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Inclination Perturbations of Polar-Orbit Satellites

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CONTENTS

ABSTRACT

This report describes an investigation of the ability to predict long-term orbital elements for medium-height (satellite orbiting the earth at altitudes above the regions of appreciable drag), earth-orbiting satellites. The particular types of satellites studied are those having nearly circular, polar orbits. The primary element of interest is orbital inclination.

A theoretical equation for predicting the change in inclination caused by the gravitational attraction of the sun and moon is given. Expressions for the orbital elements of the sun and moon required in this inclination-predicting equation are developed. A FORTRAN program listing and computation procedure for determining the inclination of a satellite at any time after a chosen epoch is in the appendix.

Nine satellites were selected for study. For these satellites, comparison plots of observed and predicted inclinations over a 3-year time interval are shown. The agreement between predicted and observed inclinations is very good for most of the satellites. Theoretical plots of the inclination of Satellite 902 over time intervals of up to 300 years are included.

PROBLEM STATUS

This is an interim report; work is continuing on the problem.

AUTHORIZATION

NRL Problem R04-16
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INCLINATION PERTURBATIONS OF POLAR-ORBITING SATELLITES

INTRODUCTION

With the advent of modern satellite navigation systems, the ability to generate a satellite ephemeris which is valid for a long period of time has become important. Much effort is now being directed toward the development of techniques for accurately predicting the position of a satellite at any instant of time. Sophisticated models of the earth's gravitational field and of the upper atmosphere have been developed to explain the observed complex motion of an orbiting satellite. Much work is left to be done in this area of predicting satellite orbits.

In many cases, knowledge of the short-term satellite position is not the only important factor. Long-term orbit predictions can be as important in applications wherein station keeping is a necessity or wherein knowledge of the maximum deviations of orbital elements from their initial values is required. Examples of such applications are satellites in synchronous or subsynchronous orbit or satellites required to traverse a specified constant or nearly constant ground track over a long period of time.

In line with the above purposes, it is desirable to check the validity of existing theory on the long-term prediction of orbits and to ascertain the important factors which must be controlled or compensated for in order to maintain any desired satellite ephemeris.

COORDINATE SYSTEM AND TERMINOLOGY

The position of a satellite is described by the six conventional, elliptical, orbital elements. The position of the orbital plane along the equator is defined by the right ascension of the ascending node Ω and is measured eastward from Aries (the vernal equinox). The angle between the equatorial and orbital planes is defined as the orbital inclination i. The position of the ellipse in its own plane is defined by the argument of perigee ω. The size and obliquity of the orbit are defined by the semimajor axis a and the eccentricity e, respectively. The instantaneous position of the satellite in the orbital plane is defined by the mean anamoly M measured from the argument of perigee. Figure 1 illustrates the above definitions.

THEORY

Of the six orbital elements defined in the last section, orbital inclination is one of the easiest to work with from the standpoint of both ease of prediction and physical interpretation. Therefore, in the following work, inclination will be the element of interest.

For high, circular-orbiting satellites, the primary forces affecting satellite motion are variations in the gravitational field of the earth and the gravitational attraction of the sun and moon. For satellites with a large surface-area-to-mass ratio, solar radiation pressure may also be important. The relative importance of the above influences vary according to the orbital element considered. For example, the most prominent influence on right ascension and argument of perigee is related to irregularities in the earth's

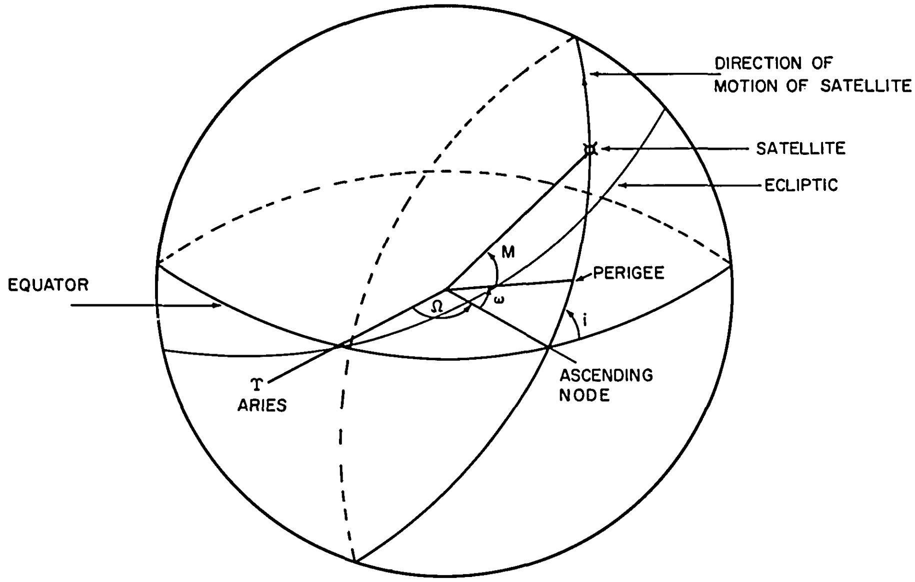

Fig. 1 - Elliptical coordinate system

gravitational field, while the major effect on orbital inclination is caused by the lunisolar forces. Perturbations in inclination are only indirectly caused by the earth through changes in the other satellite elements (e.g., right ascension, argument of perigee, etc.).

Theoretical expressions for the rates of change of orbital elements for satellites under the influence of lunisolar gravity forces have been developed by several authors (1,2,3). A report by G.E. Cook (1), for example, lists expressions for computing first-order rates of change in orbit parameters. An extension of Cook's work in a paper by Myrna Lewis (2) gives the pertinent expressions in a form more suitable for computation; the equation listed for inclination is the one used to obtain predictions in this report. One of the most comprehensive papers to date on perturbations of satellite orbits is by R.H. Gooding (3). This report contains higher-order terms, as well as the primary ones appearing in the above reports, for computing the rates of change of satellite orbital elements. Several other possible perturbations are also discussed in his report. Since the purpose of this report is not to study the theory of satellite perturbations, but to determine its validity and usefulness, only the appropriate results of these theoretical developments will be given.

The rate of change of orbital inclination may be predicted, fairly accurately, by Eq. 118 in Ref. 2. If the satellite orbit of interest may be regarded as circular, those terms in the equation multiplied by e^2 may be eliminated. Also, for polar-orbit satellites, terms involving $\cos i$ may also be omitted. The resulting, simplified version of this equation is given below:

$$\frac{di}{dt} = \frac{3K}{4n\,(1-e^2)^{1/2}}\left(1+\frac{3}{2}e^2\right)(\sin i)\left\{\sin^4\frac{i_d}{2}\left[\sin 2\left(\beta+\omega_d+M_d\right) - \frac{e_d}{2}\sin 2\left(\beta+\omega_d+\frac{M_d}{2}\right)+\frac{7e_d}{2}\sin\left(2\beta+2\omega_d+3M_d\right)\right]\right. \qquad (1)$$

(Equation (1) Continued)

$$+\cos^4\frac{i_d}{2}\left[\sin 2\left(\beta-\omega_d-M_d\right)-\frac{e_d}{2}\sin 2\left(\beta-\omega_d-\frac{M_d}{2}\right)\right.$$
$$\left.+\frac{7e_d}{2}\sin\left(2\beta-2\omega_d-3M_d\right)\right]+1/2\sin^2 i_d\sin^2 2\beta\left[1+3e_d\cos M_d\right]\Bigg\}. \tag{1}$$

In the above equation, mean orbital elements are to be used (short-period terms, i.e., those related to time intervals of less than one satellite revolution are integrated out). Assumptions used in the development of this equation are that the perturbing body does not move during one revolution of the satellite, and that the semimajor axis of the satellite orbit is much smaller than the distance from the earth to the moon—at least smaller than a tenth of the earth-moon distance. In this expression, variables having a subscript d refer to the orbital elements of the disturbing body (sun or moon). Two quantities appearing in the equation need some clarification. The variable β is the right ascension (Ω) of the satellite if di/dt due to the sun is being calculated, and $\beta = \Omega - \Omega_d$ if di/dt due to the moon is being computed (Ω_d for the sun is by definition, zero). The constant K is defined as

$$K = \frac{\mu_d}{a_d^3}, \tag{2}$$

where μ_d is the product of the universal gravitation constant G and the mass of the disturbing body. The remaining variables in the equation require no special mention but are defined in the glossary at the end of the report.

APPLICATION

In order to use Eq. (1) for inclination prediction, the orbital elements of the sun and moon and certain satellite parameters must be determined. The elements of the sun present little problem. They are directly available from "The American Ephemeris and Nautical Almanac" (4) for any particular year of interest. Approximations of the equations listed in Ephemeris for predicting orbital elements of the sun are given below. These equations yield valid predictions over several years (from 1966).

$$L \text{ (mean longitude)} = 279^\circ\!.6967 + 0^\circ\!.98564734\ d \tag{3}$$

$$\Gamma \text{ (mean longitude of perigee)} = 281^\circ\!.22083 + 0^\circ\!.00004707\ d \tag{4}$$

$$L - \Gamma \text{ (mean anomaly)} = 358^\circ\!.47583 + 0^\circ\!.9856002\ d \tag{5}$$

$$e_s \text{ (eccentricity)} = 0.01673 \tag{6}$$

$$\epsilon \text{ (mean obliquity or inclination)} = 23^\circ\!.4432 \tag{7}$$

In the above equations, d is in units of ephemeris days measured from the epoch 1900 January 0.5 E.T.

Similar expressions exist for the orbital elements of the moon and are listed here.

$$☾ \text{ (mean longitude)} = 270^\circ\!.434 + 13^\circ\!.17639653\ d \tag{8}$$

$$\Gamma' \text{ (mean longitude of perigee)} = 334^\circ\!.3294 + 0^\circ\!.11140408\ d \tag{9}$$

$$\Omega\ (\text{right ascension}) = 259^\circ.1833 - 0^\circ.05295392\ d \tag{10}$$

$$e_M\ (\text{eccentricity}) = 0.054900489 \tag{11}$$

Unfortunately, these elements are measured from the mean equinox of date along the ecliptic and then along the lunar orbit. Therefore, they must be transformed to the coordinate system defined at the beginning of this paper (see Fig. 1). This is easily accomplished by the use of spherical trigonometry. Figure 2 (a modified form of a diagram appearing in Cook's paper (1)) illustrates the definitions of the moon's elements listed above and the required transformed elements. Expressions for obtaining the desired elements derived with the aid of Fig. 2 are

$$i\ (\text{inclination}) = \cos^{-1}(\cos\epsilon\cos\alpha - \sin\epsilon\sin'\alpha\ \cos\Omega)$$

and

$$\omega\ (\text{argument of perigee}) = \sigma + \Gamma - \Omega,$$

$$\text{where } \sigma = \sin^{-1}\left(\frac{\sin\epsilon\ \sin\Omega}{\sin i}\right) \text{ and } \epsilon = \text{obliquity of the ecliptic} \tag{12}$$

$$\Omega\ (\text{right ascension}) = \sin^{-1}\left(\frac{\sin\alpha\ \sin\Omega}{\sin i}\right), \text{ where } \alpha \text{ is the inclination of the lunar} \tag{13}$$

orbit to the ecliptic

$$L\ (\text{mean longitude}) = ☾ - \Omega + \sigma \tag{14}$$

$$M\ (\text{mean anomaly}) = ☾ - \Gamma \tag{15}$$

$$e_M\ (\text{eccentricity}) = \text{same as Eq. (11)}.$$

With the aid of all of the above expressions, we can now predict the orbital elements of the sun and moon for any desired time. The only remaining information that is needed is the orbital elements (other than inclination, which is required only at the chosen epoch) of the satellite as a function of time. These elements need not be known to great accuracy because, with the exception of mean motion and eccentricity, each element appears only as a sin-cos argument in the prediction equation for inclination. Two of the satellite elements, right ascension and argument of perigee, can be predicted with sufficient accuracy (i.e., sufficient for use in predicting inclination) by using the following expressions (modified forms of equations appearing in the Space Navigation Handbook (5) for their rates of change:

$$\dot{\Omega} = -\frac{2\pi n J_n \cos i}{a^2(1-e^2)^2} \tag{16}$$

and

$$\dot{\omega} = \frac{2\pi n J_n\left(2 - \frac{5}{2}\sin^2 i\right)}{a^2(1-e^2)^2}, \tag{17}$$

where J_n is the nth zonal harmonic of the earth. For the accuracy required of the predictions, only the major harmonic J_2 need be used. If n is in rev/day and a in units of earth radii, then the units of the results are rad/day. Given the right ascension and argument of perigee at any epoch, their values on succeeding days can be determined by simply adding the cumulative rates of change to the initial value. The eccentricity and the semimajor axis values (also mean motion, which may be derived from the semimajor axis), if changing appreciably during the required prediction time interval, must also be updated. It should be noted that the mean anomaly of the satellite is not required, as

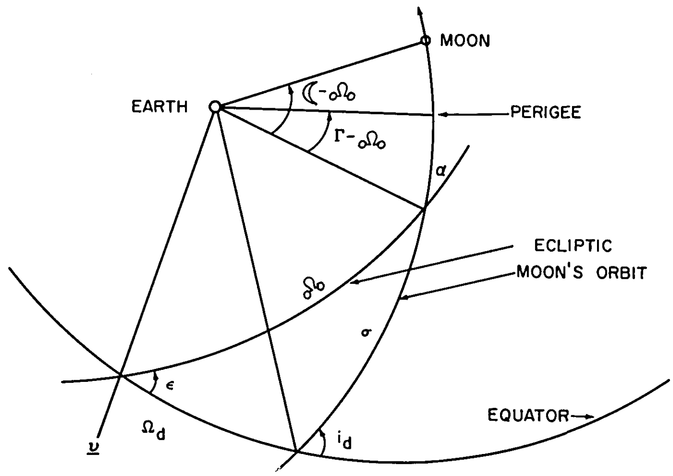

Fig. 2 - The orbital elements of the moon
(modified from Ref. 1)

it has been eliminated through integration in the development of Eq. (1). Equation (1) gives the average rate of change of inclination over one complete orbit.

PREDICTION PROGRAM

A FORTRAN program for computing the orbital inclination of a satellite was written incorporating the previous equations. This program was developed with the intent of predicting inclination only for those satellites in high, circular, polar orbits; thus, all of the terms in some of the previous equations, particularly Eq. (1), are not necessary. The general procedure in this program is first to undate the elements of the sun, and then to calculate the rate of change of inclination due to the sun. Next, the moon's elements are updated, and the rate of change of inclination due to the moon calculated. The net rate of change is then the linear combination of the two calculated rates of change. Finally, the satellite elements are updated in preparation for the computation of inclination for the next unit of time. This process is then repeated until the desired time span of predictions is reached.

The input data required by the program includes the initial satellite orbital elements with all angles in radians, semimajor axis in earth radii, and mean motion in deg/day (as noted previously, the mean anomaly of the satellite is not required). The number of ephemeris days from 1900 Jan 0.5 E.T. to the desired starting point in time is also required. Another time factor, which has not been previously discussed, appears in the program. The variable T, which is d/36525, is the time from 1900 Jan 0.5 E.T. to the prediction epoch in Julian centuries. It is only required for prediction over a time interval of many years. It appears in the more complete expressions for the orbital elements of the sun and moon in Ref. 4.

The time interval between successive predictions and corrections to all orbital elements is set for 1 day in the program. This can easily be changed, however, by simply modifying the factors added to d and T in the prediction program. The second index of the DO loop may also be altered to allow coverage of any desired prediction time span. The appendix contains the FORTRAN listing of this prediction program.

PREDICTIONS

It was decided that the best approach to evaluating the prediction capabilities of the program was to compare computed inclinations on certain existing satellites with available data obtained from the U.S. Naval Space Surveillance System. To examine this capability, nine polar-orbiting satellites were chosen. The epoch for predictions and data was chosen to be January 1, 1966 E.T. This epoch was chosen because the data prior to this time was not of sufficient accuracy to be useful for accomplishing the stated task. Improvements in the WS-434, the U.S. Naval Space Surveillance System in late 1965, vastly improved the data. The mean orbital elements of the nine selected satellites on January 1, 1966 appear in Table 1.

Table 1
Orbital Elements of Nine Satellites on January 1, 1966

Satellite	Period (min)	Period Rate (min/day)	Inclination (deg)	Eccentricity	Semimajor Axis (earth radii)	Right Ascension (deg)	Right Ascension Rate (deg/day)	Argument of Perigee (deg)	Argument of Perigee Rate (deg/day)
509	99.0553	-0.00005	90.658	0.00279	1.1114	253.602	0.079	130.206	-3.427
671	107.3350	-0.00000	89.900	0.00441	1.1725	4.956	-0.010	85.880	-2.844
704	107.1210	-0.00000	89.947	0.00331	1.1710	98.706	-0.005	290.443	-2.857
801	103.0663	-0.00001	90.505	0.00651	1.1412	42.382	0.055	102.182	-3.125
902	106.6006	-0.00000	89.900	0.00168	1.1672	327.698	-0.010	151.513	-2.890
959	106.3125	-0.00000	89.969	0.00396	1.1651	143.377	-0.003	287.878	-2.908
1314	111.5066	-0.00000	90.205	0.00279	1.2027	218.570	0.019	242.541	-2.602
1420	106.9138	-0.00001	89.976	0.00788	1.1695	311.469	-0.002	114.996	-2.871
1514	108.1308	-0.00000	90.012	0.00694	1.1783	355.603	0.001	267.997	-2.796

Several prediction runs were made for the nine satellites when the program debugging was completed. One of the first plots made of theoretical and observed data for Satellite 902 (the 10-kg, 14-in.-diameter NRL calsphere (6) is shown in Fig. 3. The observed and predicted data appear to agree well during most of 1966 and early 1967, but the two curves tend to drift apart in late 1967 and continue to separate during 1968. The initial conclusions made on the basis of this plot were that either an error existed in the computer program or some factor affecting inclination either had not been incorporated into the prediction theory, or had been incorrectly included.

After much labor in checking the computer program for errors, and checking the derivations of both the theoretical equations developed by Myrna Lewis (2) and the expressions for predicting the sun, moon, and satellite elements, it was concluded that no errors existed in the theory being used or in its application. Other factors not included in the theory which may affect inclination were also considered and found to be insignificant for the type of errors found (further mention of these neglected factors will occur later). The only other item left to check was the WS-434 data.

After studying available literature on WS-434 computation methods and through communication with presonnel at Space Surveillance System headquarters at Dahlgren, Virginia, it was finally determined that the problem was with the coordinate system references used. The theoretical data were referenced to the true equinox of date, while the WS-434 data were referenced to two different mean equinoxes during the 3-year time interval considered. The WS-434 data collected during 1966 were referenced to the mean equinox of January 1, 1966. All data collected since the end of 1966 have been referenced

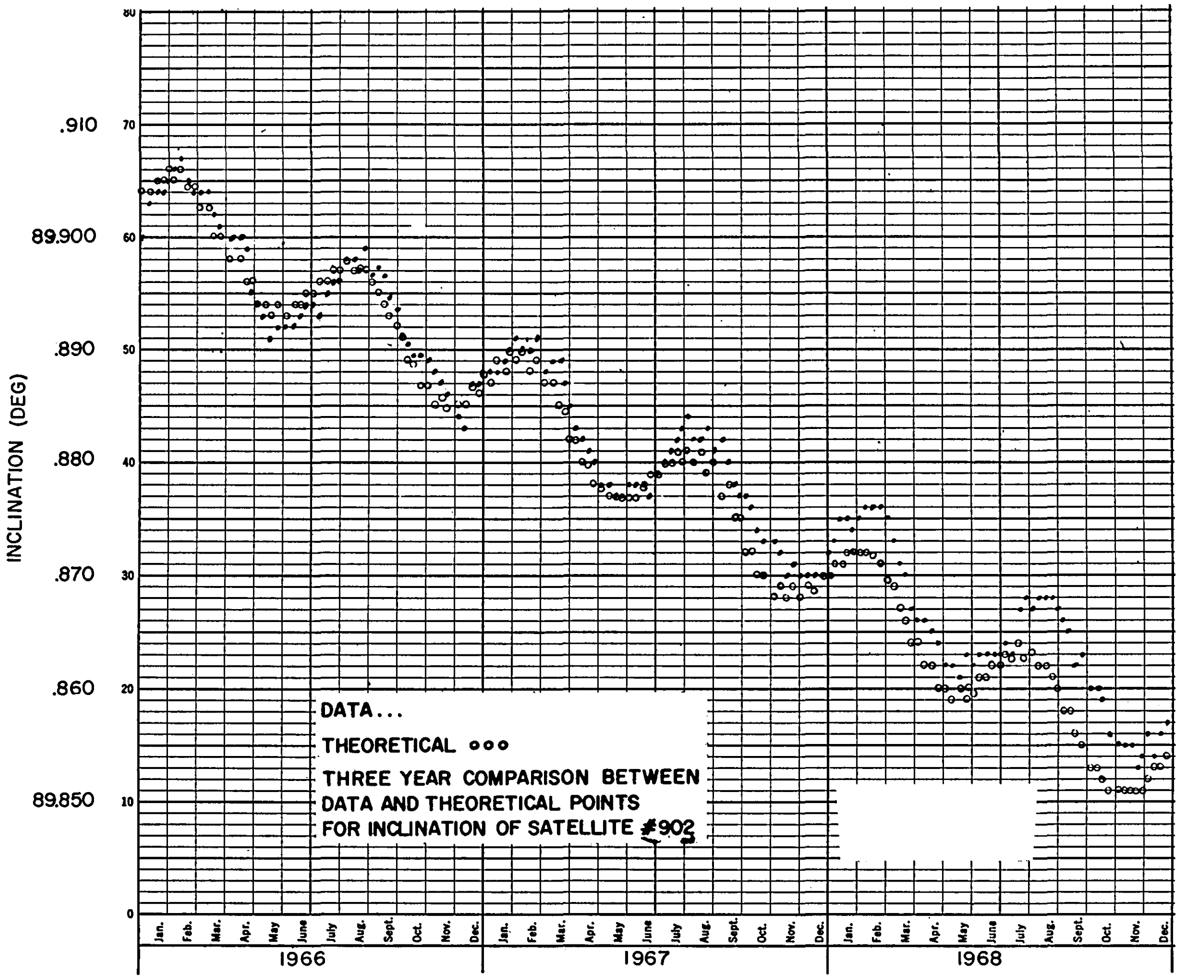

Fig. 3 - Theoretical and observed inclination for Satellite 902 before correction for precession and nutation

to the mean equinox of January 1, 1967. Since a true equinox reference is required, it was necessary to correct the data by adding the nutational effects and the cumulative precessional effects occurring after each reference epoch.

ADJUSTING DATA COORDINATE SYSTEM

General precession consists of motion of the vernal equinox backwards along the ecliptic at the approximate rate of 50."3 per year (the full period is about 26,000 years). This is the only type of precession of any significance for the purposes of this paper (there exist other precessional terms of much smaller magnitude). There are also two nutational terms affecting the position of the reference or the orbital elements of the satellite, sun, and moon. Nutation in longitude involves motion of the equinox along the ecliptic, as does precession. Nutation in obliquity causes a change in the obliquity of the ecliptic (inclination of the sun's apparent orbit). Precessional and nutational effects are caused by the gravitational attraction of the sun and moon on the equatorial bulge of the earth.

The motion of the coordinate system reference and its effect on satellite inclination is illustrated by Fig. 4 for the case of a satellite having a right ascension in the first quadrant. With the aid of this figure, an expression may be developed for calculating the change in inclination Δi, for any particular precession or nutation, which must be added to the observed inclination data. From a computational point of view, it is not necessary to have only a single equation for calculating Δi. A set of short, ordered equations is of equal value, and is much easier to develop. Such a series of equations for the case illustrated in Fig. 4 follows:

$$\cos\theta = \cos\epsilon \cos i + \sin\epsilon \sin i \cos\Omega, \tag{18}$$

$$\sin\theta = \sin(\arccos(\cos\theta)), \tag{19}$$

$$\sin R = \frac{\sin i \sin\Omega}{\sin\theta}, \tag{20}$$

$$R = \arcsin(\sin R'), \tag{21}$$

$$\cos(i + \Delta i_2) = \cos(\epsilon + \Delta\epsilon)\cos\theta - \sin(\epsilon + \Delta\epsilon)\sin\theta\cos(R + P), \tag{22}$$

$$i + \Delta i_2 = \arccos\cos(i + \Delta i_2), \tag{23}$$

and

$$\Delta i_2 = \Delta i = i + \Delta i_2 - i. \tag{24}$$

Care must be taken when using this method of analysis in connection with spherical trigonometry, as the same equations may not be valid for angles in quadrants other than the one being considered.

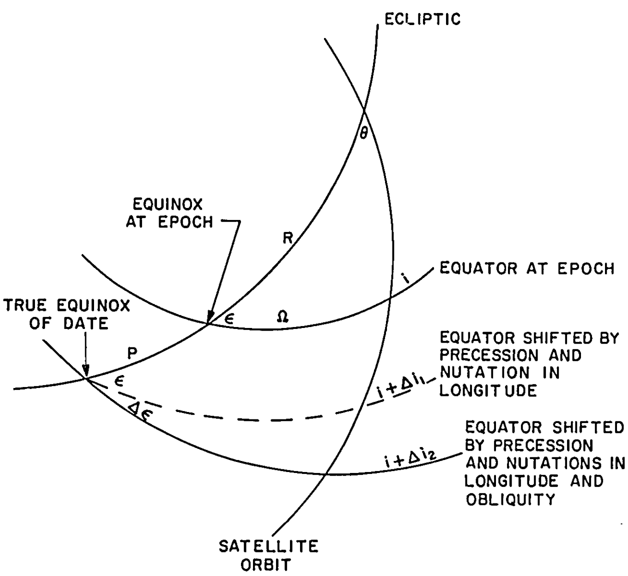

Fig. 4 - Precession and nutation effects on inclination

A simple equation is given in a paper by Newton (7) for determining the apparent change in inclination due to precession and nutation. It is listed here for reference:

$$\Delta i = -(P + Nl) \sin I_s \sin \Omega_s + (\Delta I_s) \cos \Omega_s, \tag{25}$$

where

P = total precession since an epoch,

NL = nutation in longitude,

I_s = obliquity of the ecliptic,

Ω_s = right ascension of the satellite,

and

ΔI_s = nutation in obliquity.

This equation has been evaluated experimentally by comparison with inclination corrections obtained using the set of equations given above, and it appears to be a very good approximation.

The values of the nutational terms may be found for any given day in "The American Ephemeris and Nautical Almanac" (4). It is much simpler to calculate them, however, when their values are required in a computer program. Equations for calculating the approximate values of nutation in longitude and obliquity are given by Smart (Ref. 8, p. 247). The major terms of these expressions are

$$NL = -17\overset{''}{.}234 \sin \Omega + 0\overset{''}{.}209 \sin (2\Omega) - 1\overset{''}{.}272 \sin (2L) - 0\overset{''}{.}204 \sin (2☾) \tag{26}$$

and

$$\Delta\epsilon = 9\overset{''}{.}210 \cos \Omega + 0\overset{''}{.}551 \cos (2L) + 0\overset{''}{.}088 \cos 2☾, \tag{27}$$

where

NL is nutation in longitude

$\Delta\epsilon$ is nutation in obliquity

L is the sun's mean longitude,

☾ is the moon's longitude,

and

Ω is the moon's right ascension.

Figure 5 shows a plot of the theoretical inclination and the observed inclination corrected using the scheme developed above for Satellite 902. The agreement between two curves is now excellent over the 3-year time interval considered. All significant perturbing influences seem to have been incorporated into the prediction theory. Figures 6 through 13 show similar plots for the eight other satellites studied. Most of the satellite predictions hold as well as they do for Satellite 902. Three of the satellites (numbers 704, 959, and 1314) exhibited a peculiar behavior near the end of 1966 and in early 1967. During

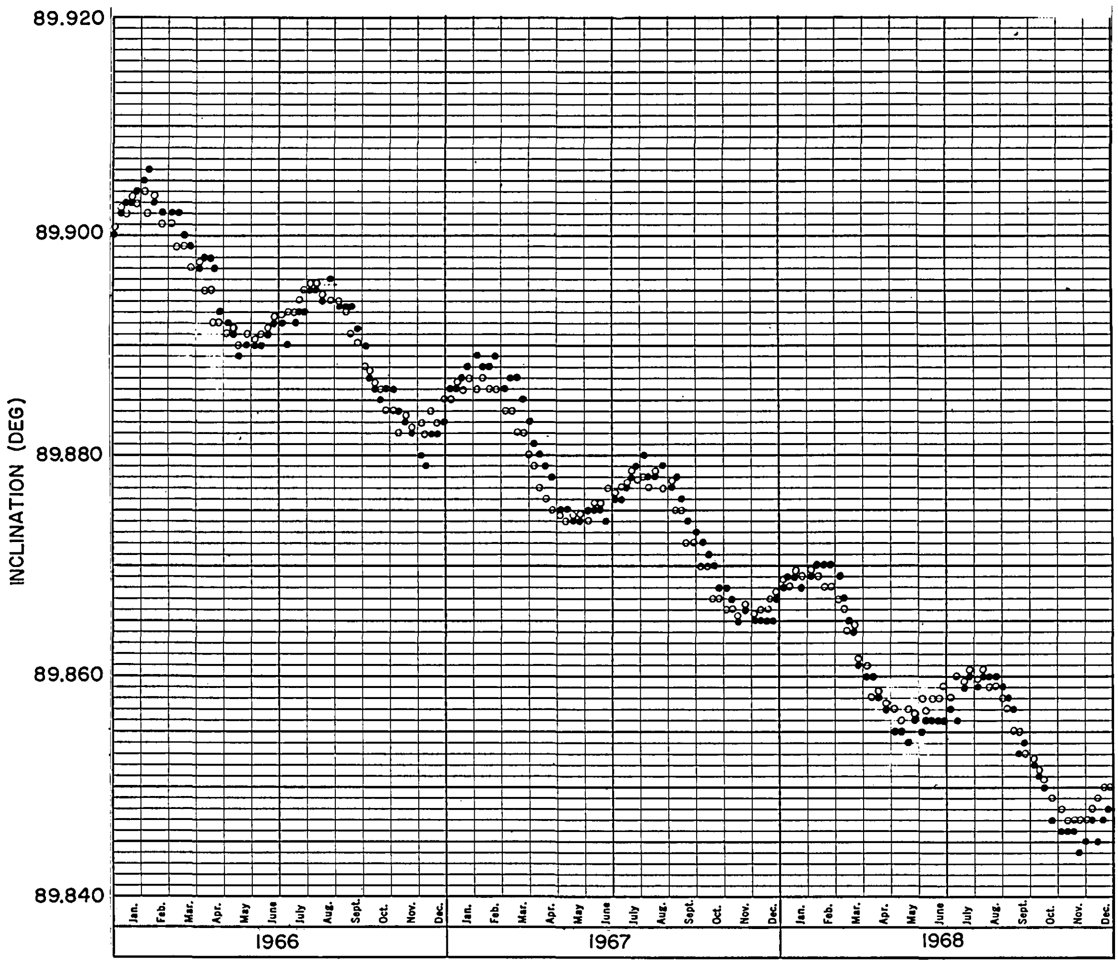

Fig. 5 - Theoretical and observed inclination for Satellite 902 after correction for precession and nutation

● Data

o Theoretical

this period, the predicted and observed data (corrected for precession and nutation) plots tend to separate and then remain nearly parallel during the remainder of the 3-year period. The exact cause of this effect is not clear; however, the peculiar behavior seems to be associated with the WS-434 orbital elements for these satellites rather than with the theory. There are several reasons for this conclusion. First, since the two curves (theoretical and observed) remain nearly parallel after the beginning of 1967, simply changing the epoch to this data would allow nearly perfect prediction for the remaining two years. Also, the three satellites exhibiting this effect should be well behaved in their respective orbits. No known active propulsion sources exist for these satellites, and atmospheric drag, solar radiation pressure, and resonance effects should not be of importance for the degree of prediction accuracy being considered. Finally, since the same prediction program and theory are used to predict all nine satellite inclinations, an error in theory or program should be evident in all nine comparison plots. Thus, it appears that an anomaly exists in the orbital computations made from the observations for these three satellites during 1966 and early 1967.

PHYSICAL INTERPRETATION OF RESULTS

The various perturbations in inclination indicated by the nine comparison plots can be easily associated with a particular change in one or more orbital elements of the sun or moon. For example, an oscillatory change in inclination having a period of about 6 months is evident in each plot. This is related to the apparent motion of the sun about the earth in 1 year (the orbital element changing is the mean anomaly of the sun). It should be noted that the orbital elements of the sun or moon appear as multiple-frequency terms in the inclination prediction equation, hence, the oscillation frequencies evident in the graphs will be integral multiples of the frequencies of particular sun, moon, or satellite elements.

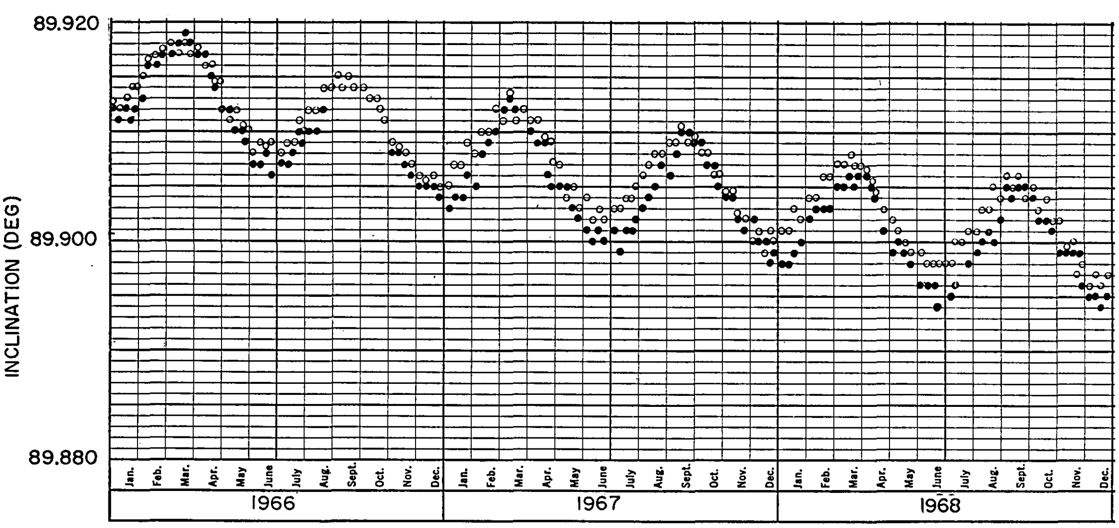

Fig. 6 - Theoretical and observed inclination for Satellite 671
● Data
o Theoretical

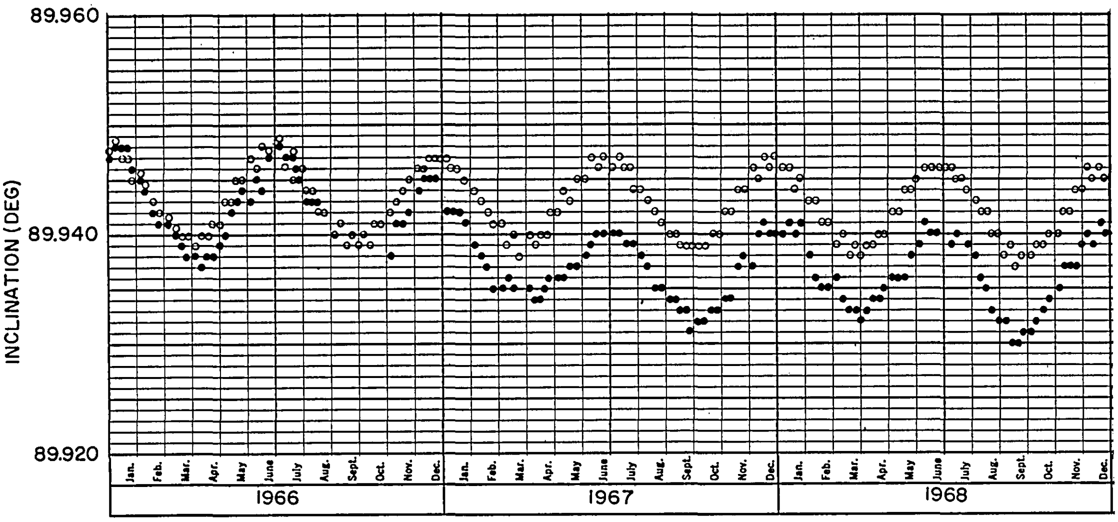

Fig. 7 - Theoretical and observed Inclination for Satellite 704
● Data o Theoretical

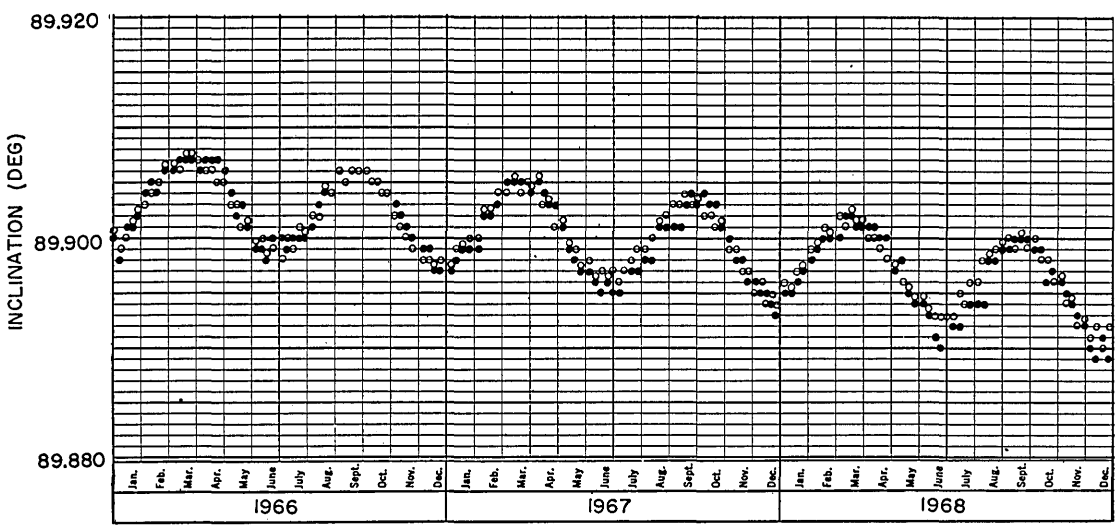

Fig. 8 - Theoretical and observed inclination for Satellite 1514
● Data o Theoretical

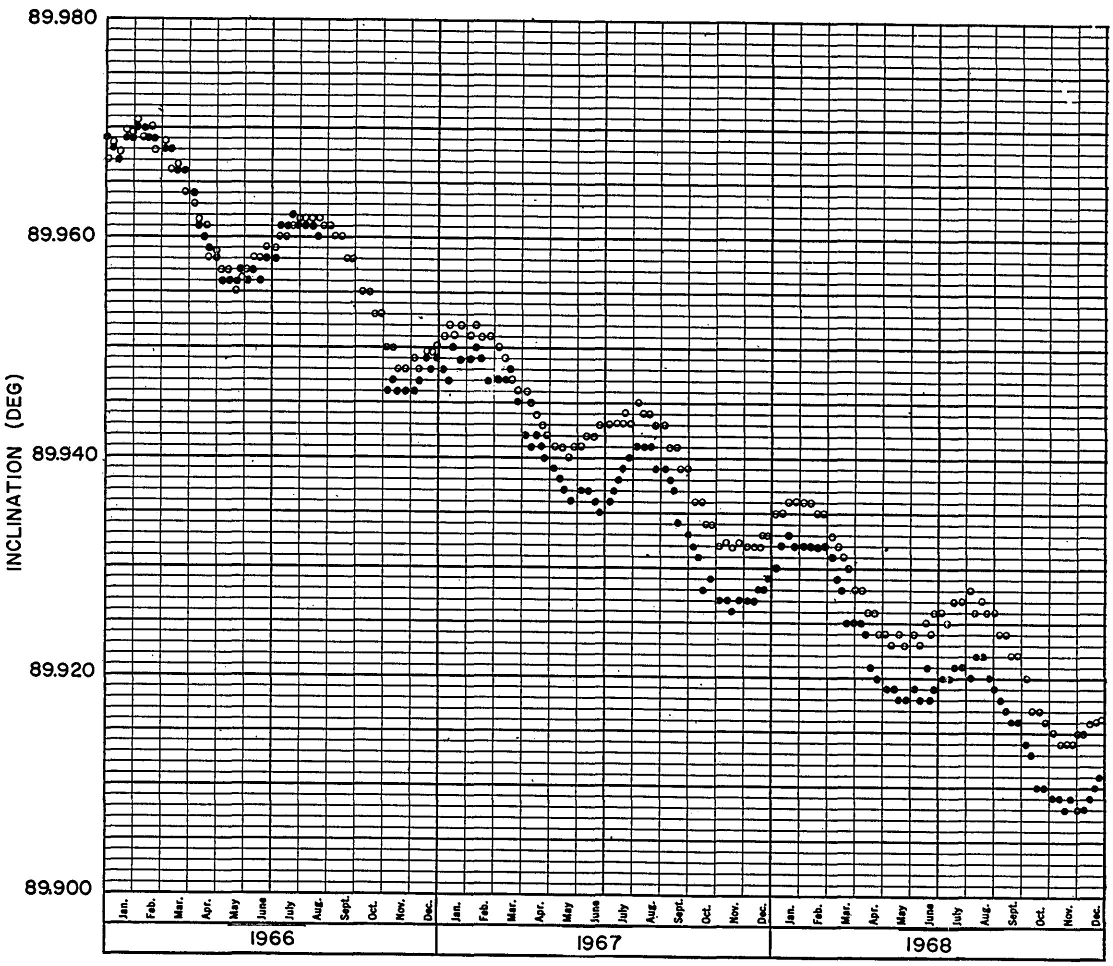

Fig. 9 - Theoretical and observed inclination for Satellite 959. ● Data o Theoretical

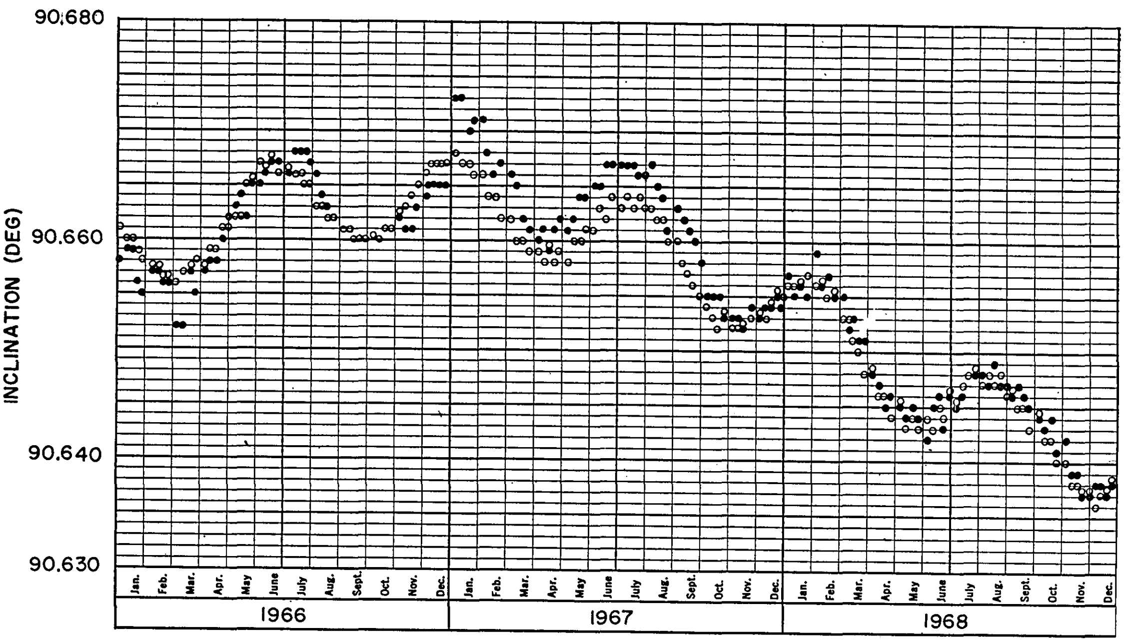

Fig. 10 - Theoretical and observed inclination for Satellite 509. ● Data o Theoretical

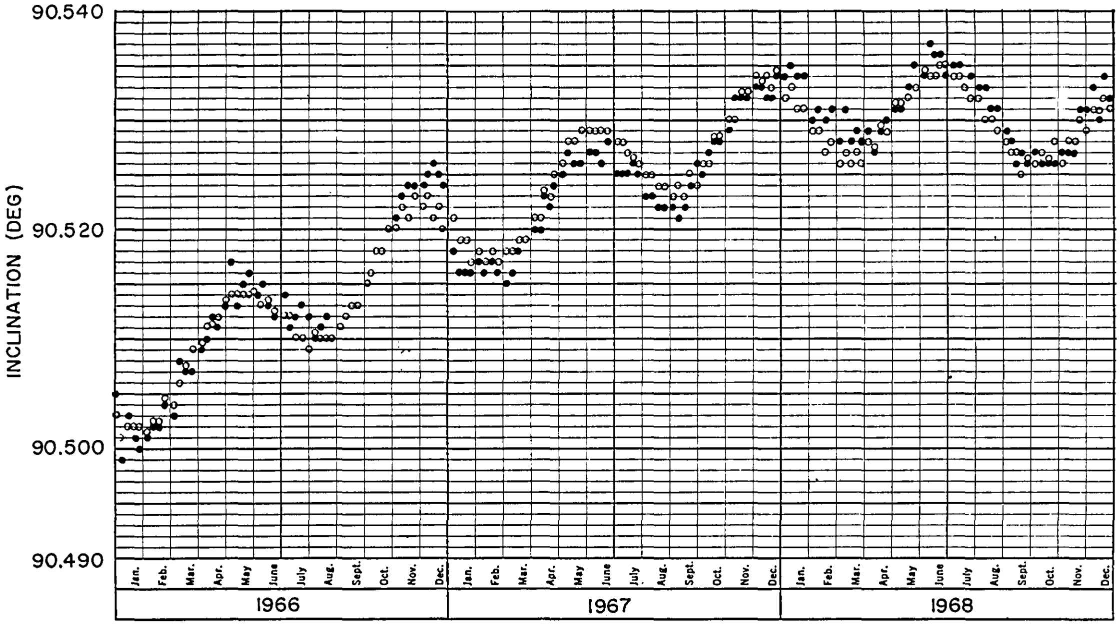

Fig. 11 - Theoretical and observed inclination for Satellite 801. ● Data o Theoretical

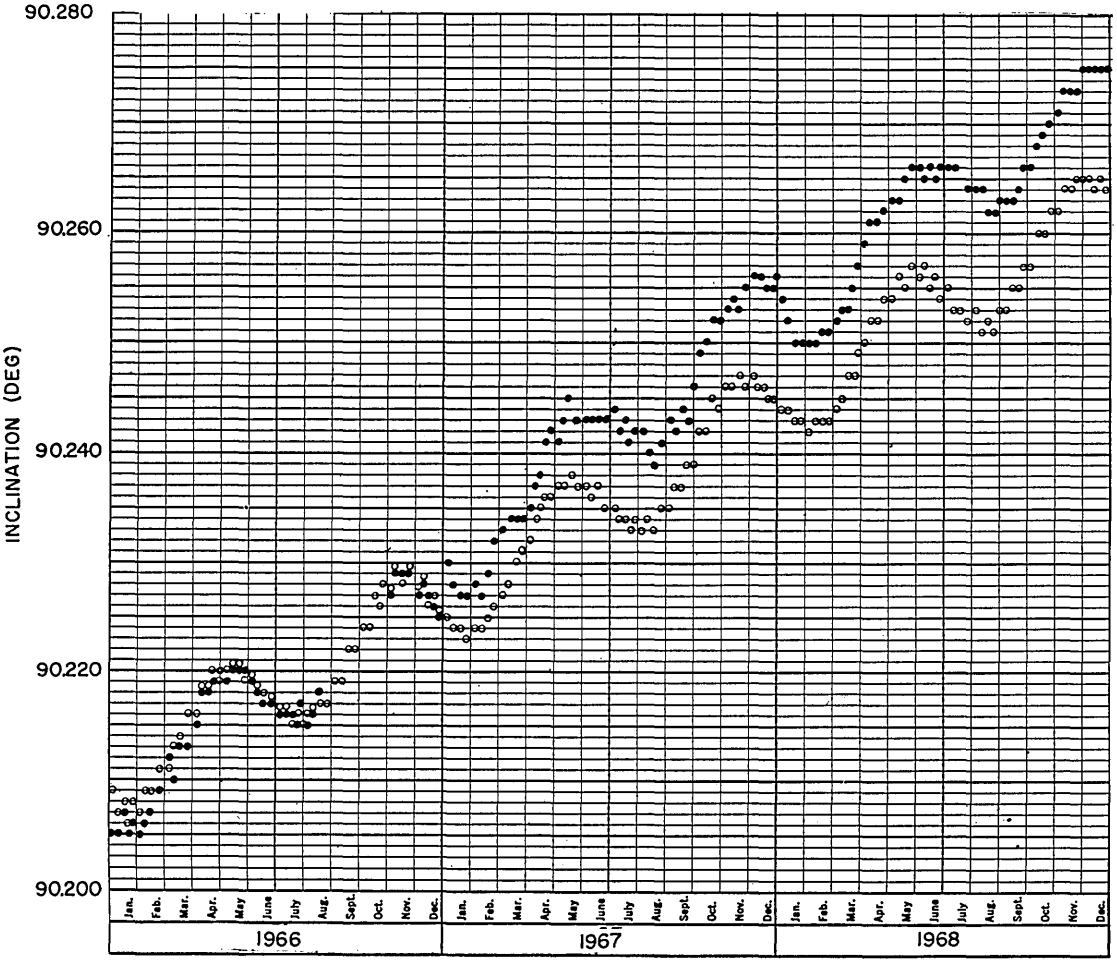

Fig. 12 - Theoretical and observed inclination for Satellite 1314. ● Data o Theoretical

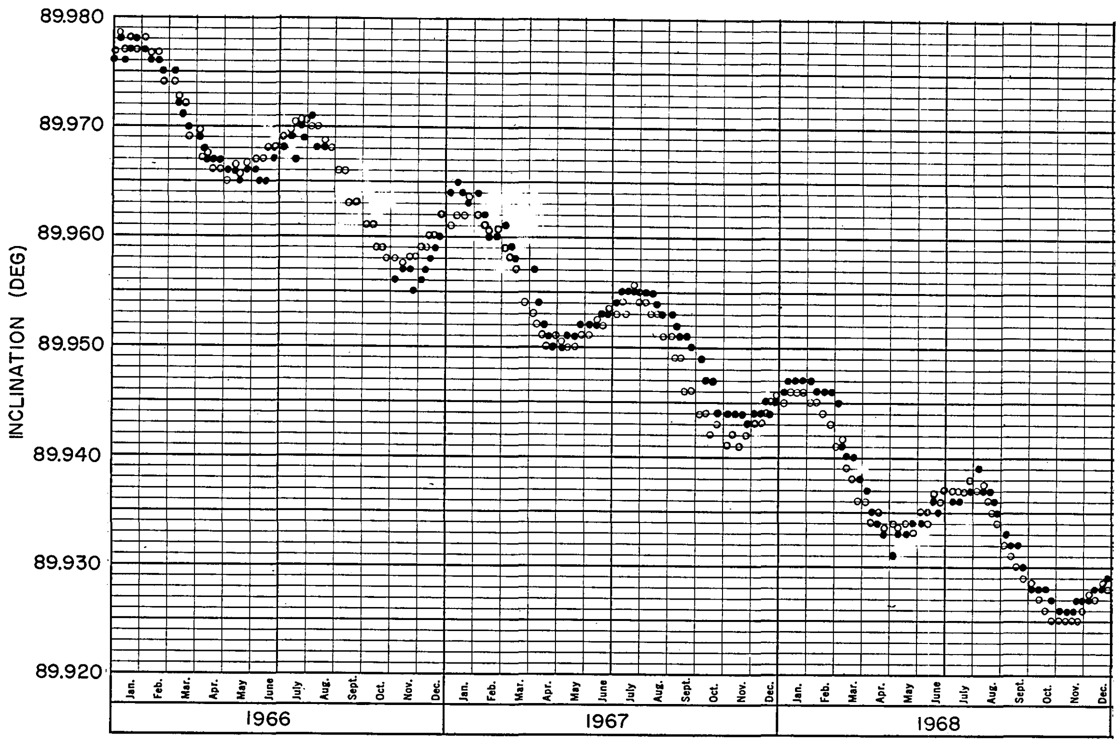

Fig. 13 - Theoretical and observed inclination for Satellite 1420. ● Data o Theoretical

A daily plot of predicted inclination for Satellite 902 is shown in Fig. 14. An oscillation having a 14-day period is evident in the graph. This is associated with the motion (mean anomaly is the element changing) of the moon around the earth in 28 days. This oscillation accounts for the apparent noiselike, short-period variations in inclination on the 3-year plots. Figures 15, 16, and 17 show long-term predictions in inclination for Satellite 902. Two major oscillatory components having periods of 56 years and 26 years are evident in Fig. 17. The first component is due to the rotation of the moon's orbital plane in 18.6 years, which is the change in right ascension. The second is related to the combined motions of lunar perigee and the satellite right ascension.

CONCLUSIONS

The theoretical expression used in this report for predicting the inclination of a satellite orbit appears to yield fairly accurate results over a long period of time. The predicted data are believed to be accurate to +0.002° for these particular satellites. To obtain predictions accurate to four decimal places in inclination, it is necessary to increase the complexity of the predicting program. Higher order terms that were neglected in the development of the basic predicting equation (Eq. (1)) must now be included. For those satellites having a high surface-area-to-mass ratio, it may be necessary to correct for solar radiation pressure. The effects of solar tide also become important for this degree of accuracy (see Ref. 7).

Inclination is one of the easiest satellite parameters to predict. Prediction of right ascension and argument of perigee are two of the most difficult. In addition to the effects of the sun and moon on these parameters as with inclination, the effects of the earth's gravitational irregularities complicate the situation. Though, as the satellite altitude

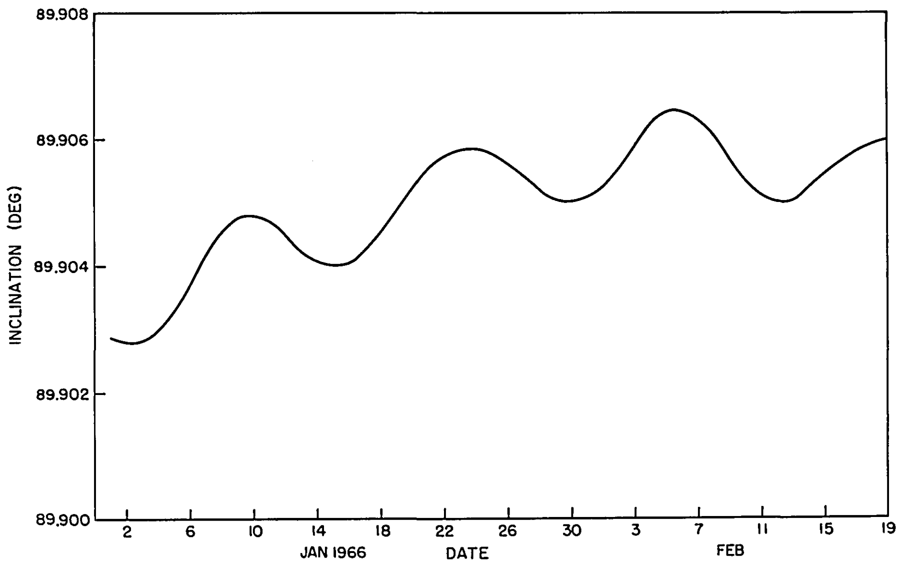

Fig. 14 - Short-term predicted inclination for Satellite 902

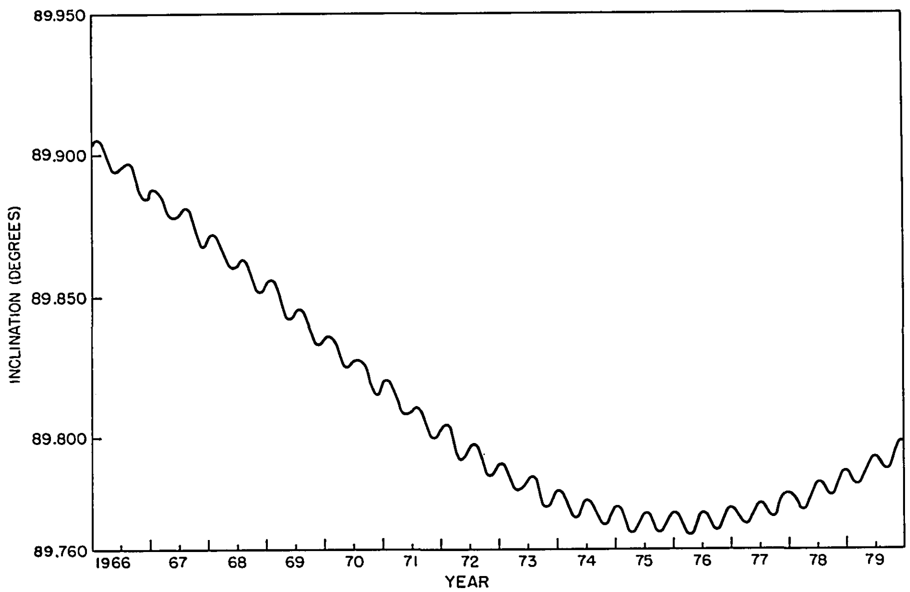

Fig. 15 - Long-term variations in inclination of Satellite 902

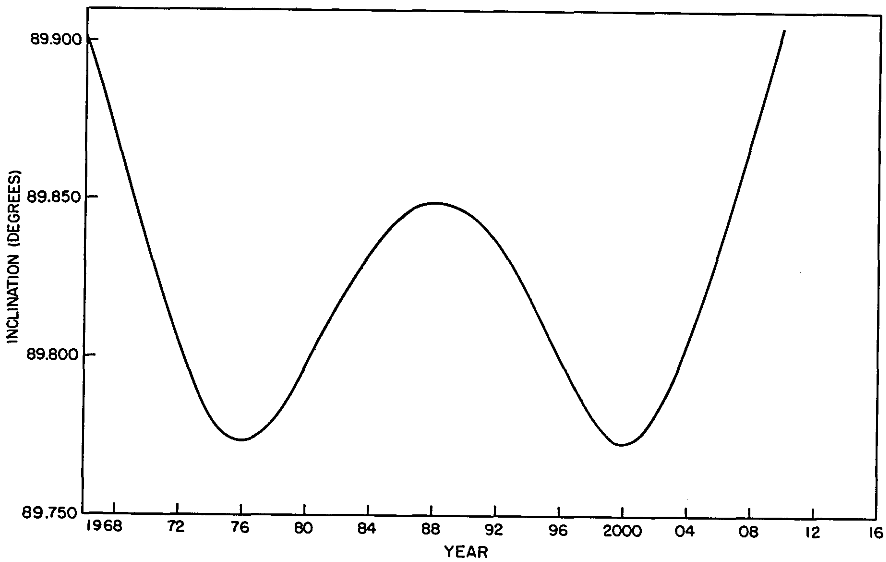

Fig. 16 - Smoothed 40-year variations in inclination of Satellite 902

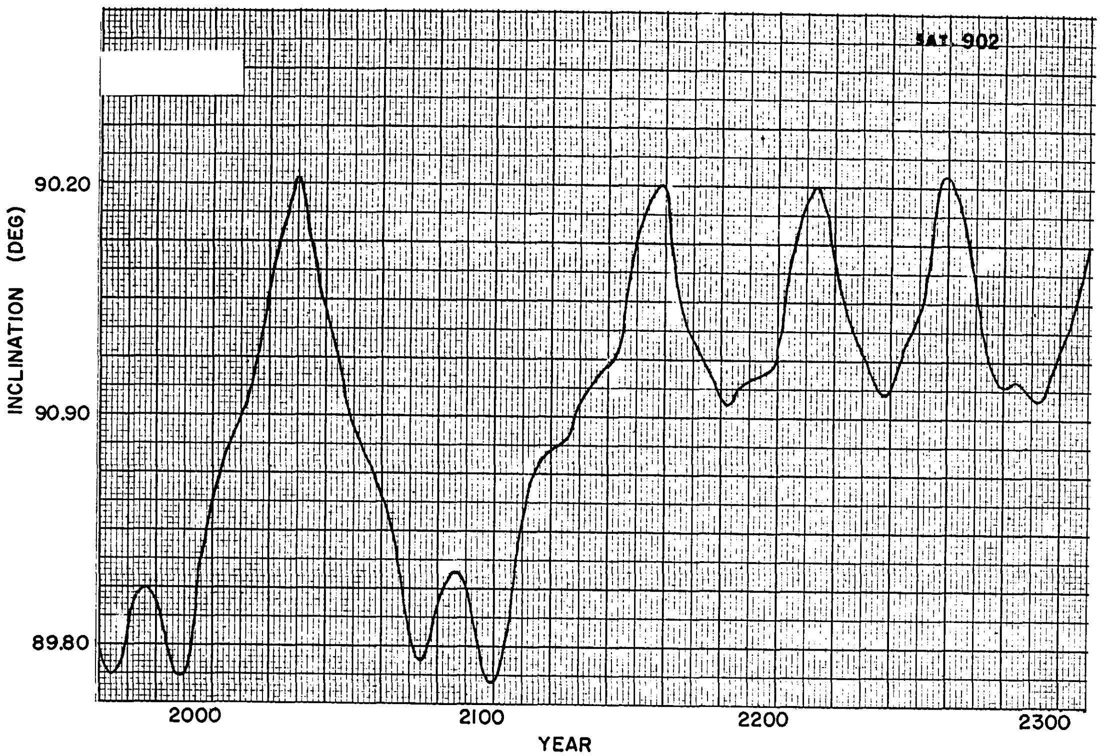

Fig. 17 - Long-term (300-year) predicted inclination for Satellite 902

becomes high, these two variables become easier to predict. Long-term prediction accuracy for these two parameters will increase as greater knowledge of the earth's gravitational structure is obtained.

SYMBOLS

a	Semimajor axis of a satellite orbit
a_d	Semimajor axis of lunar or solar orbit
B	Ω or $(\Omega - \Omega_d)$
d	Time in ephemeris days
e	Eccentricity of a satellite orbit
e_d	Eccentricity of lunar or solar orbit
e_m	Eccentricity of the lunar orbit
e_s	Eccentricity of the sun
G	Universal gravitational constant
I_s	Obliquity of the ecliptic
i	Inclination of a satellite orbit
i_d	Inclination of the lunar or solar orbit
J_n	nth zonal harmonic of the earth
L	$\frac{\mu_d}{a_d^3}$
L	Mean longitude of the sun
$(L - \Gamma)$	Mean anomaly of the sun
M	Mean anomaly of a satellite
M_d	Mean anomaly of the sun or moon
NL	Nutation in longitude
n	Mean motion of a satellite
P	Total precession since a given epoch
T	Time in Julian centuries (1 Julian century = 36,525 ephemeris days)
α	Inclination of the lunar orbit to the ecliptic
Γ	Mean longitude of perigee of the sun

Γ' Mean longitude of lunar perigee measured along the ecliptic and then along the lunar orbit

ΔI_s Nutation in obliquity

ϵ Mean obliquity (inclination) of the ecliptic

μ_d Product of G and the mass of the sun or moon

ω Argument of perigee of a satellite orbit

ω_d Argument of perigee of lunar or solar orbit

$\dot{\omega}$ Rate of change of argument of perigee

Ω Right ascension of the ascending node of a satellite orbit

Ω_d Right ascension of the sun or moon

$\dot{\Omega}$ Rate of change of right ascension

☾ Mean longitude of the moon measured along the ecliptic and then along the lunar orbit

☊ Right ascension of the lunar orbit measured along the ecliptic

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Appendix

FORTRAN PROGRAM FOR PREDICTION CALCULATIONS

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```
      PROGRAM INCRATE
      DIMENSION DIDT(1000),DIDT1(1000),DIDT2(1000),SAT1(1000)
     C,DELTA(1000)
      DR = 3.1415926/180.
      T = 24106.5/36525. - 1./36525.
      D = 24106.5 - 1.
      VS1 = SIN(5.145*DR)
      VC1 = COS(5.145*DR)
      DNT = 0.
      PK = 50.25/3600.*DR
      KM = 0
      KG = 0
      DELE = 0.
      ME = 0
      VA = 0
      L = 0
      IW = 0
      SEMI = 1.2027
      W = 242.541*DR
      B = 218.570*DR
      E = .00279
      AI = 90.2085*DR
      AN = 12.914*360.
      DO 10 K3 = 1,10,1
      DO 10 K = 1,16263,1
      T = T + 1./36525.
      ME = ME + 1
      D = D + 1.
      AID=(23.452294-0.0130125*T-0.000150*(T**2)-0.000003*(T**3))*D
      ED=(0.01675104-0.00004180*T-0.000000126*(T**2))
      WD=(281.22083+0.0000470684*D+0.000453*(T**2)+0.000003*(T**3))
      WD = WD + DNT
      AMD=(358.47583+0.985600267*D-0.000150*(T**2)-0.000003*(T**3))
      RDD = 1.496E011
      AK = .97142
      SLONG = (279.69688 + 0.9856473354*D + 0.000303*T**2)*DR
      SLONG = SLONG + DNT
      L = L + 1
      IW = IW + 1
      BS = B
      VW = 0.
      IF(365-KG)40,41,40
   41 KM = 0
   40 KM = KM + 1
      KG = KG + 1
      P  = PK*KM/365.
      FA = 0.
      F = 1.
      BP = B
      IF(B-3.14159)43,43,42
   42 BP  = 2.*3.1415926-B
      FA = 1.
   43 CONTINUE
      CTH = COS(AID)*COS(AI)+SIN(AID)*SIN(AI)*COS(BP)
      STH = SIN(ACOS(CTH))
      SIR = SIN(AI)*SIN(BP)/STH
```

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```
      R = ASIN(SIR)
      R = R - FA*2.*P
      EPN = ACOS(COS(AID+DELE)*CTH-F*SIN(AID+DELE)*STH*COS(R+P))
      DELTA(L) = (EPN-AI)/DR
      AID = AID + DELE
      VS2 = SIN(AID)
      VC2 = COS(AID)
    2 CONTINUE
      HAID = AID/2.
      S1 = SIN(HAID)**4
      S2 = COS(HAID)**4
      AC = 7*ED/2.
      A1 = 2.*(B + WD + AMD)
      A2 = 2.*(B+WD+AMD/2.)
      A3 = 2*B + 2*WD +3*AMD
      HAI = AI/2.
      AE4 = 2.*B + 2.*WD + 4.*AMD
      BR1 = (1.-5./2.*ED**2)*SIN(A1)-ED/2.*SIN(A2)+AC*SIN(A3)+17./2.*ED
     C **2*SIN(AE4)
      A4 = 2*(B-WD-AMD)
      A5 = 2*(B-WD-AMD/2)
      A6 = 2*B-2*WD-3*AMD
      AE6 = 2.*B - 2.*WD - 4.*AMD
      BR2 = (1.-5./2.*ED**2)*SIN(A4)-ED/2.*SIN(A5)+AC*SIN(A6)+17./2.*ED
     C **2*SIN(AE6)
      P1=SIN(AI)*(S1*BR1 + S2*BR2 +.5* (SIN(AID)**2)*SIN(2.*B)*(1.+3.*
     C ED*COS(AMD) + 3./2.*ED**2 + 9./2.*ED**2*COS(2.*AMD)))
      P2 = 0.
      T1 = (1.+3./2.*E**2-2.*E*COS(AMD)-.5*E**2*COS(2.*AMD))*P1+5./4.*E
     C **2*P2
      T1 = T1 + COS(AI)*SIN(AID)*((SIN(HAID)**2)*SIN(B+2.*WD+2.*AMD)
     C -(COS(HAID)**2)*SIN(B-2.*WD-2.*AMD) + COS(AID)*SIN(B))
      TF = (3.*AK/(4.*AN*(1.-E**2)**.5))*T1
      GO TO (4,5),VW + 1
    4 CONTINUE
      VW = VW + 1
      DIDT1(L) = TF
      MODIFY MOON PARAMETERS
      ED = 0.054900489
      WD = (334.329556+0.1114040803*D-0.010325*T**2-0.000012*T**3)*DR
      WD = WD + DNT
      ZL = (270.434164+13.1763965268*D-0.001133*T**2+0.0000019*T**3)*DR
      ZL = ZL + DNT
      RAS=(259.183275-0.0529539222*D+0.002078*T**2+0.000002*T**3)*DR
      RAS = RAS + DNT
      DNT=(-(17.234+0.017*T)*SIN(RAS)+0.209*SIN(2*RAS)-1.272*SIN(2*SLONG
     C )    -0.204*SIN(2*ZL))/3600.*DR
      DELE=((9.210+0.0009*T)*COS(RAS)-0.090*COS(2*RAS)+.551*COS(2*
     C SLONG)+0.088*COS(2*ZL))/3600.*DR
      SRAS = SIN(RAS)
      CRAS = COS(RAS)
      AID = ACOS(VC2*VC1 - VS2*VS1*CRAS)
      SIAID = SIN(AID)
      RTAS = ASIN(VS1*SRAS/SIAID)
      B = B - RTAS
      GAM = WD
```

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```
      WD = ASIN(VS2*SRAS/SIAID) + GAM - RAS
      AMD = ZL - GAM
      AK = 2,1093
      RDD = 3.844E008
      GO TO 2
    5 CONTINUE
      DIDT2(L) = TF
      DIDT(L) = DIDT1(L) + DIDT2(L)
C     MODIFY SAT PARAMETERS
      WDOT = ((6,28318*0,0016238*(2,-5.*SIN(AI)**2/2.))/(SEMI**2*
     C (1,-E**2)**2))*AN/360,
      BDOT = (6,28318*3,*,00054115*COS(AI)/(SEMI**2*(1,-E**2)**2))
     C *AN/360,
      BDOT = -BDOT
      AI = AI + DIDT(L)*DR
      SATI(L) = AI/DR
      W = W + WDOT
      B = BS + BDOT
  609 CONTINUE
  100 CONTINUE
      IF(1000-L)8,8,10
    8 CONTINUE
      PRINT 6,(SATI(M),M=1,1000,7)
      PRINT 6,(DELTA(M),M=1,1000,7)
    6 FORMAT(1X,10(1X,F11,6))
      L = 0
   10 CONTINUE
      END
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13. ABSTRACT

This report describes an investigation of the ability to predict long-term orbital elements for medium-height (satellite orbiting the earth at altitudes above the regions of appreciable drag), earth-orbiting satellites. The particular types of satellites studied are those having nearly circular, polar orbits. The primary element of interest is orbital inclination.

A theoretical equation for predicting the change in inclination caused by the gravitational attraction of the sun and moon is given. Expressions for the orbital elements of the sun and moon required in this inclination-predicting equation are developed. A FORTRAN program listing and computation procedure for determining the inclination of a satellite at any time after a chosen epoch is in the appendix.

Nine satellites were selected for study. For these satellites, comparison plots of observed and predicted inclinations over a 3-year time interval are shown. The agreement between predicted and observed inclinations is very good for most of the satellites. Theoretical plots of the inclination of Satellite 902 over time intervals of up to 300 years are included.

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