NRL Report 6306

Filter Pack Technique for Classifying Radioactive Aerosols by Particle Size

Part 4 - Mathematical Considerations

ROBERT M. MASON, PETER C. RYAN, AND SARAH L. HILL

Applied Mathematics Staff

August 9, 1965

U.S. NAVAL RESEARCH LABORATORY
Washington, D.C.

Previous Reports in this Series

"Part 1 – Preliminary Report and Evaluation," L.B. Lockhart, Jr., and R.L. Patterson, Jr., NRL Report 5970, August 16, 1963

"Part 2 – Isotopic Fractionation with Particle Size," L.B. Lockhart, Jr., R.L. Patterson, Jr., and A.W. Saunders, Jr., NRL Report 6164, October 28, 1964

"Part 3 – The Size Distribution of Airborne Fission Products During 1963 and 1964," L.B. Lockhart, Jr., R.L. Patterson, Jr., and A.W. Saunders, Jr., NRL Report 6305, July 30, 1965

Clearinghouse for Federal Scientific and Technical Information (CFSTI)
Sills Building, 5285 Port Royal Road
Springfield, Virginia 22151
$1.00

Filter Pack Technique for Classifying Radioactive Aerosols by Particle Size

Part 4 – Mathematical Considerations

ROBERT M. MASON, PETER C. RYAN, AND SARAH L. HILL

Applied Mathematics Staff

Passing an air stream through a sequence of filters having different retentivities effects some classification of radioactive particles by size. Airborne radioactivity has been related to particle size by a system of four simultaneous linear algebraic equations in four unknowns. Each equation expresses the total radioactivity collected on a given filter in terms of the total activity for each size group and the probability for particles of that group being trapped on that filter. A preliminary set of filter pack collection data was used to determine a coefficient matrix for the system based on these probabilities. The system was then applied to new data.

This procedure can be applied to a variety of filter pack constructions for relating other characteristics of atmospheric particles to particle size.

INTRODUCTION

In the method described in previous reports of this series (Refs. 1-3), an air stream is drawn sequentially through a pack of filters having different retentivities to effect some classification of radioactive particles by size. The present report develops a general mathematical procedure which has been applied to the filter pack problem to obtain a coefficient matrix for relating the radioactivity contribution to particle size.

Suppose that both radioactive and nonradioactive particles (assumed spherical) are classified by their diameters into n arbitrary, but fixed, size groups. Any exposed m-filter pack can then be related to the following system of simultaneous linear equations:

$$\begin{aligned} a_{11}X_1 + a_{12}X_2 + \cdots + a_{1n}X_n &= B_1 \\ a_{21}X_1 + a_{22}X_2 + \cdots + a_{2n}X_n &= B_2 \\ &\cdots \\ a_{m1}X_1 + a_{m2}X_2 + \cdots + a_{mn}X_n &= B_m \end{aligned} \qquad (1)^*$$

where B_i is the measured total radioactivity of the filter occupying the ith layer of the filter pack ($i = 1, \cdots, m$), X_j is the total radioactivity attributable to particles in the jth size group ($j = 1, \cdots, n$), and a_{ij} is the probability that a particle in the jth size group will be trapped by the ith filter. With this interpretation, the equations of system (1) express the total radioactivity of each filter as the sum of the n activities contributed to it by the various particle size groups.

The filter pack used in the radioactivity determinations described in Part 3 (Ref. 3) contained four filters, so system (1) can be specialized to four equations in four unknowns. Then, either of the two sums

$$B_1 + B_2 + B_3 + B_4 \qquad (2)$$

or

$$X_1 + X_2 + X_3 + X_4 \qquad (3)$$

gives the total activity of the filter pack, which will be denoted by B_T.

Dividing each equation of system (1) by B_T, and setting

$$b_i = \frac{B_i}{B_T} \qquad (i = 1, 2, 3, 4) \qquad (4)$$

$$x_j = \frac{X_j}{B_T} \qquad (j = 1, 2, 3, 4) \qquad (5)$$

NRL Problems A02-13 and F04-05; Projects RR-004-02-42-5151, RR-009-03-45-5802, and AEC AT(49-7)-2435. This is an interim report; work on this problem is continuing. Manuscript submitted June 7, 1965.

*The difference in notation between this report and Part 3 should be noted.

yields the following system:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 &= b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 &= b_4 \end{aligned} \tag{6}$$

where the b_i and x_j values represent fractional or relative rather than absolute activities.

By their nature, the quantities appearing in system (6) are nonnegative; that is

$$a_{ij} \geqslant 0, \tag{7}$$

$$x_j \geqslant 0, \tag{8}$$

and

$$b_i \geqslant 0. \tag{9}$$

Also

$$\sum_{i=1}^{4} b_i = 1, \tag{10}$$

and

$$\sum_{j=1}^{4} x_j = 1, \tag{11}$$

because of Eqs. (2) through (5).

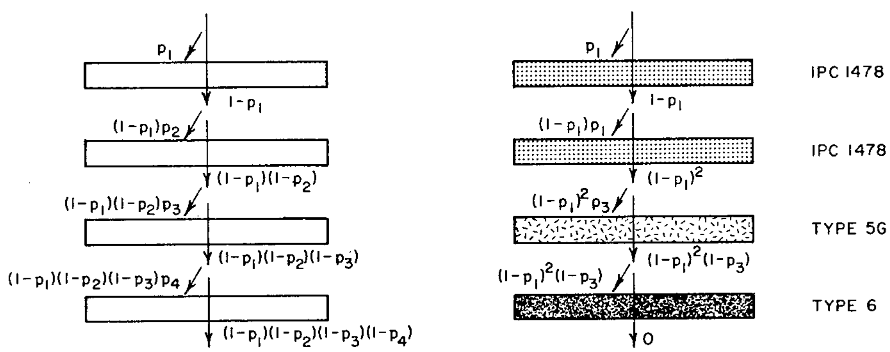
Experimentation with identically constructed filter packs has shown that the b_i values vary widely from day to day. Part of this variation occurs because filters of the same type cannot be manufactured to have precisely the same characteristics. Part of it comes from changes in the air flow. But variations due to these two sources are expected to be small compared to the variation that is associated with a genuine shift in the distribution of the radioactive content and particle sizes of the air samples. This latter variation is the object of the investigation; it is revealed by the behavior of the x_j for successive air samples.

Thus, for a particular filter pack design, it is assumed that the probability matrix $A = (a_{ij})$ is constant in time. If the fixed elements of this matrix can be determined, they will serve as a standard set of coefficients in system (6) for all future applications. In theory then, having determined the a_{ij} values, system (6) may be solved for the x_j describing each new air sample, provided only that a set of measured values for the constants b_i is given.

An empirical attempt to determine such an A matrix for the four-filter pack depicted in Fig. 1(b), succeeded only in establishing approximate ranges for the matrix elements a_{ij} (see Table 1). Calibration difficulties made an exact determination of the elements impossible by measurement alone. Certain theoretical considerations, however, have led to a mathematical procedure – presented in the next section – which enables sufficiently exact determinations of the elements to be made.

THEORY

Suppose momentarily that a four-filter pack is used to sample an air mass containing particles belonging solely to the jth size group (j fixed). Suppose further that the probabilities of capture by the filters (taken singly, before arranging in a pack) for particles of this size are p_1, p_2, p_3, and p_4.



(a) General probabilities (b) Specific probabilities

Fig. 1 – Capture probabilities by individual filters in four-filter packs

Table 1
Capture Probability Bounds

Layer (i)	Capture Probability Bounds for Size Groups (j)*				Filter
	1 (1.1μ)	2 (0.6μ)	3 (0.3μ)	4 (0.15μ)	
1	.40 − .95	.20 − .40	.05 − .15	.00 − .04	IPC 1478
2	.0475 − .24	.16 − .24	.0475 − .1275	.00 − .0384	IPC 1478
3	.0025 − .358	.358 − .625	.6936 − .74	.05 − .4608	TYPE 5G
4	.000 − .002	.002 − .015	.0289 − .1625	.4608 − .95	TYPE 6

*Numbers in parentheses refer to particle diameters.

Then the probability that a particle will lodge in the second filter layer, for example, is the product of the probability that it is *not* trapped by the first layer times the probability that it *is* trapped by the second, or

$$(1 - p_1)p_2.$$

A complete description of these probabilities is given in Fig. 1(a).

In the particular filter pack under consideration, the top two layers consist of identical filters, so that $p_2 = p_1$; the bottom layer contains a so-called "ultimate" filter (that is, one for which $p_4 = 1$). The modified probabilities for this arrangement are given in Fig. 1(b).

The probability relations serve as constraints on the elements of the jth column of the A matrix. Specifically, the top two elements of each column are related by the expression

$$a_{2j} = (1 - a_{1j})a_{1j}, \qquad j = 1, 2, 3, 4. \qquad (12)$$

Also,

$$\sum_i a_{ij} = 1, \qquad j = 1, 2, 3, 4 \qquad (13)$$

since

$$\begin{aligned} p_1 + (1 - p_1)p_1 + (1 - p_1)^2 p_3 + (1 - p_1)^2(1 - p_3) \\ = p_1 + (1 - p_1)p_1 + (1 - p_1)^2(p_3 + 1 - p_3) \\ = p_1 + (1 - p_1)(p_1 + 1 - p_1) \\ = 1. \end{aligned}$$

Equation (12) is a constraint resulting from the specific type of filter pack used. However, Eq. (13) has a more general validity if it is understood that the quantities in system (6) refer to *captured* particles, for the probability that such particles lodge somewhere within the pack is, of course, unity.

From Fig. 1(b) one might think that

$$a_{3j} \leqslant (1 - a_{1j})^2$$

would have to be imposed. But this follows from

$$a_{3j} + a_{4j} = (1 - a_{1j})^2, \qquad (*)$$

which also can be read off from the figure. One might think that Eq. (*) has to be explicitly posed, but it actually is a consequence of Eqs. (12) and (13), since

$$\begin{aligned} a_{3j} + a_{4j} &= 1 - a_{1j} - a_{2j} = 1 - a_{1j} - (1 - a_{1j})a_{1j} \\ &= 1 - 2a_{1j} + a_{1j}^2 = (1 - a_{1j})^2. \end{aligned}$$

Considered together, the equations of system (6) and Eq. (11) form a system of five equations in the four unknowns x_1, x_2, x_3, and x_4. Notice that these five equations constitute a dependent set because Eq. (11) is obtainable by adding the left and right sides of the equations of system (6) and using the relations for the constants and coefficients given by (10) and (13).

So it is possible to summarize the preceding information from a different point of view as follows, using vector notation

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} x_1 + \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} x_2 + \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} x_3 + \begin{pmatrix} a_{14} \\ a_{24} \\ a_{34} \end{pmatrix} x_4 = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \qquad (14)$$

where

$$a_{ij} \geqslant 0, \qquad \sum_{i=1}^{3} a_{ij} \leqslant 1, \qquad (j = 1, 2, 3, 4) \tag{15}$$

$$b_i \geqslant 0, \qquad \sum_{i=1}^{3} b_i \leqslant 1, \tag{16}$$

$$x_j \geqslant 0, \qquad \sum_{j=1}^{4} x_j = 1. \tag{17}$$

The column vectors

$$\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

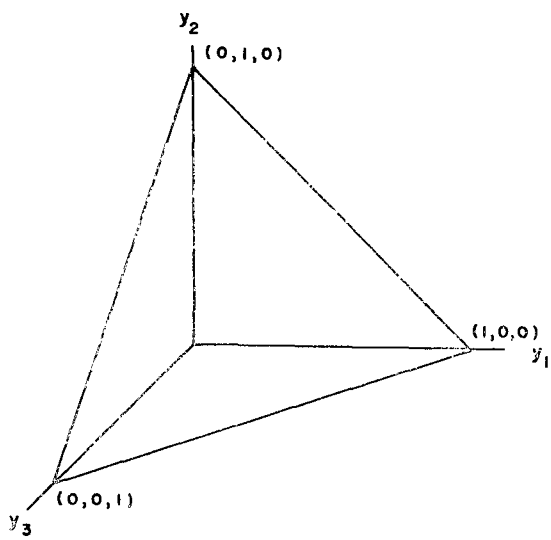
may be regarded as the coordinates of the five points $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, $\mathbf{a}_4$, and $\mathbf{b}$ in three-dimensional Euclidean space. Accordingly, Eqs. (15) and (16) state that these points lie inside or on the boundary of the *unit simplex* (see Ref. 4), that is, the closed solid figure which in three dimensions is determined by the three coordinate planes

$$y_1 = 0, \qquad y_2 = 0, \qquad y_3 = 0,$$

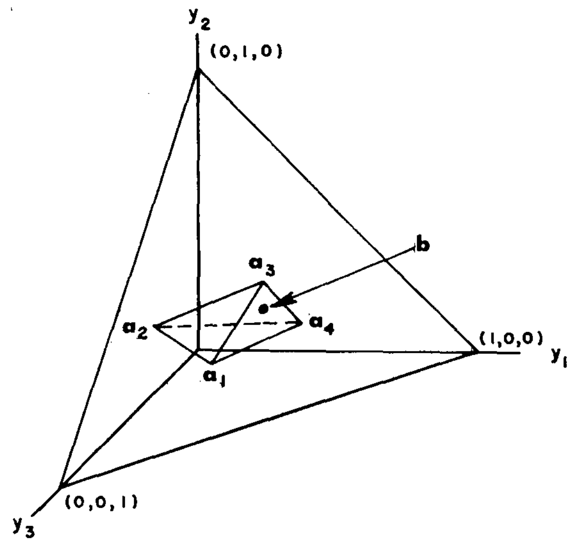
and the plane

$$y_1 + y_2 + y_3 = 1.$$

This unit simplex is shown below:



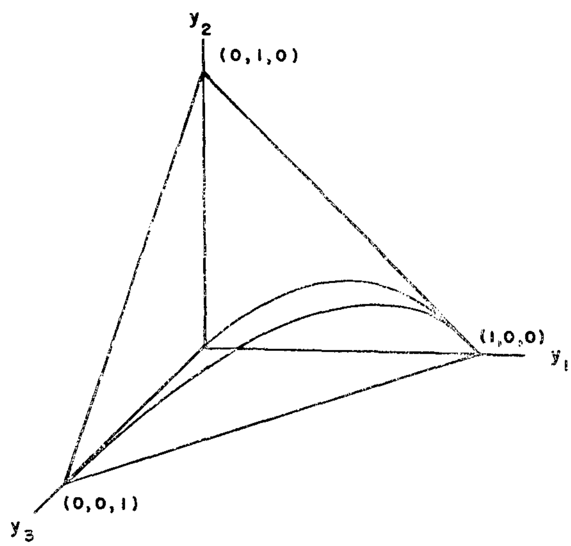
Also, Eqs. (14) and (17) state that $\mathbf{b}$ can be expressed as a *convex combination* (see Ref. 5) of the $\mathbf{a}_j (j = 1,2,3,4)$ where the x_j are the coefficients in the combination; or that $\mathbf{b}$ is in the *convex hull* (again see Ref. 5) of the $\mathbf{a}_j$; or, more simply, that $\mathbf{b}$ is inside or on the boundary of the tetrahedron having the $\mathbf{a}_j$ as vertices. This can be shown as



Geometrically speaking Eq. (12), which imposes another condition on each $\mathbf{a}_j$, restricts the corners of the tetrahedron to the surface

$$y_2 = (1 - y_1)y_1 = y_1 - y_1^2.$$

The portion of this surface interior to the unit simplex is sketched in the following diagram:



Finally, the expected extremities of the ranges for the elements a_{ij} of the matrix A are given in Table 1.

The first three ranges appearing in the jth column of this table define a rectangular parallelepiped in three-dimensional space within which the point $\mathbf{a}_j$ must be found.

PROCEDURE

The data used in the initial determination of the A matrix resulted from 40 separate collections made over a two-year period using filter packs of the type illustrated in Fig. 1(b). For each collection, the relative counts (that is, the b_i values in system (6)) for each of the four filters in the pack were provided.

The problem, reformulated in accordance with the preceding theory, reduces to finding four three-dimensional points, $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, $\mathbf{a}_4$, which satisfy the following conditions:

(1) Each $\mathbf{a}_j$ must be interior to the unit simplex in three dimensions,

(2) Each $\mathbf{a}_j$ must lie on the constraint surface defined by Eq. (12),

(3) Each $\mathbf{a}_j$ must fall within the regions defined in Table 1, and

(4) The tetrahedron having $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, $\mathbf{a}_4$ as corners must enclose all 40 of the given $\mathbf{b}$'s.

First observe that any tetrahedron which encloses the outermost points of a given cluster of $\mathbf{b}$ values will necessarily enclose the entire cluster. Mathematically, these "outermost" points are termed the *extreme points* (see Ref. 5) of the convex hull of the $\mathbf{b}$ values.

Use of a NAREC program called CONVEX HULL (see Appendix) determined that 18 of the 40 given $\mathbf{b}$ values are extreme points. The three-dimensional graph appearing in Fig. 2 shows these extreme points. The shaded area indicates the portion of the constraint surface given by Eq. (12) that lies inside the unit simplex. An acceptable tetrahedron must both enclose these 18 points and have its corners lying on the constraint surface.

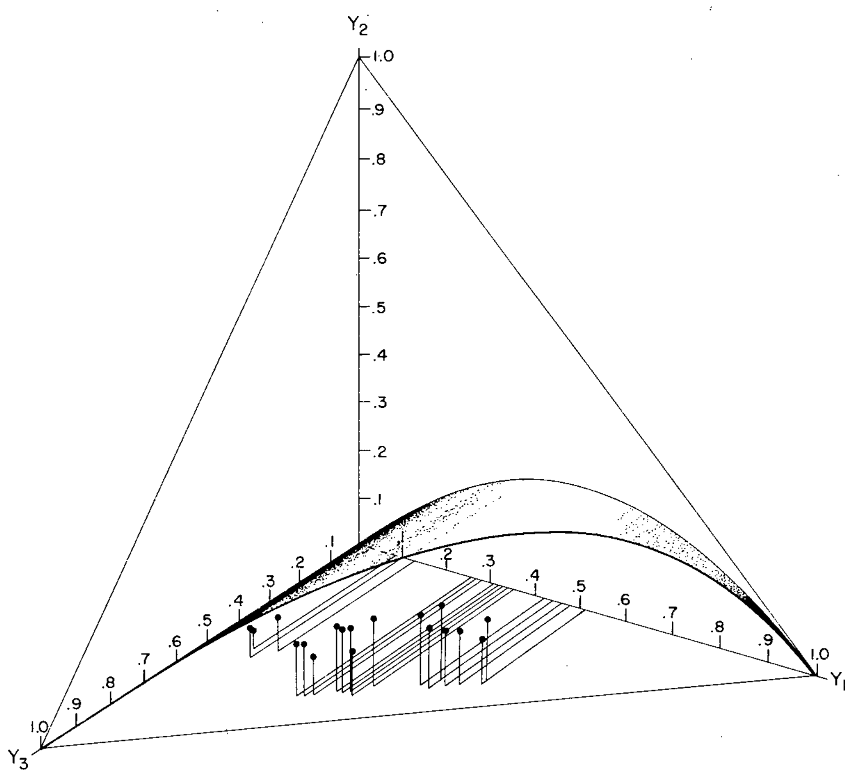


Fig. 2 — Extreme points of the convex hull of the 40 given $\mathbf{b}$ values

A trial-and-error procedure was applied to find the vertices of such a tetrahedron. Using Fig. 2 and the bounds for the elements of the matrix A given in Table 1, likely sets of coordinates for the corners of each trial tetrahedron were checked by invoking another NAREC program called POLYHEDRON, with $p = 18$ and $q = 3$ (see Appendix). The 75th candidate set investigated was found to specify an acceptable tetrahedron. The vertices of this tetrahedron are as follows:

$$\mathbf{a}_1 = \begin{pmatrix} .95 \\ .0475 \\ .0025 \end{pmatrix}, \qquad \mathbf{a}_2 = \begin{pmatrix} .30 \\ .21 \\ .488 \end{pmatrix},$$

$$\mathbf{a}_3 = \begin{pmatrix} .10 \\ .09 \\ .778 \end{pmatrix}, \qquad \text{and} \qquad \mathbf{a}_4 = \begin{pmatrix} .02 \\ .0196 \\ .38 \end{pmatrix}.$$

This yields the following matrix of capture probabilities:

$$A = \begin{pmatrix} .95 & .30 & .10 & .02 \\ .0475 & .21 & .09 & .0196 \\ .0025 & .488 & .778 & .38 \\ .00 & .002 & .032 & .5804 \end{pmatrix}. \tag{18}$$

A final NAREC program, SIMLINEQ (see Appendix), was then used to solve system (6) 40 times (that is, once for each $\mathbf{b}$). As expected, the answers x_1, x_2, x_3, and x_4 were found to be nonnegative and to sum to one.

The general applicability of matrix (18) was checked later when the results of 22 additional collections made on the same type of filter packs became available. System (6) was solved 22 more times and all but one of the 88 answers proved to be nonnegative. The negative answer is attributable to counting error.

The probability matrix (18) obtained by this procedure is not unique, since certain small perturbations of the corners of any tetrahedron so determined will yield other legitimate tetrahedrons. Because the problem is so highly constrained, however, it is felt that no other tetrahedron satisfying the aforementioned constraints could give rise to appreciably different x_i values.

It should be emphasized that no data from packs containing more than four layers have been analysed by this method thus far. For the more general m-filter pack there is no simple analog to Fig. 2, which here played an important role in suggesting coordinates for the corner points of the trial tetrahedrons. Whether or not it is possible to develop an analogous procedure for the determination of such coordinates in higher dimensional Euclidean space is presently uncertain.

Although the requirement here was to determine the radioactivities associated with various size groups, nothing in the described method would prevent an investigation of other measurable characteristics of atmospheric particles using the same techniques. For example, mass, chloride or sulfate ion content, fluorescence, magnetism, or other bulk properties of aerosols could be related to particle size.

It is hoped that this report will suggest further applications which will be amenable to solution by the technique described.

ACKNOWLEDGMENT

Appreciation is due to Dr. Alan J. Goldman, of the National Bureau of Standards, who read the manuscript and made suggestions to improve the correctness and clarity of the presentation and to extend the investigation.

REFERENCES

1. Lockhart, L.B., Jr., and Patterson, R.L., Jr., "Filter Pack Technique for Classifying Radioactive Aerosols by Particle Size. Part 1 – Preliminary Report and Evaluation," NRL Report 5970, Aug. 16, 1963

2. Lockhart, L.B., Jr., Patterson, R.L., Jr., and Saunders, A.W., Jr., "Filter Pack Technique for Classifying Radioactive Aerosols by Particle Size. Part 2 – Isotopic Fractionation with Particle Size," NRL Report 6164, Oct. 28, 1964

3. Lockhart, L.B., Jr., Patterson, R.L., Jr., and Saunders, A.W., Jr., "Filter Pack Technique for Classifying Radioactive Aerosols by Particle Size. Part 3 – The Size Distribution of Airborne Fission Products During 1963 and 1964," NRL Report 6305, July 30, 1965

4. Gass, S.I., "Linear Programming," 2nd ed., New York: McGraw-Hill, p. 30, 1964

5. Spivey, W.A., "Linear Programming – An Introduction," New York: Macmillan, 1963

Appendix
DESCRIPTION OF COMPUTER PROGRAMS

The computations were performed on the NAREC (Naval Research Electronic Computer), a medium-sized, medium-speed digital computer located in the Research Computation Center, U.S. Naval Research Laboratory.

The programs were written in NELIAC-N, a problem-oriented compiler language for the NAREC. NELIAC-N is a dialect of ALGOL.

The four NELIAC programs used in the solution of the filter-pack problem are described below:

(1) COMB, used as a subprogram in the two following programs, produces the ${}_pC_q$ combinations of p things taken q at a time, corresponding to the integers from 1 to ${}_pC_q$, $q \leq 15$. For example, in producing the ${}_5C_3$ combinations of 5 things taken 3 at a time given the integers one through ten, the program will yield the following results:

Input	Output
1	1,2,3
2	1,2,4
3	1,2,5
4	1,3,4
5	1,3,5
6	1,4,5
7	2,3,4
8	2,3,5
9	2,4,5
10	3,4,5

(2) CONVEX HULL takes a small number of q-dimensional ($q \leq 15$) Euclidean vectors and determines the extreme points of their convex hull.

A combinatorial procedure is used and the amount of computer time necessary to find the extreme points increases rapidly with the number of vectors involved. For example, 1-1/2 minutes of computer time are required for ten three-dimensional points; 69 minutes are required for thirty-five three-dimensional points.

This program also produces the equations of the bounding hyperplanes of the convex hull and the number of bounding hyperplanes incident at each vertex.

(3) POLYHEDRON inputs an array of $q + 1$ q-dimensional points ($q \leq 15$) and an array of p q-dimensional points, then determines if the p points are interior to the simplex determined by the $q + 1$ points.

The output of this program is the perpendicular distance of each of the p points from each face of the simplex determined by the $q + 1$ points. A negative distance indicates that a given point is exterior to the simplex.

In addition, other numerical descriptions of the simplex are given. Specifically, k-dimensional measures of the simplex are computed ($k = 1, \cdots, q$). For a triangle (a two-dimensional simplex), the one-dimensional measure corresponds to its perimeter, the two-dimensional measure to its area. For a tetrahedron (a three-dimensional simplex), the one-dimensional measure corresponds to the sum of its edge lengths, the two-dimensional measure to the sum of the areas of its faces, and the three-dimensional measure to its volume.

(4) SIMLINEQ* is a routine to solve a system of simultaneous linear algebraic equations. It finds the values of the unknowns for a given coefficient matrix and stores them in a set of specified locations.

This routine will detect a singular matrix or an inconsistent system of equations and will indicate the error.

In addition, the value of the determinant and the re-evaluated constants obtained by substituting the answers for the unknowns in the original system may be printed.

*Mason, Janet P., "NELIAC Functions to Solve Systems of Simultaneous Linear Equations," NELIAC Bulletin #8.

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
U.S. Naval Research Laboratory Washington, D.C. 20390	Unclassified
	2b. GROUP

3. REPORT TITLE

Filter Pack Technique for Classifying Radioactive Aerosols by Particle Size. Part 4 – Mathematical Considerations

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

An interim report on one phase of the problem.

5. AUTHOR(S) *(Last name, first name, initial)*

Mason, Robert M., Ryan, Peter C., and Hill, Sarah L.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 9, 1965	10	5

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
NRL Problems A02-13 and F04-05	NRL Report 6306
b. PROJECT NO. RR-004-02-42-5151 RR-009-03-45-5802 AEC Project AT(49-7)-2435	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES

Unlimited availability
Available at CFSTI

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Department of the Navy (Office of Naval Research) and AEC

13. ABSTRACT

Passing an air stream through a sequence of filters having different retentivities effects some classification of radioactive particles by size. Airborne radioactivity has been related to particle size by a system of four simultaneous linear algebraic equations in four unknowns. Each equation expresses the total radioactivity collected on a given filter in terms of the total activity for each size group and the probability for particles of that group being trapped on that filter. A preliminary set of filter pack collection data was used to determine a coefficient matrix for the system based on these probabilities. The system was then applied to new data.

This procedure can be applied to a variety of filter pack constructions for relating other characteristics of atmospheric particles to particle size.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Mathematical analysis						
Computer programming						
NAREC						
Coefficient matrix						
Filter pack technique						
Atmospheric radioactivity						
Particle size distribution						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.