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Absolute Electric Field Measurements Using Field Mills

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ABSTRACT

In order to measure absolute values of electric field, an accurate method of calibration of field meters must be obtained. One purpose of calibration is to relate electric output from an inverted field meter to the true field. One method of calibration allows absolute values of electric field from a flush-mounted field meter. A new way of combining the geometry of the field meter mount and atmospheric space charge results in only a slight improvement in measurement error over a former method of calibration involving only time averages. Nevertheless, the experiment points out that atmospheric space charge affects in a complicated manner measurements made with inverted field mills.

PROBLEM STATUS

This is an interim report; work on this and other phases of the problem is continuing.

AUTHORIZATION

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ABSOLUTE ELECTRIC FIELD MEASUREMENTS USING FIELD MILLS

INTRODUCTION

The science of atmospheric electricity has been studied intermittently since the famous kite experiment of Benjamin Franklin in 1751. Many questions still remain unanswered. Atmospheric electricity work at NRL has made creditable contributions toward the advancement of the science. Airborne and ground-based instrumentation has been designed to study the physical processes inherent in atmospheric electricity, and the knowledge gained has been applied to relationships of atmospheric electrical parameters and meteorological phenomena.

At the time that NRL initiated studies in atmospheric electricity there were no commercially available instruments suitable for electric field measurements. NRL scientists developed and have continued to improve field-mill-type meters, one type of which (1) is presently being used at several locations. In general, physical instruments affect the parameter being measured, and the degree of error introduced into the measurement depends largely on the design of the instrument. Atmospheric electric field meters are no exception to this general rule. An important job of the investigator is to correct his results for systematic errors.

From experience it was found that by inverting the NRL field meter problems associated with rain, dust, snow, etc. were eliminated; but then there were problems concerned with measurements of absolute values associated with the geometry of the instrumentation. Factors had to be found which, when used to correct the indicated values, would reduce the errors due to instrumentation geometry.

For lack of standardized terminology in reference to these factors, this report defines the "reduction factor" as the ratio between the true electric field measured on a flat surface area of the earth and a simultaneous measurement of the field indicated by an instrument of specific geometry and known sensitivity located nearby.

In the past, experimenters have mounted field measuring devices in the most convenient positions without regard to field distortion and have relied on a constant reduction factor to compute the local undistorted field in the vicinity. This reduction factor, when once obtained, was usually assumed constant for a particular instrument. An experiment was designed to test for the constancy of the reduction factor by setting up instrumentation at the NRL Blue Ridge Station.

FLUSH-MOUNTED FIELD METER CALIBRATIONS

An electric field meter (EFM) produces an output voltage v which is proportional to the sum of the external field E applied to the face of the meter and its residual field R:

$$v = k(E + R). \tag{1}$$

The residual field is the output voltage of the field meter when a zero external field is applied to the face of the meter. It is produced by surface charges in the field meter head itself and varies somewhat with humidity and other atmospheric changes. The field meter is usually provided with a circuit to buck out this field. However, because R may vary with time, the bucking must be checked periodically.

The purpose of calibration is to relate the field meter output voltage v to the true field $E(0)$ that would exist on a flat surface of the earth were the field meter not there. A field meter is flush mounted in the earth's surface in such a way that a single calibration allows a knowledge of $E(0)$ from its output voltage v. The positioning and calibration of this flush-mounted field meter is carried out as follows. As defined, its reduction factor is $r = E(0)/E$, that is, the ratio of the true field to the field at the surface of the field meter head. Using f as a subscript to refer to the flush-mounted meter and substituting $E_f = E(0)/r_f$ for E in Eq. (1) the output voltage of this meter is written in terms of this reduction factor and the true field as

$$v_f = (k_f/r_f)\,E(0) + k_f R_f \tag{2}$$

and

$$E(0) = (r_f/k_f)\,(v_f - k_f R_f)\,. \tag{3}$$

For calibration a standard one-meter-square metal plate is mounted above the field meter parallel to the earth's surface. When it is at ground potential, the plate acts as an electrostatic shield and $E(0) = 0$. Then the output voltage is due only to the residual field R_f, which may now be bucked out. The term r_f/k_f must be determined if $E(0)$ is to be expressed in terms of v_f.

In order to understand how the term r_f/k_f is affected by the geometry of the field meter mount and the spacing between the calibration plate and the ground plane, a series of measurements are made. A field meter is mounted as shown in Fig. 1 in an adjustable cylindrical mount with its sensing area perpendicular to the axis of the cylinder. The height of the field meter face with respect to the ground plane is a parameter δ which is varied to take on both positive and negative values (a positive δ being shown in Fig. 1). The sensitivity of the field meter k_f is kept constant throughout the experiment. At each value of δ the distance d between the field meter face and the calibration plate is varied between 0 and 0.35 meter. The applied voltage V_{cal} and the output voltage v_f are measured for each value of d. By substitution of $R_f = 0$ and $E(0) = -V_{cal}/(d+\delta)$ into Eq. (3) the following expression is obtained for r_f/k_f :

$$r_f/k_f = -V_{cal}/v_f(d+\delta)\,. \tag{4}$$

Figure 2 is a plot of d and the values of r_f/k_f calculated from Eq. (4) for several values of δ. Thus for various values of δ, the variation of r_f/k_f with d is obtained experimentally. This is important because when measuring the real atmospheric electric field $E(0)$, the source of the field is far away from the field meter compared to the dimensions of the field meter mount. To duplicate this situation exactly with a known artificial field is impossible. A finite sized calibration plate can be used to generate a known field, and the real case can be closely approximated as long as the spacing d is large compared with the dimension δ of the field meter mount. As d and δ become comparable in length, however, the situation changes quite drastically. It can be seen in Fig. 2 that for a mount definitely protruding above the ground plane (positive δ), the value of r_f/k_f at a particular d decreases with increasing δ. On the other hand, when the field meter is definitely mounted in a hole below the ground plane (negative δ), r_f/k_f at a particular d increases for increasing depth of cavity. It is logical to assume that for one particular value of δ the value of r_f/k_f will remain essentially constant for decreasing d. This is the ideal case where the field meter and it mounting do not effectively distort the field lines and the field meter is now "flush" with the ground plane and $E_f = E(0)$. Since there is no simple way of determining this value of δ at which to mount the field meter so that it will

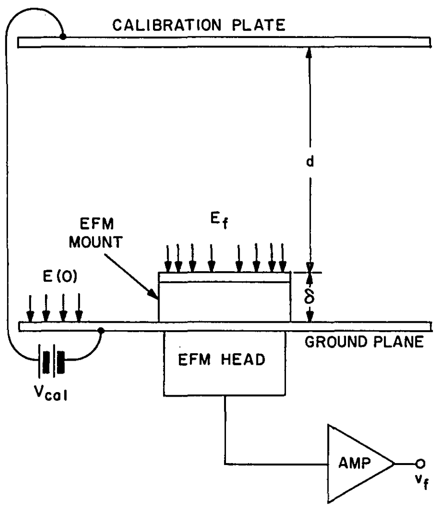

Fig. 1 - Schematic diagram of the calibration hardware for a ground field meter

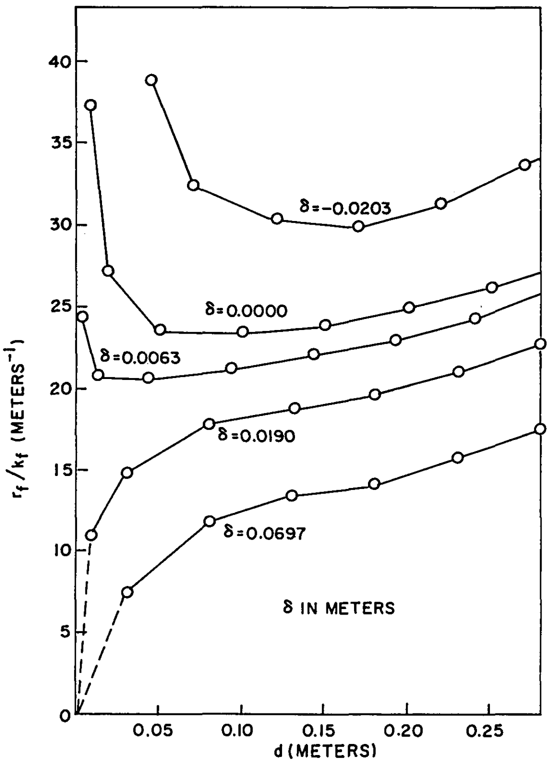

Fig. 2 - Variation of the reduction factor of the flush-mounted field meter for changes in calibration parameters

be flush with the ground plane, several values of δ were tried. A glance at Fig. 2 indicates that for this field meter $\delta = 0.0063$ m closely approaches the ideal case. Therefore, in the experiment, in order to have $E(0) = E_f$, the field meter was mounted with its face 0.0063 m above the ground.

A striking feature of these plots is that r_f/k_f has a general increase with large values of d for all values of δ. This feature is attributed to the finite size of the calibrating plate. As the distance d increases, more and more fringing takes place. The solid angle formed by the plate at the field meter decreases as d increases. The field measured by the field meter is in transition between the $1/d$ form for the parallel plate capacitor and the $1/d^2$ form for the point charge field. One is thus forced to use extremely large calibrating plates to simulate the earth's field or to correct existing calibration hardware for this effect.

The curve in Fig. 2 shows that when $\delta = 0.0063$ m, there is little distortion near the ground plane, and it is concluded that the general tendency for r_f/k_f to increase with increasing d is a result of the finite size of the calibration plate. For calibrations with $\delta = 0.0063$ m, a spacing of d between 0.02 and 0.06 m gives values of r_f/k_f that are not noticeably affected by the point charge effect. Thus for each calibration with our standard calibration plate, δ is 0.0063 m and d lies between 0.02 and 0.06 m. The reduction factor r_f is now 1, and the sensitivity k_f of the flush-mounted meter may be easily obtained after bucking out the residual field.

INVERTED FIELD METER CALIBRATIONS

At the NRL Blue Ridge Station an inverted field meter was located on a boom 1 m above the ground near an instrumentation building as shown in Fig. 3. The field meter mounted flush with ground (not shown) was about 20 m from the building in such a position that the building did not distort the true field. Calibrations of the meters were performed at least every 6 hours throughout 169 hours of data collection. The sensitivity of the inverted meter is maintained constant by periodically placing a flat metal plate at a constant distance from the sensing area of the field meter, applying zero voltage to the plate to allow zeroing any residual field, and then applying a known voltage to the plate. Although the actual value of the reduction factor is not germane to this report, the constancy of this number is important. Analysis of the data showed that there was a standard deviation of ± 19.3 percent in its value. Such a variation indicates that it was desirable to improve the determination.

Benndorf and Hess (2) describe a reduction factor in terms of a geometrical factor ξ and an average space charge factor η in the following way:

$$R = \xi(1 + \eta). \qquad (5)$$

If there were no space charge in the atmosphere, then $R = \xi$, which is a number which theoretically can be calculated from Laplace's equation for the particular geometry involved. However, when there is space charge, the situation becomes more complicated and Poisson's equation holds. The actual atmospheric space charge distribution enters the picture and depends on many variables including atmospheric turbulence and pollution.

Consider a large charged parallel plate capacitor of infinite extent and large separation. Let one of the plates be called the ground plane and let a calibrated field meter be flush mounted in that plane. Assume that there is a uniform layer of space charge of finite thickness at a distance h above the ground plane (Fig. 4a). Let the value of the field measured be $E(0) = \sigma_0/\epsilon_0$, where σ_0 is the surface charge density at the ground plane and ϵ_0 is the permittivity of free space. Next, let this layer of space charge become adjacent to the ground plane (Fig. 4b). The value of $E(0)$ remains the same in both cases. Thus in this particular model, the actual distance between the space charge layer and the ground plane does not affect the measurement.

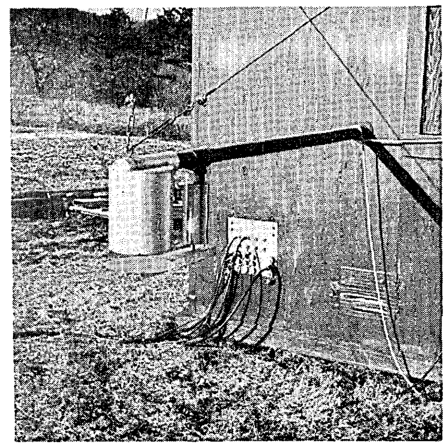

Fig. 3 - Inverted field meter during calibration

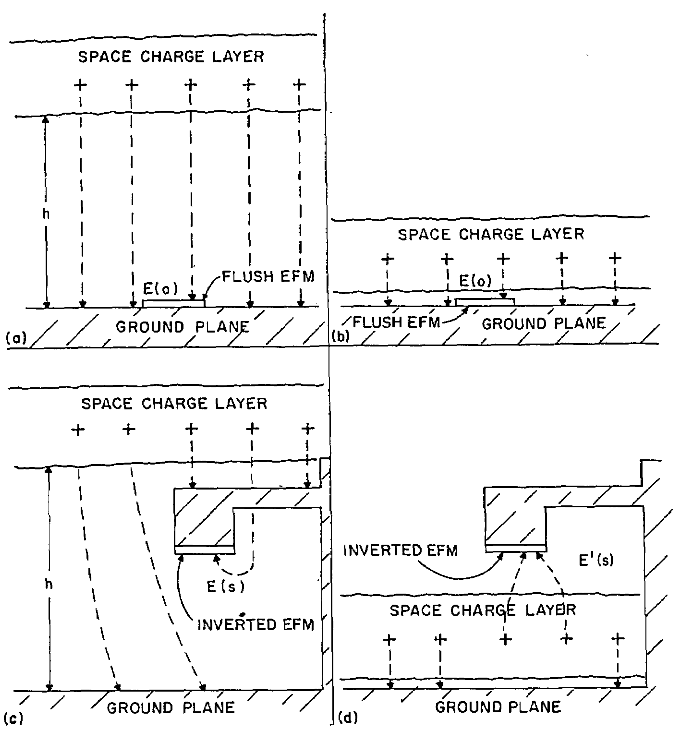

Fig. 4 - Effect of space charge distributions on electric field

Consider, on the other hand, a more complex surface geometry. Let an inverted field meter be superimposed on the ground plane as in Fig. 4c. With the layer of space charge a distance h away from the plane, the field meter will register a value $E_s = \sigma_s/\epsilon_0$. If this layer of charge is brought nearer to this plane, the field meter will read differently (Fig. 4d). A new E_s, symbolized as E_s', is obtained and is expressed for convenience in terms of the previous surface charge density and a new surface charge density σ_s' which is due to the proximity of the space charge layer to the inverted field meter. Then

$$E_s' = \frac{\sigma_s + \sigma_s'}{\epsilon_0}.$$

The reduction factor is defined as $R \equiv E(0)/E_s'$. If the fields are expressed in terms of the surface charges induced at the point of measurement

$$R = \frac{\sigma_0}{\sigma_s + \sigma_s'} = \frac{\sigma_0}{\sigma_s} \frac{1}{\left(1 + \frac{\sigma_s'}{\sigma_s}\right)}.$$

It should be noted that σ_0/σ_s is the ratio $E(0)/E_s$ when space charge is not present. Let this ratio, often referred to as a geometric factor, be defined as ξ. By substitution an exact expression for the reduction factor can be written:

$$R = \xi\left(1 - \frac{\sigma_s'}{\epsilon_0 E_s'}\right) \tag{6}$$

where $-\sigma_s'/\epsilon_0 E_s'$ is the space charge term expressed by Benndorf and Hess in Eq. (5) as η.

The question is raised as to the exact dependence of σ_s' on the distribution of the space charge ρ. The simplest approximation is that of proportionality,

$$\sigma_s' \approx \epsilon_0 \mu \rho$$

When this approximation is applied to Eq. (6), we obtain

$$R = \xi\left(1 + \mu \frac{\rho}{-E_s'}\right) \tag{7}$$

and this equation is linear in the variables R and $\rho/-E_s'$. If the approximation is at all correct, there should be a definite correlation between R and $\rho/-E_s'$ and it should be possible to obtain the constants μ and ξ from a scatter diagram of points.

A second experiment involved the two field measurements and a measure of space charge density ρ at 1 m above the ground with an Obolensky type filter (3). The experiment was designed to use Eq. (7) in order to obtain a more reliable reduction factor and give more confidence to the correction of measurements from an inverted field meter.

The time averages of $E(0)/E_s' \equiv R_{av}$ and of $\rho/-E_s'$ were plotted as point A in Fig. 5. Equation 7 represents a line passing through this point and intersecting the ordinate at $R = \xi$. where ξ is yet to be determined. The value of μ can be written as

$$\mu = \frac{R_{av} - \xi}{\xi} \frac{1}{(\rho/-E_s')_{av}}$$

With an assumed value of ξ, solve for

$$R = \xi\left[1 + \mu\,(\rho/-E_s')\right].$$

By substituting this value of R in $R \equiv E(0)/E'_s$, "corrected" values of E'_s are obtained. Calculations of the rms of the error between the corrected values of E'_s and $E(0)$ over 169 data hours for various values of ξ were plotted against the selected values of ξ. Figure 6 shows the minimum error to be at the intercept point of $\xi = 0.784$. A line is defined now by the two points, A and $\xi = 0.784$, $\rho/E'_s = 0$ on Fig. 5. Once this line is determined by the minimum error method of evaluating ξ, no improvement in the value of the reduction factor can be made by this simple approximation method.

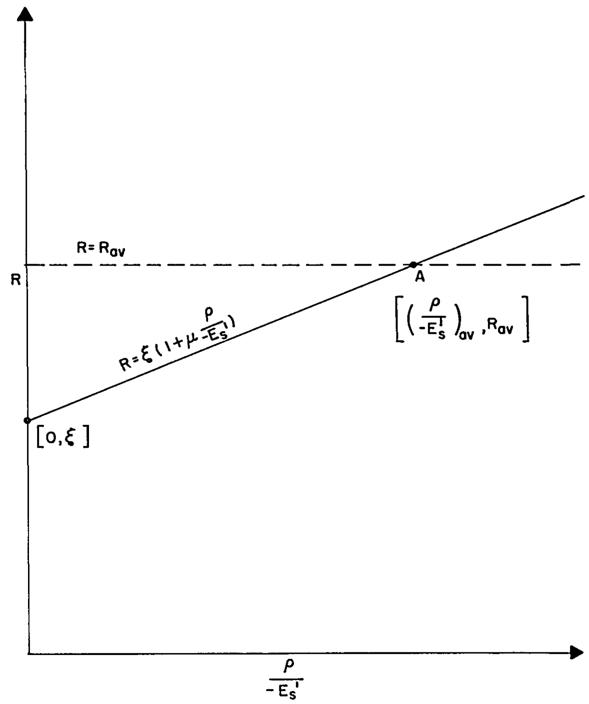

Fig. 5 - Graph of Equation (7)

CONCLUSIONS

The error when using an average reduction factor in the first experiment was ±19.3 percent. By using the method outlined immediately above, the error was ±17.8 percent, showing only a slight improvement. This improvement does not seem to warrant the added instrumentation and computations. Nevertheless, the experiment does point out that the existence of atmospheric space charge affects measurements made with inverted field mills in a complicated manner, and that this phenomenon does not lend itself to simple analyses. It can be concluded therefore that accurate measurements of electrostatic fields at the surface of the earth can be made only with a flush-mounted meter.

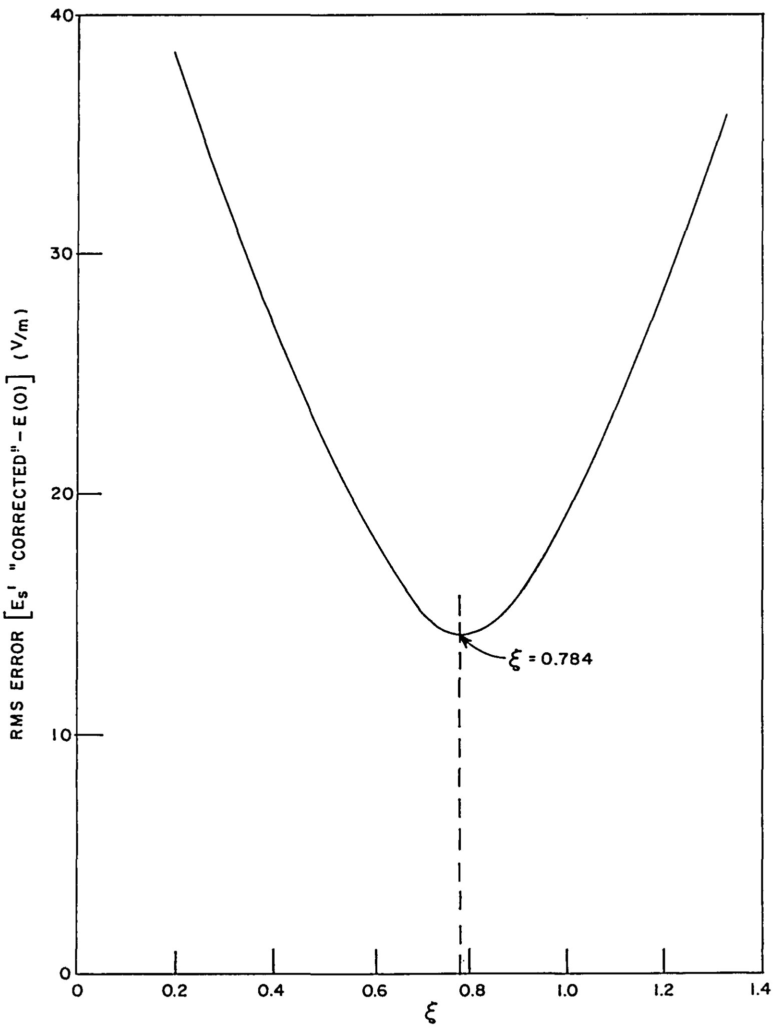

Fig. 6 - Plot of rms error as a function of ξ

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13. ABSTRACT

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