

Theory on Optimum Performance of Modern Jet Ejectors

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FOREWORD

Studies of plasma properties in the Flame and Plasma Laboratory of the Radar Division are directed in part toward a determination of the electrical properties of the exhausts from propulsion systems and their interaction with detection and control systems. These properties and interactions vary widely with the ambient pressure, i.e., with altitude. High pumping rates are required to pump rocket motor exhaust gases to simulate the wide range of altitude encountered in operational systems. This study of jet ejector performance was conducted to aid in the design of the NRL space simulation system used in such studies, the assembly of which is now approaching completion. It is also hoped that this study will form an analytical basis for the design of future ejectors for use under less restricted operation conditions.

ABSTRACT

A theoretical investigation was made of the optimum performance of a single-stage jet ejector with allowances made for the differences in temperature and molecular weight of the motive gas and the suction gas. Constant-pressure mixing of the motive and suction gases was assumed in the analytical model, and the task of optimization was carried out by rigorous mathematical operation. The analysis considers the case in which supersonic flow and hence normal shock occurs in the injector and the case of flow without normal shock. Numerical calculations of results were performed on an electronic digital computer for four selected typical examples of practical importance in the ranges of operating conditions of modern jet ejectors. These examples involved the following respective combinations of motive gas and suction gas: steam and air, silicone oil and air, steam and steam, and air and air. The results were found to compare rather satisfactorily with reported performance characteristics of a jet ejector.

PROBLEM STATUS

This is an interim report on a continuing problem.

AUTHORIZATION

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THEORY ON OPTIMUM PERFORMANCE OF MODERN JET EJECTORS

INTRODUCTION

One-dimensional analyses have been in the past successfully developed for the prediction of the performance of jet ejectors with various restrictions. Keenan, Neumann, and Lustwerk [1], Flügel [2], Keenan and Neumann [3], Elrod [4], Dotterweich and Mooney [5], and Fletcher [6] applied the equations for an adiabatic and frictionless flow to the cases in which the motive gas is the same as the suction gas with respect to molecular weight, temperature, and specific heat. Their calculated optimum performance curves of jet ejectors agree closely with the corresponding experimental results.

Experimental work reported later by Work and Haedrich [7], Holton [8], and Holton and Schulz [9] shows the effects on ejector performance if the molecular weight and temperature of the motive and suction gases are different from each other. DeFrate and Hoerl [10] developed an extended one-dimensional theory which includes these observed effects. Their calculations for optimum design curves based on the assumption of constant-area mixing between the motive and suction gases were carried out on a digital computer over typical ranges of molecular weights and operating conditions of practical interest to chemical plants. Their method of optimization was strictly machine iteration rather than the more rigorous mathematical processing.

However, these ranges of molecular weights and operating conditions covered by the previous investigations are found inadequate for vacuum facilities used in modern altitude simulation installations. Furthermore, as pointed out by Keenan, Neumann, and Lustwerk [1], in most cases it is possible to achieve better performance with constant-pressure mixing than with constant-area mixing of the motive and suction gases, and the commercially available ejectors are usually of the constant-pressure type. In fact, at higher molecular weight ratios, computed performance curves of DeFrate and Hoerl [10] are found to disagree very significantly from the corresponding experimental findings of Refs. 7, 8, and 9. An analysis of the optimum performance of jet ejectors based on the assumption of constant-pressure mixing of the motive and suction gases for extended ranges of molecular weights and operating conditions seems in order.

METHOD OF ANALYSIS

It will be assumed that the gases are ideal gases with constant specific heats and that the flow is adiabatic and reversible, with the motive and suction gases mixing at constant pressure. A general description of the analytical model is sketched in Fig. 1. The motive gas a from a high pressure source expands in a convergent-divergent nozzle to its exit plane (1), at which the suction gas b starts to be entrained into the flow leaving the nozzle. The motive and suction gases are further mixed in a section of the flow passage whose cross-sectional area is so varied that the static pressure is maintained constant. The mixing is assumed to be completed long before the entrance to the diffuser section of the flow passage (2). The homogeneous mixture then slows down to a negligible velocity at the exit plane of the diffuser section of the flow passage (3). However, if the gas mixture velocity should have become supersonic before the entrance to the diffuser section, a normal shock is assumed to exist somewhere before the entrance to the diffuser section, where mixing is already completed, as shown in Fig. 2. In this case, the pressure in the region of the mixing section before the normal shock is assumed to be constant.

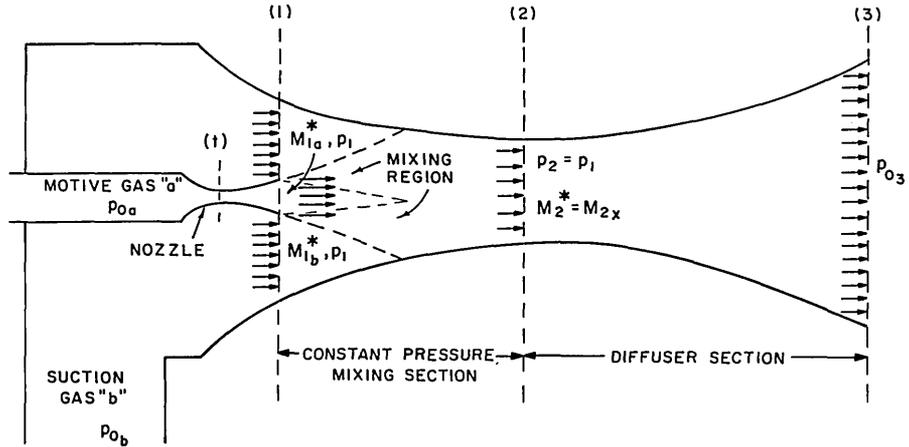


Fig. 1 - Analytical model for flow without normal shock

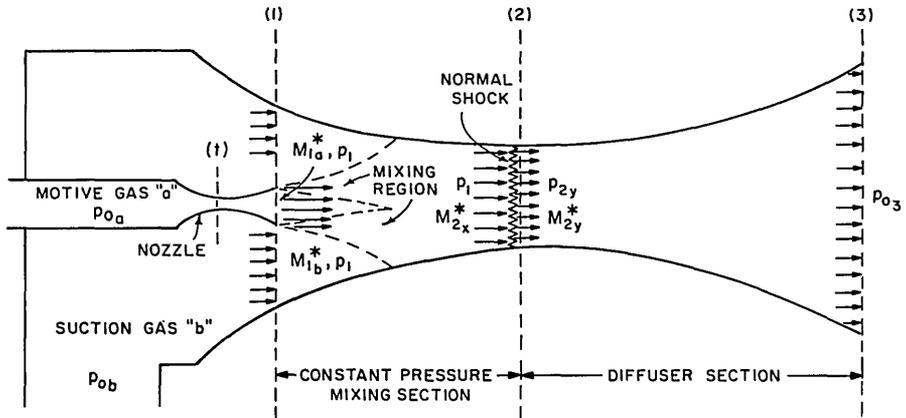


Fig. 2 - Analytical model for flow with normal shock

CASE 1: FLOW WITH NORMAL SHOCK (FIG. 2)

Nozzle Flow Analysis

From one-dimensional adiabatic nozzle flow analysis of an ideal gas, critical Mach numbers of the motive gas a and the suction gas b at the exit plane (1) can be obtained respectively as

$$M_{1a}^* = \alpha_2^{1/2} (1 - \beta_1^{\alpha_1} X^{\alpha_1})^{1/2} \quad (1)$$

and

$$M_{1b}^* = \alpha_4^{1/2} (1 - X^{\alpha_3})^{1/2} \quad (2)$$

where the symbols (which are also listed in the last section of the report) are

$$\alpha_1 = (\gamma_a - 1) / \gamma_a$$

$$\alpha_2 = (\gamma_a + 1) / (\gamma_a - 1)$$

$\beta_1 = P_{0_b} / P_{0_a}$, the ratio of suction pressure to motive pressure

$X = P_1 / P_{0_b}$, the expansion ratio

$$\alpha_3 = (\gamma_b - 1) / \gamma_b$$

$$\alpha_4 = (\gamma_b + 1) / (\gamma_b - 1)$$

$\gamma_a = C_{p_a} / C_{v_a}$, the ratio of the specific heats of the motive gas

$\gamma_b = C_{p_b} / C_{v_b}$, the ratio of the specific heats of the suction gas

$M_{1_a}^* = V_{1_a} / C_a^*$, the critical Mach number of the motive gas at the nozzle exit plane

$M_{1_b}^* = V_{1_b} / C_b^*$, the critical Mach number of the suction gas at the nozzle exit plane

V_{1_a} , the velocity of the motive gas at the nozzle exit plane

V_{1_b} , the velocity of the suction gas at the nozzle exit plane

$C_a^* = \sqrt{2g \left(\frac{\gamma_a}{\gamma_a + 1} \right) \frac{R}{M_a} T_{0_a}}$, the critical sonic velocity of the motive gas

$C_b^* = \sqrt{2g \left(\frac{\gamma_b}{\gamma_b + 1} \right) \frac{R}{M_b} T_{0_b}}$, the critical sonic velocity of the suction gas

M_a , the molecular weight of the motive gas

M_b , the molecular weight of the suction gas

T_{0_a} , the stagnation temperature of the motive gas

T_{0_b} , the stagnation temperature of the suction gas

R , the universal gas constant

and where the ideal gas law

$$P = \rho \frac{R}{M} T \quad (3)$$

for both the motive and suction gases has been used.

Constant-Pressure Mixing Analysis

The governing one-dimensional equations of conservation of mass, momentum, and energy for adiabatic ideal gas flows integrated between cross sections (1) and (2_x) are

$$\rho_{1_a} A_{1_a} V_{1_a} + \rho_{1_b} A_{1_b} V_{1_b} = \rho_{2_x} A_{2_x} V_{2_x} \quad (\text{mass}) \quad (4)$$

$$\frac{W_a}{g} V_{1a} + \frac{W_b}{g} V_{1b} + P_1 A_2 = \frac{W_2}{g} V_{2x} + P_2 A_2 \quad (\text{momentum}) \quad (5)$$

$$h_{1a} + \frac{V_{1a}^2}{2g} + h_{1b} + \frac{V_{1b}^2}{2g} = h_{2x} + \frac{V_{2x}^2}{2g} \quad (\text{energy}) \quad (6)$$

where the symbols are

$$h_{1a} = \left(\frac{\gamma_a}{\gamma_a - 1} \right) \frac{R}{M_a} T_{1a}, \text{ the specific enthalpy of the motive gas at cross section (1)}$$

$$h_{1b} = \left(\frac{\gamma_b}{\gamma_b - 1} \right) \frac{R}{M_b} T_{1b}, \text{ the specific enthalpy of the suction gas at cross section (1)}$$

$$h_{2x} = \left(\frac{\gamma_2}{\gamma_2 - 1} \right) \frac{R}{M_2} T_{2x}, \text{ the specific enthalpy of the gas mixture at cross section (2}_x\text{) just before shock}$$

subscript 2_x, the cross section just before shock

A_{1a}, the cross-sectional area of the exit plane of the motive gas nozzle

A_{1b}, the cross-sectional area of the exit plane of the suction gas passage

A₂, the cross-sectional area of the flow passage at the entrance of the diffuser section

W_a, the mass flow rate of the motive gas

W_b, the mass flow rate of the suction gas

W₂ = W_a + W_b, the mass flow rate of the gas mixture

ρ_{1a}, the density of the motive gas at cross section (1)

ρ_{1b}, the density of the suction gas at cross section (1)

ρ_{2x}, the density of the gas mixture at cross section (2_x)

γ₂, the ratio of the specific heats of the gas mixture

M₂, the molecular weight of the gas mixture.

Equations (1) through (6) can be combined to yield the result for the critical Mach number of the gas mixture at cross section (2_x):

$$M_{2x}^* = \frac{M_{1a}^* + M_{1b}^* \beta_2 \alpha_5}{\alpha_6 (1 + \beta_2)} \quad (7)$$

where

β₂ = W_b/W_a, the entrainment ratio

$$\alpha_5 = C_b^*/C_a^*$$

$$\alpha_6 = C_2^*/C_a^*$$

$$C_2^* = \sqrt{2g \left(\frac{\gamma_2}{\gamma_2 + 1} \right) \frac{R}{M_2} T_{0_2}}, \text{ the critical sonic velocity of the gas mixture.}$$

From the normal shock relations the ratio of pressures at cross sections (2_x) and (2_y) immediately before and after the shock respectively, P_{2_y}/P_1 , can be obtained as

$$F = \frac{\alpha_7 \left(M_{2_x}^* \right)^2 - 1}{\alpha_7 - \left(M_{2_x}^* \right)^2} \quad (8)$$

where

$F = P_{2_y}/P_1$, the pressure ratio across the normal shock

$$\alpha_7 = (\gamma_2 + 1)/(\gamma_2 - 1).$$

Diffuser Flow Analysis

From one-dimensional adiabatic diffuser flow analysis of an ideal gas, the diffuser compression ratio, P_{0_3}/P_{2_y} , can be obtained as

$$G = \left[1 - \frac{1}{\alpha_7} \frac{1}{\left(M_{2_x}^* \right)^2} \right]^{-\alpha_8} \quad (9)$$

where

$G = P_{0_3}/P_{2_y}$, the diffuser compression ratio for this case of flow with normal shock

$$\alpha_8 = \gamma_2/(\gamma_2 - 1).$$

Mixture Properties

From the analysis of the properties of a mixture of two ideal gases the following results can be obtained:

$$\frac{T_{0_2}}{T_{0_a}} = \frac{\frac{1}{\alpha_1} + \frac{\beta_2 \alpha_9 \alpha_{10}}{\alpha_3}}{\frac{1}{\alpha_1} + \frac{\beta_2 \alpha_9}{\alpha_3}} \quad (10)$$

$$\gamma_2 = \frac{\frac{1}{\alpha_1} + \frac{\beta_2 \alpha_9}{\alpha_3}}{\left(\frac{1 - \alpha_1}{\alpha_1} \right) + \left(\frac{1 - \alpha_3}{\alpha_3} \right) \beta_2 \alpha_9} \quad (11)$$

$$\alpha_6 = \left[\left(\frac{\gamma_2}{\gamma_2 + 1} \right) (2 - \alpha_1) \frac{M_a}{M_2} \frac{T_{0_2}}{T_{0_a}} \right]^{1/2} \quad (12)$$

$$\frac{M_a}{M_2} = \frac{1 + \beta_2 \alpha_9}{1 + \beta_2} \quad (13)$$

where

$$\alpha_9 = M_a / M_b$$

$$\alpha_{10} = T_{0_b} / T_{0_a}$$

Overall Consideration

Combining Eqs. (1) through (3) and (7) through (13) yields the area ratio of the cross section (2) to the throat cross section of the motive gas nozzle, A_2/A_t :

$$\beta_3 = \left\{ (1 + \beta_2) \left[\frac{2(1 - \alpha_1)}{2 - \alpha_1} \right]^{(1 - \alpha_1) / \alpha_1} \left(\frac{1}{\beta_1} \frac{1}{X} \right) \frac{M_a}{M_2} \frac{T_{0_2}}{T_{0_a}} \frac{1}{\alpha_6} \right\} \quad (14)$$

where $\beta_3 = A_2/A_t$ is the area ratio and M_a/M_2 , T_{0_2}/T_{0_a} , and α_6 are given by Eqs. (13), (10), and (12) respectively.

Optimization

By the definition of the overall compression ratio, P_{0_3}/P_{0_b} , we can write

$$Y = XFG \quad (15)$$

where $Y = P_{0_3}/P_{0_b}$, the overall compression ratio.

The optimum design condition is achieved when the overall compression ratio Y is maximized as a function of the expansion ratio X by differentiation:

$$\frac{\partial Y}{\partial X} = 0 = X \left[F \frac{\partial G}{\partial X} + G \frac{\partial F}{\partial X} \right] + FG$$

which can be simplified to

$$\frac{1}{X} = - \frac{1}{G} \frac{\partial G}{\partial X} - \frac{1}{F} \frac{\partial F}{\partial X} \quad (16)$$

From Eqs. (9) and (8) we can get

$$\left(\frac{1}{G} \frac{\partial F}{\partial X} \right) = - \frac{2\alpha_8}{\alpha_7} \frac{\partial M_{2_x}^*}{\partial X} \left(M_{2_x}^* \right)^{-3} \left[1 - \frac{1}{\alpha_7} \left(M_{2_x}^* \right)^{-2} \right]^{-1} \quad (17)$$

and

$$\left(\frac{1}{F} \frac{\partial F}{\partial X} \right) = \left[2(\alpha_7^2 - 1) \right] M_{2_x}^* \frac{\partial M_{2_x}^*}{\partial X} \left[\alpha_7 \left(M_{2_x}^* \right)^2 - 1 \right]^{-1} \left[\alpha_7 - \left(M_{2_x}^* \right)^2 \right]^{-1} \quad (18)$$

respectively. From Eq. (7), which is

$$M_{2x}^* = \frac{M_{1a}^* + M_{1b}^* \beta_2 \alpha_5}{\alpha_6 (1 + \beta_2)} \quad (7)$$

we can get

$$\frac{\partial M_{2x}^*}{\partial X} = \alpha_6^{-1} (1 + \beta_2)^{-1} \left(\frac{\partial M_{1a}^*}{\partial X} + \beta_2 \alpha_5 \frac{\partial M_{1b}^*}{\partial X} \right). \quad (19)$$

Finally, from Eqs. (1) and (2), which are

$$M_{1a}^* = \alpha_2^{1/2} \left[1 - \beta_1^{\alpha_1} X^{\alpha_1} \right]^{1/2} \quad (1)$$

and

$$M_{1b}^* = \alpha_4^{1/2} \left[1 - X^{\alpha_3} \right]^{1/2}. \quad (2)$$

we can get

$$\frac{\partial M_{1a}^*}{\partial X} = \left(-\frac{1}{2} \alpha_2^{1/2} \alpha_1 \beta_1^{\alpha_1} \right) X^{\alpha_1-1} \left(1 - \beta_1^{\alpha_1} X^{\alpha_1} \right)^{-1/2} \quad (20)$$

and

$$\frac{\partial M_{1b}^*}{\partial X} = \left(-\frac{1}{2} \alpha_4^{1/2} \alpha_3 \right) X^{\alpha_3-1} \left(1 - X^{\alpha_3} \right)^{-1/2} \quad (21)$$

respectively. With the expressions from Eqs. (17), (18), (7), (19), (1), (2), (20), and (21) used, Eqs. (14), (15) and (16) form the governing equations relating the various parameters and variables under the optimum design conditions.

Computational Procedure

1. For a fixed pair of values of the area ratio β_3 and the entrainment ratio β_2 assume a trial value for the expansion ratio X . Compute the ratio of suction to motive pressures β_1 from Eq. (14). The compositions and initial conditions of both the motive and suction gas are assumed given.

2. These values of $\beta_1, \beta_2, \beta_3$, and X are then used to compute the two sides of Eq. (16). If the computed values of the two sides of Eq. (16) are found not to agree with each other, assume a different trial value for X and repeat the computation until Eq. (16) is satisfied within the required degree of accuracy. Then compute the value of the overall compression ratio γ from Eq. (15).

CASE 2: FLOW WITHOUT NORMAL SHOCK (FIG. 1)

Equations (1) through (13), with the exceptions of Eqs. (8) and (9), derived for the case of flow with normal shock can be directly used in the case of flow without normal shock. From the definition of the overall compression ratio Y we can write for this case

$$Y = XG' \quad (22)$$

where $G' = P_{0_3}/P_2$ is the diffuser compression ratio for the case without normal shock. The optimum design condition is achieved when the overall compression ratio Y is maximized as a function of the expansion ratio X by differentiation:

$$\frac{\partial Y}{\partial X} = 0 = X \frac{\partial G'}{\partial X} + G'$$

which can be simplified to

$$\frac{1}{X} = - \frac{1}{G'} \frac{\partial G'}{\partial X} \quad (23)$$

From a diffuser analysis similar to the one in the previous case we can get

$$G' = \left[1 - \frac{1}{\alpha_7} \left(M_{2_x}^* \right)^2 \right]^{-\alpha_8} \quad (24)$$

and

$$\frac{\partial G'}{\partial X} = (-\alpha_8) \left[1 - \frac{1}{\alpha_7} \left(M_{2_x}^* \right)^2 \right]^{-\alpha_8-1} \left(\frac{-2}{\alpha_7} M_{2_x}^* \frac{\partial M_{2_x}^*}{\partial X} \right) \quad (25)$$

With the expressions from Eqs. (24), (25), (7), (19), (1), (2), (20), and (21) used, Eqs. (14), (22), and (23) form the governing equations relating the various parameters and variables under the optimum design conditions for this case.

The computational procedure for this case is exactly the same as that for the previous case except that Eqs. (15) and (16) are replaced by Eqs. (22) and (23) respectively. In an actual computation, the value of the critical Mach number, $M_{2_x}^*$, is evaluated at an early stage. If this value is found to be larger than unity, the computation is continued following the analysis developed for the previous case with normal shock. If this value is found to be smaller than unity, the computation is continued following the analysis developed for this case without normal shock.

RESULTS

Numerical calculations are performed on an electronic digital computer for the four following selected typical examples of practical importance in the ranges of operating conditions of modern jet ejectors:

In example A the motive gas is steam ($M_a = 18$ lb/mole, $\gamma_a = 1.3$) saturated at 93 psig ($P_{0_a} = 107.7$ psia, $T_{0_a} = 793.21^\circ\text{R}$) and the suction gas is air ($M_b = 28.8$ lb/mole, $\gamma_b = 1.4$) at 70°F ($T_{0_b} = 530^\circ\text{R}$). The results ($\gamma_a = 1.3$, $\gamma_b = 1.4$, $M_b/M_a = 1.6$, $T_{0_a}/T_{0_b} = 1.49$) are plotted in Fig. 3.

In example B the motive gas is Dow-Corning 704 silicone oil ($M_a = 546$ lb/mole, $\gamma_a = 1.01$) at 450°F ($T_{0_a} = 910^\circ\text{R}$) and the suction gas is air ($M_b = 28.8$ lb/mole, $\gamma_b = 1.4$) at 70°F ($T_{0_b} = 530^\circ\text{R}$). The results ($\gamma_a = 1.01$, $\gamma_b = 1.4$, $M_b/M_a = 0.053$, $T_a/T_b = 1.7$) are plotted in Fig. 4.

In example C the motive gas is steam ($M_a = 18$ lb/mole, $\gamma_a = 1.3$) saturated at 100 psig ($P_{0_a} = 114.7$ psia, $T_{0_a} = 797.88^\circ\text{R}$) and the suction gas is steam ($M_b = 18$ lb/mole,

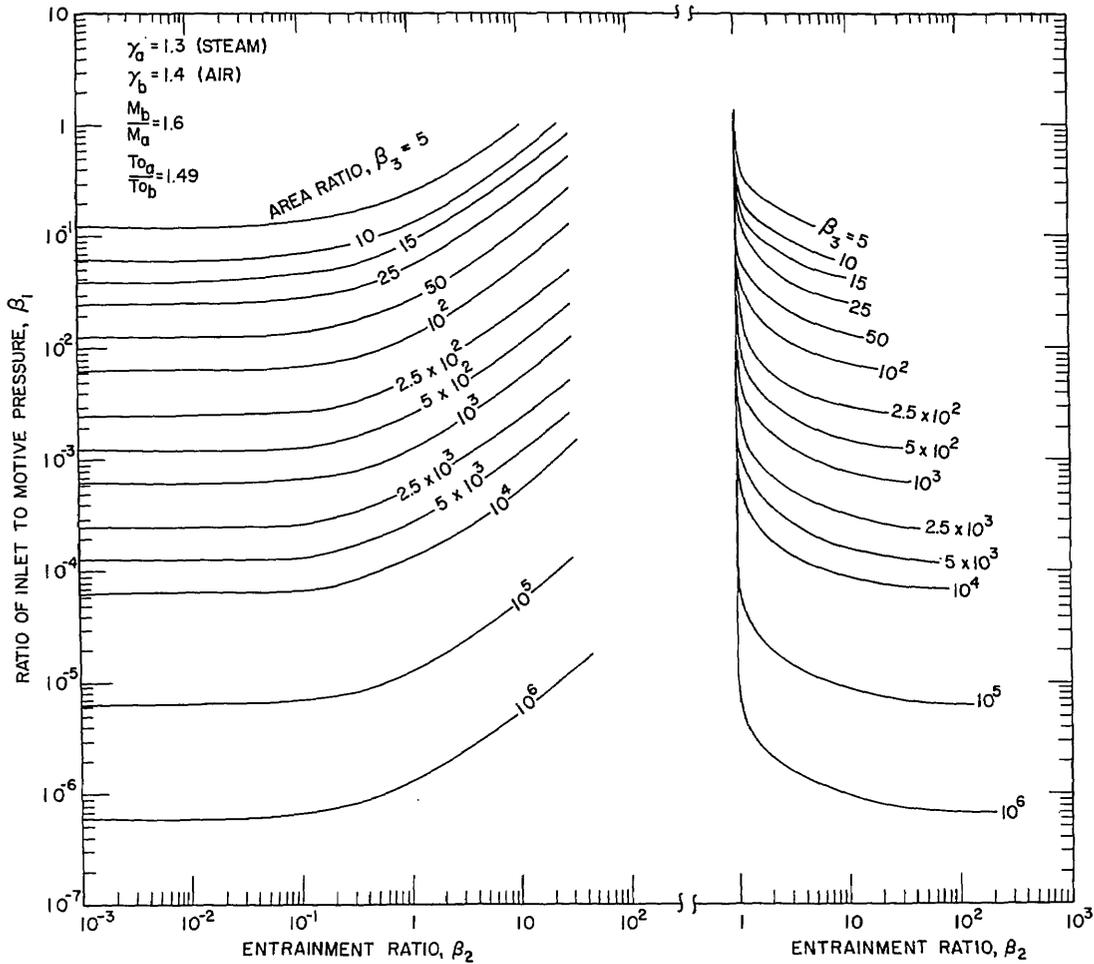


Fig. 3 - Theoretical results for an ejector using steam as the motive gas and air as the suction gas

$\gamma_b = 1.3$) at 70°F ($T_{0b} = 530^\circ\text{R}$). The results ($\gamma_a = 1.3$, $\gamma_b = 1.3$, $M_b/M_a = 1$, $T_{0a}/T_{0b} = 1.5$) are plotted in Fig. 5.

In example D the motive gas is air ($M_a = 28.8$ lb/mole, $\gamma_a = 1.4$) at the same temperature as the suction gas, which is also air ($M_b = 28.8$ lb/mole, $\gamma_b = 1.4$). The results ($\gamma_a = 1.4$, $\gamma_b = 1.4$, $M_b/M_a = 1$, $T_{0a}/T_{0b} = 1$) are plotted in Fig. 6.

COMPARISON WITH A REPORTED JET EJECTOR PERFORMANCE

The reported [11] performance characteristics of an Elliott four-stage steam jet ejector are shown in Fig. 7. The ejector consists of four single steam jet ejectors connected in series. The curves respectively show the performance of the system under the desired operation conditions when only the fourth stage, the fourth and third stages, the fourth, third, and second stages, and all four stages are in operation. The reported performance curve for the system when only the fourth stage is in operation is in a suitable form for comparison with the present theoretical results.

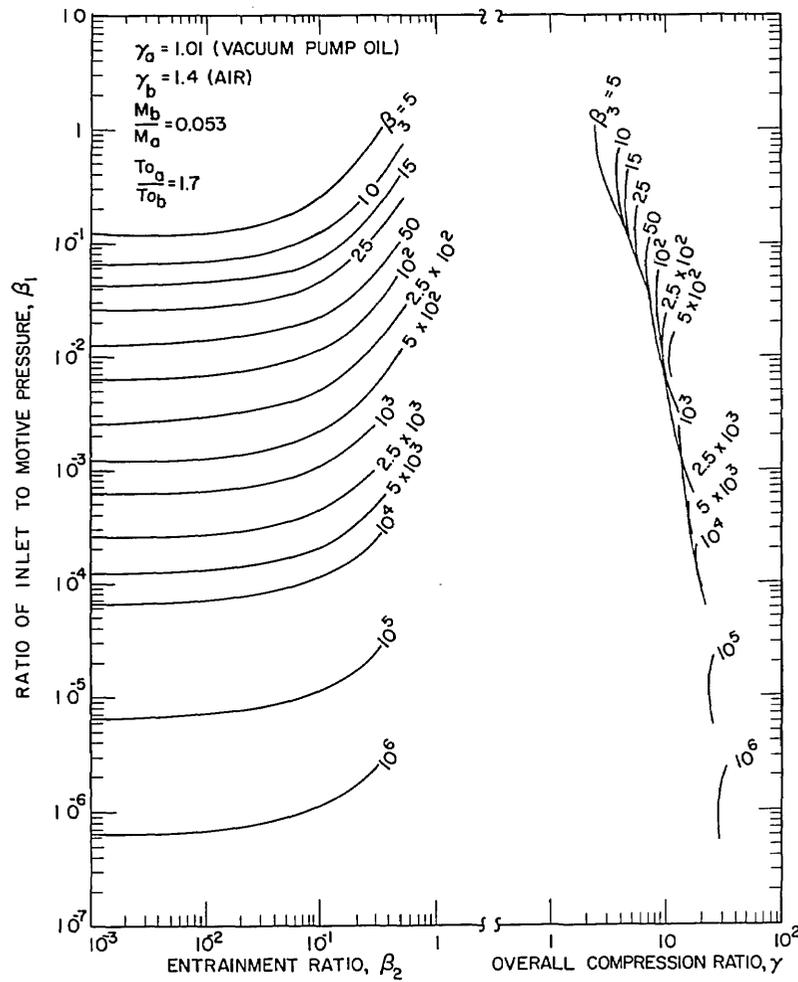


Fig. 4 - Theoretical results for an ejector using Dow-Corning 704 silicone oil as the motive gas and air as the suction gas

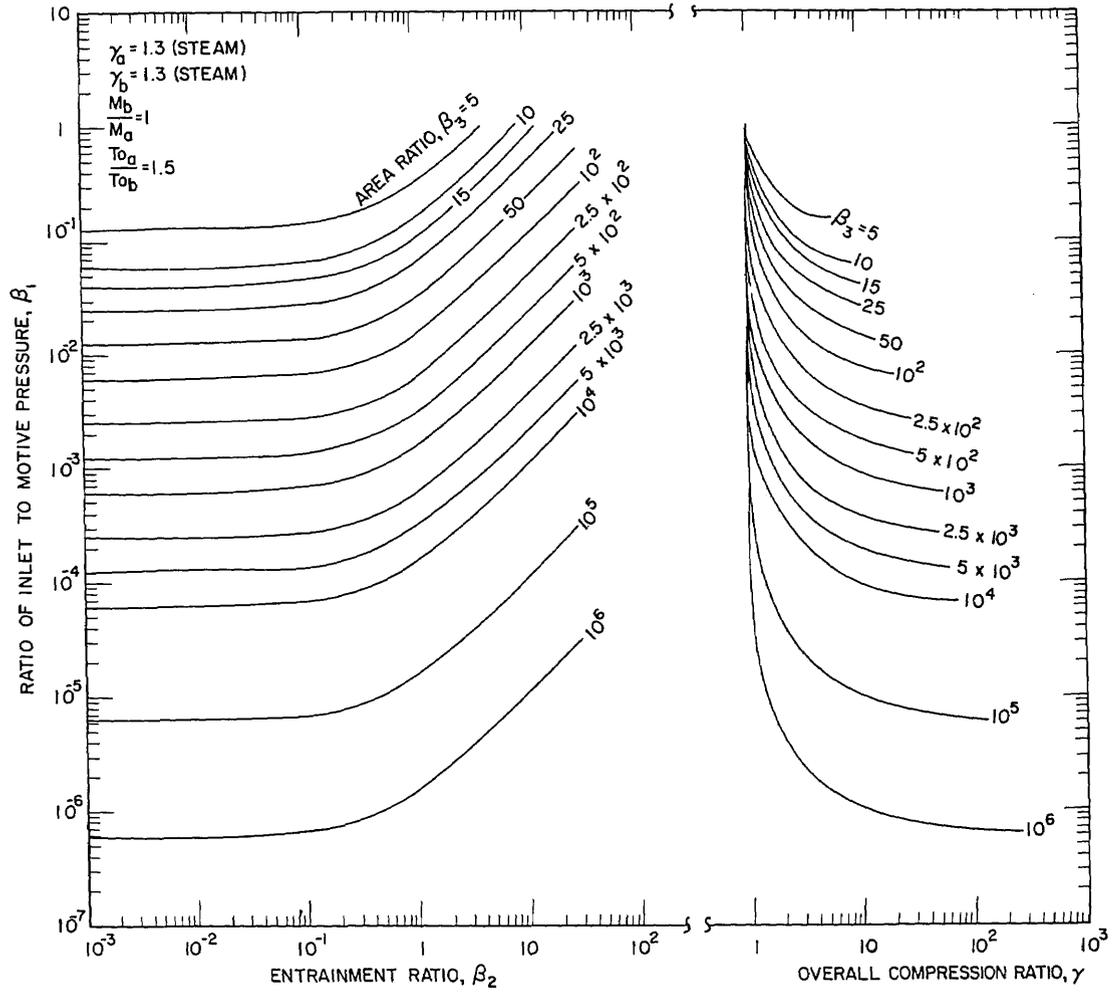


Fig. 5 - Theoretical results for an ejector using steam as both the motive and the suction gas

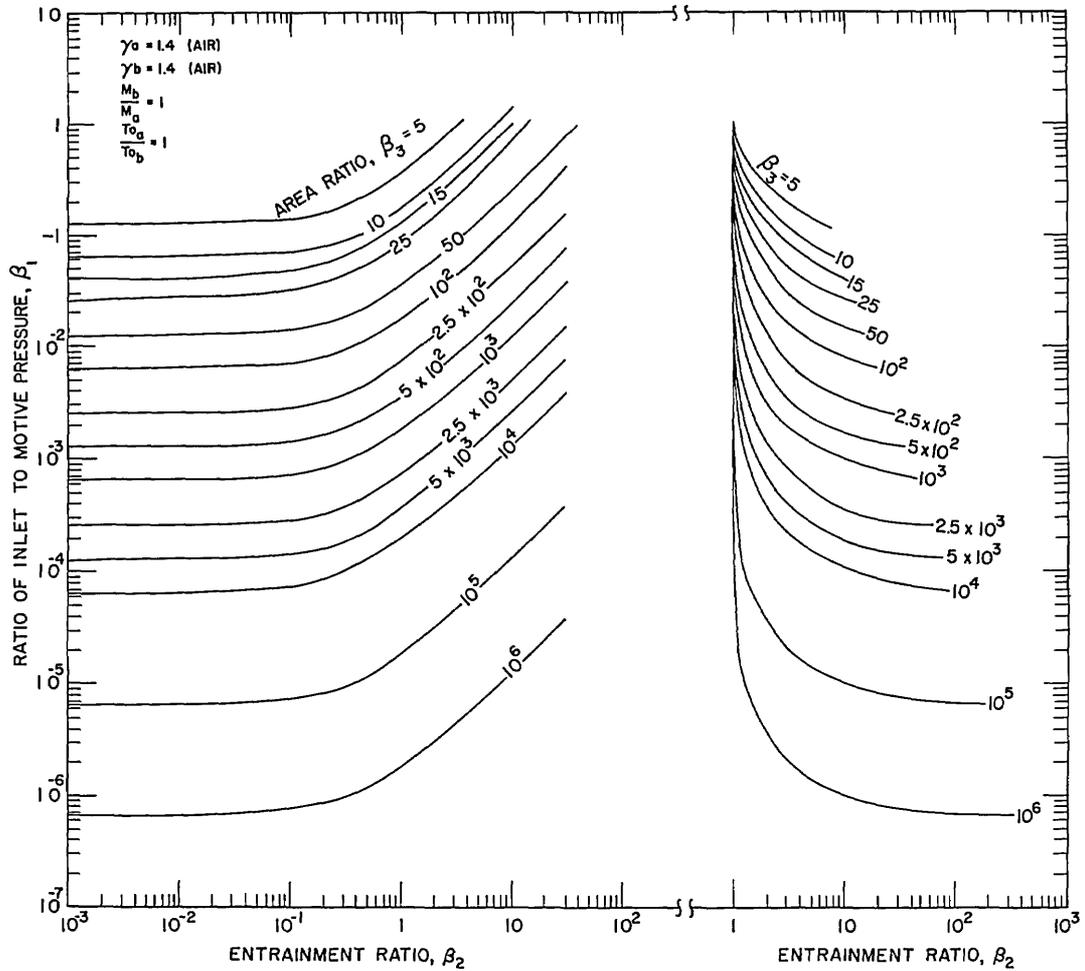


Fig. 6 - Theoretical results for an ejector using air as both the motive and the suction gas

The given operational conditions are as follows: The motive gas is steam ($M_a = 18$ lb/mole, $\gamma_a = 1.3$) saturated at 93 psig ($P_{0_a} = 107.7$ psia, $T_{0_a} = 793.21^\circ\text{R}$).

The suction gas is air ($M_b = 28.8$ lb/mole, $\gamma_b = 1.4$) at 70°F ($T_{0_b} = 530^\circ\text{R}$). The exit pressure is $P_{0_3} = 70$ mm Hg abs. The mass flow rate of the motive gas is $W_a = 15,280$ lb/hr. The area ratio is $A_2/A_t = \beta_3 = 368$.

The performance curve gives the values of the mass flow rate of the suction gas (air) W_b for given values of the suction pressure P_{0_3} . Therefore, for each point on the curve the values of the overall compression ratio Y , the ratio of suction to motive pressure β_1 , and the entrainment ratio β_2 can be evaluated. These values over the range of operational conditions plotted against the corresponding theoretical results from Fig. 3 are shown in Fig. 8. We can regard this comparison to be rather satisfactory.

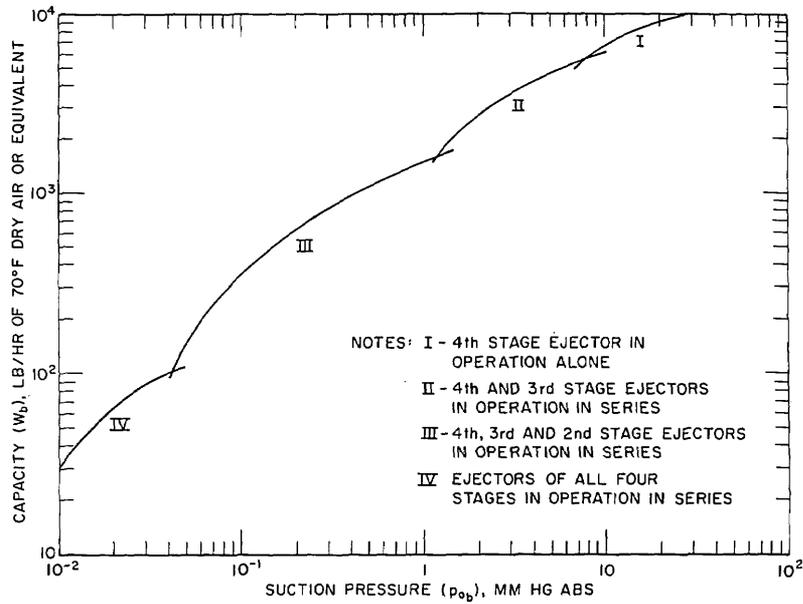


Fig. 7 - Expected performance of an Elliott four-stage steam jet ejector

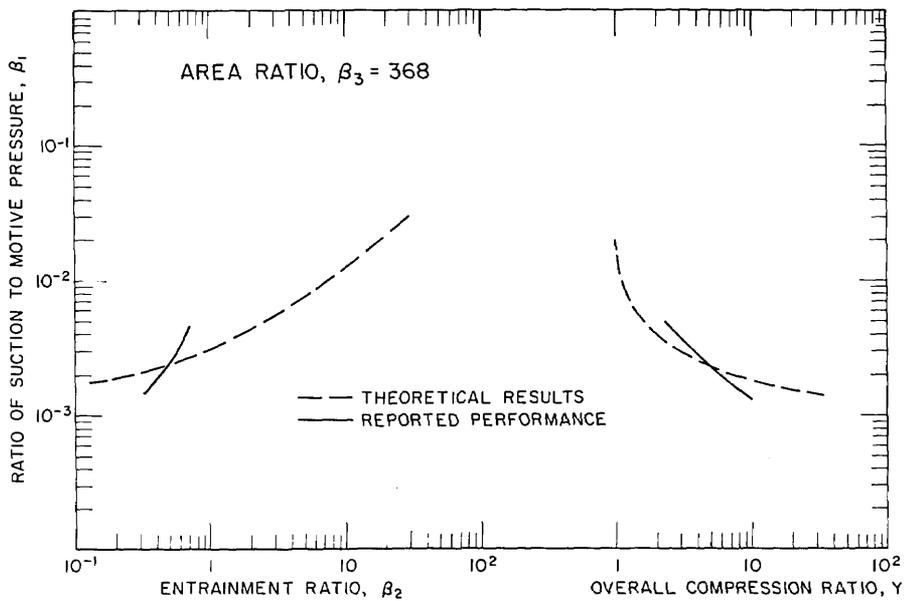


Fig. 8 - Comparison of the theoretical results with a reported performance (the fourth-stage ejector of the Elliott four-stage steam jet ejector)

LIST OF SYMBOLS

A_{1a}	the cross-sectional area of the exit plane of the motive gas nozzle
A_{1b}	the cross-sectional area of the exit plane of the suction gas passage
A_2	the cross-sectional area of the flow passage at the entrance of the diffuser section
A_t	the cross-sectional area of the throat of the motive gas nozzle
C_{p_a}, C_{p_b}	the specific heats at constant pressure of the motive gas and the suction gas respectively
C_{v_a}, C_{v_b}	the specific heats at constant volume of the motive gas and the suction gas respectively
C_a^*, C_b^*, C_x^*	the critical sonic velocities of the motive gas, the suction gas, and the gas mixture respectively
$F = P_{2y} / P_1$	the pressure ratio across the normal shock
$G = P_{0_3} / P_{2y}$	the diffuser compression ratio for the case of flow with normal shock
$G' = P_{0_3} / P_2$	the diffuser compression ratio for the case of flow without normal shock
g	the gravitational acceleration
h_{1a}, h_{1b}, h_{2x}	the specific enthalpies of the motive gas and the suction gas at the nozzle exit plane and of the gas mixture just before normal shock before the entrance to the diffuser section respectively
M	the molecular weight
M_a, M_b, M_x	the molecular weights of the motive gas, the suction gas, and the gas mixture respectively
$M_{1a}^*, M_{1b}^*, M_{2x}^*$	the critical Mach numbers of the motive gas and the suction gas at nozzle exit plane and of the gas mixture just before normal shock before the entrance to the diffuser section respectively
P	the static pressure
P_1, P_2	the static pressures at the nozzle exit plane and at the entrance to the diffuser section respectively
P_{2x}, P_{2y}	the static pressures before and after the normal shock respectively
P_{0a}, P_{0b}, P_{0_3}	the stagnation pressures of the motive gas, the suction gas, and the gas mixture at the exit of the diffuser section respectively
R	the universal gas constant
T	the static temperature

$T_{1_a}, T_{1_b}, T_{2_x}$	the static temperatures of the motive gas and suction gas at the nozzle exit plane and of the gas mixture just before normal shock before the entrance to the diffuser section respectively
$T_{0_a}, T_{0_b}, T_{0_2}$	the stagnation temperatures of the motive gas, the suction gas, and the gas mixture respectively
$V_{1_a}, V_{1_b}, V_{2_x}$	the velocities of the motive gas and suction gas at the nozzle exit plane and of the gas mixture just before normal shock before the entrance to the diffuser section respectively
W_a, W_b, W_2	the mass flow rates of the motive gas, the suction gas, and the gas mixture respectively
$X = P_1/P_{0_b}$	the expansion ratio
$Y = P_{0_3}/P_{0_b}$	the overall compression ratio
$\alpha_1 = (\gamma_a - 1)/\gamma_a$	by definition
$\alpha_2 = (\gamma_a + 1)/(\gamma_a - 1)$	by definition
$\alpha_3 = (\gamma_b - 1)/\gamma_b$	by definition
$\alpha_4 = (\gamma_b + 1)/(\gamma_b - 1)$	by definition
$\alpha_5 = C_b^*/C_a^*$	by definition
$\alpha_6 = C_2^*/C_a^*$	by definition
$\alpha_7 = (\gamma_2 + 1)/(\gamma_2 - 1)$	by definition
$\alpha_8 = \gamma_2/(\gamma_2 - 1)$	by definition
$\alpha_9 = M_a/M_b$	by definition
$\alpha_{10} = T_{0_b}/T_{0_a}$	by definition
$\beta_1 = P_{0_b}/P_{0_a}$	the ratio of suction pressure to motive pressure
$\beta_2 = W_b/W_a$	the entrainment ratio
$\beta_3 = A_2/A_t$	the area ratio
$\gamma_a = C_{p_a}/C_{v_a}$	the specific heat ratio of the motive gas
$\gamma_b = C_{p_b}/C_{v_b}$	the specific heat ratio of the suction gas
γ_2	the specific heat ratio of the gas mixture
ρ	the density
$\rho_{1_a}, \rho_{1_b}, \rho_{2_x}$	the densities of the motive gas and the suction gas at the nozzle exit plane and of the gas mixture just before normal shock before the entrance to the diffuser section respectively.

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13. ABSTRACT A theoretical investigation was made of the optimum performance of a single-stage jet ejector with allowances made for the differences in temperature and molecular weight of the motive gas and the suction gas. Constant-pressure mixing of the motive and suction gases was assumed in the analytical model, and the task of optimization was carried out by rigorous mathematical operation. The analysis considers the case in which supersonic flow and hence normal shock occurs in the injector and the case of flow without normal shock. Numerical calculations of results were performed on an electronic digital computer for four selected typical examples of practical importance in the ranges of operating conditions of modern jet ejectors. These examples involved the following respective combinations of motive gas and suction gas: steam and air, silicone oil and air, steam and steam, and air and air. The results were found to compare rather satisfactorily with reported performance characteristics of a jet ejector.			

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