NRL Report 8219

Statistical Properties of a Staggered-PRF MTI System

J. K. Hsiao

Target Characteristics Branch
Radar Division

April 25, 1978

NAVAL RESEARCH LABORATORY
Washington, D.C.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NRL Report 8219	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): STATISTICAL PROPERTIES OF A STAGGERED-PRF MTI SYSTEM	5. TYPE OF REPORT & PERIOD COVERED: Interim report on a continuing NRL problem.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): James K. Hsiao	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Research Laboratory, Washington, D.C. 20375	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NRL Problem R12-51, Project RR014-09-41, Element 611153N-21
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Arlington, VA 22217	12. REPORT DATE: April 25, 1978
	13. NUMBER OF PAGES: 14
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Moving-target indicators
Clutter rejection
Staggered-PRF MTI

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report the statistical properties of improvement factors of a staggered-PRF MTI system are presented for cases in which optimal filter weights are used and in which binomial filter weights are used. It is shown that the degree of deviation of samples is a function of both the standard deviation of the clutter spectrum density function and the amount of variation of interpulse durations.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

CONTENTS

STATISTICAL PROPERTIES OF A STAGGERED—PRF MTI SYSTEM

INTRODUCTION

To reject unwanted clutter, a radar usually transmits a sequence of pulses. When the returns of these pulses are properly weighted and summed, stationary clutter can be filtered out. In a conventional radar system, the interpulse durations (or sampling frequencies) are held constant. Targets having a doppler frequency which is an integer multiple of this sampling frequency will be seen as a stationary target and be filtered out. This target is said to have a blind velocity. To alleviate this problem, a staggered-PRF system has been proposed. In that system the interpulse durations are varied from pulse to pulse; hence this blind velocity phenomenon is avoided. A number of papers dealt with the design problem of this system [1-4]. However no known analytic method can be used to select a set of interpulse durations to achieve a desired MTI performance. In this report the effects of variation of the interpulse durations on the MTI improvement factor are investigated. A Monte Carlo approach is used to derive the statistical properties of this improvement factor in a staggered-PRF MTI system.*

IMPROVEMENT FACTOR AND INTERPULSE DURATION

To set up a common reference for the convenience of comparison, a criterion to measure the performance of an MTI system will be presented here. One widely accepted measurement parameter is the so-called improvement factor, which is defined as the expected value of the ratio of the output target-signal-to-clutter ratio to the input target-signal-to-clutter ratio. This improvement factor is

$$I = \frac{\sum_i a_i^2}{\sum_i \sum_j a_i a_j R_{ij}}, \tag{1}$$

where the a_i's are the MTI filter weights and R_{ij} is the clutter correlation function at times t_i and t_j. This correlation function is the Fourier transform of the clutter spectrum density function $G(f)$:

$$R_{ij} = \int G(f) e^{j2\pi f(T_i - T_j)}\, df. \tag{2}$$

In deriving Eq. (1) it is assumed that the target doppler has a uniform distribution function.

Manuscript submitted February 28, 1978.

*Part of this report has been presented as a paper at the 1977 IEEE International Conference on Acoustic, Speech and Signal Processing, Hartford, Connecticut, May 9-11, 1977.

For a constant-PRF MTI system, one may normalize the doppler frequency f by the radar PRF (the reciprocal of interpulse duration T), and Eq. (2) becomes

$$R_{ij} = \int \frac{G(f)}{T} e^{j2\pi f'(i-j)} df'. \tag{3}$$

Under this assumption the improvement factor I is not a function of the radar interpulse duration T. However, the clutter spectrum density function may have to be modified due to this transformation. For example, if the clutter spectrum density function is a Gaussian function

$$G(f) = \frac{1}{\sqrt{2\pi\sigma}} e^{-f^2/2\sigma^2}, \tag{4a}$$

then

$$R_{ij} = e^{-2\pi^2\sigma^2 T^2 (i-j)^2}. \tag{4b}$$

If one lets $f' = fT$ and $\sigma' = \sigma/T$, one has

$$G'(f') = \frac{1}{\sqrt{2\pi\sigma'}} e^{-f'^2/2\sigma'^2} \tag{5a}$$

and

$$R_{ij} = e^{-2\pi^2\sigma'^2(i-j)^2}. \tag{5b}$$

One notices that the standard deviation σ of the spectrum density is modified. However, the spectrum density remains unchanged. This formulation has the advantage that the radar PRF is not directly involved in the computation of the improvement factor. In a staggered-PRF MTI system the interpulse durations vary from pulse to pulse. To accommodate this situation and for the convenience of comparison, a basic interpulse duration T is defined which is the shortest interpulse time among all pulses in a staggered PRF system. The interpulse time between any two successive pulses is then

$$T_i - T_{i-1} = (1 + \alpha_i)T, \tag{6}$$

where $\alpha_i \geqslant 0$ and

$$G(f') = \frac{1}{\sqrt{2\pi\sigma'}} e^{-f'^2/2\sigma'^2}, \tag{7}$$

which is identical to Eq. (5a). In other words, as long as the basic interpulse time is the same, the normalized spectrum density functions are the same for both the constant-PRF and the staggered-PRF cases. The correlation function R_{ij} however becomes

$$R_{ij} = \exp\left\{-2\pi^2\sigma'^2\left[\sum_{k=i}^{j}(1+\alpha_k)\right]^2\right\}. \tag{8}$$

When this relation is inserted into Eq. (1), one sees that the variation of interpulse duration α_k influences the MTI improvement factor. However, one may see intuitively that R_{ij} reduces in the case of a staggered-PRF system, because the correlation time becomes longer. Naturally, the MTI performance is degraded, and the improvement factor is reduced.

OPTIMAL MTI PERFORMANCE

The conclusion has been drawn that the MTI filter can be so chosen that it yields a best improvement factor for a given clutter spectrum density. Hsiao [5] showed that for a staggered-PRF system this optimal improvement factor is bounded by two limits. The upper bound is the improvement factor of a constant-PRF system with a PRF that is equivalent to the shortest interpulse duration of the staggered system, and the lower bound is the improvement factor of a constant-PRF system which has a PRF equivalent to the longest interpulse duration of the staggered system.

The preceding conclusion is drawn from investigations of a large number of samples. Each sample has a randomly chosen interpulse duration. However in each case the filter weights are so chosen that the improvement factor is optimized. This approach is useful in determining the performance bounds. In practice, however, one may be more interested in keeping the filter weights fixed while varying the interpulse durations. Some statistical properties of such systems are as follows.

Figure 1 shows the statistical distribution of the improvement factor of a three-pulse, staggered-PRF MTI system. The filter weights are initially chosen for optimal performance for a constant-PRF system assuming that the clutter spectrum density function is Gaussian having a normalized standard deviation σ (normalized with respect to PRF). The improvement factor of this MTI system is then computed assuming that the interpulse duration varies from T to $T + \alpha T$ where α is a random variable with a uniform distribution. In Fig. 1 four sets of curves are plotted, for normalized standard deviations $\sigma = 0.03, 0.05, 0.07$ and 0.1. Within each set of curves the limit of the variation of the interpulse time varies from 0.1 to 0.6. The improvement factor of each sample is computed when the interpulse durations of that sample are chosen randomly (with a uniform distribution) with a maximum limit as mentioned above. The cumulative probability of the improvement factor of these samples is plotted for each different σ and α.

Several interesting points may be observed:

- Since the sample having the smallest interpulse duration is the one which has a constant PRF, the highest improvement factor for various α values occurs at the same point (of the constant-PRF case) no matter what α is chosen.

- The variation of the improvement factor is small for small α but increases as α increases.

- The spreading of the samples is also a function of σ, the standard deviation of the clutter spectrum density function. As σ increases, the spreading of the samples reduces.

The results shown in Fig. 1 are summarized in Table 1. When $\sigma = 0.03$, the difference of the improvement factor varies from 1.5 dB to 8.5 dB as α varies from 0.1 to 0.6. When $\sigma = 0.1$, the variation is limited to 1.2 to 6.2 dB.

Figure 2 shows the same curves for the case of a four-pulse canceler. These curves have similar properties as those shown in Figure 1. However, the spread of the samples in general is more pronounced, particularly for high improvement factors. This means that if one has a high-performance MTI system, with four or more pulses for rejection of clutter with small spectral spread, one should be more careful in choosing the interpulse time when a staggered-PRF system is used, particularly when the variation of interpulse duration is large. On the other hand, if the maximum variation of interpulse duration is small and the designed MTI system has a smaller improvement factor, the choice of interpulse duration is not important. Probably any randomly selected combination of interpulse durations may yield just about the same result as that of a carefully selected one.

The results shown in Fig. 2 are summarized in Table 2. One notices that in general the spreading of samples is more pronounced in this case than in the three-pulse case.

Figure 3 shows the statistical properties of the improvement factor of a three-pulse staggered-PRF MTI system. In the figure the average value and the standard deviation (or RMS deviation from mean) are plotted as a function of the percent of variation of interpulse delay. The average improvement factor is almost a linear function of the percent of variation of interpulse delay. As the percent of delay variation increases, the improvement factor reduces. This improvement factor is also very sensitive to the σ value. The RMS deviation increases as the percent of variation of delay increases, but its value remains small (the deviation curves in Fig. 3 being plotted to an expanded scale relative to average-value scale). The significance of this is that by a random choice of any combination of interpulse durations the amount of improvement-factor variation is small. For example, for a case of $\sigma = 0.03$, when the delay variation of the staggered PRF system is set at 0.5%, by any choice of a combination of interpulse duration, the RMS deviation from the mean of all these samples is no more than 1.8 dB.

Figure 4 shows the same statistic properties of the improvement factor for a four-pulse staggered system. This figure exhibits properties similar to those exhibited in Fig. 3.

BINOMIALLY WEIGHTED FILTER

In the previous examples, optimal filter weights are used. In practice, however, filter weights are often set according to the binomial distribution. Therefore it is of interest to investigate the effect of staggering on the MTI system for such cases. The distribution of improvement factors for a three-pulse and four-pulse staggered-PRF MTI system using binomial weights are respectively shown in Figs. 5 and 6. The clutter spectrum density function is again assumed to be Gaussian with a normalized standard deviation σ, with the variation of interpulse duration being randomly distributed from T to $T + \alpha T$ similar to the variation in the previous examples. Comparing these two figures with Figs. 1 and 2, one sees that these curves have almost the same shape. Therefore the properties discussed in the preceding section apply to these cases. In general, for the same σ and α, the improvement factor which can be achieved by a MTI system with optimal weights is slightly better than that of a binomial case. However the difference is not that much.

Figure 7 and Figure 8 show respectively the statistic properties of the improvement factor of a three-pulse and four-pulse staggered-PRF MTI system. These figures show a similar properties of that of an optimally weighted MTI.

REFERENCES

1. O. J. Jacomini, "Weighting Factor and Transmission Time Optimization in Video MTI Radar," *IEEE Trans. on AES* AES-8, (July 1972).
2. P. J. A. Prinsen, "Elimination of Blind Velocities of MTI Radar by Modulating the Interpulse Period," *IEEE Trans. on AES* AES-9, 714-724 (Sept. 1973).
3. J. K. Hsiao and F. F. Kretschmer, Jr., "Design of a Staggered PRF Moving Target Indication Filter," *Radio and Electronic Engineer* 43 689-693 (Nov. 1973).
4. G. W. Ewell and A. M. Bush, "Constrained Improvement MTI Radar Processors," *IEEE Trans. on AES* AES-11, 768-780 (Sept. 1975).
5. J. K. Hsiao, "On the Optimization of MTI Clutter Rejection," *IEEE Trans. on AES* AES-10, 622-629 (Sept. 1974).

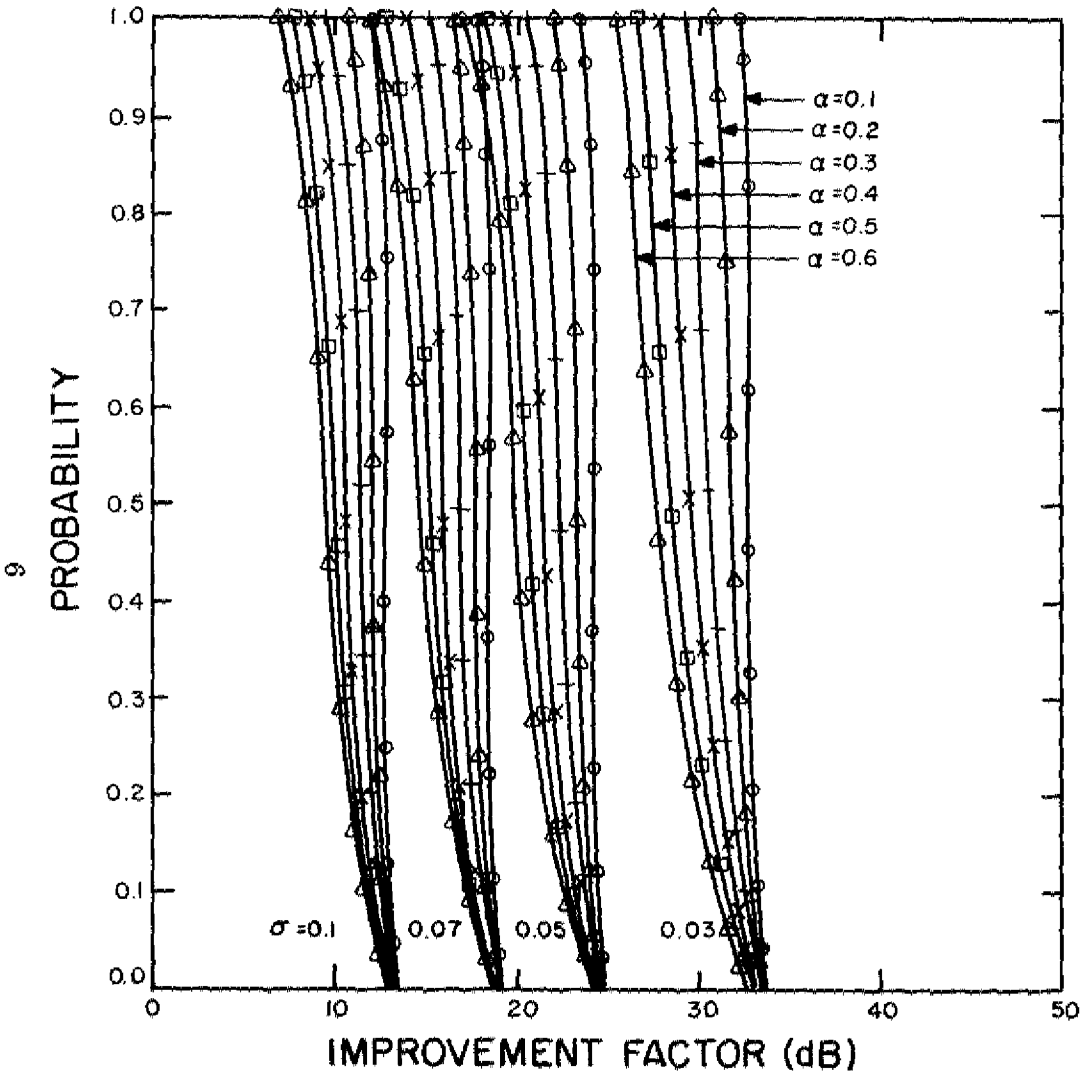

Fig. 1 — Distribution of the improvement factor for a three-pulse staggered-PRF MTI system using optimal filter weights

Table 1 — Results in Fig. 1

σ	α	Improvement Factor (dB)		
		High	Low	Difference
0.03	0.1	33.5	32	1.5
	0.2	33.5	30.5	3.0
	0.3	33.5	29.0	4.5
	0.4	33.5	27.7	5.8
	0.5	33.5	26.5	7.0
	0.6	33.5	25	8.5
0.05	0.1	24.5	23.5	1.0
	0.2	24.5	22.0	2.0
	0.3	24.5	20.5	4.0
	0.4	24.5	19.0	5.5
	0.5	24.5	18.0	6.5
	0.6	24.5	17.0	7.5
0.07	0.1	19.0	18.0	1.0
	0.2	19.0	16.5	2.5
	0.3	19.0	15.0	4.0
	0.4	19.0	13.8	5.5
	0.5	19.0	12.8	6.2
	0.6	19.0	11.8	7.2
0.1	0.1	13.2	12.0	1.2
	0.2	13.2	11	2.2
	0.3	13.2	9.5	3.7
	0.4	13.2	8.5	4.7
	0.5	13.2	7.5	5.7
	0.6	13.2	7.0	6.2

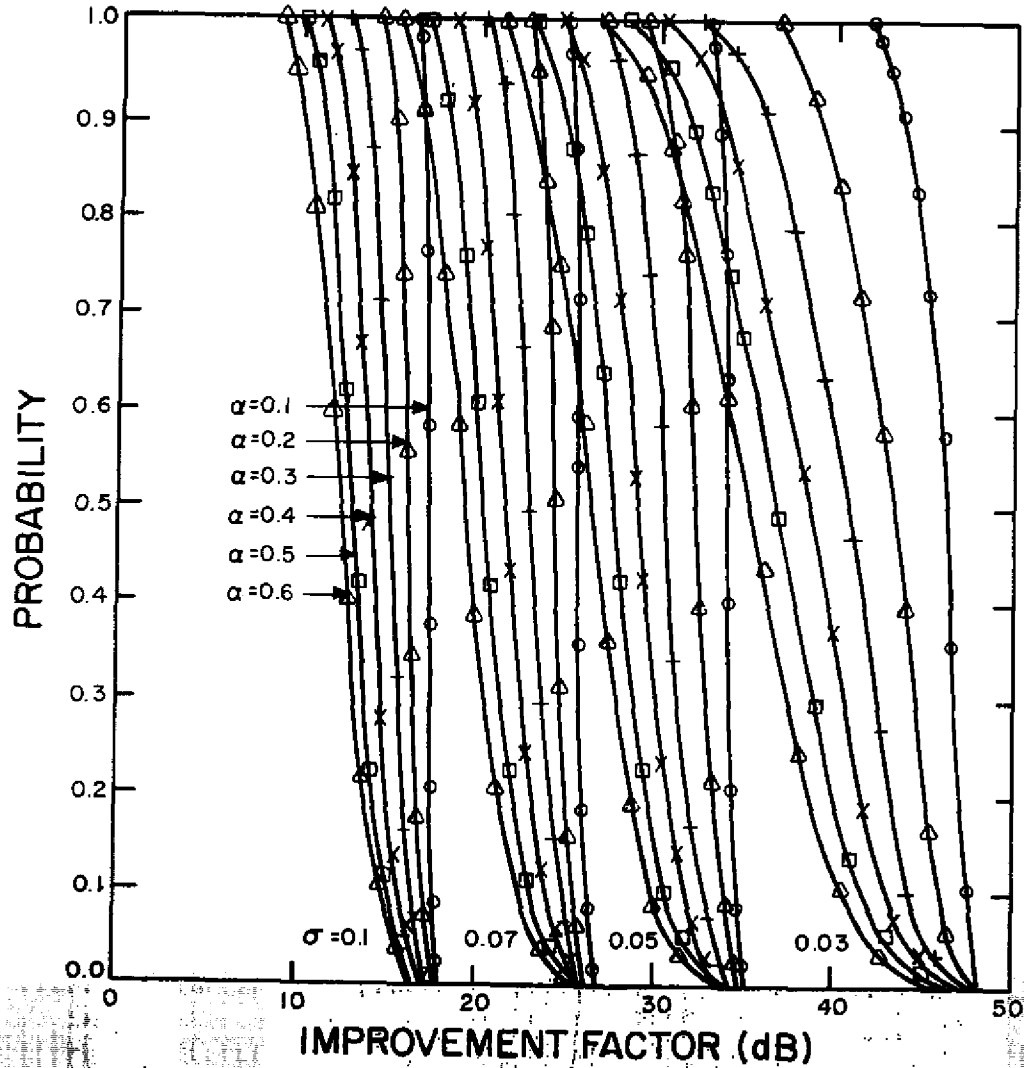

Fig. 2 — Distribution of the improvement factor for a four-pulse staggered PRF MTI system using optimal filter weights

Table 2 — Results in Fig. 2

σ	α	Improvement Factor (dB)		
		High	Low	Difference
0.03	0.1	48.5	41.0	7.5
	0.2	48.5	36.0	12.5
	0.3	48.5	32.5	16.0
	0.4	48.5	30.5	18.0
	0.5	48.5	28.5	20.0
	0.6	48.5	27	21.5
0.05	0.1	35.5	32.5	3.0
	0.2	35.5	29.5	6.0
	0.3	35.5	27.0	8.5
	0.4	35.5	24.5	11.0
	0.5	35.5	23.0	12.5
	0.6	35.5	21.5	14.0
0.07	0.1	27.0	24.5	3.5
	0.2	27.0	22.5	4.5
	0.3	27.0	20.5	6.5
	0.4	27.0	18.5	8.5
	0.5	27.0	17.0	10
	0.6	27.0	15.5	11.5
1	0.1	18.0	16.5	1.5
	0.2	18.0	14.5	3.5
	0.3	18.0	12.8	5.2
	0.4	18.0	11.2	6.8
	0.5	18.0	10.0	8.0
	0.6	18.0	9.0	9.0

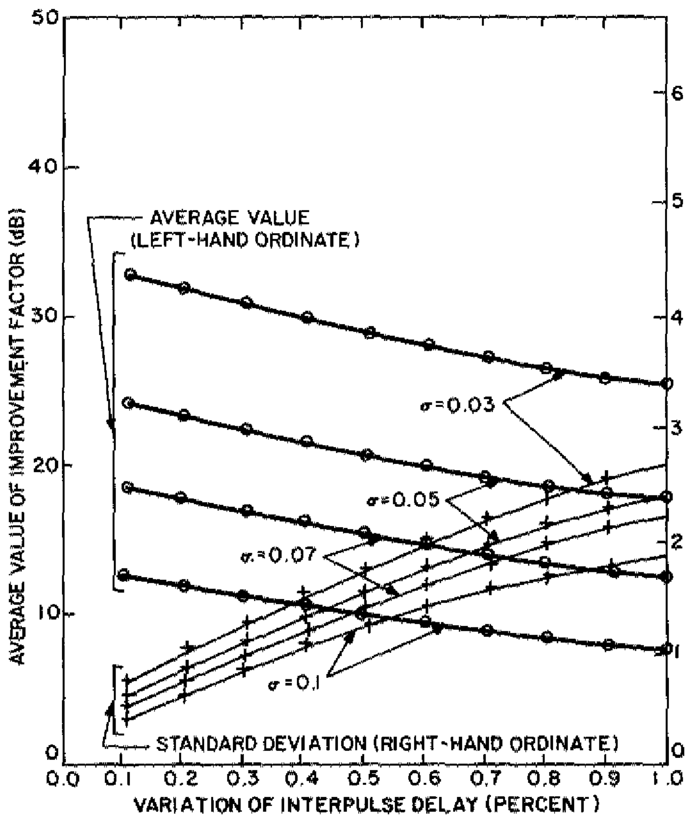

Fig. 3 — Statistical properties of the improvement factor of a three-pulse staggered-PRF MTI system using optimal filter weights

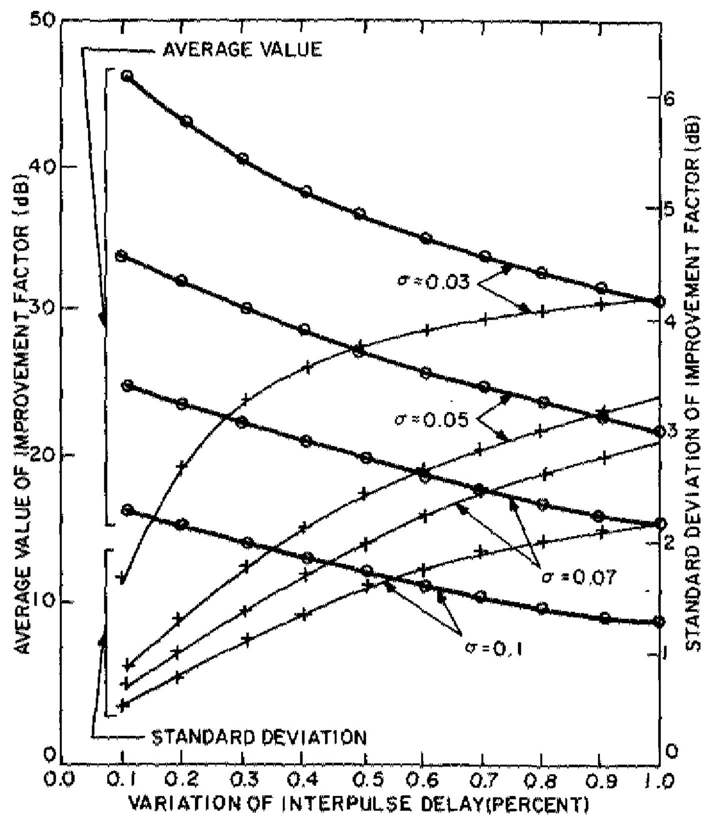

Fig. 4 — Statistical properties of the improvement factor of a four-pulse staggered-PRF MTI system using optimal weights

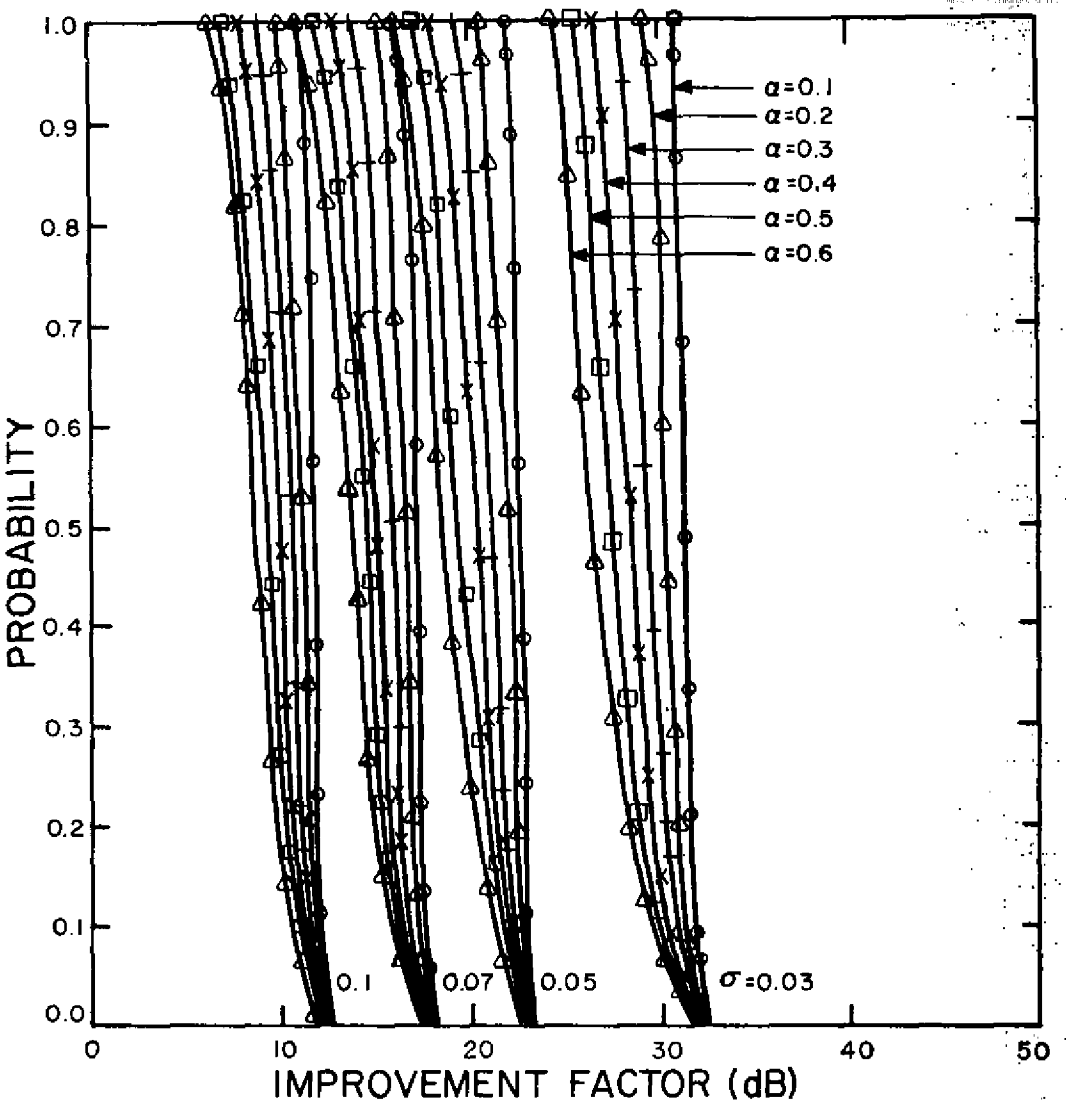

Fig. 5 — Distribution of improvement factor for a three-pulse staggered-PRF MTI system using binomial filter weights

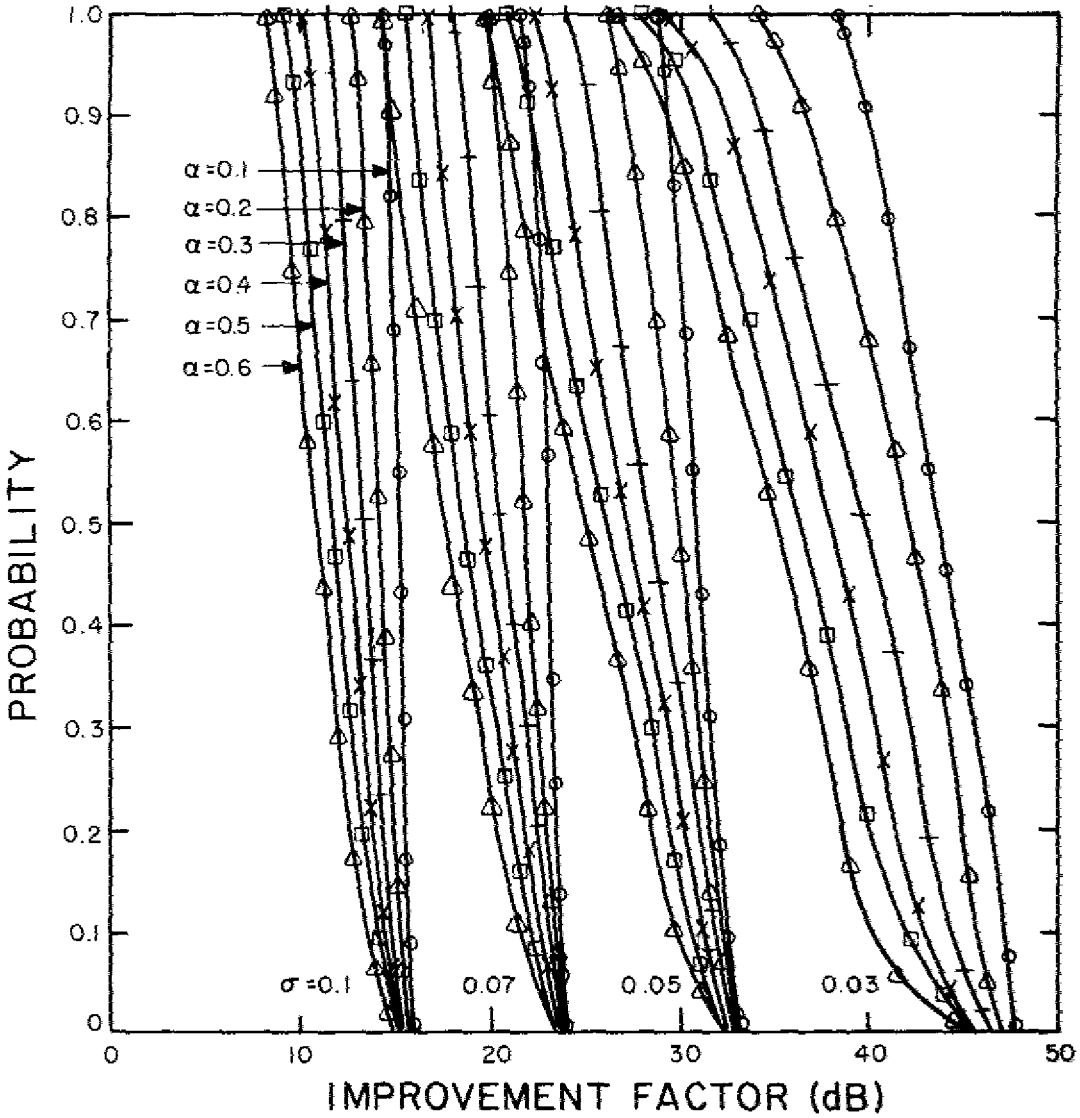

Fig. 6 — Distribution of improvement factor for a four-pulse staggered-PRF MTI using binomial filter weights

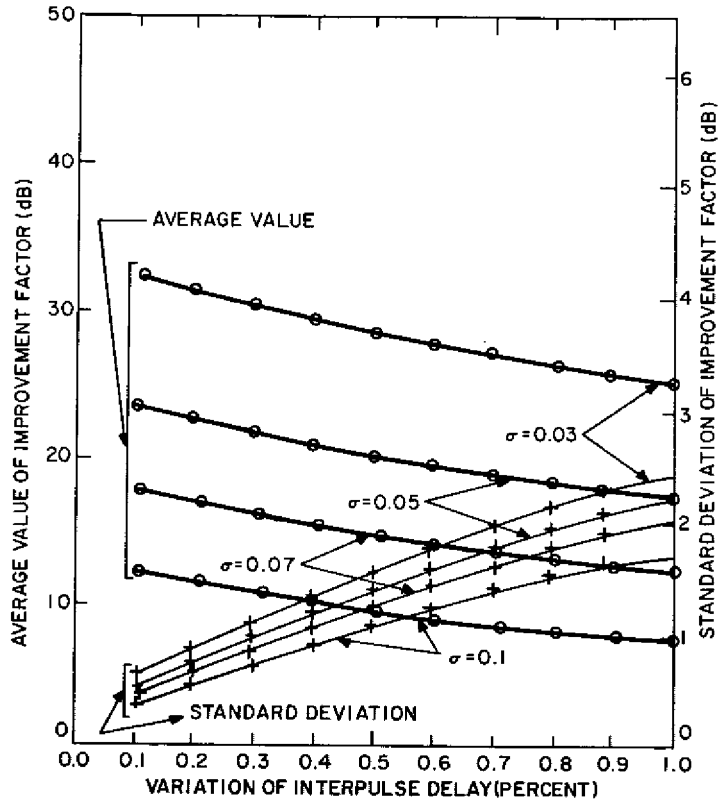

Fig. 7 — Statistic properties of the improvement factor of a three-pulse staggered-PRF MTI system using binomial filter weights

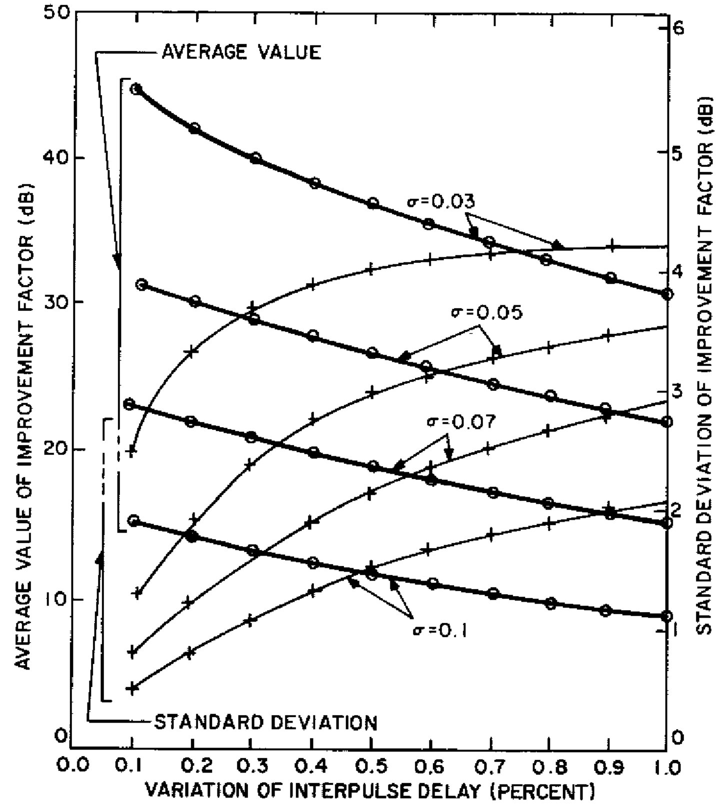

Fig. 8 — Statistic properties of the improvement factor of a four-pulse staggered-PRF MTI system using binomial filter weights