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The Scattered-Field Response of A Linear Array to a Step-Function Illumination

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ADDENDUM

Immediately after the second sentence in the section entitled "Theoretical Model" in NRL Report 6823, introduce a footnote which should read as follows:

The angle of incidence ϕ_{inc} shown in Fig. 1 is negative in value; ϕ_{inc} assumes positive values on the opposite side of the normal to the plane of the array.

CONTENTS

Abstract	ii
Problem Status	ii
Authorization	ii
SUMMARY	1
INTRODUCTION	1
THEORETICAL MODEL	2
FOURIER REPRESENTATION OF THE TRANSIENT SCATTERED FIELD	2
SYNTHESIS OF A TRANSIENT FIELD SCATTERED IN ANY DIRECTION FROM THE TRANSIENT FIELD SCATTERED IN THE REFLECTED DIRECTION	4
CALCULATION OF THE TRANSIENT SCATTERED FIELD OF A MODEL ARRAY	5
CONCLUSIONS	25
ACKNOWLEDGMENT	26
REFERENCES	26
APPENDIX A — A Comparison of the Synthesis Formula With the Fourier Integral Representation of the Scattered Field	27

ABSTRACT

A limited investigation is made of the scattered-field response of a linear array of center-loaded cylindrical elements illuminated by a plane electromagnetic wave with a step function of time variation. The theoretical approach utilizes Fourier's theorem to express the transient response as a sum of the steady-state responses to the individual sine-wave components of the step-function illumination.

The number of array elements and their spacing can be determined simply from the time variation of the backscattered field, provided the angle of incidence of the illumination is known and adequately large to separate the component transient waves.

In the case of an N-element array, it is found that the transient field scattered in any arbitrary direction in the H plane can be represented as a summation of N appropriately time-delayed components, each with the same time variation as the field scattered in the reflected direction but with an amplitude reduced by a factor of $1/N$.

PROBLEM STATUS

This is the second report on the final phase of the problem; a third and final report on the problem is now in progress.

AUTHORIZATION

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THE SCATTERED-FIELD RESPONSE OF A LINEAR ARRAY TO A STEP-FUNCTION ILLUMINATION

SUMMARY

The scattered-field response of a linear array of cylindrical elements illuminated by a plane electromagnetic wave with a step function of time variation has been investigated. In the theoretical model the array consists of N thin, uniformly spaced, center-loaded elements in free space. The transient wave of illumination, polarized with its electric vector parallel to the axes of the elements, is incident in the H plane of the array. The theoretical formulation uses the theorem of Fourier to express the transient scattered field in the far zone as an integral of the steady-state scattered-field responses to the individual sine-wave components of the illumination.

From an approximate expression for the steady-state scattered field of a linear array of N elements developed in a previous study (1) we find that the transient field scattered in any arbitrary direction in the H plane is a summation of N appropriately time-delayed components, each with the same time variation as the field scattered in the reflected direction but with an amplitude reduced by a factor $1/N$. It follows from this synthesis concept that the amplitude of the transient response reaches a maximum in the reflected direction, where the time delay between the component fields vanishes. Also, the transient scattered wave has a maximum time of duration in the backscattered direction of observation.

The transient scattered field of a model array of eight elements was calculated from the continuous, sine-wave, scattered-field response of the model array for various conditions of loading and illumination.

When the reflected transient fields are well damped or nearly periodic in character, during several cycles of oscillation, the number of array elements and their spacing can be determined simply from an observation of the transient backscattered field as a function of time, provided the angle of incidence of the illumination is known and made sufficiently large to separate the component transient waves.

It is found that the largest H-plane scattered energy occurs when the center loads of the elements represent approximately a conjugate match to the effective internal impedance of the array elements over the frequency spectrum of the transient illumination.

INTRODUCTION

Subsequent to 1960, a number of papers on transient phenomena in antennas have appeared in the literature. Polk (2) treated the transient behavior of aperture antennas. He pointed out that the transient conditions in the radiation pattern of an antenna depend upon antenna size, the carrier frequency, the duration of the time signal, and the amplitude and phase distributions over the aperture. Polk's procedure involved the evaluation of infinite integrals, which is a difficult operation even for the simplest types of time signal and aperture distribution. Tseng and Cheng (3) present a simplified approach for the analytical determination of the transient pattern characteristics of an aperture-type antenna for arbitrary time signals and arbitrary aperture distributions. In their method

it is not necessary to take the Fourier transform of the time signal or to evaluate complicated inverse transforms.

Several authors (4-7) have investigated various kinds of transient phenomena in particular types of simple cylindrical antennas. The investigation of the scattering of a transient-type of illumination by a linear array of cylinders has not appeared, to this author's knowledge, in the antenna literature. In this report a numerical solution is presented to the problem of the scattering of a plane electromagnetic wave with a step function of time variation by a linear array of cylindrical elements. The theoretical formulation makes use of the Fourier theorem to express the transient response as an integral of the responses to the individual sine-wave components of the step function of illumination.

THEORETICAL MODEL

The theoretical model is shown in Fig. 1. The model consists of a linear array of N parallel, center-loaded, cylindrical elements in free space illuminated by a step function (plane wave) incident in the H plane of the array at an angle ϕ_{inc} to the normal of the plane of the array. The polarization of the illumination is such that its electric vector lies parallel to the axes of the array elements, which are of length $2h$, of radius a and are spaced uniformly at a distance d. The radius of the elements is assumed thin relative to their length and to the shortest significant wavelength appearing in the spectrum of incident illumination. In our theoretical model, only the axial currents induced in the elements are significant; consequently the induced currents maintain a vector potential having only a z component.

FOURIER REPRESENTATION OF THE TRANSIENT SCATTERED FIELD

The steady-state scattered field observed in the far zone of the array illuminated by a plane wave of time dependence $\exp(j\omega t)$ incident in the H plane of the array can be expressed in the form

$$E_\theta^s(\omega) = [A(\omega h/c) + jB(\omega h/c)]\ \frac{60}{r}\exp(-j\omega r/c)\ , \tag{1}$$

where r is the distance from the center of the first element of the array to the point of observation in the far zone, c is the velocity of electromagnetic propagation in free space, and ω is the angular frequency of the incident wave. The quantities $A(\omega h/c)$ and $B(\omega h/c)$ are real functions of ω only, provided the physical dimensions of the array, the angle of incidence of the illumination, the direction of observation of the scattered field, and the character of the load impedance functions $Z_i(\omega)$ are fixed.

When we have a transient type of illumination of the array, the scattered field is likewise transient in character. Let the incident electric field be a step function of magnitude E_0; then according to Papoulis (8), the Fourier transform of the incident field is $E_0[\pi\delta(\omega) + 1/j\omega]$, where $\delta(\omega)$ is the Dirac delta function. It follows from Fourier's theorem and Eq. (1) that the transient field resulting from the step-function illumination may be represented by the following Fourier integral

$$e_\theta^s(t - r/c) = \frac{30}{\pi}\int_{-\infty}^{\infty}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]\left[\frac{A(\omega h/c) + jB(\omega h/c)}{r}\right]\exp[j\omega(t - r/c)]\ d\omega\ . \tag{2}$$

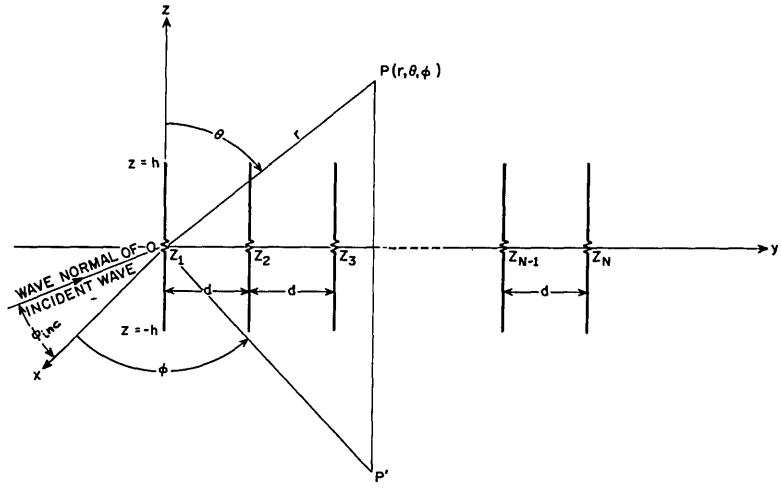


Fig. 1 - Theoretical model

From an integral property of the Dirac delta function, Eq. (2) may be cast in the form

$$e_\theta^s(\tau) = \frac{30}{r}\left\{\frac{1}{\pi}\int_{-\infty}^{\infty}\left[\frac{A(\omega h/c) + jB(\omega h/c)}{j\omega}\right]\exp(j\omega\tau)\,d\omega \right.$$

$$\left. + \lim_{\omega\to 0}\,[A(\omega h/c) + jB(\omega h/c)]\,\exp(j\omega\tau)\right\}, \tag{3}$$

where $\tau = t - r/c$.

To evaluate the limit, a numerical investigation for small values of $\omega h/c$ shows that $A(\omega h/c)$ and $B(\omega h/c)$ have the following $\omega h/c$ dependence:

$$A(\omega h/c) \to K'(\omega h/c)^2 \tag{4}$$

and

$$B(\omega h/c) \to K''(\omega h/c)^3\,, \tag{5}$$

where K' and K'' are constants. It is apparent from Eqs. (4) and (5) that both $A(\omega h/c)$ and $B(\omega h/c)$ vanish in the limit as ω goes to zero; hence Eq. (3) may be written

$$e_\theta^s(\tau) = \frac{30}{\pi r}\left\{\int_{-\infty}^{\infty}\frac{1}{\omega}\,[B(\omega h/c)\cos\omega\tau + A(\omega h/c)\sin\omega\tau]\,d\omega \right.$$

$$\left. + j\int_{-\infty}^{\infty}\frac{1}{\omega}\,[B(\omega h/c)\sin\omega\tau - A(\omega h/c)\cos\omega\tau]\,d\omega\right\}. \tag{6}$$

For a physical transient field, the individual parts of the last integral in Eq. (6) must each vanish under all conditions of impedance loading, angle of incidence, etc. Thus, it

is evident that $A(\omega h/c)$ and $B(\omega h/c)$ must be respectively even and odd functions of $\omega h/c$. Accordingly, the expression for the transient scattered field may be put in the form

$$e_\theta^s(\tau) = \frac{60}{\pi r}\int_0^\infty \left[\frac{B(\omega h/c)}{\omega}\cos\omega\tau + \frac{A(\omega h/c)}{\omega}\sin\omega\tau\right] d\omega \ . \tag{7}$$

Note that $t = 0$ or $\tau = -r/c$ in Eq. (7) corresponds to the instant of arrival of the step-function illumination at the first element of the array (Fig. 1).

Because of the finite velocity of electromagnetic disturbances, the transient scattered field should be zero for values of retarded time τ less than zero. To meet this physical requirement the following causality condition must hold true:

$$\int_0^\infty \frac{B(\omega h/c)}{\omega}\cos\omega\tau\, d\omega = \int_0^\infty \frac{A(\omega h/c)}{\omega}\sin\omega|\tau|\, d\omega \ . \tag{8}$$

One can conclude from Eqs. (7) and (8) that the complete scattering behavior of an array (or any antenna for that matter) in the time domain can be found theoretically provided either the real or imaginary part of the steady-state sinusoidal scattering response is known over all frequencies.

SYNTHESIS OF A TRANSIENT FIELD SCATTERED IN ANY DIRECTION FROM THE TRANSIENT FIELD SCATTERED IN THE REFLECTED DIRECTION

At this juncture we will digress and demonstrate that the transient scattered field in any direction in the H plane can be synthesized approximately from the time variation of the field scattered in the reflected direction.

From Sledge (1) the following expression is given for the H-plane scattered field observed in the direction ϕ (Fig. 1) in the far zone when the array is illuminated by a plane wave of time dependence $\exp(j\omega t)$ incident in the H plane of the array:

$$E_\theta^s(\omega) = j\left[\frac{60}{r}\exp\left(-j\frac{\omega}{c}r\right)\right]\sum_{i=1}^{N} G_i \exp\left[j(i-1)\frac{\omega}{c}d\sin\phi + j\gamma_i\right] , \tag{9}$$

where G_i and γ_i are the magnitude and phase of the complex effective scattering coefficient of the ith element of the array. In general G_i and γ_i are functions of $\omega h/c$, ϕ_{inc}, and the impedance function of the load. From extensive numerical calculations (see Appendix B of Ref. 1) it is found that G_i remains reasonably constant across the array and that γ_i increases approximately in uniform increments of $(\omega d/c)\sin\phi_{inc}$ radians as the element number i progresses from 1 to N. By virtue of this property of the scattering coefficients, Eq. (9) can be approximated by

$$E_\theta^s(\omega) = \frac{1}{N}E_\theta^r(\omega)\sum_{i=1}^{N}\exp[j(i-1)\psi] , \tag{10}$$

where $\psi = (\omega d/c)(\sin\phi + \sin\phi_{inc})$ and $E_\theta^r(\omega)$ is the complex scattered field in the reflected direction ($\phi = -\phi_{inc}$). Using Eq. (10) and Fourier's theorem it can be shown that the transient scattered field resulting from a step-function illumination of the array is given by the sum

$$e_\theta^s(\tau) = \frac{1}{N}\sum_{i=1}^{N} e_\theta^r[\tau - (i-1)\Delta t] , \tag{11a}$$

where $e_\theta^r(\tau)$ is the transient scattered field observed in the reflected direction and

$$\Delta t = -(d/c)\ (\sin\phi + \sin\phi_{inc}) \tag{11b}$$

is the time delay of the $(i + 1)$th component transient field relative to the ith component. (Appendix A gives a quantitative comparison of Eq. (11) with the Fourier integral form to be given as Eq. (12).) It is apparent from Eq. (11) that the scattered H-plane field consists of N components, each an appropriately delayed version of $e_\theta^r(\tau)/N$. The ith element of the array radiates the ith-component scattered field.

Thus, we observe that once the basic transient field scattered in the reflected direction has been determined, it becomes a relatively simple matter to synthesize the transient field scattered in any arbitrary direction in the H plane. Also, it is seen from Eq. (11a) that the transient scattered-field response of an array possesses its largest amplitude when the field is observed in the reflected direction where Δt vanishes. Also, it follows from Eq. (11b) that Δt is a maximum value in the backscattered direction; hence the time duration of the transient field is largest in this direction of observation.

CALCULATION OF THE TRANSIENT SCATTERED FIELD OF A MODEL ARRAY

In this section we will make a limited study of the scattering characteristics of a model array of eight, identical, center-loaded elements illuminated by a step function of electromagnetic field. In our calculations we will keep the uniform separation between the array elements equal to the length of the elements (a commonly used spacing in broadside arrays). We will assume that the ratio of the length to the radius of an element is fixed at 1000 and that the strength of the incident illumination remains constant such that $E_0 h = 30$. To make our calculated results independent of the distance from the array to the point of observation of the field, we will suppress the factor $60/r$ appearing in Eq. (7). Also, the transient scattered field becomes a function of relative retarded time α if we change the variable of integration from ω to $\omega h/c$.

The modifications of Eq. (7) result in the following formula for the normalized transient scattered field

$$[e_\theta^s(\alpha)]_n = \frac{1}{\pi}\int_0^\infty \left[\frac{A(x)}{x}\sin\alpha x + \frac{B(x)}{x}\cos\alpha x\right] dx , \tag{12}$$

where $x = \omega h/c$ and

$$\alpha = \frac{c}{h}\tau = \frac{c}{h}(t - r/c).$$

As a practical matter the upper limit in the integral of Eq. (12) must be finite. In the numerical approximation of Eq. (12) the limits of integration were from 0.05 to 4.05 in x. The upper limit is determined by the extent of the validity of the steady-state scattered field calculations of Sledge (1), the source of the values of $A(x)$ and $B(x)$.

Filon's quadrature formula with subintervals of 0.05 in x is used to numerically evaluate the truncated version of the integral appearing in Eq. (12). For each combination

of ϕ_{inc}, load impedance, and ϕ the calculation of the transient field is made in α steps of 0.1 over a sufficient range in α.

We will present in Figs. 2 through 6 examples of the scattered-field response of the model array to a step-function illumination, all calculated using Eq. (12). One thing in common to all the transient fields shown in this report is the existence of a time variation of the field prior to $x = 0$. This condition cannot happen in a physical system, as was pointed out earlier. Two reasons account for the precursors exhibited in our calculated transient fields. First, the steady-state fields from which the transient fields are calculated are not truly physical fields that satisfy Maxwell's equations and the boundary conditions on the array but are unphysical, mathematical approximations to the true physical fields. Second, the limits of the Fourier integral representation of the transient fields are truncated of necessity at both the upper and lower ends of the frequency spectrum. Intuitively it is felt that the high-frequency cutoff at $\omega h/c = 4.05$ and the nature of the load impedance function principally determine the character of all the transient fields in the negative α regions.

It is interesting to note that the rise time of all our calculated transient fields seems to be related, as it should be, to the high-frequency cutoff at $\omega h/c \approx 4$. The rise time of an ideal low-pass filter with a cutoff at $\omega = \omega_c$ is simply π/ω_c. In our field calculations, $\omega_c = 4c/h$; therefore the rise time of our transient fields should be of the order of $\pi h/4c$. This formula appears to agree with our calculated results, provided we define the rise time of the fields to be the positive increment in time required for the transient field to increase from zero to 90% of the first large positive peak. According to this definition, the rise time of our graphs runs between 0.70 and 0.80 in α.

Our calculated results are limited to the scattered-field responses of the model array observed in either the backscattered or the reflected direction.

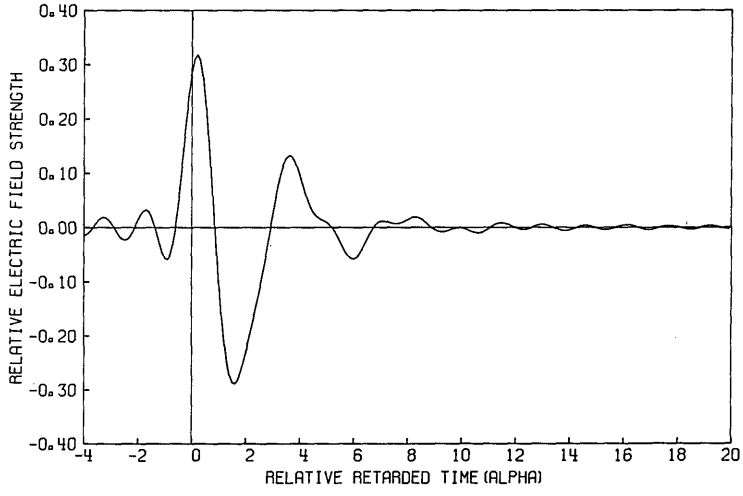
Figure 2 presents several scattered-field responses of the model array uniformly loaded with $Z_i(\omega) = 240$ ohms. Figures 2a, 2b, 2d, and 2f show the basic field responses observed in the reflected direction for the illumination incident at angles of 0, -20, -40, and -60 degrees respectively. We see that the reflected transient fields resemble each other fairly closely over a wide range of incidence angles. Figures 2c, 2e, and 2g show the backscattered-field responses for the illumination incident at -20, -40, and -60 degrees respectively. In Fig. 2c the first large peak is predominantly due to the large positive peak of the component transient radiation from the first element of the model array, and the large peak at $\alpha = 11$ results primarily from the large negative peak of the component transient field radiated by the eighth (last) element.

The smaller amplitude oscillations lying between the two large peaks result from the partial annulment of the component transient radiations in this region of α. When the illumination is incident at -40 degrees or more, the separate components of the backscattered transient fields are easily discernible, as one can see in Figs. 2e and 2g. From Eq. (11b) it is seen that the α delay of the $(i + 1)$th transient field component relative to the ith component field when observed in the ϕ direction is given by

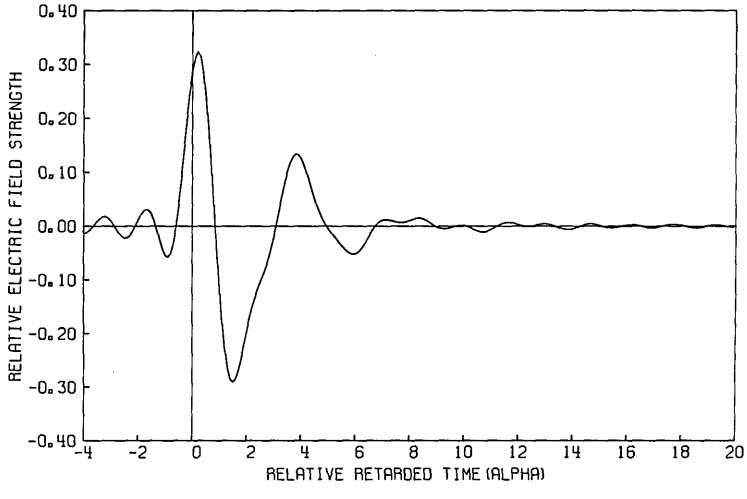
$$\Delta\alpha = -\left(\frac{d/c}{h/c}\right)(\sin\phi + \sin\phi_{inc}). \tag{13}$$

For the model array with $d = 2h$, the $\Delta\alpha$ of the field observed in the backscattered direction is given by

$$\Delta\alpha = -4\sin\phi_{inc} \tag{14}$$

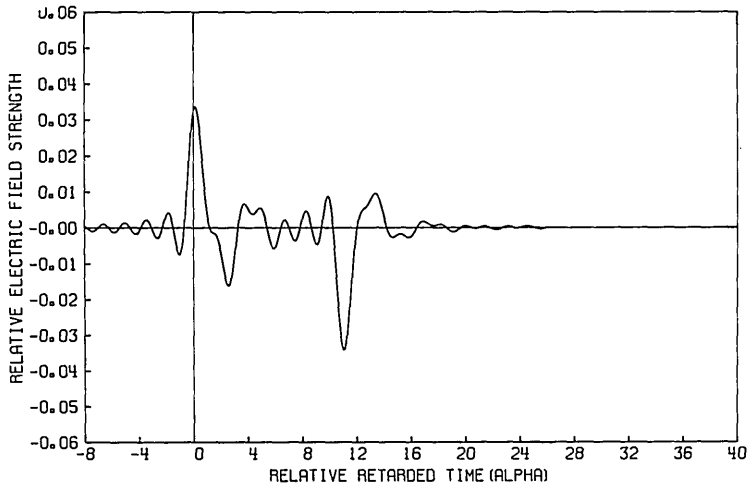


(a) Reflected field for normal incidence

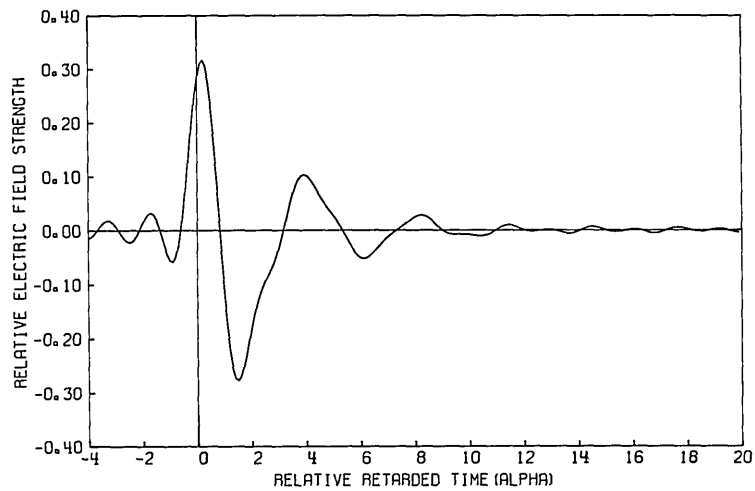


(b) Reflected field for incidence at -20 degrees

Fig. 2 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 240 + j0 ohms.

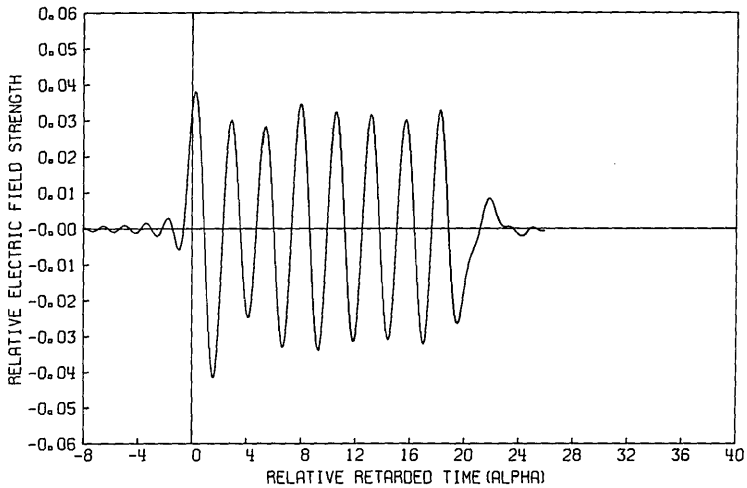


(c) Backscattered field for incidence at -20 degrees

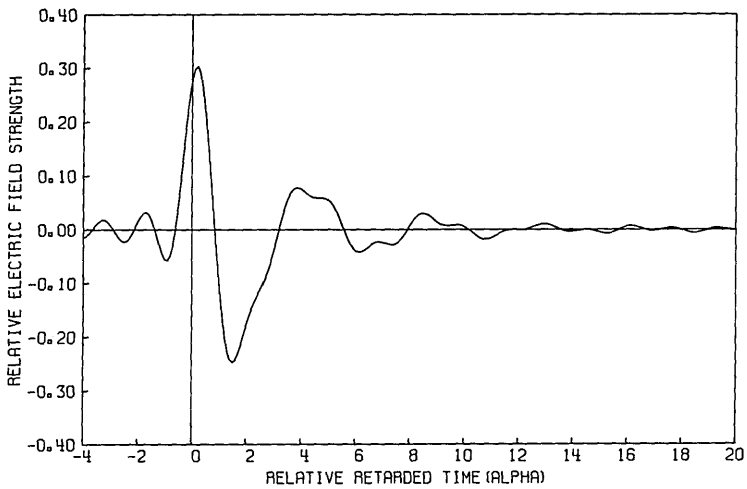


(d) Reflected field for incidence at -40 degrees

Fig. 2 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 240 + j0 ohms. (Continued)

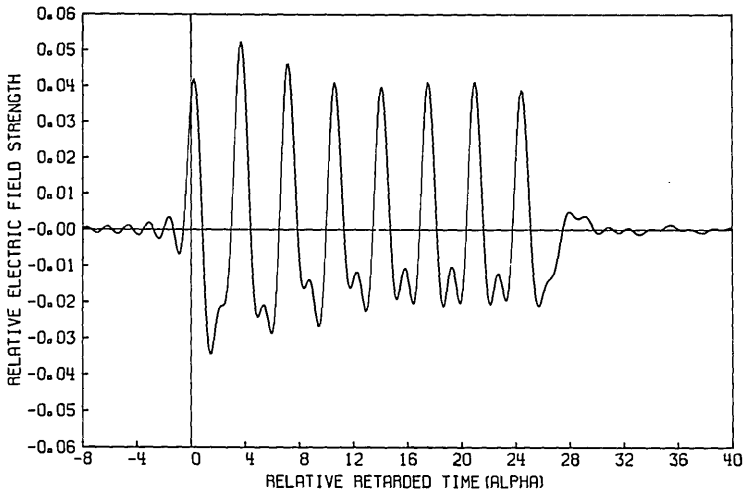


(e) Backscattered field for incidence at -40 degrees



(f) Reflected field for incidence at -60 degrees

Fig. 2 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 240 + j0 ohms. (Continued)



(g) Backscattered field for incidence at -60 degrees

Fig. 2 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 240 + j0 ohms. (Continued)

From Eq. (14), $\Delta\alpha$ is 2.57 for ϕ_{inc} = -40 degrees and 3.46 for ϕ_{inc} = -60 degrees. The values agree with the α separation of the positively going zeros of the transient fields portrayed in Figs. 2e and 2g respectively. In fact, the number of elements in the model array is easily determined from either Fig. 2e or Fig. 2g by simply counting the large positive peaks of the transient waves. It is apparent from Eq. (13) that the spacing of the elements of an array can be found from the backscattered field, provided the angle of incidence of the illumination is known and made sufficiently large to allow the measurement of the time displacement between the component transient fields. In the case of our model array with element spacing $2h$, an angle of incidence of 40 degrees is apparently adequate for this measurement with the exception of the case of Fig. 3, where the reactive load impedance $72 + j(300\ \omega h/c - 740\ c/\omega h)$ presents a poor match to the internal impedance of the elements of the array over the frequency range of interest ($0.05 \leq \omega h/c \leq 4.05$). In fact, as shown in Ref. 9, at normal incidence the reactance of the load referred to above is approximately equal to the internal reactance of the elements in the range between 1.0 and 2.0 in $\omega h/c$. The basic reflected transient fields of Fig. 3 are relatively long in duration, and as a result the beginning of the component transients are not easily discernible in the backscattered transient waves, because the component transients overlap considerably in time. The reflected transient waves of Figs. 3a, 3b, 3d, and 3f exhibit oscillations at two frequencies: the lower one relates to the fundamental element resonance at $\omega h/c = \pi/2$, and the higher frequency relates to a resonance of the elements at $\omega h/c = 3.55$, where the load reactance tunes out the element reactance.

A closer examination of Fig. 3g reveals the existence of some constructive interference between the component waves. The time delay between successive transient waves matches the time separation of the first and second large positive peaks of the component transient form shown in Fig. 3f. Because of the nearly periodic character of the component wave between -0.6 and 6.7 in α, we can, by a suitable choice of the angle of incidence, cause the strength of the backscattered field to be a maximum, as shown in Fig. 3g. Notice in Fig. 3g that the first large peak of the ith transient component approximately coincides in time with the second large peak of the $(i - 1)$th component, where i also is the numeral appearing on the large peaks of the transient wave. The

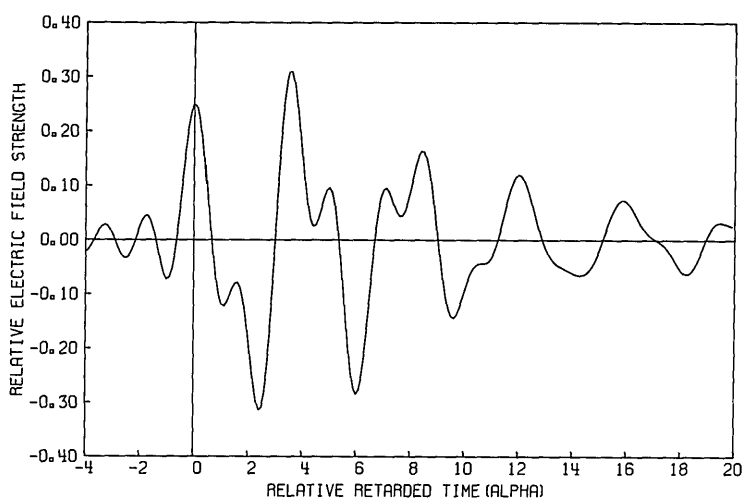
numbered large peaks are separated almost uniformly by an amount $\Delta\alpha = 3.5 \approx -4 \sin \phi_{inc}$, the α delay between successive transient components. The first two large positive (or negative) peaks of the component transients are separated by approximately 3.5 units in α (Fig. 3f).

In general the discussion of Fig. 2 applies as well to explain the character of the graphs of Figs. 4, 5, and 6. However, a few additional remarks about Figs. 4 and 5 seem appropriate.

In Figs. 4a, 4b, 4d, and 4f the transient waves appear to have a damped oscillatory tail with a period approximately equal to the reciprocal of the fundamental resonant frequency of the isolated elements of the array.

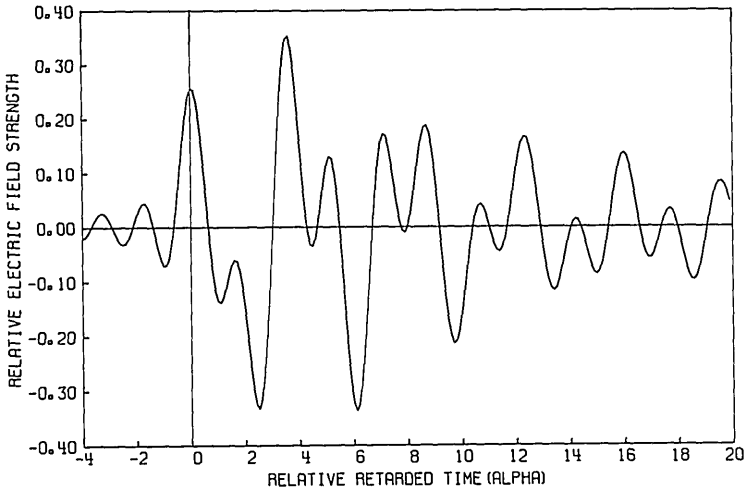
The impedance function of the load in Fig. 5 presents approximately a conjugate match to the effective internal impedance of the elements of the array at normal incidence over a range in $\omega h/c$ extending from 1 to 2.2. Apparently this load impedance with its better match to the antenna impedance allows more current to flow in the elements over a broad band than is allowed by the other loads considered in this study. This accounts for the larger energy scattered by the array when using the matched load than when using the other loads.

The larger, resistive, frequency-invariant loads of 240 and 600 ohms present a mismatch at all frequencies contained in our Fourier integral with finite limits. The model array when using these two loads scatters less energy than the array with center loads of 72 ohms resistance, because the induced currents in the array are smaller for the larger load impedances.

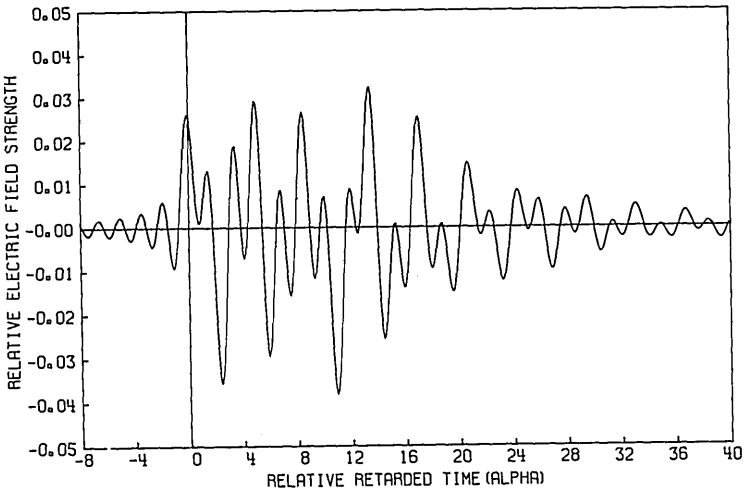


(a) Reflected field for normal incidence

Fig. 3 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $72 + j(300\omega h/c - 740c/\omega h)$ ohms.

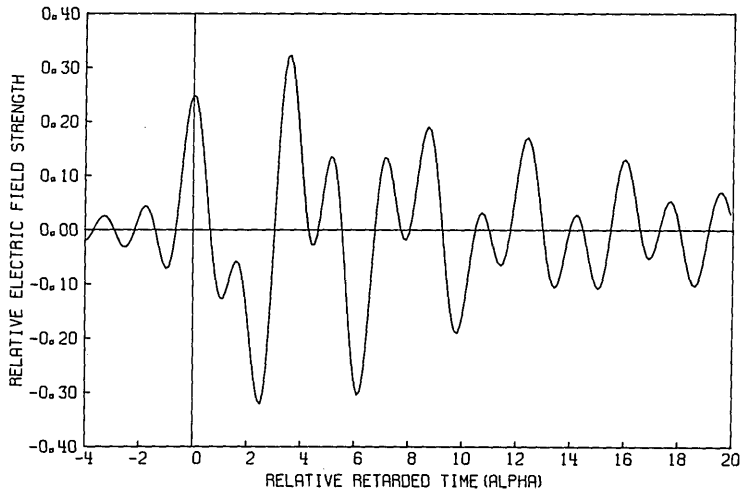


(b) Reflected field for incidence at -20 degrees

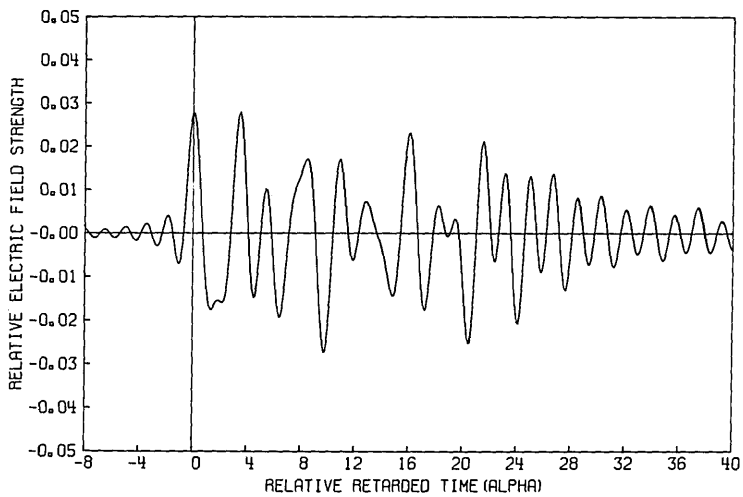


(c) Backscattered field for incidence at -20 degrees

Fig. 3 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $72 + j(300\omega h/c - 740c/\omega h)$ ohms. (Continued)

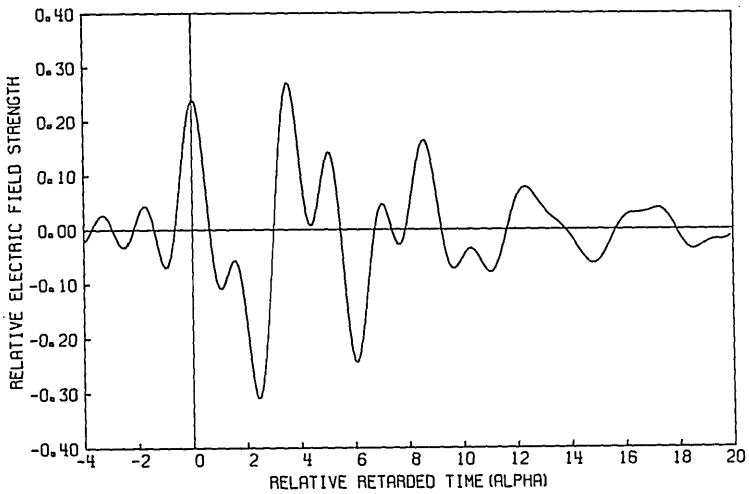


(d) Reflected field for incidence at -40 degrees

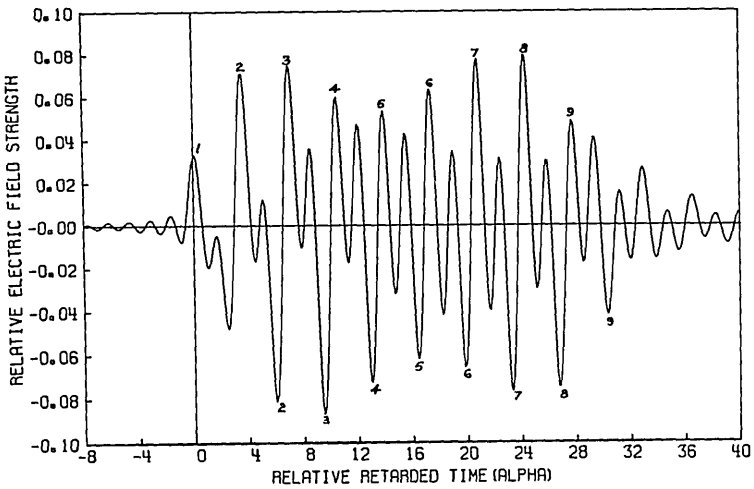


(e) Backscattered field for incidence at -40 degrees

Fig. 3 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $72 + j(300\omega h/c - 740c/\omega h)$ ohms. (Continued)

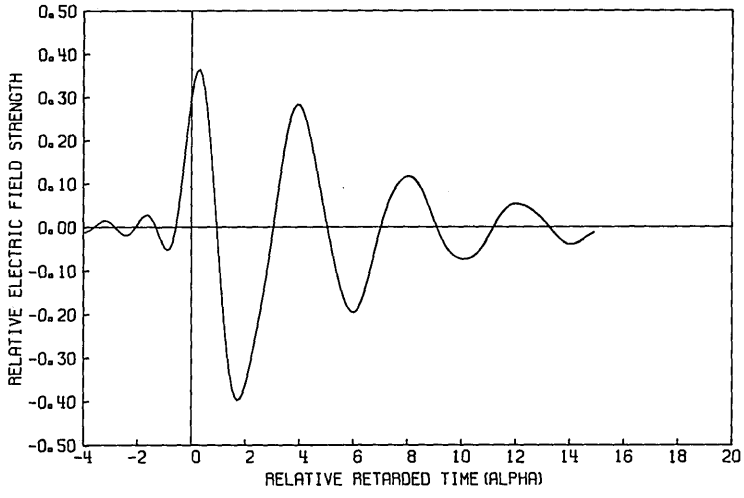


(f) Reflected field for incidence at -60 degrees

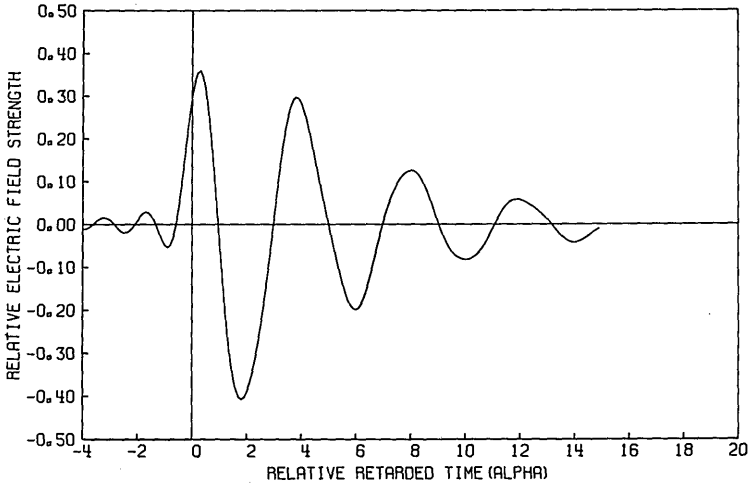


(g) Backscattered field for incidence at -60 degrees

Fig. 3 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $72 + j(300\omega h/c - 740c/\omega h)$ ohms. (Continued)

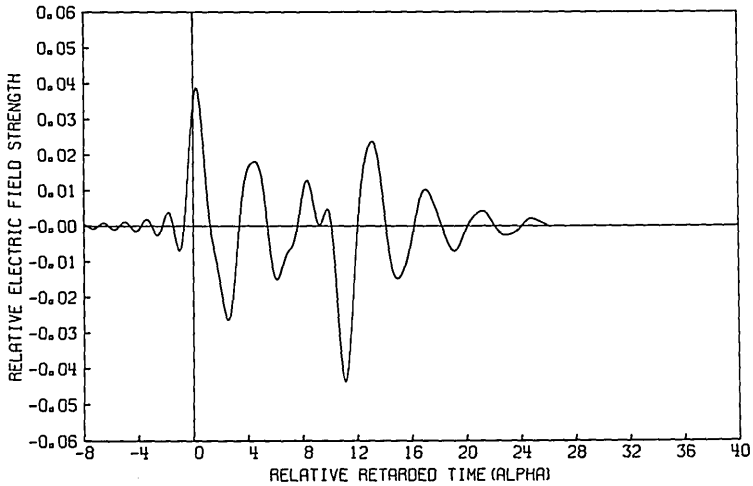


(a) Reflected field for normal incidence

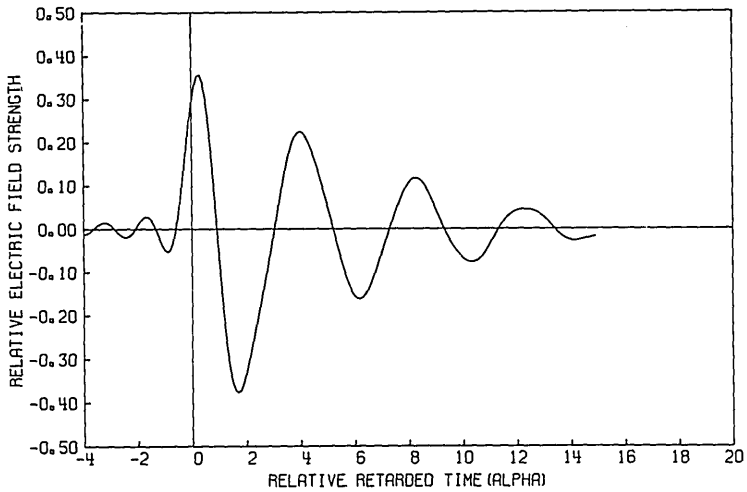


(b) Reflected field for incidence at -20 degrees

Fig. 4 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 72 + j0 ohms.

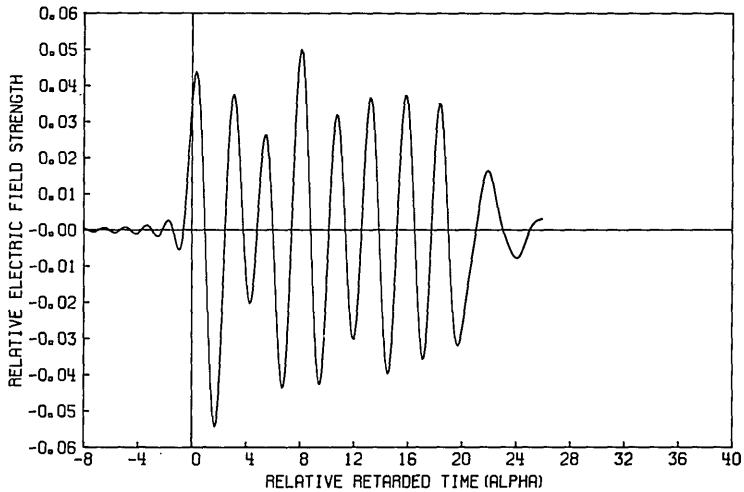


(c) Backscattered field for incidence at -20 degrees

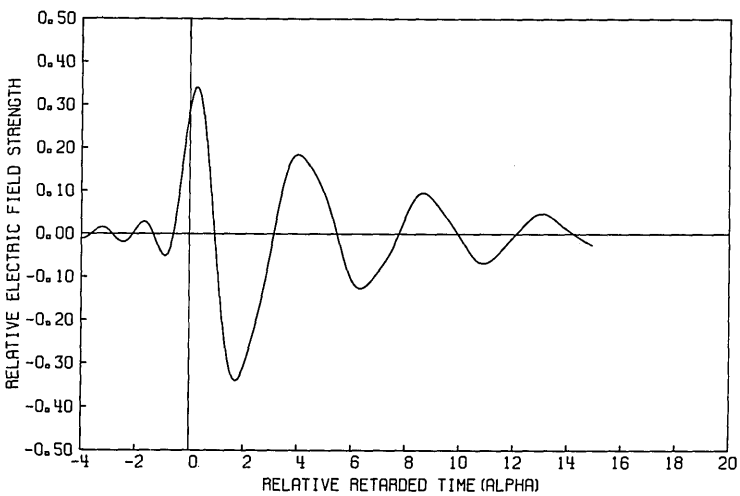


(d) Reflected field for incidence at -40 degrees

Fig. 4 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 72 + j0 ohms. (Continued)

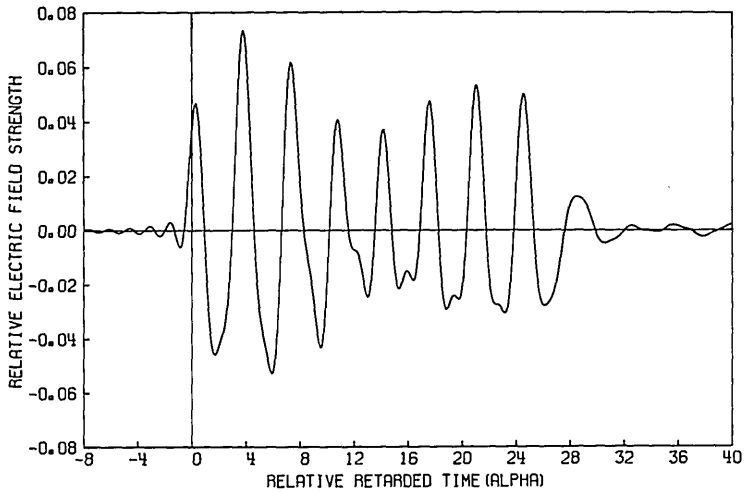


(e) Backscattered field for incidence at -40 degrees



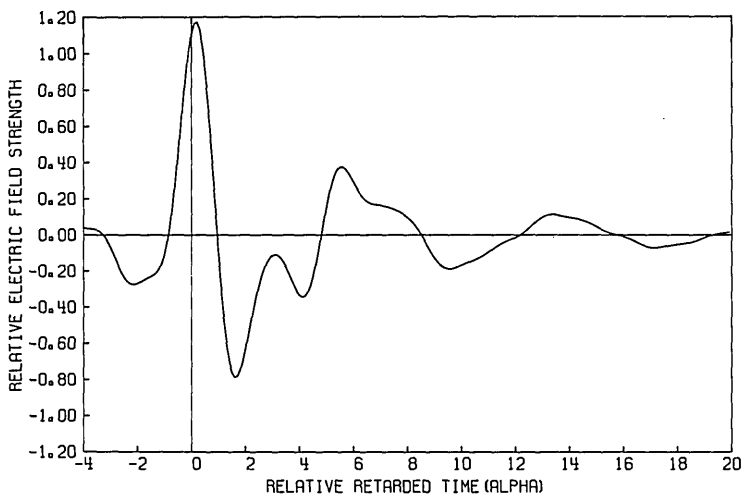
(f) Reflected field for incidence at -60 degrees

Fig. 4 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 72 + j0 ohms. (Continued)



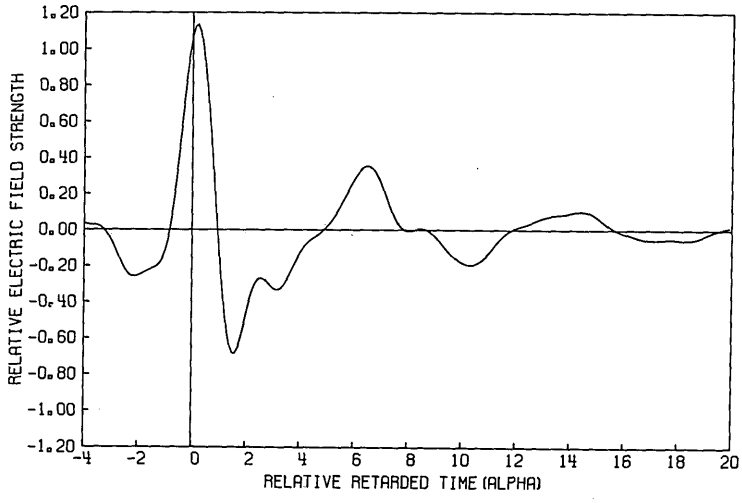
(g) Backscattered field for incidence at -60 degrees

Fig. 4 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 72 + j0 ohms. (Continued)

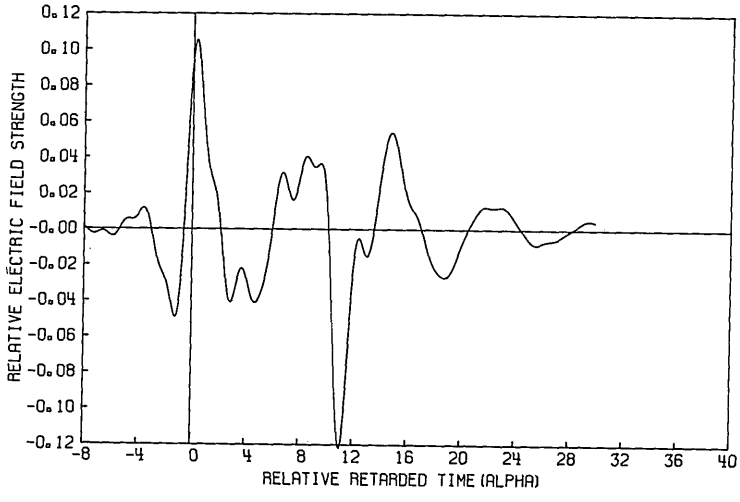


(a) Reflected field for normal incidence

Fig. 5 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $-17 + 39\omega h/c + 9\omega^2 h^2/c^2 + j(1160 - 743\omega h/c)$ ohms.

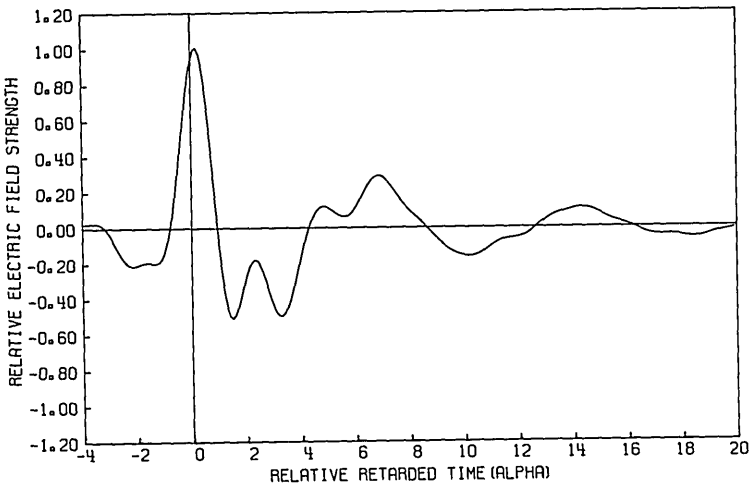


(b) Reflected field for incidence at -20 degrees

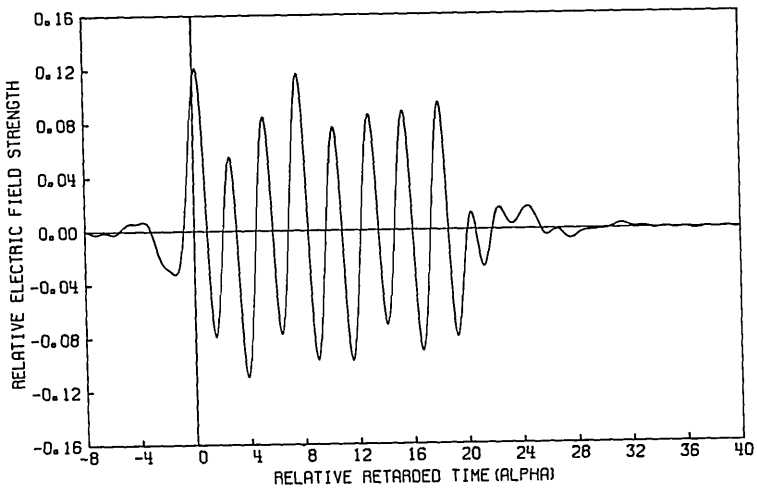


(c) Backscattered field for incidence at -20 degrees

Fig. 5 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $-17 + 39\omega h/c + 9\omega^2 h^2/c^2 + j(1160 - 743\omega h/c)$ ohms. (Continued)

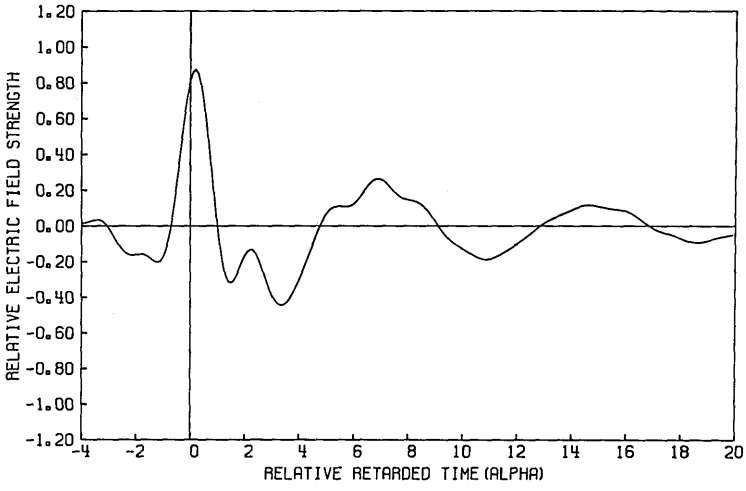


(d) Reflected field for incidence at -40 degrees

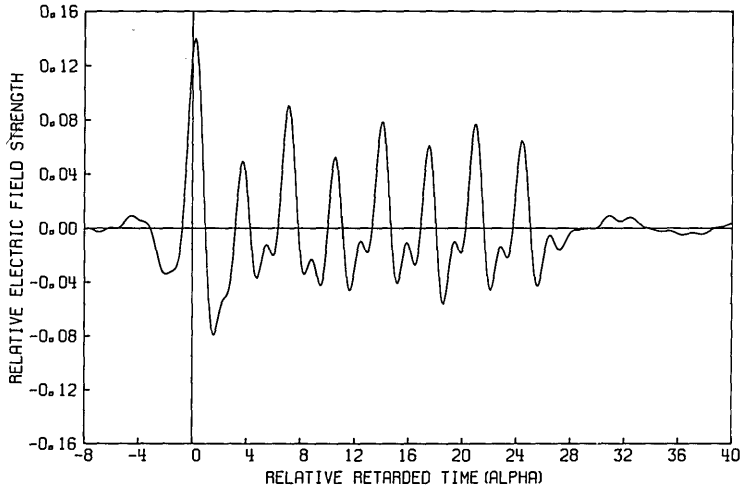


(e) Backscattered field for incidence at -40 degrees

Fig. 5 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $-17 + 39\omega h/c + 9\omega^2 h^2/c^2 + j(1160 - 743\omega h/c)$ ohms. (Continued)

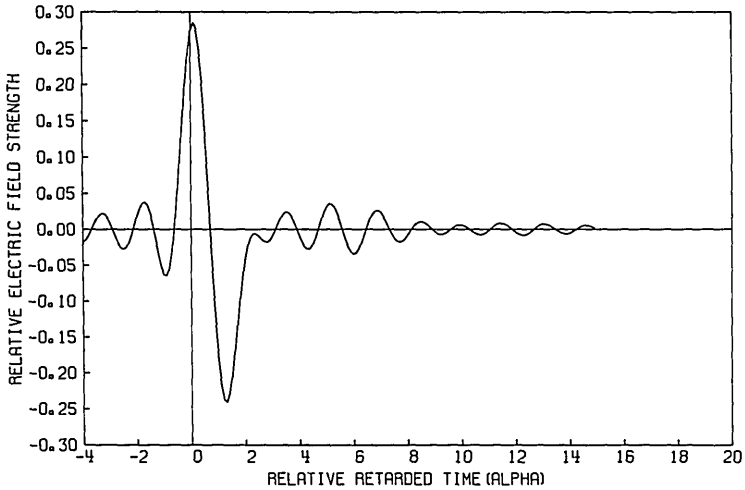


(f) Reflected field for incidence at -60 degrees

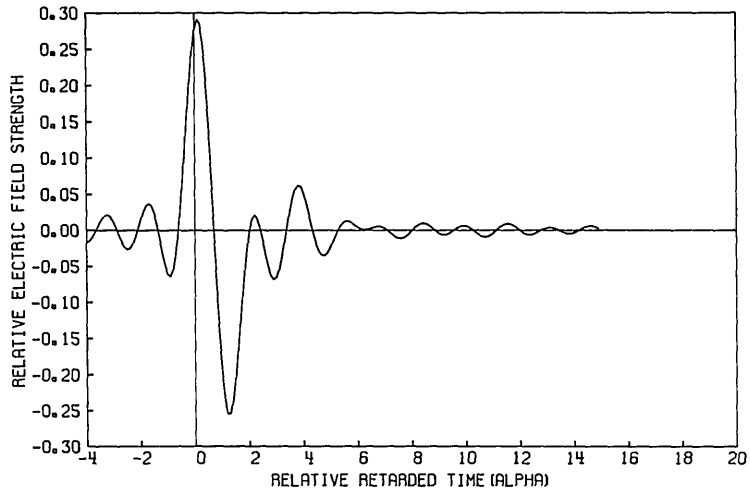


(g) Backscattered field for incidence at -60 degrees

Fig. 5 - The scattered-field response of the model array to a step-function illumination. The load impedance function is $-17 + 39\omega h/c + 9\omega^2 h^2/c^2 + j(1160 - 743\omega h/c)$ ohms. (Continued)

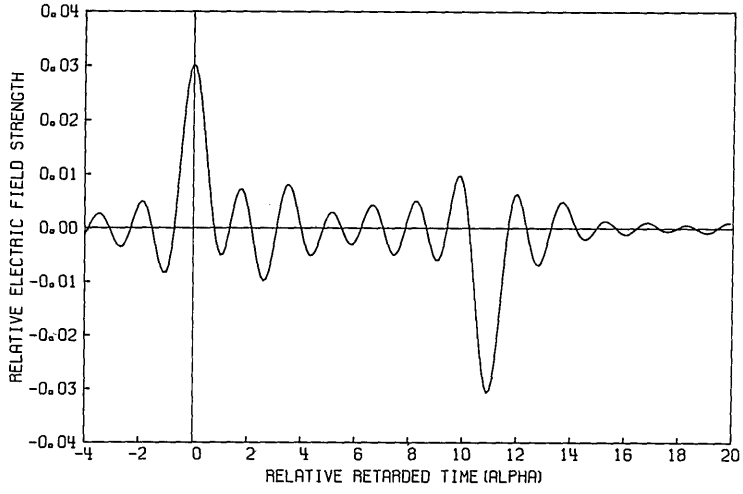


(a) Reflected field for normal incidence

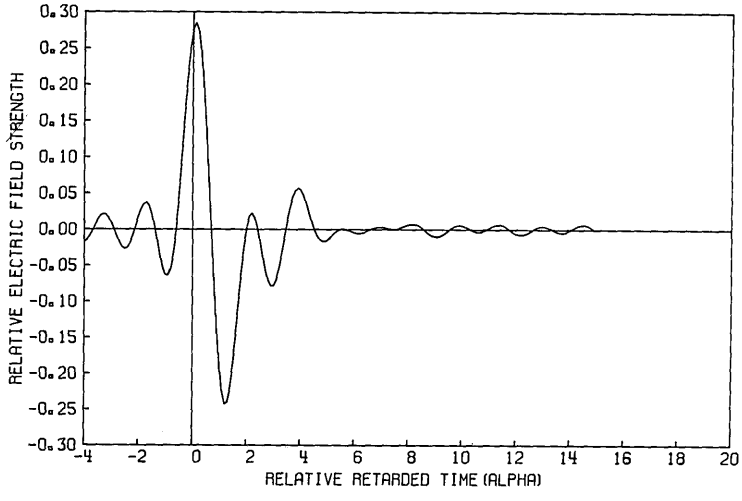


(b) Reflected field for incidence at -20 degrees

Fig. 6 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 600 + j0 ohms.

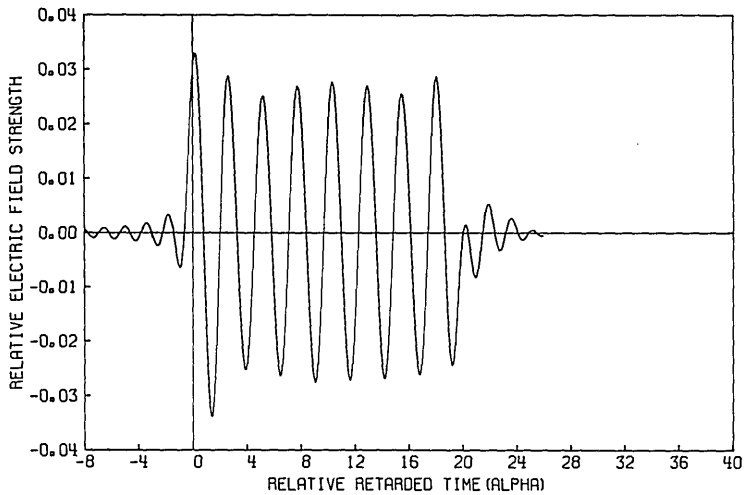


(c) Backscattered field for incidence at -20 degrees

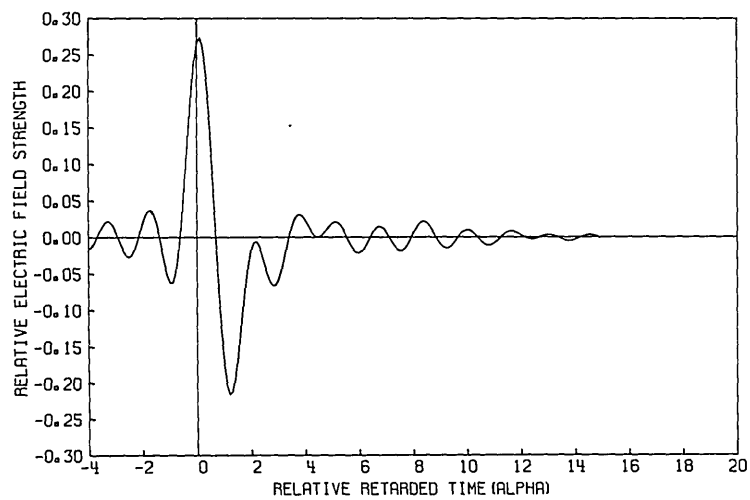


(d) Reflected field for incidence at -40 degrees

Fig. 6 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 600 + j0 ohms. (Continued)

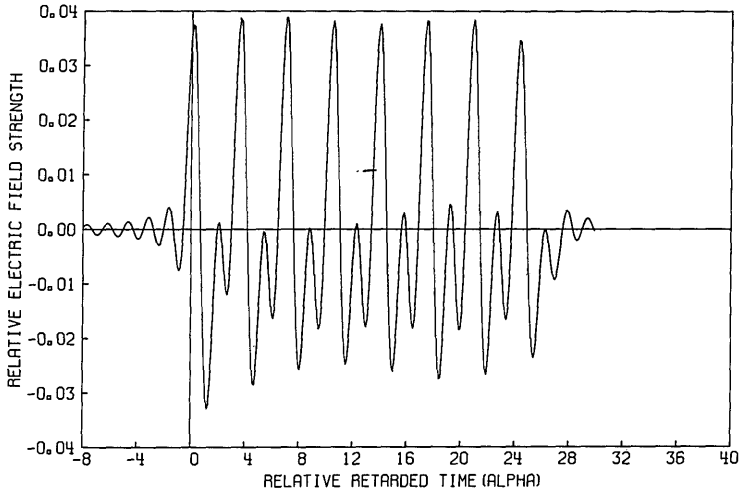


(e) Backscattered field for incidence at -40 degrees



(f) Reflected field for incidence at -60 degrees

Fig. 6 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 600 + j0 ohms. (Continued)



(g) Backscattered field for incidence at -60 degrees

Fig. 6 - The scattered-field response of the model array to a step-function illumination. The load impedance function is 600 + j0 ohms. (Continued)

CONCLUSIONS

When a linear array of N elements is illuminated by a step function of electromagnetic field, the resulting transient field scattered in any direction can be synthesized approximately from N suitably delayed replicas of $(1/N)$ times the field scattered in the reflected direction. The $(i + 1)$th-component field is delayed relative to the ith-component field by $-d/c(\sin\phi + \sin\phi_{inc})$. The scattered field reaches its maximum amplitude in the reflected direction ($\phi = -\phi_{inc}$) because the component transient fields add constructively in step with each other only in this direction. The time duration of the scattered field has a maximum value in the backscattered direction of observation. The basic field scattered in the reflected direction characterizes the array and the incident illumination. The reflected field for a given illumination manifests the interaction between the elements of the array, the effect of the impedance function of the center loads, and the physical dimensions and spacings of the cylindrical elements; however this information is very difficult to extract from an analysis of the time variation of the reflected field. When the reflected transient fields are well damped or nearly periodic in character during several cycles of oscillation, the number of array elements and their spacing can be determined simply from an observation of the transient backscattered field as a function of time, provided the angle of incidence of the illumination is known and made sufficiently large to separate the component transient waves in time.

Of necessity our study of array scattering is limited to a small number of load impedances. Among the loads considered, it is found that the smallest H-plane scattered energy occurs when the center loads of the array are nonreactive impedances of 600 ohms (invariant with ω) and that the scattered energy is largest when the load impedances represent approximately a conjugate match to the effective internal impedance of the array elements over the principal part of the frequency spectrum of the transient illumination.

ACKNOWLEDGMENT

The author is indebted to Jon D. Wilson for developing the computer programs used in this investigation

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Appendix A

A COMPARISON OF THE SYNTHESIS FORMULA WITH THE FOURIER INTEGRAL REPRESENTATION OF THE SCATTERED FIELD

To show quantitatively how well the approximate synthesis formula, Eq. (11), agrees with the more accurate formula, Eq. (12), a comparison of the two formulas is given in Table A1 at a number of salient points on the backscattered transient wave shown in Fig. 2c. Table A2 shows a similar comparison made on the wave shown in Fig. 2e, where $\phi_{inc} = -40$ degrees. The agreement is quite good at all points.

Table A1
Comparison of Eq. (11) and Eq. (12) When ϕ_{inc} is -20 Degrees and the Load Impedance Function is 240 + j0 ohms (Fig. 2c)

a	Field From Eq. (11)	Field From Eq. (12)	Point of Comparison
0.10	0.0331	0.0335	First large positive peak
2.50	-0.017	-0.016	First large negative peak
3.27	–	0.00	First zero after large negative peak
3.28	0.00	–	First zero after large negative peak
9.90	0.0083	0.0087	Small positive peak just before large negative peak
10.25	–	0.00	Zero just before large negative peak
10.30	0.00	–	Zero just before large negative peak
11.00	-0.0335	-0.0340	Last large negative peak
12.05	–	0.00	Zero immediately following large negative peak
12.10	0.00	–	Zero immediately following large negative peak

Table A2
Comparison of Eq. (11) and Eq. (12) When ϕ_{inc} is -40 Degrees and the Load Impedance Function is 240 + j0 ohms (Fig. 2e)

α	Field From Eq. (11)	Field From Eq. (12)	Point of Comparison
0.20	0.037	0.038	First large positive peak
0.87	0.00	0.00	First zero following first large positive peak
1.50	-0.0406	-0.0412	First large negative peak
2.20	0.00	–	First zero following first large negative peak
2.27	–	0.00	First zero following first large negative peak
2.90	0.0290	0.030	Second large positive peak
18.20	0.0325	0.0327	Eighth large positive peak
19.50	–	-0.0264	Last large negative peak
19.60	-0.0264	–	Last large negative peak
21.20	–	0.00	First zero after last large negative peak
21.30	0.00	–	First zero after last large negative peak

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13. ABSTRACT

A limited investigation is made of the scattered-field response of a linear array of center-loaded cylindrical elements illuminated by a plane electromagnetic wave with a step function of time variation. The theoretical approach utilizes Fourier's theorem to express the transient response as a sum of the steady-state responses to the individual sine-wave components of the step-function illumination.

The number of array elements and their spacing can be determined simply from the time variation of the backscattered field, provided the angle of incidence of the illumination is known and adequately large to separate the component transient waves.

In the case of an N-element array, it is found that the transient field scattered in any arbitrary direction in the H plane can be represented as a summation of N appropriately time-delayed components, each with the same time variation as the field scattered in the reflected direction but with an amplitude reduced by a factor of $1/N$.

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Backscattered fields						
Step-function illumination						
Reflected fields						
Angle of incidence						