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Global Position and Height Determination from Lunar Radar Range Measurements

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INTRODUCTION

Present operational requirements need a simple method to accurately determine the position and height of terrestrial locations on a worldwide basis. Recently, artificial satellites have been used as radar reference points in the sky for determining the position and height of such locations. Satellite systems such as Secor (1) have shown that it is possible to achieve accuracies of the order of a few meters under optimum conditions. However, the unknown variations of the satellite's orbit require the placing of several reference ground stations close to the unknown location, so that simultaneous observations will reduce the satellite orbit errors. If the position and height of widely separated locations are needed, the method becomes quite cumbersome, for it requires the repeated physical relocation of the ground reference stations.

The utilization of the moon as a radar reference point would greatly simplify the acquisition of the necessary information for position and height determination. Since the position and velocity of the moon are well known and can be predicted for many years in advance, the tracking and prediction of artificial satellite orbits are eliminated. Furthermore, three permanent appropriately spaced reference ground stations would provide worldwide coverage, and relatively simple dipole receivers could be employed at the unknown stations for determining their position. While the relatively slow angular motion of the moon, as seen from the earth, requires longer observation time than artificial satellite systems, it also facilitates the calibration of the bistatic radar system. On the other hand, the extended target characteristics of the moon reduces to some extent the potential measurement accuracy that could be achieved. However, with the landing of a man on the moon, it is possible to consider placing a corner reflector or transponder on the lunar surface. This would provide a radar point target on the moon and thus make possible the full utilization of the measurement accuracy of the ground equipment.

BASIC PROCEDURE FOR STATION LOCATION

Due to the earth's rotation, each point on the earth's surface describes a unique path relative to the moon. The position of a terrestrial point can be derived if its distance, and the change of its distance, relative to the moon is measured over one day. For this purpose a transmitter at a known location is made to emit radio energy at known time intervals to the moon. The energy is reflected to the earth after a time interval corresponding to the round-trip distance. A receiver located at some unknown station measures the time of arrival of the reflected energy, and knowing the time of each transmission, as well as the position of the transmitter and the moon, the distance of the station relative to the moon can be determined. By measuring the distance variations over one day, the position of the unknown station is derived by appropriate analysis. In the analysis it is assumed that the location of the reference transmitting station is known or determined independently. Worldwide coverage is achieved by placing three reference stations at moderate latitudes and separated by about 120° in longitude.

Each transmitting station would have a receiver associated with it to correct any changes in round-trip time that may be introduced by refraction in the atmosphere, deviations caused by unknown errors in the lunar ephemeris, and unknown variations of the subearth lunar radius. The correction would be applied by shifting the time of transmission, so that the observing station at the unknown location can use directly the position values predicted by the lunar ephemeris for the moon and a nominal lunar radius.

The unknown station computes its distance and motion relative to the moon, based on the lunar ephemeris and an estimate of its location on the earth. The station's receiver then searches in time for the received signal until it is acquired. The receiver is programmed to track the received signal for several hours, and the series of distance measurements are either stored or reduced directly in an on-line computer. By comparing the computed distances with the measured distances, the correction for the three coordinates of the unknown station is obtained from a least-square analysis.

BASIC REQUIREMENTS OF A BISTATIC LUNAR RADAR SYSTEM

The basic considerations and requirements of a bistatic lunar radar system as applied to a navigational system have been previously discussed (2). Since the present measurements are to be made from a stable platform, some of the rather stringent requirements of a shipboard navigational system can be relaxed. Thus, for example, longer integration times can be allowed in ground-based bistatic radar operations. But the other parameters of the shipboard navigational system have been used as points of departure. A dipole receiver with a 1000°K system noise temperature is assumed at the unknown position, and a signal-to-noise (S/N) ratio of 100 is prescribed at the receiver output. To reduce the effects of ionospheric refraction, and at the same time preserve sufficient sensitivity at the dipole receiver, a wavelength of 30 cm is chosen.

With these parameters, three different cases will be considered. For moderate accuracies the moon will be used directly as the radar reflecting target (Case 1) and the required transmitter characteristics will be derived. For extremely accurate position and height determination, a system configuration which utilizes a lunar corner reflector (Case 2) and a lunar transponder (Case 3) will also be considered.

TRANSMITTER REQUIREMENTS WITH MOON AS RADAR TARGET (Case 1)

The parameters of a transmitter that will supply sufficient energy to a dipole receiver are summarized in Table 1. The parameters have been based on the lunar radar system employed at NRL for several years. The parameters satisfy the S/N requirement at the receiver output, which are derived from the basic radar equation

$$S/N = \frac{P_t D_t^2}{128\pi R^4} \frac{\sigma}{KTB} \sqrt{B\tau}\, L \approx 100$$

where R is the distance to the moon ($\approx 3.8\times10^8$ m) and the other symbols are as defined in Table 1. The corresponding range resolution is given by

$$\sigma_r = \frac{c\delta}{\sqrt{S/N}} = \pm 30\ m$$

Table 1 (C)
Bistatic Radar System Parameter Values
with Moon as Radar Target

Transmitter:	
Transmitted Power P_t	2 MW
Antenna Diameter D_t	30 m
Antenna Efficiency η	50 percent
Polarization	circular
Wavelength λ	30 cm
Antenna Beamwidth Ω	21 arc minutes
Pulse Width δ	1 μs
Pulse Compression Ratio	2×10^5
Radar Cross-section of Moon (1 μs) σ	3×10^9 m^2
Receiver:	
System Noise Temperature T	1000K
Antenna	dipole
Signal Bandwidth B_s	1 MHz
Detection Bandwidth B	5 Hz
Output Time Constant τ	150 sec
System Loss L (includes depolarization, atmospheric absorption, waveguide losses, etc.)	-10 db
Output S/N	100
Range Resolution σ_r	±30 m

where c is the velocity of light, and $\sqrt{S/N} \approx 10$. A tradeoff can be made between the transmitted power and the antenna size. For example, if the transmitted power is increased to 8 MW, the antenna diameter can be decreased to 15 m. The decrease in antenna size would reduce the angular tracking requirements.

SYSTEM REQUIREMENTS FOR A LUNAR CORNER REFLECTOR (Case 2)

The size of the corner reflector is determined by the power incident on the corner reflector and the additional requirement that the radar cross section of the corner reflector should exceed the lunar radar cross section by at least a factor of 100. If the transmitting system described above is used, the radar cross section of the corner reflector has to be made equal to at least the lunar radar cross section, i.e.,

$$\sigma_{CR} = \sigma = 3\times10^9\ m^2.$$

The effective area of a corner reflector as shown in Ref. 3 is $A_{eff} \approx 0.3\ \ell^2$, where ℓ is the length of the edge of the corner reflector. Then since

$$\sigma_{CR} = \frac{4\pi\ A_{eff}^2}{\lambda^2} \approx \frac{0.4\pi\ \ell^4}{\lambda^2},$$

the length of the corner reflector is given by

$$\ell = \left(\frac{\lambda^2 \sigma_{CR}}{1.2}\right)^{1/4} \approx 120 \; m.$$

The radar cross section of the moon can be reduced by 20 db if the location of the corner reflector is shifted by about 30° in selenographic longitude or latitude from the lunar center. This corresponds to a delay of 2 msec relative to the nearest point of the moon, and reduces the lunar reflectivity by 20 db for $\lambda \approx$ 30 cm, as given in Ref. 4.

By placing the corner reflector closer to the limb of the moon and by increasing the transmitted power, the size of the corner reflector could be reduced. The minimum size of the corner reflector is about 70 m, if it is placed about 13° from the lunar limb. The transmitter product $P_t D_t^2$ would have to be increased for this case by a factor of 10.

Since the corner reflector acts as a point target reflector, the signal bandwidth can be increased to 10 MHz, with a corresponding increase in effective range resolution to ±3 m. Other parameters are the same as in Case 1.

POWER CONSIDERATIONS FOR A LUNAR TRANSPONDER (Case 3)

The use of a lunar transponder considerably decreases the power requirements of the transmitting station, since the power to supply sufficient energy to a dipole receiver is now provided by the transponder. Furthermore, since no pointing should be required on the moon, which in addition to other complications would increase the power requirement, the antenna size is restricted to 1 m (21° beamwidth) or less to allow for the monthly librations of the moon. With these basic requirements, the transmitted power P_T of the transponder is obtained from

$$P_T = \frac{3200 \, R^2 \, KTB}{D_T^2 \, L \, \sqrt{B_T}} = 12 \text{ watts},$$

where D_T is the antenna diameter (1 m) of the transponder, and the other symbols are as defined previously.

Setting the S/N at the transponder to be 100, the product $P_t D_t^2$ for the ground transmitter is obtained from

$$P_t D_t^2 = \frac{6400 \, R^2 \, \lambda^2 \, KTB_T}{\pi^2 \, D_T^2 \, L} \approx 10^7$$

where B_T = 10 MHz is the bandwidth of the transponder. This relation would be satisfied if, for example, D_t = 30 m and P_t = 20 kw. The measurement accuracy would be similar to that of the lunar corner reflector.

Summarizing the requirements of the bistatic system, it has been shown that for the moon (Case 1) and a lunar corner reflector (Case 2), the size of the transmitting system corresponds to a 30-m antenna transmitting 2 MW of power for a dipole receiver. If the sensitivity of the receiver is increased, by increasing the receiver antenna size for example, the transmitter requirements would be correspondingly decreased. The above system will provide a potential location accuracy of ±30 m with the moon, and ±3 m with the corner reflector or a transponder. The transponder (Case 3) reduces considerably the power requirements of the ground transmitter, but it has to transmit about 12 W for dipole receivers.

ERROR ANALYSIS

Sources of Error

The effective accuracy of location determinations will depend on the accuracy of the individual distance measurements and the subsequent ability to separate the three coordinates of the position in the least-square analysis. The measurement accuracy is primarily limited by the radar target characteristics of the moon, the unknown variation of the propagation velocity in the earth-moon path, and the basic equipment accuracy of the ground system. On the other hand, the efficiency of the least-square analysis will depend on the geometric configuration between the unknown station and the moon at the time of observation. Since the moon revolves around the earth in a monthly period, each station will have at least one optimum period of observation each month. The effect of these errors will now be discussed in more detail.

Measurement Errors

Effects of Lunar Topography — If the moon is used directly as a passive reflector, the spread target characteristics of the reflection have to be considered. These affect the radar return in two different manners. First, the time of reflection is shifted in a somewhat random manner since the point of reflection for the transmitter and receiver is not identical. This is caused by the fact that the receiver is located at a position that is, in general, at a considerable distance from the transmitter. The maximum angular displacement of the transmitter and the receiver, as seen from the moon, may correspond to about 2/3° in selenographic longitude or latitude, which corresponds to about 15 km on the lunar surface. Because of the height variations on the lunar surface, any displacement of the reflection point will change the mean height, and thus directly alter the measured distance to the receiver. Previous NRL lunar radar measurements indicate that within any one-degree square on the moon's surface, the mean height variations amount to 200 m in mountainous regions and about 50 m in flat areas. Averaging the height variations over one day in the least-square analysis should reduce this error to a maximum of about ±30 m. The efficiency of the averaging process could be increased by extending the observations to several days.

The second effect of the extended target reflection of the moon is the spread of the radar pulse due to the complex reflection characteristics of the lunar surface. A typical lunar radar echo is shown in Fig. 1. Studies are being made now at NRL to determine the consistency with which the mean lunar height can be derived. However, based on a preliminary analysis, standard deviations of the delay time for a 20-sec sample with a 180-m range resolution is less than ±30 m on the average. For the integration time of 150 sec that is assumed in the error analysis in Appendix A, this error should be reduced to about ±15 m.

The remaining measurement errors that are discussed below apply both to the moon and the lunar corner reflector, or transponder if used.

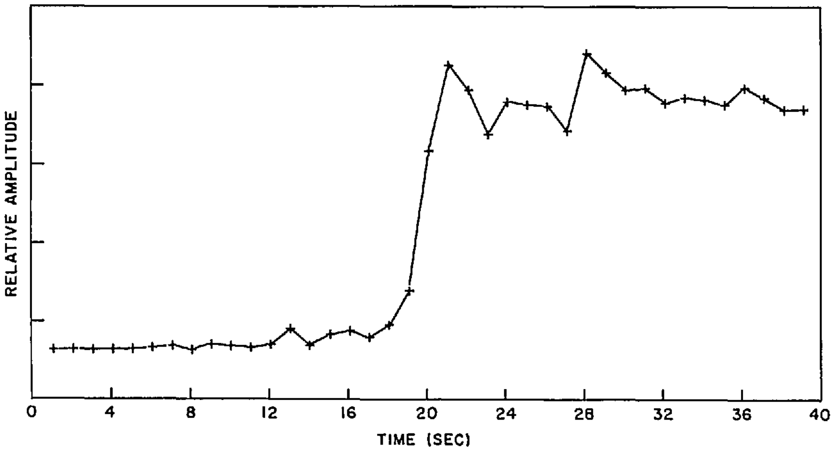

Fig. 1 - A typical lunar radar echo which illustrates the complex reflection characteristics of the moon

(U) Earth-Moon Propagation Path — The atmosphere will introduce time delays at low elevation angles that become significant for the range resolution under consideration. There are two effects on the time delay that have to be considered. The major error is introduced by the systematic atmospheric time delay, which is a function of the elevation angle. In Appendix A observations are restricted to elevation angles of 10° or more and, for the minimum elevation angle, a correction of about 14 m has to be applied (5a). Appropriate corrections for other elevation angles are required. In addition, random errors that vary over short-term periods occur, which may amount to about 10 percent of the systematic error. These are small relative to the range resolution used and are further reduced by the averaging process, and thus can be neglected. Also because of the high compression ratio used in the equipment, it will be necessary to correct for the lunar doppler shift, which at low elevation angles may amount to a maximum of about 20 Hz (5b). Dispersion effects for the 10-MHz bandwidth are negligible (5c), and a maximum attenuation of about 2 db due to atmospheric absorption (5d) has been included in the loss factor L of -10 db in the radar equation (see Table 1).

Stability of the Atomic Clock — The system accuracy will be primarily limited by the stability of the atomic clock at the receiver and the ability to synchronize it with the atomic clock at the transmitter. As pointed out in the appendix, the atomic clock can be synchronized with the transmitter clock every two weeks with an accuracy of about seven times the measurement accuracy, when observations for about 12 hours are performed. Since the measurement accuracy has to be maintained for two weeks, a stability of about 1 part in 10^{13} is required. At present, only a hydrogen maser would possibly fulfill these requirements, but it is too bulky to be used with the receiver. However, new compact time standards, such as a thallium clock, with the required time stability are being developed and should be available in the near future.

Geometric Errors

In Appendix A the effective errors of station location for various configurations of the station and the moon have been computed. From the observation equation (see Eq. A3)

$$-\Delta\rho = (\sin\delta)\Delta(r\sin\varphi) + (\cos\delta\cos\gamma)\Delta(r\cos\varphi) + (\cos\varphi\cos\delta\sin\gamma)r\Delta\lambda - c\Delta t_c,$$

where

δ is the declination of the moon,

r is the geocentric radius of the station,

φ is the geocentric latitude,

γ is the local hour angle,

c is the velocity of light, and

Δt_c is the error of the timing clock,

it is seen that in general the three coordinates of a station can be best determined when the moon is at high declination and is visible for an extended fraction of a day. The timing clock error can best be obtained near a lunar declination of zero, since the error of the variable $r\sin\varphi$ is then negligible in the observation equation.

The average error for a 12-hour observation period is given in Table 2 where σ_m is the basic measurement error and shows that only for low declination and very high latitudes will the errors exceed the measurement errors. The effective error in terms of the measurement error is tabulated in the tables of Appendix A as a function of the latitude of the station and the declination of the moon. The observation time in degrees and the correlation coefficient between $r\sin\varphi$ and $r\cos\varphi$ are also given. The tables indicate that when measurements near zero lunar declination are excluded, the effective error is less than the measurement error for more than 60 percent of the time, and less than two times the measurement error for 85 percent of the time.

Table 2 (U)
Average Effective Error of Variables
for 12-Hour Observation

Variables:	$\Delta(r\sin\varphi)$	$\Delta(r\cos\varphi)$	$r\Delta\lambda$
Error:	$\frac{\sigma_m}{7.4\sin\delta}$	$\frac{\sigma_m}{5\cos\delta}$	$\frac{\sigma_m}{12\cos\varphi\cos\delta}$

DISCUSSION AND CONCLUSIONS

The use of a corner reflector or transponder on the moon for high-accuracy position and height measurements presents certain inherent difficulties at present. In addition to the obvious effort to transport and maintain the equipment on the lunar surface, there are several other factors that may create problems in the operation of the system. The collisions of meteorites with the relatively large surface of the corner reflector may deteriorate the performance of the reflecting surface. As for the transponder, the power supply may be difficult to maintain during the lunar night, if solar batteries are used. Thus an atomic power supply would be preferred for continuous operation. Finally, the high location accuracy of 3 m that can be attained with a lunar corner reflector or transponder requires the stability of the atomic clock at the receiver and transmitter to be within 1 part in 10^{14}, a stability that is not attainable at present.

For these reasons, it would be preferable to use the moon directly, at least in the immediate future, and consider improvements in accuracy that can be attained without employing reflecting devices on the moon. As has been pointed out, the major error of the distance measurement is caused by the displacement of the lunar reflecting point relative to the receiver, whose unknown height on the earth may introduce a systematic error in the position determination. Three possible ways to reduce the systematic error are:

1. Since different areas of the moon are observed each day due to the libration of the moon, and the lunar topography can be assumed to be random for at least a few days, it will be possible to reduce this systematic error by taking observations over several days.

2. Since the transmitters will measure the distance to the moon, it can make corrections to the lunar ephemeris and the lunar radius. This can be achieved by an analysis of the data taken over long periods of time which will permit the separation of lunar topography variations from the lunar ephemeris errors, and thus essentially map the subearth (i.e., earth-facing) region of the moon in terms of the height variations of the lunar surface. However, several years will be required for the complete description of the lunar subearth topography.

3. A more rapid calibration of the subearth topography could be attained if a lunar orbiter with a radar system is utilized to map the height distribution of the subearth region.

With available calibrations for the lunar topography and a concurrent improvement of atomic clock stability, measurement accuracies of the order of ± 15 m should be attainable.

With new technological developments in the future, it will be possible to consider placing active devices on the moon, and thus reduce the ground transmitter requirements as well as increase further the measurement accuracy. Furthermore, if rapid position and height determinations are necessary, the lunar concept could be extended by placing additional active or passive devices at the stable equilibrium points of the moon's orbit. This approach, which has been discussed in Ref. 2, could reduce the time for a position and height determination to a few minutes, with accuracies commensurate with the clock stability and available topographic corrections.

To conclude, at present the direct use of the moon would provide earth position and height measurements with a potential accuracy of ± 30 m, and the relatively simple operational requirements should make this approach useful for worldwide applications. In the future, higher accuracies and shorter observation times could be attained by extending this concept to active or passive devices at the moon and the lunar equilibrium points.

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(References are Unclassified in entirety)

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   a. pp. 340-341
   b. p. 347
   c. pp. 366-369
   d. pp. 371-373

Appendix A

COMPUTATION OF POSITION ERROR
(This Appendix is Confidential in its entirety)

The distance from the receiver to the moon is given by

$$\rho = -b + \sqrt{s^2 + r^2 - 2sr\cos\beta} \tag{A1}$$

where

b is the mean lunar radius,

s is the center-of-earth to center-of-moon distance,

r is the geocentric radius of the unknown receiver location, and

$$\cos\beta = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos\gamma .$$

In the relation for $\cos\beta$ we have

φ, the geocentric latitude of the receiver location,

δ, the moon's declination, and

γ, the local hour angle $= GST - \lambda - \alpha$

where

GST is the Greenwich sidereal time,

λ is the longitude of the receiver, and

α is the right ascension of the moon.

The residual $\Delta\rho$, the difference between the measured and computed value of ρ, is then given to first order as

$$\Delta\rho = \rho_m - \rho_c = -\Delta(r\cos\beta) + c\Delta t_c \tag{A2}$$

since s and b are assumed to have been measured and corrected for by the transmitter.

Expanding the observation equation in terms of $r\sin\varphi$, $r\cos\varphi$, and λ, Eq. (A2) becomes

$$\begin{aligned} -\Delta\rho = (\sin\delta)\,\Delta(r\sin\varphi) + (\cos\delta\cos\gamma)\,\Delta(r\cos\varphi) \\ + (\cos\varphi\cos\delta\sin\gamma)\,r\Delta\lambda - c\Delta t_c . \end{aligned} \tag{A3}$$

The timing error Δt_c will be neglected in the preliminary analysis.

Let the coefficient of the variables be expressed as

$$K_1 = \sin\delta, \; K_2 = \cos\delta\cos\gamma, \; K_3 = \cos\varphi\cos\delta\sin\gamma,$$

and the unknown variables as

$$x_1 = r \sin\varphi, \quad x_2 = r \cos\varphi, \quad x_3 = r\Delta\lambda .$$

Performing the least-square analysis on Eq. (A3), the three normal equations become

$$\sum_j^N K_{ij} \Delta\rho_j = \sum_j^N K_{ij} K_{1j} \Delta x_1 + \sum_j^N K_{ij} K_{2j} \Delta x_2 + \sum_j^N K_{ij} K_{3j} \Delta x_3 \tag{A4}$$

where $i = 1, 2, 3$, and N is the number of observation points. The variance of the corrections $\sigma^2_{\Delta x_i}$ in terms of the variance of the measurement errors σ^2_m are given by

$$\sigma^2_{\Delta x_1} = \frac{\sigma^2_m}{\sum K_1^2 (1 - \rho_{12}^2)},$$

$$\sigma^2_{\Delta x_2} = \frac{\sigma^2_m}{\sum K_2^2 (1 - \rho_{12}^2)}, \tag{A5}$$

and

$$\sigma^2_{\Delta x_3} = \frac{\sigma^2_m}{\sum K_3^2},$$

where ρ_{12} is the correlation coefficient between the coefficients K_1 and K_2, and $\rho_{23} = \rho_{13} = 0$, as will be shown later.

The correlation between Δx_1 and Δx_2 is given by

$$\rho_{12} = \frac{\sum K_1 K_2}{\sqrt{\sum K_1^2 \sum K_2^2}}. \tag{A6}$$

To evaluate the L.S. (least-square) coefficients it is assumed that measurements are taken every 2.5 minutes for all elevations of the moon that exceed 10°. This restricts the maximum hour angle γ_o to

$$\gamma_o = \cos^{-1}\left(\frac{\sin 10° - \sin\delta \sin\varphi}{\cos\delta \cos\varphi}\right) \tag{A7}$$

for a particular lunar declination δ and geocentric latitude φ. The L.S. coefficients are then given by

$$\sum K_1^2 = \sum_{i=1}^N \sin^2\delta_i ,$$

$$\sum K_2^2 = \sum^N \cos\delta_i \cos\gamma_i ,$$

$$\sum K_3^2 = \cos\varphi \sum \cos\delta_i \sin\gamma_i ,$$

and

$$\sum K_1K_2 = \sum^{N} \sin\delta_i \cos\delta_i \cos\gamma_i,$$

$$\sum K_1K_3 = \cos\varphi \sum^{N} \sin\delta_i \cos\delta_i \sin\gamma_i,$$

$$\sum K_2K_3 = \cos\varphi \sum^{N} \cos^2\delta_i \cos\gamma_i \sin\gamma_i,$$

where $N = [2\gamma_o/(\pi/12)] \times 24$ is the number of samples over the observation period (24 samples/hour).

The standard errors $\sigma_{\Delta x_1}$, $\sigma_{\Delta x_2}$, and $\sigma_{\Delta x_3}$, the extreme hour angle γ_o and the correlation ρ_{12} have been computed for different values of φ and δ and are presented in Tables A1-A7 for the case when $\sigma_m = 1$ m.

The first column in the tables gives the latitude of the position. The second column indicates the available observation time, expressed in terms of the hour angle (H.A.) in degrees, for one day at the given position when the moon's elevation exceeds 10°. The third column presents the correlation ρ_{12} between $r \sin\varphi$ and $r \cos\varphi$ in the L.S. analysis. The 4th, 5th, and 6th columns give the effective error of the three coordinates in meters for a measurement error σ_m of 1 m and for observations over one day. The next three columns indicate the initial, average, and final declination of the moon for the given conditions, and the last column, the number of samples over one day. The asterisks, located in $\sigma_{\Delta x_1}$, $\sigma_{\Delta x_2}$, and $\sigma_{\Delta x_3}$, indicate conditions where effective errors exceed 100 times the measurement error.

To illustrate the error analysis, and compute directly a few simple cases, the following simplifying assumptions will be made:

1. The moon's slow motion relative to the rapid motion of the unknown location is neglected for the period of observation, i.e., $\delta_i = \delta_{\text{mean}}$ for one day.

2. The interval $\Delta\gamma = 2\gamma_o/N$ approaches 0 between each observation. Then the L.S. coefficients can be expressed explicitly as

$$\sum K_1^2 = N \sin^2\delta,$$

$$\sum K_2^2 = (\cos^2\delta) \frac{N}{2\gamma_o} \sum \cos^2\gamma_i \ \Delta\gamma = \frac{N}{2\gamma_o} \cos^2\delta \int \cos^2\gamma \, d\gamma$$

$$= \frac{N}{2\gamma_o} \cos^2\delta \left(\gamma_o + \frac{\sin^2\gamma_o}{2}\right),$$

$$\sum K_3^2 = \frac{N}{2\gamma_o} \cos^2\delta \cos^2\varphi \left(\gamma_o - \frac{\sin^2\gamma_o}{2}\right),$$

and

$$\sum K_1K_2 = \frac{N}{2\gamma_o} \sin\delta \cos\delta \int \cos\gamma \, d\gamma = \frac{N}{\gamma_o} \sin\delta \cos\delta \sin\gamma_o ,$$

$$\sum K_1K_3 = \frac{N}{2\gamma_o} \sin\delta \cos\delta \cos\varphi \int \sin\gamma \, d\gamma = 0 ,$$

$$\sum K_2K_3 = \frac{N}{2\gamma_o} \cos^2\delta \cos\varphi \int \cos\gamma \sin\gamma \, d\gamma = 0 .$$

Thus from Eq. (A6)

$$\rho_{12} = \frac{\sin\gamma_o}{\sqrt{\frac{\gamma_o}{2}\left(\gamma_o + \frac{\sin 2\gamma_o}{2}\right)}}$$

and $\rho_{23} = \rho_{13} = 0$ as indicated previously. The standard errors of the coordinates are obtained from Eq. (A5):

$$\sigma_{\Delta x_1} = \frac{\sigma_m}{\sqrt{N} \sin\delta \sqrt{1 - \frac{\sin^2\gamma_o}{\frac{\gamma_o}{2}\left(\gamma_o + \frac{\sin 2\gamma_o}{2}\right)}}}, \tag{A8}$$

$$\sigma_{\Delta x_2} = \frac{\sigma_m}{\sqrt{\frac{N}{2\gamma_o}} \cos\delta \sqrt{\left(\gamma_o + \frac{\sin 2\gamma_o}{2}\right)\left(1 - \frac{\sin^2\gamma_o}{\frac{\gamma_o}{2}\left(\gamma_o + \frac{\sin 2\gamma_o}{2}\right)}\right)}}, \tag{A9}$$

and

$$\sigma_{\Delta x_3} = \frac{\sigma_m}{\sqrt{\frac{N}{2\gamma_o}} \cos\delta \cos\varphi \sqrt{\gamma_o - \frac{\sin 2\gamma_o}{2}}}. \tag{A10}$$

For the simple case when $\gamma_o = \pi/2$, the values can be easily computed. The correlation coefficient becomes

$$\rho_{12} = \frac{1}{\sqrt{(\pi/4)\,(\pi/2)}} \approx 0.9$$

and

$$N = \frac{2\pi/2}{\pi/12} \times 24 = 288 .$$

For this condition

$$\sigma_{\Delta x_1} = \frac{1}{\sqrt{288}\ \sin\delta\ \sqrt{1 - 0.81}} = \frac{1}{7.4\ \sin\delta},$$

$$\sigma_{\Delta x_2} = \frac{1}{\sqrt{288/\pi}\ \cos\delta\ \sqrt{\pi/2\ (1 - 0.81)}} = \frac{1}{5.25\ \cos\delta},$$

and

$$\sigma_{\Delta x_3} = \frac{1}{\sqrt{288/\pi}\ \cos\delta\ \cos\varphi\ \sqrt{\pi/2}} = \frac{1}{12\ \cos\delta\ \cos\varphi}.$$

The equations indicate that if $\gamma_o = \pi/2$, the effective error of $\Delta x_1 = \Delta(r \sin\varphi)$ will exceed the measurement error for $\delta < 8°$. Similarly for $\varphi \geq 85°$ the effective error of $\Delta x_3 = r\Delta\lambda$ will exceed the measurement error. This will restrict the observation time to periods when the moon's declination is $\geq 8°$ and to latitudes less than 85° for optimum accuracy.

On the other hand, when the moon's declination δ approaches 0, the time of the atomic clock can be reset, since then the error of $r \sin\varphi$ is negligible relative to the time error.

To obtain the time error, a similar analysis is performed using

$$-\Delta\rho = (\cos\delta\ \cos\gamma)\ \Delta(r\ \cos\varphi) + (\cos\varphi\ \cos\delta\ \sin\gamma)\ r\Delta\lambda - c\Delta t_c$$

where Δt_c is the timing error of the atomic clock. The standard deviation of the timing error is obtained as

$$\sigma_{\Delta t} = \frac{\sigma_m}{c\sqrt{N(1 - \rho_{24}^2)}}$$

where

$$\rho_{24} = \rho_{12} = \frac{\sin\gamma_o}{\sqrt{\frac{\gamma_o}{2}\left(\gamma_o + \frac{\sin 2\gamma_o}{2}\right)}}.$$

For 12 hours of observation time

$$\sigma_{\Delta t} = \frac{\sigma_m}{7.4c}$$

where σ_m is the measurement error, as before.

Program for Solution of Eqs. A8 - A10

```
      PROGRAM    LUN POS ER
      DIMENSION    KONE(600),KTWO(600),KTHR(600)
      TYPE REAL   KONE,KTWO,KTHR
      TYPE DOUBLE   JD,RTD,DTR,PI,SINETEN
      COMMON/CETBL1/AU,RE,TPD/CETBL2/IFLAG(11)
      DATA(AU=0.),(RE=6378149.5),(TPD=86400.)
      DATA((IFLAG(I),I=1,11)=9(0),2(1))
      JD(AD,KD,BD)=DBLE(AD)+DBLE(FLOAT(KD)/24.)+DBLE(BD/24.)
  900 FORMAT (F10.2)
  901 FORMAT (*1    LATITUDE      GAMMA 0       RHO        DX        DY        DZ       DE
     *C-ST      DC-AVG      DC-END      NUMB*,///)
  902 FORMAT (*0*,7XI3,4XF8.1,2XF6.2,3(1X,F6.2),3(3XF7.2),4XI4)
      RTD = 57.295779513082321D
      DTR = 0.017453292519943296D
      SINETEN = 0.173648177666930D
      PI = 3.1415926536
      DM = 1.
    1 READ  900,   DJO
      IF(EOF,60)  3,2
    2 PRINT 901
      DO 10     I = 1,17
      LAT = 10*(1 - I) + 80
      PHI = LAT*DTR
      SINP = SIN(PHI)
      COSP = COS(PHI)
      SKONE = 0
      SKONFSQ = SKTWOSQ = SKTHRSQ = SKONETWO = 0.
      DJ = DJO
      CALL EPHNOW(JD(DJ,0,0),RR,DD,SY)
      DJ = DJ - 2.5/1440.
      CGAMMA = (SINETEN - SINP*SIN(DD))/(COSP*COS(DD))
      IF(CGAMMA .GT. -1.)    GO TO 20
      GAMMAO = -180.*DTR
      GO TO 30
   20 IF(CGAMMA .LT. 1.)    GO TO 25
      GAMMAO = NUM = DECST = DECEND = AVGDEC = 0
      DX = DY = DZ = RHO = 1000.
      GO TO 35
   25 GAMMAO =-ACOS(CGAMMA)
   30 NUM = ABSF(2.*GAMMAO*24.*12./PI - 1.)
      DO 40    J = 1,NUM
      GAMMA = ((PI/12.)*(J-1)/24.) + GAMMAO
      DJ = DJ + 2.5/1440.
      CALL EPHNOW(JD(DJ,0,0),RR,DD,SY)
      KONE(J) = SIN(DD)
      KTWO(J) = COS(DD)*COS(GAMMA)
      KTHR(J) = COS(DD)*SIN(GAMMA)*COSP
      SKONE = SKONE + KONE(J)
      SKONESQ = SKONESQ + KONE(J)**2
      SKTWOSQ = SKTWOSQ + KTWO(J)**2
      SKTHRSQ = SKTHRSQ + KTHR(J)**2
      SKONETWO = SKONETWO + KONE(J)*KTWO(J)
   40 CONTINUE
      RHO = SKONETWO/SQRTF(SKONESQ* SKTWOSQ)
      IF(RHO .LT. 1.   .AND.  RHO .GT. -1.)    GO TO 45
      DX = DY = DZ = 1000.
      DZ = SQRTF(DM**2/SKTHRSQ)
      GO TO 50
   45 DX = SQRTF(DM**2/(SKONESQ*(1. - RHO**2)))
      DY = SQRTF(DM**2/(SKTWOSQ*(1. - RHO**2)))
      DZ = SQRTF(DM**2/SKTHRSQ)
   50 AVGKONE = SKONE/NUM
      AVGDEC = ASIN(AVGKONE)*RTD
      DECST = ASIN(KONE(1))*RTD
      DECEND = ASIN(KONE(NUM))*RTD
      GAMMAO = GAMMAO*RTD
      GAMMAO = ABSF(GAMMAO)
   35 PRINT 902,  LAT,GAMMAO,RHO,DX,DY,DZ,DECST,AVGDEC,DECEND,NUM
   10 CONTINUE
      GO TO 1
    3 END
```

The symbols in the program do not conform with those used in the mathematical analysis in the appendix. The relation between the symbols are as follows:

φ = LAT	$\sigma_{\Delta x_3}$ = DZ
γ_o = GAMMA0	δ_o = DECST
ρ_{12} = RHO	δ_{av} = AVGDEC
$\sigma_{\Delta x_1}$ = DX	δ_f = DECEND
$\sigma_{\Delta x_2}$ = DY	N = NUM

Table A1 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H. A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for σ_m = 1 m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	0.0	******	******	******	******	0.00	0.00	0.00	0
70	0.0	******	******	******	******	0.00	0.00	0.00	0
60	0.0	******	******	******	******	0.00	0.00	0.00	0
50	22.3	1.00	10.92	5.81	0.93	-27.51	-27.41	-27.31	72
40	46.2	0.99	1.74	1.00	0.28	-27.51	-27.30	-27.07	148
30	58.2	0.99	0.94	0.57	0.18	-27.51	-27.24	-26.94	187
20	66.6	0.97	0.67	0.42	0.14	-27.51	-27.20	-26.84	213
10	73.1	0.96	0.52	0.34	0.12	-27.51	-27.16	-26.77	234
0	78.7	0.95	0.42	0.29	0.11	-27.51	-27.13	-26.70	252
-10	83.9	0.93	0.36	0.25	0.10	-27.51	-27.10	-26.63	269
-20	88.9	0.91	0.31	0.22	0.10	-27.51	-27.07	-26.57	285
-30	94.3	0.88	0.26	0.19	0.10	-27.51	-27.04	-26.50	302
-40	100.5	0.84	0.22	0.17	0.11	-27.51	-27.01	-26.42	322
-50	108.4	0.77	0.19	0.15	0.12	-27.51	-26.96	-26.31	347
-60	120.7	0.65	0.15	0.12	0.15	-27.51	-26.88	-26.13	387
-70	149.2	0.30	0.11	0.08	0.20	-27.51	-26.70	-25.71	478
-80	180.0	0.00	0.09	0.07	0.38	-27.51	-26.48	-25.20	577

Table A2 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H.A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for σ_m = 1 m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	0.0	******	******	******	******	0.00	0.00	0.00	0
70	0.0	******	******	******	******	0.00	0.00	0.00	0
60	19.6	1.00	19.96	6.67	1.35	-18.40	-18.14	-17.88	63
50	47.1	0.99	2.43	0.87	0.30	-18.40	-17.76	-17.12	151
40	58.8	0.98	1.37	0.51	0.19	-18.40	-17.60	-16.79	189
30	66.2	0.97	1.01	0.39	0.14	-18.40	-17.50	-16.59	212
20	71.6	0.96	0.82	0.33	0.12	-18.40	-17.42	-16.43	230
10	75.8	0.95	0.71	0.29	0.11	-18.40	-17.37	-16.32	243
0	79.5	0.94	0.63	0.26	0.10	-18.40	-17.31	-16.21	255
-10	82.7	0.93	0.57	0.24	0.10	-18.40	-17.27	-16.12	265
-20	85.8	0.92	0.51	0.22	0.10	-18.40	-17.23	-16.03	275
-30	88.9	0.90	0.47	0.20	0.10	-18.40	-17.18	-15.94	285
-40	92.3	0.89	0.43	0.19	0.11	-18.40	-17.13	-15.85	296
-50	96.4	0.86	0.38	0.17	0.13	-18.40	-17.08	-15.73	309
-60	102.1	0.82	0.33	0.15	0.15	-18.40	-17.00	-15.57	327
-70	112.3	0.73	0.27	0.13	0.21	-18.40	-16.85	-15.27	360
-80	146.4	0.34	0.17	0.08	0.36	-18.40	-16.37	-14.28	469

Table A3 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H.A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for $\sigma_m = 1$ m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	0.0	******	******	******	******	0.00	0.00	0.00	0
70	0.0	******	******	******	******	0.00	0.00	0.00	0
60	47.8	0.99	4.00	0.76	0.37	-10.42	-9.69	-8.96	153
50	60.4	0.98	2.26	0.45	0.21	-10.42	-9.49	-8.56	194
40	67.4	0.97	1.73	0.35	0.15	-10.42	-9.39	-8.34	216
30	71.9	0.96	1.46	0.31	0.12	-10.42	-9.31	-8.20	231
20	75.2	0.95	1.31	0.28	0.11	-10.42	-9.26	-8.10	241
10	77.8	0.95	1.21	0.26	0.10	-10.42	-9.23	-8.02	249
0	79.8	0.94	1.13	0.24	0.10	-10.42	-9.19	-7.96	256
-10	81.6	0.93	1.07	0.23	0.10	-10.42	-9.17	-7.91	261
-20	83.1	0.93	1.03	0.23	0.10	-10.42	-9.14	-7.86	266
-30	84.4	0.92	0.98	0.22	0.10	-10.42	-9.12	-7.81	271
-40	85.6	0.92	0.95	0.21	0.12	-10.42	-9.10	-7.77	275
-50	86.8	0.91	0.92	0.21	0.14	-10.42	-9.09	-7.74	278
-60	88.0	0.91	0.89	0.20	0.17	-10.42	-9.07	-7.70	282
-70	89.4	0.90	0.85	0.19	0.25	-10.42	-9.04	-7.65	287
-80	91.5	0.89	0.81	0.18	0.48	-10.42	-9.01	-7.59	293

Table A4 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H.A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for $\sigma_m = 1$ m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	0.0	******	******	******	******	0.00	0.00	0.00	0
70	53.4	0.89	8.64	0.19	0.45	-1.84	-1.00	-0.17	171
60	66.2	0.78	6.25	0.13	0.24	-1.84	-0.80	0.23	212
50	72.0	0.71	5.52	0.12	0.17	-1.84	-0.71	0.42	231
40	75.3	0.66	5.18	0.11	0.13	-1.84	-0.66	0.51	241
30	77.3	0.63	4.98	0.11	0.11	-1.84	-0.63	0.58	248
20	78.7	0.61	4.86	0.11	0.10	-1.84	-0.61	0.62	252
10	79.5	0.60	4.79	0.10	0.10	-1.84	-0.59	0.65	255
0	80.0	0.59	4.75	0.10	0.09	-1.84	-0.59	0.66	256
-10	80.2	0.59	4.73	0.10	0.10	-1.84	-0.58	0.67	257
-20	80.0	0.59	4.74	0.10	0.10	-1.84	-0.58	0.67	257
-30	79.5	0.60	4.79	0.10	0.11	-1.84	-0.59	0.65	255
-40	78.5	0.61	4.88	0.11	0.13	-1.84	-0.61	0.62	252
-50	76.6	0.64	5.05	0.11	0.15	-1.84	-0.64	0.56	246
-60	73.0	0.69	5.41	0.12	0.21	-1.84	-0.70	0.45	234
-70	65.2	0.79	6.40	0.14	0.35	-1.84	-0.82	0.20	209
-80	35.1	0.97	16.35	0.40	1.58	-1.84	-1.29	-0.74	113

Table A5 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H.A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for $\sigma_m = 1$ m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	77.7	0.95	1.23	0.26	0.57	8.00	9.13	10.26	249
70	82.7	0.93	1.04	0.23	0.27	8.00	9.21	10.40	265
60	83.8	0.92	0.99	0.22	0.18	8.00	9.22	10.44	269
50	84.0	0.92	0.99	0.22	0.14	8.00	9.22	10.44	269
40	83.6	0.93	1.00	0.22	0.12	8.00	9.22	10.43	268
30	83.0	0.93	1.03	0.23	0.11	8.00	9.21	10.41	266
20	82.2	0.93	1.05	0.23	0.10	8.00	9.20	10.39	264
10	81.2	0.93	1.09	0.24	0.10	8.00	9.18	10.36	260
0	79.9	0.94	1.14	0.25	0.10	8.00	9.17	10.32	256
-10	78.3	0.94	1.20	0.26	0.10	8.00	9.14	10.28	251
-20	76.2	0.95	1.30	0.27	0.11	8.00	9.11	10.21	244
-30	73.5	0.96	1.42	0.29	0.12	8.00	9.07	10.14	236
-40	69.7	0.97	1.64	0.33	0.15	8.00	9.02	10.03	224
-50	63.9	0.98	2.09	0.40	0.19	8.00	8.93	9.86	205
-60	53.6	0.99	3.31	0.59	0.31	8.00	8.78	9.56	172
-70	26.0	1.00	18.02	2.75	1.25	8.00	8.38	8.76	84
-80	0.0	******	******	******	******	0.00	0.00	0.00	0

Table A6 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H.A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for $\sigma_m = 1$ m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	132.1	0.51	0.18	0.10	0.38	16.83	18.43	19.97	423
70	107.5	0.78	0.28	0.14	0.22	16.83	18.14	19.41	344
60	99.3	0.84	0.34	0.16	0.16	16.83	18.04	19.22	318
50	94.5	0.87	0.38	0.18	0.13	16.83	17.99	19.12	303
40	91.0	0.89	0.42	0.20	0.11	16.83	17.94	19.04	292
30	88.0	0.91	0.46	0.21	0.10	16.83	17.91	18.96	282
20	85.2	0.92	0.51	0.23	0.10	16.83	17.87	18.90	273
10	82.5	0.93	0.56	0.24	0.10	16.83	17.84	18.83	264
0	79.5	0.94	0.61	0.26	0.10	16.83	17.81	18.76	255
-10	76.3	0.95	0.68	0.29	0.11	16.83	17.77	18.69	245
-20	72.4	0.96	0.79	0.32	0.12	16.83	17.72	18.59	232
-30	67.4	0.97	0.96	0.38	0.14	16.83	17.66	18.47	216
-40	60.6	0.98	1.29	0.49	0.18	16.83	17.57	18.31	194
-50	50.0	0.99	2.10	0.75	0.27	16.83	17.45	18.06	161
-60	27.5	1.00	9.48	3.05	0.81	16.83	17.17	17.51	89
-70	0.0	******	******	******	*****	0.00	0.00	0.00	0
-80	0.0	******	******	******	*****	0.00	0.00	0.00	0

Table A7 (U)
Effective Error of Three Coordinates
and Geometric Parameters of Error Analysis

Lat. φ (degrees)	Avail. H.A. γ_o (degrees)	Corr. Coeff. ρ_{12}	Effective Error (in meters) for $\sigma_m = 1$ m			Lunar Declination (degrees)			Samples/Day N
			$\sigma_{\Delta x_1}$	$\sigma_{\Delta x_2}$	$\sigma_{\Delta x_3}$	Initial δ_o	Av. δ_{av}	Final δ_f	
80	180.0	0.00	0.09	0.07	0.38	26.42	27.25	27.87	577
70	143.0	0.38	0.11	0.09	0.20	26.42	27.12	27.68	458
60	118.2	0.67	0.15	0.12	0.15	26.42	27.01	27.52	379
50	106.9	0.78	0.19	0.15	0.12	26.42	26.97	27.44	343
40	99.4	0.84	0.23	0.17	0.11	26.42	26.93	27.38	319
30	93.6	0.88	0.27	0.20	0.10	26.42	26.91	27.33	300
20	88.5	0.91	0.31	0.22	0.10	26.42	26.88	27.29	284
10	83.7	0.93	0.36	0.25	0.10	26.42	26.86	27.25	268
0	78.8	0.94	0.42	0.28	0.11	26.42	26.84	27.21	253
-10	73.5	0.96	0.51	0.33	0.12	26.42	26.81	27.17	236
-20	67.2	0.97	0.65	0.41	0.14	26.42	26.78	27.11	216
-30	59.3	0.98	0.92	0.55	0.17	26.42	26.74	27.04	190
-40	47.9	0.99	1.60	0.90	0.26	26.42	26.68	26.93	154
-50	26.6	1.00	7.18	3.72	0.71	26.42	26.57	26.72	86
-60	0.0	******	******	******	******	0.00	0.00	0.00	0
-70	0.0	******	******	******	******	0.00	0.00	0.00	0
-80	0.0	******	******	******	******	0.00	0.00	0.00	0

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13. ABSTRACT

(Unclassified)

Recently, radio methods have utilized artificial satellites as radio reference points in the sky to determine accurately the position and height of unknown locations on the earth's surface. However, an extensive tracking network is required to achieve the required accuracy. This report describes a system concept that utilizes a natural satellite, the moon, as a radio reference point for worldwide position and height determination. This method obviates the necessity for tracking and simplifies the resulting analysis, but requires longer observation periods and increased transmitted power.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Radar echoes						
Lunar topography						
Geodetic coordinates						