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A Comparison of Shock and Isentropic Heating in Light-Gas Gun Compression

J. R. Baker

Dynamics Branch
Mechanics Division

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U.S. NAVAL RESEARCH LABORATORY
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LIST OF SYMBOLS

a = initial gas sound speed

c_v = specific heat at constant volume

D^0 = compression tube diameter

L^0 = initial compression tube length

L = distance between piston and end of compression tube at start of shock compression

m = molecular weight of gas

m_p = piston mass

n = number of shock transits

N = number of moles of gas

P^0 = initial pressure

P_0 = pressure in front of initial shock wave

P_n = pressure behind nth shock wave

P_f = final pressure

R = universal gas constant

T^0 = initial temperature

T_0 = temperature in front of initial shock wave

T_n = temperature behind nth shock

T_f = final temperature

t_n = time for nth shock transit

NOTE: A prime indicates the corresponding quantity for isentropic compression.

u_p = piston velocity

U = shock wave velocity

$\bar{u}_p = u_p/a$

$\bar{U} = U/a$

$\bar{u} = u_p/U$

V^0 = initial compression tube volume

V_0 = compression tube volume at the start of shock compression

V_f = final volume

$X = (\gamma + 1)\bar{u}/2$

x_n = distance between piston and end of compression tube after nth shock transit

γ = ratio of specific heats

$\alpha = m_p u_p^2/2Nc_vT_0$

$\mu = (\gamma + 1)/(\gamma - 1)$

$\sigma_n = P_n/P_{n-1}$

$\lambda = (\mu + 1)/(\sigma_1 - 1)$

ρ_0 = gas density in front of initial shock wave

ρ_n = density behind the nth shock wave

$\eta_n = \rho_n/\rho_{n-1}$

$\kappa_n = \rho_n/\rho_0$

$\pi_n = P_n/P_0$

$\tau_n = T_n/T_0$

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An ideal model of light-gas gun operation that permits the evaluation of the effectiveness of shock heating in the light gas as opposed to the heating due to isentropic compression is hypothesized. This model assumes a shock wave generated by a piston which is instantly accelerated to some constant velocity. After the shock wave traverses the compression tube of the gun four times, it is followed by isentropic compression which absorbs all of the piston energy. The temperatures arrived at by shock compression and isentropic compression are compared when the pressures of the final states are equal. The results indicate that for reasonably fast pistons (around 6000 fps), the ideal temperature gains are on the order of 30 percent and the corresponding sound speed gains are only about 15 percent. The resulting increase in projectile velocity under these conditions cannot exceed 15 percent. Real gas effects would be expected to make this value still lower. It is concluded consequently, especially for the gun configurations currently used at NRL, that shock heating is not an important factor in the gun operation nor a particularly fruitful means of enhancing gun performance.

INTRODUCTION

Current light-gas gun technology clearly indicates that further advances in projectile velocity, and gun capability in general, can only be achieved by increasing the temperature and, thereby, the sound speed of the light gas. One possible method for obtaining this result is to generate shock waves in the gas by an appropriate piston motion. As the shock waves reflect up and down the compression tube, the gas is heated nonisentropically to a temperature higher than that which would be obtained by isentropic compression. A comparison of the final temperatures for the two cases would provide a basis for judging the value of the shock heating as a technique for improving light-gas gun performance.

The following model, which operates in three distinct phases, has been selected for the purposes of this comparison:

Phase 1 – Isentropic compression until the piston reaches maximum velocity.

Phase 2 – Shock compression by a shock wave due to a uniformly moving piston and lasting for four shock transits of the compression tube.

Phase 3 – Isentropic compression from the fourth state of phase 2 to a final state commensurate with a change of internal energy of the gas equal to the kinetic energy of the piston, which comes to rest.

The following conditions are assumed: (a) the gas is ideal, (b) there are no losses due to friction, radiation, etc., (c) the shock wave in phase 2 is due to a piston instantaneously accelerated to a given velocity and moving uniformly thereafter, and (d) the projectile is at rest until the end of the piston compression; that is, until the end of phase 3. This model is essentially the same as that used in the work reported by Stephenson (1) and Lemcke (2). A more accurate, but considerably more computationally involved model has been suggested by Winter (3).

COMPUTATIONS

Phase 1

The light-gas gun is loaded initially to a pressure P^0, temperature T^0, and volume V^0. The gas is then compressed isentropically, until the piston reaches a maximum velocity and the gas has a pressure P_0, a temperature T_0, and a volume V_0 which are given by

$$P_0 = P^0 (V^0/V_0)^\gamma \tag{1}$$

$$T_0 = T^0 (V^0/V_0)^{\gamma-1}. \tag{2}$$

NRL Problem F04-04; Project RR 009-03-45-5801. This is an interim report on one phase of the problem; work on this and other phases is continuing. Manuscript submitted January 9, 1964.

Phase 2

The piston, having reached its maximum velocity, is assumed to generate a shock wave of velocity (4)

$$\bar{U} = (X + \sqrt{4 + X^2})/2 \quad (3)$$

where $X = (\gamma+1)\,\bar{u}_p/2$, $\bar{u}_p = u_p/a$, $\bar{U} = U/a$, and a = initial gas sound speed. The strength of this shock wave is determined by the pressure ratio across it and is a function of the piston velocity:

$$\frac{P_1}{P_0} = 1 + \gamma\,\bar{u}_p\,(X + \sqrt{4 + X^2})/2. \quad (4)$$

Introducing the results of Evans and Evans (5), let $\sigma_1 = P_1/P_0$, or in general, the pressure ratio across the nth shock is

$$\sigma_n = P_n/P_{n-1} = \frac{\lambda + \mu + n}{\lambda + n - 1} \quad (5)$$

where $n = 1,2,3, \ldots$ is the number of the shock transits, $\mu = (\gamma + 1)/(\gamma - 1)$, and $\lambda = (\mu + 1)/(\sigma_1 - 1)$. Similarly, for the density ratio across the nth shock

$$\eta_n = \rho_n/\rho_{n-1} = \frac{\lambda + \mu + n - 1}{\lambda + n}. \quad (6)$$

These equations may then be used to express the conditions behind the nth shock relative to the state before the first shock; thus

$$\pi_n = \frac{P_n}{P_0} = \prod_{i=1}^{n} \sigma_i = \frac{\Gamma(\lambda)\Gamma(\lambda + \mu + n + 1)}{\Gamma(\lambda + \mu + 1)\Gamma(\lambda + n)} \quad (7)$$

$$\kappa_n = \frac{\rho_n}{\rho_0} = \prod_{i=1}^{n} \eta_i = \frac{\Gamma(\lambda + 1)\Gamma(\lambda + \mu + n)}{\Gamma(\lambda + \mu)\Gamma(\lambda + n + 1)} \quad (8)$$

$$\tau_n = \frac{T_n}{T_0} = \frac{\pi_n}{\kappa_n} = \frac{(\lambda + n)\;(\lambda + \mu + n)}{\lambda(\lambda + \mu)} \quad (9)$$

where Γ is the gamma function and is defined by

$$\Gamma(z) = \int_0^\infty e^{-t}\, t^{z-1}\, dt \quad \text{and} \quad \Gamma(z + 1) = z\Gamma(z).$$

Calculations of these quantities for the first four shock transits are shown in Table 1 for selected values of $\bar{u}_p$. Evans and Evans show that after the fourth shock transit the change of state is essentially isentropic. Furthermore, in the fourth state, the gas is at rest, as it must also be in the final state; consequently, phase 3 is assumed to begin at the end of the fourth shock transit.

Phase 3

As the shock wave reflects off the piston for the second time, all of the gas in the compression tube is in the $n = 4$ state (see Fig. 1). In the remaining compression, the entire kinetic energy of the piston is assumed to go into the internal energy of the gas, and the piston comes to rest; hence

$$m_p u_p^2/2 = N c_v\,(T_f - T_4)$$

or

$$\frac{T_f}{T_0} = \frac{T_4}{T_0} + \frac{m_p u_p^2}{2 N c_v\, T_0}. \quad (10)$$

The standard isentropic relationship $PT^{\gamma/(1-\gamma)}$ = constant divided by P_0 and T_0 gives

$$\frac{P_f}{P_0} = \frac{P_4}{P_0}\left(\frac{T_f/T_0}{T_4/T_0}\right)^{\gamma/(\gamma-1)}. \quad (11)$$

Equations (10) and (11) determine the final state of the gas for the case of shock heating. In order to compare this to the isentropic case, it is necessary to specify that either the pressure, the temperature, or the density be the same for the two cases. The other two quantities may then be compared. For the case of the light-gas gun, it is felt that the most meaningful comparison can be made when the pressures are the same. This is because, of the three quantities, the pressure most directly controls the gun's performance. For example, the pressure determines the time of projectile release. Also, the maximum capability of the gun is limited by the peak pressures it is able to contain. Therefore, choosing $P_n = P'_n$ and $P_f = P'_f$, the state of the gas for isentropic compression is given by

$$\tau'_n = \frac{T'_n}{T_0} = \left(\frac{P_n}{P_0}\right)^{(\gamma-1)/\gamma} \quad (12)$$

and

$$\frac{T'_f}{T_0} = \left(\frac{P_f}{P_0}\right)^{(\gamma-1)/\gamma}. \quad (13)$$

The quantities τ'_n also appear in Table 1.

The comparison that is of particular interest is the ratio T_f/T'_f, and for the model used here it can be shown that this ratio is the same as

TABLE 1

Relative Shock and Isentropic Temperature Ratios for $\gamma = 1.4$

$\bar{u}_p$	x	σ_1	λ	n	π_n	κ_n	τ_n	τ_n'*	τ_n/τ_n'	$(\tau_n/\tau_n')^{1/2}$
0.5	0.6	1.941	7.440	1	1.941	1.592	1.219	1.209	1.008	1.004
				2	3.550	2.436	1.458	1.436	1.015	1.007
				3	6.183	3.602	1.716	1.683	1.020	1.010
				4	10.329	5.177	1.995	1.949	1.024	1.012
1.0	1.2	3.473	2.831	1	3.473	2.305	1.506	1.427	1.056	1.027
				2	9.818	4.691	2.093	1.921	1.090	1.044
				3	24.044	8.713	2.759	2.481	1.112	1.055
				4	52.909	15.091	3.506	3.108	1.128	1.062
1.5	1.8	5.716	1.484	1	5.715	3.012	1.897	1.645	1.153	1.074
				2	21.818	7.335	2.974	2.413	1.233	1.110
				3	65.646	15.513	4.232	3.305	1.280	1.131
				4	168.12	29.656	5.669	4.324	1.311	1.145
2.0	2.4	8.734	0.905	1	8.734	3.624	2.410	1.857	1.297	1.139
				2	40.824	9.863	4.139	2.886	1.434	1.198
				3	139.19	22.490	6.189	4.097	1.511	1.229
				4	388.69	45.416	8.559	5.494	1.558	1.248

*For isentropic compression to the same pressure, *i.e.*, for the same π_n.

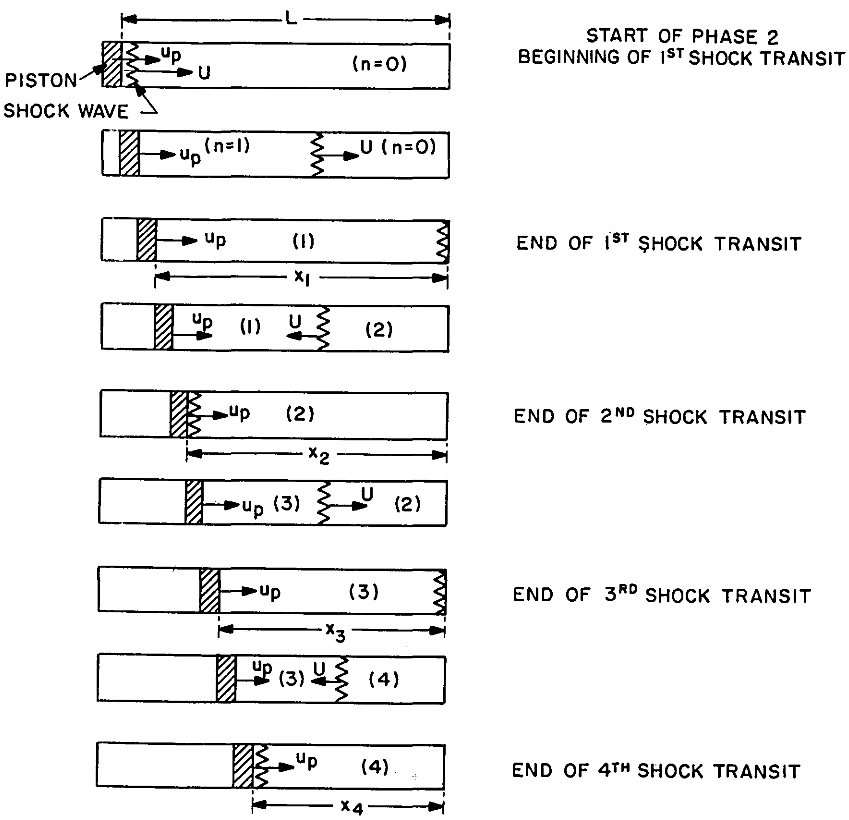

Fig. 1 — Piston position and gas state during phase 2

the ratio of the temperatures at the end of four shock transits (see Appendix A); that is,

$$\frac{T_f}{T_f'} = \frac{\tau_4}{\tau_4'}. \tag{14}$$

This ratio indicates the relative advantage of shock heating over isentropic heating. Even more meaningful, however, is the square root of this ratio, which gives the ratio of the sound speeds for the two cases; and it is the sound speed of the gas which ultimately limits the obtainable projectile velocity. These ratios are listed in Table 1 and appear graphically in Figs. 2 and 3.

The relative advantage of shock heating over isentropic heating is independent of the gun geometry and the initial gas state. The absolute values of the final state of the gas, however, do depend on both of these. Therefore, by using the following relations for an ideal diatomic gas $N = P^0V^0/RT^0$, $c_v = 5R/2$, $a^2 = \gamma RT^0/m$ we obtain from Eq. (10)

$$T_f = T_4 + \left(\frac{\gamma R}{5m}\right)\frac{m_p\bar{u}_p{}^2(T^0)^2}{P^0V^0} \tag{15}$$

and by (11) we obtain

$$P_f = P_4\left(\frac{T_f}{T_4}\right)^{\gamma/(\gamma-1)}. \tag{16}$$

Since the number of moles is constant

$$V_f = \left(\frac{T_f}{T^0}\right)\left(\frac{P^0}{P_f}\right)V^0. \tag{17}$$

Consequently, by altering the values of P^0 or V^0 it is possible to adjust the final temperature, pressure, and volume while retaining the same relative advantage of shock heating. Consider the following numerical examples:

Let $D^0 = 3.25$ in., $L^0 = 64\ D^0$, $m_p = 2270$ g, $\bar{u}_p = 1$, $P^0 = 3.447 \times 10^7$ dynes/cm^2 (500 psia), $T^0 = 300°$K, $\gamma = 1.4$, $m = 2$ g (hydrogen), and $V^0/V_0 = 2$ (the piston reaches maximum velocity halfway down the compression tube). From Eq. (15) and obtaining $T_4 = \tau_4 T_0$ from Table 1

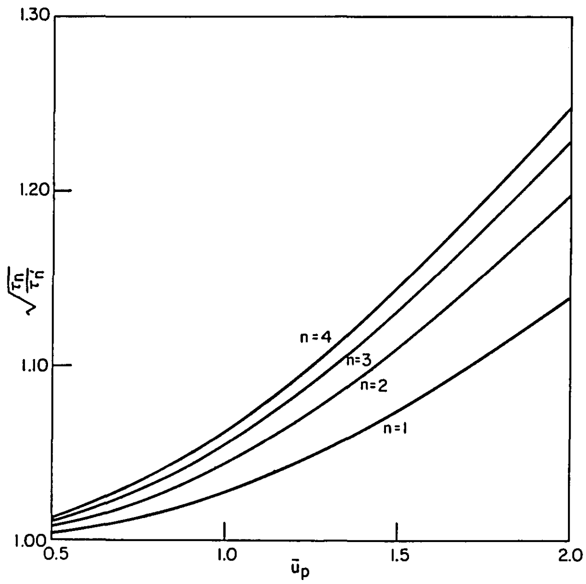

Fig. 3 – Ratio of relative shock and isentropic sound speeds vs relative piston velocity

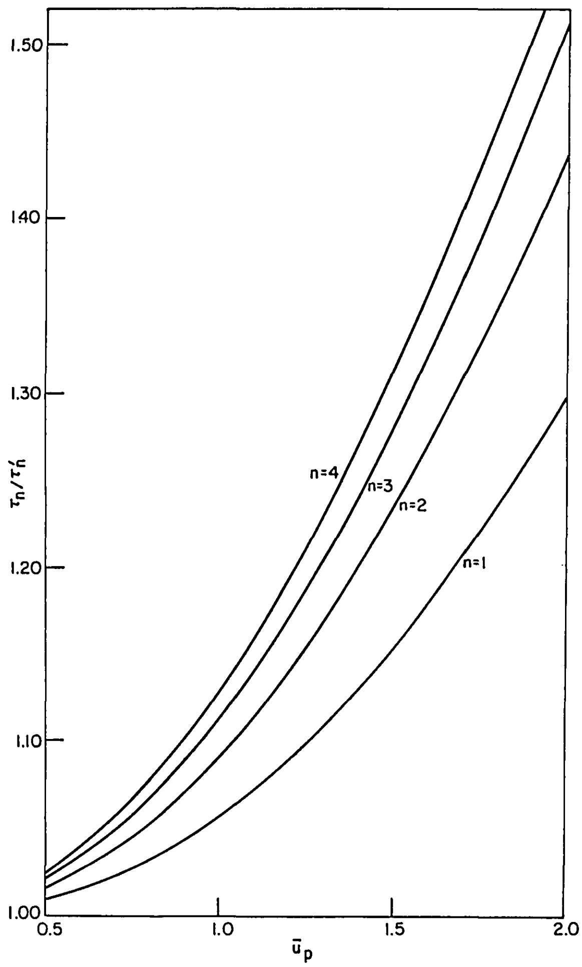

Fig. 2 – Ratio of relative shock and isentropic temperatures vs relative piston velocity

we see that $T_f = 3830°K$. Then by Eq. (16) and obtaining $P_4 = \pi_4 P_0$ from Table 1 we see that $P_f = 2.43 \times 10^6$ psi. Now if the compression tube is lengthened so that $L^0 = 256\, D^0$, then, proceeding as before, $T_f = 2000°K$ and $P_f = 0.25 \times 10^6$ psi. Note that $T_f/T_f' = 1.128$ for both cases. Thus, while the temperatures, pressures, and volumes differ for the two examples, the final temperature ratio of shock heating to isentropic compression remains the same.

In addition, it is possible to obtain an approximate equation for the location and time of travel of the piston after n shock transits. It is assumed that when the piston has reached its maximum velocity, the shock wave is created at the piston face. In reality the shock wave will be formed at some distance from the piston; consequently, empirical results should give somewhat larger t_n's and smaller x_n's than the equations below. If the piston is a distance L from the end of the compression tube at the start of phase 2, then the distance between the piston and the end of the compression tube after the nth shock transit is

$$x_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{(n-1)/2} (1-\bar{u})L \quad \text{where } n = 1,3,5,\ldots, \text{ and} \tag{18a}$$

$$x_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{n/2} L \quad \text{where } n = 2,4,6,\ldots, \tag{18b}$$

and where $\bar{u} = u_p/U$ (see Appendix B); and the time for the nth shock transit is

$$t_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{(n-1)/2} \frac{L}{U} \quad \text{where } n = 1,3,5,\ldots, \text{ and} \tag{19a}$$

$$t_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{n/2} \frac{L}{U} \quad \text{where } n = 2,4,6,\ldots. \tag{19b}$$

The total time elapsed after n shock transits is

$$T_n = \left[1 - (1+\bar{u})\left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{(n-1)/2}\right]\frac{L}{u_p} \quad \text{where } n = 1,3,5,\ldots, \text{ and}$$

$$T_n = \left[1 - \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{n/2}\right]\frac{L}{u_p} \quad \text{where } n = 2,4,6,\ldots. \tag{20}$$

Note that since $(1-\bar{u})/(1+\bar{u}) < 1$, as $n \to \infty$ then $x_n \to 0$, $t_n \to 0$, and $T_n \to L/u_p$.

CONCLUSION

It can be seen from the next to the last column of Table 1 that shock compression does provide higher final temperatures than isentropic compression for the same final pressure. With current gun configurations, however, the improvement seems to be marginal at best. Significant gains in sound speed begin to appear for piston speeds on the order of Mach 2 or greater, which, for hydrogen gas initially at 300°K, would mean speeds in excess of 8500 fps. On the other hand, the ideal model used here must be regarded as an upper limit on the improvement to be expected because of the energy losses and real gas effects that occur in the real gun. Furthermore, serious deviation from this model occurs, particularly, at low piston velocities, because the piston is not accelerated instantaneously to its maximum velocity. Consequently, the shock wave may not form or may not arise until projectile launch has begun. This, of course, would reduce any augmentation of temperature over isentropic compression.

The model also has assumed that the projectile is stationary until the piston comes to rest. In the light-gas gun this condition is controlled primarily by the projectile release pressure at which a valve or some corresponding device is activated to release the projectile. In order that the conclusions of the analysis used here be directly applicable to the light-gas gun, it is necessary that at least most of the shock heating shall have occurred before the projectile release pressure is attained. Assuming then that at least three shock transits must occur before projectile release, it can be concluded that the analysis is valid for piston velocities of less than 4000 fps with projectile release pressures as low as 5000 psi and valid for higher piston velocities with correspondingly higher release pressures. (The projectile release pressures here depend on the initial gas loading pressures, which are in the range of 100 to 500 psi for the conditions cited above.) For those cases where projectile launch occurs at the same time as shock heating, the compression process becomes considerably more complicated and further work will be necessary to correctly analyze this interaction and apply it to the light-gas gun. It seems doubtful that the motion of the projectile and its effects could enhance the shock heating *per se*.

The real gas effects on the shock heating of air has been reported by Stollery and Maull (6) and shows that serious departure from the ideal gas assumption appears at about $\bar{u} = 4$. Moreover, the deviations are of such a nature as to detract from the desirability of shock heating. For the shock heating of a real gas, not only is the temperature increase *less* than for an ideal gas, but also, the pressure increase is *greater*. It therefore seems doubtful, particularly with present gun facilities, that shock heating of the light gas is the most fruitful method for advancing gun capabilities.

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Appendix A

Ratio of the Final Shock Temperature to the Final Isentropic Temperature

By definition

$$\tau_4 = T_4/T_0 \qquad \text{and} \qquad \pi_4 = P_4/P_0.$$

From Eq. (15)

$$\frac{T_f}{T_0} = \frac{T_4}{T_0} + \frac{m_p u_p^2}{2Nc_v T_0} \equiv \tau_4 + \alpha = \tau_4(1 + \alpha/\tau_4)$$

from Eq. (11)

$$\frac{P_f}{P_0} = \frac{P_4}{P_0}\left(\frac{T_f}{T_4}\right)^{\gamma/(\gamma-1)} = \frac{P_4}{P_0}\left(\frac{T_f/T_0}{T_4/T_0}\right)^{\gamma/(\gamma-1)}$$

$$= \pi_4 (1 + \alpha/\tau_4)^{\gamma/(\gamma-1)}.$$

From Eq. (12)

$$\tau_4' = (\pi_4)^{(\gamma-1)/\gamma}$$

and from Eq. (13)

$$\frac{T_f'}{T_0} = \left(\frac{P_f}{P_0}\right)^{(\gamma-1)/\gamma} = (\pi_4)^{(\gamma-1)/\gamma}\,(1 + \alpha/\tau_4)$$

$$= \tau_4'\,(1 + \alpha/\tau_4).$$

Hence

$$\frac{T_f}{T_f'} = \frac{T_f/T_0}{T_f'/T_0} = \frac{\tau_4(1 + \alpha/\tau_4)}{\tau_4'(1 + \alpha/\tau_4)} = \frac{\tau_4}{\tau_4'}.$$

Appendix B
Piston Position and Time After nth Shock Transit

Initially the shock wave must travel a distance L to the end of the compression tube at a velocity U. This requires a time

$$t_1 = L/U$$

in which time the piston moves a distance $u_p t_1$ and is then a distance (see Fig. 1)

$$x_1 = L - u_p t_1 = (1 - \bar{u})L$$

from the end of the compression tube, where $\bar{u} = u_p/U$. The new distance the shock wave must travel back to the piston is $x_2 = x_1 - u_p t_2$ in time $t_2 = x_2/U$. Therefore

$$x_2 = \left(\frac{1-\bar{u}}{1+u}\right) L$$

and

$$t_2 = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)\frac{L}{U}.$$

Continuing in this manner

$$t_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{(n-1)/2}\frac{L}{U} \quad x_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{(n-1)/2}(1-\bar{u})L$$

$n = 1,3,5,\ldots,$ and

$$t_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{n/2}\frac{L}{U} \qquad x_n = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^{n/2} L$$

$n = 2,4,6,\ldots.$

The total time for n transits is given by

$$T_n = \sum_{k=1}^{n} t_k = \sum_i t_{2i-1} + \sum_j t_{2j}$$

where $i,j = 1,2,3,\ldots,$

$$= \sum_i \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^i \left(\frac{1+\bar{u}}{1-\bar{u}}\right)\frac{L}{U} + \sum_j \left(\frac{1-\bar{u}}{1+\bar{u}}\right)^j \frac{L}{U}.$$

Now let the upper limit be $n = 2j$, then since

$$\sum_{k=1}^{n} x^k = x\left(\frac{1-x^n}{1-x}\right)$$

$$T_{2j} = \left(\frac{1-\bar{u}}{1+\bar{u}}\right)\left[\frac{1-\left(\frac{1-\bar{u}}{1+\bar{u}}\right)^j}{1-\left(\frac{1-\bar{u}}{1+\bar{u}}\right)}\right]\left(\frac{2}{1-\bar{u}}\right)\frac{L}{U}$$

$$= \left[1-\left(\frac{1-\bar{u}}{1+\bar{u}}\right)^j\right]\frac{L}{\bar{u}U}.$$

Now since $(1-\bar{u}/1+\bar{u}) < 1$, then $(1-\bar{u}/1+\bar{u})^j < 1$ and as $j \to \infty$, $(1-\bar{u}/1+\bar{u})^j \to 0$

therefore

$$T_\infty = \lim_{j\to\infty} T_{2j} = \frac{L}{\bar{u}U} = \frac{L}{u_p}.$$