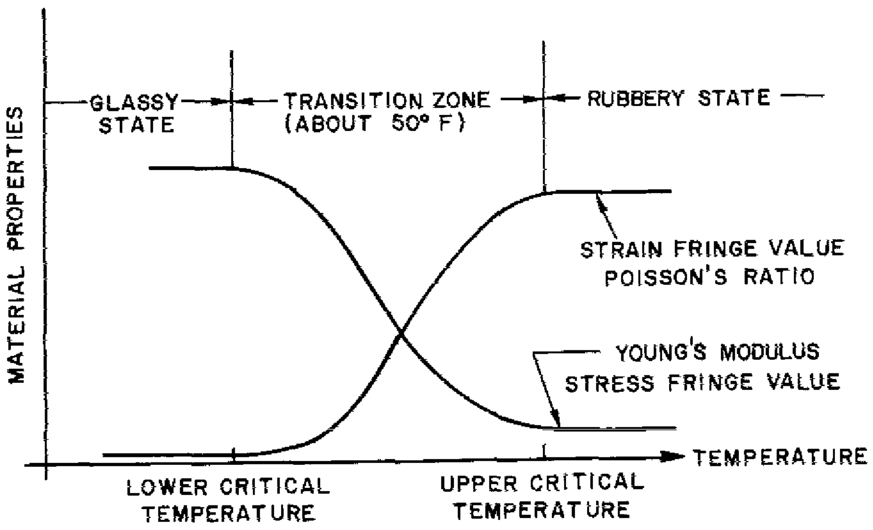

Fig. 1 — Variation of material properties through the *Transition-Zone* range of temperature

Although shrinkage continues below the lower critical temperature and produces additional photoelastic response, this photoelastic response is not locked in (is elastic) and is removed with the removal of the restraint. Indeed, if the model is cut up for analysis, it is necessary to remove the restraints, since the elastic pattern would distort the pattern in the cut pieces and must be eliminated completely.

Photoelastic materials are known to be viscoelastic in the transition zone, so that the continually increasing shrinkage load is being reduced somewhat by the relaxation of the material. To justify the analysis, this relaxation must follow the laws of linear viscoelastic behavior. For three-dimensional photoelastic materials this is found to be the case up to a certain level of strain, which is similar to and will be called the proportional limit.

To summarize the two requirements, the final response depends on spacial uniformity of temperature (zero temperature gradients) or curing, and linear viscoelastic behavior of the material throughout the transition-zone span to insure applicability to the prototype shrinkage problem. The importance of these two requirements can be appreciated by consideration of the problem of the rind effect.

Rind Effect

The rind effect is a permanent photoelastic response of up to about six fringes per inch of thickness that often occurs during curing of photoelastic materials. The effect is observed as fringes parallel to the cast surface when viewing the material in a polariscope parallel to the cast surface. It is produced in casting by thermal and curing gradients on

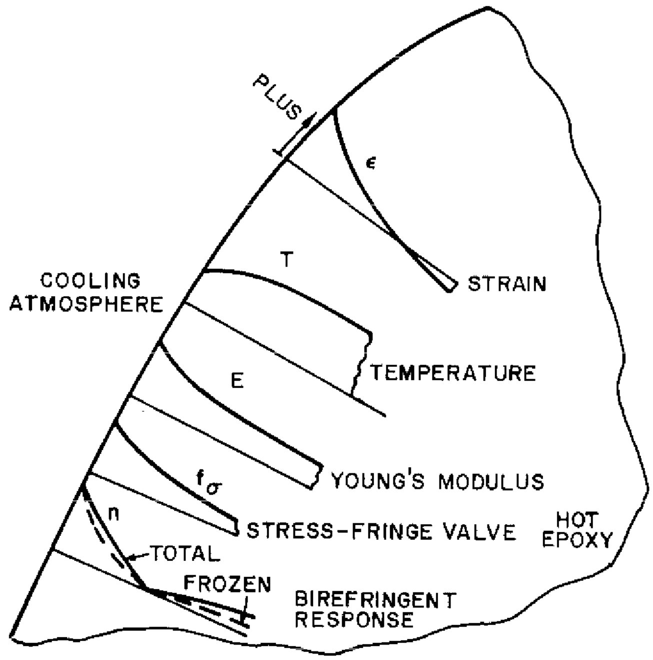

Fig. 2 — Gradients in from the surface of a material which is subjected to curing and temperature cycles such as to produce the rind effect

of Young's moduli in the various components be fixed and that Poisson's ratio of each of the components remain fixed. It is known that as the material-property proportions change, the stress and strain distributions may change. So any photoelastic results from a system with varying material properties could give an integral effect of varying stress and strain fields, with no way to separate the parts. The analysis of such a result is currently beyond photoelastic methods of anlaysis.

It might seem that this requirement creates an insurmountable barrier to any three-dimensional photoelastic analysis of shrinkage, since the photoelastic response is produced by shrinkage, and is locked-in, as the material passes through the transition zone, which also produces a great variation in Young's modulus.

One important class of problems avoids the difficulty because one material is rigid with respect to the other. So, although Young's modulus of the soft photoelastic material may change with temperature, its ratio with respect to the "rigid" material is zero, or near enough to zero. This allows appreciable variation of the soft-material modulus without affecting the stress or strain distribution.

Further research is needed to decide if the locked-in birefrigence would indeed be influenced by the varying of the material property ratio. It is conceivable that the locked-in portion of the birefringent response is associated only with the rubber phase of the material. Conceivably the birefringence produced in the transition zone is composed of two parts, one part associated with the rubbery state of the material and locked in and the other part associated with the glassy state and not locked in.

The stress distribution of the components of axisymmetric and radially symmetric composites is known not to change due to variation of material properties. The proportion of load carried may change but not the distribution of stress within each component. So this class of problems also gives valid photoelastic results. The particular result represents the integral ratio of the material properties.

THREE-DIMENSIONAL ANALYSIS

Thermal Shrinkage

In principle one could heat a three-dimensional photoelastic model to the upper critical temperature, bond it or restrain it in some other manner at that temperature, and then cool it to obtain the shrinkage strain and stress due to the particular restraint applied. However it is quite inconvenient and sometimes impossible to apply restraints to a model in the heated state. It is found however that models restrained at room temperature, heated to about the upper critical temperature, and slowly cooled, lock in a photoelastic pattern of the same distribution and sign as (but much smaller magnitude than) that of the same model restrained at the high temperature and cooled. One could argue that since the thermal expansion and contraction of a material are about the same, and since the locking-in occurs on the return, or downside, of the thermal cycle, the response should be due primarily to the expansion, reduced somewhat by the contraction. That this does not occur indicates that as the model is heated, the strains due to restrained expansion are relaxed in the soft state to the point that, on cooling, the contraction overcomes the remaining expansion and puts the body in restrained contraction which is locked in before the lower critical temperature is reached.

A significant point here is that the total expansion and contraction are the same but with different signs. Linear material behavior requires that the part of the load that does not relax and remains as the heating cycle reaches the maximum temperature produces a birefringent response proportional to the field of birefrigence that would have been produced if there were no relaxation. At some stage of cooling, contraction reduces these stresses and strains to zero throughout the field. The continued restrained contraction then produces stresses and strains of the opposite sign which increase with the continued cooling. Some part of these stresses and strains are locked in. Since the locked in effect is the integral response over a span of time, the response can be thought of as the sum of a series of responses which differ in magnitude, but which all have the same spacial proportions; therefore the locked-in part is also proportional to the total stress and strain response.

Curing Shrinkage

Typically strains occur due to curing shrinkage when a material is cast in a mold, bonds (or clings) to the mold, and is restrained from the shrinkage of curing. Cooling which occurs during curing has its own thermal contraction, which is usually considered part of the curing shrinkage. In rocket grains, photoelastic models, and many composites the material is liquid and hot when cast in the mold. The liquid may be further heated or cooled, but not until it forms a gel will shrinkage be significant in the sense used above. Once it gels and adheres to the mold, or clings to core components of the mold, there is the possibility of shrinkage, restraint of shrinkage, and strain. Observations indicate that little photoelastic response occurs in the gel state and that often the photoelastic model can be cured completely without significant photoelastic response.

Sampson in a careful study [9] of shrinkage stresses viewed the photoelastic response of a material from casting to the final state at ambient temperature with restraints removed. After the material gelled, the rate of heating was controlled to balance the curing shrinkage with thermal expansion, so that the material was cured with little or no photoelastic response. Only on cooling to ambient temperature was appreciable photoelastic response seen. It was further observed that in a nonuniform strain field due to restrained shrinkage (a disk bonded at opposite ends of its vertical diameter) the distribution of photoelastic fringes did not change with temperature.

The study also noted that on removal of restraint there was an immediate reduction in the fringe order of 40% followed during the subsequent month by about a 16% reduction. The fringe response as a function of temperature for a restrained tensile specimen following Sampson is shown in Fig. 3. Similar curves were obtained for fringes in a nonhomogenous strain field.

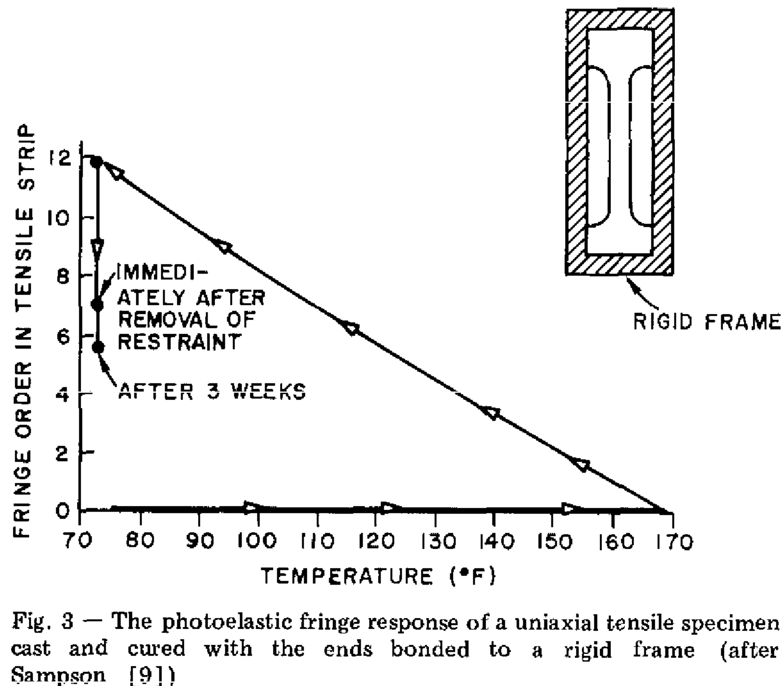

Fig. 3 — The photoelastic fringe response of a uniaxial tensile specimen cast and cured with the ends bonded to a rigid frame (after Sampson [9])

It has been shown [10] that a partially cured epoxy model loaded mechanically also responds in a manner proportional to the elastic response. Both Sampson's and Durelli's work support the hypothesis that throughout the curing and cooling cycle, above, through, and below the transition zone, the photoelastic response can always act in an elastic or linear viscoelastic manner and portions of the response can be superposed to give an integral elastic-viscoelastic response that is proportional to the elastic distribution and can be used to represent either the elastic or viscoelastic solution with proper calibration within the usual limitations of elastic theory.

Checks and Calibration

Rather than test this hypothesis on a given material by a continuous study of the photoelastic response of a material through all phases of its loading, as was done by Sampson, Durelli in all cases of shrinkage stress analysis simultaneously applied the same load cycle (curing, thermal loading, or both) to the specimen to be analyzed and to a model with a known stress distribution. The model with the known solution served both as a calibration device and as a check on the method. The check was primarily to assure that there were no thermal or curing gradients and that the material was elastic or linearly viscoelastic throughout the loading cycle.

These calibration models were rings and cylinders subjected to uniform internal or external restraint (Lame's solution), tensile bars bonded at both ends to rigid frames, tapered tensile bars, two-dimensional star shapes for which response had been obtained by actual thermal shrinkage of rubber models, and models with available numerical solutions (as the meridian plane of a hollow cylinder bonded on its outer diameter and shrunk). The method has been verified many times over by this approach. The verification came in comparing the distribution of the photoelastic response of the calibration model with the predicted response from the known solution. Subsequently the known solution was used to relate the fringe response in the calibration model to the shrinkage α and Young's modulus E. It was thus possible to obtain stress and strain concentration factors without determining actual values of α, E, or the material fringe value f_{σ}.

The stress σ and strain ϵ are usually reported in the nondimensional terms $\sigma/E\alpha$ and ϵ/α. The shrinkage symbol α, as used here, is defined as the total shrinkage due to either curing or thermal change that would occur in the material if it were not restrained. If the body is made of two materials, α is taken as the difference in shrinkage between the two materials. The value of shrinkage α is a nondimensional number similar to strain.

Although the photoelastic pattern represents the restrained shrinkage strain, it is not produced by the total shrinkage. That fraction of the load at the beginning of the locking-in process that creeps out before it can be locked in and that fraction of the load produced near the end of the locking-in process that is not locked in before the material became elastic do not contribute to the frozen-in photoelastic response. So only a fraction of the load is locked in, but it is a proportional fraction and therefore a representative fraction. This integrated response tends to mitigate the usefulness of constants such as α, E, and f_{σ} computed from the frozen-in response, since they would only represent the relations of fractions of the stress, strain, and photoelastic response accumulated over a part of the loading cycle.

SHRINKAGE IN COMPOSITES

The solid-propellent-rocket grain is essentially a two-material, two-part composite. Nearly all the techniques developed for the study of shrinkage stresses in rocket grains can be applied directly to the microscopic analysis of composite materials. By microscopic analysis is meant the study of the composite as a body composed of several materials, each of which is treated as an isotropic, homogeneous material. In many composites there are two materials: the fibers or inclusions and the surrounding matrix.

An alternative approach in the study of composites is to treat the composites as a single anisotropic homogeneous material. In this case the stresses and strains are the average value over a region large enough to encompass many elemental inclusions. Because of this restriction to a comparatively large region, the approach is called macroscopic. In macroscopic analysis there can be no shrinkage stresses or strains of the type discussed here due to uniform shrinkage. Only thermal or curing gradients would produce shrinkage stresses and strains, and this would be a different analysis than considered here.

A number of composite bodies subjected to both shrinkage and mechanical loads have been analyzed [11-15]. Figure 4 shows the photoelastic response in a transverse section of three long parallel rods in a shrunk matrix.

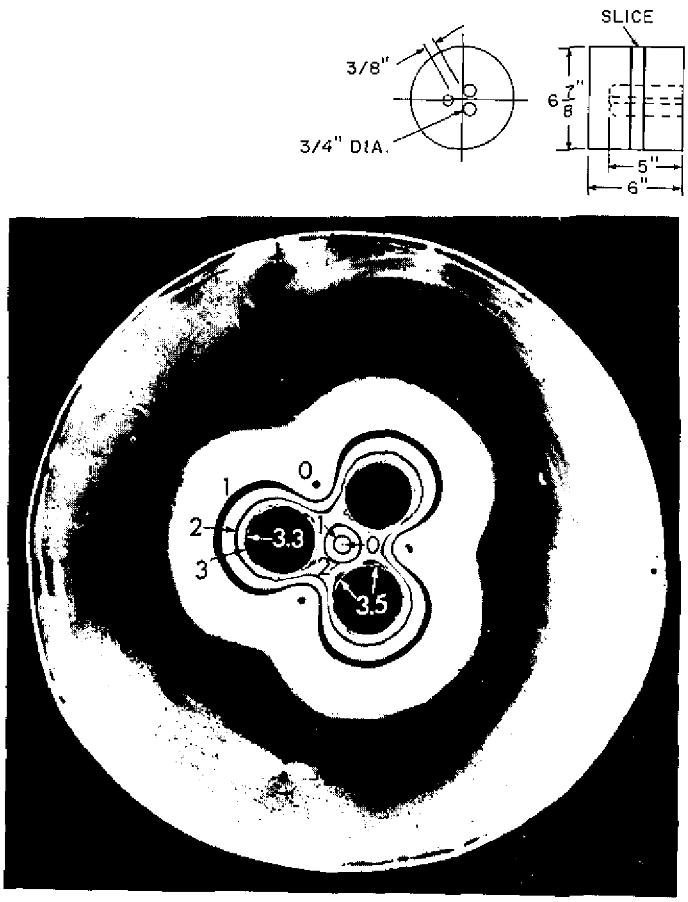

Fig. 4 — Isochromatics in a transverse cross section of a matrix shrunk about three inserts

TWO-DIMENSIONAL ANALYSIS

The methods used in three-dimensional analysis are sometimes applied in the same way to two-dimensional modeling of shrinkage stresses. Often however two-dimensional studies treat *live* photoelastic response, as opposed to the three-dimensional locked-in response. As with three-dimensional studies either curing or thermal shrinkage can be applied, but it is unnecessary to lock in or freeze the photoelastic response. Three-dimensional photoelastic studies currently use expoxy material almost exclusively. Two-dimensional studies use epoxies, other hard plastics, and transparent rubbers (primarily polyurethane).

Thermal loading is straightforward. The prototype is modeled geometrically, the model is either heated or cooled, and the response recorded (usually by photography). Thermal gradients must be avoided, and if the temperature takes the material into the viscoelastic zone, the cautions mentioned in the Special Requirements section must be exercised. For epoxy materials the viscoelastic zone is above room temperature. For polyurethane the viscoelastic zone is below room temperature.

Curing loading with epoxy is no different in two-dimensional models than in three-dimensional models except that it is not necessary to remove the restraints. If a polyurethane rubber is used as the shrinkage material in curing, then the method has one additional distinction: the rubber cures completely in the rubbery state. Whatever strains are produced due to restraint are elastic. The strains and the associated photoelastic response will vanish immediately and completely if the restraints are removed. This disappearance of response coupled with the fact that the unloaded body has a shape smaller than but geometrically similar to the mold verifies that the response produced in curing is equivalent to the thermal response. Many solutions using cured rubber models [16-23] are reported. Figure 5 is an illustration of a rubber model shrunk on rigid inserts comparable to the three-dimensional model in Fig. 4. The isoclinics are included in the pattern.

Slot [24] reports an ingenious photoelastic method to analyze certain shrinkage problems. A mechanical load is applied to an epoxy material and the material is heated to the critical temperature in the usual way and cooled to lock in a strain field. A model is cut from the prestrained epoxy material in the shape of the shrinking portion of the body to be analyzed. This piece is cemented to a second model of the portion of the body which produces the restraint. When the combined body is reheated, the prestrained body relaxes and sets up stress and strain fields. The method is primarily for two-dimensional analysis. Two-dimensional methods can also be applied to failure studies [25].

A unique difficulty arises in two-dimensional so-called plane-stress analysis of composites subjected to shrinkage (or to mechanical loads), since at the microscopic level there is no plane-stress condition in composites. Trivial cases that can be ruled out are composites composed of materials with identical properties (these can be treated as single-material bodies) and composites loaded such that the loads on the individual materials are self-equilibriating and produce no stress on material interfaces (these can be divided into separate single-material problems). What remains as a difficulty are composites with an interface that transmits stresses and across which there is a discontinuity of mechanical properties, say Young's modulus E, Poisson's ratio ν, and the shrinkage coefficient α. The normal stress and shear stress on the interface are continuous, as are the direct strains and shear strains tangential to the interface. However the normal stresses tangential to the interface can be discontinuous across the interface and be one value in one material and another in the other material.

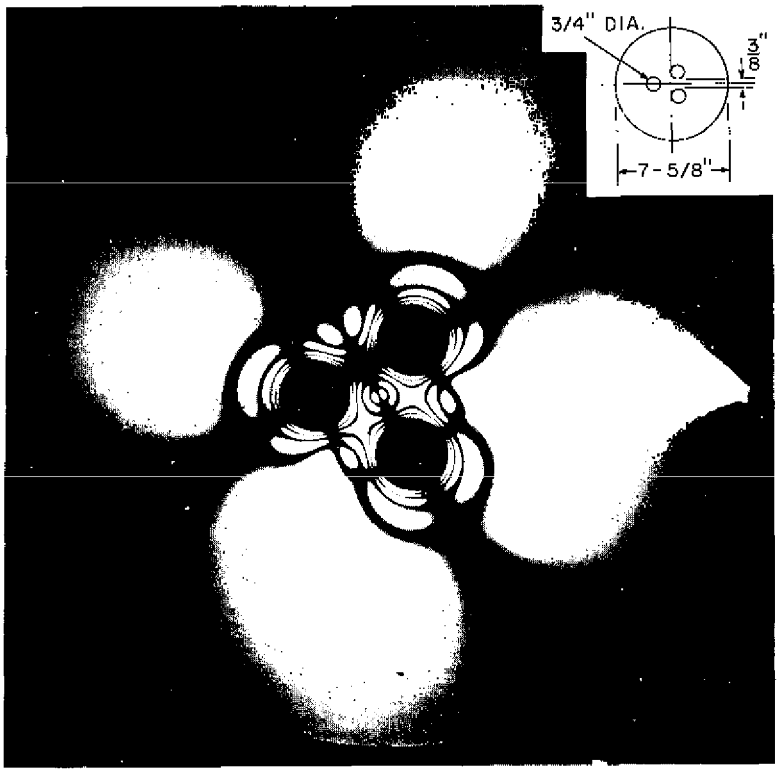

Fig. 5 — Isoclinics (θ = 30°) and isochromatics around three rigid inserts in a plate subjected to restrained shrinkage

Consider a flat composite plate having a uniform thickness that is small compared to its other dimensions. The parallel surfaces are without load; the plate is loaded in the plane of the plate by loads at its periphery. It will be assumed that interfaces between the materials must have their normals in the plane of the plate. On the interface, tangential stresses perpendicular to the free surfaces will depend on the in-plane stresses and the material properties of the two materials but will not tend to zero as the thickness tends to zero. For example, for a nonrigid material bonded to a rigid material the out-of-plane tangential stress in the nonrigid material σ_t is related to the in-plane stress perpendicular to the interface σ_n by the formula

$$\sigma_t = \nu\sigma_n$$

regardless of the plate thickness. Thus reducing the thickness of a composite plate does not insure that the out-of-plane stresses approach zero. Figure 6 is an illustration of the strains set up on a straight boundary at a line of symmetry for an incompressible material bonded to a rigid material and shrunk an amount α.

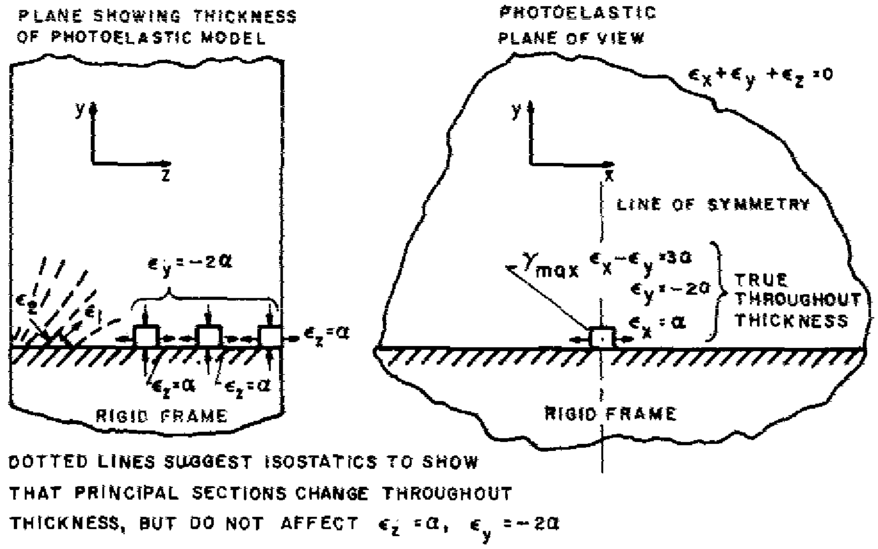

Fig. 6 — Plane views showing that at the intersection of a line of symmetry and a boundary of an incompressible material bonded to a rigid member and shrunk an amount α, the photoelastically measurable stress γ_{max} will be 3α

The discontinuity at the interface is essentially a loading condition that creates out-of-plane stress irrespective of thickness. This out-of-plane load is called *pinching* [23]. An example of pinched and unpinched models is shown in Fig. 7. Some have argued that away from the interface the plane-stress condition prevails, but this ignores the fact that it is the interface condition which makes the composite a distinct problem. In experimental work this means one cannot assume the out-of-plane stresses are zero. Photoelastic response over the interface has to be looked on as an average rather than as uniform through the thickness.

Theoretical solutions which begin by stating that the out-of-plane stress is zero and then specify two-dimensional boundary conditions on the interface continue to be published. It is not clear what physical significance they can have. Often they accompany a plane-strain analysis in which pinching is not involved. The extension of these plane-strain solutions to "plane stress" is disturbing. Equally disturbing is the argument that since there is plane stress away from the interface, and plane strain on the interface, the two-dimensional solution is approximately correct everywhere.

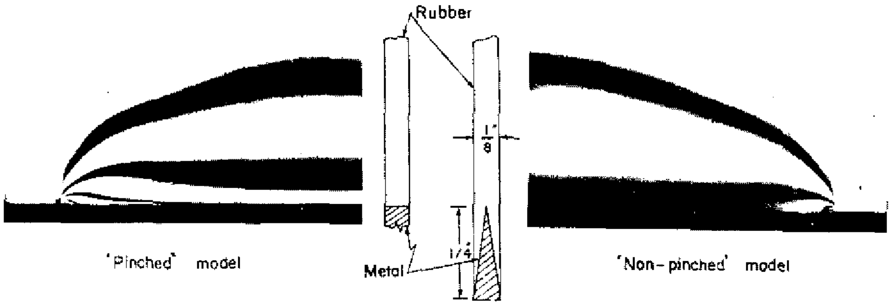

Fig. 7 — Isochromatic patterns of 1-by-2 rectangular plates bonded on the long side and subjected to shrinkage

CURRENT STATE OF THE ART

In this section the photoelastic modeling of the uniform-shrinkage-stress problem will be discussed in such a way as to indicate current available methods and see what limits are imposed by the nature of the problem. Uniform shrinkage produces stress only if the body has two or more materials with different material properties, and this is restricted to the class of materials known as composites (such as fiberglass) and to multimaterial bodies, such as the well known bimetallic strip used in thermometers. Any problem of stress analysis can be described in terms of three components: the geometry, the loading, and the material. The limitations for each of these three components and how they overlap will be discussed in the following subsections.

Geometric Limitations

Little limitation is put on geometry as such by shrinkage analysis. The geometric relationship between the prototype and model is called the scale factor and simply requires that every model dimension be the same fixed scale or proportion of the prototype. How geometry influences loading and material is taken up in the following discussions.

Loading Limitations

Both model and prototype are assumed to be subjected to slow, uniform material shrinkage, restrained the same in each case. The shrinkage need not have the same value in model and prototype. It is required only that both be small enough to avoid gross distortion (geometric nonlinearity) and both have strains below the material proportional limit (material nonlinearity).

The uniformity of shrinkage is essential to the success of the three-dimensional stress-freezing method. If the problem is one of nonuniform shrinkage, other methods are required.

The specification of loading depends to some extent on both the geometry and the material. The loading is specified as slow, uniform shrinkage. For the shrinkage to be uniform in the case of thermal change, it is necessary that the thermal gradient be negligible. Depending on the geometry, size, and thermal conductivity of the body the thermal gradient can range from a temperature change of a few degrees an hour to fractions of a degree per hour. Many large photoelastic castings take a month or more to heat and cool to satisfy this requirement. The adjective slow is added because in some situations uniform shrinkage could be obtained with a fairly fast cycle (say a thin plate with good thermal conductivity), but if the material is viscoelastic, the minimum time is restricted by the material's characteristic response time. Since more than one material is involved, it requires full response of all materials to insure that there is no redistribution of load during the cycle. This is a material requirement independent of geometry (size). It can be characterized by the time it takes to produce a certain fraction of the total response. Say a material shows 2/3 of its response to a load in 5 seconds, or 99% in 20 minutes. A measurement such as this can be used to estimate what can be considered slow.

Studies of photoelastic fringes show that in the transition zone the fringe response becomes slow. But these studies combine the frozen and nonfrozen response. It becomes difficult to decide for the complex locking-in-cooling cycle what the contribution to the integral effect will be. This phenomenon must be checked using a model with a known solution.

Material Limitations

Simulating the prototype material in the model is probably the most difficult part in setting up the experimental analysis of shrinkage stresses, and it puts the most severe limits on the analysis. For the prototype and model, materials must be assumed to be isotropic, homogeneous, and linear. The materials of the model and the prototype must be linearly elastic or linearly viscoelastic throughout its loading history in the case of the prototype and throughout the stress-freezing portion of the thermal cycle in the case of the model.

The elastic moduli do not have to be the same in the model and the prototype, but they do have to be in proportion. That is, the ratio of Young's moduli has to be the same as the ratio of the corresponding moduli of the materials in the prototype. Thus, if the prototype has materials with moduli in the ratio 1:3:5, the model must have the corresponding moduli in the ratio 1:3:5. Poisson's ratio should be the same for corresponding materials of the model and the prototype.

If the prototype materials are viscoelastic, the moduli change in time after loading, and the model moduli must change proportionally. This is a severe limitation, but one class of viscoelastic problems which circumvent (or in another sense satisfy) this limitation are prototypes composed of a nonrigid linear viscoelastic material and a rigid material. Regardless of variations of the modulus of the viscoelastic material of the prototype, the ratio with the rigid material modulus is always zero; hence the viscoelastic material of the model can have any modulus as long as it is linear and nonrigid with respect to the other, rigid model material.

Another class of viscoelastic problems which satisfy this condition, in a rather simple manner, was mentioned in the discussion of loading. This is the class of problems in which the loading is slow enough to allow the viscoelastic materials to reach the equilibrium modulus. It is then necessary only that the equilibrium moduli be proportional in the model and the prototype.

Other classes of problems which partially circumvent the relaxation difficulty are two-dimensional (plane-stress or plane-strain) problems with axisymmetry and three-dimensional radially symmetric problems. The two-dimensional group are often referred to as Lamé's problem, and radially symmetric problems are the three-dimensional counterparts of Lamé's problem. In these problems, because of different rates of relaxation there can be a redistribution of load between the various materials, but each individual stress distribution will remain proportional within each material.

Finally if the prototype material itself can be analyzed photoelastically, then there is no modeling to consider. This is not uncommon, since many composite matrices are of epoxy or other plastics. One such study was recently published [25]; The material itself must still be calibrated; that is, the birefringence must be related to stress or strain.

SUMMARY

It is felt that the photoelastic method has been developed into a viable method for both two-dimensional and three-dimensional analysis of stresses due to restrained shrinkage. Like any other method it has limitations, some of which have been discussed and which have to be considered in each application.

Because of these limitations three-dimensional photoelasticity in general, and the stress-freezing technique in particular, is primarily confined to the microanalysis of shrinkage stresses in composite materials subjected to slow curing or slow thermal changes.

Two-dimensional photoelastic analysis of composites is hampered by an inherent condition called pinching which seriously affects the analysis at the interfaces of the composite material. Despite these limitations the method has been used to solve a number of important problems and presumably will be used to solve more.

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