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Use of Attitude Sensors to Determine the Motion of a Free Rotator

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CONTENTS

ABSTRACT

From the free-body representation of Poinsot and expressions derived therefrom for the behavior of a body free from internal and external torques, the practical determination of the parameters of rocket or satellite motion has been considered. Transient perturbing torques can result in unique free motion patterns. A small number of measurements using rocket-mounted devices can determine free-body motion, with or without knowledge of the moments of inertia. These devices are magnetometers that measure the earth's field and telescopic instruments that view stars or the sun. Digital computer programs have proved useful for analyzing signals from rocket-borne attitude sensing devices. These include two programs which map stars or cities in local coordinates and which plot the sweep of a telescopic sensor across the bright object map and a program which uses magnetometer and star signals to locate the azimuth of a rocket free-body precession cone about the earth's magnetic field direction.

PROBLEM STATUS

This is an interim report on a continuing NRL problem.

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USE OF ATTITUDE SENSORS TO DETERMINE THE MOTION OF A FREE ROTATOR

MOTION OF A FREE ROTATOR

The interpretation of scientific measurements made from rocket and satellite vehicles requires knowledge of the direction that an instrument points at the time of the observation.

If the rocket or satellite body is rigid, and if the portion of the trajectory under consideration is not within a region of atmospheric drag or high gravity gradient, then the motion of the body may be considered free in that it is not subject to torques. For a free rigid body of known moments of inertia a small number of measurements of its rotational attitude at different positions in its pattern of rotational motion is sufficient to predict subsequent motion. Even without prior knowledge of the moments of inertia, the motion is still predictable if a modest number of additional attitude measurements are made.

The rotational motion of a free rigid body is uniquely determined by the values of the principal moments of inertia, the total angular momentum, and the rotational energy. Furthermore, as shown in Appendix A, all possible motions of the body can be generated by a simple mechanical model consisting of an ellipsoid which rolls without slipping on a fixed plane (Fig. 1). The center of the ellipsoid is fixed in inertial space at a distance from the plane determined by the ratio of the total energy to the total angular momentum. If the major axes of the ellipsoid are constructed inversely proportional to the square root of the respective principal moments of inertia of the body, the ellipsoid is identical to the ellipsoid of inertia of the body. If the directions of the principal axes of inertia for the body are maintained parallel to the respective major axes of the ellipsoid of inertia, the rolling motion of the ellipsoid on the fixed plane is identical to the real motion of the free body.

For motion which is free of external torques, the total angular momentum vector remains constant in time and is perpendicular to the plane on which rolls the ellipsoid. For a body in which two moments of inertia are equal, the motion consists of a uniform rotation about the odd-moment-of-inertia axis in combination with a uniform precession of that axis about the total angular momentum vector. When no two moments of inertia are equal, the rotational velocities are nonuniform with time.

The total instantaneous angular velocity of the body is the vector with orgin at the center of the ellipsoid and with its terminus at the point where the ellipsoid is tangent to the plane. The instantaneous angular velocity of the body about any direction is the projection of the total angular velocity in that direction. For bodies having an axis of symmetry, the projection of the total angular velocity on such an axis is the roll of the body about that axis.

To illustrate the usefulness of the mechanical model consider a free-falling rocket, a long thin body having an axis of symmetry running from the tail to the nose. The moment of inertia I_3 about this axis (the roll axis) is small; the moments I_1 and I_2 about the other two axes (tumble axes) are equal and large. The ellipsoid of inertia thus resembles the body itself, and its long axis is coincident with that of the rocket.

Figure 1 illustrates the rocket ellipsoid rolling on the fixed plane of tangency. The projection of the total angular velocity vector $\vec{\omega}$ on the roll axis is constant with time because of the roll-axis symmetry of the ellipsoid. Thus the precession of the axis about the total angular momentum vector $\vec{P}$ is uniform and in the same direction as the roll. The magnitude of the tumble angular velocity is also constant, and its direction also precesses uniformly about the total angular momentum. An interesting feature of this motion is that, although the roll component of the total angular velocity is constant, the projection of the tumble angular velocity on any axis fixed in the rocket varies sinusoidally with time.

As may be seen from Fig. 1, the ratio of the tumble angular momentum to the roll angular momentum is tan θ, where θ is the half-angle of the precession cone. The precession rate is identical to the tumble angular velocity only if the precession cone has a half angle of 90 degrees. Figure 1 shows the ratio of the tumble to precession rates as sin θ. It follows that the ratio of the precession to roll rates is

$$\frac{\omega_{\text{precession}}}{\omega_{\text{roll}}} = \frac{I_3}{I_1 \cos\theta}$$

Thus for a body having axially symmetric mass distribution the ratio of roll to precession rates is constant in time and is a single-valued function of the ratio of the moments of inertia and the precession cone angle.

If the free rotating body has the mass distribution of a wheel rather than a long thin body, the ellipsoid of inertia is fat rather than cigar shaped. If the inertial ellipsoid does not possess axial symmetry, but rather has three distinct principal axes of inertia, then the free motion of the body departs from the uniformity described above.

For minor departures of the inertial ellipsoid from axial symmetry and for small precession cone angles, the departure of the roll or precession rates from uniformity is small, and the roll-precession relationships for the symmetric body are good approximations for the unsymmetrical case. But for the unsymmetrical body, there is a critical half-cone angle, $\tan\theta_{\text{critical}} = [I_1(I_2 - I_3)/I_3(I_1 - I_2)]^{1/2}$, near which the nonuniformity of roll and precession becomes high (Appendix B). If this critical angle is exceeded, the roll reverses its direction periodically such that the roll position of the body about the roll axis oscillates through an angle which never exceeds 180 degrees. As the half-cone angle, which is no longer constant during precession, continues to increase toward 90 degrees, the roll decreases, the amplitude of the roll position oscillation decreases, and the magnitude and direction of the tumble angular velocity approach constant values (Appendix C). At a half-cone angle of 90 degrees, the roll is zero and the tumble is constant.

In summary, the motion of a free body can be described in terms of the degree of deviation from the uniform precession of a body rolling uniformly about a circular cone. If the region of the critical precession cone angle is avoided, the precession and roll rates approach uniformity.

EFFECT OF PERTURBING TORQUES

The rotational motion of a free body is the result of torques acting upon the body prior to its freedom from these torques. Although the history of torques is often unavailable, it is useful to know that certain general torque phenomena result in unique free motion patterns.

Consider a long slim rocket body having axial symmetry. The rocket is launched from the earth's surface in a nearly vertical direction through the atmosphere, with the atmosphere applying a torque to the rocket whenever the rocket axis deviates from the direction of the trajectory. Rockets are frequently given an initial roll so that small amounts of tumble angular momentum will result in precession cones having small cone angles, and so that the effects of minor axial assymetries in the rocket body or the propulsion system will be nullified by the roll. Thus a well behaved rocket emerges from the atmosphere with a small amount of tumble angular velocity compared with the roll. The motion is a circular precession cone of small angle, and the roll and precession are uniform. If the roll of the rocket is low, then small amounts of tumble angular velocity imparted by atmospheric torques or engine misalignment will cause a larger ratio of tumble to roll angular momentum, and the cone angle will be larger.

The attitude of the precession cone is partially determined by the attitude of the rocket as it leaves the atmosphere, which has heretofore kept the rocket axis aligned close to the rocket trajectory. Thus as the rocket exits from the atmosphere, one ray of the precession cone lies nearly along the direction of the trajectory. The azimuthal position of the cone axis in the plane perpendicular to the atmospheric trajectory is generally not predictable. However, once free of the atmosphere, the precession cone axis is fixed in inertial space.

The roll axis of a poorly behaved rocket wanders from its trajectory while within the atmosphere. A common type of aerodynamic instability allows the rocket axis to trace out a precession cone of increasing apex angle. The result is almost pure tumble in a plane perpendicular to the atmospheric trajectory. Such rocket behavior frequently produces a free-body precession cone angle close to the critical value.

For a free rigid body the pattern of the motion is invariant with time. If the body is not entirely rigid, the nature of the motion changes with time so that the direction of the shortest axis of the ellipsoid of inertia eventually becomes parallel to the total angular momentum vector. Thus the total angular momentum, which must remain constant, becomes simple rotation about the largest moment of inertia of the body, and the final configuration for the motion of a nonrigid slim body is pure tumble in the plane perpendicular to the total angular momentum vector, which itself is parallel to the axis of the initial precession cone. Energy in a non-rigid system is not conserved, and the friction which exists during the transition from the initial motion to the final configuration is dissipated as heat.

Likewise for a wheel-shaped nonrigid body, the direction of the largest moment of inertia is along the axis of symmetry of the wheel, and the precession cone angle approaches zero with time. All the initial tumble angular momentum is eventually transformed into roll.

MEASUREMENTS THAT DETERMINE FREE BODY MOTION

If a free body is equipped with devices to measure within the rocket coordinate system the directions of two objects at infinity in inertial space, then the attitude of the free body can be determined at all times. Either the earth's magnetic field or a distant object, such as the sun, the moon, or a star, serves to define a direction.

The simplest attitude-determining devices however do not provide continuous three coordinate information on the directions of sensed objects. Often such a device registers the time at which a bright object is viewed by a narrow-angle telescope mounted on the side of a rolling rocket. The amplitude of the star signal may indicate how close to the center of the field of view the bright object passes as the rocket rolls. The signals from such devices, when examined for recurrent patterns, provide accurate determination of roll rate.

Another useful attitude-sensing device is the fluxgate magnetometer, which provides a continuous monitor of the strength of the projection of the earth's magnetic field in a direction parallel to the sensitive axis of the device (Appendix D). Three mutually perpendicular magnetometers thus define the direction cosines of the magnetic field in the rocket coordinate system. The roll and precession periods of a free body are determined from the periods of the magnetometer signals. The amplitudes of the signals determine the instantaneous attitude of the rocket body with respect to one line in space, the magnetic field.

Consider the case of the free body that precesses and rolls uniformly. If its moments of inertia are known, then a measurement of the roll and precession rates determines the precession cone angle. A simultaneous measurement, with respect to the body coordinate system, of two known directions in space is sufficient to determine the attitude of the body. A second such measurement at another position on the precession cone determines the attitude of the cone itself.

Alternatively, if the moments of inertia are not known, the angle that the roll axis makes to the earth magnetic field can be determined with a magnetometer, and a measurement of this angle at two positions, when the body roll axis is closest to and farthest from being parallel to the field, determines the precession cone angle.

This measurement involves an ambiguity in that it is necessary to know whether the field lies outside or inside the precession cone. If the field lies outside, the total cone angle is the difference between the two angles mentioned above. Conversely, if the field lies within the cone, the total cone angle is the sum of the angles. If the field lies outside the precession cone, the time history of the signal from the magnetometer mounted parallel to the roll axis is indistinguishable from that for a cone wherein the field falls within the cone, provided that the maximum and minimum angles are the same for each case. However, the signal from the magnetometer mounted perpendicular to the roll axis is distinctly different in the two cases (Appendix D).

If the field lies outside the precession cone, and if the roll and precession are uniform, then when the roll axis is in the vicinity of closest approach to the field line, the period of the roll signal from the magnetometer mounted perpendicular to the roll axis is shorter than when the roll axis is farthest from the field. On the other hand, if the field is inside the cone, the period of this magnetometer signal lengthens as the roll axis precesses close to the field. If the free rotating body has one magnetometer mounted along its roll axis, and another mounted perpendicular thereto, the size of the precession cone and the aspect of any part of the body with respect to the cone axis can be determined. If a third magnetometer is fixed perpendicular to the other two, the relative phase of the signals from the two magnetometers that are perpendicular to the roll axis determines the direction of roll. An alternative method for roll direction determination is to use two magnetometers, one almost parallel to the roll axis and one perpendicular to the roll axis. The roll axis magnetometer is tilted slightly away from the roll axis in a plane whose normal is parallel to the other magnetometer.

In summary, magnetometer signals can be used to determine the attitude of a free rotating body both with respect to the axis of its precession cone and with respect to the magnetic field. The azimuth of the cone axis in a plane perpendicular to the magnetic field line is undetermined.

To determine this azimuth of the precession cone in inertial space, it is necessary only to define a time when an object at infinity (in a direction different from that of the magnetic field) has a given bearing or azimuth as measured about the roll axis in the rocket coordinate system. A narrow-angle telescope mounted perpendicular to the roll axis may see many stars in a single roll. One such star identification is sufficient to fix the azimuth location of the cone in inertial space.

Alternatively, a star telescope alone may be used to determine the motion of a free rotating body. The star pattern, fixed in the sky, offers many identifiable objects at infinity, each of which identifies a direction. Thus with only a general knowledge of the characteristic of the free body, an examination of the star signal from the telescope will result in many fixes. Because each star sight must fit into a relatively simple precession cone pattern, the determination of that pattern is simple, provided the cone angle is not in the vicinity of the critical angle (Appendix E). Even then, the identification of stars alone may be sufficient to solve a problem involving nonuniform roll and precession rates.

It is sometimes difficult to view stars during the daytime, and the sun and the magnetic field, when viewed at different times during the precession period, can determine completely the attitude of the body and its precession cone.

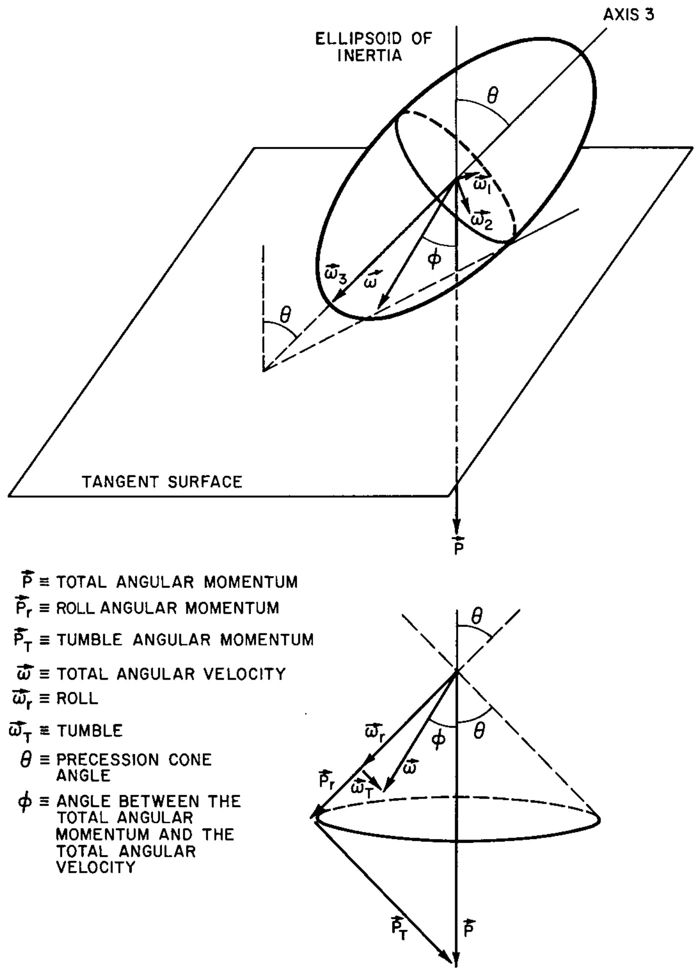

Fig. 1 - Mechanical model for the motion of a rocket and the corresponding vector diagram for the motion. The ellipsoid rolls on the plane without slipping.

An attitude device that is useful for this purpose is a telescope having an angle of view defined by two baffles. The entrance baffle is a long thin slit. The exit baffle is a small orifice in front of a light sensitive device. If the telescope is mounted on the free body with the normal to the baffle planes perpendicular to the roll axis and with the slit parallel to the roll axis, then sunlight will fall on the exit orifice each time the sun crosses the plane determined by the roll axis and the perpendicular to the baffles. Because the amount of light falling on the exit orifice is a function of the sine of the angle between the sun direction and the rocket axis, the amplitude of the signal from the light sensing device is a measure of this angle. Thus the attitude of the body with respect to the sun direction is determined once each roll period.

Another similar device uses two intersecting slits in the outer baffle with the intersection located on the line perpendicular to the baffles, which line passes through the exit orifice. During a single roll, sunlight falls twice on the exit orifice, once for each of the entrance slits. The amount of roll angle occurring between sun signals is a function of the angle between the sun direction and the roll axis. This two-slit system obviates the need for amplitude calibration.

Other solar attitude systems are considerably more elaborate. They have intricate baffles which provide unique digital sun signals for each degree or so of angle between the sun direction and the roll axis.

APPENDIX A

MECHANICAL MODEL FOR FREE BODY ROTATION (POINSOT'S REPRESENTATION*)

If the ellipsoid of inertia of a free body is constrained to rotate about its fixed center, and if the motion is further constrained such that the ellipsoid is tangent to a fixed plane with no slipping to occur between the ellipsoid and the plane, then the free body itself moves as if it were rigidly attached to its ellipsoid of inertia. The proof follows.

Let the invariance of the total energy of the free rotating body $T = \left(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2\right)/2$ define the ellipsoid of inertia having semimajor axes of

$$\omega_1 = \sqrt{2T/I_1}, \quad \omega_2 = \sqrt{2T/I_2}, \quad \omega_3 = \sqrt{2T/I_3},$$

where T is the total energy of rotation, the I's are the principal moments of inertia, and the ω's are the angular velocities about the principal axes. The total angular momentum (Fig. 1) is then

$$\vec{P} = I_1\vec{\omega}_1 + I_2\vec{\omega}_2 + I_3\vec{\omega}_3 \; .$$

Both $\vec{P}$ and $\vec{\omega} \cdot \vec{P} = 2T$ are invariant, so that $\vec{\omega} \cdot \vec{P}/|\vec{P}| = |\vec{\omega}| \cos \phi$ is also invariant and is the projection of $\vec{\omega}$ on $\vec{P}$.

Let us investigate the nature of the surface that is tangent to the ellipsoid at $\vec{\omega}$. The normal to this surface is also normal to the ellipsoid; the direction cosines of the normal to the ellipsoid $[T - (I_1\omega_1^2/2) - (I_2\omega_2^2/2) - (I_3\omega_3^2/2) = 0,\ F(\vec{\omega}) = 0]$ are $F'(\vec{\omega}_1)$, $F'(\omega_2)$ and $F'(\omega_3)$ or $I_1\omega_1$, $I_2\omega_2$, and $I_3\omega_3$. These components of the normal to the surface are identical to the components of $\vec{P}$, so the normal to the surface of tangency is constant in direction and the surface is a plane.

There can be no slipping between the ellipsoid and the plane because the point of tangency is on the instantaneous rotation axis of the body. The fixed distance of the ellipsoid center from the plane of tangency is $|\vec{\omega}| \cos \phi$.

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*Poinsot, "Theorie Nouvelle de la Rotation des Corps," Paris, 1834.

Appendix B

CRITICAL ANGLE FOR THE PRECESSION CONE OF A FREE ROTATING BODY

For a free rotating body having principal moments of inertia $I_1 > I_2 > I_3$ the distance d from the center of the ellipsoid to the plane of tangency (Poinsot representation, Appendix A) is $\vec{\omega} \cdot \vec{P}/|\vec{P}|$. From the geometry of the Poinsot representation it is apparent that $\sqrt{2T/I_1} < d < \sqrt{2T/I_3}$. If $d > \sqrt{2T/I_2}$, then the precession motion of the ellipsoid involves unidirectional rolling about axis 3. If $d < \sqrt{2T/I_2}$, the roll about this axis oscillates through a maximum angle of 180 degrees. The critical value of d that divides these two regions of behavior is $d = \sqrt{2T/I_2}$. We define the critical precession cone angle θ_c at $d = \sqrt{2T/I_1}$ and $\omega_2 = 0$.

Invariance of the total rotational energy and the total angular momentum gives $I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2 = 2T$ and $I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + I_3^2 \omega_3^2 = |\vec{P}|^2$. The relationship of $\vec{P}$ to T is easily determined for the special case where $d = \sqrt{2T/I_2}$, $\omega_2 \neq 0$, and where $\omega_1 = \omega_3 = 0$. In this case the equations above reduce to $I_2 \omega_2^2 = 2T$ and $I_2^2 \omega_2^2 = |\vec{P}|^2$. Therefore the relationship between $\vec{P}$ and T is $|\vec{P}|^2 = 2I_2T$. Using this relationship at θ_c, the energy and momentum equations reduce to

$$\frac{\omega_1^2}{\omega_3^2} = \frac{I_3 (I_2 - I_3)}{I_1 (I_1 - I_2)}. \tag{B1}$$

With the ellipsoid of inertia in a position such that $\omega_2 = 0$ (Fig. B1), the ellipsoid planar cross section which contacts the plane of tangency, and within which lies ω_3, is described by

$$I_1 \omega_1^2 + I_3 \omega_3^2 = 2T. \tag{B2}$$

From inspection of Fig. B1 it follows that

$$\tan \theta = -\frac{d\omega_3}{d\omega_1} \tag{B3}$$

Upon differentiating Eq. (B2) with respect to ω_1, and eliminating the derivative term from the result by use of Eq. (B3), θ can be expressed as a function of I and ω:

$$\tan \theta = \frac{I_1}{I_3} \left(\frac{\omega_1}{\omega_3}\right) \tag{B4}$$

Substitution of (B1) in (B4) eliminates the ω dependence, and

$$\theta_c = \arctan \sqrt{\frac{I_1 (I_2 - I_3)}{I_3 (I_1 - I_2)}}$$

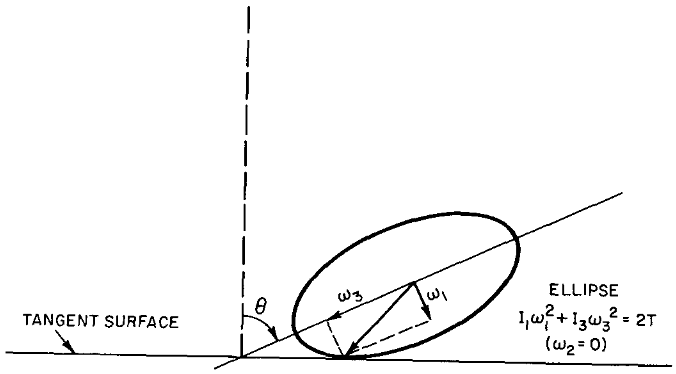

Fig. B1 - Notation defining θ

Appendix C

NONUNIFORM ROLL FOR A RIGID FREE ROTATING BODY

For a free body having moments of inertia $I_1 > I_2 > I_3$ it is useful to determine the amount of nonuniformity in roll about axis 3. Nonuniformity is defined as the ratio of ω_3 when $\omega_1 = 0$ to ω_3 when $\omega_2 = 0$.

From Eq. (B4) the tangent of the half angle of the precession cone is $\tan\theta = [(I_1/I_3)(\omega_1/\omega_3)]$ Because $\tan\theta$ is proportional to ω_1/ω_3, it is convenient to express it in terms of $(\omega_1/\omega_3)_{\theta_c}$. From Eq. (B1)

$$\left(\frac{\omega_1}{\omega_3}\right)^2_{\theta_c} = \frac{I_3\,(I_2 - I_3)}{I_1\,(I_1 - I_2)}.$$

If

$$\left(\frac{\omega_1}{\omega_3}\right)^2_{\theta} \equiv \alpha^2 \left(\frac{\omega_1}{\omega_3}\right)^2_{\theta_c} \equiv \alpha^2 B, \text{ where } \alpha = \frac{\tan\theta}{\tan\theta_c},$$

then at $\omega_2 = 0$ the invariance of the total energy

$$I_1\,\omega_1^2 + I_2\,\omega_2^2 + I_3\,\omega_3^2 = 2T$$

becomes

$$I_1\,\omega_3^2\,\alpha^2 B + I_3\,\omega_3^2 = 2T,$$

so that

$$(\omega_3)^2_{\omega_2 = 0} = \frac{2T}{I_3 + I_1\,\alpha^2 B} \equiv A. \tag{C1}$$

The invariance of the total angular momentum

$$I_1^2\,\omega_1^2 + I_2^2\,\omega_2^2 + I_3^2\,\omega_3^2 = P^2$$

becomes

$$P^2 = I_1^2\,A\alpha^2 B + I_3^2\,A. \tag{C2}$$

Let $\omega_1 = 0$. By eliminating ω_2 from the energy and momentum equations the expression for ω_3 at $\omega_1 = 0$ becomes

$$(\omega_3)^2_{\omega_1 = 0} = \frac{P^2 - 2T\,I_2}{I_3\,(I_3 - I_2)}. \tag{C3}$$

By using Eqs. (C1), (C2), and (C3), the expression for nonuniformity becomes

$$\frac{(\omega_3)^2_{\omega_1 = 0}}{(\omega_3)^2_{\omega_2 = 0}} = 1 - \alpha^2,$$

where α is the ratio of the tangent of the precession cone angle under consideration to the tangent of the critical cone angle.

Appendix D

SIGNAL FROM MAGNETIC-FIELD SENSORS AFFIXED TO A RIGID FREE ROTATING BODY

The characteristic motion for a free body having moments of inertia $I_1 = I_2 > I_3$ is uniform roll about axis 3, which itself precesses uniformly around a circular cone in the same direction as the roll.

Let a set of coordinate axes be affixed to the body with unit vector 3′ parallel to inertial axis 3 of the body. Let another set of axes be fixed in inertial space such that unit vector 3 of this fixed coordinate system is parallel to the axis of the precession cone and such that at $t = 0$ unit vector 1′ in the body-mounted coordinate system is in the plane defined by unit vectors 1 and 3 in the fixed coordinate system as shown in Fig. D1. Let the magnetic field unit vector lie in the same plane. As shown in Fig. D2, β is the half angle of the precession cone, γ is the angle between the magnetic field $\vec{F}$ and fixed unit vector 3, α is the angle that the body has precessed around the precession cone, and θ is the roll angle that axis 1′ has rotated about the body-oriented coordinate axis 3′ away from the plane determined by axes 3′ and 3.

The three components of the magnetic field unit vector in the fixed coordinate system are

$$\left.\begin{aligned} F_{11} &= \sin\gamma, \\ F_{12} &= 0, \\ F_{13} &= \cos\gamma. \end{aligned}\right\} \qquad \text{(D1)}$$

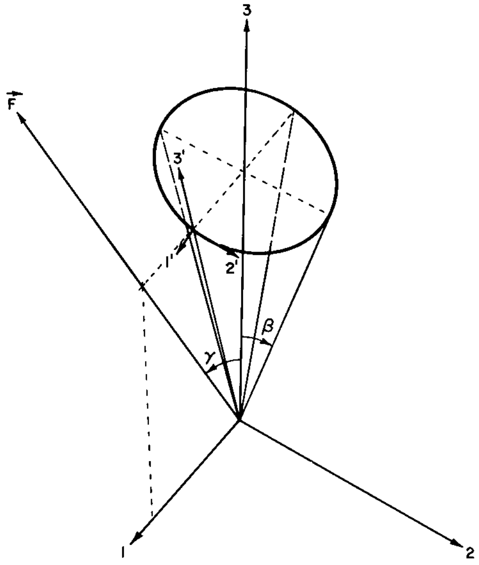

Fig. D1 - Coordinate systems for a rigid free rotating body in a magnetic field $\vec{F}$

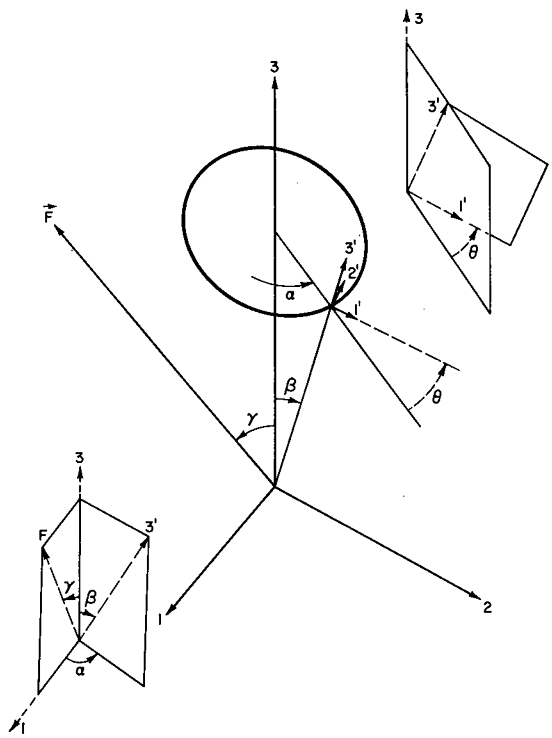

Fig. D2 - Additional notation for the motion of the body in Fig. D1

The values of the three components of this unit vector as expressed in the coordinate system affixed to the body will be identical to signals from magnetometers mounted parallel to the respective coordinate axes. To derive the components of the field line in the body-mounted coordinate system, it is convenient to define the field components successively in each of several coordinate systems; each coordinate system differs from its predecessor by a single rotation about one coordinate axis.

The field is defined in the fixed coordinate system, which shall be labeled system (1). Let us define the field in a new coordinate system (2) wherein coordinate axis 3 is parallel with that in system (1) but wherein axis 1 lies in the plane defined by body coordinate axis 3 and axis 3 of system (1). System (2) differs from system (1) by a rotation of angle α which has taken place about axis 3. The magnetic field in system (2) is

$$\begin{vmatrix} F_{21} \\ F_{22} \\ F_{23} \end{vmatrix} = \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} F_{11} \\ F_{12} \\ F_{13} \end{vmatrix}.$$

Let coordinate system (3) have its coordinate axis 3 parallel to axis 3 of the body-mounted system but with its axis 1 still in the plane defined by the axis 3 of system (1) and axis 3 of the body-mounted system. System (3) differs from system (2) by a single rotation through angle β about axis 2. The magnetic field in system (3) is

$$\begin{vmatrix} F_{31} \\ F_{32} \\ F_{33} \end{vmatrix} = \begin{vmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{vmatrix} \times \begin{vmatrix} F_{21} \\ F_{22} \\ F_{23} \end{vmatrix}$$

System (4) is the body-mounted system itself, and it differs from system (3) by a single rotation of angle θ about the axis 3$'$. The magnetic field in system (4) is

$$\begin{vmatrix} F_{41} \\ F_{42} \\ F_{43} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} F_{31} \\ F_{32} \\ F_{33} \end{vmatrix}$$

The result of these three matrix multiplications is the magnetic field direction expressed in the body-mounted coordinate system:

$$F_{41} = \cos\theta\cos\beta\cos\alpha\sin\gamma - \cos\theta\sin\beta\cos\gamma - \sin\theta\sin\alpha\sin\gamma,$$

$$F_{42} = -\sin\theta\cos\beta\cos\alpha\sin\gamma + \sin\theta\sin\beta\cos\gamma - \cos\theta\sin\alpha\sin\gamma,$$

$$F_{43} = \sin\beta\sin\gamma\cos\alpha + \cos\beta\cos\gamma.$$

Since β and γ are constant, then, as may be seen from inspection, the component of the magnetic field along axis 3$'$ is periodic with α, and the components along axes 1$'$ and 2$'$ are each periodic with α and θ.

It is interesting to note that for two distinctly different configurations of the precession cone and the field, the component of the magnetic field along axis 3$'$ is identical for all values of α. In one configuration the field line lies within the precession cone; in the other the field is outside. The relationship between these two components is the interchanging of the respective values of β and γ. Inspection of the expression for the field components along the body-mounted axes reveals that the component along axis 3$'$ is indeed independent of the order of appearance of β and γ; this is not true for the field components along the other axes.

Appendix E

COMPUTER PROGRAMS THAT AID IN DETERMINING ROCKET ATTITUDE

In conjunction with the matrix multiplication concept outlined in Appendix D three digital computer programs have proved useful in matching a signal from an attitude-sensing telescope on the rocket to a specific free body motion.

Program 1 produces a map of stars or cities plotted about the local zenith or nadir. Program 2 plots the path of the star- or city-sensing telescope as it sweeps through the sky or over the earth; it superimposes on this sweep identification marks at times when the telescope indicates a bright-object signal. Thus with proper manipulation of the parameters of the free body motion, the map of bright-object signals can be made to fit the map of stars or cities.

The input to Program 1 consists of a table of celestial coordinates of bright stars, a table of the latitudes and longitudes of big cities, and the latitude, longitude, right ascension, and altitude above the earth surface of the free rotating axially symmetric rocket.

The program plots the stars in spherical coordinates centered on the local zenith, plots the cities as seen from altitude in spherical coordinates centered about the local nadir, and plots a map of cities as seen from the center of the earth. The number one axis of the local coordinate system points north, the number two axis points west, and the number three axis points to the local zenith.

Program 3 is an aid to acquiring an initial estimate of the position of a precession cone. The input to Program 3, which can be derived from magnetometer signals alone, is a set of parameters which define the precession cone, its position in the local coordinate system, the roll and precession rates, and the initial angular position of the rocket body in roll and precession. More specifically the required input is the precession cone angle, the roll and precession periods, the initial roll angle with respect to the outward perpendicular to the surface of the precession cone, the initial precession angle measured around the cone from the point on the cone closest to the magnetic field line, and the times at which star signals are seen.

This program produces a table of spherical coordinates for the star signals. This table, when transferred to a sphere, is a map of the sky, even through its orientation has been determined only with respect to the magnetic field line and the cone center. Rotation of this sphere about the magnetic field will bring it into coincidence with the actual star field; thus the absolute orientation of the cone center is established. The cone position can now be fed into Programs 1 and 2 for high-accuracy matching.

The following FORTRAN programs were written for use on an NRL CDC-3100 digital computer. Expressions which may not be standard on other machines are listed below:

Expression	Definition
CALL PLOTF(x, y, 1)	an entry into a subroutine which plots a graph. This command sets the plotter sensitivity to x data units per inch on the x axis and y units per inch on the y axis.
CALL PLOTF(a, b, 2)	sets the origin of the coordinate system at x = a and y = b.
CALL PLOTF(x, y, 3)	plots data point x, y.
CALL PLOTF(x, y, 4)	plots a straight line to point x, y.
SIND(THETA)	sin θ, where θ is expressed in degrees.
COSD(THETA)	cos θ, where θ is expressed in degrees.

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      PROGRAM C3136
C     JUNE 12, 1970.  J. C. HOLMES
C
C     PLOTS OR OUTPUTS IN TABULAR FORM THE COORDINATES, IN A SPHERICAL
C     COORDINATE SYSTEM, OF THE DIRECTION OF A SENSOR ON THE SIDE OF A
C     ROCKET WHICH PRECESSES IN A REGULAR CYLINDRICAL CONE.
C     THE SENSOR VIEW DIRECTION IS PERPENDICULAR TO THE ROCKET AXIS.
C
C     CARD NUMBER 1 CARRIES FOUR ENTRIES.
C     CA=HALF CONE ANGLE.
C     CAZ=AZIMUTH OF THE CONE AXIS MEASURED CCW FROM THE NUMBER ONE
C       COORDINATE AXIS TOWARD COORDINATE AXIS NUMBER 2.
C     CEL=ELEVATION ANGLE OF THE CONE AXIS ABOVE THE PLANE CONTAINING
C       COORDINATE AXES 1 AND 2.
C     ANG=ANGULAR POSITION OF THE ROCKET AXIS MEASURED AROUND THE
C       PRECESSION CONE CCW FROM THE LINE ON THE CONE SURFACE WHICH IS
C       CLOSEST TO THE NUMBER 3 COORDINATE AXIS.
C
C     CARD NUMBER 2 CARRIES SIX ENTRIES.
C     RAO=INITIAL ROLL ANGLE OF THE ROCKET SENSOR MEASURED CCW ABOUT THE
C       ROCKET AXIS. ZERO ROLL ANGLE IS A LINE EXTENDING FROM THE
C       ROCKET AXIS TO THE CONE AXIS.
C     DELT=TIME INCREMENT BETWEEN SUCCESSIVE COMPUTATIONS OF SENSOR
C       POSITION.
C     TNOT=INITIAL TIME AT WHICH ANG AND RAO ARE DEFINED.
C     TROLL=ROLL PERIOD DEFINED AS THE TIME INTERVAL BETWEEN TWO
C       SUCCESSIVE PASSES OF THE SENSOR THROUGH THE ZERO ROLL POSITION.
C     TPREC=PRECESSION PERIOD DEFINED AS TIME OF ONE PRECESSION OF THE
C       ROCKET AXIS AROUND THE PRECESSION CONE.
C     TMAX=TIME AT WHICH PROGRAM STOPS.
C
C     THE NEXT SET OF FIVE CARDS CONTAINS THE TIMES, TI, AT WHICH OCCUR
C     A DEVIATION OF ROLL ANGLE FROM THAT WHICH IS COMPUTED ON THE BASIS
C     OF CONSTANT ROLL RATE.
C
C     THE NEXT SET OF FIVE CARDS CONTAINS THE VALUES OF ROLL ANGLE
C     DEVIATIONS, VI, WHICH CORRESPOND RESPECTIVELY TO THE TI ABOVE.
C
C     THE NEXT SET OF CARDS CONTAIN, ONE ENTRY PER CARD, THE SPECIFIC
C     TIMES (T) AT WHICH DATA IS TO BE OUTPUT OR PLOTTED.
C
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C     THE PROGRAM MODIFIES THE COMPUTED ROLL POSITION WITH A CORRECTION
C     WHICH IS A SEGMENTED STRAIGHT LINE CONNECTING THE SET OF VI.
C     NEXT A PAUSE OCCURS DURING WHICH TIME THE SENSE SWITCHES ARE SET.
C     SS1 ALLOWS THE RESULTS TO BE OUTPUT.
C     SS2 WHEN DEPRESSED PROVIDES OUTPUT DATA AT ONLY SPECIFIC TIMES.
C       IF NOT DEPRESSED, DATA OUTPUT OCCURS AT EVERY DELT.
C     SS3 ENABLES THE PLOTTER.
C     SS4, IF DEPRESSED, MAKES A PLOT CENTERED ABOUT THE NEGATIVE NUMBER
C       3 AXIS.  WHEN NOT DEPRESSED, THE PLOT IS CENTERED ABOUT THE
C       POSITIVE NUMBER 3 AXIS.
C
C     THE SENSITIVITY OF THE PLOTTER IS NEXT TYPED IN.
C     S IS MEASURED IN ANGULAR DEGREES/INCH.  THE NUMBER 3 AXIS IS IN
C     THE CENTER OF THE PLOT, SO THAT IF, FOR INSTANCE, S WAS SET EQUAL
C     TO 25.0, THE EDGE OF THE PLOT ON THE TEN INCH WIDE CALCOMP PLOTTER
C     WOULD BE 5X25.0 = 125 DEGREES ZENITH ANGLE.
C
C     NEXT IS TYPED IN A FIXED POINT SINGLE DIGIT, N.
C     1=TYPWRITER OUTPUT
C     4=PUNCHED CARD OUTPUT
C
C     THE PROGRAM WILL NOW OUTPUT TYPED OR PUNCHED TABULATIONS OF TIME,
C     AZIMUTH (CCW), AND THE ELEVATION OF THE SENSOR VIEW VECTOR.  IT
C     WILL DO THIS AT TNOT AND EVERY DELT THEREAFTER UP TO TMAX.  OR, IT
C     WILL OUTPUT THE DATA AT EVERY SPECIFIED TIME ON THE T CARDS.
C
C     THE PROGRAM WILL ALSO PLOT THE DATA CONNECTED BY STRAIGHT LINES
C     FOR EVERY DELT, OR IT WILL PLOT THE DATA AT EACH SPECIFIC TIME
C     AS AN X.  IF THE LAST T CARD HAS A ZERO ENTRY, THE PLOTTER WILL
C     AUTOMATICALLY RETURN TO THE CENTER OF THE PLOT.  THE PLOTTER
C     SHOULD ALWAYS BE MANUALLY CENTERED BEFORE STARTING THE PROGRAM.
C
      REAL PA
      DIMENSION A(36), UI(3), U(3), TI(30), VI(30)
  100 FORMAT(6F10.4)
  101 FORMAT(F10.2, 2F10.1)
  102 FORMAT(I1)
      READ(3,100) CA, CAZ, CEL, ANG
      READ(3,100) RAO, DELT, TNOT, TROLL, TPREC, TMAX
      READ(3,100) TI
      READ(3,100) VI
      PAUSE
      READ(1,100) S
C     TYPE IN S
      CALL PLOTF(S,S,1)
      CALL PLOTF(0., 0., 2)
      READ(1,102) N
C     TYPE IN N.
      WRITE(N,100) CA, CAZ, CEL, ANG
      WRITE(N,100) RAO, DELT, TNOT, TROLL, TPREC, TMAX
      A(1)=COSD(CA)
      A(2)=0.
      A(3)=SIND(CA)
      A(4)=0.
      A(5)=1.
      A(6)=0.
      A(7)=-SIND(CA)
      A(8)=0.
      A(9)=COSD(CA)
      A(19)=SIND(CEL)
      A(20)=0.
      A(21)=-COSD(CEL)
      A(22)=0.
      A(23)=1.
      A(24)=0.
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      A(25)=COSD(CEL)
      A(26)=0.
      A(27)=SIND(CEL)
      A(28)=-COSD(CAZ)
      A(29)=SIND(CAZ)
      A(30)=0.
      A(31)=-SIND(CAZ)
      A(32)=-COSD(CAZ)
      A(33)=0.
      A(34)=0.
      A(35)=0.
      A(36)=1.
C     THE A'S ARE THE MATRIX ELEMENTS FOR CONVERSION FROM THE ROCKET
C     SYSTEM TO THE EARTH SYSTEM.
      M=3
      T=TNOT
      J=1
    1 IF(T-TI(J+1)) 210, 220, 220
  220 J=J+1
      GO TO 1
  210 IF(T-TI(J)) 230, 240, 240
  230 J=J-1
      GO TO 210
  240 THC=VI(J)+(VI(J+1)-VI(J))*(T-TI(J))/(TI(J+1)-TI(J))
      RA=RAO+THC+(T-TNOT)*360./TROLL
C     RA=ROLL ANGLE.
      PA=ANG+(T-TNOT)*360./TPREC
C     PA=PRECESSION ANGLE
      UI(1)=COSD(RA)
      UI(2)=SIND(RA)
      UI(3)=0.
C     THE UI'S ARE THE DIRECTION COSINES OF THE VIEW AXIS OF THE ROCKET
C     ATTITUDE SENSOR.  THE ROCKET COORDINATE SYSTEM IN WHICH THE UI'S
C     ARE EXPRESSED HAS ITS NUMBER 3 COORDINATE AXIS PARALLEL TO THE
C     ROCKET AXIS OF ROTATION AND ITS NUMBER 1 AXIS PERPENDICULAR TO THE
C     OUTER SURFACE OF THE PRECESSION CONE.
      A(10)=COSD(PA)
      A(11)=-SIND(PA)
      A(12)=0.
      A(13)=SIND(PA)
      A(14)=COSD(PA)
      A(15)=0.
      A(16)=0.
      A(17)=0.
      A(18)=1.
      K=1
      L=36
      CALL MATRIX(UI, A, U, K, L)
C     SUBROUTINE MATRIX PERFORMS MATRIX MULTIPLICATIONS OF 3X3 MATRICES
C     ON THREE COMPONENT VECTOR UI.  THE NUMBER OF MATRIX MULTIPLICATIONS
C     PERFORMED IS (L-K+1)/9.  THE MATRIX ELEMENTS (A) ARE LISTED IN ORDER,
C     AND THE MATRIX MULTIPLICATIONS ARE PERFORMED SEQUENTIALLY USING
C     BLOCKS OF 9 MATRIX ELEMENTS FOR EACH MULTIPLICATION.  THE VECTOR
C     HAVING COMPONENTS UI IN THE INITIAL COORDINATE SYSTEM HAS COMPONENTS
C     U IN THE FINAL COORDINATE SYSTEM.
      TOT=SQRT(U(1)*U(1)+U(2)*U(2))
      ELEV=57.29578*ATANF(U(3)/TOT)
      IF(SENSE SWITCH 1) 70,71
   70 AZCCW=ATAN2PI(U(2),U(1))
C     SUBROUTINE ATAN2PI CONVERTS A Y AND AN X (U(2) AND U(1)) TO THE
C     APPROPRIATE ANGLE BETWEEN 0 AND 360 DEGREES.
      WRITE(N,101) T, AZCCW, ELEV
   71 IF(SENSE SWITCH 4) 73,72
   73 ELEV=-ELEV
   72 IF(90.-ELEV-5.*S) 61,61,60
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60 M=3
   GO TO 65
61 IF(SENSE SWITCH 3) 63,65
63 X=(90.-ELEV)*U(1)/TOT
   Y=(90.-ELEV)*U(2)/TOT
   IF(SENSE SWITCH 2) 80,81
81 CALL PLOTF(X,Y,M)
   M=4
   GO TO 10
80 CALL PLOTF(X+.05*S, Y, 3)
   CALL PLOTF(X-.05*S, Y, 4)
   CALL PLOTF(X+.05*S, Y, 4)
   CALL PLOTF(X, Y+.05*S, 3)
   CALL PLOTF(X, Y-.05*S, 4)
   CALL PLOTF(X, Y+.05*S, 4)
   GO TO 20
65 IF(SENSE SWITCH 2) 20,10
10 T=T+DELT
21 IF(T-TMAX) 1,1,2
20 READ(3,100) T
   IF(T) 2,2,21
 2 CALL PLOTF(0.,0.,3)
   END

   SUBROUTINE MATRIX(UI, A, U, K, L)
   DIMENSION UI(3), A(36), U(3), T(3)
   T(1)=UI(1)
   T(2)=UI(2)
   T(3)=UI(3)
   N=0
   K=K-3
11 DO 10 I=1,3
   U(I)=0.
   K=K+3
   DO 10 J=K,K+2
36 M=J-N
   IF(M-3) 10,10,34
34 N=N+3
   GO TO 36
10 U(I)=U(I)+A(J)*T(M)
   IF(L-2-K) 1,1,2
 2 T(1)=U(1)
   T(2)=U(2)
   T(3)=U(3)
   GO TO 11
 1 CONTINUE
   RETURN
   END

   PROGRAM CONPLOT
   JUNE 12, 1970   J. C. HOLMES
   PLOTS SPHERICAL COORDINATE GRAPH PAPER CENTERED ABOUT EITHER THE
   ZENITH OR NADIR.  PLOTS ON THIS GRAPH EITHER STARS OR CITIES.
   CITIES MAY BE PLOTTED EITHER IN COORDINATES CENTERED AT THE
   CENTER OF THE EARTH, OR THEY MAY BE PLOTTED IN A ROCKET CENTERED
   SYSTEM.  IN THIS LATTER CASE, THE NADIR IS THE CENTER OF THE PLOT.

   ON A ZENITH CENTERED COORDINATE SYSTEM PLOT, THIS PROGRAM
   WILL ALSO PLOT THE HORIZON WHICH IS OF COURSE A FUNCTION OF THE
   ALTITUDE OF THE ROCKET.

   THE SENSITIVITY S IS ENTERED FROM THE TYPWRITER.  S IS THE NUMBER
   OF ELEVATION DEGREES/INCH.

   SENSE SWITCHES ARE SET AS FOLLOWS.
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C
C     SS1 ENABLES THE GRAPH PAPER PLOT ROUTINE.  IF SS1 IS ACTIVATED,
C     THE NUMBER OF ELEVATION CIRCLES/INCH (AA) AND THE NUMBER OF RADIAL
C     LINES/CIRCLE (BB) ARE ENTERED VIA THE TYPWRITER.
C
C     SS2, IF ACTIVATED, PLOTS CITIES AS SEEN FROM ALTITUDE ON A
C     HEMISPHERE CENTERED ON THE NADIR.  IF SS2 IS NOT ACTIVATED, STARS
C     ARE PLOTTED IN SPHERICAL COORDINATES CENTERED ON THE ZENITH.
C     CITIES MAY BE PLOTTED ABOUT THE ZENITH IN THIS MODE.  IN THIS
C     CASE, CITIES APPEAR AS SEEN FROM THE CENTER OF THE EARTH.
C
C     SS3 SHOULD NOT BE ACTIVATED IF SS2 IS ACTIVATED.  SS3 ALLOWS FOR
C     THE PLOTTING OF AN HORIZON ON A PLOT CENTERED ABOUT THE ZENITH.
C
C     SS4, IF ACTIVATED, ALLOWS THE STAR OR CITY POSITIONS TO BE
C     PUNCHED ON CARDS.
C
C     SS5, IF ACTIVATED, ENABLES THE PLOTTER TO PLOT STARS AND CITIES.
C
C     FOR PLOTS OF CITIES FROM ALTITUDE, THE ALTITUDE (ALT) OF THE
C     ROCKET MUST BE ENTERED FROM THE TYPWRITER.
C
C     NEXT THE RA AND LAT OF THE LOCAL ZENITH IS ENTERED FROM A CARD
C     IF STARS ARE TO BE PLOTTED ABOUT THE LOCAL ZENITH.
C
C     IF CITIES ARE TO BE PLOTTED ABOUT THE LOCAL ZENITH, THE LONGITUDE
C     OF THE SUB ROCKET POINT IS ENTERED IN PLACE OF RA.
C
C     NEXT COMES A SET OF CARDS BEARING THE RA AND DEC OF STARS.
C     IF CITIES ARE TO BE PLOTTED INSTEAD, THE CARDS CONTAIN THE
C     EAST LONGITUDE AND LATITUDE OF THE CITIES.
C
C     NEITHER STARS OR CITIES ARE PLOTTED IF THEY FALL BEYOND THE EDGE
C     OF THE GRAPH PAPER.
C
      REAL LAT
      DIMENSION A(36), U(3), UI(3), I(15)
  100 FORMAT(2F10.4,15A4)
  101 FORMAT(2F9.2, 2X, 15A4)
      PAUSE
      READ(1,100) S
C     TYPE IN S.
      IF(SENSE SWITCH 1) 60,61
   60 READ(1,100) AA,BB
C     TYPE IN AA AND BB.
      CALL CPLOT(AA,BB)
C     CPLOT IS A SUBROUTINE FOR PLOTTING SPHERICAL COORDINATE GRAPH PAPER.
   61 CALL PLOTF(S, S, 1)
      CALL PLOTF(0., 0., 2)
      PAUSE
      IF(SENSE SWITCH 3) 25,24
   24 IF(SENSE SWITCH 2) 25,23
   25 READ(1,100) ALT
C     TYPE IN ALT.
      R=6400.
    ' RO=57.29578*ATANF(SQRT(2.*ALT*R+ALT*ALT)/R)
   23 IF(SENSE SWITCH 3) 35,7
   35 CALL PLOTF(RO,0.,3)
C     STATEMENTS 35 THROUGH 27 COMPRISE THE HORIZON PLOTTING ROUTINE.
      TH=0.
      DO 27 K=1,180
      TH=TH+6.2832/180.
      Y=RO*SIN(TH)
      X=RO*COS(TH)
   27 CALL PLOTF(X,Y,4)
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      PAUSE
      IF(SENSE SWITCH 2) 18, 7
   18 PAUSE
    7 READ(3,100) RASCEN, LAT
C     RASCEN AND LAT ARE THE RIGHT ASCENSION AND LATITUDE OF THE POINT
C     ON OR ABOVE THE EARTH THAT IS TO SERVE AS THE ZENITH OR NADIR OF A
C     STAR OR CITY PLOT.
    8 READ(3,100) THETA, FI, I
C     THETA, FI, AND I ARE RESPECTIVELY THE RIGHT ASCENSION, DECLINATION,
C     AND IDENTITY OF A STAR.  OR THEY ARE RESPECTIVELY THE EAST LONGITUDE,
C     LATITUDE, AND IDENTITY OF A CITY.
      IF(THETA)22,50,22
   22 UI(1)=COSD(FI)*COSD(THETA)
      UI(2)=COSD(FI)*SIND(THETA)
      UI(3)=SIND(FI)
C     THE UI'S ARE THE DIRECTION COSINES OF THE STARS OR CITIES IN THE
C     RASCEN-DEC OR THE LONGITUDE-LAT COORDINATE SYSTEMS.
      A(1)=-COSD(RASCEN)
      A(2)=-SIND(RASCEN)
      A(3)=0.
      A(4)=SIND(RASCEN)
      A(5)=-COSD(RASCEN)
      A(6)=0.
      A(7)=0.
      A(8)=0.
      A(9)=1.
      A(10)=SIND(LAT)
      A(11)=0.
      A(12)=COSD(LAT)
      A(13)=0.
      A(14)=1.
      A(15)=0.
      A(16)=-COSD(LAT)
      A(17)=0.
      A(18)=SIND(LAT)
C     THE A'S ARE THE MATRIX ELEMENTS USED FOR COORDINATE TRANSFORMATION
C     FROM THE RA-DEC SYSTEM OR THE LAT-LONG SYSTEM TO A NORTH-WEST-ZENITH
C     SYSTEM CENTERED AT A PARTICULAR POINT ON OR ABOVE THE EARTH.
   30 K=1
      L=18
   33 CALL MATRIX(UI, A, U, K, L)
C     SUBROUTINE MATRIX PERFORMS MATRIX MULTIPLICATIONS OF 3X3 MATRICES
C     ON THREE COMPONENT VECTOR UI.  THE NUMBER OF MATRIX MULTIPLICATIONS
C     PERFORMED IS (L-K+1)/9.  THE MATRIX ELEMENTS (A) ARE LISTED IN ORDER,
C     AND THE MATRIX MULTIPLICATIONS ARE PERFORMED SEQUENTIALLY USING
C     BLOCKS OF 9 MATRIX ELEMENTS FOR EACH MULTIPLICATION.  THE VECTOR
C     HAVING COMPONENTS UI IN THE INITIAL COORDINATE SYSTEM HAS COMPONENTS
C     U IN THE FINAL COORDINATE SYSTEM.
      AZ=ATAN2PI(U(2),U(1))
C     SUBROUTINE ATAN2PI CONVERTS A Y AND AN X (U(2) AND U(1)) TO THE
C     APPROPRIATE ANGLE BETWEEN 0 AND 360 DEGREES.
      M=1
  209 ELEV=57.29578*ATANF(U(3)/SQRT(U(1)*U(1)+U(2)*U(2)))
      GO TO (212,211),M
  212 B=ELEV
      IF(SENSE SWITCH 2) 200, 201
  201 IF(SENSE SWITCH 4) 206, 203
  206 WRITE(4,101) AZ, ELEV, I
      GO TO 203
  200 U(3)=U(3)-1.-ALT/6400.
      M=2
      GO TO 209
  211 IF(SENSE SWITCH 4) 207, 204
  207 WRITE(4,101) AZ, ELEV, I
  204 IF(90.-B-RO) 205, 8, 8
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  205 ELEV=-ELEV
  203 IF(90.-ELEV-5.*S) 1, 1, 8
    1 X=(90.-ELEV)*COSD(AZ)
      Y=(90.-ELEV)*SIND(AZ)
      IF(SENSE SWITCH 5) 90,8
   90 CALL PLOTF(X+.03*S,Y,3)
      DO 14 N=1,2
      CALL PLOTF(X,Y+.03*S,4)
      CALL PLOTF(X-.03*S,Y,4)
      CALL PLOTF(X,Y-.03*S,4)
   14 CALL PLOTF(X+.03*S,Y,4)
      GO TO 8
   50 CALL PLOTF(0., 0., 3)
      STOP
      END
C
      SUBROUTINE CPLOT(A,B)
      CALL PLOTF(1.,1.,1)
      CALL PLOTF(0.,0.,2)
      R=0.
      M=5.*A
      DO 70 I=1,M
      R=R+1./A
      TH=0.
      CALL PLOTF(R,0.,3)
      DO 60 J=1,180
      TH=TH+6.2832/180.
      X=R*COS(TH)
      Y=R*SIN(TH)
   60 CALL PLOTF(X,Y,4)
   70 CONTINUE
      TH=0.
      N=B
      DO 80 I=1,N
      TH=TH+6.2832/B
      X=R*COS(TH)
      Y=R*SIN(TH)
      IF(TH-6.284) 93,93,91
   93 CALL PLOTF(X,Y,3)
      CALL PLOTF(0.,0.,4)
      TH=TH+6.2832/B
      X=R*COS(TH)
      Y=R*SIN(TH)
      IF(TH-6.284)80,80,91
   80 CALL PLOTF(X,Y,4)
   91 CALL PLOTF(0.,0.,3)
      RETURN
      END
C
      SUBROUTINE MATRIX(UI,A,U,K,L)
      DIMENSION UI(3), A(36), U(3), T(3)
      T(1)=UI(1)
      T(2)=UI(2)
      T(3)=UI(3)
      N=0
      K=K-3
   11 DO 10 I=1,3
      U(I)=0.
      K=K+3
      DO 10 J=K,K+2
   36 M=J-N
      IF(M-3) 10,10,34
   34 N=N+3
      GO TO 36
```

```
10 U(I)=U(I)+A(J)*T(M)
   IF(L-2-K) 1,1,2
 2 T(1)=U(1)
   T(2)=U(2)
   T(3)=U(3)
   GO TO 11
 1 CONTINUE
   RETURN
   END

   FUNCTION ATAN2PI(R,S)
   IF(S) 3,4,5
 5 Q=ATANF(R/S)
   IF(Q) 9,8,8
 9 Q=Q+6.2832
   GO TO 8
 3 Q=ATANF(R/S)+3.1416
   GO TO 8
 4 IF(R) 6,6,7
 6 Q=4.7124
   GO TO 8
 7 Q=1.5708
   GO TO 8
 8 Q=Q*57.29578
   ATAN2PI=Q
   RETURN
   END

   PROGRAM STAR
C
C     PRODUCES A STAR MAP WHEN STAR SITE TIMES ARE ENTERED.  THE INPUT
C     DATA REQUIRED ARE ENTERED IN THE FOLLOWING ORDER.  FIRST ARE ENTERED
C     THE STAR SITE TIMES, ONE PER CARD WITH A F10.2 FORMAT.  AFTER THE
C     LAST STAR SITE CARD IS INSERTED  A BLANK CARD.   NEXT IS ENTERED
C     ON A SINGLE CARD, IN ORDER, THE FOLLOWING DATA, FORMAT(4F10.4).
C     CA = HALF ANGLE OF THE PRECESSION CONE.
C     BETA = ANGLE BETWEEN CONE AXIS AND MAGNETIC FIELD.
C     TROLL = ROLL PERIOD MEASURED WITH RESPECT TO CONE CENTER.
C     TPREC = PRECESSION PERIOD.
C     THE NEXT CARD HAS ENTRIES IN THE FOLLOWING ORDER, SAME FORMAT.
C     RAO = INITIAL ROLL ANGLE AT TNOT.  THIS ANGLE IS MEASURED CCW FROM THE
C     OUTWARD PERPENDICULAR TO THE CONE SURFACE.  THE ROLL DIRECTION IS
C     CCW LOOKING TOWARD THE NOSE OF THE ROCKET.
C     PAO = INITIAL PRECESSION ANGLE AROUND THE CONE FROM THE ZERO POSITION
C     WHICH IS THE POINT ON THE PRECESSION CONE NEAREST THE FIELD LINE.
C     THIS ANGLE IS POSITIVE IN THE CCW DIRECTION LOOKING DOWN INTO THE
C     CONE FROM THE NOSE OF THE ROCKET.
C     TNOT = INITIAL TIME AT WHICH THE ROLL AND PRECESSION ANGLES ARE EQUAL
C     TO RAO AND PAO RESPECTIVELY.
C
C     PROGRAM PRINTS OUT ON PUNCHED TAPE THE STAR SITE TIME, THE AZIMUTH
C     ANGLE OF THE STAR, AND THE DECLINATION OF THE STAR.   THE
C     COORDINATE SYSTEM IS RIGHT HANDED WITH THE AZIMUTH ANGLE MEASURED CCW
C     LOOKING DOWN AXIS 3 TOWARD THE ORIGIN.  THE NUMBER ONE AXIS POINTS AWAY
C     FROM THE CONE AXIS TOWARD THE FIELD LINE.  THE NUMBER ONE AXIS THUS
C     LIES IN THE PLANE OF THE CONE AXIS AND FIELD LINE.
C     THE NUMBER 3 AXIS LIES ALONG THE FIELD LINE.
C
C     THIS PROGRAM IS USEFUL FOR PLOTTING STARS WHEN THE IDENTIFICATION
C     OF THE STARS IS COMPLETELY UNKNOWN.  THE ONLY INPUT DATA NEEDED ARE THE
C     ANGLES MENTIONED ABOVE.  THUS A MAP OF BRIGHT OBJECTS IS PRODUCED WHICH
C     CAN BE ROTATED ABOUT THE MAGNETIC FIELD LINE TO MATCH THE BRIGHT
C     OBJECTS WITH ACTUAL STARS.
C
```

```
      DIMENSION A(27), UI(3), U(3), T(50)
  100 FORMAT(5F10.4)
  101 FORMAT(F10.2,2F10.1)
      N=0
    1 N=N+1
      READ(3,100) T(N)
      IF(T(N)) 1,2,1
    2 N=N-1
      M=N
      READ(3,100) CA, BETA, TROLL, TPREC
      READ(3,100)RAO, PAO, TNOT
      WRITE(4,100) CA, BETA, TROLL, TPREC
      WRITE(4,100) RAO, PAO, TNOT
      RAO=RAO/57.29578
      PAO=PAO/57.29578
      BETA=BETA/57.29578
      CA=CA/57.29578
      N=0
   10 N=N+1
      PA=PAO+(T(N)-TNOT)*6.2832/TPREC
      RA=RAO+(T(N)-TNOT)*6.2832/TROLL
      UI(1)=COS(RA)
      UI(2)=SIN(RA)
      UI(3)=0.
      A(1)=COS(KA)
      A(2)=0.
      A(3)=SIN(CA)
      A(4)=0.
      A(5)=1.
      A(6)=0.
      A(7)=-SIN(CA)
      A(8)=0.
      A(9)=COS(CA)
      A(10)=COS(PA)
      A(11)=-SIN(PA)
      A(12)=0.
      A(13)=SIN(PA)
      A(14)=COS(PA)
      A(15)=0.
      A(16)=0.
      A(17)=0.
      A(18)=1.
      A(19)=COS(BETA)
      A(20)=0.
      A(21)=-SIN(BETA)
      A(22)=0.
      A(23)=1.
      A(24)=0.
      A(25)=SIN(BETA)
      A(26)=0.
      A(27)=COS(BETA)
      K=1
      L=27
      CALL MATRIX (UI, A, U, K, L)
      THETA=ATAN2PI(U(2),U(1))
      DEC=57.29578*ATANF(U(3)/SQRT(U(2)*U(2)+U(1)*U(1)))
      WRITE(4,101) T(N), THETA, DEC
      IF(N-M) 10,11,11
   11 STOP
      END
C
      SUBROUTINE MATRIX(UI,A,U,K,L)
      DIMENSION UI(3), A(36), U(3), T(3)
      T(1)=UI(1)
      T(2)=UI(2)
```

```
      T(3)=UI(3)
      N=0
      K=K-3
   11 DO 10 I=1,3
      U(I)=0.
      K=K+3
      DO 10 J=K,K+2
   36 M=J-N
      IF(M-3) 10,10,34
   34 N=N+3
      GO TO 36
   10 U(I)=U(I)+A(J)*T(M)
      IF(L-2-K) 1,1,2
    2 T(1)=U(1)
      T(2)=U(2)
      T(3)=U(3)
      GO TO 11
    1 CONTINUE
      RETURN
      END
C
      FUNCTION ATAN2PI(R,S)
      IF(S) 3,4,5
    5 Q=ATANF(R/S)
      IF(Q) 9,8,8
    9 Q=Q+6.2832
      GO TO 8
    3 Q=ATANF(R/S)+3.1416
      GO TO 8
    4 IF(R) 6,6,7
    6 Q=4.7124
      GO TO 8
    7 Q=1.5708
      GO TO 8
    8 Q=Q*57.29578
      ATAN2PI=Q
      RETURN
      END
```

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13. ABSTRACT

From the free-body representation of Poinsot and expressions derived therefrom for the behavior of a body free from internal and external torques, the practical determination of the parameters of rocket or satellite motion has been considered. Transient perturbing torques can result in unique free motion patterns. A small number of measurements using rocket-mounted devices can determine free-body motion, with or without knowledge of the moments of inertia. These devices are magnetometers that measure the earth's field and telescopic instruments that view stars or the sun. Digital computer programs have proved useful for analyzing signals from rocket-borne attitude sensing devices. These include two programs which map stars or cities in local coordinates and which plot the sweep of a telescopic sensor across the bright object map and a program which uses magnetometer and star signals to locate the azimuth of a rocket free-body precession cone about the earth's magnetic field direction.

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