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Cryogenic Loop Antennas for VLF Reception in Seawater

DAVID L. GUERRINO

Communication Systems Branch
Communications Sciences Division

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The signal-to-noise ratio used to receive vlf transmissions will improve as the antenna temperature is lowered, even when the antenna is enclosed in an insulating radome and submerged in seawater. Noise (or losses) coupled into the antenna from the seawater will limit the improvement if the received atmospheric noise is small compared to the coupled sea noise. The coupled losses can be reduced by increasing the size of the radome. The loop does not have to be superconducting to achieve important gains in its signal-to-noise ratio.

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20. Abstract (Continued)

Several small 16-gauge copper wire loops were analyzed. Calculations showed that the signal-to-noise ratio of a 5.08-cm-diameter, untuned, single-turn loop, enclosed in a 30.48-cm-diameter radome and submerged in 4-mho/m water, will improve by a factor of about 104 (40 dB) as the temperature of the antenna is lowered from 295 K to 20 K. The calculation was made at 20 kHz, and the noise level was assumed to be established by the real part of the antenna impedance. The signal level was held constant.

A preamplifier using a superconducting quantum interference device (SQUID) appears to be the best suited for use with a small cryogenic loop antenna. SQUID preamplifiers at the present time are capable of low-noise operation only at extremely low frequencies, but there is evidence that their low-noise capabilities can be extended to include the vlf band.

CONTENTS

CRYOGENIC LOOP ANTENNAS FOR VLF RECEPTION IN SEAWATER

INTRODUCTION

This report is a study of the effects of cooling on the signal-to-noise ratio (S/N) of electrically-small loop antennas used to receive very-low-frequency (vlf) transmissions when the antenna is submerged in seawater. The efficiency of an electrically-small antenna can be increased by cooling because the antenna terminal resistance is reduced [1]. Terminal resistance is the real part of the antenna impedance measured at the output terminals of the antenna. The thermal noise generated by the terminal resistance will be less, and as a result the antenna signal-to-noise ratio will tend to improve, assuming the signal level is held constant. It has also been pointed out that the aperture or cross section of a lossless, electrically-small receiving antenna is independent of its physical size and that the power available from such an antenna is constant [2]. That is to say, a lossless, electrically-small receiving loop can be reduced to a vanishingly small size, and the power available from the antenna will not change. Thus by cooling a small loop antenna submerged in seawater, its size could hopefully be reduced and, more importantly, its signal-to-noise ratio improved.

The capability to cool an antenna to low temperatures has existed for many years, but only since the relatively recent discovery of superconducting quantum interference devices (SQUIDs) has the application of small, cooled antennas appeared practical. A SQUID (a low-impedance device) provides a means of matching the low impedance of the antenna. The continuing development of preamplifiers using SQUIDs capable of low-noise performance at vlf has prompted this study.

The second section of this report studies the properties of small loop antennas at low temperatures. An expression is developed for the signal-to-noise ratio of a small loop conjugately matched to a load, and the concept of reducing the antenna loss resistance to improve the signal-to-noise ratio is introduced by an example. The third section identifies the losses associated with a small loop immersed in seawater. The fourth section derives an expression for the signal-to-noise ratio of a small loop conjugately matched to a load and submerged in seawater. Several small loops are analyzed, and their signal-to-noise ratio as a function of temperature is calculated. An expression is also derived for the signal-to-noise ratio of a small, submerged loop operating broadband.

ANTENNA PROPERTIES AT LOW TEMPERATURES

The electrically-small loop antenna is a magnetic dipole whose dimensions are small compared with the wavelength at which it is operated. If the antenna is a multiturn coil in the form of a solenoid, its extended length must be small with respect to the operating wavelength. The magnetic dipole is usually pictured as a tiny, single-turn, circular loop.

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The equations which describe the circuit properties and the radiation fields of a small loop in nondissipative media will now be considered to determine to what extent the loop characteristics are affected by a cryogenic environment. The equations will be listed here, however their derivations are given in standard textbooks such as Kraus [2] and Schelkunoff [3]. Rationalized mks units are used throughout this report.

Consider a spherical coordinate system with a small circular loop centered at the origin and lying in the xy plane (Fig. 1). The radiation properties of the loop can be characterized by the ϕ component of the electric field intensity, E_ϕ, together with the θ and r components of magnetic field intensity, H_θ and H_r. The field equations E_ϕ, H_θ, and H_r are expressed in terms of the phase constant β, the current I in the loop, the area A of the loop, the intrinsic impedance γ_0 of the medium, and the distance r from loop center to the field point. The antenna resistance does not enter into any of the field equations.

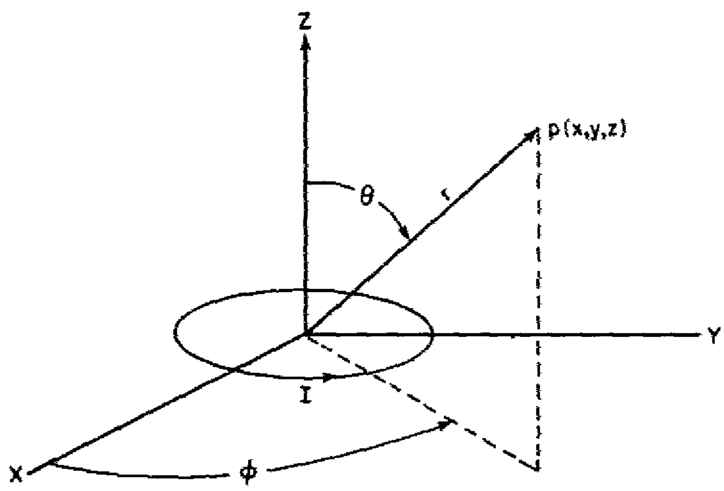


Fig. 1—Coordinates for the loop antenna

The radiation resistance of a single-turn loop is

$$R_a = 320\pi^4 \frac{A^2}{\lambda^4}, \tag{1}$$

where λ is the wavelength. If the loop consists of N turns, the radiation resistance will be N^2 times as large as for a single turn.

The inductance of a single-turn loop is approximated as

$$L = \mu b \log \frac{b}{a}, \tag{2}$$

where b is the radius of the loop, a is the wire radius, and μ is the permeability of the medium.

The capacitance of a single-turn loop is approximated as

$$C = \frac{\pi^2 \epsilon_0 b}{3 \log \left(\frac{b}{a}\right)} , \tag{3}$$

where ϵ_0 is the dielectric constant of free space.

The effective height of a single-turn loop is

$$h = \frac{2\pi A}{\lambda} . \tag{4}$$

For N turns the effective height will be N times as large as for a single turn.

The voltage induced in a single-turn loop is

$$|V| = \frac{2\pi \gamma_0 H_0 A}{\lambda} \cos \Psi , \tag{5}$$

where Ψ is the angle between the normal to the plane of the loop and the magnetic field intensity H_0. The induced voltage will increase directly as a function of N.

The ohmic resistance of the loop conductors does not enter into Eqs. (1) through (5). Superconduction will not alter any steps in their derivation, and consequently the equations are valid regardless of the antenna temperature. The preceding equations indicate that as the temperature of a small loop is lowered, its radiation resistance, inductance, capacitance, and effective height will theoretically not change. Slight changes in the antenna characteristics may occur, however, as a result of dimensional changes in the antenna and its supports. The radiation pattern described by E_ϕ, H_θ, and H_r is also temperature independent. An experiment where the radiation pattern of a superconducting loop was measured [1] showed that the antenna displayed classical loop characteristics.

A loop antenna can be operated either tuned or broadband. In the tuned case, a capacitor is placed in series or in parallel with the inductive loop, and the value of the capacitor is chosen so that resonance occurs at the operating frequency. Parallel resonance will maximize the voltage appearing across the antenna terminals, and series resonance will maximize the current in the antenna. Tuning the antenna will usually restrict the bandwidth of the antenna to a narrow range of frequencies centered about the resonant frequency. A broadband antenna is operated without addition of any capacitive elements which would severely narrow the bandwidth.

A small, tuned loop antenna will experience a significant reduction in bandwidth as the temperature of the antenna is reduced. Bandwidth is defined by the half-power points on the frequency response of the antenna and is given by

$$B = \frac{f}{Q} , \tag{6}$$

where f is the frequency and Q is the quality factor of the antenna. The quality factor is defined as

$$Q = \frac{2\pi f L}{R}, \tag{7}$$

where L is the inductance and R is the resistance in series with the antenna terminals. The terminal resistance R is composed of the radiation resistance R_a and the loss resistance R_0. The radiation resistance is a physical constant of the antenna, and its magnitude cannot be changed by cryogenic techniques. The radiation resistance establishes the maximum possible value for the unloaded-antenna Q. The loss resistance includes the ohmic resistance of the conductor, dielectric losses, and capacitive and magnetic coupled losses. If the loss resistance is contained within the dewar, its magnitude can be reduced. Examples of reduceable losses are the ohmic resistance of the conductors and the dielectric losses in the antenna support structures. Teflon and polyethylene have been found to exhibit low losses at cryogenic temperatures [4].

Coupled losses are the result of lossy material located in the near field. If the lossy material is located outside the dewar, the losses are not reduceable by cooling but can be reduced only by increased separation or by using other decoupling methods. In addition the temperature of the coupled resistance will be the same as the temperature of the material from which it originates. It will be shown in a later section that for a small loop submerged in seawater, the coupled resistance is significant.

The bandwidth and efficiency of a small, tuned loop are conveniently related. Antenna efficiency is defined as

$$\eta = \frac{R_a}{R_a + R_0}. \tag{8}$$

If B_0 is the bandwidth of a tuned, lossy antenna, then the bandwidth of the same antenna with its losses eliminated (an ideal, lossless antenna) is B_a and is given by

$$B_a = \eta B_0. \tag{9}$$

Unlike the tuned case, the bandwidth of a broadband antenna is relatively independent of temperature effects. As Eqs. (2) and (3) indicate, the self-resonant frequency of a small loop is independent of temperature. Most broadband receiving systems that use small loops operate at frequencies much lower than the self-resonant frequency of the antenna, and the sensitivity of the system is ultimately determined by a narrowband detector. The frequency coverage of such a system that uses a small, cooled loop will be independent of temperature effects if the detector does not tune the antenna.

The signal-to-noise ratio of an electrically-small antenna will increase as the temperature of the antenna decreases. The effective height or capture area of the antenna is not a function of temperature, as Eq. (4) shows. But since the temperature and magnitude of the noise-generating loss resistance is reduced in a cryogenic environment, it is intuitive that if the signal level is held constant, the signal-to-noise ratio will increase as the temperature of the antenna is lowered. The degree of improvement, however, depends on the irreduceable coupled losses and the temperature of the radiation resistance. In an area where the atmospheric noise is high or where the coupled losses are large, cooling the antenna to reduce ohmic losses will have minimal effect on the signal-to-noise ratio.

The following example illustrates the way in which the S/N of a small, lossy loop antenna located in free space improves as its losses are reduced. The antenna is conjugately matched to a load (the antenna is series tuned) for maximum power transfer, and the signal level is held constant. Calculation of the S/N is straightforward for a series-tuned antenna because the antenna reactance is cancelled by that of the tuning capacitor. The expressions developed here will be useful in a later section where the signal-to-noise improvement is calculated for a small loop submerged in seawater.

The signal power that a lossy, electrically-small loop will deliver to a conjugately matched load will be

$$S_{re} = \frac{V^2}{4(R_a + R_0)} , \tag{10}$$

where V is the antenna open-circuit voltage, R_a is the antenna radiation resistance, and R_0 is the antenna loss resistance. The antenna terminal resistance R is the sum of R_a and R_0, as before. The subscript re indicates a real antenna.

It is common practice to assign an effective temperature to the radiation resistance such that the noise power developed by the resistance is equivalent to the available atmospheric noise power delivered by the antenna. The effective temperature is fictitious because of the nonthermal origin of the noise. The atmospheric noise, like the radiation resistance, is not reduced by cooling the antenna. The available noise power from a real antenna is given by

$$N_{re} = kTB_0 ,$$

where k is Boltzmann's constant = 1.38×10^{-23} J/K, T is the effective noise temperature of the terminal resistance R, and B_0 is the bandwidth. The effective noise temperature T of the terminal resistance R can be computed from the temperature of the radiation resistance R_a at temperature T_a in series with the loss resistance R_0 at temperature T_0 and is given by

$$T = \eta T_a \left(1 + \frac{T_0 R_0}{T_a R_a}\right) ,$$

where η is the antenna efficiency defined in Eq. (8). With the use of Eq. (9) and the preceding expression for T, the available noise power can be written as

$$N_{re} = kT_a \left(1 + \frac{T_0 R_0}{T_a R_a}\right) B_a . \tag{11}$$

From Eqs. (10) and (11) the S/N will be

$$(\mathrm{S/N})_{re} = \frac{V^2}{4kT_aB_aR_a}\left[\frac{\eta^2}{\eta + (1-\eta)\dfrac{T_0}{T_a}}\right] . \tag{12}$$

Consider now the same antenna with its losses R_0 reduced to zero. The signal power that the ideal, lossless antenna will deliver to a conjugately matched load will be

$$S_{id} = \frac{V^2}{4R_a} , \tag{13}$$

where the subscript *id* indicates an ideal antenna. The available noise power from the antenna will be

$$N_{id} = kT_aB_a , \tag{14}$$

and from Eqs. (13) and (14), the S/N will be

$$(\mathrm{S/N})_{id} = \frac{V^2}{4kT_aB_aR_a} . \tag{15}$$

To demonstrate the manner in which $(\mathrm{S/N})_{re}$ approaches $(\mathrm{S/N})_{id}$ as the losses are reduced, a normalized S/N is defined [5] as

$$\frac{(\mathrm{S/N})_{re}}{(\mathrm{S/N})_{id}} = \frac{\eta^2}{\eta + (1-\eta)\dfrac{T_0}{T_a}} . \tag{16}$$

A plot of the normalized S/N is shown in Fig. 2. When the atmospheric or external noise is very high ($T_0/T_a << 1$), the S/N of the two antennas will not differ greatly, and little will be gained by increasing the efficiency of the lossy antenna. If the external noise is very low ($T_0/T_a >> 1$), the noise level is established by the loss resistance, and significant improvement in the antenna S/N is obtained as the efficiency approaches one. Under these conditions, Fig. 2 shows that the normalized S/N is small for values of η as high as 0.8. However, the normalized S/N increases rapidly for values of η near 1.

ANTENNA LOSS RESISTANCE

The antenna series terminal resistance consists of the radiation resistance R_a and the loss resistance R_0, where R_0 is composed of several resistive elements each of which generates noise. If the origin of the loss is contained in the dewar, its effect can be reduced as the temperature of the antenna is lowered. Losses that are coupled to the antenna from elements located outside the dewar are represented by hot, noisy resistances that cannot be reduced by cooling the antenna.

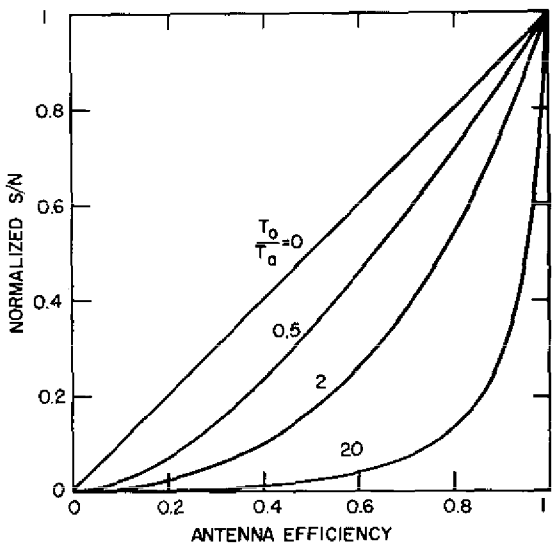


Fig. 2—Normalized signal-to-noise ratio for an electrically-small antenna

The conductor resistance can be approximated by the dc resistance formula if the radius of the wire is not more than 1.5 times the depth of penetration of the rf current, that is, if $a/\delta < 1.5$, where a is the radius of the circular conductor and δ is the depth of penetration in the conductor [6]. For a 16-gauge copper wire (a = 0.645 mm) at 20 kHz, a/δ = 1.38. The conductor resistance of a single-turn loop for which $a/\delta < 1.5$ is given by

$$R_c = \frac{2b}{a^2\sigma}, \tag{17}$$

where b is the radius of the loop and σ is the conductivity of the wire (a and δ were defined previously). The electrical conductivity of most metals increases with temperature. For example, the conductivity of copper will increase from 5.88×10^7 mho/m at 295 K to 1.25×10^{11} mho/m at 20 K or by a factor of over 2000 [7]. The ohmic resistance of a small single-turn loop two inches in diameter and wound with 16-gauge copper wire will be 2.07×10^{-3} ohms at 295 K and 9.77×10^{-7} ohms at 20 K.

Dielectric losses due to the supporting and structure and insulating elements of the loop will contribute to the antenna loss resistance. It was stated previously that certain dielectrics such as Teflon and polyethylene have a very low loss tangent at low temperatures. The loss tangent of teflon and polyethylene at 4.2 K is estimated to be less than 2×10^{-7}. The use of these materials to insulate and support the antenna will reduce the dielectric losses to insignificant levels in the present application.

For a submerged loop, the largest source of noise will be the resistance coupled from the seawater. The temperature of this resistance will be the same as the temperature of the surrounding seawater. The magntiude of the coupled resistance can be reduced

reduced by enclosing the small loop in an insulating radome. As the radius of the radome increases, the sea water is further removed from the vicinity of the antenna, and the coupled resistance will decrease. Wait [8, 9] and Row [10] have investigated the effect of an insulating cavity on the impedance of a circular loop immersed in a conducting medium of infinite extent. They have shown that the impedance of the loop inside the cavity can be expressed as

$$Z = Z_0 + \Delta Z ,$$

where Z_0 is the impedance of the loop in free space and ΔZ is the incremental change due to the finite size of the insulating cavity. ΔZ is given as

$$\Delta Z = \frac{2\pi}{3\sigma r_0}\left(\frac{b}{\delta}\right)^4 \left[1 + \frac{9}{280}\left(\frac{b}{r_0}\right)^4 + \frac{5}{704}\left(\frac{b}{r_0}\right)^8 + \ldots\right] , \tag{18}$$

where b is the radius of the loop, r_0 is the radius of the insulating cavity ($r_0 > b$), δ is the depth of penetration (skin depth) in the lossy medium, and σ is the conductivity of the lossy medium in mho/m. The expression is valid for a single-turn loop symmetrically placed in a free-space spherical cavity of radius $r_0 << \delta$. Displacement currents must be negligible in the lossy medium, which is the case for seawater. If the loop has N turns, then ΔZ will increase by N^2.

SIGNAL-TO-NOISE RATIO IN SEAWATER

The single most important figure of merit for antennas used for submerged reception is the signal-to-noise ratio. The depth to which the antenna will receive a transmitted signal depends entirely on its submerged S/N. It will now be shown that by cooling an electrically-small loop antenna, its submerged S/N does improve. The present analysis considers only the antenna.

The expression for the coupled resistor ΔZ is valid for a spherical cavity of radius $r_0 << \delta$, where the depth of penetration δ is 1.779 m at 20 kHz in 4 mho/m water. If r_0 is taken to be 0.1δ, the radius of the cavity is restricted to 17.78 cm. Also the expansion for ΔZ can be limited to three terms with negligible error by requiring $(b/r_0)^8 << 1$. This requirement also establishes the minimum cavity radius.

Single-turn loop antennas with radii of 1.27 cm, 2.54 cm, and 7.62 cm were analyzed. Larger antennas, it was believed, would be difficult to mount in a dewar. The radiation resistance in free space, conductor resistance, inductance, and effective height for each of the antennas were calculated at 20 kHz using Eqs. (1), (17), (2), and (4) respectively and are presented in Table 1. The resistance ΔZ coupled into the antennas when submerged in 4-mho water was also calculated at 20 kHz, and the results are plotted in Fig. 3a through 3c. ΔZ is given only for those values of r_0 that are valid with respect to the previously mentioned constraints.

Table 1
Parameters for Single-Turn Loop Antennas

Loop Radius (cm)	Radiation Resistance (ohm)	Conductor Resistance (ohm)	Loop Inductance (H)	Effective Height (m)
1.27	1.58×10^{-19}	1.08×10^{-3}	2.07×10^{-8}	2.12×10^{-7}
2.54	2.52×10^{-18}	2.17×10^{-3}	5.11×10^{-8}	8.48×10^{-7}
7.62	2.04×10^{-16}	6.51×10^{-3}	1.99×10^{-7}	7.64×10^{-6}

NOTE:

1. Loop material: 1.27-mm-diameter copper wire.
2. λ = free-space wavelength = 15×10^3 m.
   A = loop area (m^2).
   μ = permeability = $4\pi \times 10^{-7}$ H
   σ = conductivity = 5.8×10^7 mho/m for copper at 295 K.
   a = conductor radius = 0.635 mm $\approx$ number-16 gauge.
   b = loop radius (m).
3. All calculations were made at 20 kHz.

An expression will now be derived for the S/N of a small, lossy loop antenna submerged in seawater. An expression was derived earlier for such an antenna in free space, and the same approach will be used here. The antenna is conjugately matched to a load and can be represented by a series RLC circuit, as before. The signal power the antenna will deliver to the conjugately matched load will be

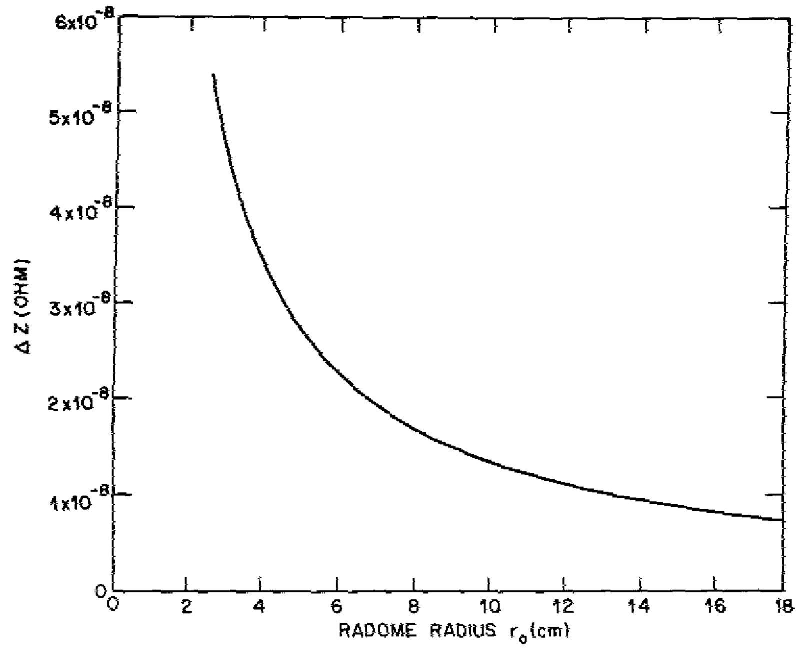
$$S = \frac{V^2}{4(R_a + \Delta Z + R_c)},$$

where V is the antenna open-circuit voltage and R_a is the radiation resistance of the antenna located in free space. But since R_a is many orders of magnitude less than ΔZ and R_c (see Table 1 and Figs. 3a through 3c), S can be rewritten as

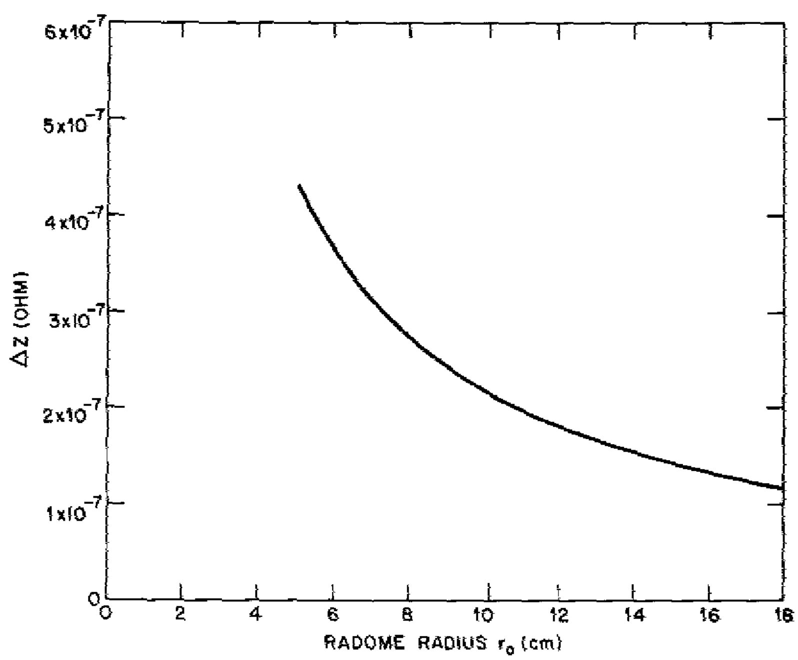
$$S = \frac{V^2}{4(\Delta Z + R_c)}. \tag{19}$$

The temperature of the antenna terminal resistance ($R = \Delta Z + R_c$) will be the temperature of ΔZ at temperature T_w (T_w is the temperature of the seawater) in series with R_c at temperature T_0 and is given by

$$T = \frac{\Delta Z T_w}{\Delta Z + R_c} + \frac{R_c T_0}{\Delta Z + R_c} = T_w \zeta \left[1 + \frac{R_c T_0}{\Delta Z T_w}\right],$$

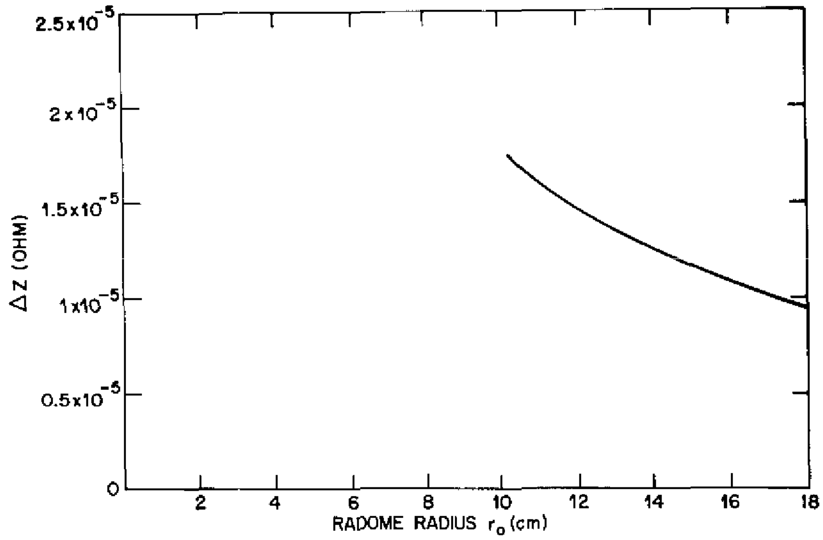


(a) 1.27-cm loop radius



(b) 2.54-cm loop radius

Fig. 3a—Resistance coupled into a loop antenna at 20 kHz when enclosed in a radome and submerged in 4-mho/m water



(c) 7.62-cm loop radius

Fig. 3a—Resistance coupled into a loop antenna at 20 kHz when enclosed in a radome and submerged in 4-mho/m water (continued)

where ζ is the seawater coupling factor, defined as

$$\zeta = \frac{\Delta Z}{\Delta Z + R_c} \ .$$

The available noise power will be

$$N = k\zeta T_w \left(1 + \frac{R_c T_0}{\Delta Z T_w}\right) B_0 \ ,$$

where B_0 is the bandwidth in hertz.

If the conductor resistance is eliminated (by cooling), the bandwidth of the antenna will be determined only by ΔZ. The bandwidth B_0 is related to the bandwidth of an antenna whose terminal resistance is ΔZ by

$$B_\Delta = \zeta B_0 \ . \tag{20}$$

Substituting Eq. (20) back into the preceding equation, it follows that

$$N = kT_w \left(1 + \frac{R_c T_0}{\Delta Z T_w}\right) B_\Delta \ . \tag{21}$$

The S/N of the antenna, using Eqs. (19) and (21), will be

$$\mathrm{S/N} = \frac{V^2}{4kT_w B_\Delta \Delta Z}\left[\frac{\zeta^2}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right] . \qquad (22)$$

Equation (22) is similar to the S/N expression derived earlier for an antenna in a nondissipative medium (Eq. (12)). As the copper losses are reduced, the term in brackets in Eq. (22) will approach 1 and the S/N will vary inversely with respect to the coupled resistance ΔZ. The signal power the antenna will deliver to a matched load will also decrease as ΔZ increases (see Eq. (19)). It is therefore desirable for the present application to minimize ΔZ by placing the antenna in as large an insulating radome as practical.

The S/N expression given in Eq. (22) was derived by assuming that the noise level was established by the antenna terminal resistance $R = R_c + \Delta Z$. As the antenna is cooled and R_c is made smaller, the improvement in S/N as a function of ΔZ can easily be determined. The validity of Eq. (22) can be insured by requiring the signal-to-atmospheric-noise ratio to be much greater than 1 because then, as the antenna is submerged deeper into the water, the atmospheric noise power received by the antenna will eventually be small compared to the noise power produced by the antenna terminal resistance.

A signal-to-noise improvement factor (SNIF) for a conjugately matched antenna can be defined as

$$(\mathrm{SNIF})_{\mathrm{cm}} = \frac{(\mathrm{S/N})_{\text{cooled antenna}}}{(\mathrm{S/N})_{\text{uncooled antenna}}} = \frac{\left[\dfrac{\zeta^2}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right]_{\text{cooled antenna}}}{\left[\dfrac{\zeta^2}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right]_{\text{uncooled antenna}}} , \qquad (23)$$

where the subscript cm indicates the antenna is conjugately matched to a load. A similar expression was developed in Ref. 5 for a small antenna located in free space. Equation (23) is valid if the noise level is established by the antenna terminal resistance. The $(\mathrm{SNIF})_{\mathrm{cm}}$ was calculated at a frequency of 20 kHz for three loops whose radii were 1.27 cm, 2.54 cm, and 7.62 cm (Table 1). The loops were single turn and wound with 1.27-mm-diameter copper wire. The calculations were based on the loops being contained in a 30.48-cm-diameter insulating aradome. Table 2 lists the conductor resistance R_c, seawater coupling factor ζ, and bandwidth B_0 at selected temperatures for each of the loops. The signal-to-noise improvement factors are about the same for temperatures above 50 K (Fig. 4). At the lower temperatures, $(\mathrm{SNIF})_{\mathrm{cm}}$ increases sharply (at 20 K, $(\mathrm{SNIF})_{\mathrm{cm}}$ is about 3×10^7 for the two smaller loops and 2.4×10^5 for the 7.62-cm-radius loop) but not without a significant loss in bandwidth (Table 2).

Table 2
Parameters for Single-Turn Loop Antennas at Selected Temperatures

Temperature (k)	Conductivity of copper Mhos/m	Conductor Resistance (ohm)			Seawater Coupling factor			Antenna Bandwidth (Hz)		
		b = 1.27 cm	b = 2.54 cm	b = 7.62 cm	b = 1.27 cm	b = 2.54 cm	b = 7.62 cm	b = 1.27 cm	b = 2.54 cm	b = 7.62 cm
295	5.88×10^{7}	1.07×10^{-3}	2.14×10^{-3}	6.43×10^{-3}	8.33×10^{-6}	6.63×10^{-5}	1.78×10^{-3}	8163	6671	5162
273.15	6.45×10^{7}	9.77×10^{-4}	1.95×10^{-3}	5.86×10^{-3}	9.12×10^{-6}	7.28×10^{-5}	1.95×10^{-3}	7456	6075	4712
250	7.14×10^{7}	8.82×10^{-4}	1.76×10^{-3}	5.29×10^{-3}	1.01×10^{-5}	8.06×10^{-5}	2.16×10^{-3}	6732	5487	4254
200	9.43×10^{7}	6.68×10^{-4}	1.33×10^{-3}	4.01×10^{-3}	1.33×10^{-5}	1.06×10^{-4}	2.85×10^{-3}	5112	4172	3224
150	1.42×10^{8}	4.43×10^{-4}	8.87×10^{-4}	2.66×10^{-3}	2.01×10^{-5}	1.60×10^{-4}	4.30×10^{-3}	3383	2764	2137
100	2.85×10^{8}	2.21×10^{-4}	4.42×10^{-4}	1.32×10^{-3}	4.03×10^{-5}	3.21×10^{-4}	8.63×10^{-3}	1687	1377	1064
80	4.76×10^{8}	1.32×10^{-4}	2.64×10^{-4}	7.94×10^{-4}	6.75×10^{-5}	5.37×10^{-4}	1.42×10^{-2}	1007	823	647
50	2.00×10^{9}	3.15×10^{-5}	6.30×10^{-5}	1.89×10^{-4}	2.83×10^{-4}	2.24×10^{-3}	5.70×10^{-2}	240	197	161
20	1.25×10^{11}	5.04×10^{-7}	1.00×10^{-6}	3.02×10^{-6}	1.73×10^{-2}	1.24×10^{-1}	7.92×10^{-1}	3.93	3.56	11.6

NOTE:

1. Conductivity values are from Ref. 7, p. 15.
2. a = loop conductor radius = 0.635×10^{-3} m
   b = loop radius (m)
   ΔZ = coupled resistance (ohm) for 30.48-cm-diameter insulated radome (Figs. 4 through 6) (ohm).
   B_{Δ} = $\Delta Z / 2\pi L$ (Hz); see table 1 for loop inductance L.
3. All calculations were made at 20 kHz.

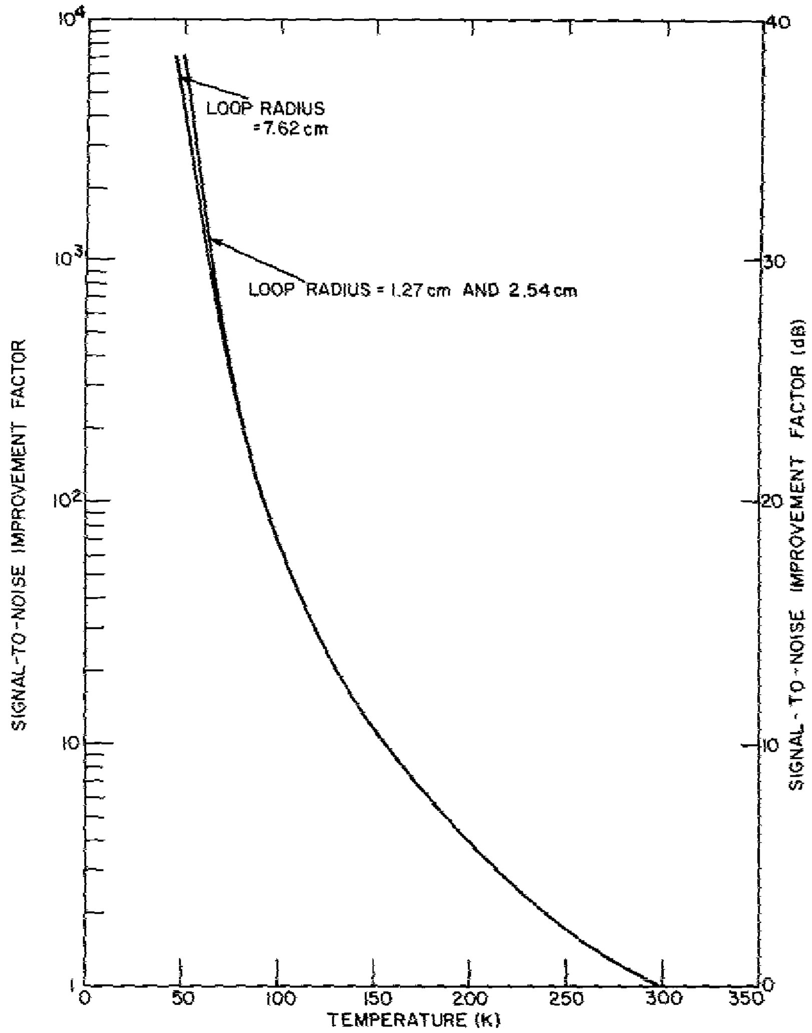


Fig. 4—Signal-to-noise improvement factor at 20 kHz for a conjugately matched loop antenna enclosed in a 30.48-cm-diameter radome and submerged in 4-mho/m water

It is important to show that the improvement in the S/N is not due entirely to narrowing the bandwidth. Equation (22) can be written as

$$\mathrm{S/N} = \frac{V^2}{4kB_0(\Delta Z T_w + R_c T_0)} ,$$

and it is seen that the S/N of a conjugately matched antenna varies inversely with respect to its bandwidth.

The improvement in S/N that results from reducing the antenna losses (and not from narrowing the bandwidth) can be demonstrated by the signal-to-noise-ratio and bandwidth product [11]. Multiplying both sides of Eq. (22) by B_0 and substituting $B_\Delta = \zeta B_0$ (Eq. (20)) into the right side, it follows that

$$(S/N)B_0 = \frac{V^2}{4kT_w \Delta Z} \left[\frac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}} \right].$$

The improvement that results from cooling the antenna is given by the signal-to-noise-ratio and bandwidth product improvement factor (SNBIF), which is defined as

$$\text{SNBIF} = \frac{[(S/N)B_0]_{\text{cooled antenna}}}{[(S/N)B_0)]_{\text{uncooled antenna}}}.$$

For the conjugately matched antenna, this improvement factor will be

$$\text{SNBIF} = \frac{\left[\dfrac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}} \right]_{\text{cooled antenna}}}{\left[\dfrac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}} \right]_{\text{uncooled antenna}}}$$

The SNBIF can be written in terms of $(\text{SNIF})_{cm}$ as

$$\text{SNBIF} = \frac{[B_0]_{\text{cooled antenna}}}{[B_0]_{\text{uncooled antenna}}} (\text{SNIF})_{cm}. \tag{24}$$

Equation (24) is plotted vs temperature in Fig. 5. The calculations were made using data from Fig. 4, which shows $(\text{SNIF})_{cm}$, and from Table 2, which lists the bandwidths B_0. The antenna performance improves substantially at the lower temperatures. The use of larger wire in the loop will lower the antenna initial resistance, and less improvement will be realized as the antenna is cooled. The SNBIF will approach a constant value as the antenna temperature is lowered to the point where the coupled resistance ΔZ is large compared to the conductor resistance R_c. This effect is apparent in the curve for the 7.62-cm-radius loop in Fig. 5. Figure 5 can also be taken to be $(\text{SNIF})_{bb}$ vs temperature when the antenna is operating broadband (see the appendix).

CONCLUSIONS

An electrically-small loop antenna when exposed to a cryogenic environment will experience insignificant changes in most of its operating characteristics. Consequently its radiation resistance, inductance, capacitance, effective height, and radiation pattern will remain relatively unchanged (except for slight variations brought about by dimensional changes). However, its quality factor Q will increase to a value ultimately determined by it radiation resistance and any irreductible coupled resistance. Bandwidth restrictions will be severe at the lower temperatures if resonances are present (antenna conjugately matched

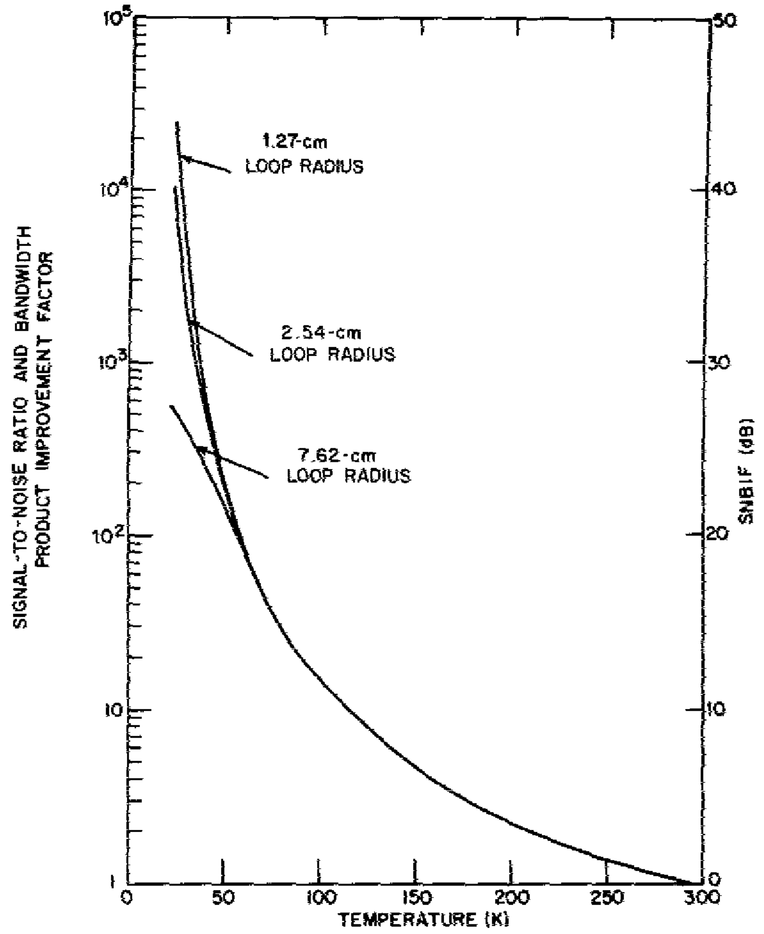


Fig. 5—Signal-to-noise ratio and bandwidth product improvement factor at 20 kHz for a conjugately matched loop antenna enclosed in a 30.48-cm-diameter radome and submerged in 4-mho/m water

to a load). However, when the antenna is operating broadband, the bandwidth is relatively independent of any temperature effects.

Resistance will be coupled into a loop antenna whenever it is submerged in seawater. Cooling the antenna does not affect the coupled resistance, however, its magnitude can be reduced by enclosing the antenna in an insulating radome.

The signal-to-noise ratio of a small loop will improve significantly as its temperature is lowered, even when the antenna is enclosed in an insulating radome and submerged in seawater, if the noise level is established by the antenna terminal resistance. The degree of improvement is dependent on the size of the radome in which the antenna is enclosed. The loop does not have to be superconducting to achieve important gains in its signal-to-noise ratio.

The practical application of small, cooled loops for the reception of vlf depends on the availability of suitable low-noise preamplifiers. The terminal impedance of such antennas is much less than 1 ohm, and conventional solid-state preamplifiers (that require source impedances many times higher) cannot provide the required sensitivity. A preamplifier using a superconducting quantum interference device (SQUID) appears to be the best suited for the present application. Optimum noise matches for source resistances less than about 10^{-6} ohm are possible with SQUID galvanometers used as low-noise preamplifiers, and with the use of superconducting transformers, higher impedance matches are possible [12]. SQUID preamplifiers at the present time are capable of low-noise operation only at extremely low frequencies, but there is evidence that their low-noise capabilities can be extended to include the vlf band [13, 14].

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LIST OF SYMBOLS
(in order of appearance)

E_ϕ	=	amplitude of the ϕ component of the electric field intensity
H_θ	=	amplitude of the θ component of the magnetic field intensity
H_r	=	amplitude of the r component of the magnetic field intensity
β	=	phase constant
I	=	current
A	=	area of a loop
γ_0	=	intrinsic impedance of a medium
r	=	distance from the loop center to a field point
R_a	=	radiation resistance
π	=	3.14150 . . .
λ	=	wavelength
N	=	number of loop turns
L	=	inductance
a	=	wire radius
b	=	loop radius
μ	=	permeability of a medium
C	=	capacitance
ϵ_0	=	dielectric constant of free space
h	=	effective height
$\lvert V \rvert$	=	absolute value of the signal voltage induced in a loop
H_0	=	magnetic field intensity
B	=	bandwidth
f	=	frequency
Q	=	quality factor
R	=	antenna terminal resistance
R_0	=	antenna loss resistance
η	=	antenna efficiency
B_a	=	bandwidth of a lossless antenna

B_0	= bandwidth of a lossy antenna
S_{re}	= available signal power from a real, lossy antenna
N_{re}	= available noise power from a real, lossy antenna
k	= Boltzmann's constant = 1.38×10^{-23} J/K
T	= total temperature
T_a	= temperature of radiation resistance
T_0	= temperature of the loss resistance
$(\mathrm{S/N})_{re}$	= signal-to-noise ratio of a real, lossy antenna
S_{id}	= available signal power from an ideal, lossless antenna
N_{id}	= available noise power from an ideal, lossless antenna
$(\mathrm{S/N})_{id}$	= signal-to-noise ratio of an ideal, lossless antenna
δ	= depth of penetration (skin depth)
R_c	= conductor resistance
σ	= conductivity
K	= degrees Kelvin
Z	= impedance of a loop inside an insulated cavity and submerged in seawater
Z_0	= impedance of a loop in free space
ΔZ	= coupled resistor
r_0	= radius of an insulating cavity
T_w	= temperature of seawater
ζ	= seawater coupling factor
N	= available noise power from a loop antenna submerged in seawater
B_Δ	= bandwidth of an antenna whose only loss is ΔZ
$(\mathrm{SNIF})_{\mathrm{cm}}$	= signal-to-noise improvement factor for a conjugately matched antenna
SNBIF	= signal-to-noise-ratio and bandwidth product improvement factor
$(\mathrm{SNIF})_{\mathrm{bb}}$	= signal-to-noise improvement factor for a broadband antenna
B_d	= detector bandwidth

Appendix A

BROADBAND SIGNAL-TO-NOISE IMPROVEMENT FACTOR

The signal-to-noise improvement factor for an electrically-small loop antenna is defined as

$$\text{SNIF} = \frac{(\text{S/N})_{\text{cooled antenna}}}{(\text{S/N})_{\text{uncooled antenna}}} .$$

The noise level is established by the antenna terminal resistance, and signal level is held constant. A subscript indicates whether the antenna is conjugately matched to a load, $(\text{SNIF})_{\text{cm}}$, or if the antenna is untuned and operated broadband, $(\text{SNIF})_{\text{bb}}$. The improvement indicated by $(\text{SNIF})_{\text{cm}}$ results from reducing the antenna losses and by narrowing the bandwidth of the antenna. That part of the improvement that results only from reducing the antenna losses (and not from narrowing the bandwidth) is given by the signal-to-noise-ratio and bandwidth product improvement factor (SNBIF). This improvement factor, which normalizes the antenna bandwidth, applies only to a conjugately matched antenna and is defined as

$$\text{SNBIF} = \frac{(B_0)_{\text{cooled antenna}}}{(B_0)_{\text{uncooled antenna}}} (\text{SNIF})_{\text{cm}} .$$

This appendix derives the $(\text{SNIF})_{\text{bb}}$ and shows that it is equal to the SNBIF.

The ratio of signal power to noise power that an electrically-small loop antenna will deliver to any resistive load can be expressed as

$$\text{S/N} = \frac{V^2}{4kTB_dR} , \tag{A1}$$

where V is the antenna open-circuit voltage, k is the Boltzmann constant, R is the antenna terminal resistance, T is the temperature of R in degrees Kelvin, and B_d is the bandwidth in hertz. B_d is considered to be much narrower than the bandwidth of the antenna, and the noise level is assumed to be established by R.

If the antenna is enclosed in an insulating radome and submerged in seawater, R will be

$$R = R_a + \Delta Z + R_c , \tag{A2}$$

where R_a is the antenna radiation resistance in free space, ΔZ is the resistance coupled from the seawater, and R_c is the conductor resistance, all in ohms. R_a is many orders of magnitude less than ΔZ and therefore can be neglected.

When the antenna is cooled, R_c will decrease, and its temperature will be that of the antenna, T_0. The coupled resistance ΔZ cannot be reduced, and its temperature will be the same as the temperature of the surrounding seawater, T_w. The temperature of R will be

$$T = \frac{\Delta Z T_w}{\Delta Z + R_c} + \frac{R_c T_0}{\Delta Z + R_c}. \qquad \text{(A3)}$$

Substituting Eqs. (A2) and (A3) into Eq. (A1), the signal-to-noise ratio can be written as

$$\mathrm{S/N} = \frac{V^2}{4kB_d(\Delta Z T_w + R_c T_0)}. \qquad \text{(A4)}$$

The signal-to-noise improvement factor can be written for the broadband antenna as

$$(\mathrm{SNIF})_{\mathrm{bb}} = \frac{[\Delta Z T_w + R_c T_0]_{\text{uncooled antenna}}}{[\Delta Z T_w + R_c T_0]_{\text{cooled antenna}}},$$

where the subscript indicates the broadband mode of operation. For the uncooled antenna, $T_0 = T_w$, and it follows that

$$(\mathrm{SNIF})_{\mathrm{bb}} = \frac{[T_w(\Delta Z + R_c)]_{\text{uncooled antenna}}}{[\Delta Z T_w + R_c T_0]_{\text{cooled antenna}}}. \qquad \text{(A5)}$$

The signal-to-noise ratio and bandwidth product for a conjugately matched, small loop antenna is

$$(\mathrm{S/N})\, B_0 = \frac{V^2}{4kT_w \Delta Z} \left[\frac{\zeta}{\zeta + (1 - \zeta)\dfrac{T_0}{T_w}} \right], \qquad \text{(A6)}$$

where B_0 is the bandwidth defined by the half-power points on the frequency response of the antenna and where ζ, the seawater coupling factor, is defined as

$$\zeta = \frac{\Delta Z}{\Delta Z + R_c}.$$

The improvement in the signal-to-noise ratio that results from reducing the antenna losses (and not from narrowing the bandwidth) is given by the signal-to-noise ratio and bandwidth product (improvement factor (SNBIF) which is defined as

$$\mathrm{SNBIF} = \frac{[(\mathrm{S/N})B_0]_{\text{cooled antenna}}}{[(\mathrm{S/N})B_0]_{\text{uncooled antenna}}}.$$

For the conjugately matched antenna, SNBIF is

$$\mathrm{SNBIF} = \frac{\left[\dfrac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right]_{\text{cooled antenna}}}{\left[\dfrac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right]_{\text{uncooled antenna}}} . \tag{A7}$$

For the uncooled antenna, $T_0 = T_w$, and the denominator of Eq. (A7)

$$\left[\frac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right]_{\text{uncooled antenna}} = [\zeta]_{\text{uncooled antenna}} .$$

Substituting $\zeta = \Delta Z/\Delta Z + R_c$ into the numerator of Eq. (A7) gives

$$\left[\frac{\zeta}{\zeta + (1-\zeta)\dfrac{T_0}{T_w}}\right]_{\text{cooled antenna}} = \left[\frac{T_w \Delta Z}{T_w \Delta Z + T_0 R_c}\right]_{\text{cooled antenna}} .$$

Equation (A7) can now be written as

$$\mathrm{SNBIF} = \frac{[T_w(\Delta Z + R_c)]_{\text{uncooled antenna}}}{[T_w \Delta Z + T_0 R_c]_{\text{cooled antenna}}} . \tag{A8}$$

Equation (A8) is the same as Eq. (A5), the expression derived previously for $(\mathrm{SNIF})_{\mathrm{bb}}$.