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The Effective Length and Internal Impedance of the Elements of a Receiving Array

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CONTENTS

ABSTRACT

An approximate method has been developed for determining both the effective length and the internal impedance of the individual elements of a receiving array from the currents induced by the incident illumination. With this information and Thévenin's theorem, one can easily arrive at the load currents for any type of impedance loading of the array.

PROBLEM STATUS

This is a final report in a series on the problem; this completes the work on the problem. Unless otherwise notified the problem will be considered closed 30 days after the issuance of this report.

AUTHORIZATION

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THE EFFECTIVE LENGTH AND INTERNAL IMPEDANCE OF THE ELEMENTS OF A RECEIVING ARRAY

SUMMARY

In a recent report (1) a method of determining the steady-state currents induced in the elements of a linear array was developed for a given illumination and loading of the array. In the present report we have described an approximate method of calculating both the effective length and the internal impedance of the individual elements of a receiving array from known induced currents.

The method is not limited to linear arrays. For convenience our theoretical model consists of a linear array of N parallel, thin, cylindrical elements illuminated by a plane wave incident in the H plane of the array. Employing the concepts of effective length and internal impedance of the elements and Thévenin's theorem, one can express the load current of an element in terms of these quantities and the electric field incident on the element. It is assumed that the effective length and internal impedance are approximately independent of the load impedance of the element. Given the load currents (obtained from independent calculations) for two different load impedances, the effective length (h_e) and the impedance (Z_a) of the element can be calculated from the two equations obtained by twice using the Thévenin's relation.

As an illustrative example we calculated the h_e and Z_a of several elements of a model array of eight elements with uniform loading and interelement spacing for various conditions of illumination.

As a check on the validity of the assumption that h_e and Z_a of an element are independent of the load impedance of the element, we have compared the load currents of the model array calculated using h_e and Z_a in the Thévenin's relation with the load currents calculated by the more lengthy and more rigorous method of Ref. 1 for various conditions of illumination and loading of the model array. The comparison shows that the h_e and Z_a of an element calculated by the approximate method described in the present report allow the load currents to be determined from the Thévenin relation with good engineering accuracy for a wide range of impedance loading.

INTRODUCTION

In Ref. 1 a method of calculating the steady-state currents induced in the elements of a linear array was developed for a given illumination and loading of the array. We will describe here a method of determining both the effective length and the internal impedance of the individual elements of a receiving array from the induced currents. With this information about the array, one can easily arrive at the load currents for any type of loading desired including nonuniform loading. Thus, in conducting a study of the effect on the load currents of a wide variation in the load impedances of a receiving array, it becomes unnecessary to repeat the lengthy calculation of the induced currents described in Ref. 1 for each set of load impedances.

THE THEORETICAL MODEL

The theoretical model (Fig. 1) consists of a linear array of N parallel cylindrical elements illuminated by a plane wave incident at an angle ϕ_{inc} to the normal to the plane of the array. The electric vector of the incident wave is parallel to the axes of the elements. For simplicity, the radii of the elements are assumed small relative to both the wavelength of the illumination and the length of the elements ($2h$). The central load impedances Z_i are lumped in character.

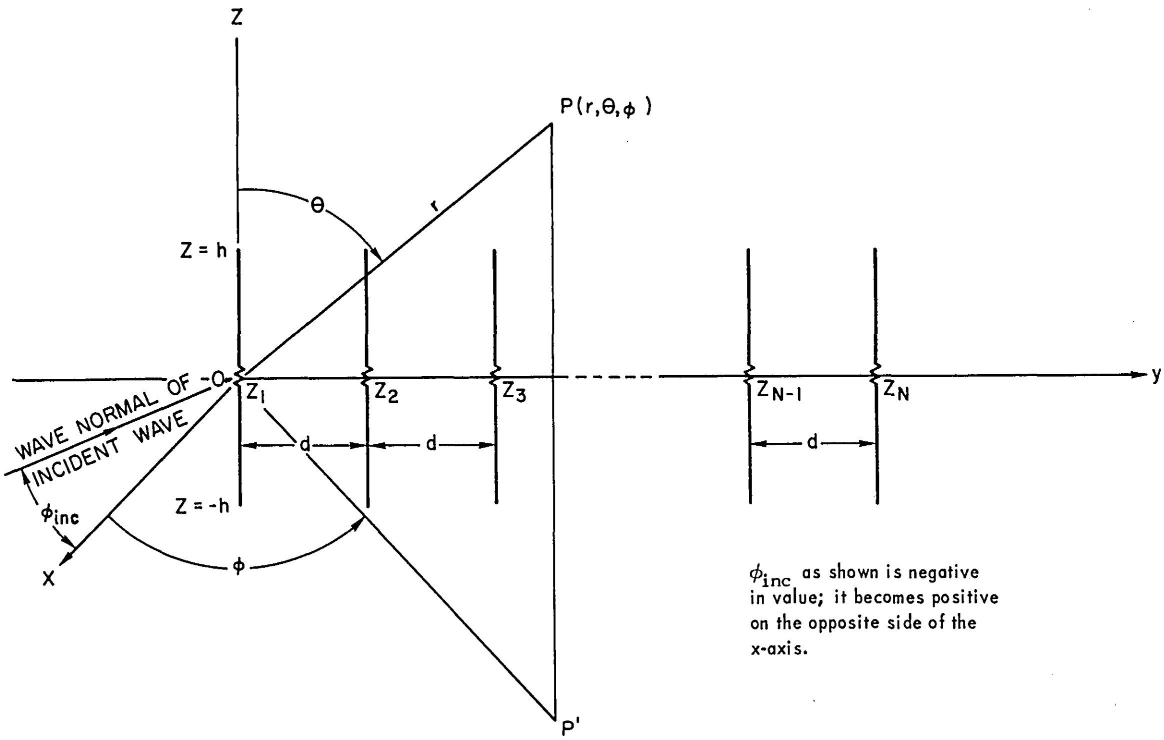

Fig. 1 - The theoretical model

METHOD OF ANALYSIS

According to Thévenin's theorem, for a given angle of incidence and frequency of the steady-state sinusoidal illumination, the load current of the ith element is given by

$$I_i = \frac{V_{0i}}{Z_{ai} + Z_i}, \tag{1}$$

where Z_{ai} and Z_i are respectively the internal impedance and the load impedance of the ith element and V_{0i} is the induced open circuit voltage at the terminals of the ith element. By the definition of the effective length $2h_{ei}$ the open circuit voltage is given by

$$V_{0i} = -2h_{ei}E_z^i, \tag{2}$$

where $E_z^{\,i}$ is the incident electric field of the ith element. The incident field may be written in the form

$$E_z^i = E_0 \exp[j(i-1)\beta_0 d \sin \phi_{inc}] , \qquad (3)$$

where E_0 is the real amplitude of the incident field, $\beta_0 = \omega/c$ is the free-space phase constant, ω is the angular frequency of the illumination, c is the velocity of propagation of EM waves in free space, and d is the interelement spacing of the array.

Figure 2 shows the equivalent circuit and the senses of positive load current and incident electric field. Upon combining Eqs. (1), (2), and (3) we obtain the following expression for the load current:

$$I_i = \frac{2E_0 h(h_{ei}/h) \exp[j(i-1)\beta_0 d \sin \phi_{inc}]}{Z_{ai} + Z_i} . \qquad (4)$$

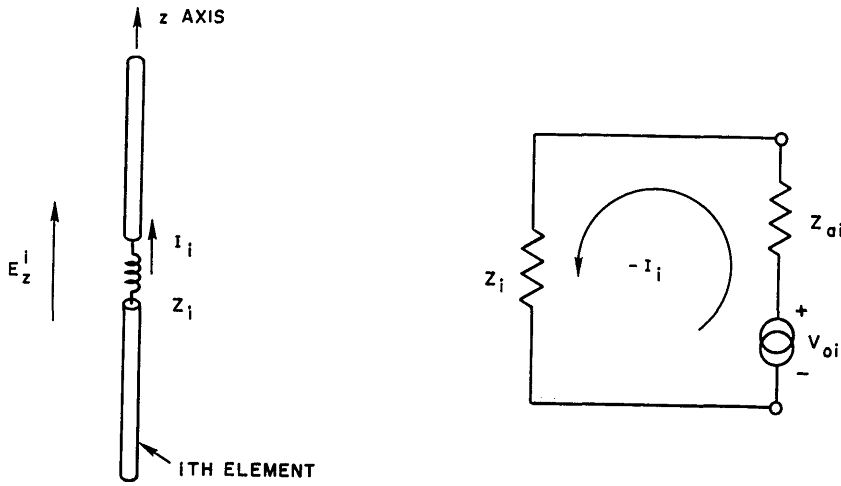

Fig. 2 - Equivalent circuit of the ith element

We assume that h_{ei} and Z_{ai} are essentially independent of the load impedance of the element; then h_{ei} and Z_{ai} can be calculated, for a given illumination, using Eq. (4) with two different values of the load impedances. The complex load current corresponding to each load impedance must be determined from the integral equations of the array, thus accounting for the mutual coupling between the elements. Reference 1 describes such a technique for calculating the induced currents.

CALCULATIONS FOR A MODEL ARRAY

As an illustrative example, we shall calculate the h_{ei} and Z_{ai} of several elements of a model array of eight elements with uniform load impedances and a constant interelement spacing equal to the length of the elements (a commonly used spacing in broadside arrays). In this study physical dimensions of the array are fixed, while the electrical length of the elements $\beta_0 h$ changes linearly with the frequency of the incident illumination. We keep the strength of the illumination constant by setting $E_0 h = 30$.

The ratio of the length to the radius of the elements is fixed at 1000. The technique described in Ref. 1 is used to calculate the load currents.

We calculate h_{ei} and Z_{ai} from a load impedance pair A and again for a load impedance pair B. The load impedance pairs used in this example are

$$Z_\ell \text{ - Pair } A = \begin{bmatrix} 60 - jX \\ 240 + jX \end{bmatrix}$$

and

$$Z_\ell \text{ - Pair } B = \begin{bmatrix} 60 + jX \\ 240 - jX \end{bmatrix},$$

where $X = 1150 - 740\ \beta_0 h$ (note that $X = 0$ at approximately $\beta_0 h = \pi/2$ where an isolated element resonates).

Our final results, which we present in the figures to follow, are the arithmetic average of the values of h_{ei} and Z_{ai} obtained from the two load impedance pairs A and B.

RESULTS

In Fig. 3 we present the resistance and reactance of the first element of the array as functions of the angle of incidence of the illumination with the electrical half length of the elements $\beta_0 h$ held fixed at various values.

We observe from Fig. 3 that the resistance and reactance vary rather slowly with ϕ_{inc} for values of $\beta_0 h$ of $\pi/2$ or less. For values of $\beta_0 h$ of 1.8 or more the resistance varies rapidly at the higher angles of incidence. The reactance changes quite a bit in this region also but not as rapidly as the resistance. Figure 3 also shows the real part (h_e'/h) and the imaginary part (h_e''/h) of the relative effective length (h_e/h) of the first element plotted in like manner. It is apparent that the real part of h_e/h remains a relatively slowly varying function of ϕ_{inc} even at the higher values of $\beta_0 h$. The imaginary part of h_e/h for $\beta_0 h = 2$ appears to change considerably with ϕ_{inc}; but for the lower values of $\beta_0 h$ the imaginary part of h_e/h remains rather small in value and almost invariant with ϕ_{inc}.

In Figs. 4, 5, and 6, we present the calculated impedance and effective length information for elements 3, 5, and 8, respectively. Because of mutual coupling effects the results for the various elements of the array differ in certain respects.

In Table 1 we present a comparison of the load currents calculated from Eq. (4) with the load currents arrived at by the more lengthy method of Ref. 1. The comparison is made for several load impedances, none of which is a member of the reference pair of load impedances used in determining the h_e and Z_a of the elements.

An examination of Table 1 shows that the largest phase difference (-2.94 degrees) occurs in the first element and that the current ratio deviates from unity by the largest amount (6.5 percent) in element 7, both taken from Table 1a, where $\beta_0 h$ = 1.57 and the load impedance is (480 - j480) ohms.

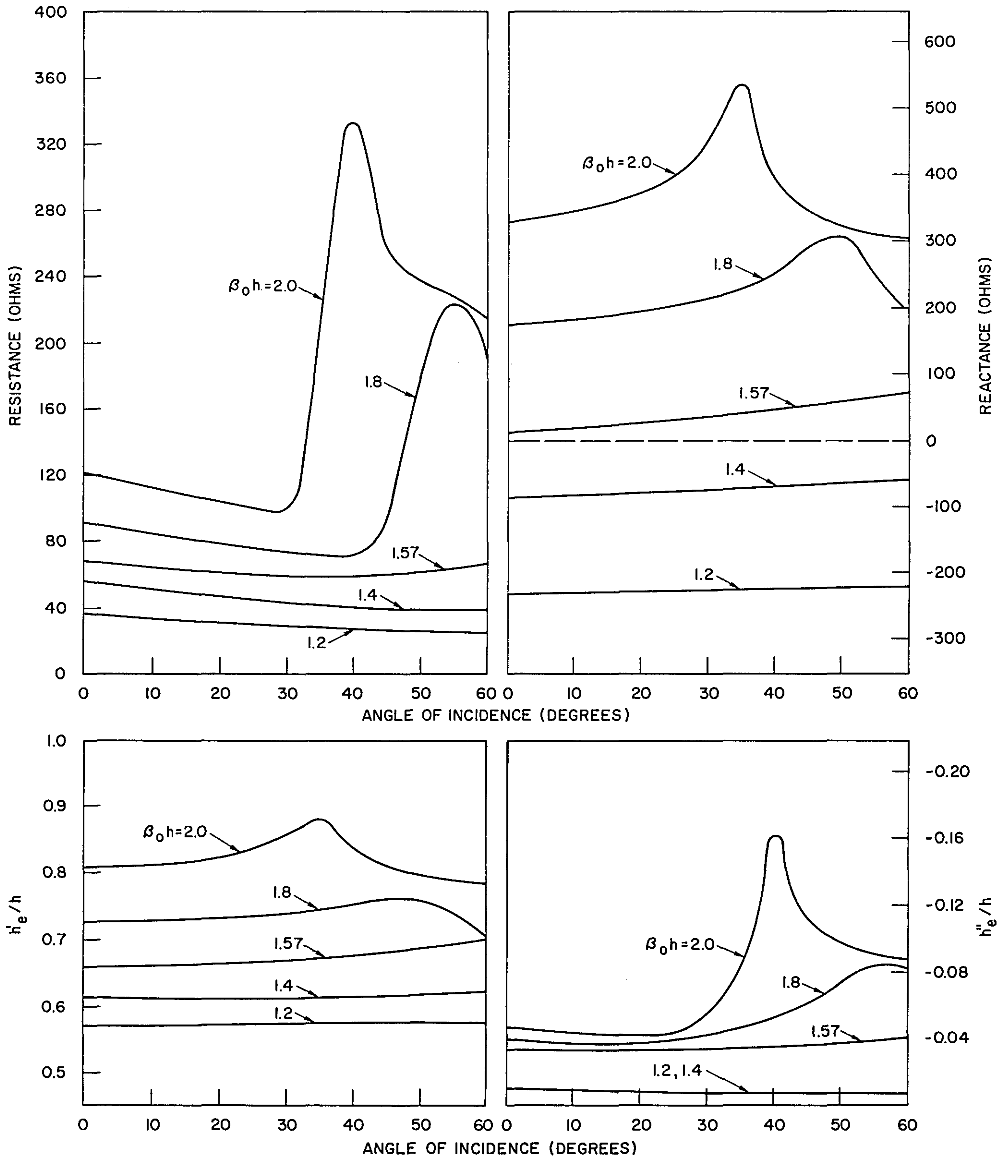

Fig. 3 - The internal impedance and effective half-length of the first element as a function of angle of incidence. The upper left gives the internal resistance; the upper right, the internal reactance; the lower left, the real part of h_e/h; and the lower right, the imaginary part of h_e/h.

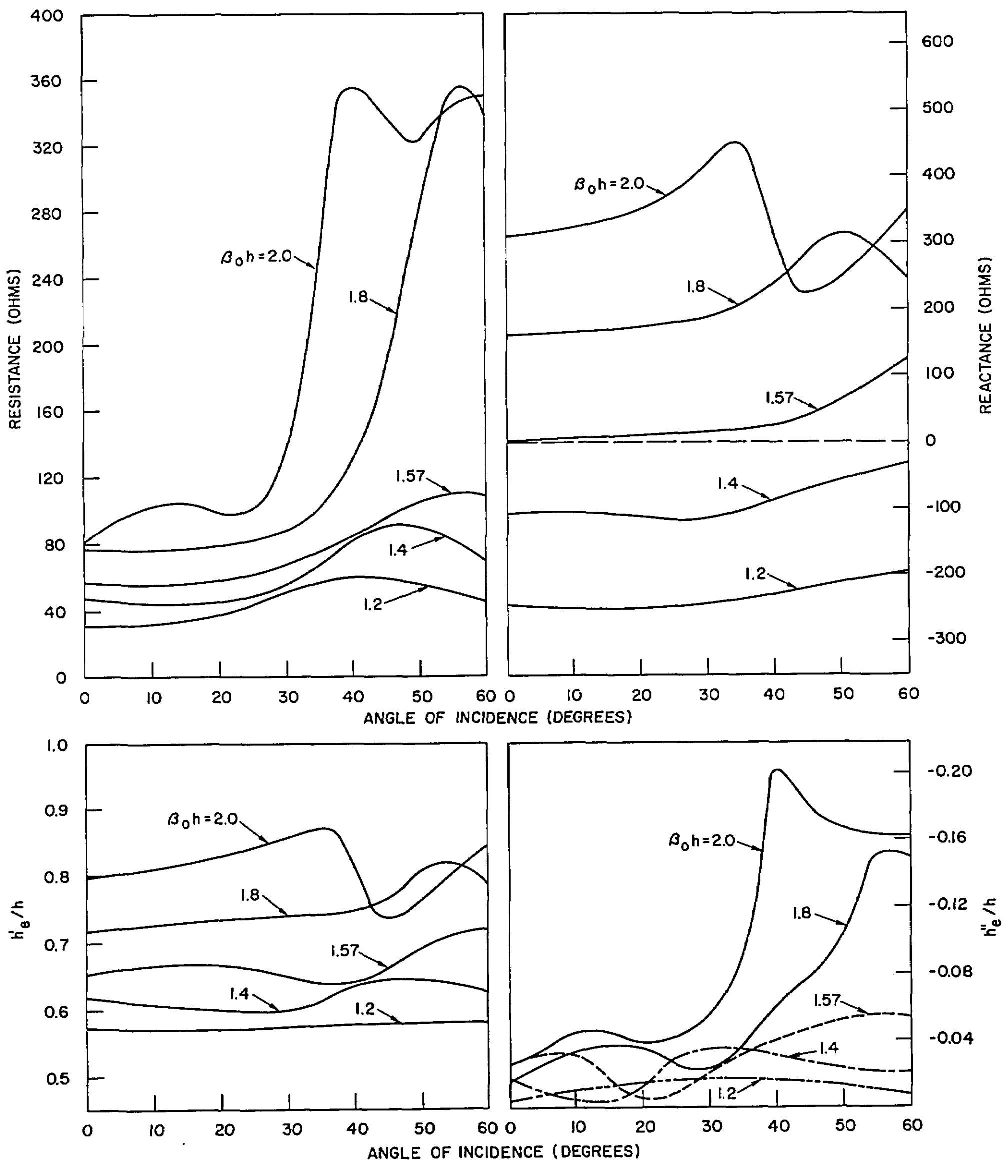

Fig. 4 - The internal impedance and effective half-length of the third element as a function of angle of incidence. The upper left gives the internal resistance; the upper right, the internal reactance; the lower left, the real part of h_e/h; and the lower right, the imaginary part of h_e/h.

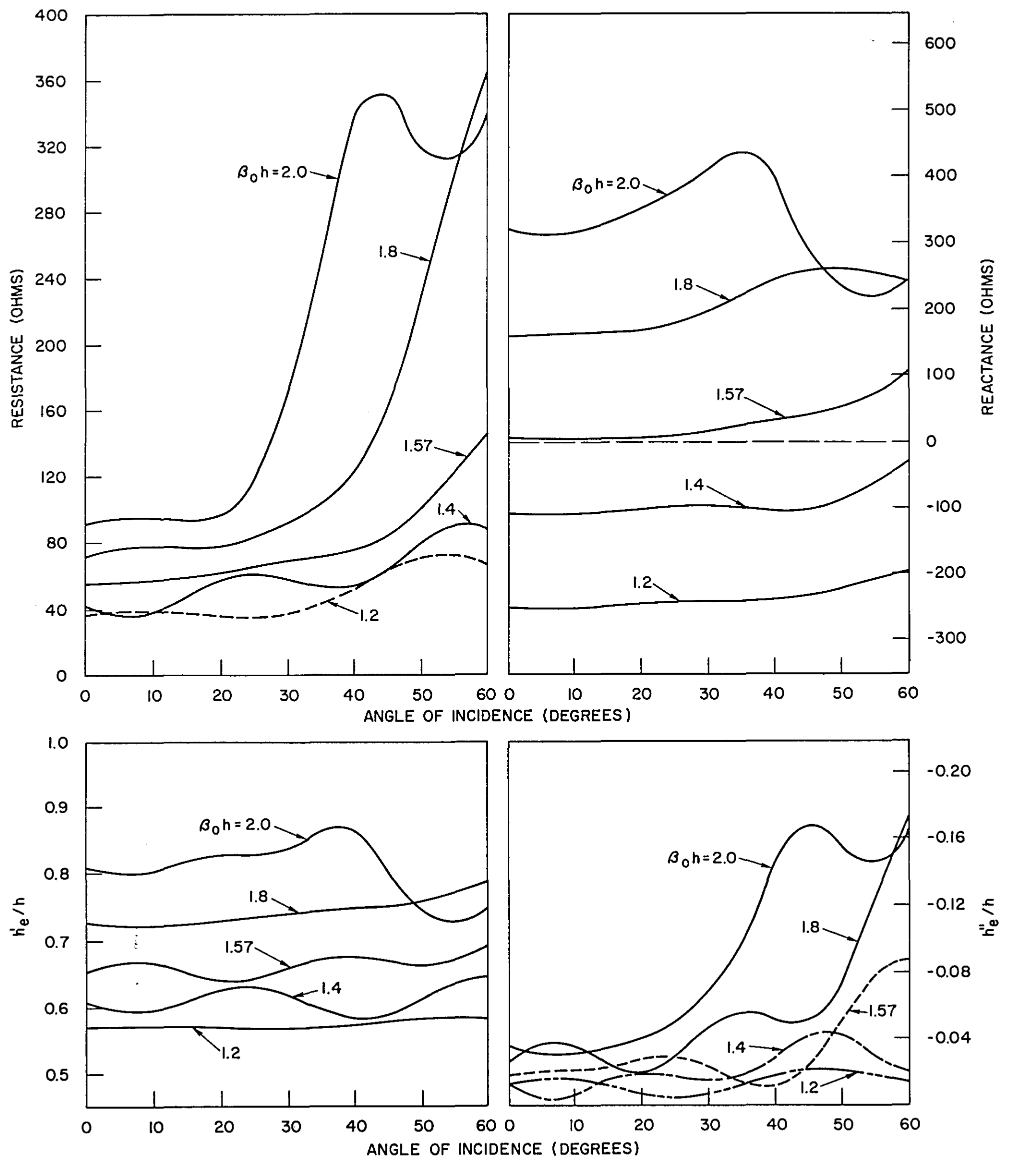

Fig. 5 - The internal impedance and effective half-length of the fifth element as a function of angle of incidence. The upper left gives the internal resistance; the upper right, the internal reactance; the lower left, the real part of h_e/h; and the lower right, the imaginary part of h_e/h.

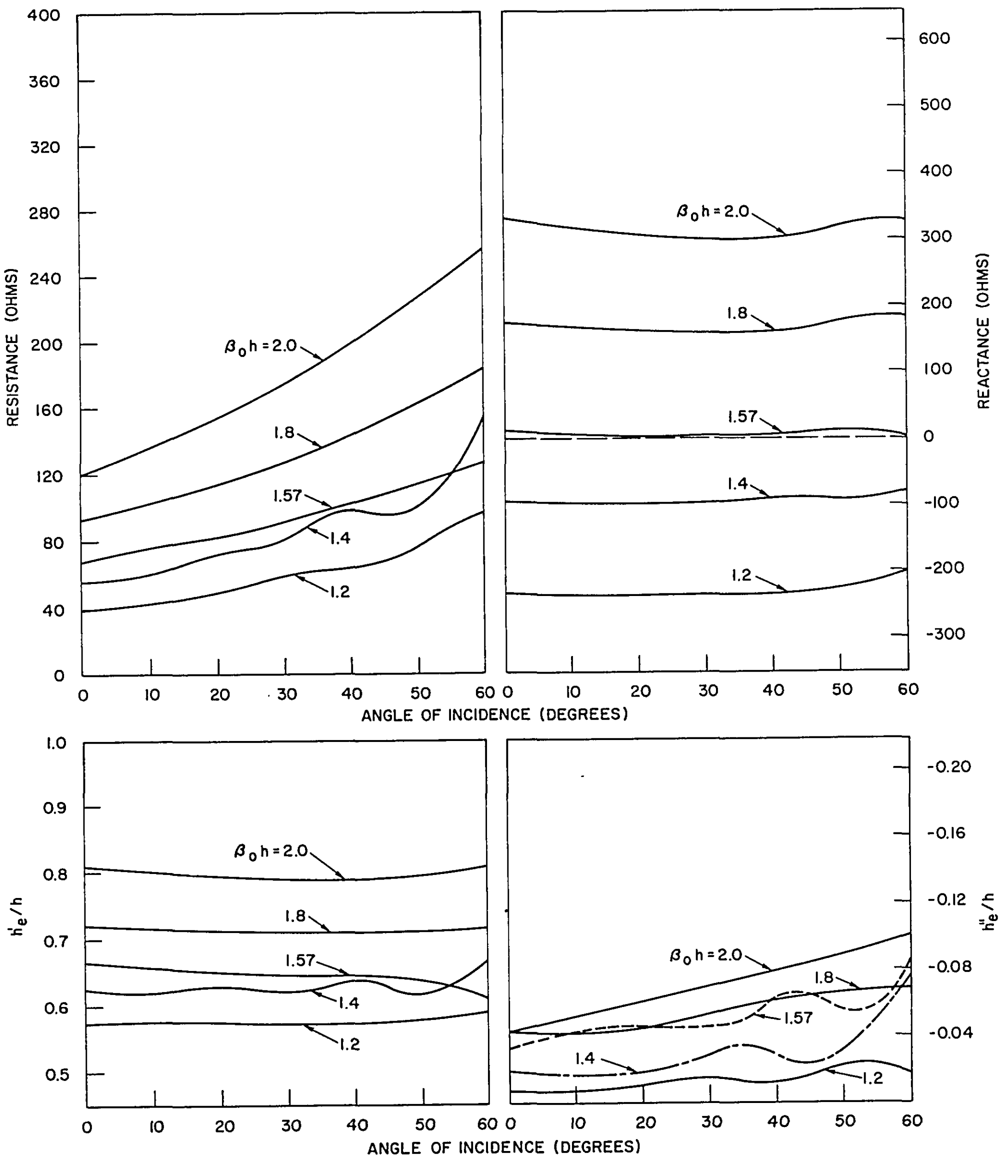

Fig. 6 - The internal impedance and effective half-length of the eighth element as a function of angle of incidence. The upper left gives the internal resistance; the upper right, the internal reactance; the lower left, the real part of h_e/h; and the lower right, the imaginary part of h_e/h.

Table 1a - Comparison of Load Currents From Eq. (4) With Those From the Method of Ref. 1 for $\beta_0 h$ = 1.57 and Load Impedances 480 - j480 ohms

Element Number	Current Ratio	Phase Difference (degrees)
ϕ_{inc} = 0 degree		
1	0.994	-0.43
2	1.008	0.44
3	0.995	-0.22
4	1.001	0.06
5	1.001	0.06
6	0.995	-0.22
7	1.008	0.44
8	0.994	-0.43
ϕ_{inc} = -20 degrees		
1	0.993	-1.03
2	0.978	0.98
3	1.021	0.27
4	0.995	-1.10
5	0.984	0.74
6	1.018	0.50
7	1.003	-1.23
8	0.981	-0.09
ϕ_{inc} = -40 degrees		
1	0.991	-1.41
2	0.982	-0.91
3	0.979	1.06
4	1.014	1.32
5	1.025	-0.32
6	1.006	-1.44
7	1.030	-1.17
8	0.968	-0.36
ϕ_{inc} = -60 degrees		
1	0.993	-2.94
2	0.996	-1.28
3	0.997	1.29
4	0.988	-1.59
5	0.971	-1.48
6	0.955	-0.74
7	0.945	0.43
8	0.936	1.02

Table 1b - Comparison of Load Currents From Eq. (4) With Those From the Method of Ref. 1 for $\beta_0 h$ = 1.57 and Load Impedances 120 - j120 ohms

Element Number	Current Ratio	Phase Difference (degrees)
ϕ_{inc} = -40 degrees		
1	0.991	0.81
2	0.999	0.69
3	1.015	0.01
4	1.001	-0.88
5	0.986	-0.37
6	0.987	0.50
7	0.999	1.04
8	1.008	0.92
ϕ_{inc} = -60 degrees		
1	0.981	1.52
2	0.997	1.51
3	0.994	0.46
4	0.995	0.91
5	1.004	1.30
6	1.018	1.31
7	1.031	0.95
8	1.032	0.80

Table 1c - Comparison of Load Currents From Eq. (4) With Those From the Method of Ref. 1 for $\beta_0 h$ = 1.40 and Load Impedances 480 - j480 ohms

Element Number	Current Ratio	Phase Difference (degrees)
ϕ_{inc} = -40 degrees		
1	1.001	0.23
2	1.012	0.40
3	1.015	-0.63
4	0.993	-1.32
5	0.973	0.16
6	0.998	1.34
7	1.016	0.42
8	1.020	-0.36
ϕ_{inc} = -60 degrees		
1	1.004	0.03
2	0.997	0.59
3	1.003	0.98
4	1.013	0.92
5	1.021	0.43
6	1.023	-0.18
7	1.003	-0.65
8	1.032	-1.71

Table 1d - Comparison of Load Currents From Eq. (4) With Those From the Method of Ref. 1 for $\beta_0 h$ = 1.80 and Load Impedances 480 - j480 ohms

Element Number	Current Ratio	Phase Difference (degrees)
ϕ_{inc} = -40 degrees		
1	0.964	0.38
2	0.982	0.07
3	0.984	0.82
4	0.996	0.77
5	0.996	0.50
6	0.992	0.68
7	0.995	1.27
8	1.004	1.38
ϕ_{inc} = -60 degrees		
1	1.000	2.04
2	1.011	0.28
3	0.999	-1.55
4	0.986	-2.56
5	0.988	-2.33
6	1.003	-1.10
7	1.016	0.51
8	1.007	1.85

CONCLUSIONS

From our experience with a number of reference load impedance pairs, it is recommended that the nonreactive load impedance pair [(60 + j0), (72 + j0)] be used as the reference pair in calculating h_e and Z_a of the elements of an array. Note that this choice makes the two members of the pair much more nearly alike than were the members of pairs A and B of this report. Admittedly this is an afterthought, but this reference pair should lead to better accuracy in the calculation of both h_e and Z_a, because the current distribution in the elements remains essentially the same for both members of the reference pair of load impedances. Also the information in Appendix A of Ref. 1 suggests that the above reference impedance pair is superior to the impedance pairs A and B used in this report.

The authors have not worked out the details, but it seems likely that one can use (h_e, Z_a) information on idealized arrays to acquire an insight into the behavior of phased arrays. To achieve this end in the transmitting mode of operation it will be necessary to fully develop the equivalent $(N + 1)$-port linear bilateral network of the antenna system involving the N-element array and a far-zone probe antenna.

REFERENCE

1. Sledge, O.D., "The Scattering of a Plane Electromagnetic Wave by a Linear Array of Center-Loaded Cylinders," NRL Report 6681, June 1968

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13. ABSTRACT

An approximate method has been developed for determining both the effective length and the internal impedance of the individual elements of a receiving array from the currents induced by the incident illumination. With this information and Thévenin's theorem, one can easily arrive at the load currents for any type of impedance loading of the array.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Effective length of array element Internal impedance of array element Receiving array antenna Thévenin's theorem for receiving arrays Cylindrical elements Angle of incidence Electrical half-length of elements Plane-wave illumination Analysis						