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Instantaneous Position Fixing from Measurement of Satellite Range and Doppler

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ABSTRACT

A worldwide satellite navigation system (TIMATION) has been proposed that will yield instantaneous and continuous position fixes by suitable satellite configurations and navigator equipment. The TIMATION project is conducted by NRL under the sponsorship of the Navy Space Project Office (PM-16) and the Naval Air Systems Command. This report addresses the instantaneous position-fixing results obtained by using range and doppler measurements obtained with the TIMATION II satellite and a navigation receiver employing a relatively precise frequency standard. The results are corrected for first-order ionospheric refraction and indicate a two-dimensional position-fixing capability of 105 m RMS (60 m CEP).

PROBLEM STATUS

This is an interim report; work continues.

AUTHORIZATION

NRL Problem R04-16
Project A370 538 265 2C1W 3411 0000

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INSTANTANEOUS POSITION FIXING FROM MEASUREMENT OF SATELLITE RANGE AND DOPPLER

BACKGROUND

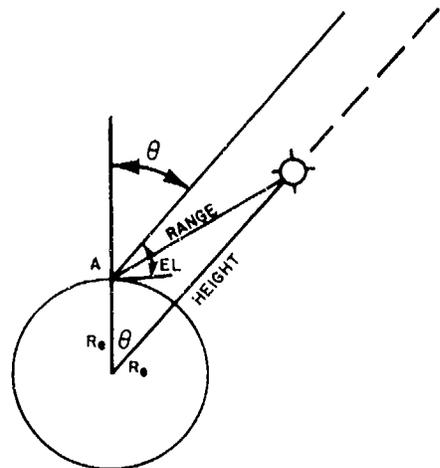
A significant advantage of satellites as sources of navigation fixes is that it is possible to determine a two-dimensional position fix from a single satellite almost instantaneously (1). Both the range and the doppler shift are measured simultaneously. This report presents the results of a large number of navigation fixes using the TIMATION II satellite.

The technique, which provides the navigator an instantaneous position with measurements from a single satellite, assumes that the passive user has four known parameters: (a) navigator's velocity, (b) time difference between satellite and user clocks, (c) frequency difference between satellite and ground frequency standards, and (d) navigator's height above the geoid. If these parameters are not known, the navigator can make additional measurements to determine the unknowns.

RANGE NAVIGATION THEORY

A method of describing the theory of satellite navigation is to compare it to celestial navigation. Figure 1 shows the relationship between satellite range measurements and celestial-navigation measurements. In this figure it is seen that the central angle, θ , is a measure of the distance of the navigator from the subsatellite point. This angle, which is determined by measuring the three sides of the triangle formed by the positions of the navigator, the satellite, and the center of the earth, is the same as the angle measurement made in celestial navigation. However, the satellite provides greater accuracy, since the central angle of the triangle can be obtained to greater precision by the three-side-measurement method than by the direct-sighting method.

Fig. 1 — Transform from satellite ranging to celestial navigation. A satellite range measurement can be converted to a line of position. The measurement of range to the satellite is analogous to the measurement of a zenith angle to a star having the same geographical position as the satellite, by using a measurement of distance to measure an angle. In celestial navigation, the observer measures the elevation angle (EL) with a sextant. The angles θ and EL are complements. The elevation angle is also called the altitude angle in celestial-navigation terminology.



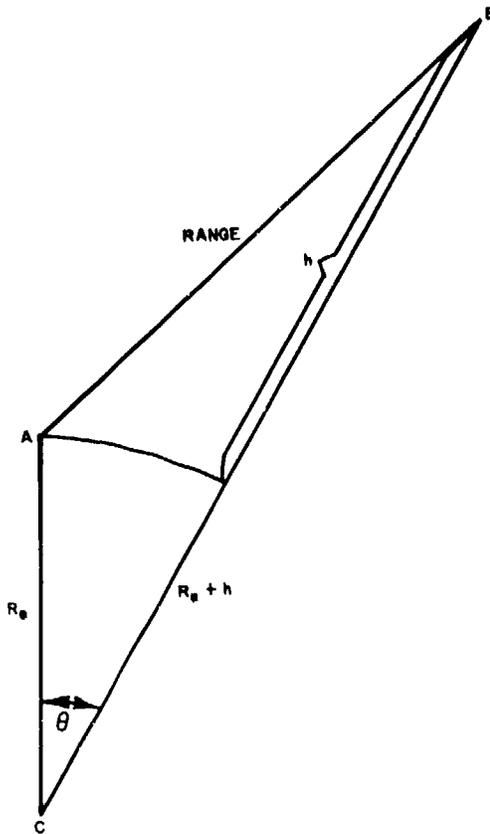


Fig. 2 — The triangle used to convert each range measurement to an angle measurement for use with celestial-navigation techniques.

Since the range-measurement method is analogous to celestial navigation, the intercept chart designed by St. Hilaire (2) can be used for both techniques. Figure 2 shows the triangle involved. Line AB is measured by the ranging technique, AC is the radius of the earth, and BC is the distance of the satellite from the center of the earth. Angle θ can now be determined from the cosine law. The complement of θ is called the altitude angle in celestial-navigation terminology; it is perhaps more commonly known as the elevation angle (EL). The elevation angle, calculated from a range observation, can be used with the observer's assumed position (A) and the local hour angle of the observer and the satellite to enter the H0214 tables (2) and to construct an intercept chart. The H0214 tables are usually accurate to 0.1 n. mi., but occasionally the error may be as large as 0.3 n. mi. This interpolation error is bypassed by calculating the required parameters for intercept-chart construction without interpolation.

The angle α (Fig. 3), which is required in calculating the range azimuth line, is found by using the spherical triangle ANG, where N is the North Pole, A is the assumed position of the navigator, and G is the geographical position of the satellite. Then the angle α may be obtained by solving Eq. (1), which is the cosine-law equation for spherical triangles, using the measured angle θ , the navigator's colatitude AN, and the satellite colatitude GN as the known quantities.

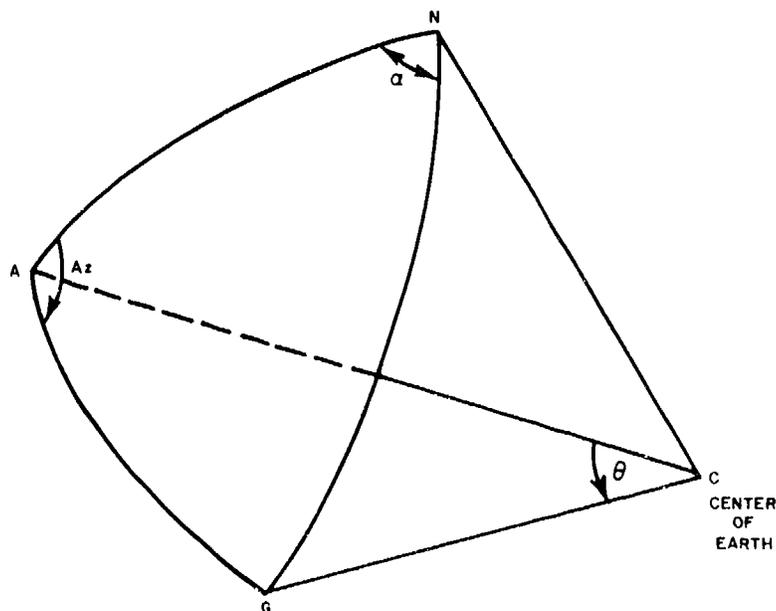


Fig. 3 — Spherical triangle NAG, where N is the North Pole, A is the navigator's assumed position, and G is the geographical position of the satellite. The arcs AN and NG are the colatitudes of the navigator and the satellite G, respectively.

$$\cos \theta = [\cos (AN) \cos (NG) + \sin (AN) \sin (NG) \cos \alpha] \quad (1)$$

The azimuth angle A_z from AN to AG is found by Eq. 2.

$$A_z = \sin^{-1} \left[\frac{\sin (NG) \sin \alpha}{\sin (AG)} \right]. \quad (2)$$

The range sensitivity, the rate of change of range in the observer's horizontal plane, is given by the factor $1/\cos EL$. The point of reference for the range measurements is the calculated range obtained by using the navigator's assumed position A and the satellite position at the time of measurement, with a small correction for time aberration.

Once the range azimuth and the range sensitivity are calculated, the intercept chart can be constructed. Figure 4 shows a typical intercept chart for a satellite pass. The distance from A to G is stated in either miles or microseconds, and the tick marks are made along the azimuth line and are marked as shown. When the reading is made, the appropriate point on the azimuth line is found, and a perpendicular is drawn to the azimuth line at this point. This line is a line of position, and the navigator's determined position is on this line, provided there is no instrument error. Figure 5 shows a perfect fix. Figure 6 shows an intercept chart in which the fix is at the center of curvature of the arc shown. The radius of the arc corresponds to instrument error in celestial navigation,

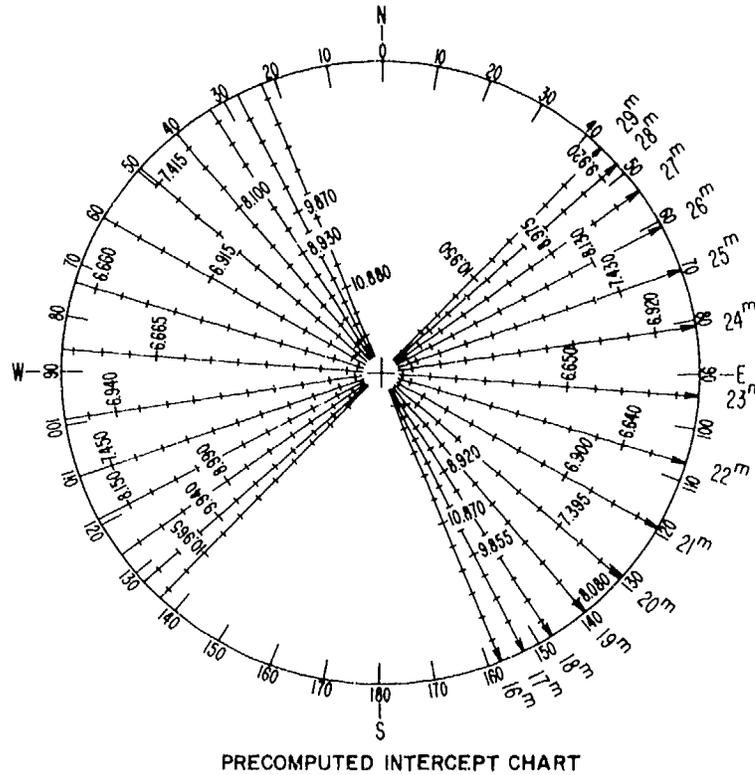


Fig. 4 — An example of an intercept chart for a typical TIMATION II satellite pass. The chart is centered at the navigator's assumed position A, and the arrows point to the satellite's shifting ground point G. The ranges are marked off in milliseconds of time delay.

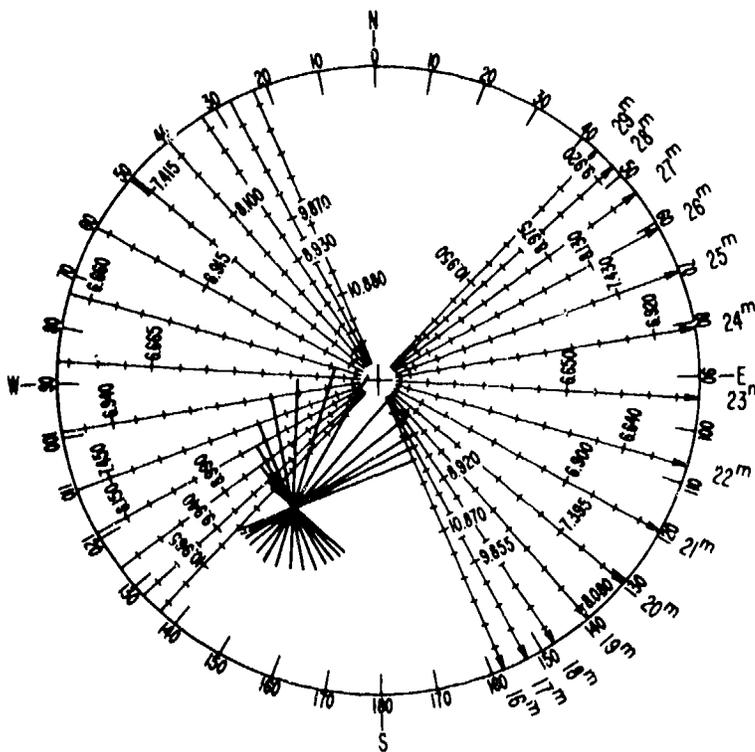
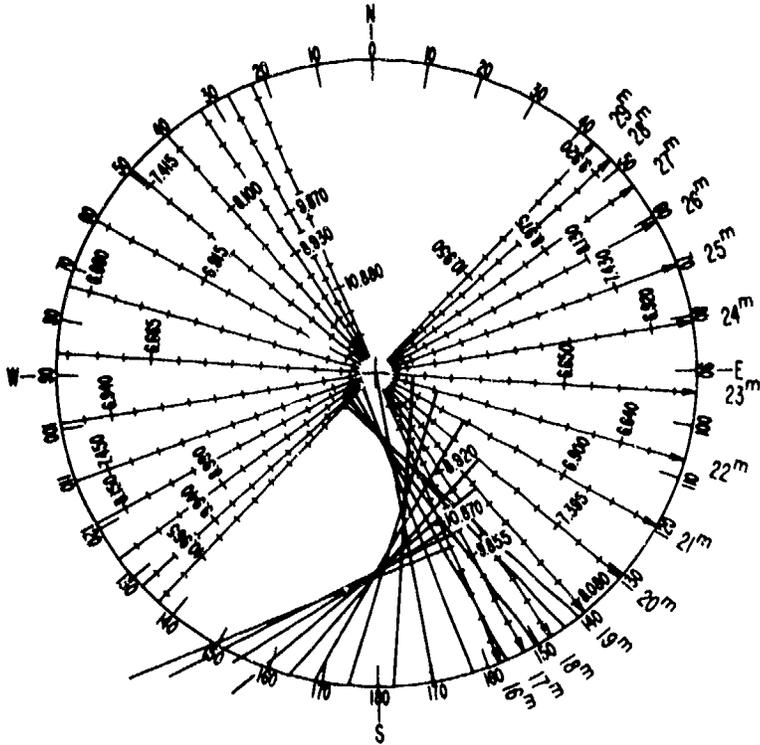


Fig. 5 — A position fix obtained using range measurements with no error. The lines of position intersect at the navigator's position.



and in satellite navigation to a time difference between the navigator and satellite clocks. Assuming that the satellite clock time is known, the error in the user clock can be found. If time-difference measurements are made at two different locations, the separate user-clock errors can be determined as referred to the satellite clock. The times at the two locations can then be compared, and a time transfer via satellite can be accomplished.

DOPPLER NAVIGATION THEORY

An intercept chart can also be constructed to obtain a navigation position fix using doppler measurements; however, the doppler azimuth lines do not point to the subposition G of the satellite. One method of calculating the direction and sensitivity of the doppler is by using the range-doppler equations described in Ref. 3.

Figure 7 shows the doppler-frequency contours of a low-altitude satellite transmitting at a constant frequency. The contours are due to the satellite velocity and the resulting doppler. An observer located at a point in view of the satellite will measure the doppler shift (Fig. 8) as the satellite passes by.

If the navigator knows his frequency as related to the satellite, he can determine which of the constant doppler lines he is on, and hence construct a line of position as determined by the doppler. Any user velocity will change the received doppler frequency and, unless corrected, will introduce an error in the doppler line of position.

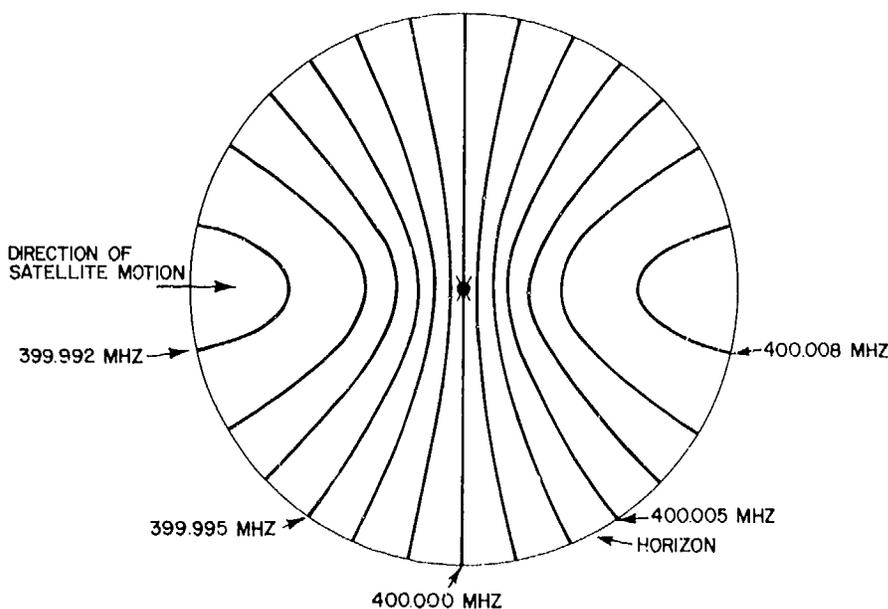


Fig. 7 — Received doppler frequency contours for a 400-MHz transmitted frequency from a satellite in a 500-mile circular orbit. The satellite is at the center of the circle and traveling from left to right.

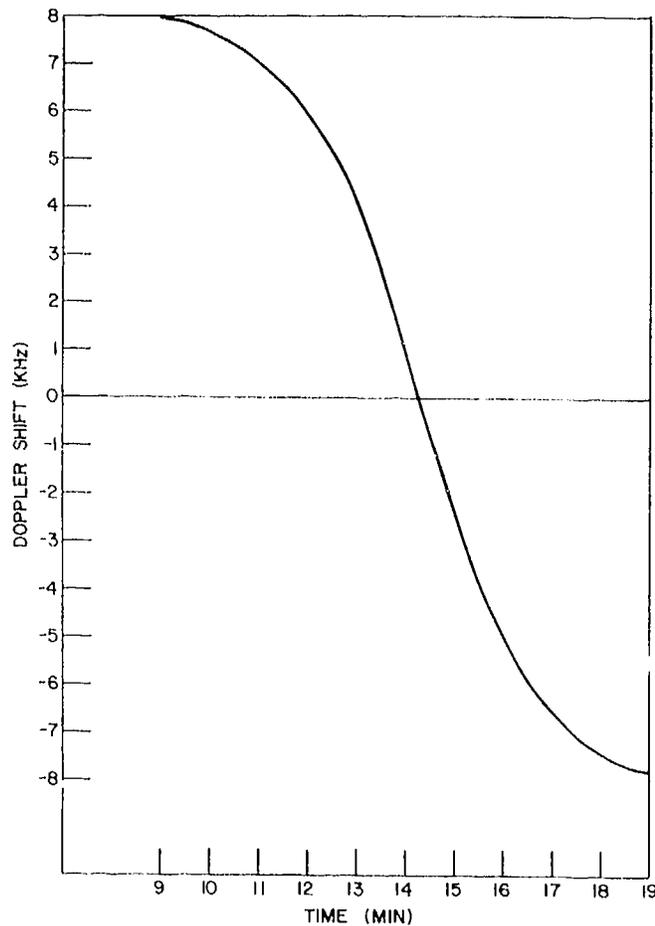


Fig. 8 — Doppler frequency shift versus time for a 400-MHz transmitted frequency from a satellite in a 500-mile circular orbit

RANGE-DOPPLER NAVIGATION THEORY

An intercept chart for range-doppler position fixes can be constructed by combining the intercept charts for range and doppler. The lines of position (LOP) of the ranging case are spheres about the satellite position. These spheres result in circles when they intersect the earth. When the ranging LOPs are combined with the doppler LOPs, Fig. 9 results. One can expand a small portion of Fig. 9 to give the combined range-doppler intercept chart shown in Fig. 10. This technique can be used to provide fixes, especially in the ranging case, to the operator who has minimum equipment. The intercept chart for range is easier to construct, because the range contour lines are perpendicular to the azimuth line between the navigator's assumed position and the satellite ground point, while the doppler LOPs are not (in general) orthogonal to the range LOPs.

The sensitivity of the range-doppler solutions to errors in measurement depends on the relative location of the satellite with respect to the navigator. The mathematical equations used for the range-doppler solution, which are given in Ref. 3, depend on the solution of a system of simultaneous nonlinear equations. The equations contain information on the sensitivity of the solution which can be verified by graphical techniques. Reference to Fig. 7 shows that the doppler contours are closely spaced for a navigator with an elevation angle near the navigator's zenith; the doppler contour lines are more widely spaced and skewed for a low elevation angle. The numerical value of the sensitivity is about 0.1 n. mi./Hz (185 m/Hz) for elevation angles near the navigator's zenith and about 0.3 n. mi./Hz (555 m/Hz) for a 10-degree elevation angle. The range sensitivity is inversely proportional to the cosine of the elevation angle and varies from 0.16 n. mi./ μ sec (1 m/m) at 0-degree elevation angle to 0.32 n. mi./ μ sec (2 m/m) at a 60-degree elevation angle.

A unique solution to the range-doppler equations for a two-dimensional position fix may be obtained by using a reasonable estimate for the navigator's assumed position within an open region defined graphically in Fig. 9 by the satellite ground track and the horizon. This estimate results in two open regions where a unique solution may be obtained, with the satellite ground track as a dividing line. The numerical uncertainty in assumed position may be quite large (on the order of 100 to 200 km), except as the navigator's position approaches the satellite ground track, where the estimate must place the navigator on the proper side of the ground track.

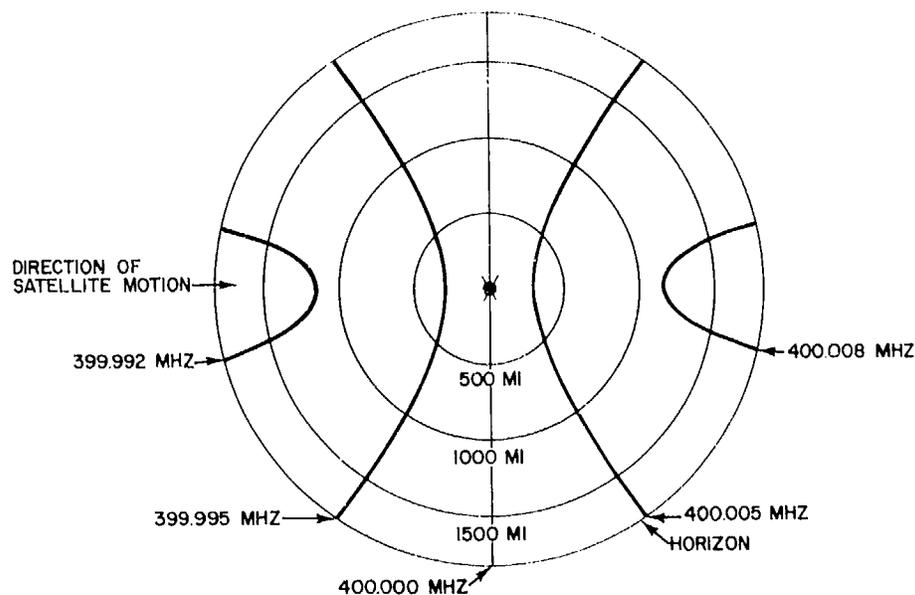


Fig. 9 — Received range-doppler contours for a 400-MHz transmitted frequency from a satellite in a 500-mile circular orbit. The satellite is at the center of the circle and traveling from left to right.

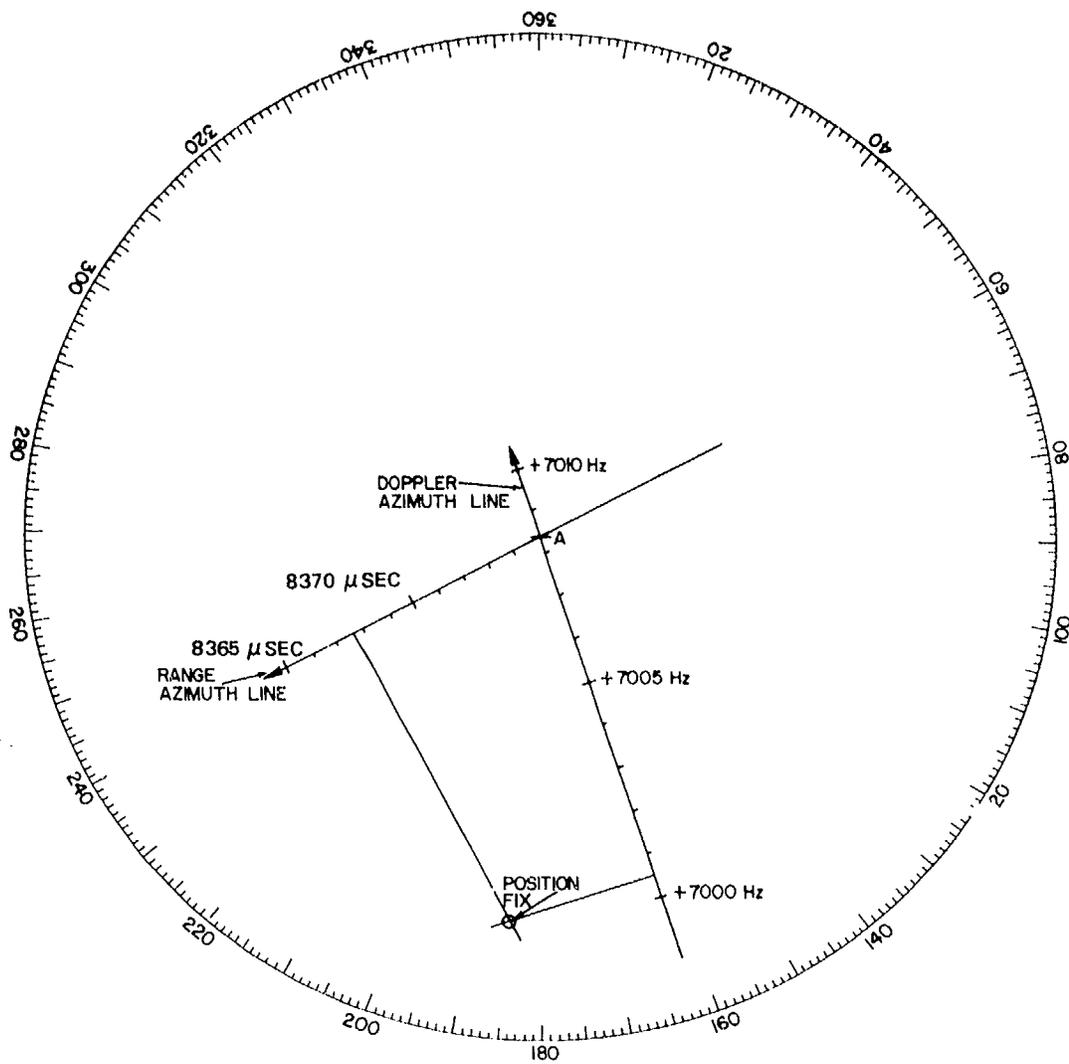


Fig. 10 — An instantaneous fix using range and doppler

POSITION-FIX RESULTS

The results of 43 satellite passes taken at the Chesapeake Bay Division field station in the summer of 1972 are presented here. The passes were first processed using range-only and doppler-only navigation solutions, then edited to remove spurious data; the remaining data were matched, and 406 points of simultaneous range and doppler data were obtained. This method yields an average of nine points of simultaneous range and doppler data per pass. By making measurements at 150 and 400 MHz, the data were corrected for first-order range and doppler ionospheric refraction.

Figure 11 shows the navigation fix results using range-only measurements. Each point on Fig. 11 is the result of using an entire pass, with a range measurement (line of position) taken every minute. In effect the ten lines of position are then averaged to obtain one best point for each pass. The mathematical technique used for range and doppler is described in Ref. 3. As shown in Fig. 11, the RMS of the range-only navigation is 64 meters, 70 yards, 210 feet, or 0.035 n. mi.

Figure 12 shows similar results for the doppler-only navigation fixes. A doppler count was taken for each minute of the pass. The time required for a count varied between one and three seconds. Experimental results on TIMATION II indicate that this is

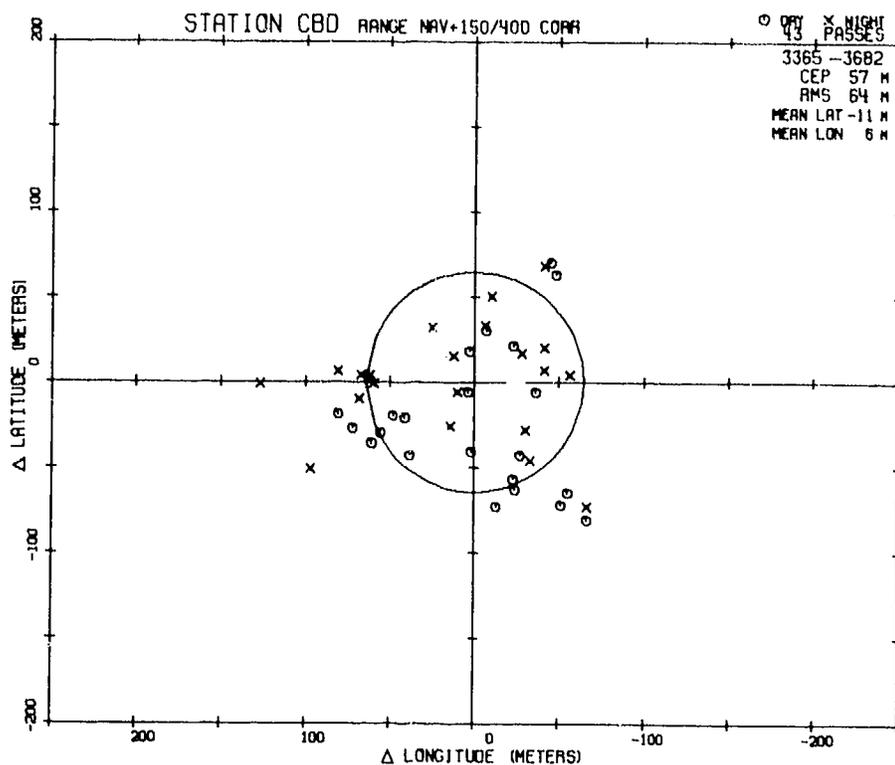


Fig. 11 — Station CBD navigation fix results using range-only measurements

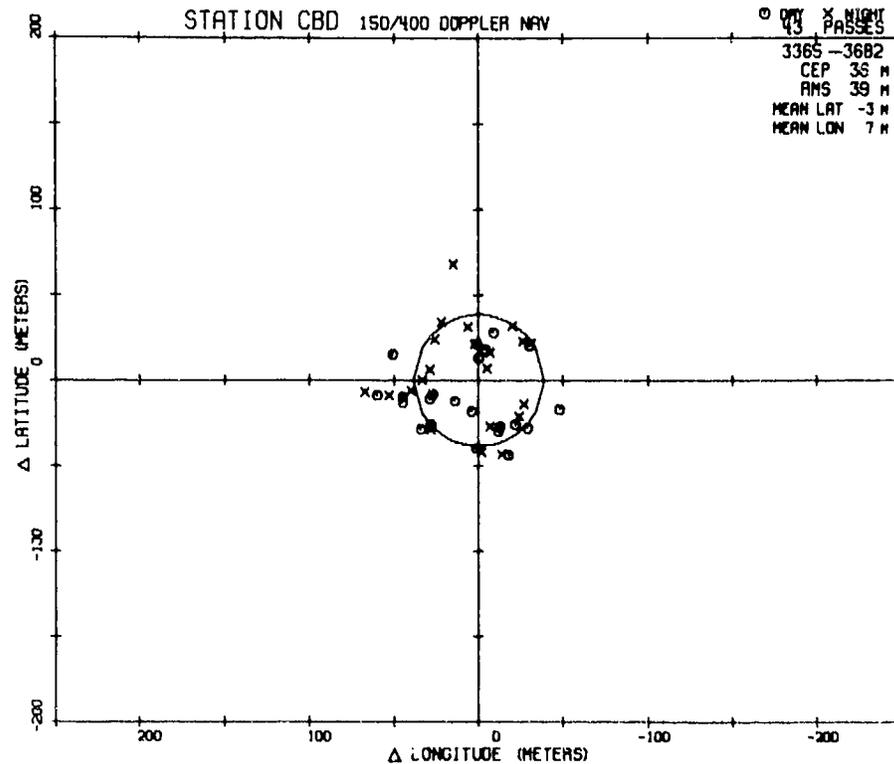


Fig. 12 — Station CBD navigation fix results using doppler-only navigation fixes

near the optimum count time for short doppler frequency measurements. For times in excess of five seconds, the nonlinear doppler correction becomes large, while for short time intervals less than one second, the short-term frequency stability causes the measurement noise to increase. Each count (one per minute) resulted figuratively in a line of position. The doppler-only navigation program then calculated a best fix for the entire pass. The RMS of the navigation fixes is slightly better than for the ranging case, with a value of 39 meters, 43 yards, 128 feet, or 0.021 n. mi.

Figure 13 shows the results obtained for 406 range-doppler position fixes using data corrected for first-order ionospheric refraction. Each point corresponds to the intersection of a single range line of position and a single doppler line of position. The navigation algorithm used is described in Ref. 3. As expected, the number of points is greater than those shown in Fig. 11 and 12. The RMS of the range-doppler position fixes is greater than the RMS of either the range-only or doppler-only position fixed. The expected ratio is approximately 2.1 for a Gaussian distribution $[(\text{number of minutes per pass})/2]^{1/2}$. The actual measured ratio is 1.6 for ranging and 2.7 for doppler (Table 1). The RMS of the instantaneous range-doppler position fixes is 105 meters, 115 yards, 344 feet, or 0.057 n. mi. Figure 13 shows the results for the instantaneous position fixes. The distribution of the fixes indicates a tendency toward larger longitude errors. Figure 14 gives 406 range-doppler position fixes without the ionospheric-refraction correction. The RMS of 161 meters indicates that the dominant error for this case is due to ionospheric refraction. The errors are again greater in longitude than in latitude.

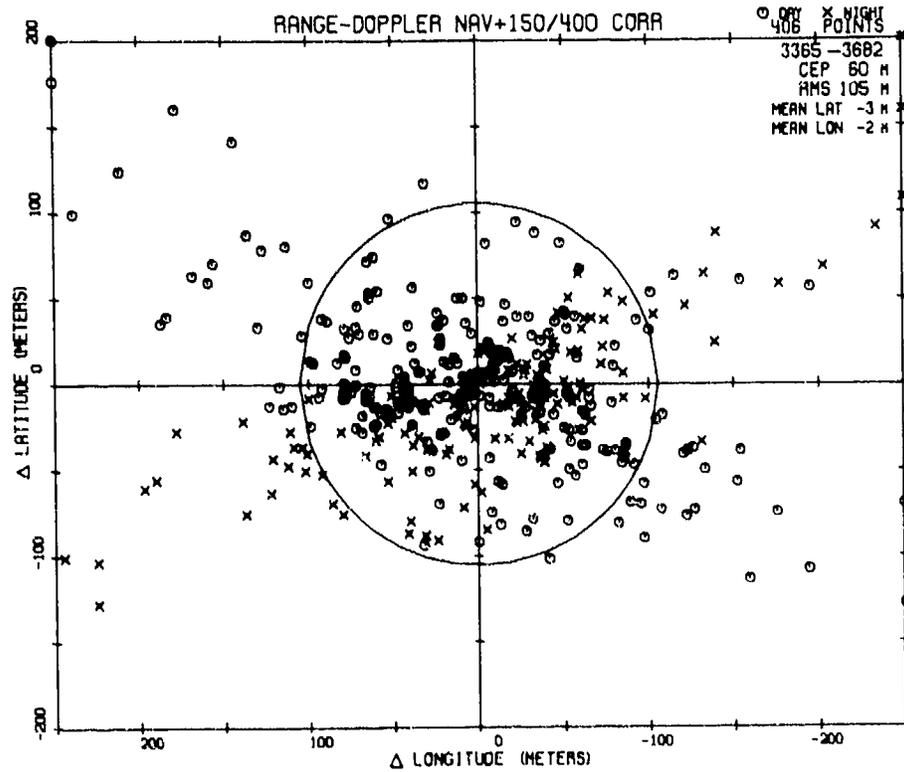


Fig. 13 — Station CBD navigation fix results for 406 range-doppler position fixes corrected for ionospheric refraction

Table 1
Two-Dimensional Navigational Accuracy
for 43 Passes at CBD
Passes 3365 through 3682

Measurements	CEP (meters)	RMS (meters)	Mean Navigational Error	
			Lat. (meters)	Long. (meters)
Range Rate	36	39	-3	7
Range	57	64	-11	6
Range-Range Rate	60	105	-3	-2

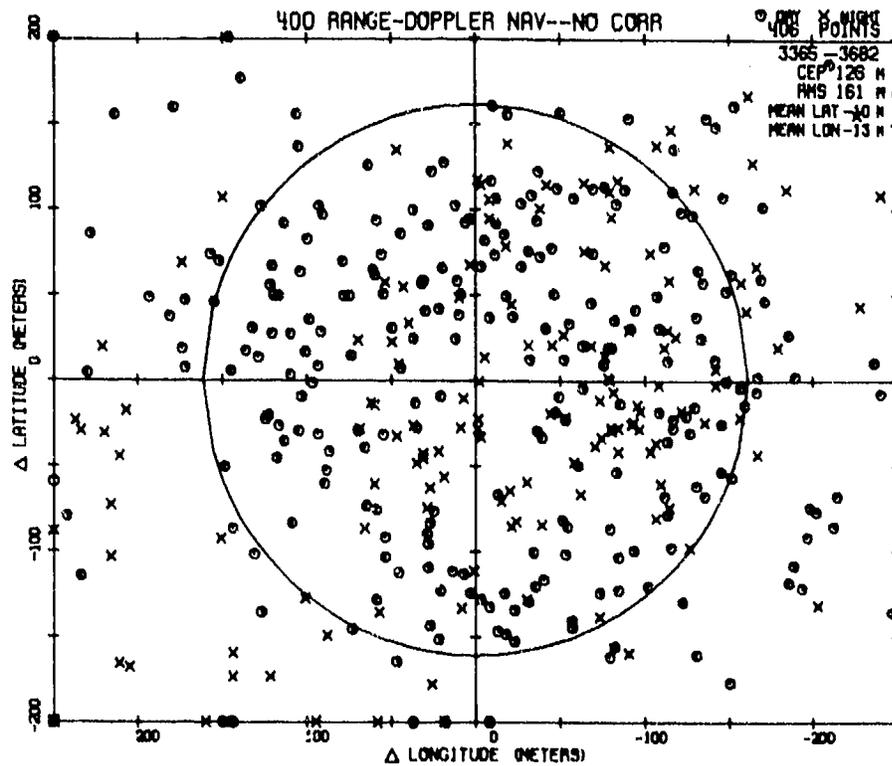


Fig. 14 — Station CBD navigation fix results for 406 range-doppler position fixes without correction for ionospheric refraction

It is instructive to look at the fixes on an individual pass. Figure 15 shows a pass using 400-MHz for ranging and doppler without correction for ionospheric refraction. It is seen that the points are not randomly distributed; they follow a somewhat smooth curve, with the best points near the center of the pass, corresponding to the points which have the 90-degree crossings for the two LOP's, the minimum ionospheric effect, the minimum atmospheric refraction, and the minimum range-rate, giving the optimum range readings. Figure 16 is for the same pass as Fig. 15, with the inclusion of the ionospheric-refraction correction, which results in improved position fixes. The distribution of the navigated points in Fig. 14 can now be explained by the systematic error introduced by lack of ionospheric-refraction correction, which appears to create random scatter in the composite of all 406 points.

Instantaneous position fixes for the remaining 42 satellite passes are given in the 84 charts in Appendix A. Comparisons can be made between position fixes calculated with and without an ionospheric-refraction correction for the range and doppler measurements.

CAVEATS

It should be realized that for all of these fixes the optimum conditions were assumed. The satellite orbit was computed after the fact, giving an orbital accuracy of 10 m, the

satellite clock was corrected using data taken both before and after the navigation fixes, and the satellite frequency was likewise known from both pre- and post-fix data. In addition, the ranging data were obtained using sidetones up to and including 1.0 MHz, for which the range resolution is approximately 10 feet. The doppler equipment used is capable of a frequency resolution on the order of 1×10^{-11} .

CONCLUSIONS

Near-instantaneous navigation fixes can be achieved using simultaneous range and doppler measurements at 150 and 400 MHz from the TIMATION II satellite, with an RMS accuracy of 105 m. An RMS accuracy of 161 m may be achieved by using the 400-MHz range and doppler measurements: in this case the principal error source is ionospheric refraction. The position fixes exhibit a tendency toward larger longitude errors than latitude errors. The position fixes with the largest error occur at the beginning or end of the satellite pass.

ACKNOWLEDGMENTS

The authors wish to acknowledge the technical guidance of Mr. D.W. Lynch, Head, Advanced Techniques and Systems Analysis Section, Space Metrology Branch. The authors also acknowledge Miss J.E. Thompson for programming the computer plot routines. Much credit is due to the other members of the Space Metrology Branch who participated in the design and construction of the TIMATION equipment, and to the Bendix personnel who operate the CBD field station.

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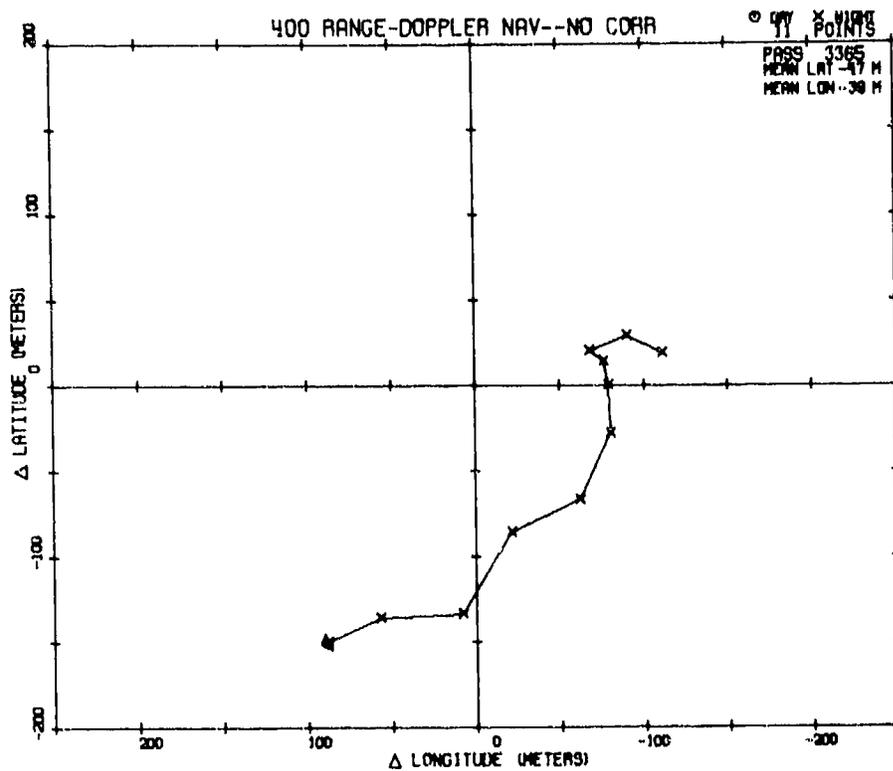


Fig. 15 — Station CBD navigation fix results for one satellite pass using 400 MHz for ranging without correction for ionospheric refraction

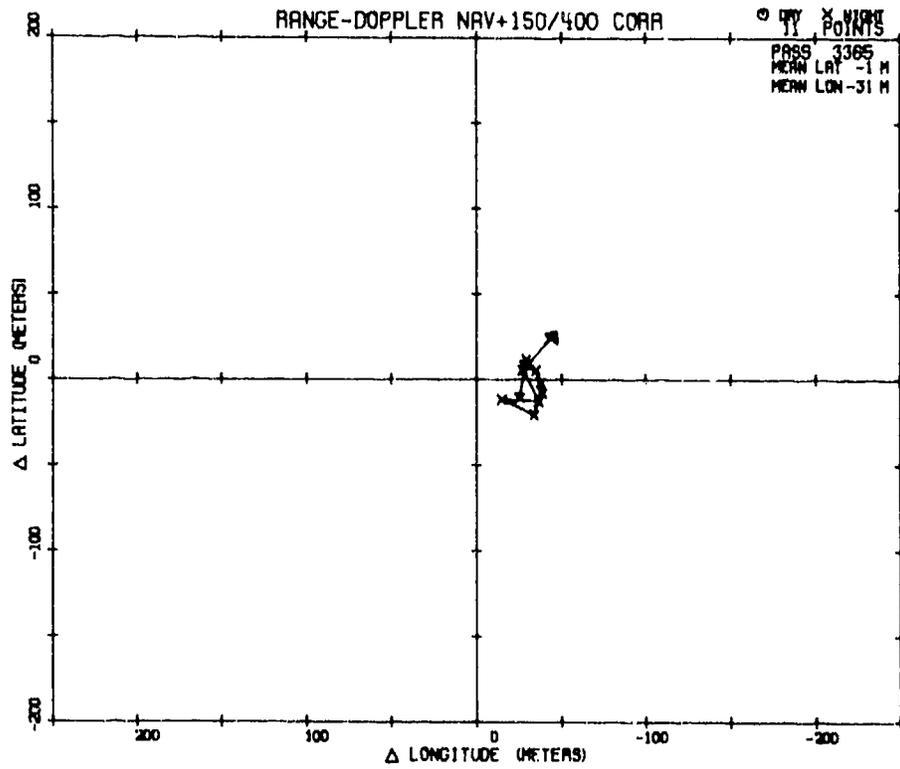


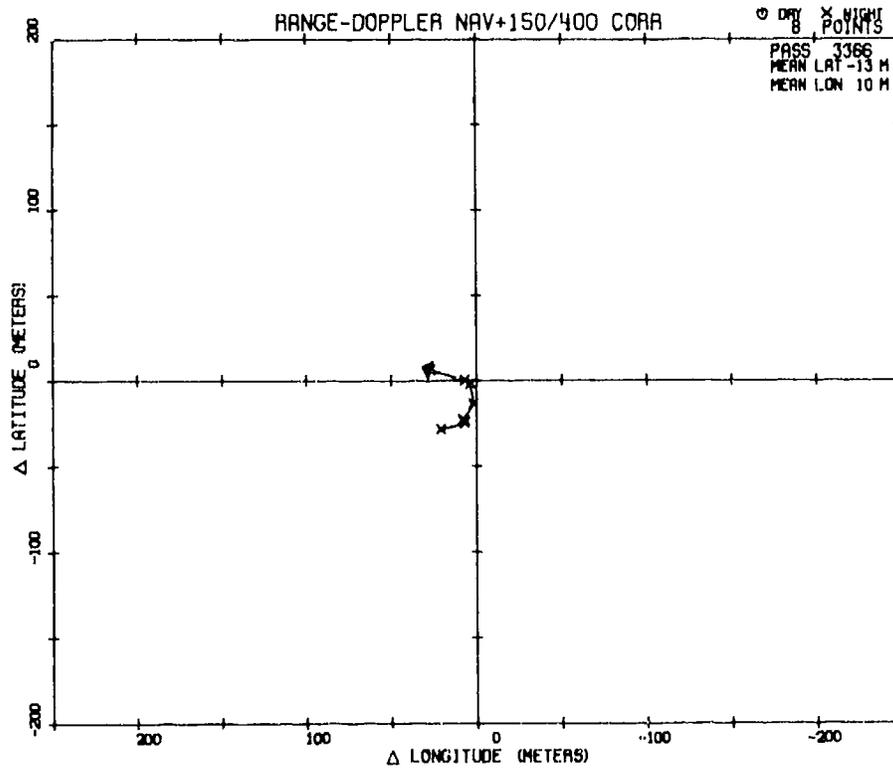
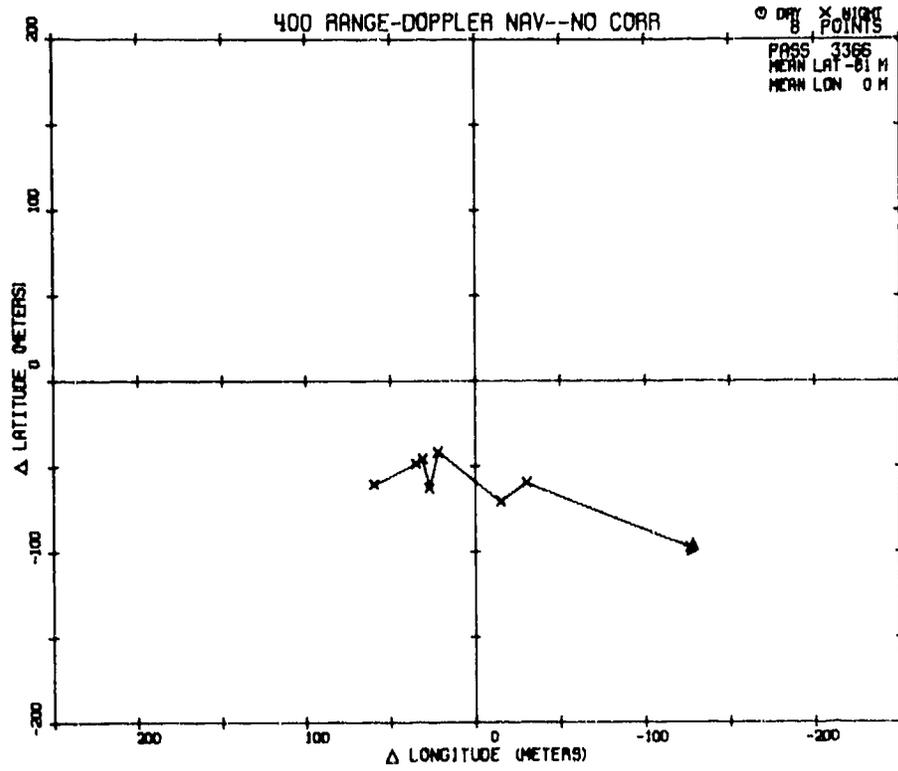
Fig. 16 — Station CBD navigation fix results for the same satellite pass given in Fig. 15, but with the inclusion of correction for ionospheric refraction

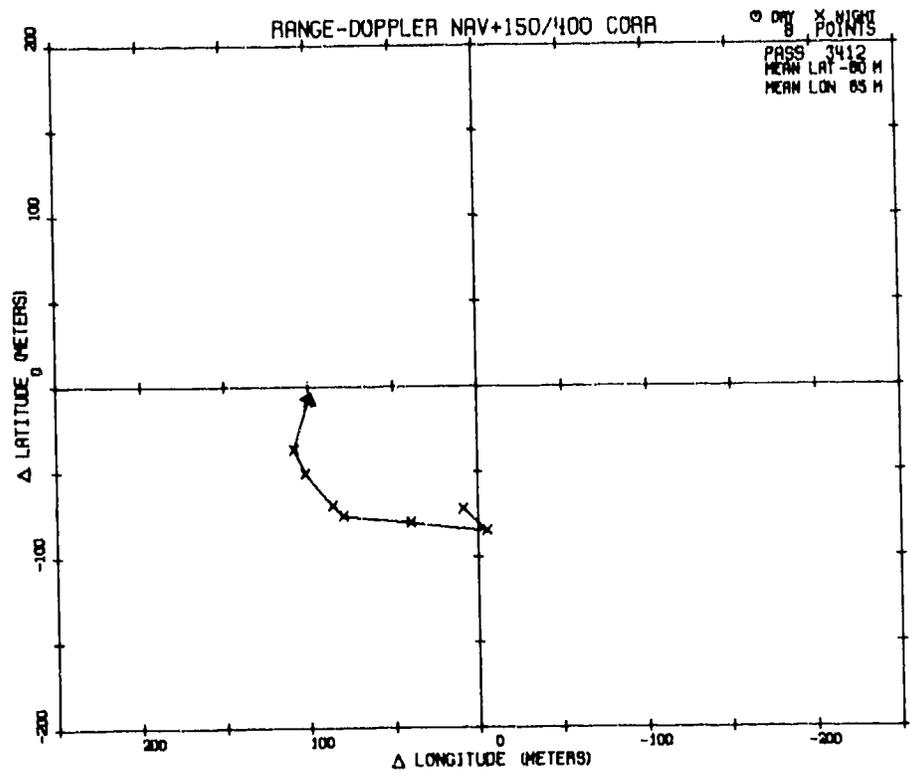
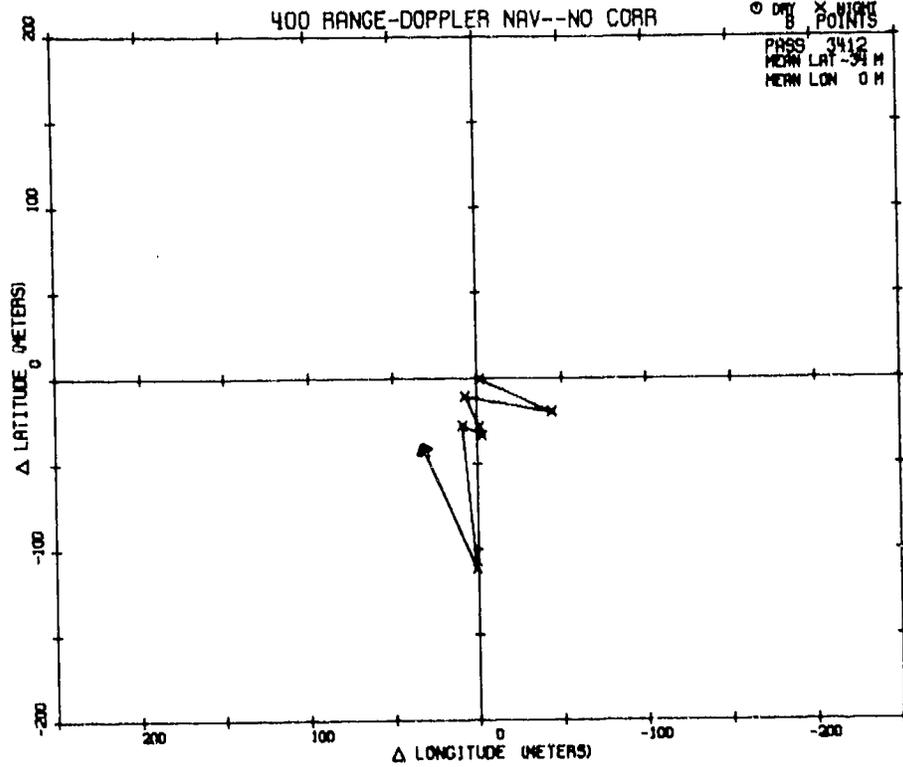
Appendix A

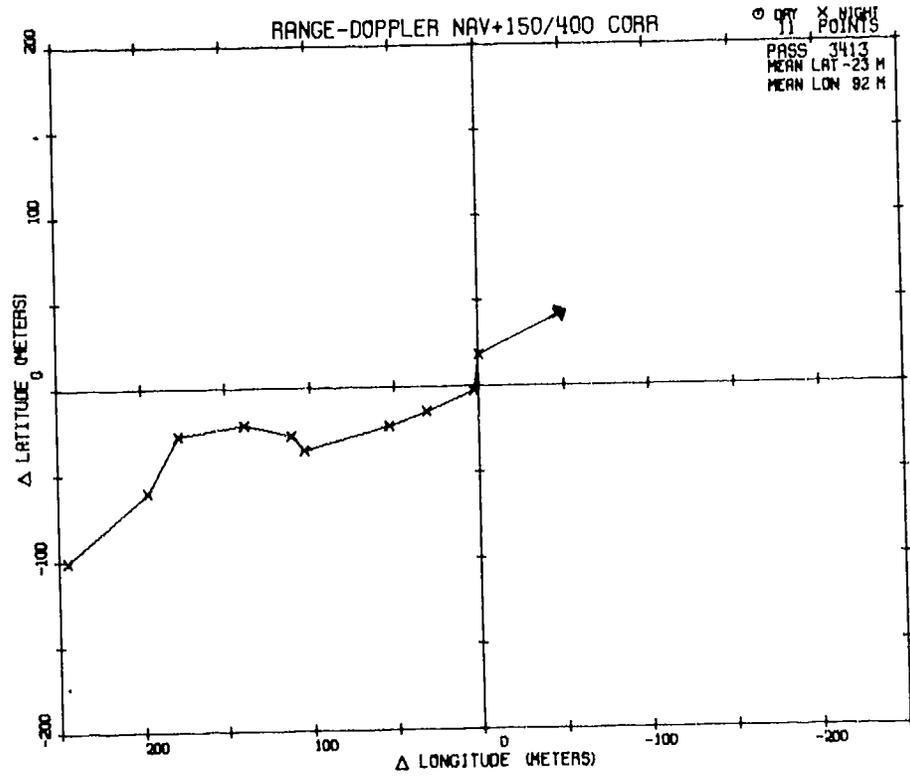
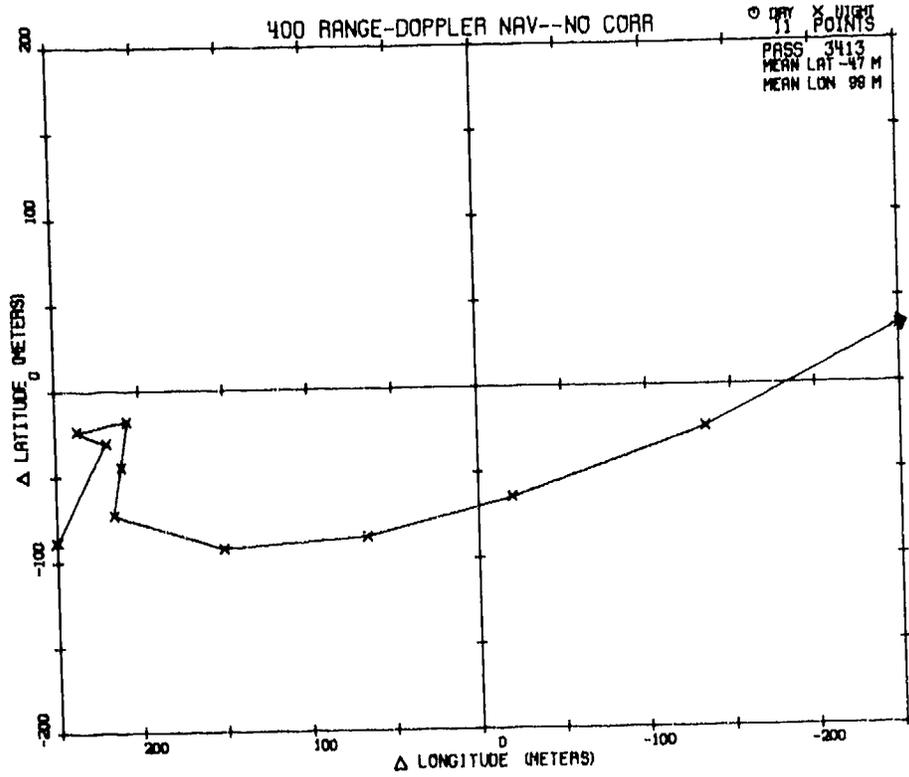
NAVIGATION FIX RESULTS FOR 42 SATELLITE PASSES

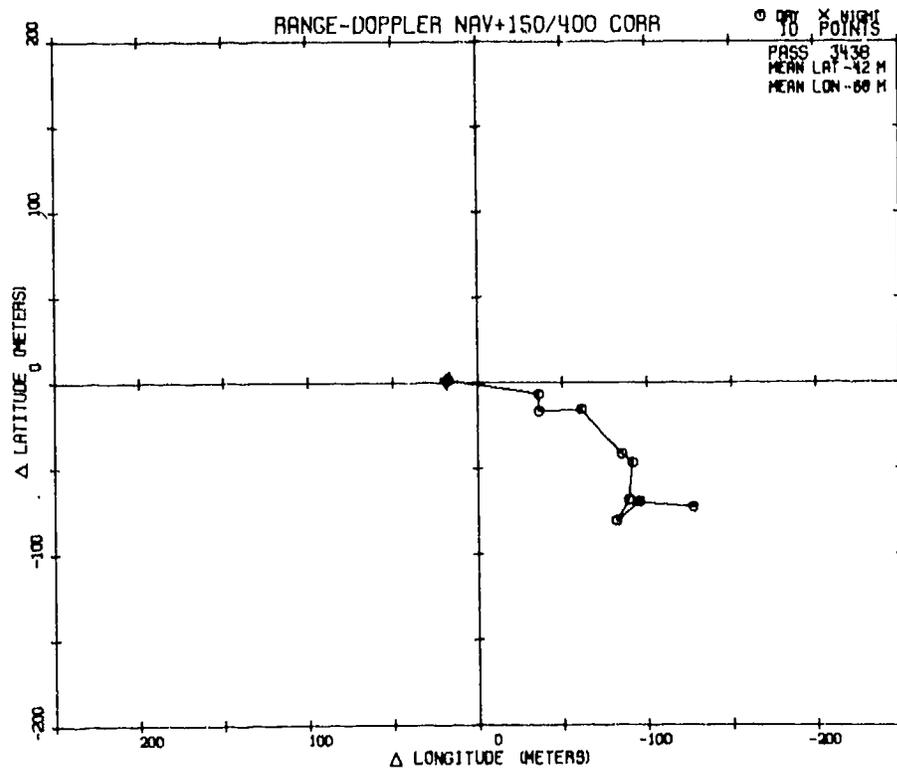
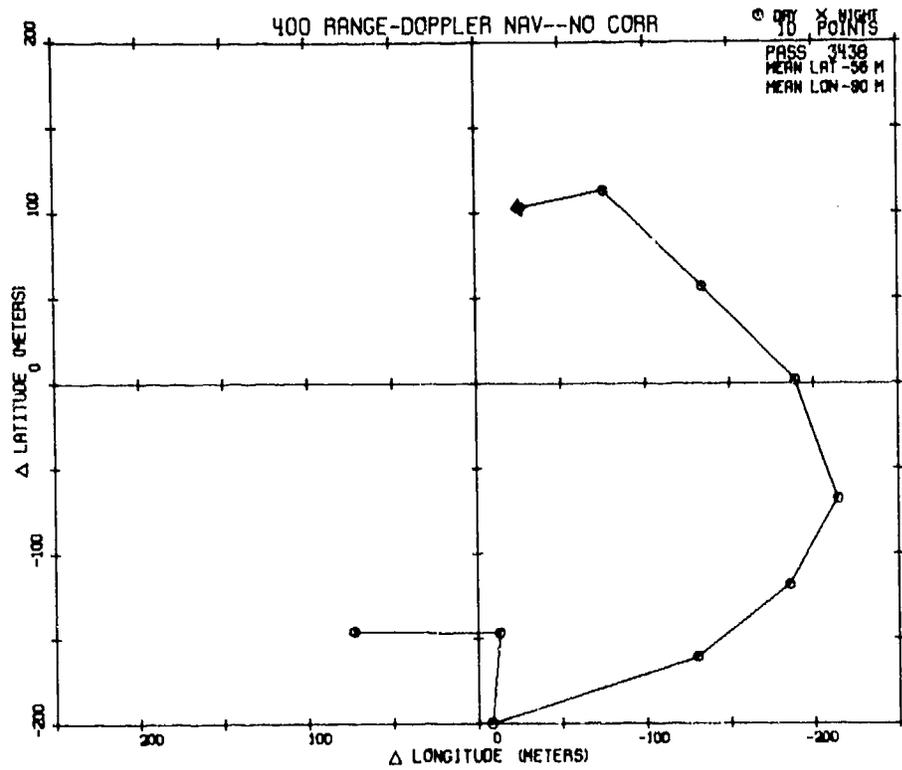
**RANGE-DOPPLER NAVIGATION FIX RESULTS USING 400-MHz
SATELLITE TRANSMITTER FREQUENCY WITHOUT CORREC-
TION FOR IONOSPHERIC REFRACTION (UPPER CHARTS)**

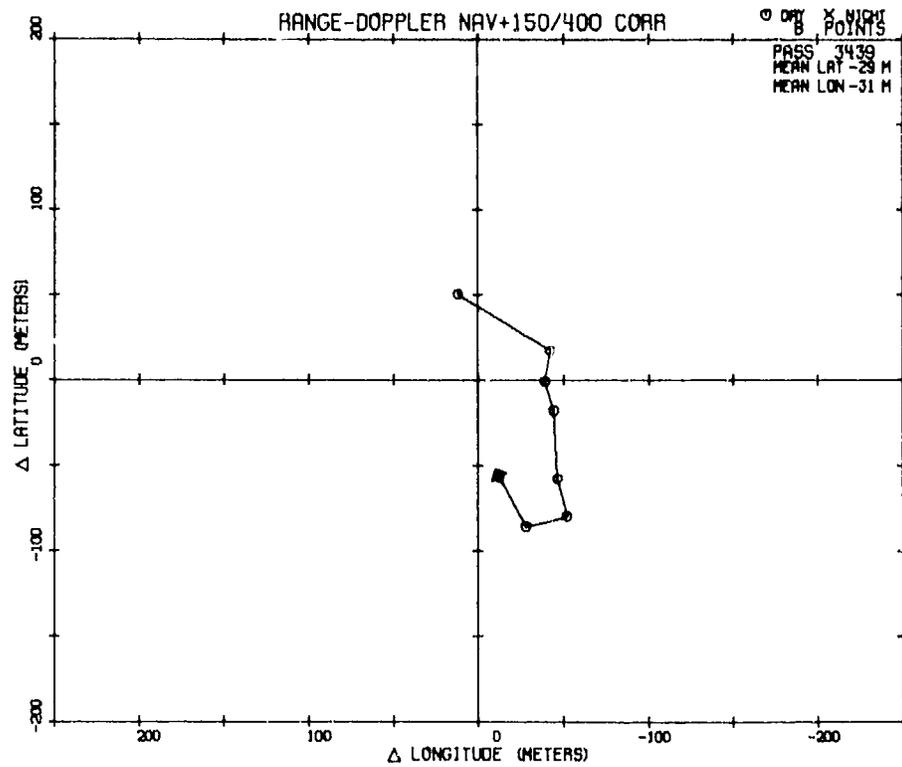
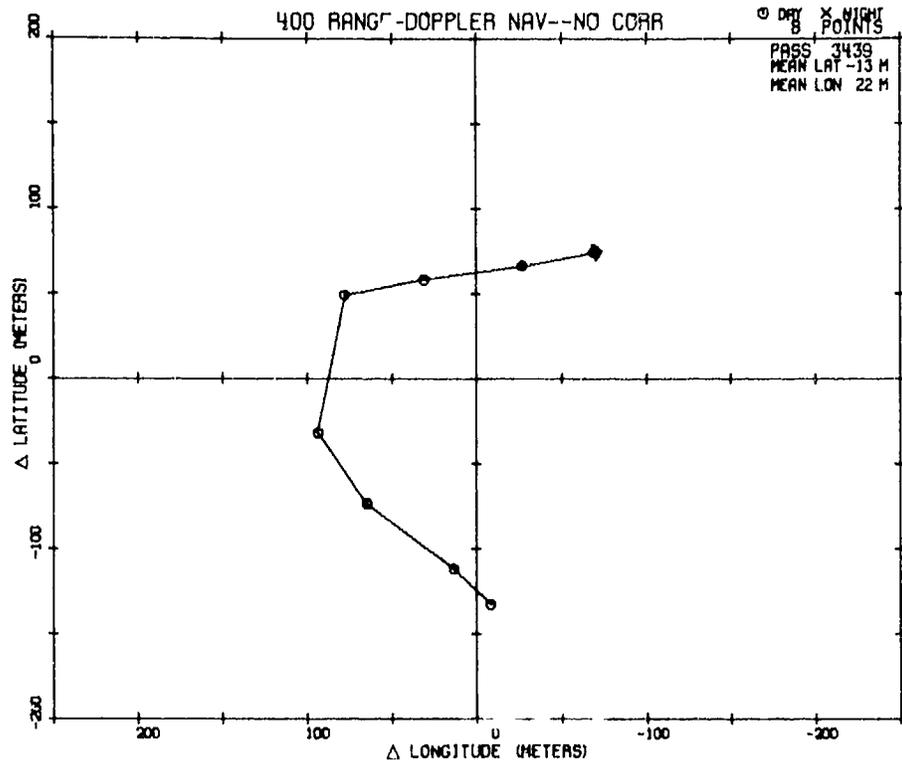
**RANGE-DOPPLER NAVIGATION FIX RESULTS USING 150- AND
400-MHz SATELLITE TRANSMITTER FREQUENCY TO CORRECT
FOR IONOSPHERIC REFRACTION (LOWER CHARTS)**

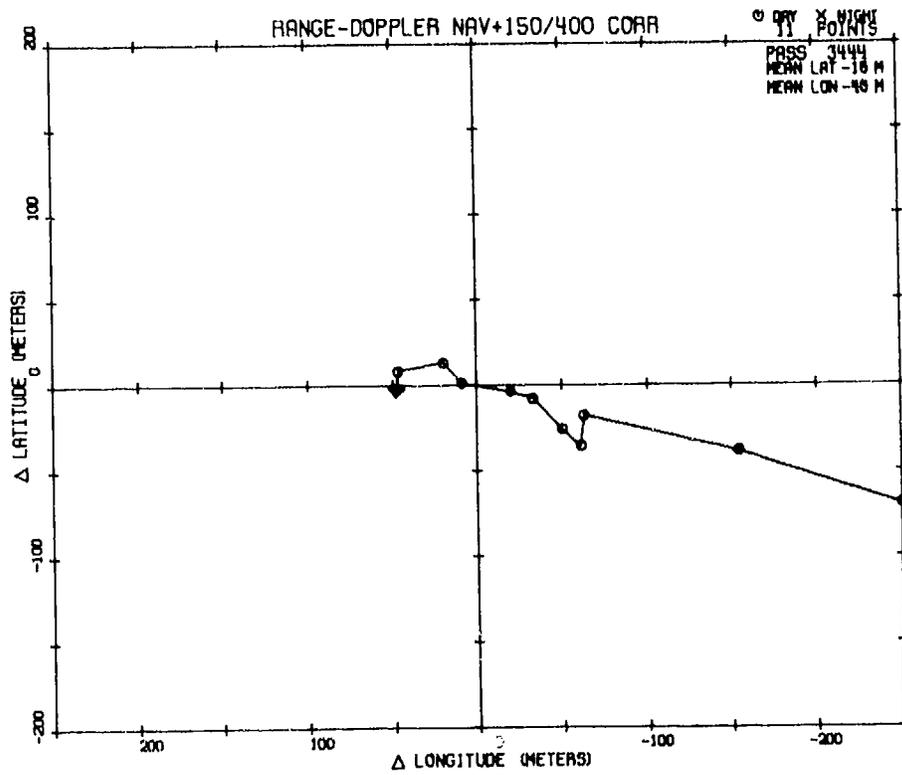
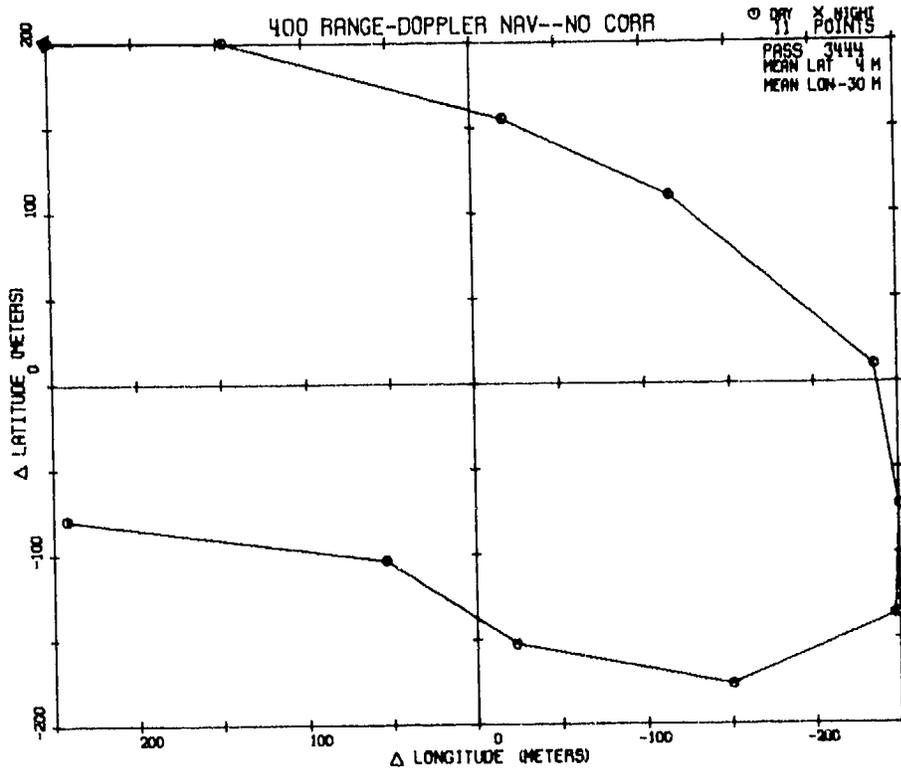


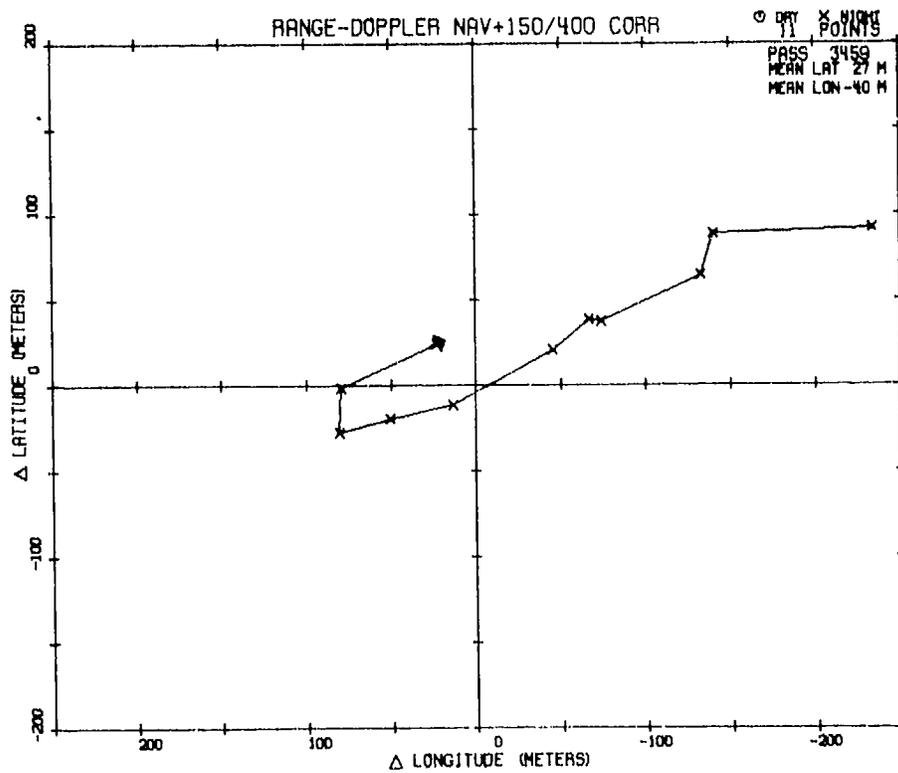
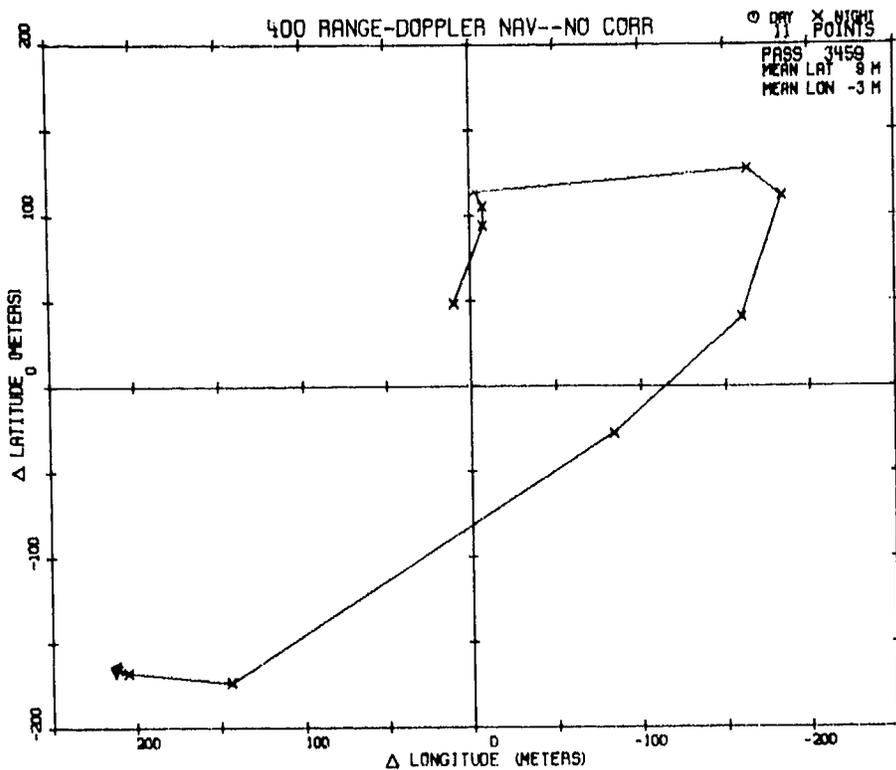




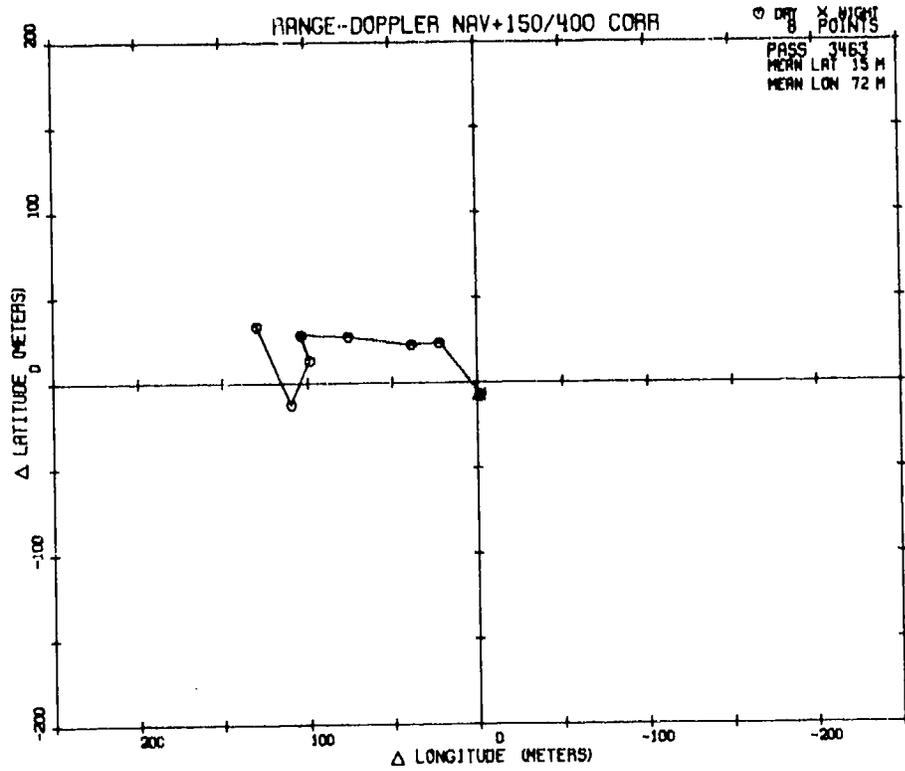
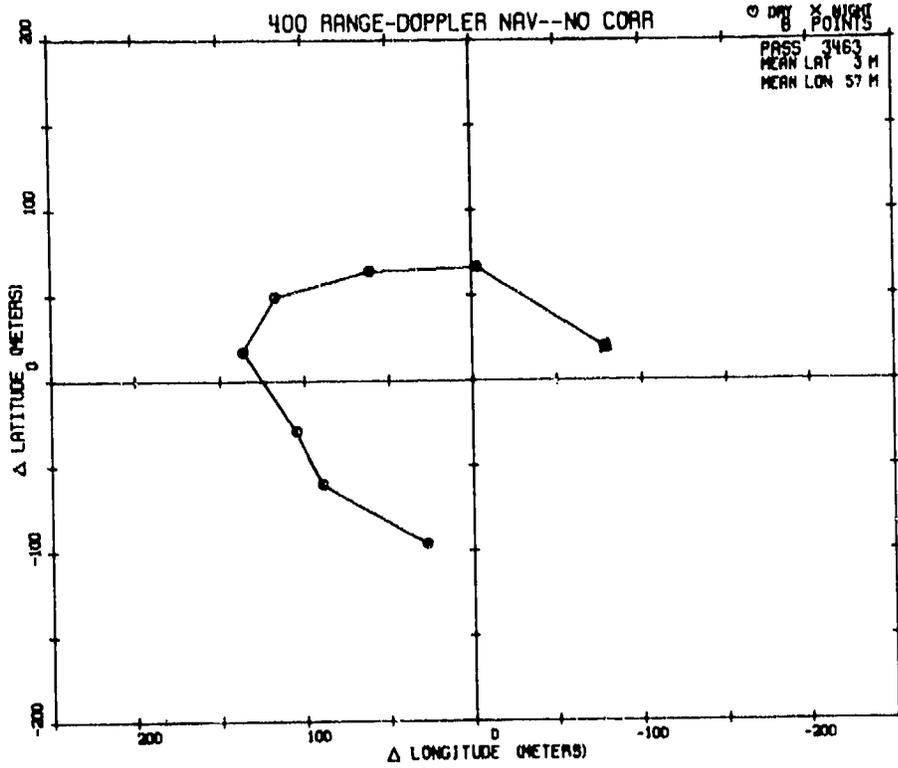


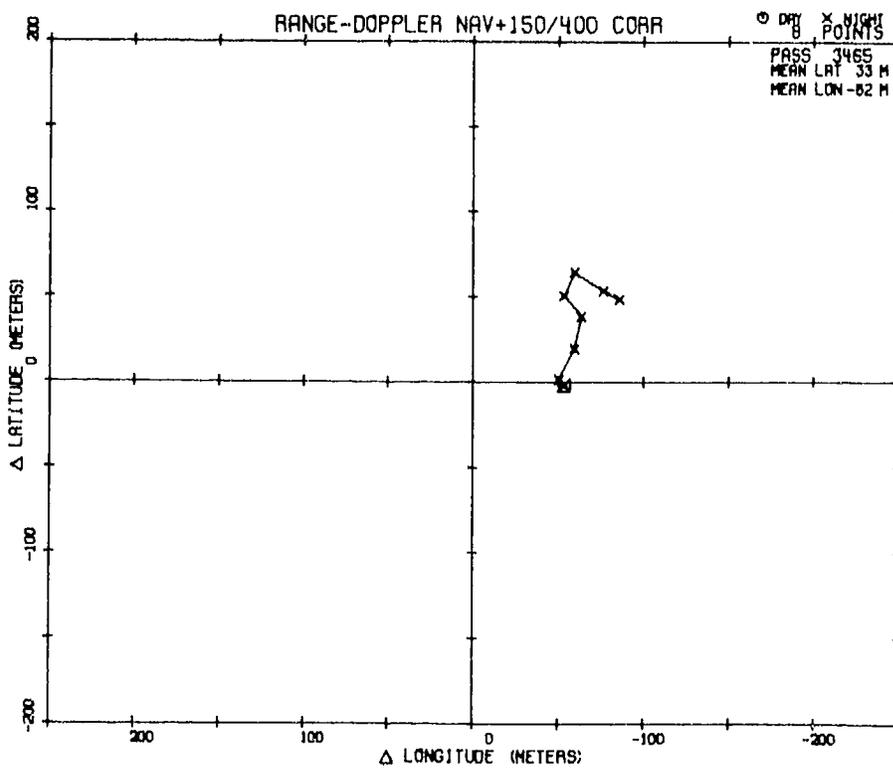
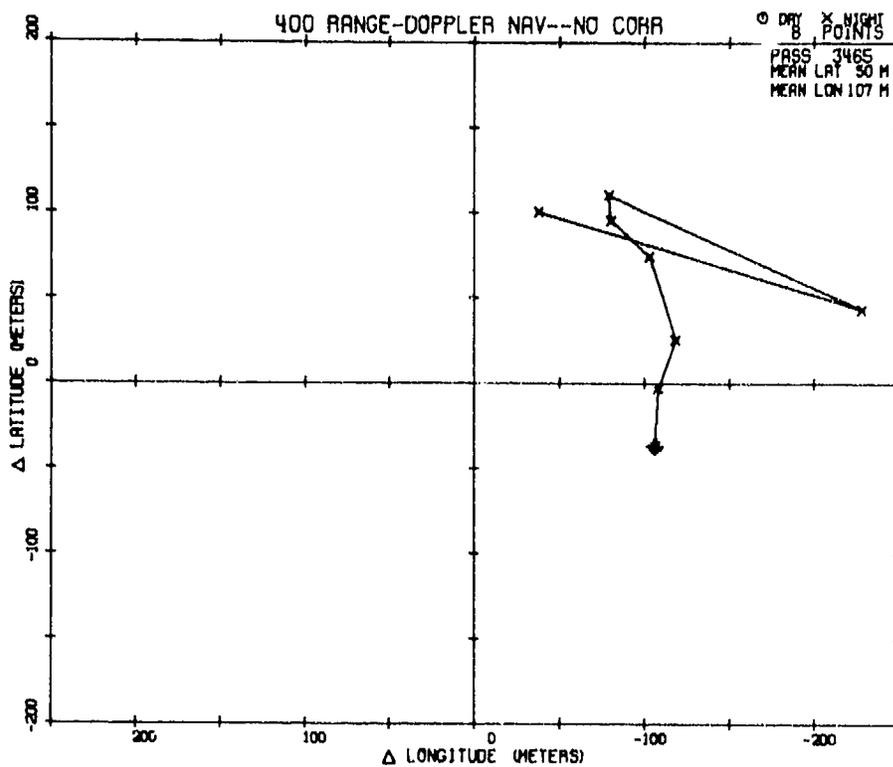




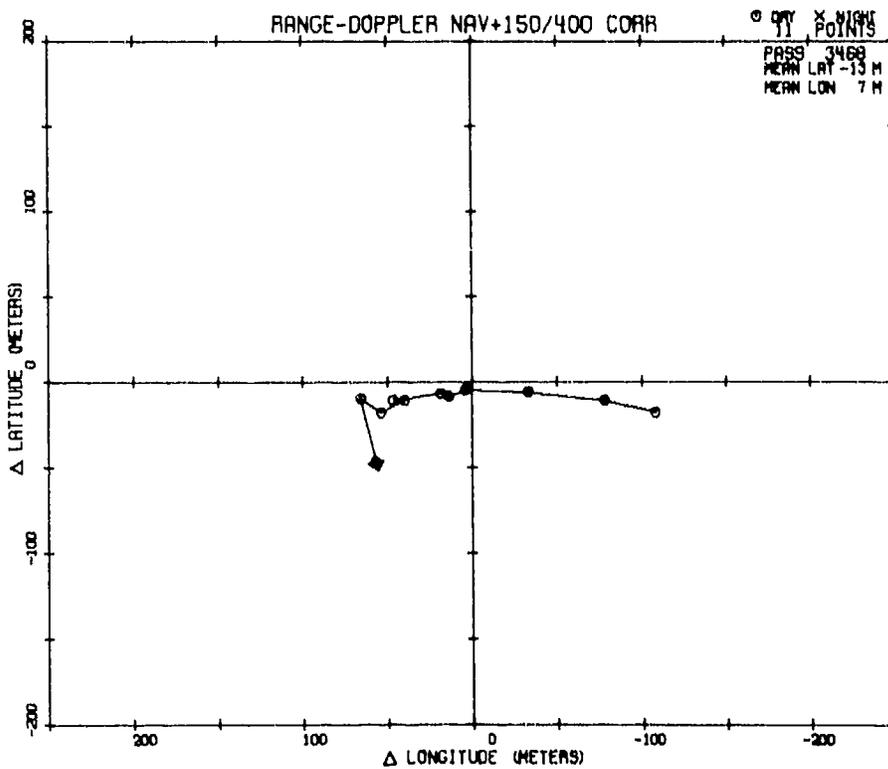
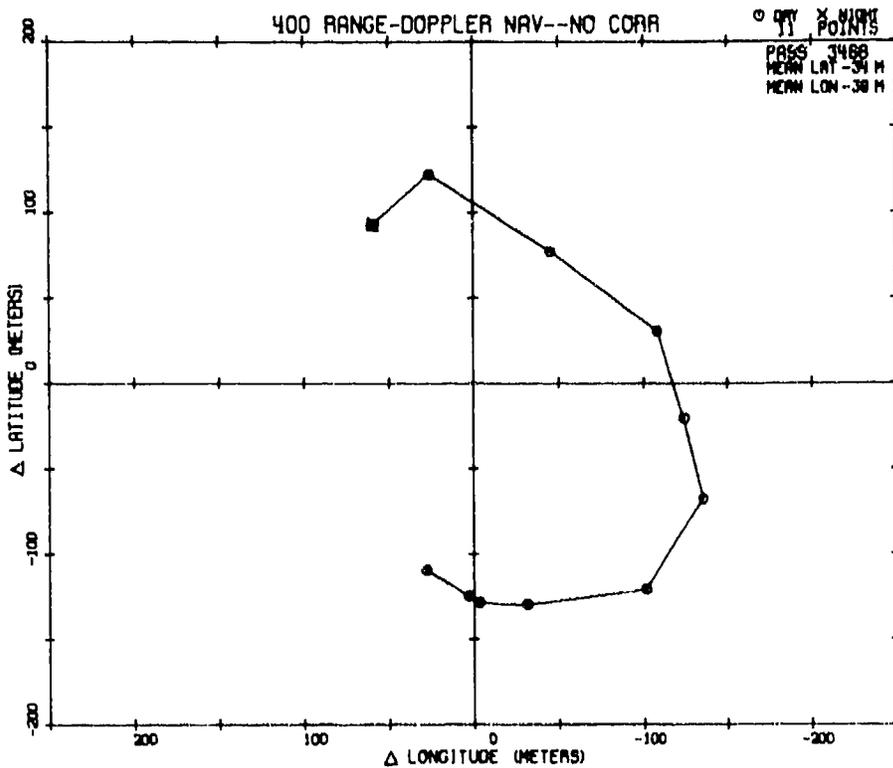


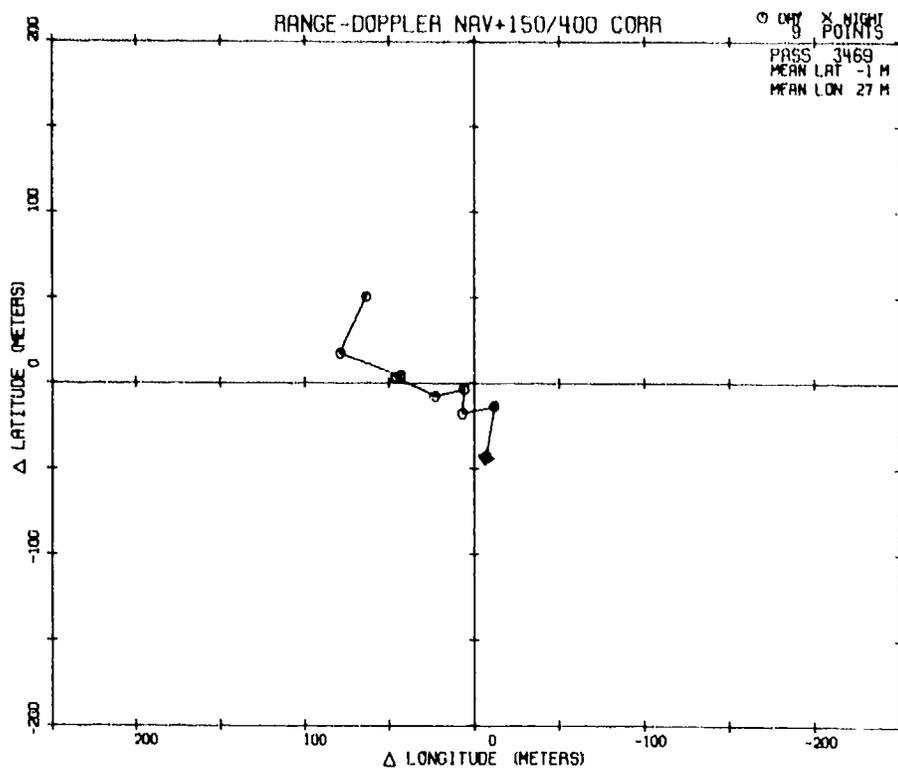
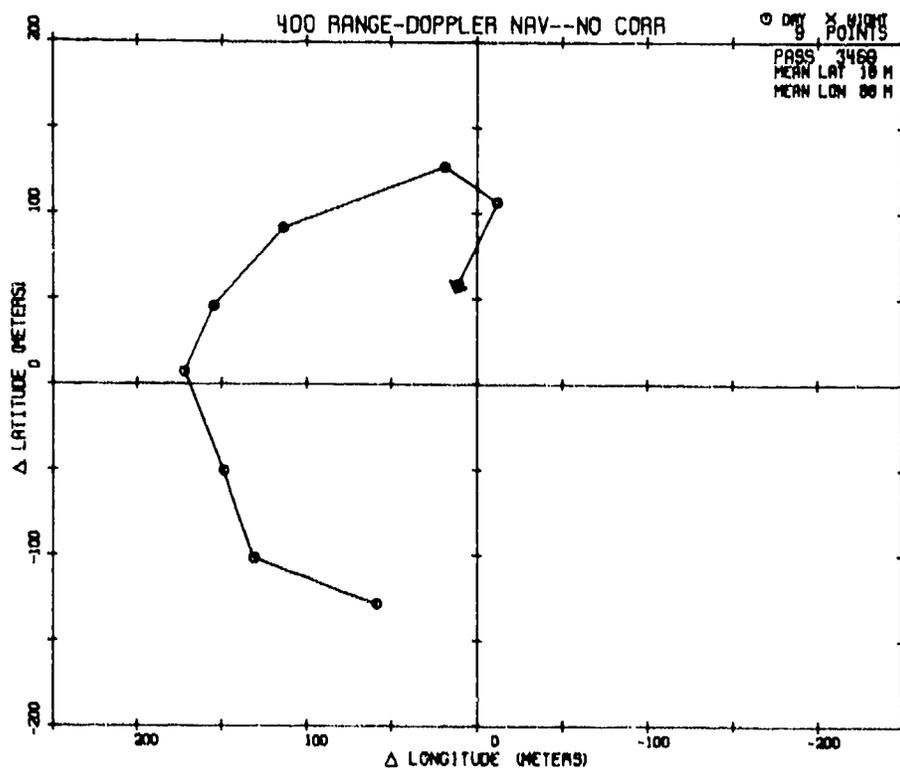
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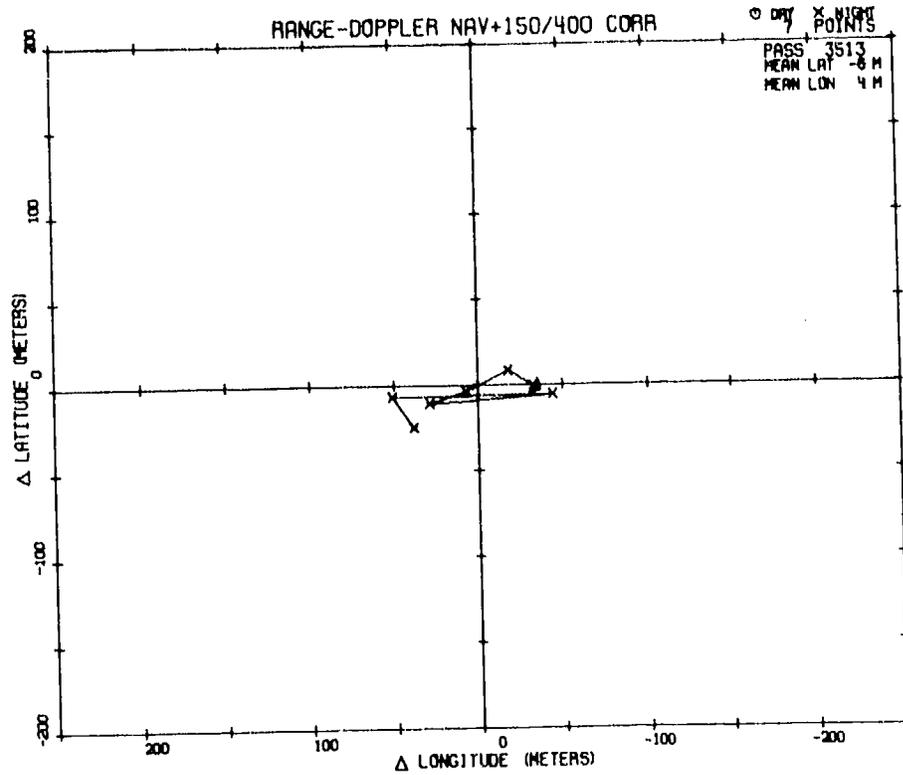
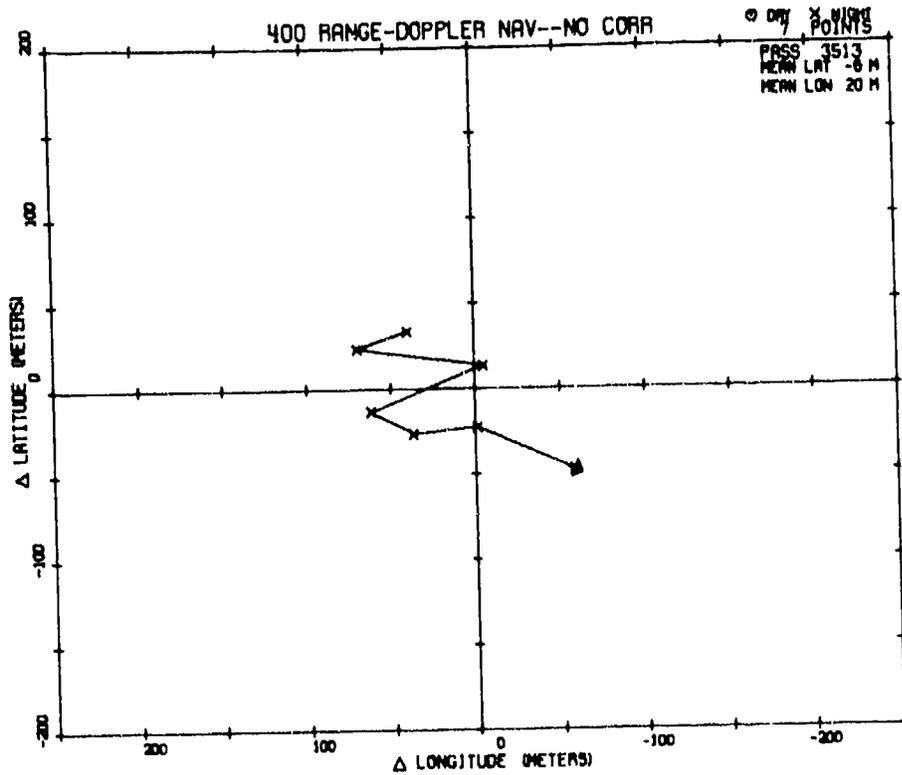


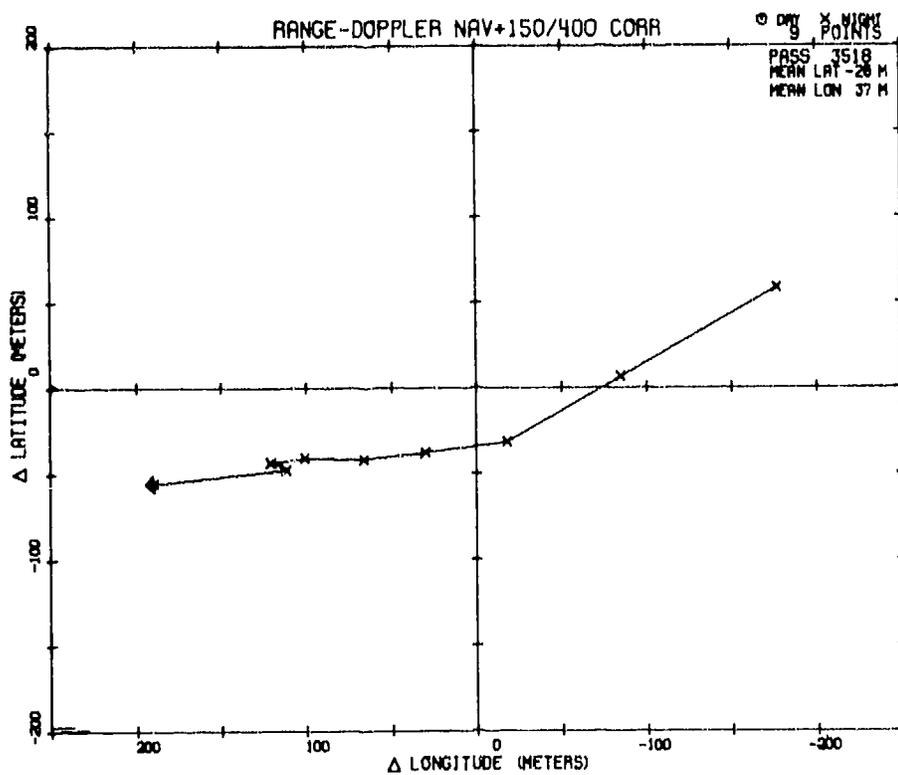
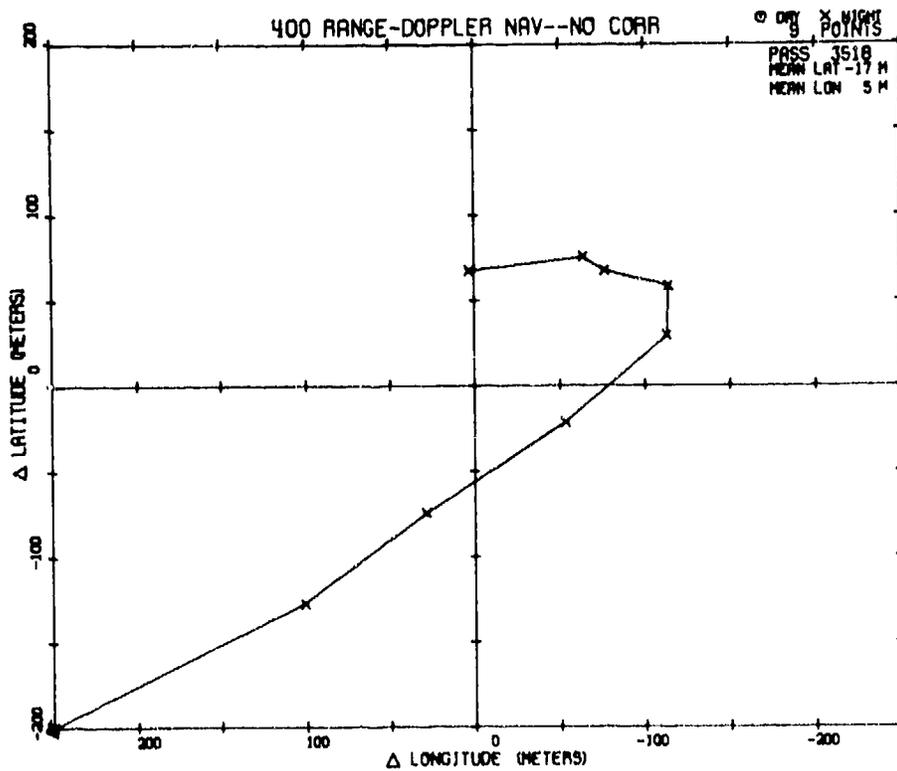
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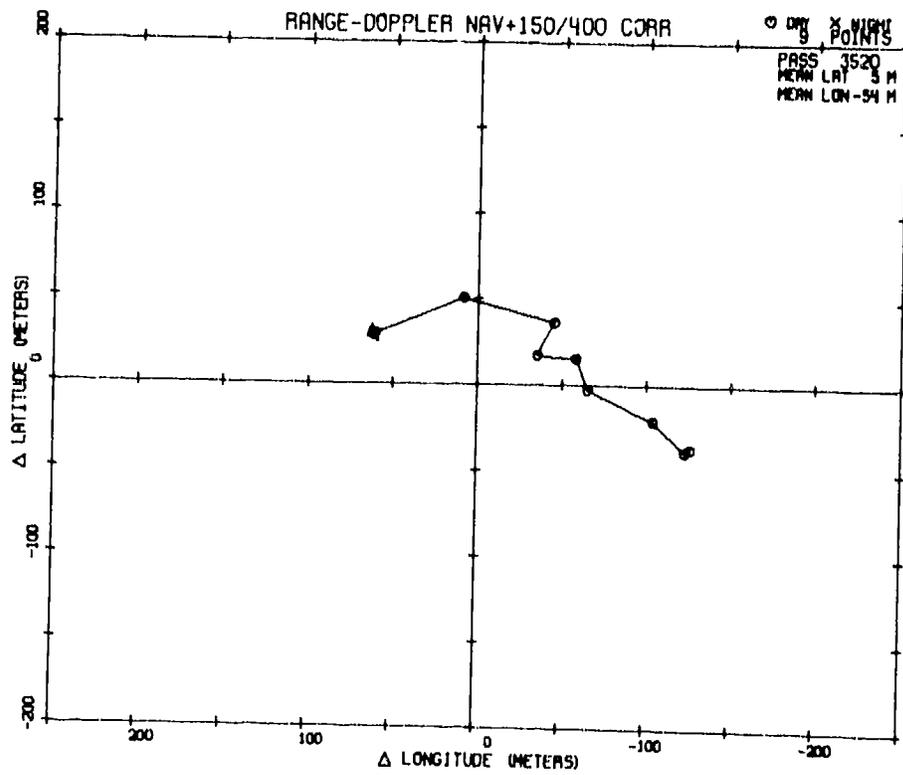
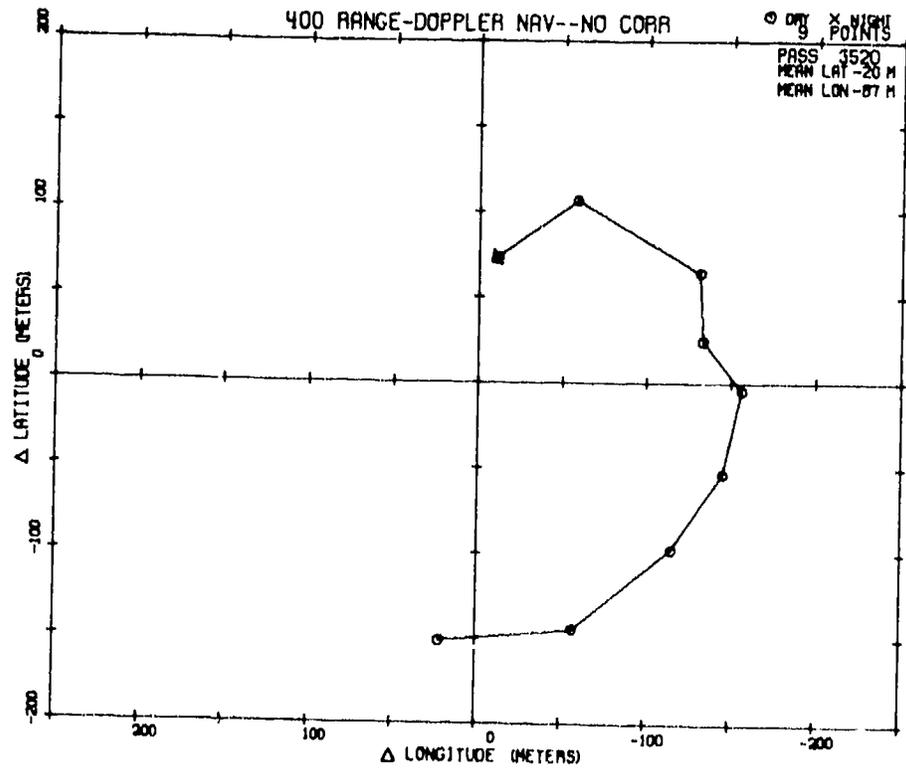


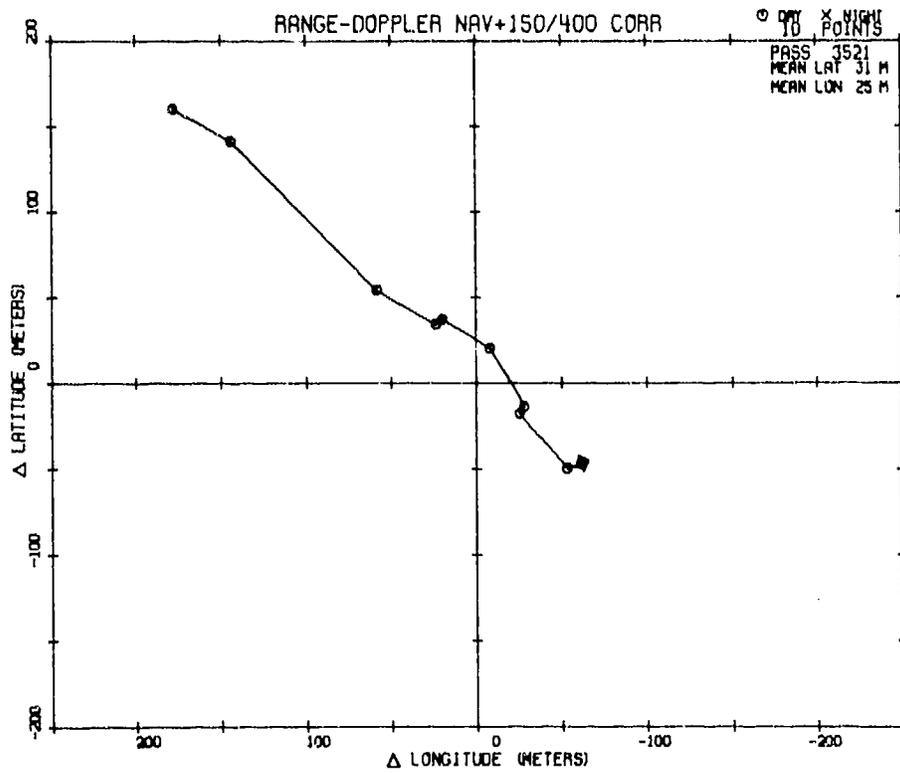
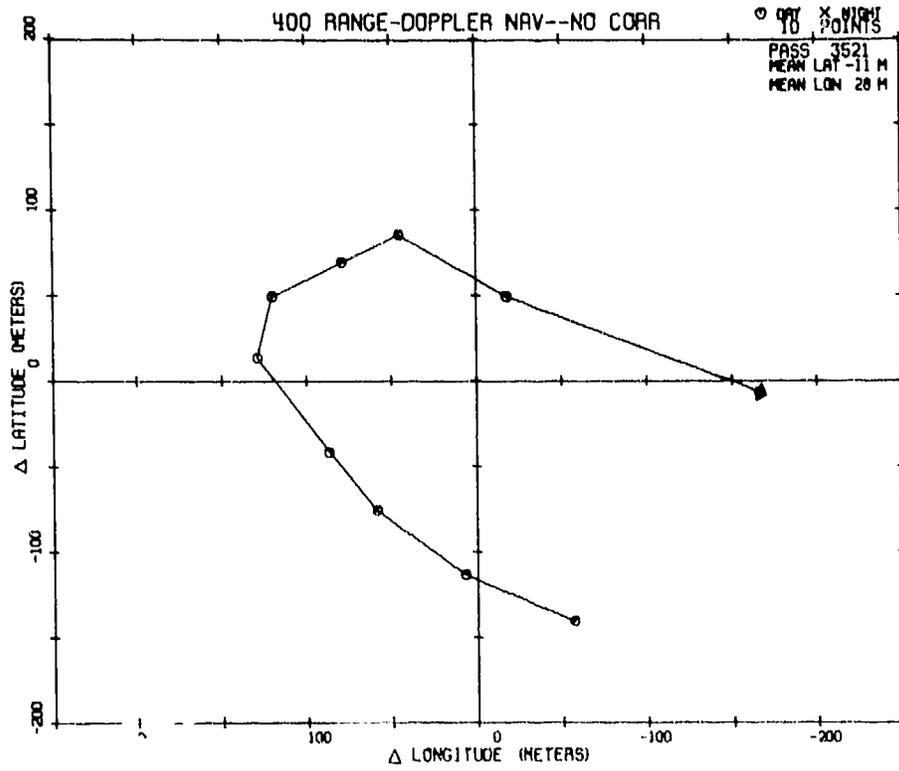
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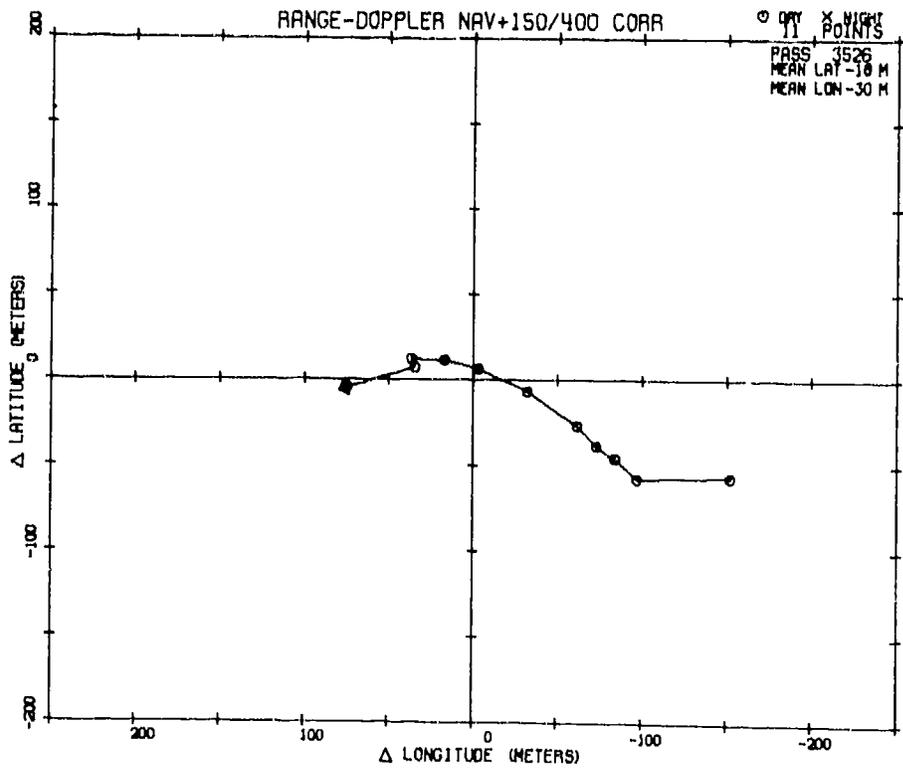
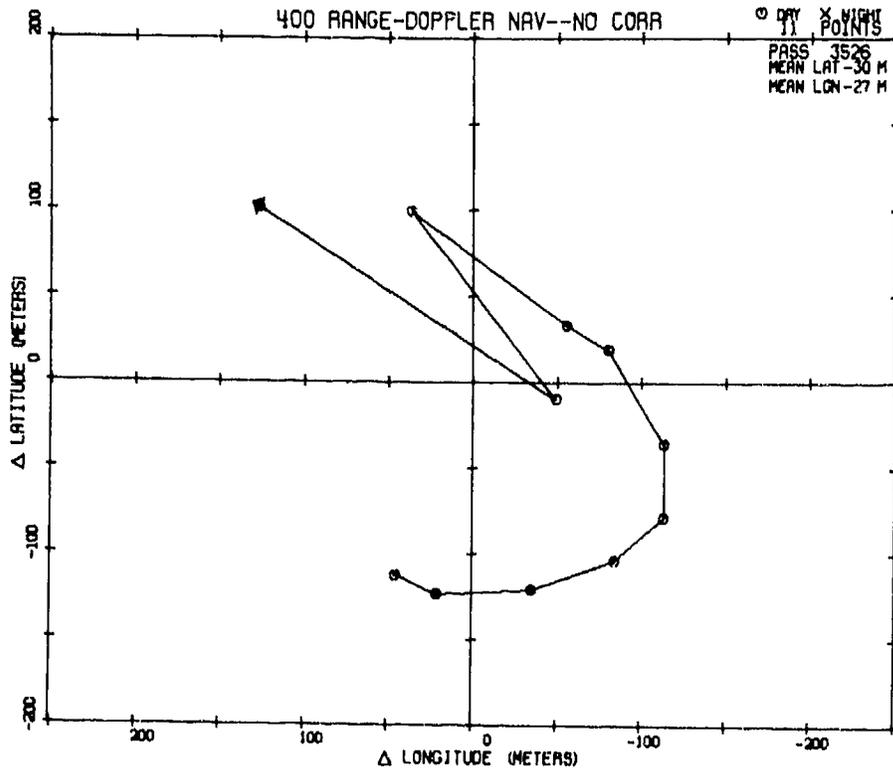


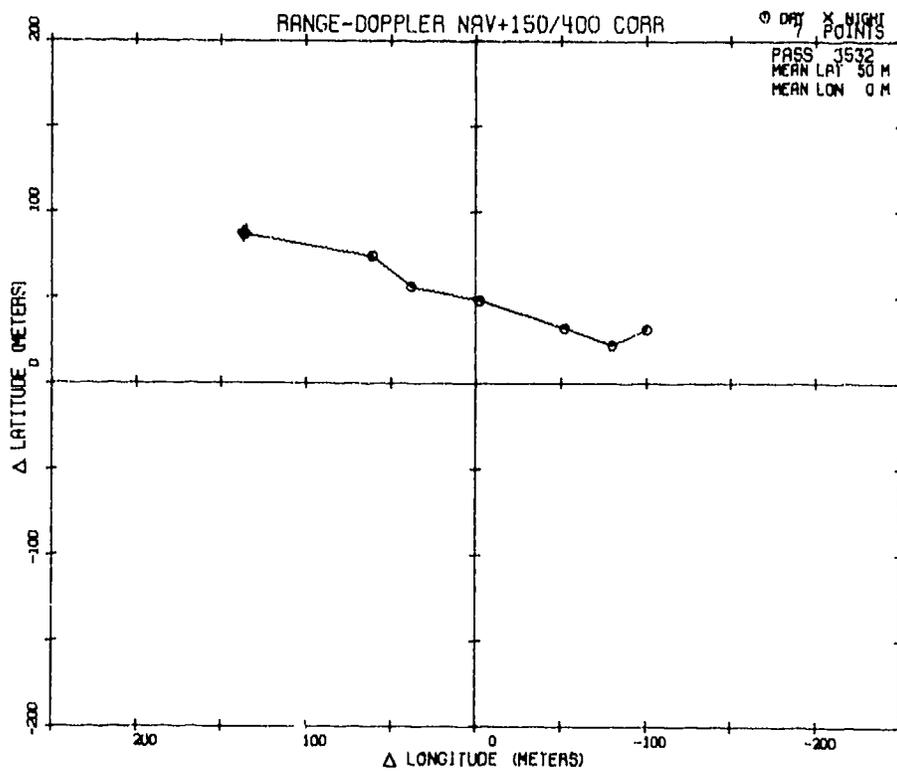
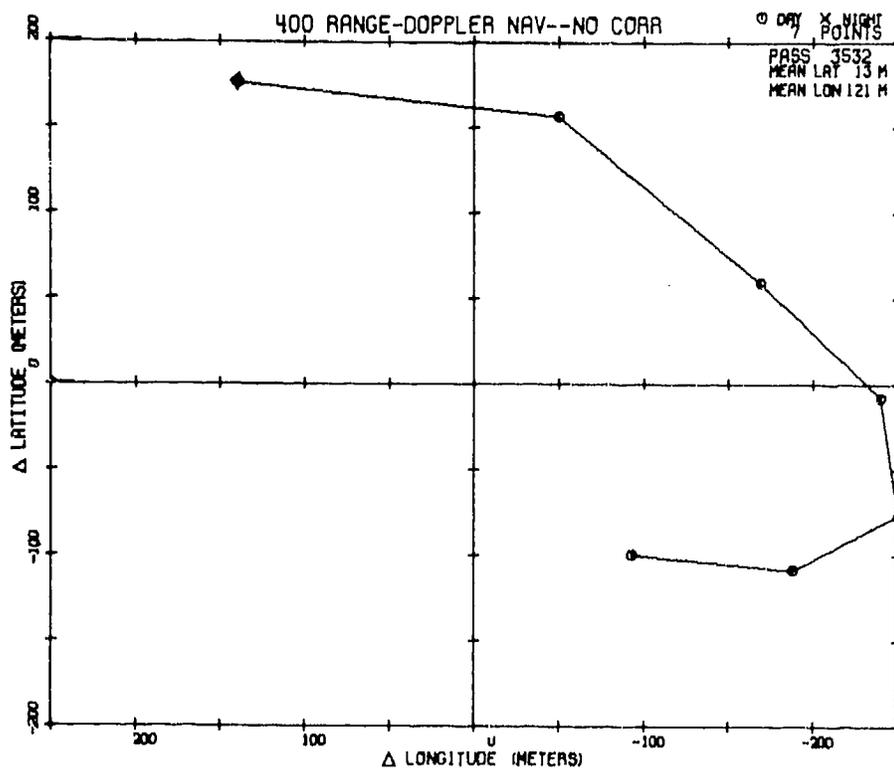
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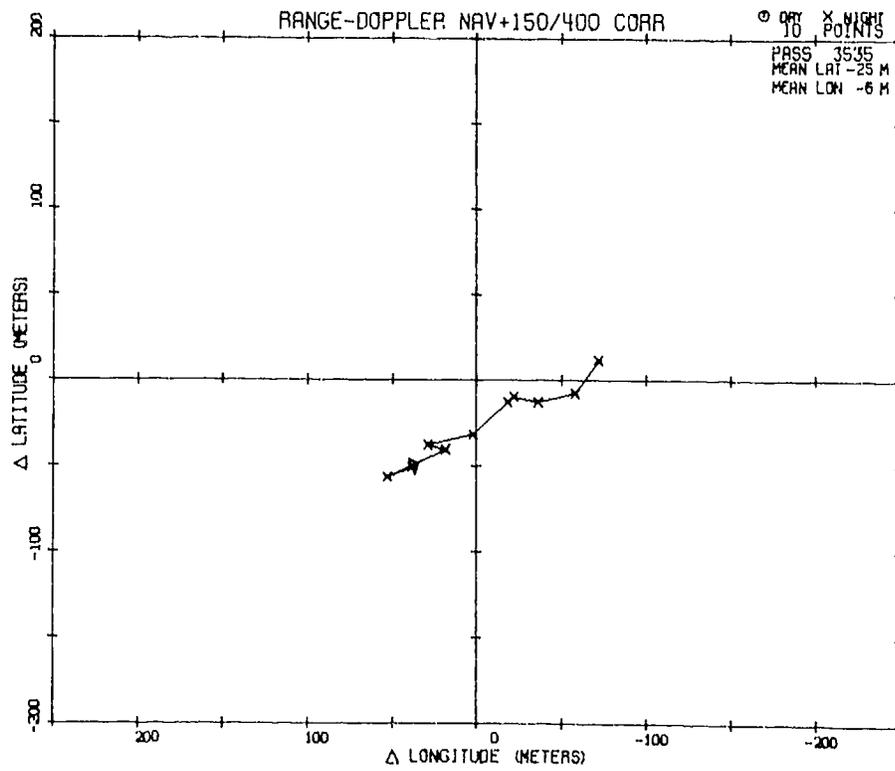
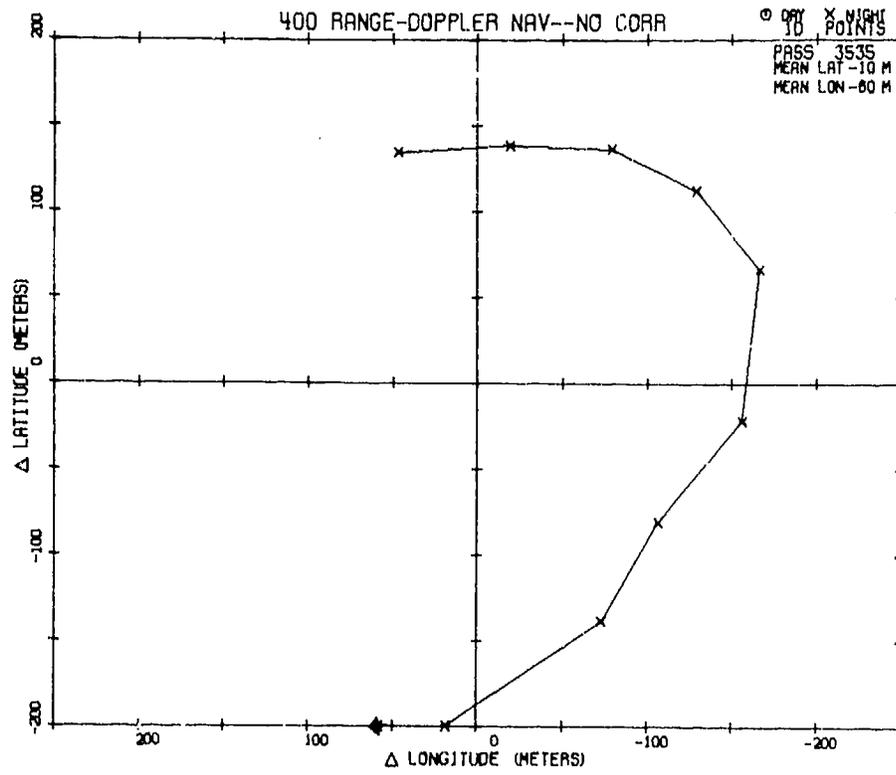


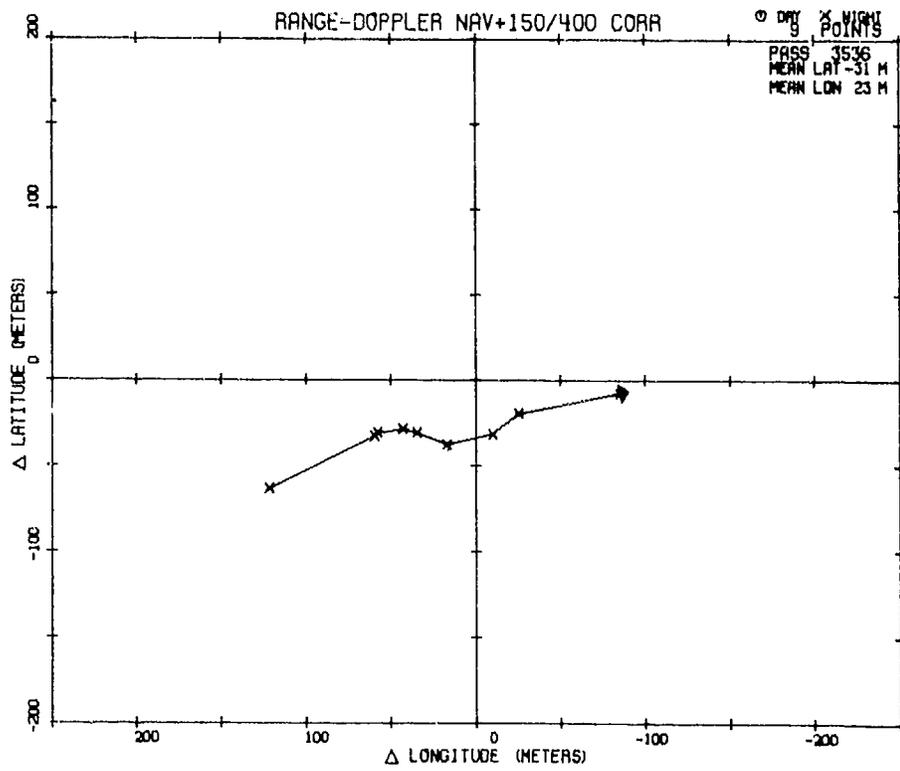
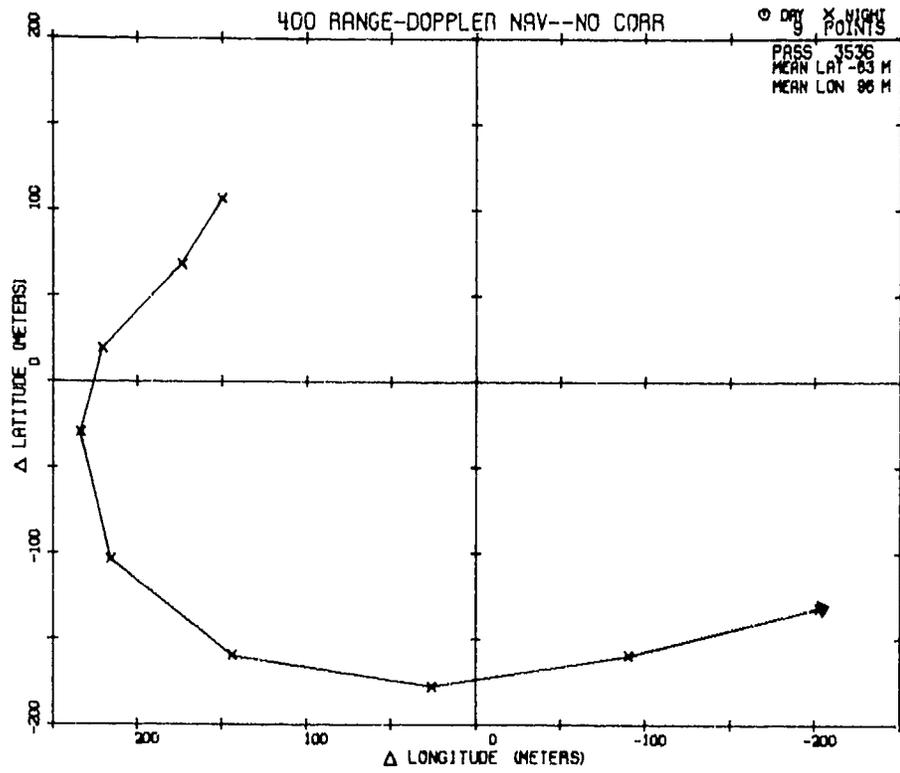
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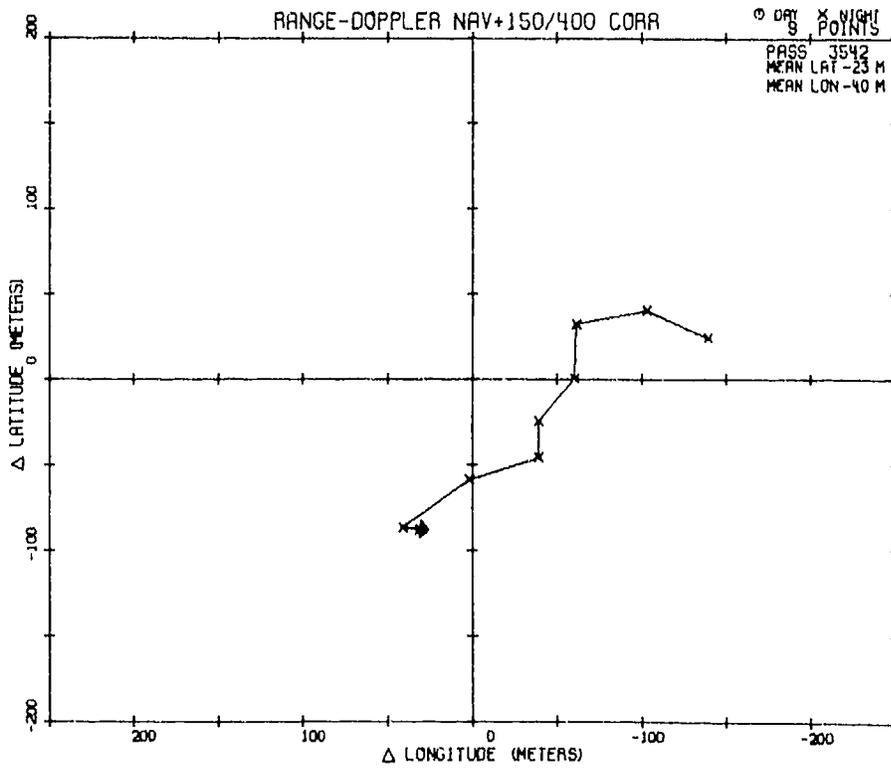
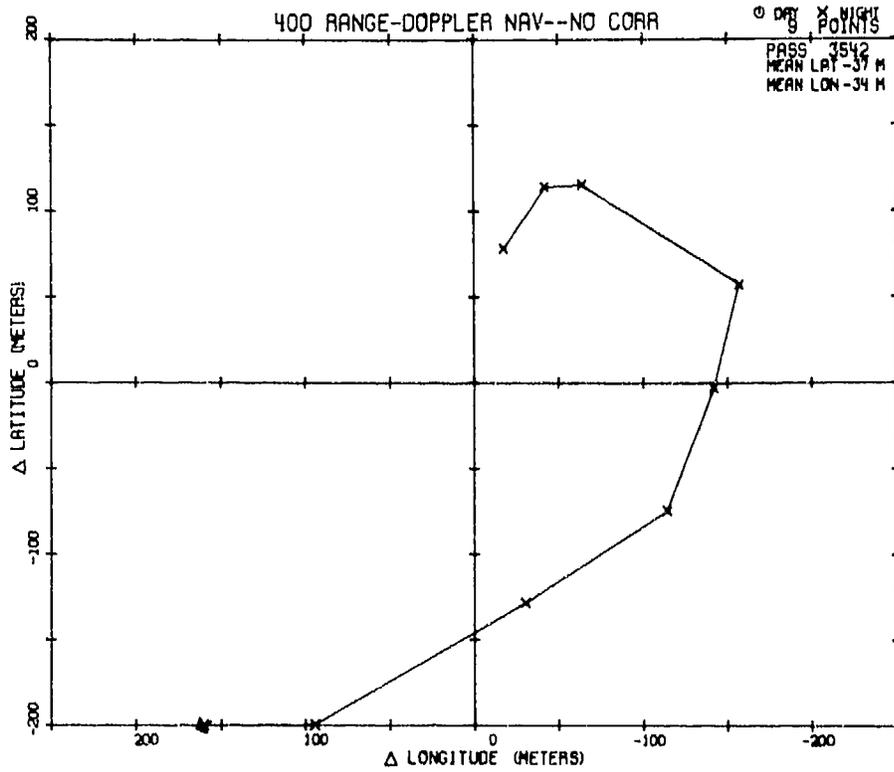


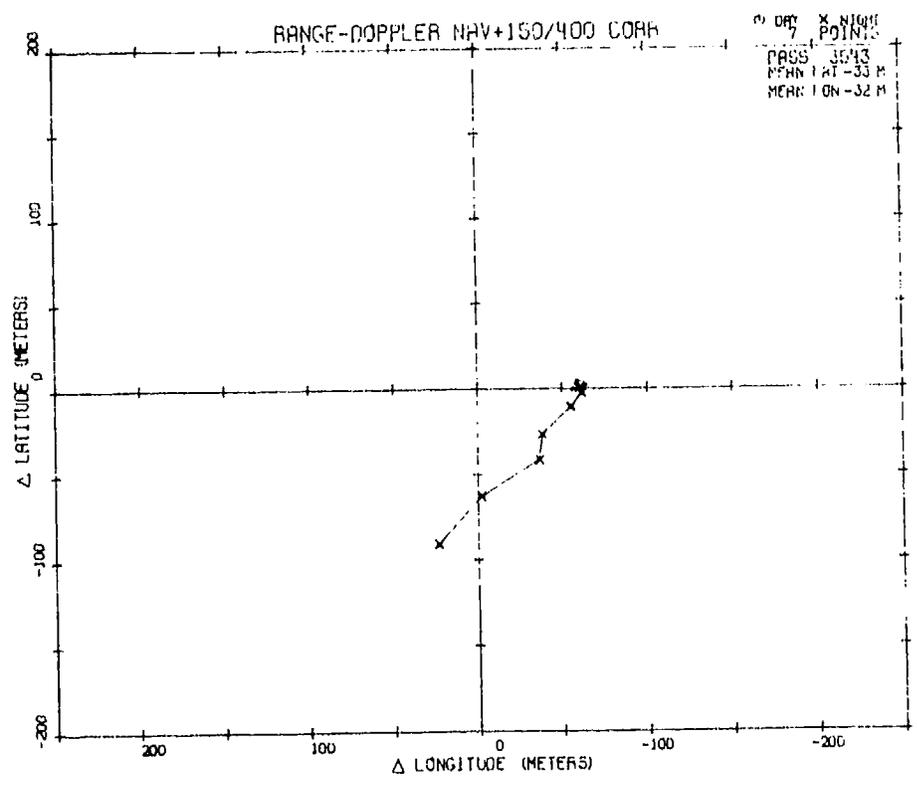
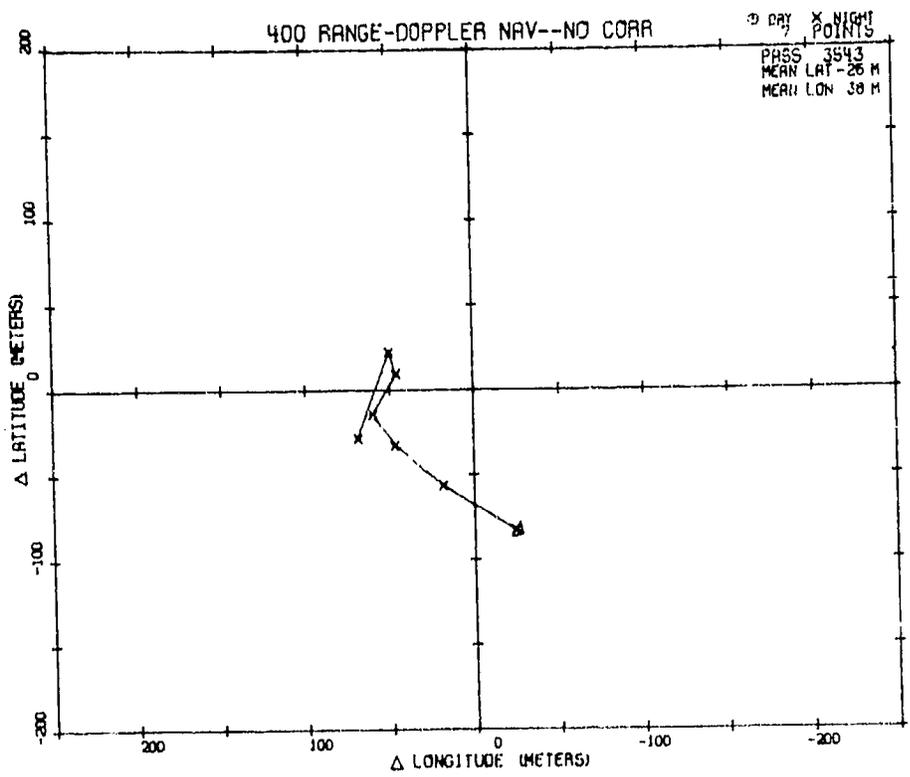
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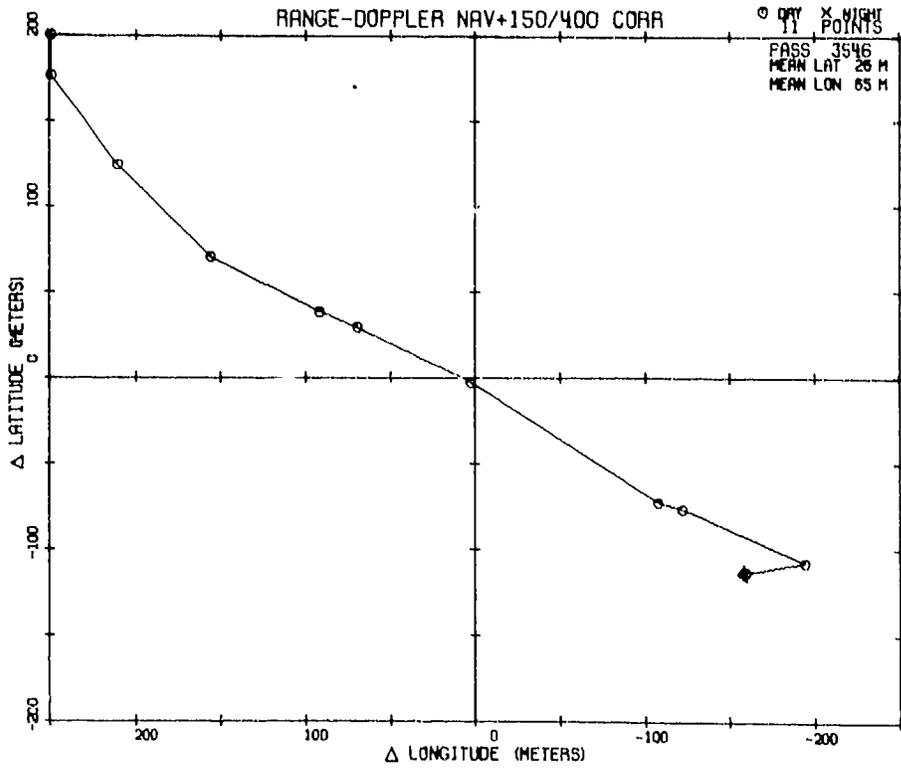
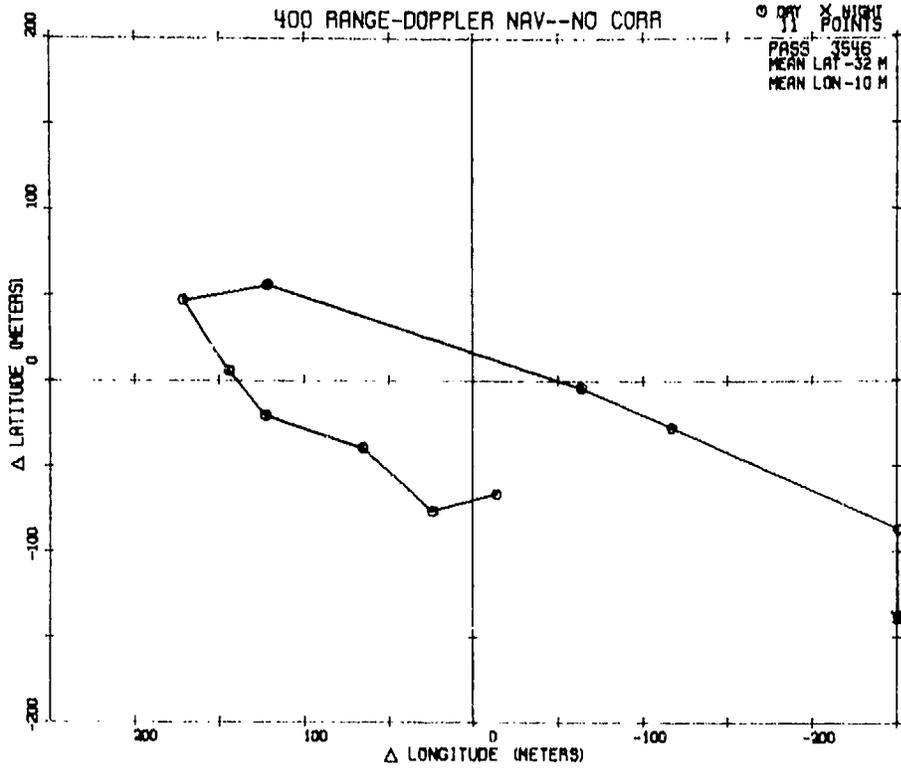


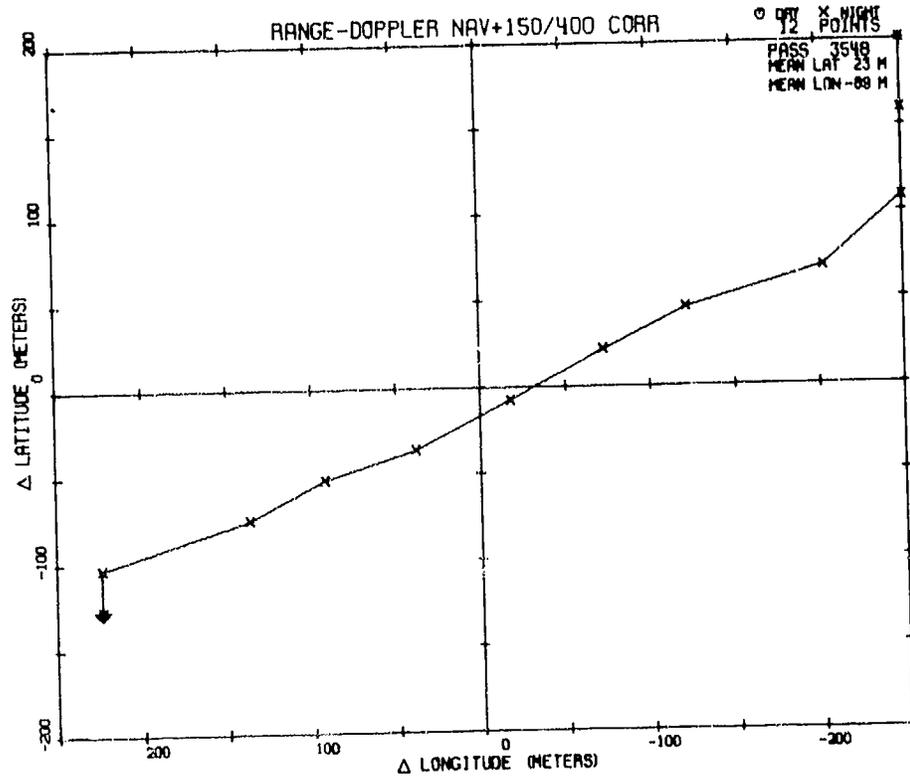
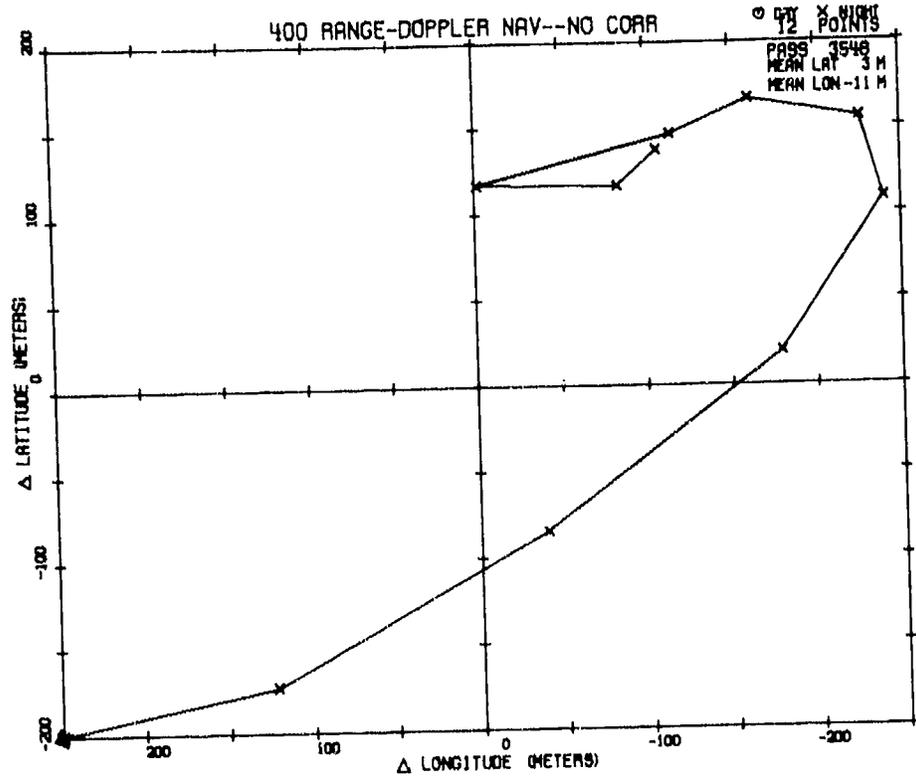


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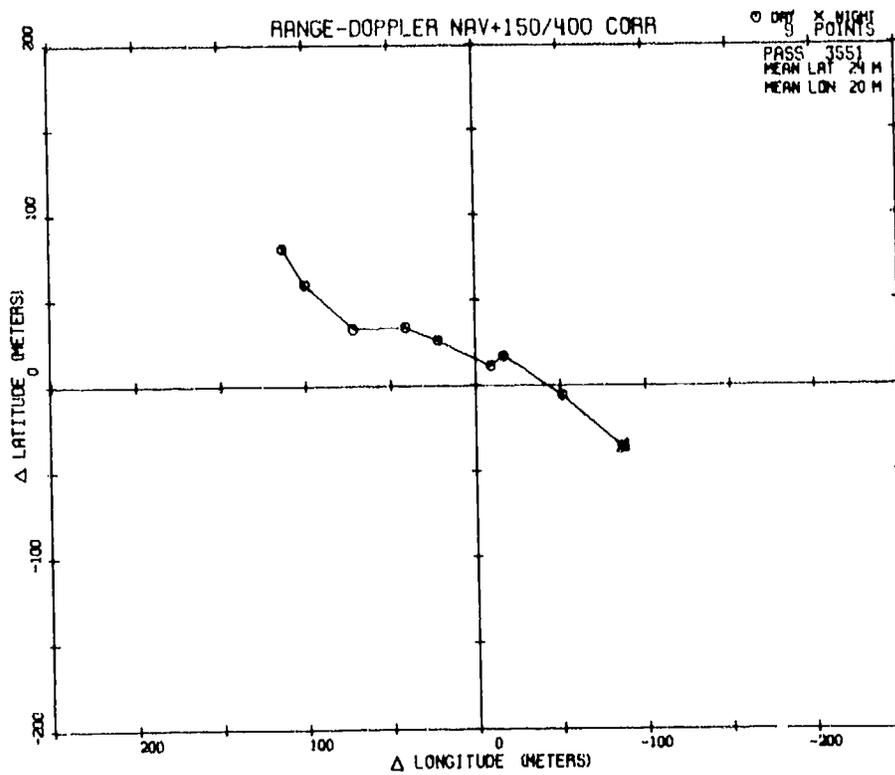
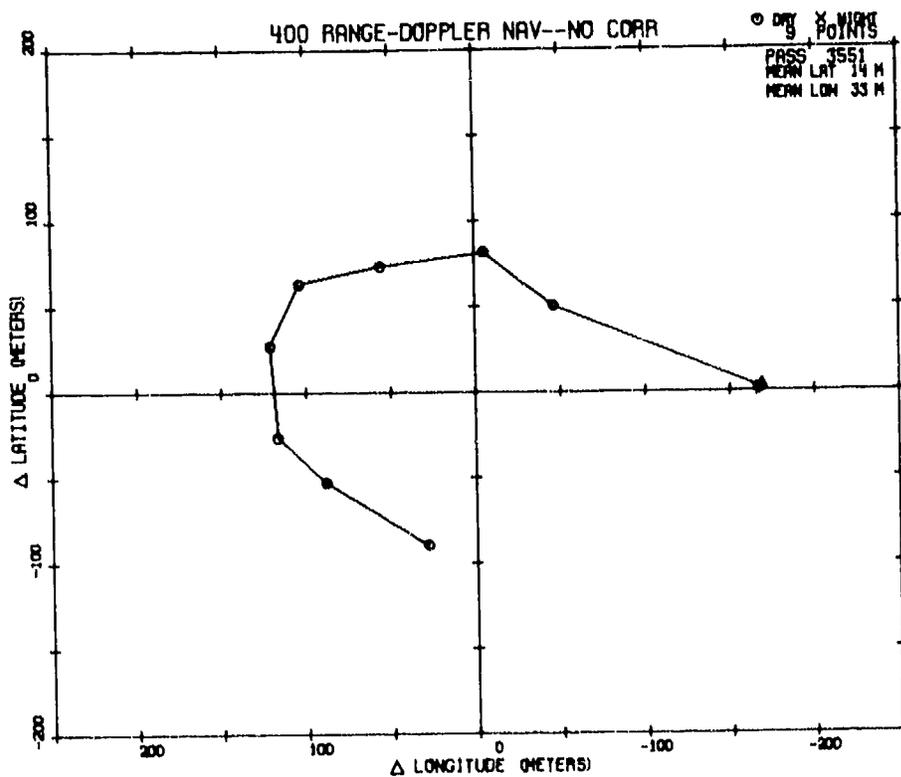


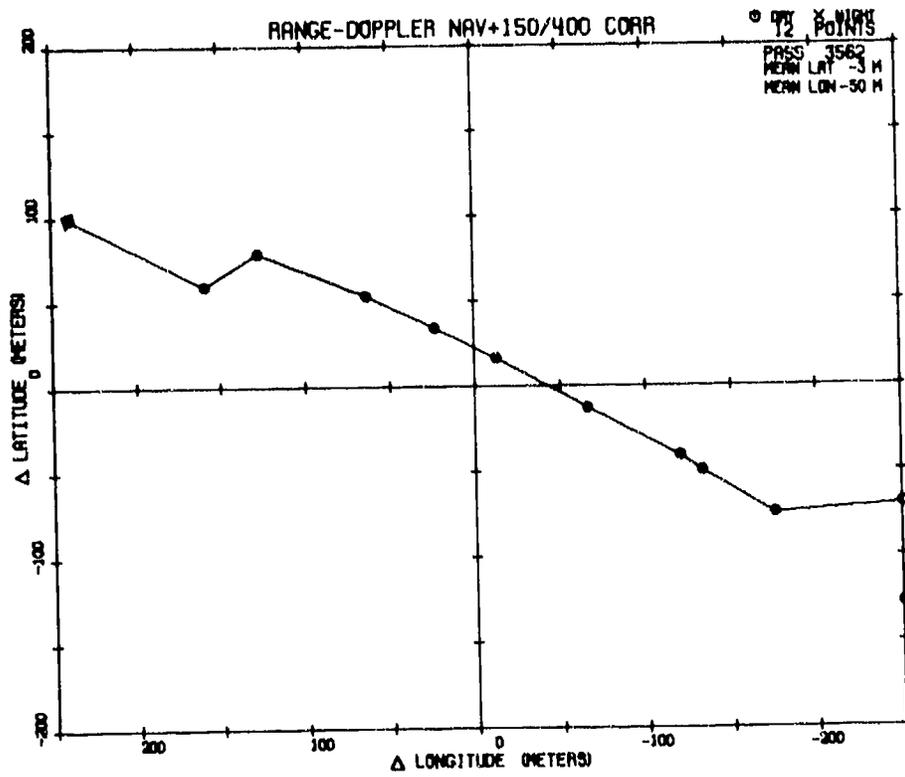
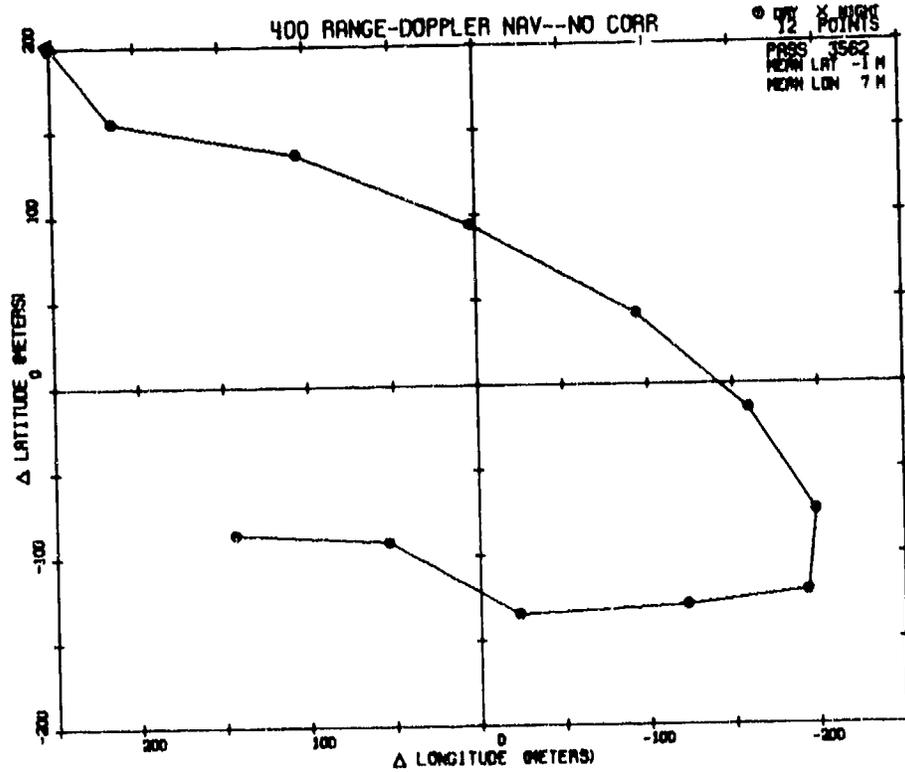




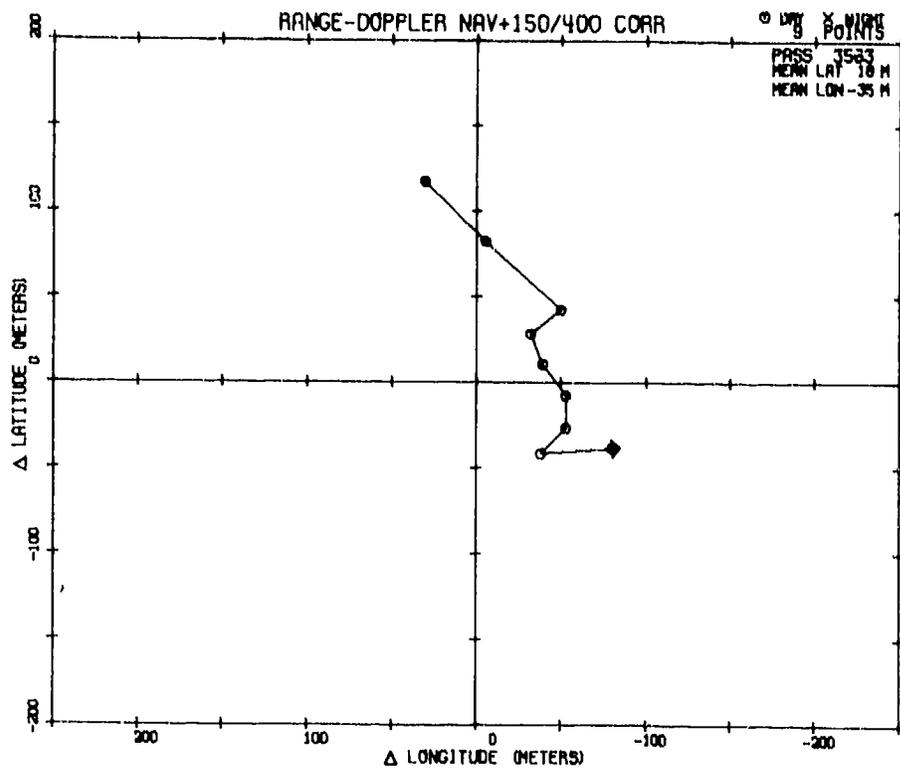
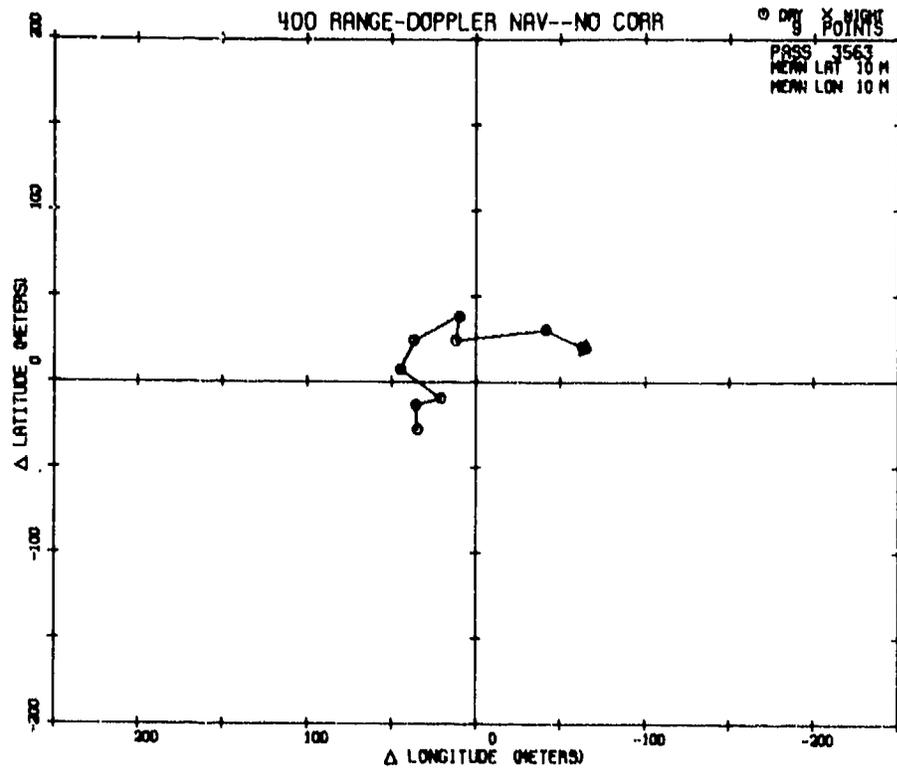


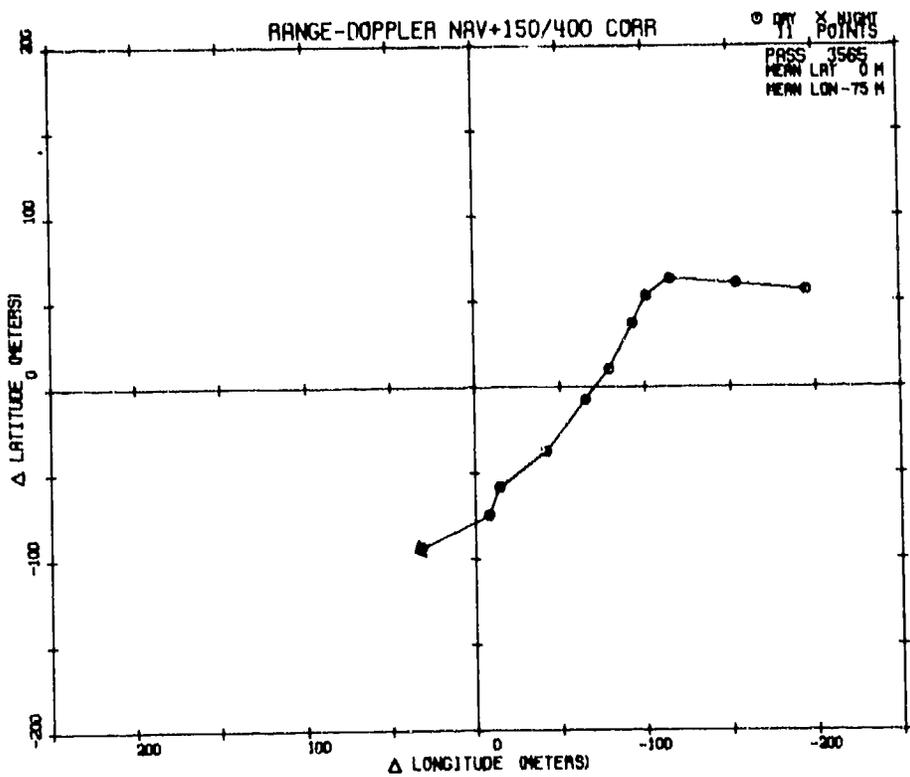
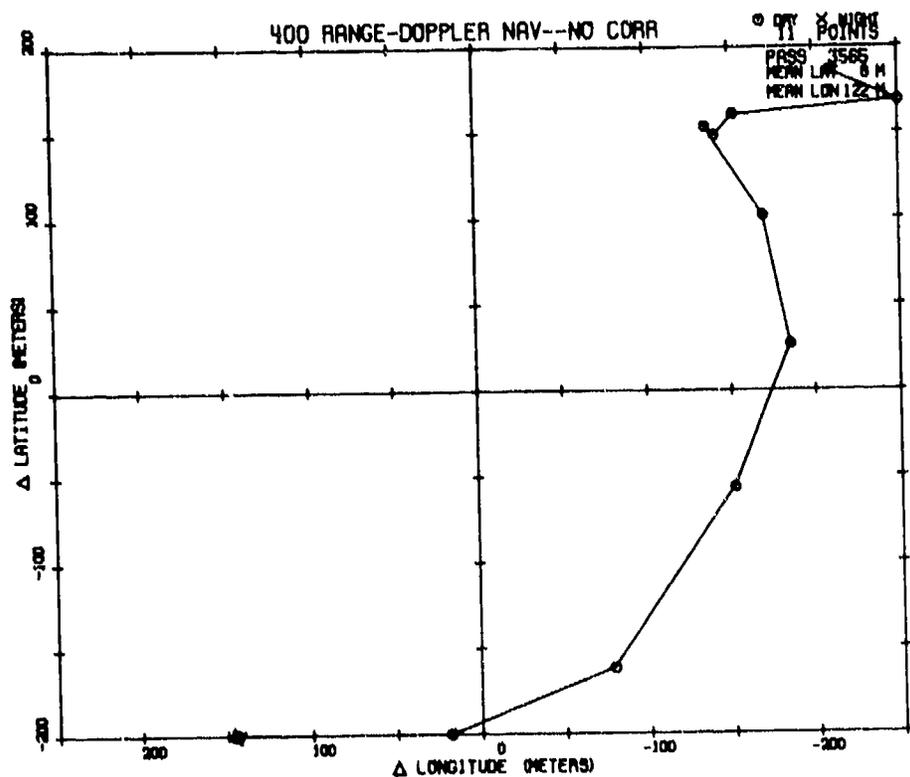
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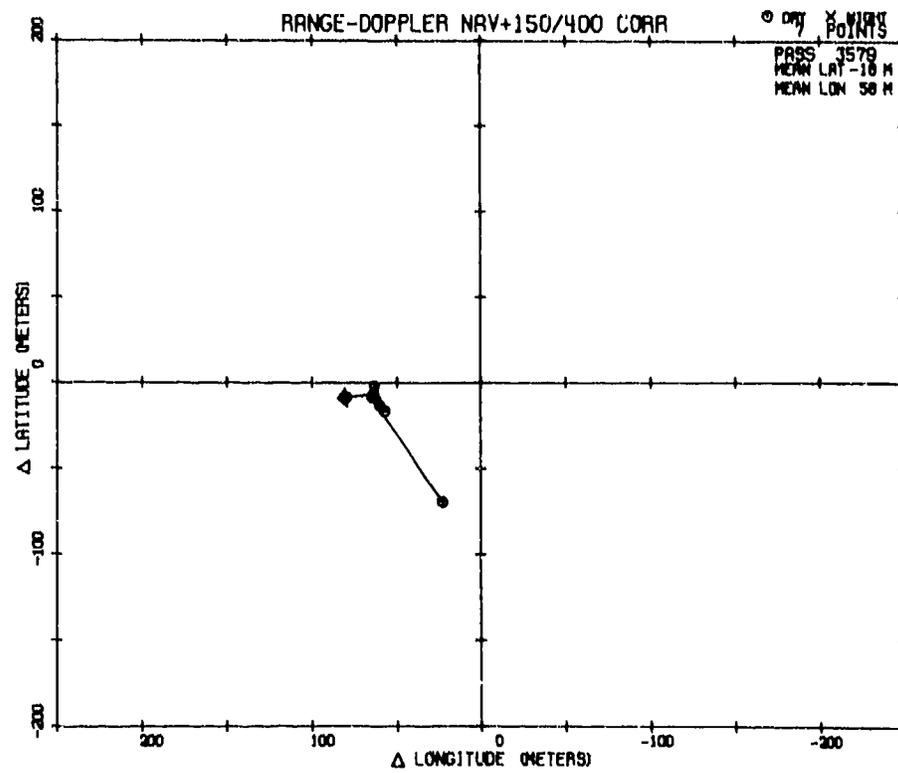
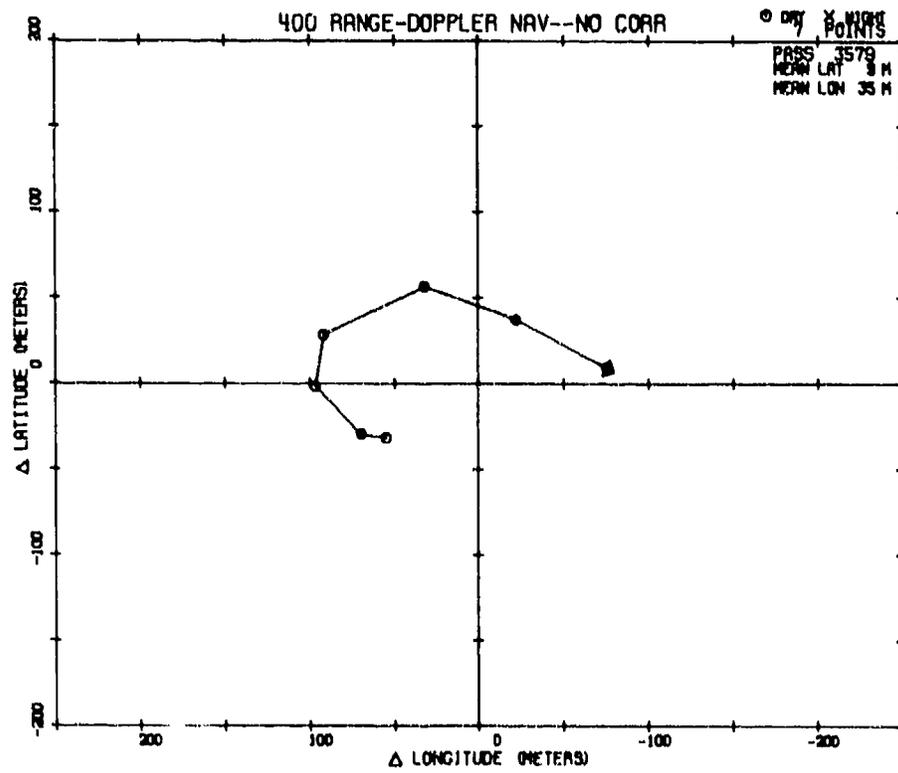


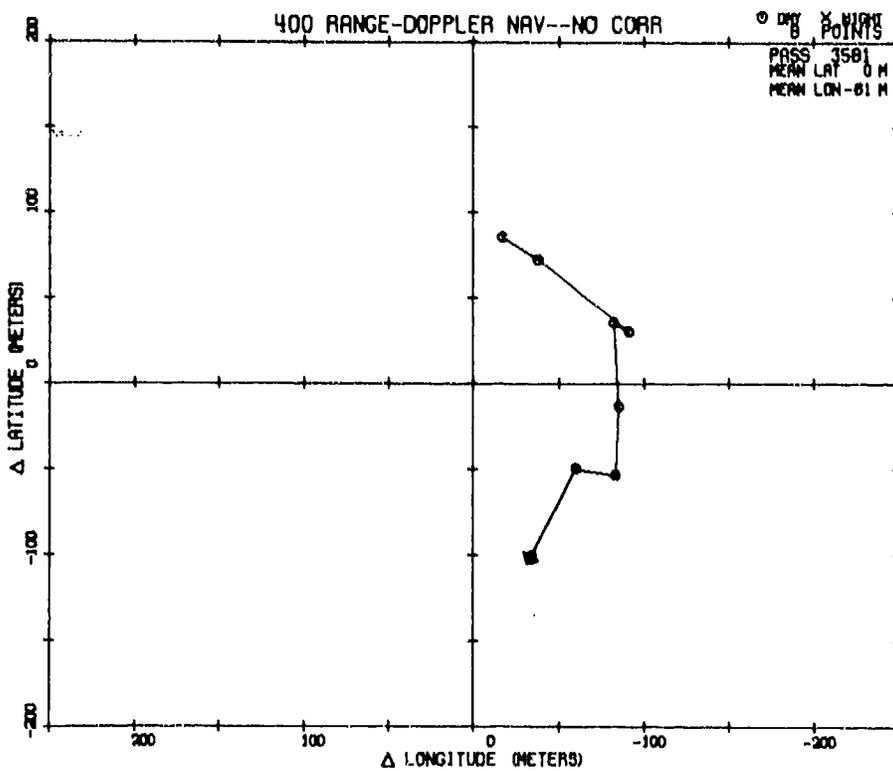
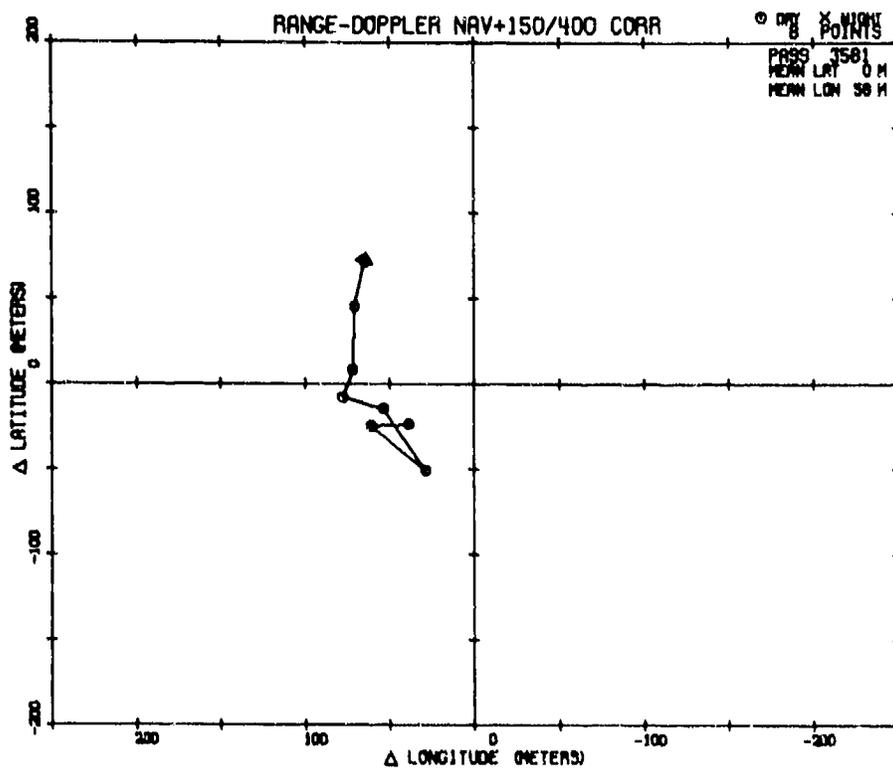


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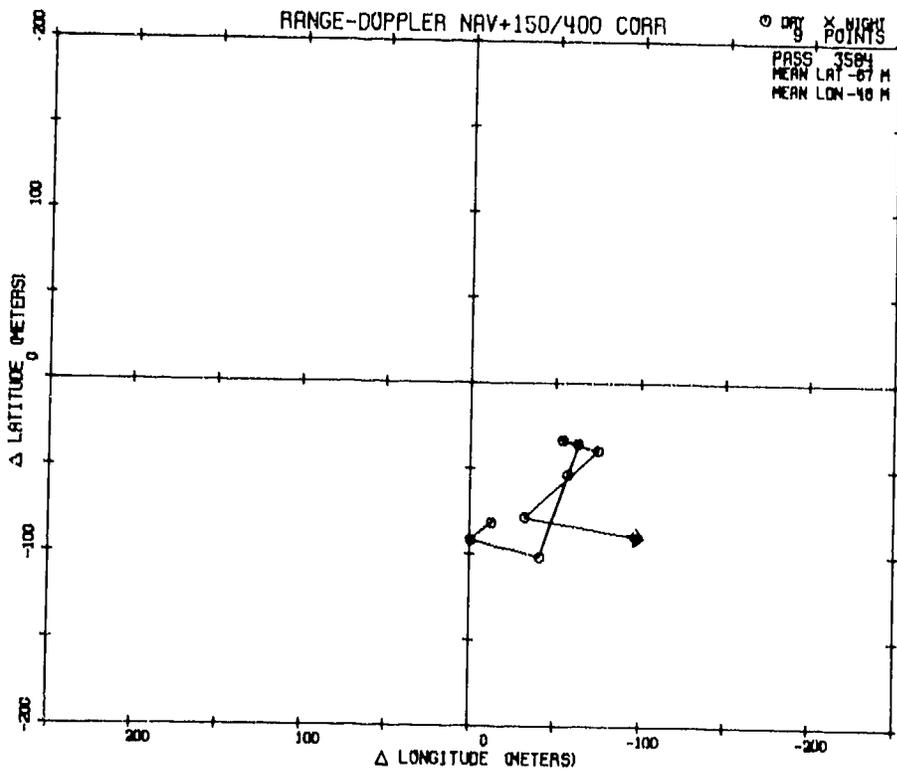
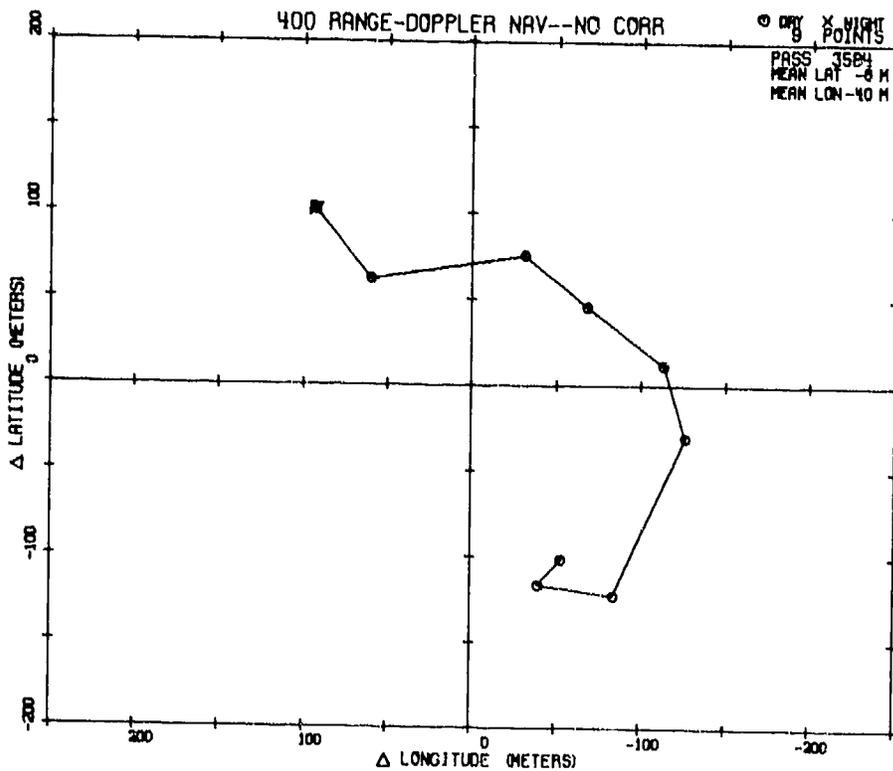


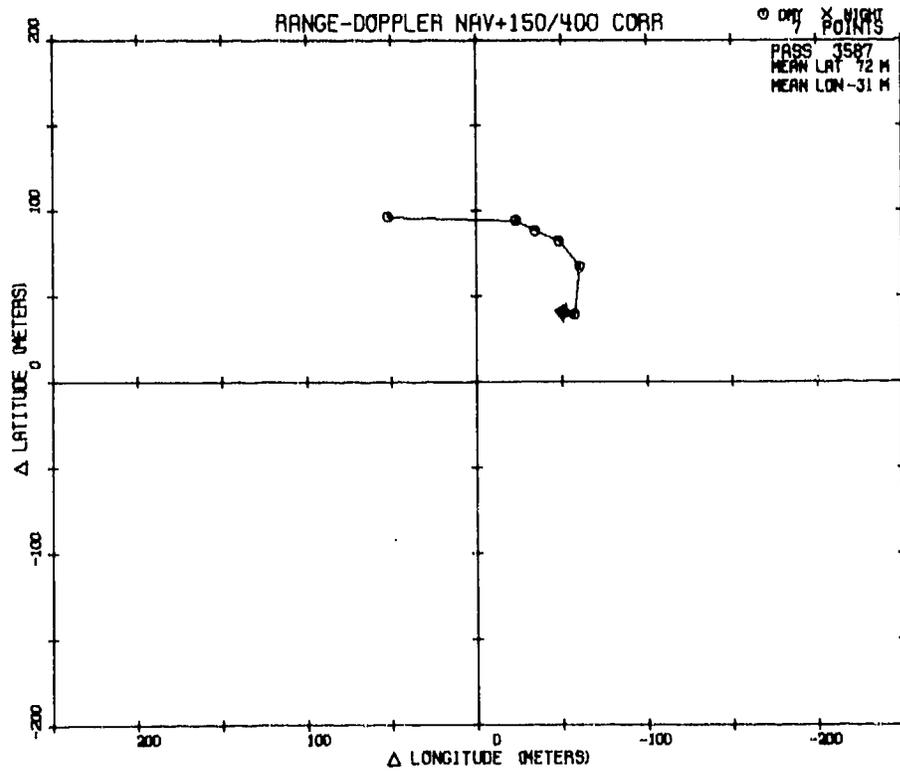
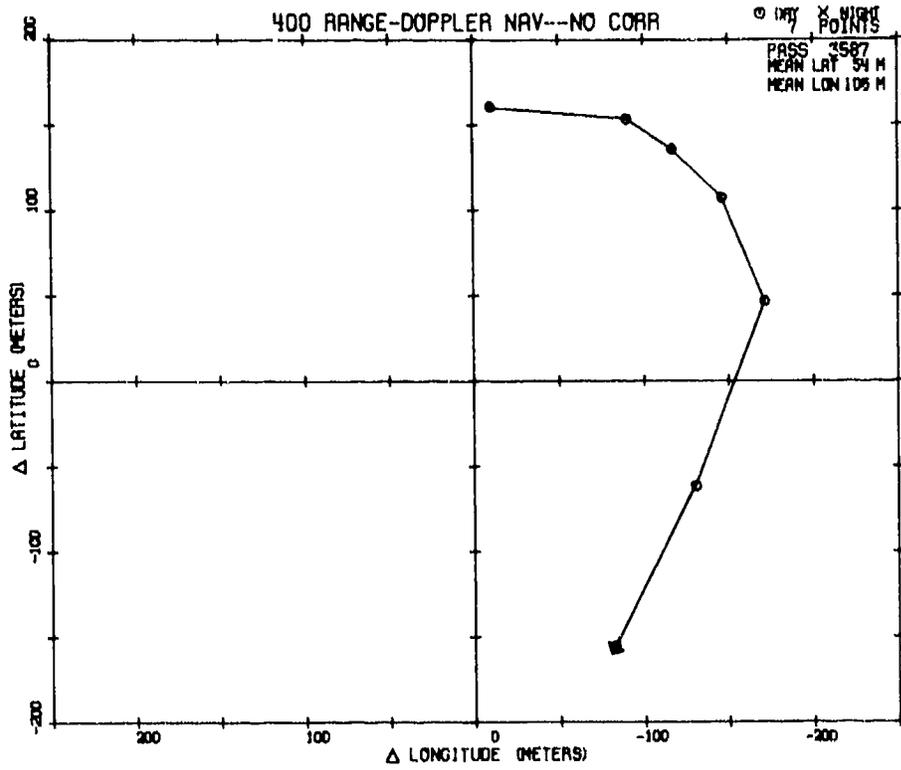


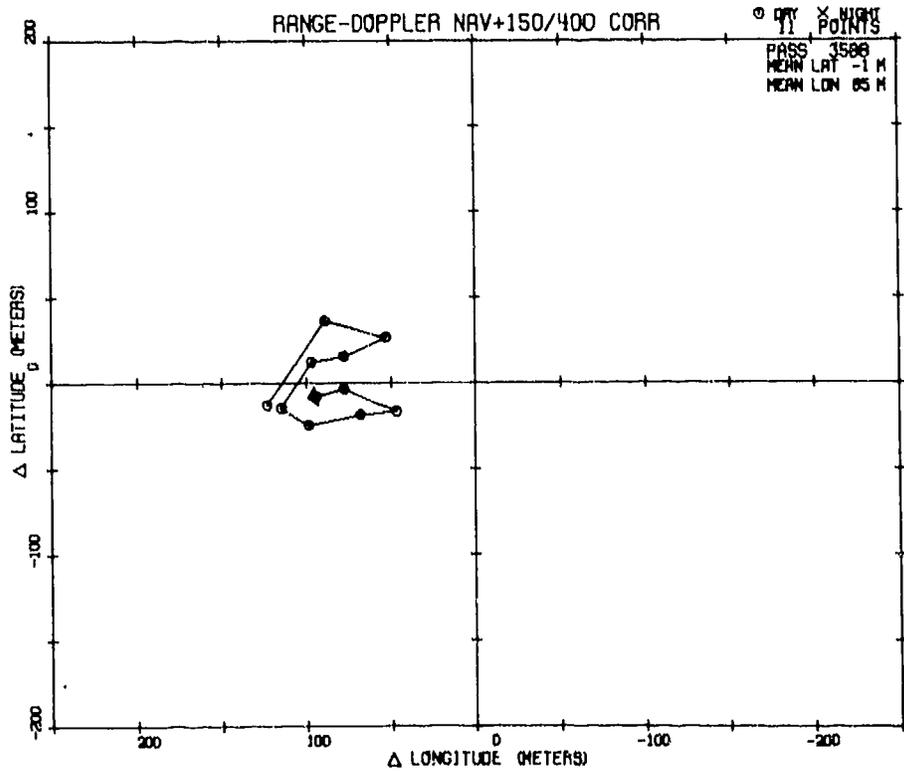
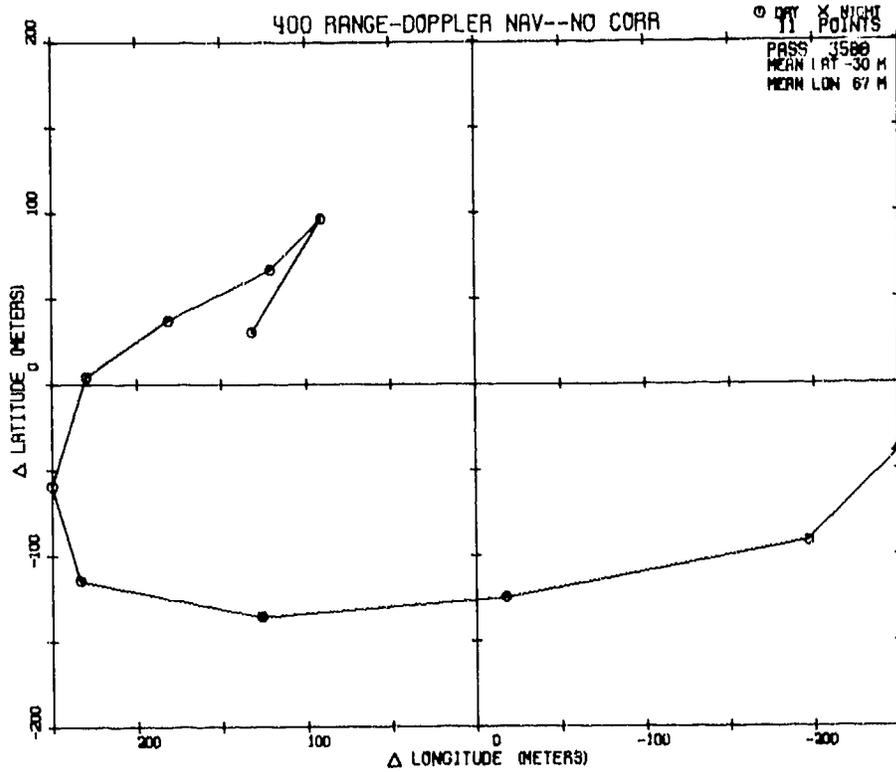


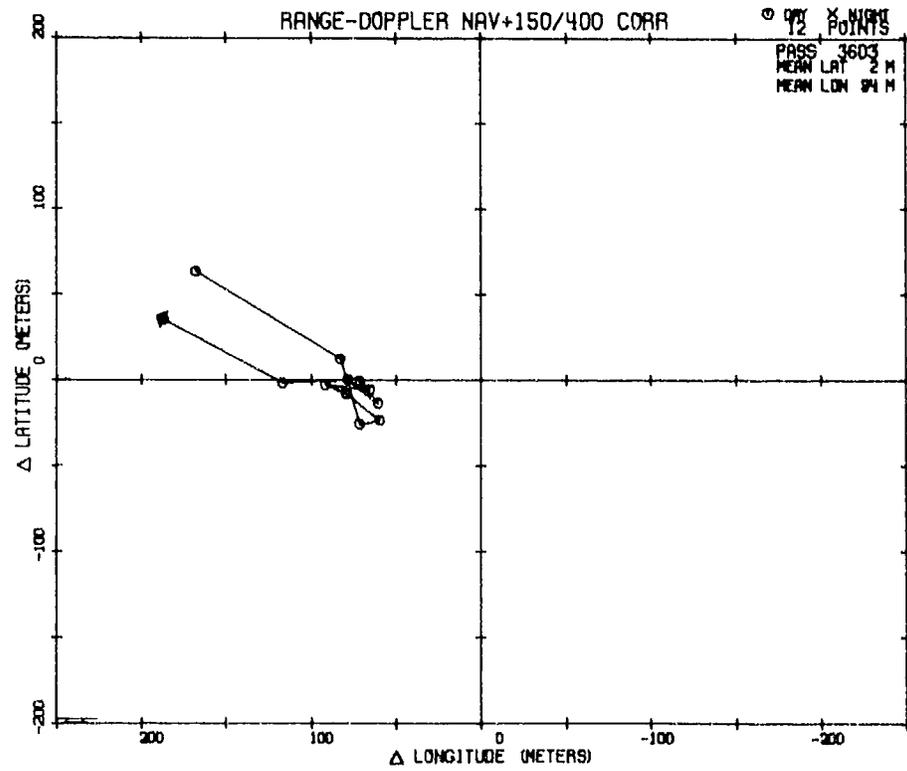
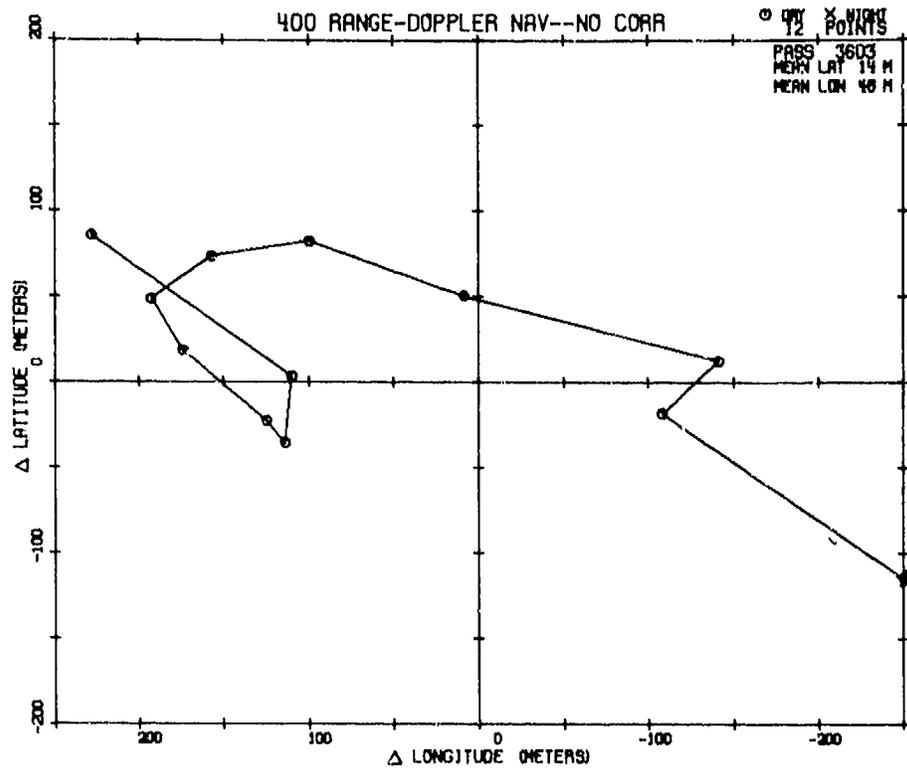


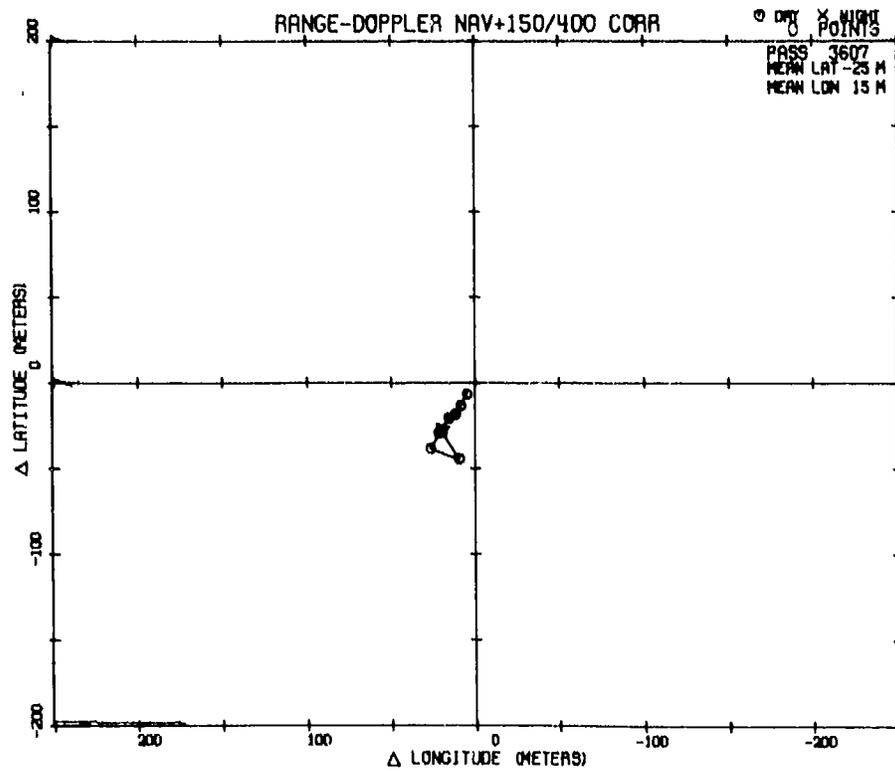
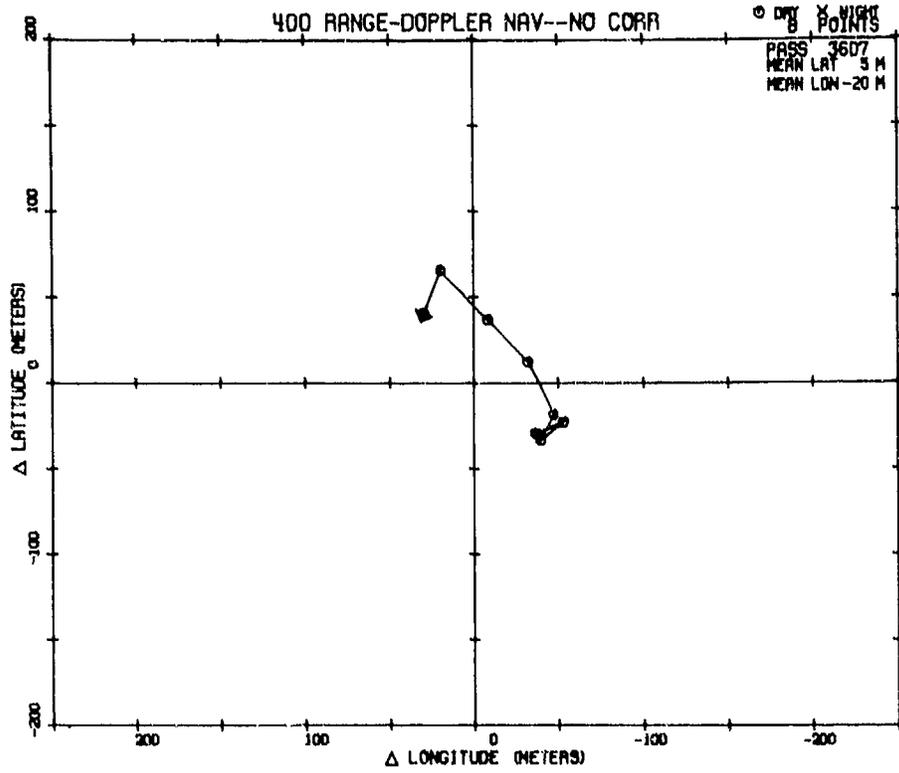
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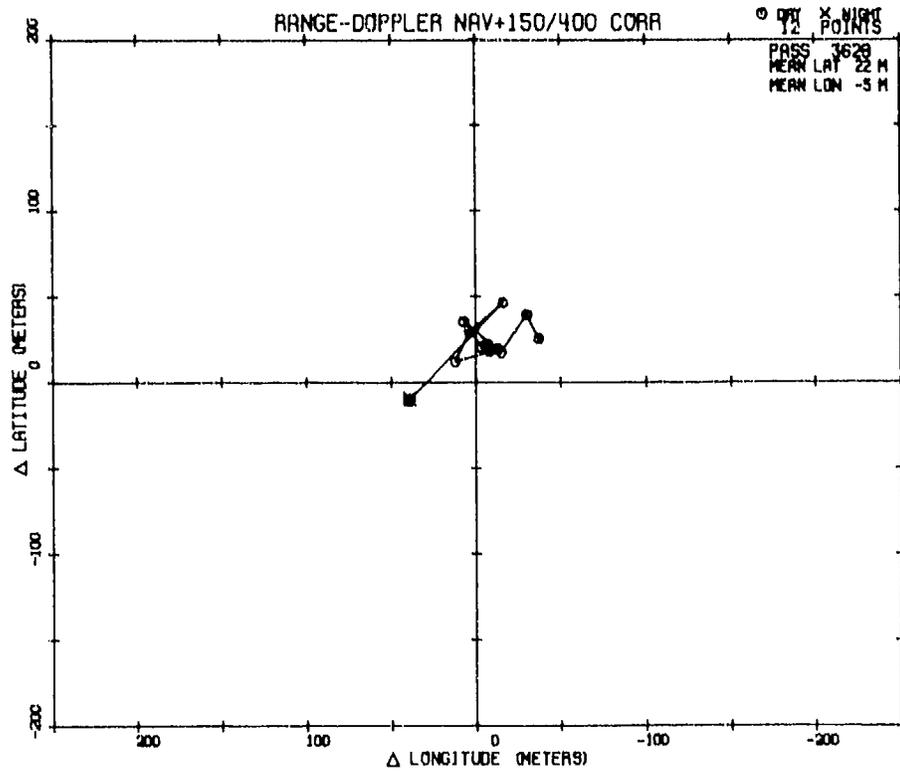
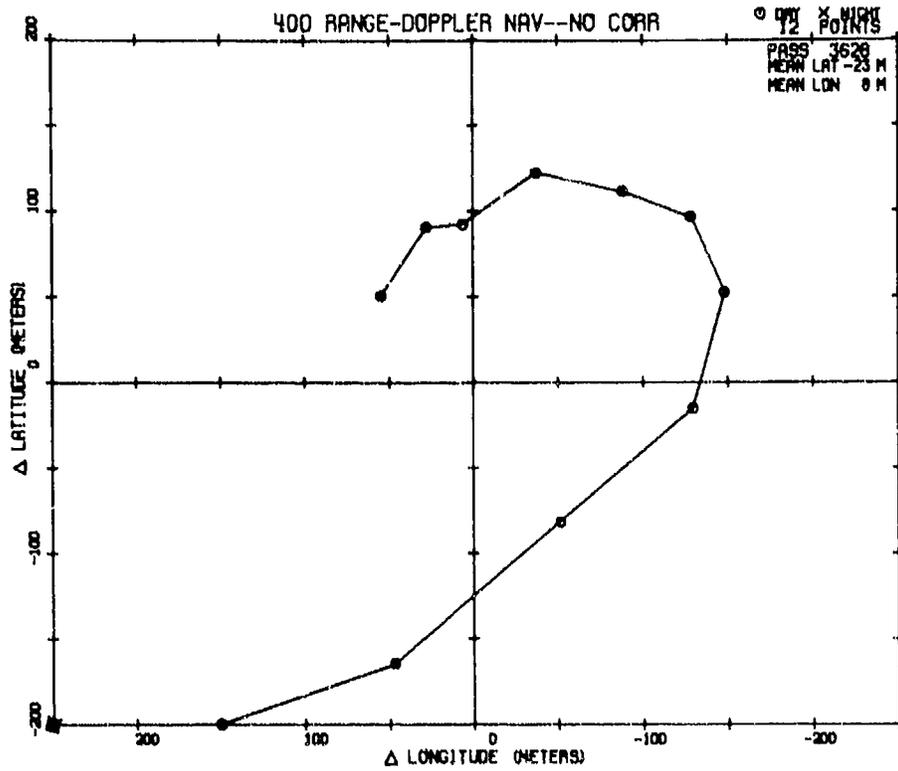




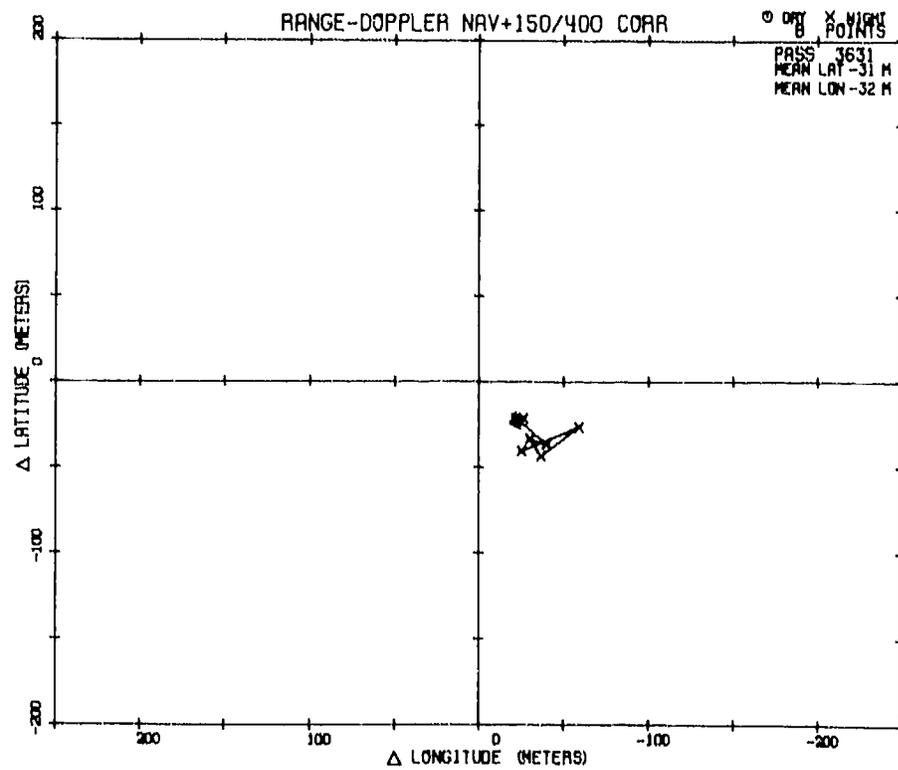
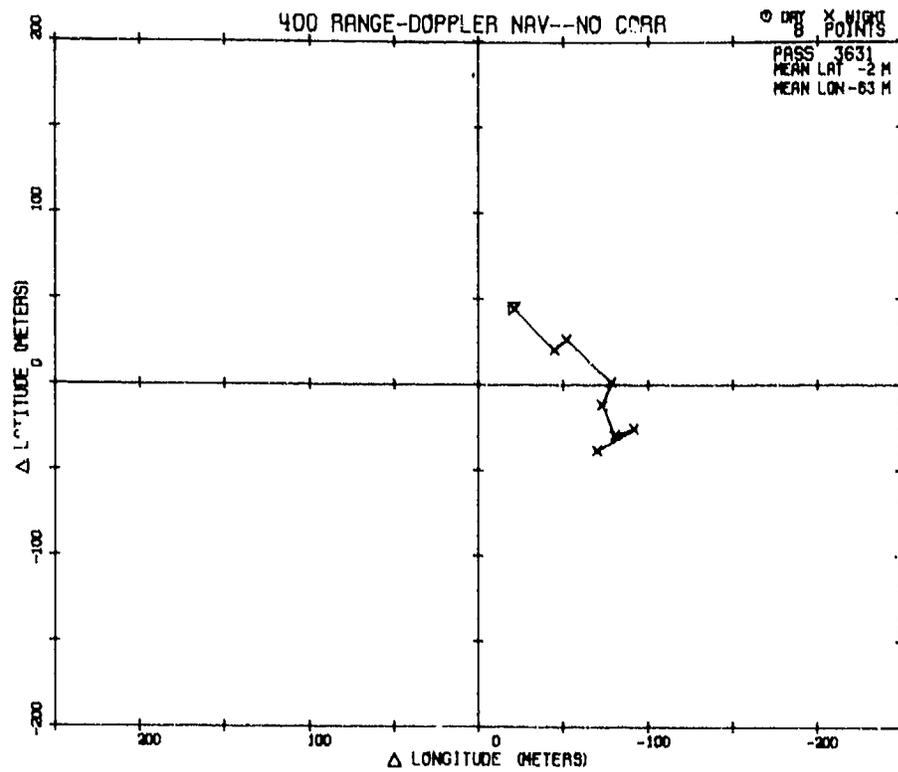


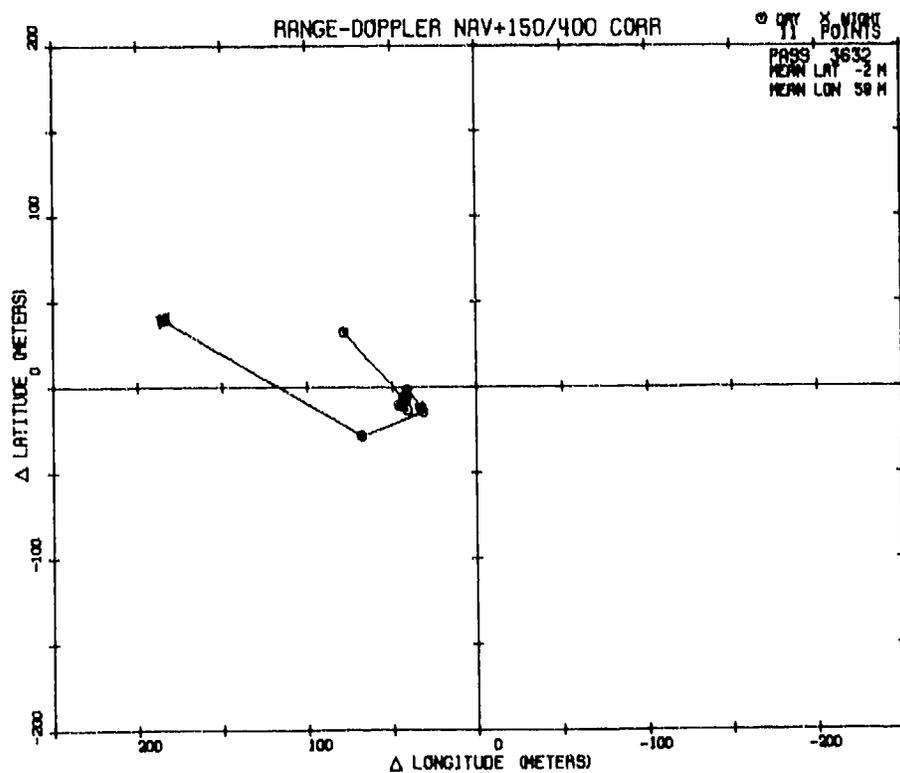
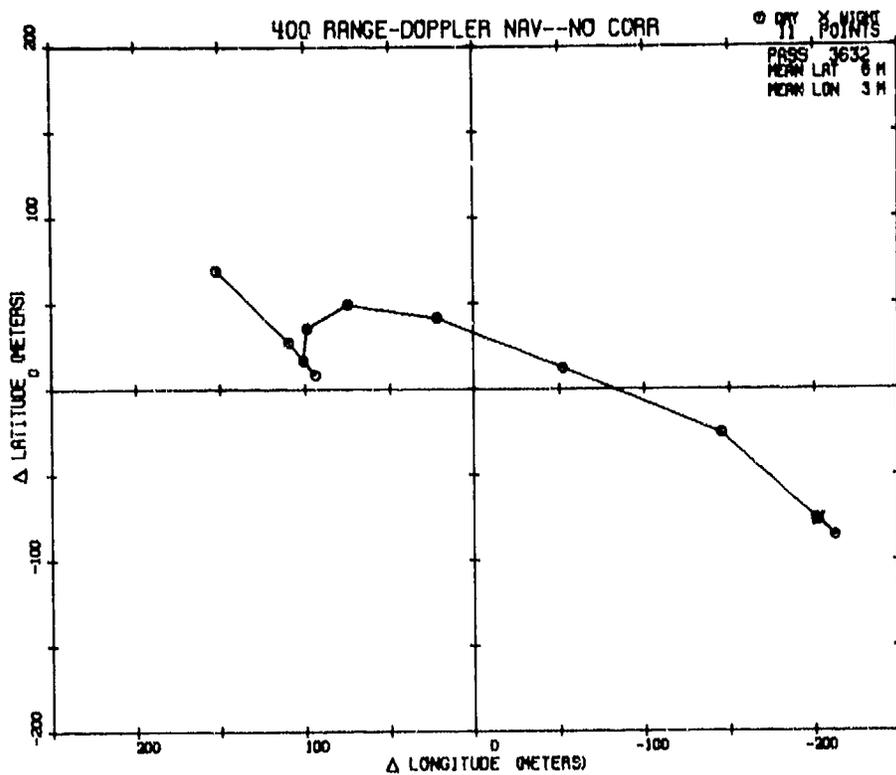




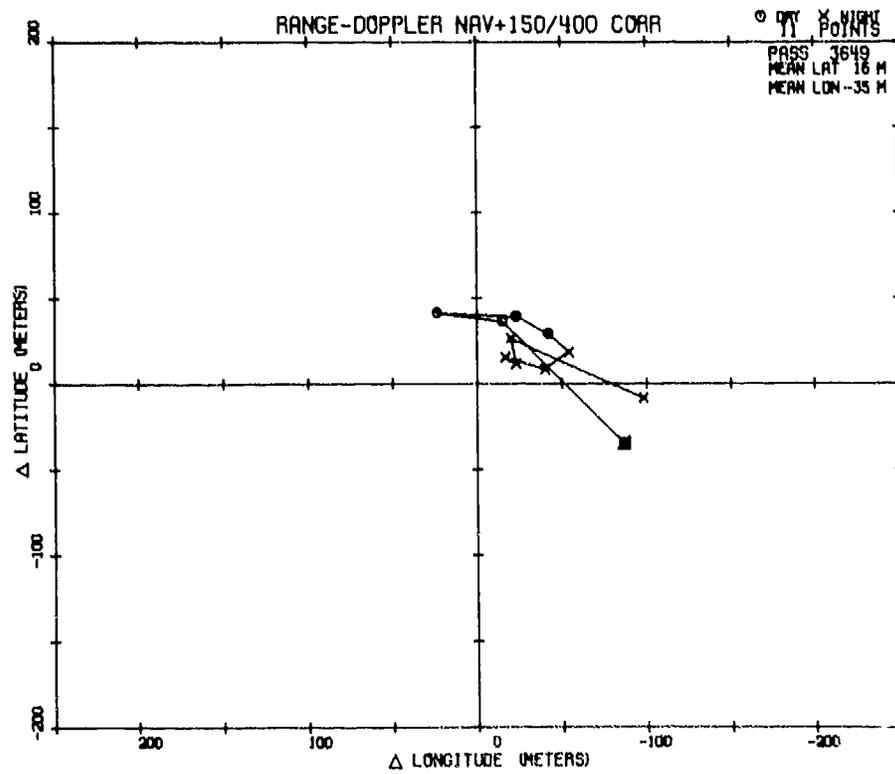
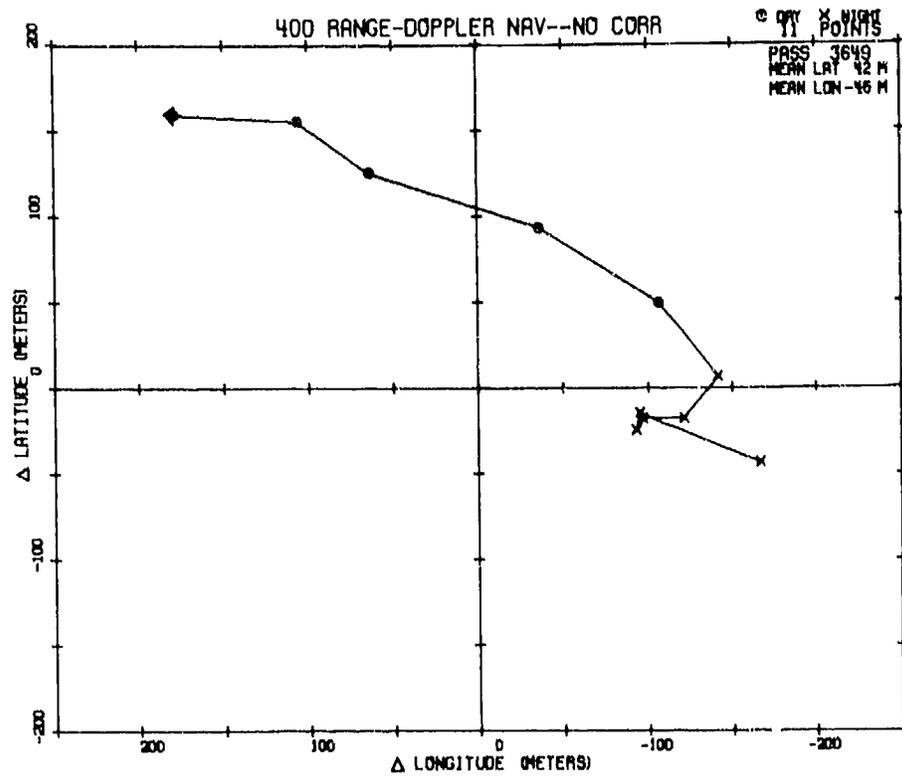


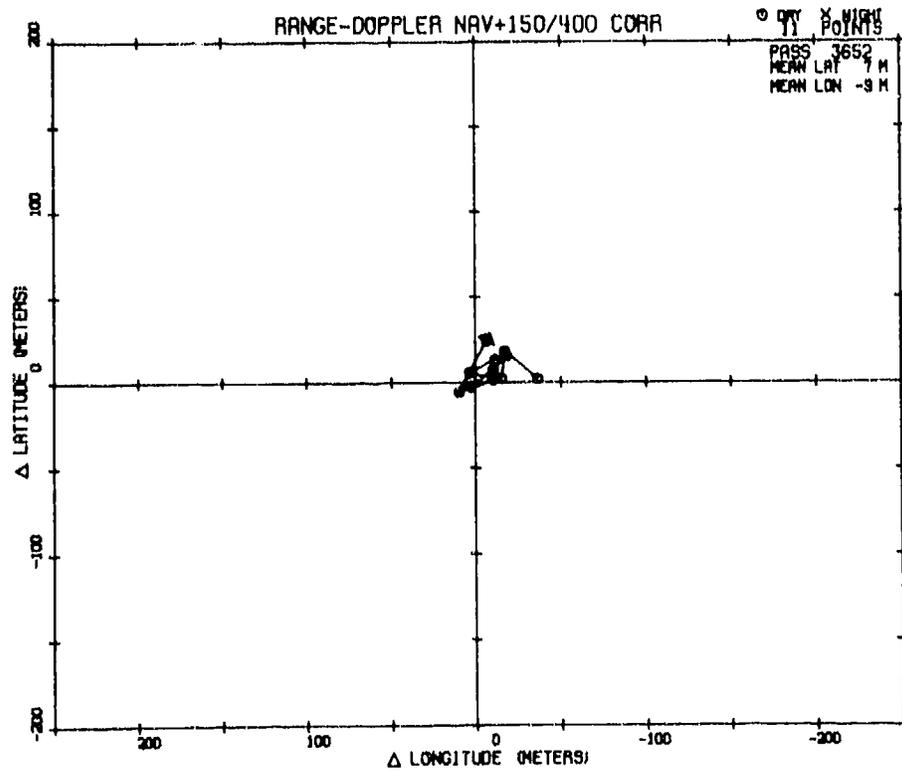
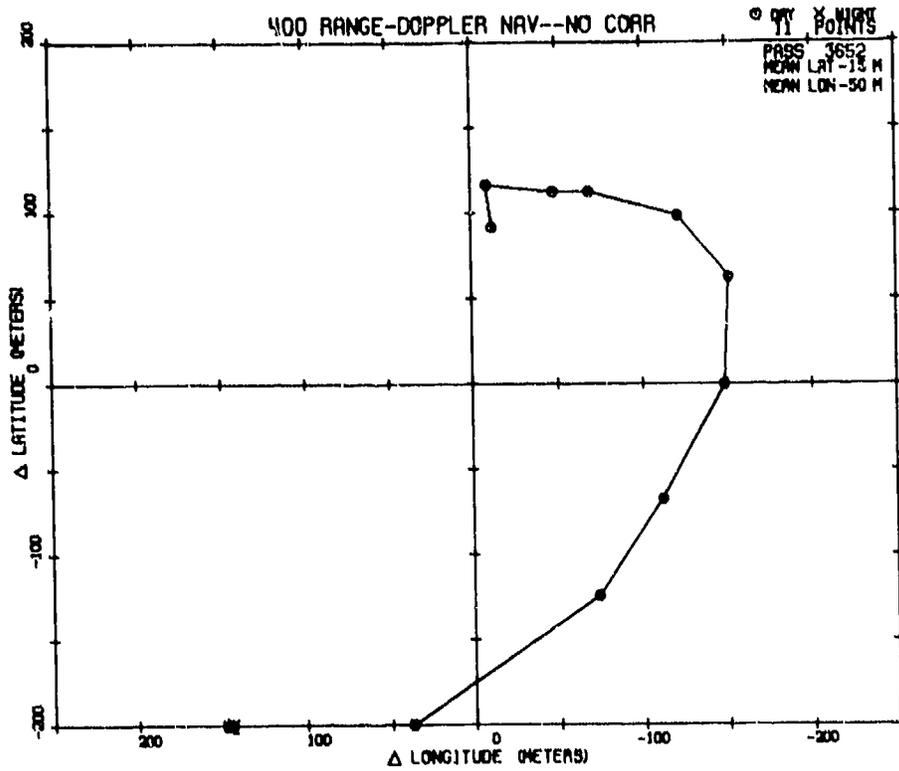
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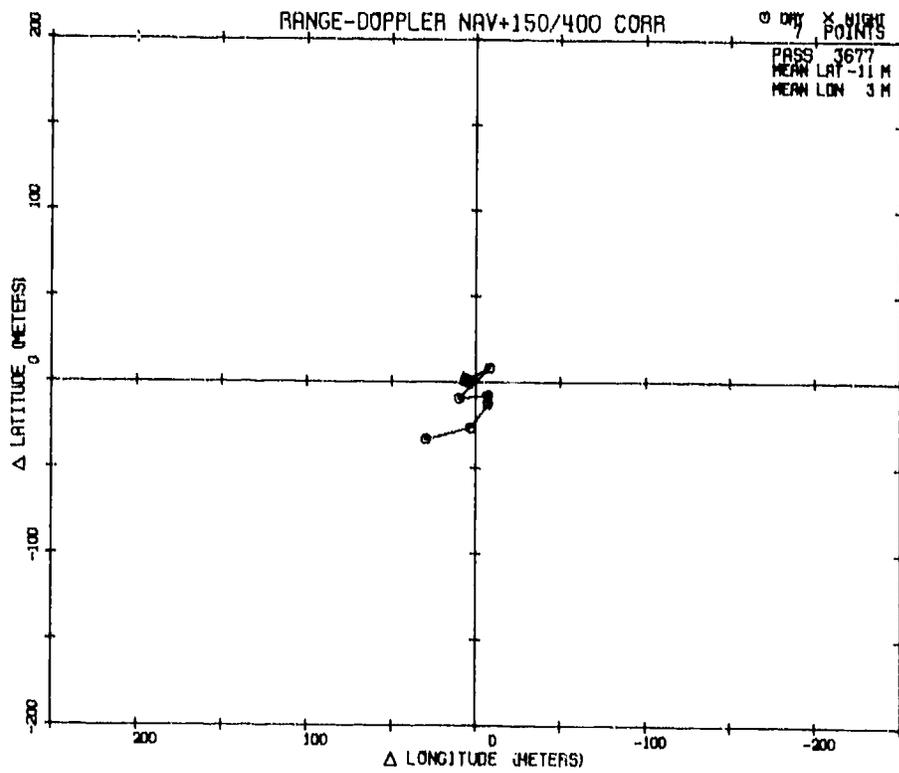
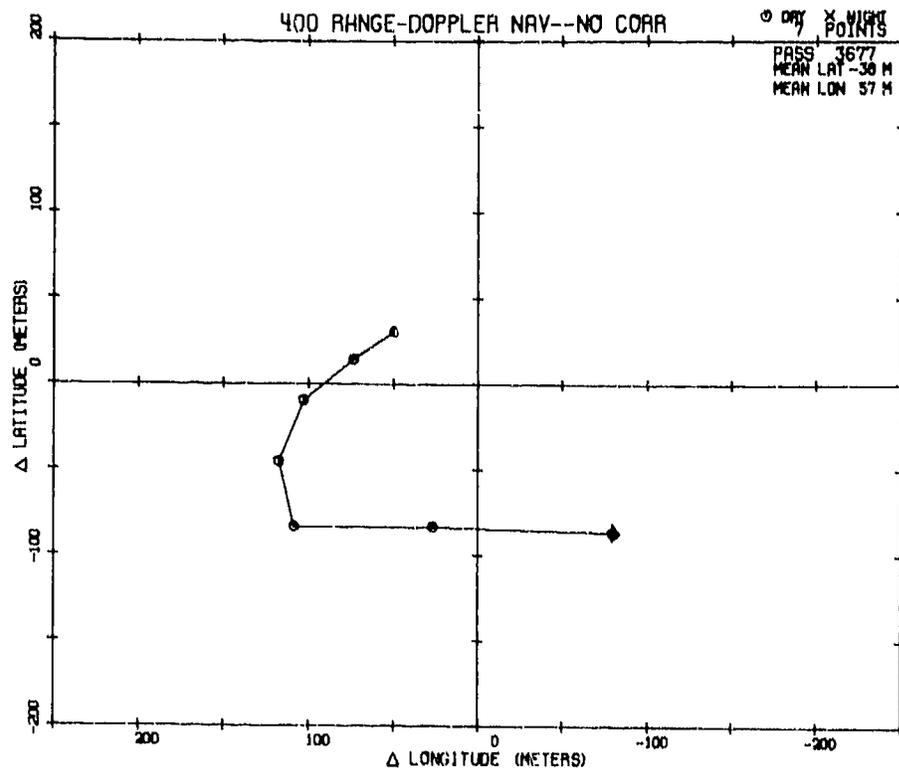


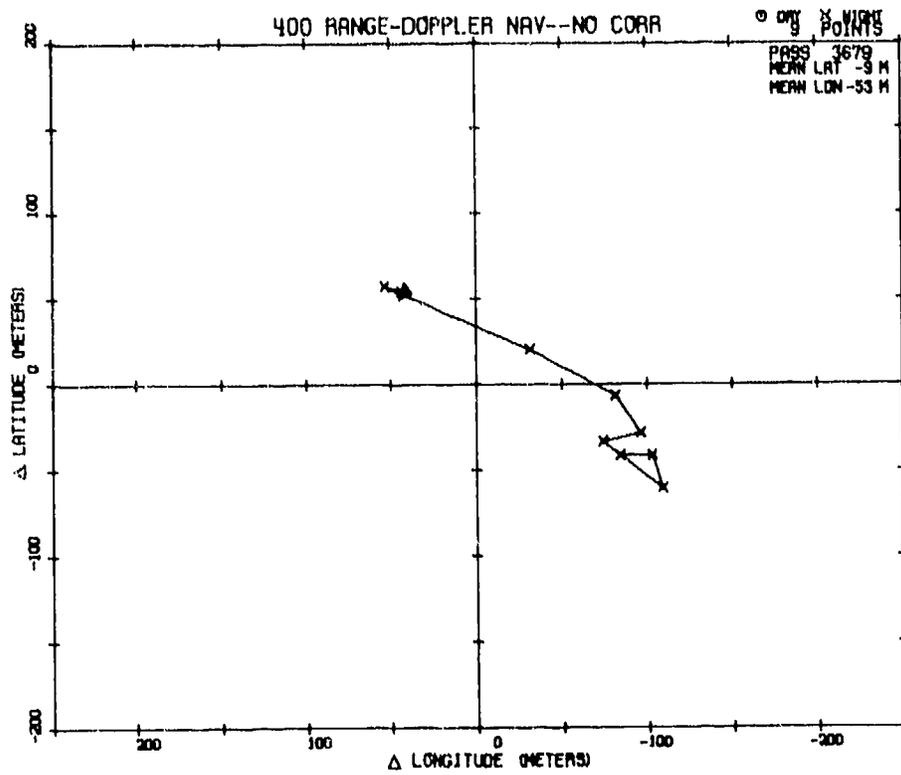
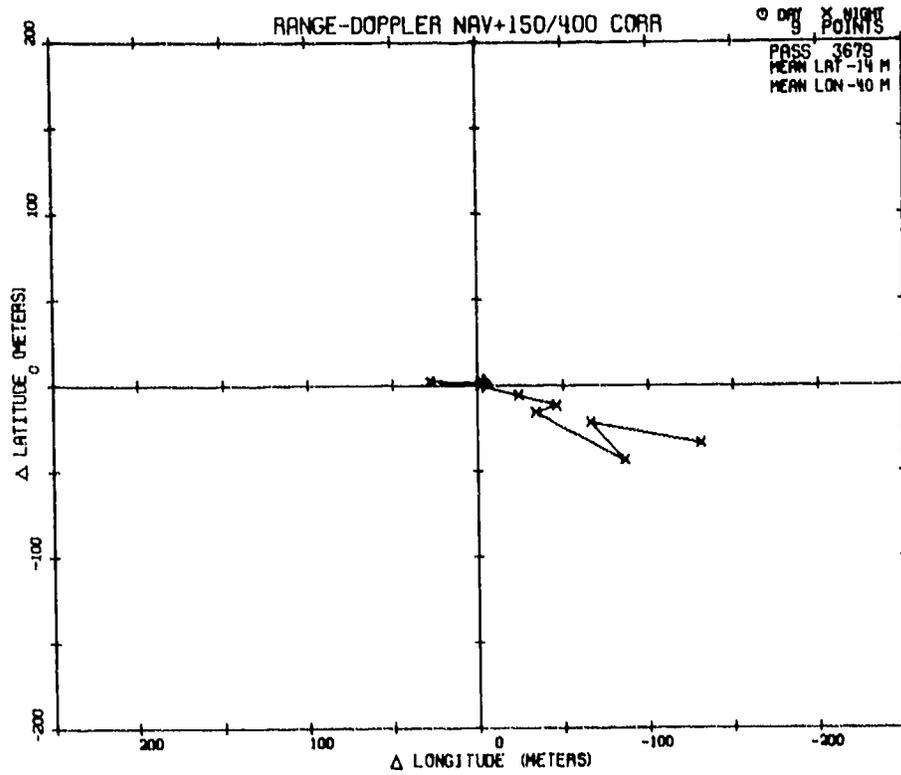
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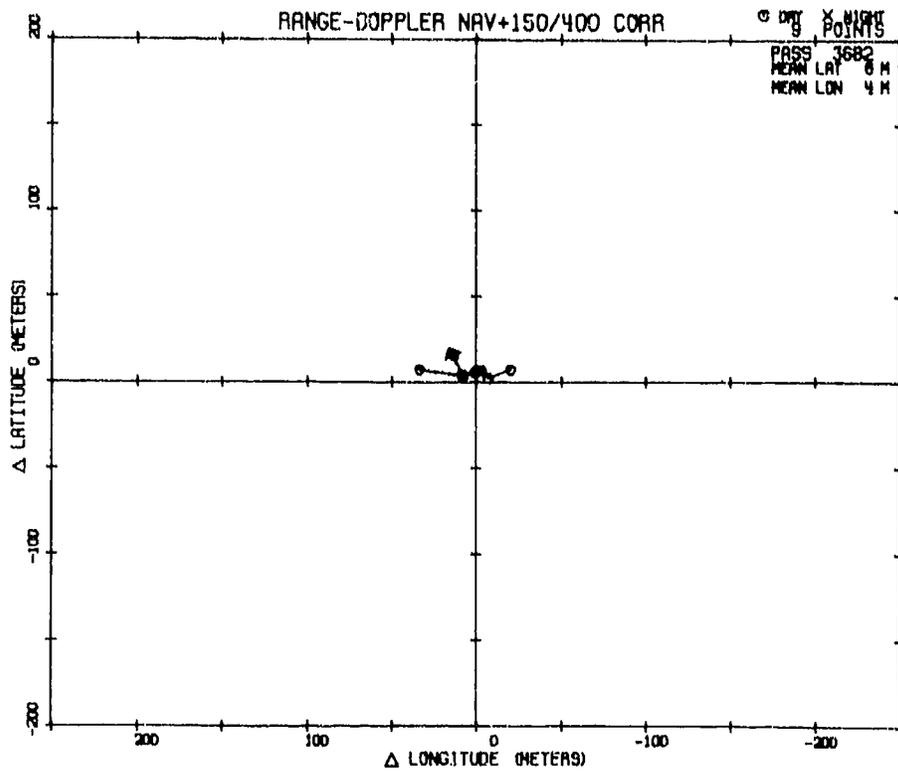
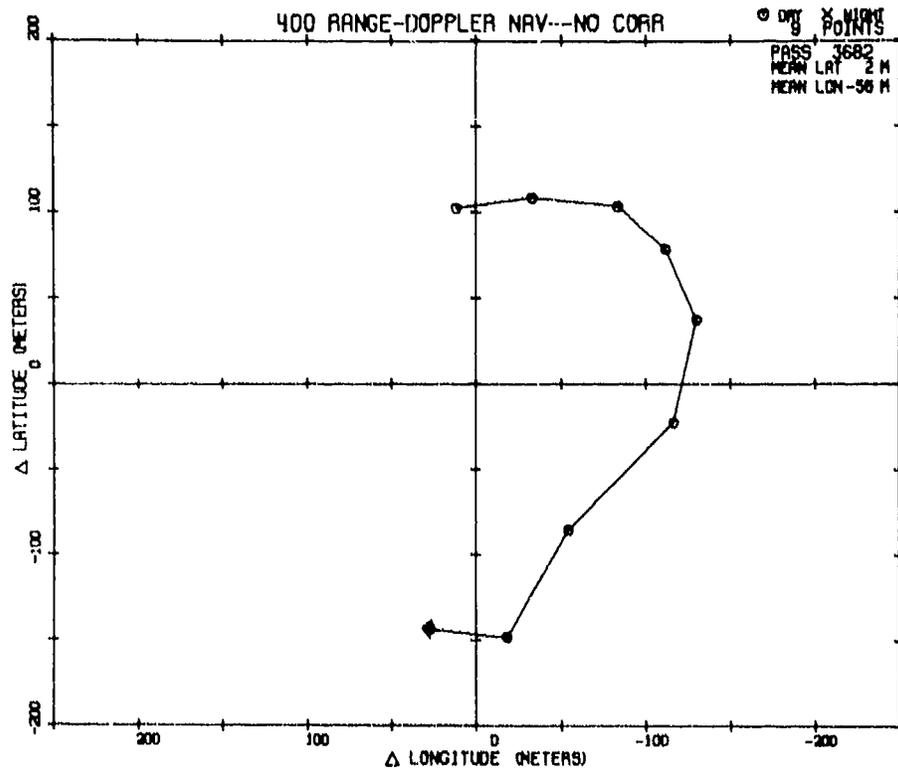


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13. ABSTRACT A worldwide satellite navigation system (TIMATION) has been proposed that will yield instantaneous and continuous position fixes by suitable satellite configurations and navigator equipment. The TIMATION project is conducted by NRL under the sponsorship of the Navy Space Project Office (PM-16) and the Naval Air Systems Command. This report addresses the instantaneous position-fixing results obtained by using range and doppler measurements obtained with the TIMATION II satellite and a navigation receiver employing a relatively precise frequency standard. The results are corrected for first-order ionospheric refraction and indicate a two-dimensional position-fixing capability of 105 m RMS (60 m CEP).			

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Satellite navigation Instantaneous position fixes Simultaneous range and doppler measurements Ionospheric refraction						

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