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## ABSTRACT

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## AUTHORIZATION


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# DOPPLER SHIFT CONSIDERATIONS IN NAVAL SATELLITE COMMUNICATION SYSTEMS EMPLOYING SYNCHRONOUS SATELLITES

## INTRODUCTION

In any satellite communication system, doppler shift will be introduced by the motion of the satellite repeater relative to a fixed point on the Earth. In the case of Naval satellite communication, the radial velocity due to the motion of the satellite is augmented by the line-of-sight components of ship headway and other ships motion (principally roll and pitch). The frequency shift due to the motion of a "stationary" satellite in an equatorial orbit can amount to approximately 825 Hz at 8 GHz when the orbit has an eccentricity of 0.01. This shift can be more than doubled when the effects of total ship motion and orbital inclination add to the relative spacecraft motion.

Modems being presently considered for satellite communication systems include pseudo-noise (PN) systems, earlier versions of which are the AN/URC-61 and the AN/URC-55, and multiple frequency shift keying modems (frequency hop) such as Tactical Transmission System (TATS). At low data rates, such as 75 bits/second, the acquisition time for PN systems may become quite long unless compensation is made for doppler shift. Similarly, a frequency hop system such as TATS requires doppler compensation to aid in signal tracking under conditions of platform acceleration. It is very advantageous to use doppler compensation for both types of modems to reduce acquisition time and doppler tracking problems.

Doppler correction requires provision for shifting the receiving system local oscillators and the transmitter oscillator in opposite directions by amounts proportional to the respective frequencies. While the residual error resulting from the simple implementations of doppler correction will be small, this error is of critical importance to the design of shipboard terminals.

In this report the values of the "range rate" or doppler shift that can occur for typical satellite and ship parameters are estimated and some techniques for automatically deriving the necessary corrections to the oscillator chains are examined.

DOPPLER SHIFT DUE TO MOTION OF STATIONARY SATELLITES

The orbits of so-called stationary communications satellites are ellipses of small eccentricity lying in planes slightly inclined to the equatorial plane with orbital periods equal to the Earth's rotational period. If the eccentricity and inclination are not identically zero, the satellite does not appear at a stationary point in the sky; it appears to oscillate back and forth, and the orbital velocity increases and decreases with respect to the average as the satellite moves from perigee to apogee. This movement through perigee to apogee while moving north and south of the equator causes the satellite suborbital point to trace a figure "8" on the Earth's surface. Although the attendant variations in antenna pointing angles are recognized, the accompanying doppler shifts due to changes in the range from the Earth terminal to the satellite have sometimes been overlooked. The computed doppler shifts will be effected by orbital perturbations due to the Sun and Moon and to asphericity of the Earth's gravitational field. But, since the variation of the shift introduced by these effects is much smaller than the shift introduced by the orbit's eccentricity and inclination, they will be neglected.

The geocentric motion of the satellite in a simple elliptical orbit is described by the six orbital elements which when combined with the three coordinates of Earth terminal give nine independent parameters to specify the instantaneous value of the radial velocity, $v_r$, relative to the Earth terminal. A determination of the combination of parameters that yields the maximum value of $v_r$ is awkward to obtain explicitly so that a somewhat heuristic analysis will be applied to estimate the maximum value of $v_r$ to be used for terminal design.

The method consists of treating an orbit of zero inclination and eccentricity $\epsilon$ to obtain $v_r(i=o)$ and an orbit of zero eccentricity and inclination i to obtain $v_r(\epsilon=o)$ and assuming that the many parameters provide enough freedom that at some time the maximum values of these two components will occur simultaneously. Anticipating this method, it is possible to simplify the general problem by setting the longitude of the ascending node to $90^\circ$ and the argument of the perigee ($\Omega$) to $-90^\circ$. The resulting geometry and the various components are shown in Fig. 1. This fixing of the argument of perigee is not an additional restriction in this approach. When determining the doppler shift due to eccentricity, the assumed orbit is equatorial; so the variation of station location will cover all situations. When determining the doppler shift due to inclination, the orbit is assumed circular, so any point could be considered perigee.

The position vector $\underline{R}$ of the satellite from Fig. 1 is

$$\underline{R} = r\,\{\cos\psi\,[\underline{x}\cos i - \underline{z}\sin i] + \underline{y}\sin\psi\}, \tag{1}$$

and the vector velocity* of the satellite is

$$\dot{\underline{R}} = \mu\,\frac{\underline{\underline{q}}}{q}\;X\;(\underline{r} + \epsilon\,\underline{p}); \tag{2}$$

where

$$r = a\,(1-\epsilon^2)/[1+\epsilon\cos\psi], \tag{3}$$

$$q = \sqrt{\mu\,a\,(1-\epsilon^2)}\;, \tag{4}$$

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* Equations (2), (3), and (4) correspond to Eqs. (A-8), (A-12) and (A-11) respectively derived in Appendix A.

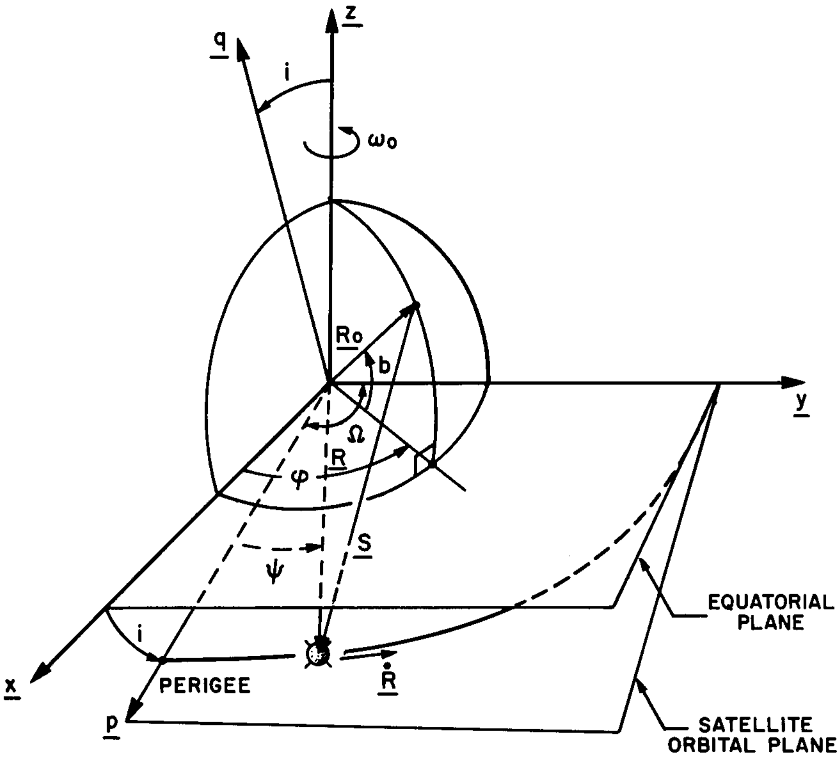


Fig. 1 - Satellite-Earth geometry

and

| | |
|---|---|
| $a$ | is the semi-major axis of the ellipse, |
| $\psi$ | is the true anomaly (angular distance in the orbital plane between the satellite position and the perigee), |
| $i$ | is the inclination of the satellite orbital plane to the equatorial plane, |
| $\epsilon$ | is the satellite orbital eccentricity, |
| $\mu$ | is the gravitational constant of Earth ($= 5.165 \times 10^{12}$ km$^3$ hr$^{-2}$), |
| $\underline{q}$ | is a unit vector perpendicular to the satellite orbital plane, |
| $q$ | is the magnitude of the vector $\underline{Q}$ defined in Appendix A, |
| $r$ | is the magnitude of $\underline{R}$, |
| $\underline{r}$ | is the unit vector in the direction of $\underline{R}$, |
| $\underline{x},\underline{y},\underline{z}$ | are unit vectors of the rectangular geocentric coordinate system, and |
| $\underline{p}$ | is the unit vector lying in the orbital plane directed from the center of the Earth toward the perigee. |

The period of the orbit (T) from Kepler's third law of planetary motion is

$$T = 2\pi \left(a^3/\mu\right)^{\frac{1}{2}}$$

For a synchronous satellite, $T = 2\pi/\omega_o$, where $\omega_o$ is the angular velocity of the Earth or about $2\pi/23.934$ radians per hour. Hence, for a synchronous orbit

$$a = \left(\mu/\omega_o^{\,2}\right)^{\frac{1}{3}} \approx 4.22 \times 10^4 \text{ km}$$

Expansion of Eq. (2) using Eqs. (1) and (4) gives

$$\dot{R} = \eta \left\{\sin\psi \left[-\underline{x}\cos i + \underline{z}\sin i\right] + \underline{y}\left[\cos\psi + \epsilon\right]\right\}$$

where

$$\eta = \frac{u}{q} = \left[\mu/\left\{a\left(1-\epsilon^2\right)\right\}\right]^{\frac{1}{2}} .$$

By inspection (Fig. 1) the geocentric position vector of the Earth terminal is

$$\underline{R}_o = a_e \{\cos b\, [\underline{x} \cos \varphi + \underline{y} \sin \varphi] + \underline{z} \sin b\}$$

where

$a_e$ = equatorial radius of Earth = 6 378 km,

$b$ = terminal latitude, and

$\varphi$ = terminal longitude.

The terminal longitude, $\varphi$, is actually a pseudo-longitude, measured from the stationary (x,y,z) coordinates not from a fixed point on the Earth. The rotation of the Earth causes $\varphi$ to increase linearly with time, so that $\varphi = \omega_o t$. Thus,

$$\dot{\underline{R}}_o = a_e \omega_o \cos b\, [-\underline{x} \sin \varphi + \underline{y} \cos \varphi] \ .$$

The topocentric position vector is

$$\begin{aligned} \underline{S} = \underline{R} - \underline{R}_o = \underline{x}\, &[r \cos \psi \cos i - a_e \cos b \cos \varphi] \\ + \underline{y}\, &[r \sin \psi - a_e \cos b \sin \varphi] \\ - \underline{z}\, &[r \cos \psi \sin i + a_e \sin b], \end{aligned}$$

and the magnitude, s, is given by

$$\begin{aligned} |S|^2 = s^2 = r^2 + a_e^2 - 2r\, a_e \{&\cos b\, [\cos \varphi \cos \psi \cos i \\ &+ \sin \psi \sin \varphi] - \cos \psi \sin i \sin b\} \end{aligned} \tag{5}$$

The velocity of the satellite relative to the observer is

$$\begin{aligned} \dot{\underline{S}} &= \dot{\underline{R}} - \dot{\underline{R}}_o \\ &= \underline{x}\, \{-\eta \sin \psi \cos i + a_e \omega_o \cos b \sin \varphi\} \\ &+ \underline{y}\, \{\eta\, [\cos \psi + \epsilon] - a_e \omega_o \cos b \cos \varphi\} \\ &+ \underline{z}\, \eta \sin \psi \sin i. \end{aligned}$$

The desired radial velocity, $v_r$, is the component of $\dot{\underline{S}}$ along the line of sight from the terminal to the satellite

$$v_r = \dot{\underline{S}} \cdot \underline{S}/s$$

where

$$\dot{\underline{S}} \cdot \underline{S} = -\eta r \sin\psi \cos\psi$$
$$+ a_e \eta \sin\psi [\cos b \cos\varphi \cos i - \sin b \sin i]$$
$$+ a_e \omega_o r \cos b [\cos i \cos\psi \sin\varphi - \cos\varphi \sin\psi]$$
$$+ \eta [\cos\psi + \epsilon][r \sin\psi - a_e \cos b \sin\varphi] \; . \qquad (6)$$

DOPPLER SHIFT DUE TO ECCENTRICITY

Consider the case in which the orbital plane coincides with the equatorial plane and the eccentricity is non-zero $v_r(i=o) = \dot{\underline{S}} \cdot \underline{S}/s \Big|_{i=o}$, where from Eq. (6),

$$\dot{\underline{S}} \cdot \underline{S}\Big|_{i=o} = \epsilon \eta r \sin\psi + a_e \cos b [(r \omega_o - \eta) \sin(\varphi - \psi) - \eta \epsilon \sin\varphi]$$

and from Eq. (5):

$$s\Big|_{i=o} = [r^2 + a_e^2 - 2r a_e \cos b \cos(\varphi - \psi)]^{\frac{1}{2}} \; .$$

Doppler shift in Hz/GHz is equal to 0.926 $v_r$ where the range rate is in km/hr. The resulting doppler shift for an orbit with eccentricity of 0.01 is presented in Fig. 2 as a function of the true anomaly (and hence time) for various combinations of station latitude b and relative longitude c, where c = $(\varphi - \psi)$ for an equatorial orbit. The maximum value of $v_r(i=o)$ for this case is about 112 Hz/GHz occurring near $\psi = \pi/2$ and $3\pi/2$. For the stationary satellite $\psi$ changes by $2\pi$ in a time period of slightly less than one day. As shown in Fig. 2, the dependence of the coordinates of the station is a second order effect influencing the amplitude and phase of the maximum only slightly.

Since this is the case an approximate equation for the range rate can be obtained by setting b=o and $(\varphi - \psi) = 0$ making

$$v_r(i=o) \approx \frac{\epsilon \eta r \sin\psi}{(r^2 + a_e^2 - 2 r a_e)^{\frac{1}{2}}}$$

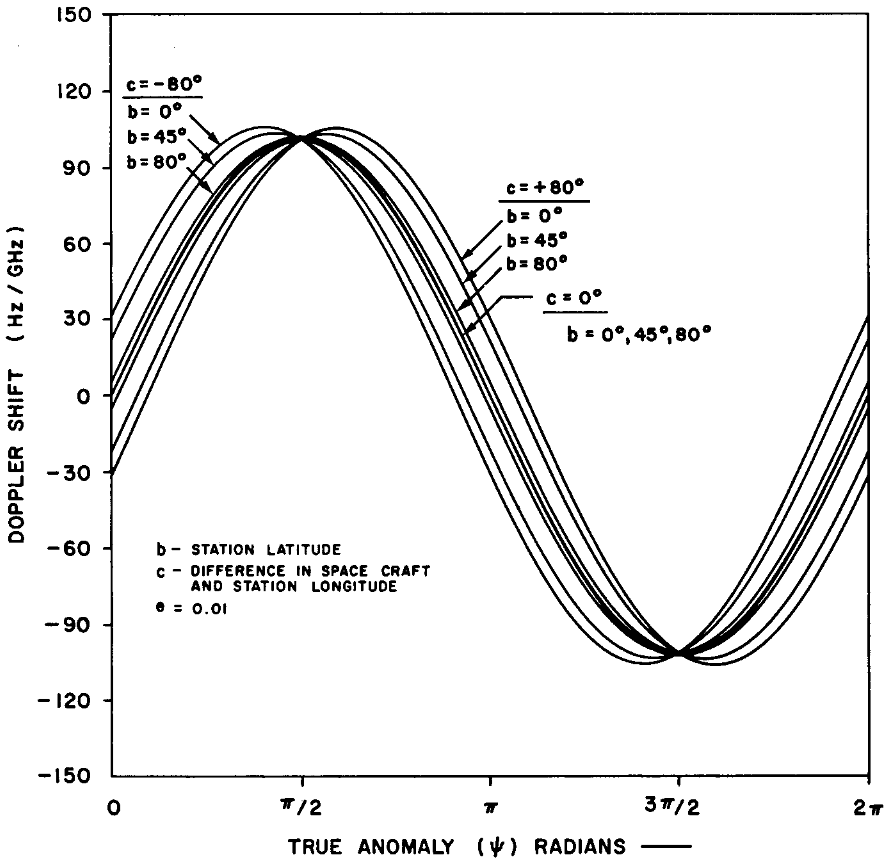


Fig. 2 - Doppler shift versus true anomaly for various terminal coordinates

which for the synchronous orbit where $r \approx 6.3\ a_e$ becomes

$$v_r\,(i=o) \approx \epsilon\ \eta \sin \psi$$

which has a maximum value

$$v_r\,(i=o)_{max} \approx \epsilon\ \eta\ . \qquad (7)$$

If $\epsilon$ is not too large (i.e., $\epsilon \leq 0.01$)

$$v_r\,(i=o) \approx 11000\ \epsilon \sin \psi \text{ km/hr}$$

and the corresponding maximum doppler shift from this approximation is found to be

$$\approx 10300\ \epsilon\ (\text{Hz/GHz})\ .$$

The exact computation of Fig. 2 for an eccentricity of 0.01 has a maximum doppler shift of about 112 Hz. The approximate equations is modified slightly to correct for this under estimation and the following will be used for the rest of the discussion:

$$\text{DOPPLER SHIFT}\Big|_{\substack{max \\ i=o}} \approx 11200\ \epsilon\ (\text{Hz/GHz})$$

and

$$v_r\,(i=o) \approx 12100\ \epsilon \sin \psi \text{ km/hr}\ .$$

In order to determine the largest eccentricity that might occur for a synchronous orbit, the orbital elements of the Defense Satellite Communication System (DSCS) Phase I satellites and the synchronous satellites were examined. A few Phase I satellites had an eccentricity of 0.008. Thus, a value of 0.01 was chosen as the upper limit in eccentricity likely to be encountered in future geostationary orbits. Such an upper limit must be used for terminal design if sufficient tolerance is to be permitted for future launches of stationary satellites. For other values of $\epsilon$ the approximately linear dependence of $v_r$ on $\epsilon$ can be used to estimate $v_r$ for other orbits. Table I gives typical eccentricities for existing satellites and the approximate maximum range rates in Hz/GHz of carrier frequency for stationary orbit satellites with these eccentricities.

Table 1

Typical Eccentricities and Corresponding Maximum Doppler Shifts for Satellite of Zero Inclination

| Satellite | $\epsilon$ | Maximum Doppler (Hz/GHz) |
|---|---|---|
| SYNCOM III | 0.0003 | 3.1 |
| TACSAT I | 0.0015 | 15.4 |
| INTELSAT (F3) | 0.0004 | 4.1 |
| INTELSAT (F2) | 0.0004 | 4.1 |
| ATS-E | 0.0004 | 4.1 |
| INTELSAT-2 | 0.0003 | 3.1 |
| Worst case | 0.01 | 112.0 |

## DOPPLER SHIFT OF A STATIONARY SATELLITE WITH CIRCULAR INCLINED ORBIT

To estimate the effect of orbital inclination, the general equations (6) and (5) for $\dot{\underline{S}} \cdot \underline{S}$ and s can be simplified by setting $\epsilon = 0$; that is, assume the case of a circular orbit. In this case, of course, $\psi$ cannot be defined relative to the perigee since a circular orbit has none. The angle retains its geometrical significance and may be interpreted for definiteness as the complement of the geocentric angle between the satellite and the ascending node. For a circular orbit

$$\eta = \{\mu/a\}^{\frac{1}{2}} \text{ and } r = a, \text{ so that } \omega_o\, r = \eta \text{ and } v_r(\epsilon = o) = \dot{\underline{S}} \cdot \underline{S}/s \Big|_{\epsilon = o}$$

where

$$\dot{\underline{S}} \cdot \underline{S}\Big|_{\epsilon = o} = a_e\, \eta \{\cos b\, [\cos i - 1] \sin (\varphi + \psi) - \sin b \sin \psi \sin i\}$$

$$s\Big|_{\epsilon = o} = a_e \left(\frac{a^2}{a_e^{\,2}} + 1 - \frac{2a}{a_e} \{\cos b\, [\cos \varphi \cos \psi \cos i + \sin \psi \sin \varphi] - \cos \psi \sin i \sin b\}\right)^{\frac{1}{2}} .$$

For small i

$$\dot{\underline{S}} \cdot \underline{S}\Big|_{\epsilon = o} \approx -a_e\, \eta\, i \sin b \sin \psi$$

and

$$s\Big|_{\epsilon = o} \approx a_e \left\{\frac{a^2}{a_e^{\,2}} + 1 - \frac{2a}{a_e} [\cos b \cos (\varphi - \psi)]\right\}^{\frac{1}{2}} .$$

$v_r(\epsilon = o)$ is slightly over-estimated by the approximation

$$v_r(\epsilon = o) \approx - \eta \; i \sin \psi \sin b / \left( \frac{a}{a_e} - 1 \right) \tag{8}$$

The satellite is visible to the surface terminal for values of longitude (b) of up to about $80^o$. The maximum doppler shift will occur when $b = 80^o$ and $\psi = 90^o$

$$v_r(\epsilon = o) \Big|_{MAX} \approx 0.985 \; \eta \; i / \left( \frac{a}{a_e} - 1 \right)$$

$$\approx 2060 \; i \; km/hr$$

with the corresponding doppler shift of 1910 i Hz/GHz. The inclination i is in radians. For $i = 3^o$, the maximum value for any existing or planned satellites,

$$\left| v_r \right|_{max} \approx 108 \; km/hr = 100 \; Hz/GHz \; .$$

The maximum probable doppler shift due to the motion of a "stationary" satellite with an orbital inclination of $3^o$ is then the sum of the shift from Table I and 100 Hz/GHz.

DOPPLER SHIFT DUE TO SHIPS MOTION

The pitch, roll, and other ship motions even for much less than maximum sea states contribute velocity components along the line of sight to the satellite which are comparable to those introduced by the ship headway. The pitch and roll motion can be approximated by a sinusoid of the form

$$\theta = \Theta \sin (2\pi \; t/\tau) \tag{9}$$

or

$$\theta = \Theta \sin \omega \; t$$

where

$\theta$ is the angular displacement from the vertical,

$\Theta$ is the maximum excursion,

$\tau$ is the period,

$\omega$ is the angular frequency, and

t is time in seconds.

A table of "typical" values of "amplitudes" and periods for pitch and roll are given in reference 1. Although "typical" is left undefined with respect to sea state and the use of the terms "maximum excursion", "double amplitude", and "amplitude" is not clear, it appears from other data that the amplitude is $2\,\Theta_{roll}$ or $2\,\Theta_{pitch}$ and that "typical" means values not at all unusual. An abbreviated form of the table is reproduced as Table 2.

Table 2

Roll and Pitch Parameters

| Ship Type | Period of Roll (sec) $\tau_{roll}$ | Double Amplitude of Roll (degrees) $2\,\Theta_{roll}$ | Period of Pitch (sec) $\tau_{pitch}$ | Double Amplitude of Pitch (degrees) $2\,\Theta_{pitch}$ |
|---|---|---|---|---|
| Destroyer | 9.5 | 25 | 5 | 5 |
| Destroyer Escort | 8.0 | 25 | 5 | 5 |
| Light Cruiser | 12.0 | 20 | 7 | 4 |
| Heavy Cruiser | 12.0 | 20 | 7 | 4 |
| Carrier | 16.0 | 15 | 7 | 4 |

The greatest contribution of the velocity due to roll and pitch motion to the line of sight velocity occurs when the satellite is on the horizon. The geometry of the problem in this case is given in Fig. 3. By inspection, the displacement d due to ship motion is given by

$$\begin{aligned} d &= h \sin \theta \\ &= h \sin [\Theta \sin \omega t] \end{aligned}$$

where h is the height of the antenna above the center of motion.

The velocity is

$$\dot{d} = h\,\omega\,\Theta \cos \theta \cos \omega t$$

or using Eq. (9)

$$\dot{d} = h\,\omega\,\Theta \left\{1 - \left(\frac{\theta}{\Theta}\right)^2\right\}^{\frac{1}{2}} \cos \theta$$

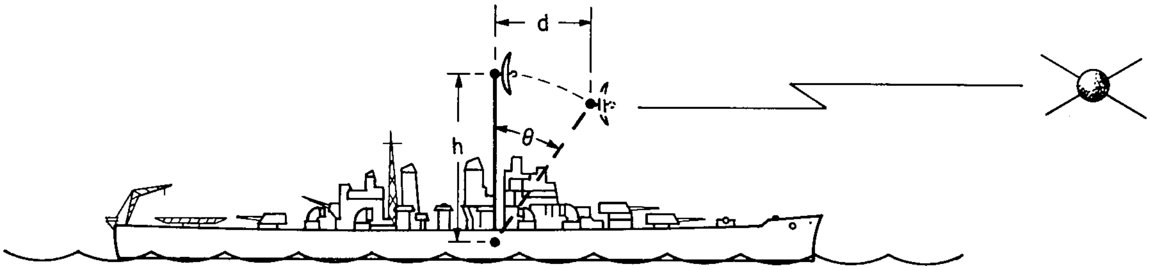


Fig. 3 - Ship and antenna roll-pitch geometry

The maximum value occurs at $\theta = 0$ and is given by

$$\dot{d}_{max} = h\,\omega\,\Theta \quad .$$

If it is assumed that

$$\Theta_{roll} = 30^{o},\ \tau_{roll} = 7^{s},\ h = 30 \text{ meters}$$

$$\Theta_{pitch} = 6^{o},\ \tau_{pitch} = 3^{s}$$

is an extreme case beyond the values in Table II, upper limits may be placed on the velocities and accelerations due to ship motion:

$$\left|\dot{d}_{max}\right| \leq 14 \text{ m/s} = 51 \text{ km/hr for roll}$$

$$\left|\dot{d}_{max}\right| \leq 6.6 \text{ m/s} = 24 \text{ km/hr for pitch.}$$

The velocities due to roll and pitch motions may be expected to be considerably less than the maximum expected due to satellite motion (~230 km/hr).

The largest possible total radial velocity occurs when the vector sum of the velocities due to satellite and ship motion is maximum. If the satellite is on the horizon at a bow angle of $\theta_B$, the total velocity is

$$v_{total} = (v_{headway} + v_{pitch}) \cos \theta_B$$
$$+ v_{roll} \sin \theta_B$$

An upper limit on $v_{headway}$ for a ship in a following current can be taken as 55 knots or 102 km/hr. The maximum value of $v_{total}$ in this case will occur at $\theta_B$ equal to $22^{o}$ and will be 136 km/hr. If the satellite were at a higher elevation, the ship velocity contribution would be reduced by the cosine of the elevation. Thus, the total ship motion velocity is maximum when the satellite is on the horizon. The maximum range rate caused by ship motion to be used for terminal design is

$$\left|v_{s\,m}\right|_{max} = 136 \text{ km/hr}$$
$$= 126 \text{ Hz/GHz.}$$

TOTAL COMBINED DOPPLER SHIFT

Although the total combined doppler shift may rarely be the maximum value, satellite communication equipment should be designed to operate within these limits. For a synchronous orbit in the equatorial plane, the approximate radial velocity relative to the ship for small eccentricities given by Eq. (7) is independent of the location of the ship, and is directly proportional to the eccentricity. When the orbital plane is tilted to the equatorial plane, the radial velocity relative to the ship for small inclinations is given by Eq. (8) for the case where the eccentricity is zero. Since the argument of the perigee, ascending node, and true anomaly are unspecified, they may be located so the radial motion due to eccentricity with zero inclination and the radial motion due to the inclination with the eccentricity equal zero may add to provide an approximate maximum total radial velocity, $v_{rs}$

$$v_{rs} \approx 12100\, \epsilon - 2060\, i \sin b$$

This has a peak value for a given inclination and eccentricity when the inclination or sin b has a negative sign and the latitude, b, has reached its maximum value. At the maximum latitude ($b \sim \pm 80^{\circ}$), the satellite will be directly north or south of the ship and lying on the horizon. At this point it is possible for the ship motion to add directly to the radial velocity of the satellite. This provides a total velocity, $v_t$, between the ship and satellite as

$$v_t = 12100\, \epsilon - 2060\, i \sin b \quad + \quad \left| v_{sm} \right|_{max}$$

The total velocity is maximized when the ship is headed towards or away from the satellite so its velocity is additive to the radial velocity. With the maximum values of eccentricity equal to 0.01, inclination equal to 3 degrees, $\left| v_{sm} \right|_{max}$ = 136 km/hr, the peak design velocity becomes equal to 365 km/hr or 338 Hz/GHz of carrier frequency.

## AUTOMATIC DOPPLER COMPENSATION METHODS

In a satellite communication system, the relative shift in frequency due to the doppler effect can be found from the difference between received and actual satellite beacon frequencies. The practical automatic implementation of the necessary frequency correction in communication receiver and transmitter gives rise to two independent problems. The first is the development of a phase lock loop of sufficient loop bandwidth to track the variation of the beacon signal for all possible platform accelerations without introducing excessive phase noise. The second instrumentation problem is the derivation of unequal and opposite shifts for transmitting and receiving frequencies that may be considerably different in frequency from the beacon signal.

### General Considerations

Since the doppler shift, $\Delta f$, is in a sense opposite to the radial velocity, $v_r$, regardless of the direction of propagation, the beacon, the communication transmit and receive frequencies are all shifted in the same direction. That is

$$\Delta f \approx \frac{-f\, v_r}{v_c}$$

where

$f$ = transmitted frequency, and

$v_c$ = velocity of light $1.08 \times 10^9$ km/hr.

It is desirable that the uplink signal appear at the correct (unshifted) frequency at the satellite receiver. Since the typical satellite translates the received signal by a fixed amount before retransmission, it is necessary to transmit a signal from the Earth terminal that is "pre-shifted" by an amount equal and opposite to the doppler shift. This shift is unequal and opposite to the correction that must be applied to the receiver local oscillator (LO) to receive the downlink signal.

In principle, perfect correction may be obtained by observing the received beacon frequency and scaling the observed doppler correction for the appropriate transmitter and receiver local oscillators. If $f_{be}$ is the beacon frequency as received at the Earth terminal, and $f_{bs}$ the (unshifted) beacon frequency at the satellite, $f_{be} = f_{bs}\ (1 + \alpha)$, where $\alpha$ is the shift coefficient $-v_r/v_c$. The received communication signal $f_{re}$ will be $f_{re} = f_{rs}\ (1 + \alpha) = f_{rs}\ f_{be}/f_{bs}$ where $f_{rs}$ is the downlink transmitted frequency at the satellite. If a signal of frequency $f_{te}$ is transmitted from Earth, the frequency $f_{ts}$, at the satellite receiver input will be $f_{ts} = f_{te}\ (1 + \alpha)$. To obtain the correct frequency, $f_{ts}$, at the satellite $f_{te} = f_{ts}/(1 + \alpha) = f_{ts}\ f_{bs}/f_{be}$ must be transmitted.

In practice, perfect correction would require more complex frequency synthesis to derive the transmit and receive local oscillator frequencies from the beacon frequency doppler shift. This is not justified if approximate correction with a simple local oscillator multiplication chain meets the requirements.

Correction with Variable Conversion Oscillations

One method is to derive the beacon receiver LO, the communication receiver LO, and the transmitter oscillator frequencies from a common voltage controlled crystal oscillator (VCXO) that is phase locked to the beacon signal. This technique can leave a residual error in the communication frequencies that introduces a very slight penalty in modem acquisition time at low data rates.

Before estimating the residual errors, consider first a typical multiple conversion chain (Fig. 4). In the case of a receiver, f is the received signal frequency, $f_r$, $f_m{}'$ is the first LO, $f_n$ the first intermediate frequency (IF), and so on to the final IF $f_1$. If each conversion is a "down-conversion",

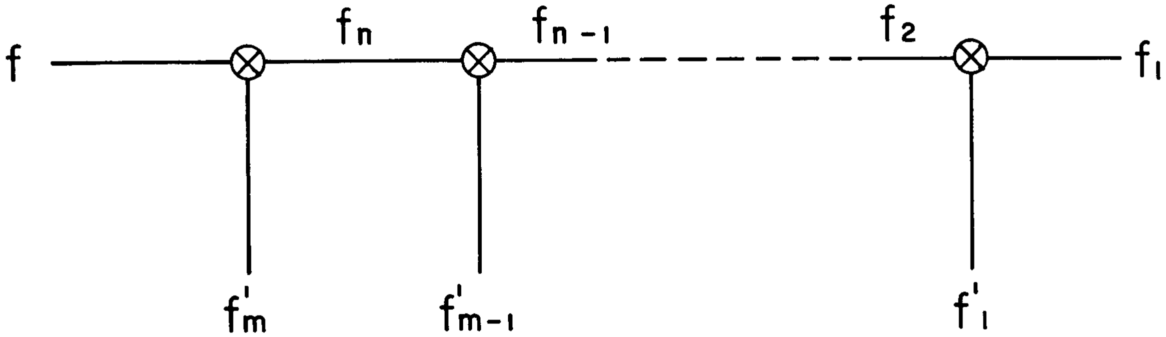


Fig. 4 - Typical multiple conversion chain

$$f = f_n + f_m' ,$$

$$f_n = f_{n-1} + f_{m-1}' ,$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$f_2 = f_1 + f_1 ,$$

and

$$f = f_1 + \sum_k{}_1^m f_k' .$$

If each $f_k'$ is derived by multiplying the common VCXO frequency $f_o$ by a constant factor $x_k$, $f_k' = x_k f_o$ and $f = f_1 + u f_o$, where $u = \sum_k{}_1^m x_k$. Thus, when a common frequency source is used for the several local oscillators, the multiple conversion chain may be represented as a single conversion. In the transmitting case, the chain in Fig. 4 is operated from right to left so that $f_1$ is the lowest IF ($f_{ift}$) in the transmit chain and f is the transmit frequency, and a similar result is obtained. This can be summarized as:

$$f_t = f_{ift} + u_t f_o$$

$$f_r = f_{ifr} + u_r f_o$$

$$f_b = f_{ifb} + u_b f_o$$

where $f_t$, $f_r$, and $f_b$ are the frequencies that the transmitter, communication channel, and beacon channel are tuned to, and where the different factors are distinguished by subscripts, t for transmitter channel, r for receiver channel, and b for beacon channel.

If the beacon frequency ($f_{bs}$) is received with no doppler shift, $f_{be} = f_{bs} = f_{ifb} + u_b f_o$. When the frequency is shifted by doppler effect, $f_{be} = f_{bs} (1 + \alpha)$ and the effect of phase locking $f_b$ to $f_{be}$ will be to shift the VCXO frequency by $\beta$ such that

$$f_{bs}(1+\alpha) = f_{ifb} + u_b(1+\beta) f_o$$

and

$$\beta = \alpha f_{bs}/u_b f_o \quad .$$

If the shift of the VCXO frequency is applied directly to the receiver and oppositely to the transmitter through their multiplier chains, the terminal frequencies with approximate doppler correction are obtained:

$$f'_r = f_{ifr} + u_r(1+\beta) f_o$$

$$f'_t = f_{ift} + u_t(1-\beta) f_o \quad .$$

The frequencies to which the channels should be tuned to perfectly compensate for the doppler shift are:

$$f_{re} = f_{rs}(1+\alpha)$$

$$f_{te} = f_{ts}/(1+\alpha) \simeq (1-\alpha) f_{ts} \text{ for } \alpha \ll 1$$

where $f_{rs}$ is the frequency transmitted from the satellite and $f_{ts}$ is the correct frequency the satellite is to receive from the transmitting ground station. The residual errors due to uncorrected doppler shifts on the received and transmitted signals are

$$E_r = f'_r - f_{re}$$

$$E_t = f'_t - f_{te} \quad .$$

With $f_{rs} = f_{ifr} + u_r f_o$ and $f_{ts} = f_{ift} + u_t f_o$, and some manipulation,

$$E_r = \alpha \left[\frac{u_r}{u_b} f_{ifb} - f_{ifr}\right] \tag{10}$$

$$E_t \approx \alpha \left[f_{ift} - \frac{u_t}{u_b} f_{ifb}\right] \tag{11}$$

are obtained.

In general, it is more convenient if all the IF amplifiers are the same frequency; that is

$$f_{ifr} = f_{ifb} = f_{ift} = f_{if} \quad .$$

The error equations then become:

$$E_r = \alpha\, f_{if} \left[\frac{u_r}{u_b} - 1\right] \tag{12}$$

$$E_t \approx \alpha\, f_{if} \left[1 - \frac{u_t}{u_b}\right] \quad . \tag{13}$$

The ratio $u_r/u_b$ and $u_t/u_b$ are the ratios of the equivalent single conversion LO frequencies:

$$u_t\, f_o = f_t - f_{if} \quad ,$$

$$u_r\, f_o = f_r - f_{if} \quad , \text{ and}$$

$$u_b\, f_o = f_b - f_{if} \quad , \text{ so that}$$

Equations (12) and (13) may be written as

$$E_r = \alpha\, f_{if} \left(\frac{f_r - f_{if}}{f_b - f_{if}} - 1\right)$$

$$E_t = \alpha\, f_{if} \left(1 - \frac{f_t - f_{if}}{f_b - f_{if}}\right)$$

A different doppler correction system is diagrammed in Fig. 5. The beacon frequency is used to phase lock an oscillator at frequency $f_o$ that will be used in this case to generate only the frequency of the lowest LO of each conversion chain for the transmit and receiving channels. The common VCXO frequency is multiplied by a factor K for the beacon receiver, a factor K' for the transmitter channel, and a factor K" for the communication receive channel. The beacon and communication receiving channels are inverted at one stage so that $f_r^* = f_{cr} - f_{2r}$ and $f_b^* = f_{cb} - f_{2b}$ where $f_{cr}$ and $f_{cb}$ are fixed oscillators such that $f_{cr} > f_{2r}$, $f_{cb} > f_{2b}$ while the transmitter frequency is not spectrum inverted.

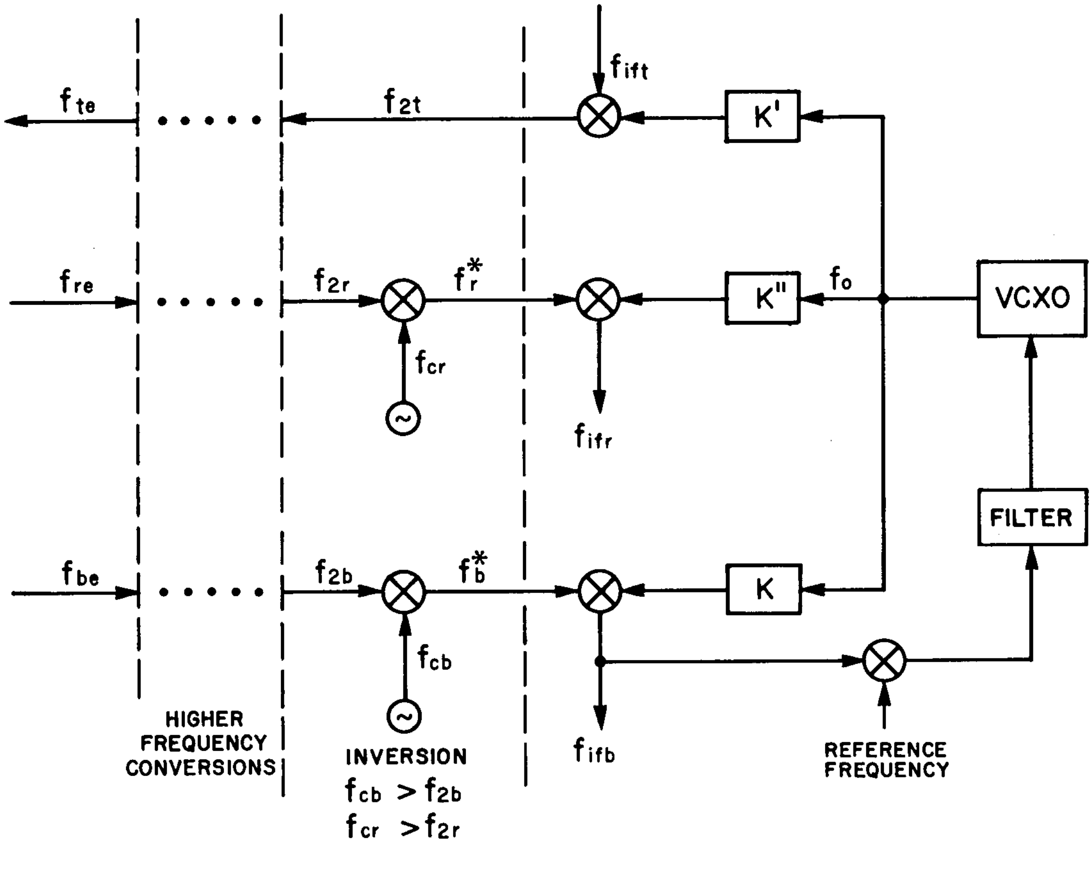


Fig. 5 - An approximate doppler correction scheme

The beacon receiver IF conversion is $f_{ifb} = f_b^* - K f_o = f_{cb} - f_{2b} - K f_o$. When doppler shift occurs, the frequency, $f_{2b}$, will be shifted by $\alpha f_{bs}$. When the VCXO is phase locked to the beacon signal, the doppler shift is corrected so that $f_{ifb}$ is constant, $f_{ifb} = f_{cb} - f_{2b} - K f_o = f_{cb} - (f_{2b} + \alpha f_{bs}) - K (1 + \beta) f_o$ and $\beta = -\alpha f_{bs}/K f_o$. Again, the transmit frequency $f_{te}$ should be preshifted to compensate for the doppler effect so that

$$f_{te} = \frac{f_{ts}}{1 + \alpha} \approx f_{ts} (1 - \alpha) \quad .$$

Thus, the shift required for the transmitter oscillator is $-\alpha f_{ts}$. Since $f_{2t} = f_{ift} + K' f_o$, the corrected transmitter frequency is actually $f_{2t}' = f_{ift} + K' f_o (1 + \beta)$ and the actual shift is $(f_{2t}' - f_2)$.

The error $E_t$, in correcting the transmitter frequency, is

$$E_t = (f_{2t}' - f_{2t}) - \alpha f_{ts}$$

or

$$E_t = \beta K' f_o + \alpha f_{ts}$$

and

$$E_t = -\alpha f_{bs} \left( \frac{K'}{K} - \frac{f_{ts}}{f_{bs}} \right) \quad . \tag{14}$$

Similarly, since $f_{2r} = f_{cr} - f_r^* = f_{cr} - f_{ifr} - K'' f_o$, it can be shown that the doppler correction error, $E_r$, for a communication channel of frequency, $f_{rs}$, becomes

$$E_r = \alpha f_{bs} \left( \frac{K''}{K} - \frac{f_{rs}}{f_{bs}} \right) \quad . \tag{15}$$

where

K" = communication channel multiplication factor.

To obtain a small or zero error the ratio of the K"/K and K'/K will have to be about equal to the ratio of the transmit or receive frequency to the beacon frequency.

## IMPLEMENTATION OF COMPENSATION AND EXPECTED ERRORS

Two methods of compensating doppler shifts based on the general analysis of the previous sections will be discussed. They are not necessarily optimum in selection of frequencies and configuration but are intended to show the considerations that should lead to an optimized design using these approaches.

Some systems will require a 1 dB bandwidth up to 40 to 50 MHz centered on 70 MHz. To convert this to any point in the 500 MHz band of the 7-8 GHz region will require at least two conversions. Probably the first translation of the 70 MHz IF would be to a frequency somewhere between 300 to 1000 MHz and then to the 7 or 8 GHz. In the first approach (Fig. 6), the highest receiver local oscillator (6.55 - 7.05 GHz) is corrected for doppler shift and the two highest frequency transmitter LO (6.55 - 7.05 GHz and 650 MHz) are corrected. There is no inversion of any of the IF spectrums and the mixing frequencies are chosen so that the synthesizers, which will be one of the more expensive items in the converter chain, will be the same for both transmitter and receiver. This choice of frequency probably is not too satisfactory because the second harmonic of the IF will fall into the next IF bandpass. The choice of these frequencies does not permit the transmitter converter chain or receiver local oscillators to generate any frequencies in the receive band, 7.25 to 7.75 GHz.

If the doppler correction were implemented in all oscillators down to the 700 MHz IF, the maximum worst case doppler correction error for the transmitter occurs (Eqs. (12) and (13)) when the transmitter is at 8.4 GHz and the beacon is at 7.250 GHz. This error will be about -17 Hz per kHz of doppler shift at 7.250 GHz. A receiver on a ship located close to the transmitting ship would experience a maximum worst case error under the same conditions as for the transmitting ship of about 7.4 Hz per kHz beacon doppler shift when the receiver communication frequency is 7.750 GHz and the beacon frequency is 7.250 GHz. This error is additive to the transmit error so that a maximum worst case system

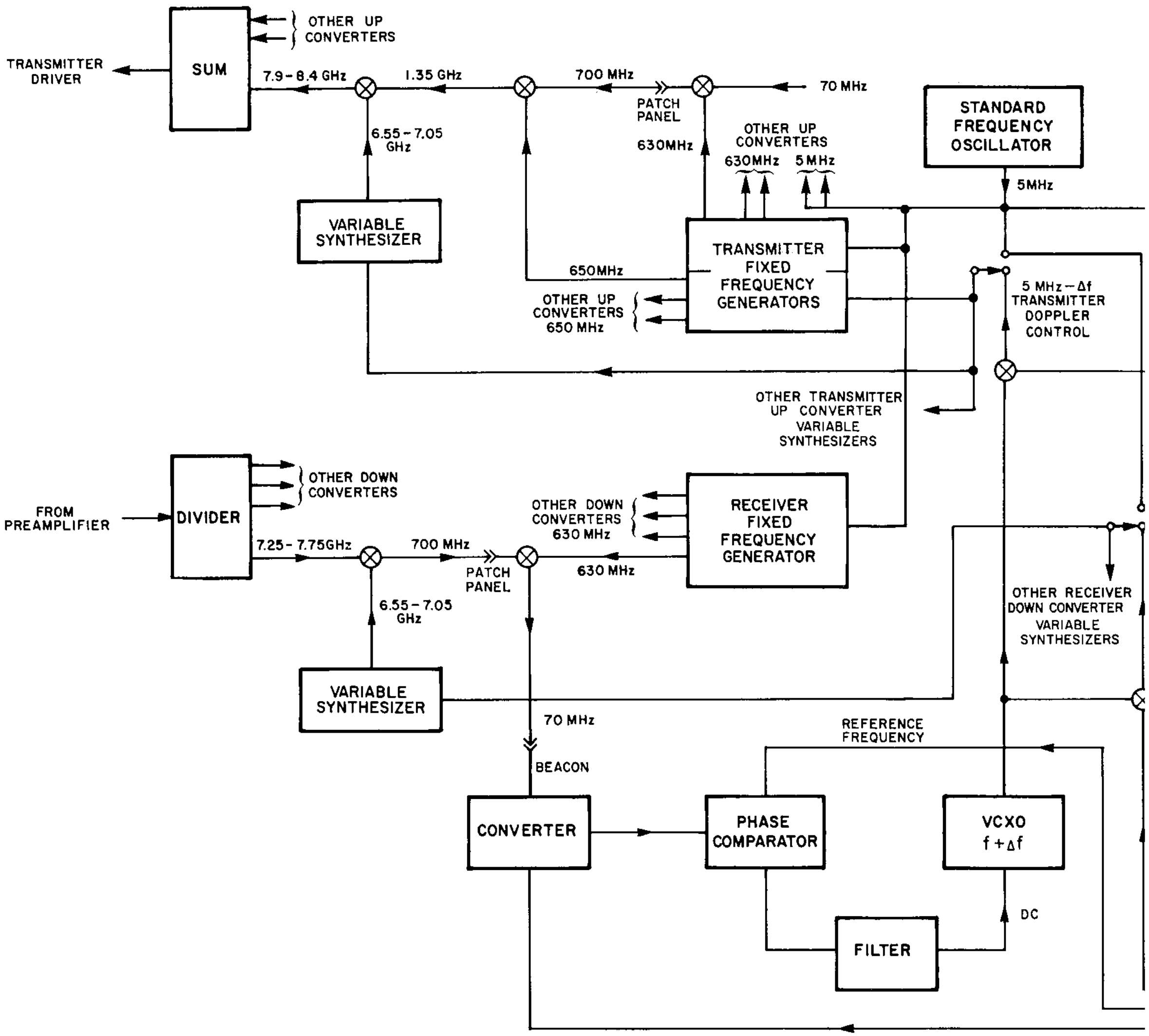


Fig. 6 - A possible doppler correction system

error of about 24.4 Hz per kHz doppler shift could exist.

The maximum possible doppler shift would occur once every 24 hours but the probability is rather slight since the ship must be operating with the above frequency combination (which probably will not exist for a particular satellite), must be located on the same longitude as the satellite with essentially zero degree elevation, the headway motion must be additive to that of the satellite motion and the satellite orbit parameters must be related so that the effects of inclination and eccentricity are maximized. Under conditions of an eccentricity of 0.01 and orbital inclination of 3 degrees maximum, total worst case doppler shifts for the beacon frequency (7.250 GHz) of approximately 2.5 kHz may occur for synchronous orbit satellites. This could result in a worst case error up to about 61 Hz which is significant but not probable. A greater portion of the modem frequency search range in acquisition should be reserved for errors due to satellite beacon frequency error and synthesizer frequency increments. If less error is required, doppler correction control of all local oscillators in the up and down converters down to the 70 MHz IF could be used. Under the conditions of LO correction down to the 70 MHz rather than the 700 MHz (that is, adding doppler correction to the 630 MHz LO of both the transmitter and receiver), the system doppler correction error for the same beacon, received communication and transmitted frequencies, becomes 2.4 Hz per kHz of doppler shift at 7.250 GHz for an IF frequency of 70 MHz.

If the satellite is well launched, as was TACSAT I ($\epsilon = 0.0015$ and $i = 0.6^{\circ}$), but the effects of eccentricity and inclination maximized, the total maximum worst case doppler shift would be reduced from the above 2.5 kHz with its maximum system correction error of about 61 Hz to a total doppler shift of 1.18 kHz with a peak doppler correction error of about 29 Hz. The greater part is contributed by total relative ship motion of 136 km/hr.

The transmitter local oscillators must be shifted in the direction opposite to the receiver local oscillator shift for doppler correction. One method to obtain this shift reversal is shown in Fig. 6. Here the VCXO is exactly centered between two frequencies when no doppler shift exists. When a doppler shift occurs, the VCXO which is phase locked to the beacon signal will shift, causing the two mixed frequency outputs from which the transmit and receive LO are derived to shift in opposite directions. It should be noted that a degenerate form of this configuration occurs when the mixing frequency from the frequency standard is reduced to zero. In this case, the VCXO becomes the one reference frequency while the other reference frequency with the doppler shift of opposite sign is derived by mixing with a frequency derived from the standard frequency which is twice the frequency of the VCXO with no doppler shift. It is doubtful that this can be satisfactorily implemented because of the spurious frequency of $f_o + \Delta f_o$ which will be at the mixer output for $f_o - \Delta f$. This presents a particularly difficult problem in this case since the signals are multiplied about 1300 times. This points out the care that must be exercised in choosing mixing frequencies that will reduce spurious responses. The phase locked VCXO must be capable of operating with a bi-phase modulated carrier for presently proposed satellites.

Figure 5 of the previous section is a block diagram of a second possible method of implementing doppler compensation when only the lowest frequency local oscillators of the up and down converters are derived from a common oscillator that is phase locked to the beacon signal. If each communication channel had its own multiplier K" and the multiplication ratios were adjusted so the ratio of K"/K are the same as the ratios of the communication carrier frequency to the beacon frequency, the compensation would be exact. Unfortunately, this will lead to a complex local oscillator doppler shift synthesizer. For the frequencies discussed above, an approximate doppler compensation can be obtained by choosing a factor of 9 for all receiver channels (beacon and communication) and 10 for the transmitter channels. Other sets of ratios such as 10 and 11 or 11 and 12 might be desirable, but more difficulty is encountered in making a satisfactory times 11 multiplier. The maximum doppler correction error for the receiver (Eq.(15)), becomes 69 Hz per 1 kHz of doppler shift at the beacon frequency. This will occur when the beacon frequency is 7.25 GHz and the communication frequency is at 7.75 GHz. The maximum transmitter error (Eq. (14)), becomes 92 Hz per 1 kHz of doppler shift of the beacon carrier frequency. This occurs with the beacon carrier frequency at 7.75 GHz and the transmitter at 7.9 GHz. This instrumentation for the doppler shift compensation is not satisfactory for use with the presently planned modems and will not be further considered.

A form of the first approach appears to offer a solution to the doppler shift correction problem. If the errors are reduced as much as possible by having the modem operate at the 70 MHz IF only with doppler correction applied down to the 70 MHz IF, the correction errors will be insignificant. Another more complicated approach is to use the corrected VCXO in a manner that furnishes doppler shift correction not only to the terminal LO but to the modem LO as well so as to minimize the error as computed in Eqs. (1) and (11).

BEACON FREQUENCY UNCERTAINTIES

The doppler shift compensation errors, just discussed, must be kept to low values since the greatest part of the total frequency error budget should be reserved for the frequency uncertainty of the satellite beacon signal. It is assumed that the frequency steps on the local oscillator synthesizers are small enough to not cause a significant error. If not, an error for the beacon receiver synthesizer must be included in the beacon frequency uncertainty. For the received communication signal, the largest possible frequency error caused by the synthesizers for the transmitter and receiver would be one half the least frequency step in the synthesizer. This assumes that the minimum frequency steps for the transmitter and receiver are the same. Let us consider the case where the beacon radiated frequency has drifted $\chi$ Hz higher than the frequency to which the beacon receiver synthesizer is set. To simplify the discussion, it will be assumed that there is no relative motion between the terminal and the satellite which could give rise to doppler shift correction error when the doppler correction is implemented. The receiver tuning will be shifted up in frequency by $\chi\, u_r/u_b$ and the transmitter will be shifted down in frequency by $\chi\, u_t/u_b$. Since the translation frequency in the satellite is generally derived from the same oscillator as the beacon carrier frequency, its frequency will be increased by $\chi \frac{u_{st}}{u_{sb}}$ where $u_{sb}$ is the multiplication factor of the oscillator frequency to provide the beacon carrier and $u_{st}$ is the multiplication factor to provide the translation frequency. If both stations of the circuit operate in the same manner, their receivers will be tuned to a frequency in error by $\chi\,(u_r/u_b + u_t/u_b + u_{st}/u_{sb})$. The ratios of $u_r/u_b$ and $u_t/u_b$ for the X band communication satellite frequencies have maximums of 1.076 and 1.176, respectively, for a 700 MHz IF. The beacons are expected to drift less than $5 \times 10^{-8}$ parts per month. If the beacon receiver synthesizer was offset from the beacon radiated frequency by a part in $10^{-8}$ with translation and beacon frequencies of 0.725 and 7.25 GHz respectively, the communication receiver could be mistuned from the desired frequency by about 171 Hz. This is too great an error and it is desirable that the radiated beacon frequency, and in turn the translation frequency, be known to a few parts in $10^{-9}$. It may be possible to furnish this information to the operators on a periodic basis, as other orbital parameters now furnished for the DSCS Phase I satellites. How well this can be done will be dependent on the stability of the beacon oscillator drift which probably will be poorest at those times when the satellite emerges from eclipse.

An operator can determine the beacon frequency by transmitting a signal and receiving his signal with the doppler compensation equipment operating. The difference between the frequency that he receives and that to which his receiver was tuned is $\chi\{(u_r + u_t)/u_b + u_{st}/u_{sb}\}$. Since $u_r$, $u_t$, $u_b$, $u_{st}$, and $u_{sb}$ are all known, $\chi$ can be immediately determined. From this the operator corrects his beacon synthesizer by $\chi/u_b$. When doppler shift occurs some error is caused by the incomplete doppler compensation as given in Eqs. (10) and (11). This must be kept small by measuring the beacon frequency when the expected total doppler shift is low enough to not render this correction useless. Also, the frequency steps in the synthesizer must be sufficiently small so as to reduce the synthesizer frequency errors to tolerable limits.

Using these corrections it may be possible to rate the satellite oscillator drift and to predict future corrections based on past data when measurements cannot be made. Rating of the oscillator drift may become erratic during periods of eclipse.

## EFFECT OF FREQUENCY STANDARD ERRORS

The previous derivations assumed an errorless frequency standard furnishing the fixed frequencies for the translations not corrected for doppler shift. Figure 7 is a functional diagram of Fig. 6 for determining the total error in receiver tuning due to doppler correction error and frequency offset of the standard frequency. The method of analysis is similar to the above. With both doppler shift and offset in the frequency standard of $\gamma$ present

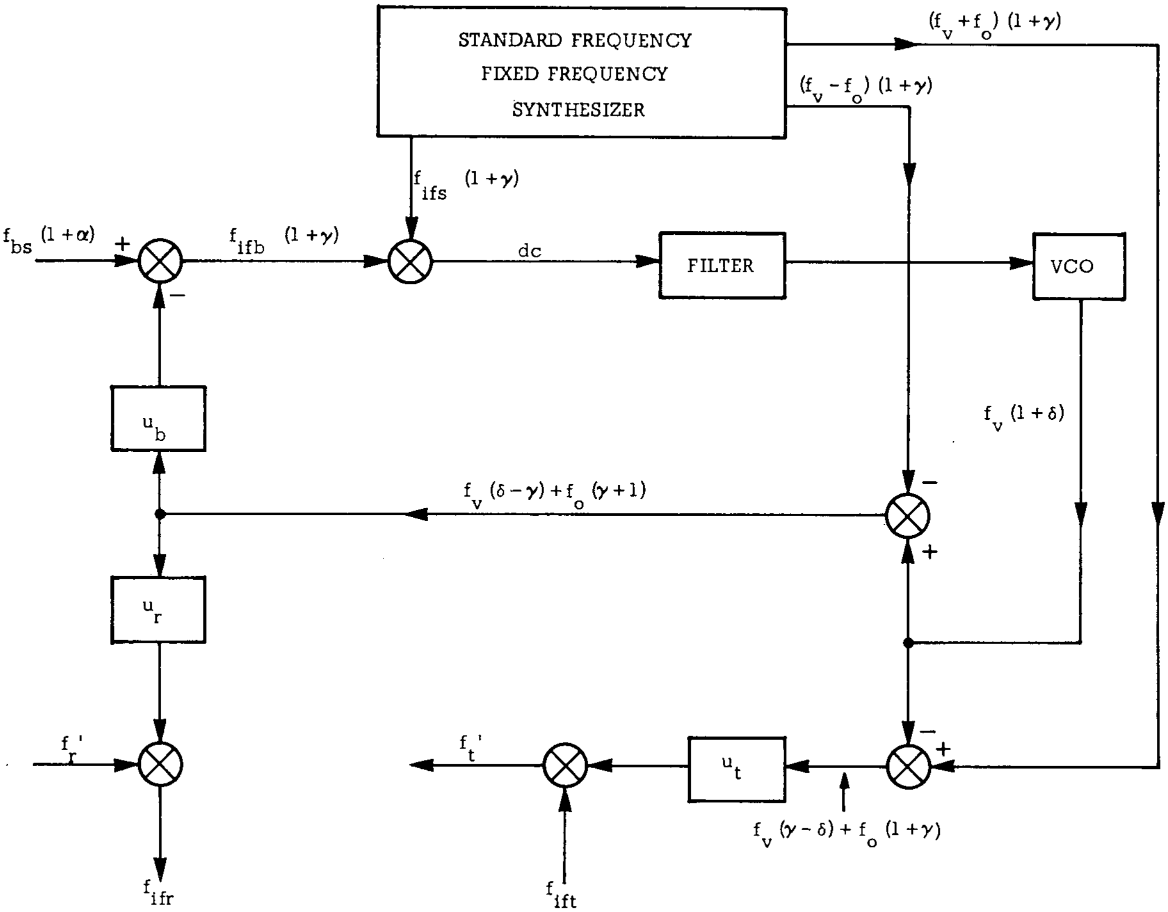


Fig. 7 - Function diagram of doppler correction system shown in Fig. 6

$$f_{bs}(1+\alpha) = f_{ifb}(1+\gamma) + u_b\,[f_v(\delta-\gamma) + f_o(\gamma+1)]$$

where $f_v$ is the voltage control oscillator center frequency in Fig. 6 in the absence of doppler shift and frequency standard offset. The quantity $\delta$ is the fraction the VCO frequency must be shifted to obtain lock when doppler shift and frequency standard offset exist. The other terms are defined in the previous analysis for the condition of doppler correction only.

The frequency, $f_r'$, to which the communication receiver is tuned is

$$f_r' = f_{ifr} + u_r\,[f_v(\delta-\gamma) + f_o(\gamma+1)]$$

or

$$f_r' = f_{ifr} + \frac{u_r}{u_b}\,[f_{bs}(1+\alpha) - f_{ifb}(1+\gamma)] \quad .$$

The total error $E_{rt}$ in communication receiver tuning for doppler correction and frequency standard offset is the difference between $f_r'$ and the frequency $f_{re}$ of the satellite signal when received by the terminal. From the previous analysis

$$f_{re} = f_{rs}(1+\alpha)$$

and with no doppler shift or frequency standard offset

$$f_{bs} = f_{ifb} + u_b f_o$$

$$f_{rs} = f_{ifr} + u_r f_o$$

The total error becomes

$$\begin{aligned} E_{rt} &= f_r' - f_{re} \\ &= f_{ifr} + \frac{u_r}{u_b}\,[f_{bs}(1+\alpha) - f_{ifb}(1+\gamma)] - f_{rs}(1+\alpha) \end{aligned}$$

With substitution and some manipulation:

$$E_{rt} = \alpha\left[\frac{u_r}{u_b} f_{ifb} - f_{ifr}\right] - \gamma\frac{u_r}{u_b} f_{ifb}$$

For the transmitter the radiated frequency $f_t'$ is

$$f_t' = f_{ift} + u_t \left[ f_v (\gamma - \delta) + f_o (1 + \gamma) \right]$$

The total transmitter frequency error, $E_{tt}$, due to doppler and standard frequency offset becomes

$$E_{tt} = f_t' - f_{te}$$

Using the previous relationships of

$$f_{te} = (1 - \alpha) f_{ts}$$

and under no doppler shift or standard frequency offsets

$$f_{bs} = f_{ifb} + u_b f_o$$

$$f_{ts} = f_{ift} + u_t f_o$$

it can be shown that

$$E_{tt} = \alpha \left[ f_{ift} - \frac{u_t}{u_b} f_{ifb} \right] + 2 u_t f_o \gamma + f_{ifb} \gamma \frac{u_t}{u_b}$$

It is interesting to note that the error on the communication receive channel is equal to only the doppler shift of the IF. The major error is contributed by the term $2 u_t f_o \gamma$ in the error for the transmitter chain. For frequencies in the 8 GHz region, it is evident that $\gamma$ should be kept to the order of $10^{-10}$ or less if the error due to the standard frequency offset is not going to become a large part of the total allowed error in received frequency.

OTHER PROBLEM AREAS

If doppler correction is to be employed, a number of areas of potential difficulty should be investigated in the early design of the frequency converters in the beacon receiver. As it is necessary to close the phase lock loop on the beacon through the synthesizer, the synthesizer must have a well behaved phase response characteristic or the loop will not operate properly if at all. While it is anticipated that this problem will not occur in existing synthesizers, this is unknown at the present.

A second closely related problem is the phase noise of the beacon receiver that is injected into the reference oscillator from which all the other up and down converters obtain their doppler corrected LO frequencies. This noise, if of a certain spectral distribution, will degrade the performance of any modem which makes use of phase information of the signal as in PSK operation. In general, to reduce the effect of this noise, the phase lock loop corner frequency or natural frequency is reduced. Unfortunately, if the corner frequency is made too small, the phase lock loop will not be capable of tracking varying doppler shifts due to platform acceleration. It has been shown (2) that for a second order loop:

$$e = \frac{\dot{f}}{2\pi f_n^2}$$

where

$e$ = steady-state phase error in radians due to the acceleration,

$\dot{f}$ = rate of change of input frequency, and

$f_n$ = loop natural frequency.

The instantaneous frequency, $f_r$, received from a moving platform, is

$$f_r = f\left(1 - \frac{v_r}{v_c}\right) \quad ,$$

where the doppler shift is in the sense opposite to the velocity. Under conditions of acceleration where the transmitted frequency, f, is essentially constant with time

$$\dot{f}_r = -f\,\dot{v}_r/v_c$$

Combining this with the equation for phase error results in

$$f_n = \{f\left|\dot{v}_r\right| g/(2\,\pi\, e\, v_c)\}^{\frac{1}{2}}$$

where $|\dot{v}_r|$ is now expressed in units of g, the acceleration due to gravity ($9.8\ m/sec^2$). The value of e for a 1 dB degradation in threshold* is about 0.5 radian. Thus, the loop should probably be designed with a 1 dB threshold degradation for a platform acceleration of about 2 g, the maximum anticipated value. The loop natural frequency or corner frequency of the phase lock loop for a carrier frequency of 7.9 GHz and 2 g acceleration would be 12.9 Hz.

For a high gain, second order loop the loop bandwidth, $B_L$, is

$$B_L = \pi f_n (\zeta + \frac{1}{4\zeta}) \text{ Hz}$$

where $\zeta$ is the damping factor, $B_L = 3.33 f_n$ for $\zeta = 0.707$. The oscillator mean square phase noise $\overline{\theta^2_{no}}$, in radians for a squaring phase lock is given by:

$$\overline{\theta^2_{no}} = \omega_i B_L / p_s \tag{16}$$

where

$p_s$ = input signal power

$\omega_i$ = the input noise power spectral density.

Gardner (2) states that this expression is valid for root mean square values, $(\overline{\theta^2_{no}})^{\frac{1}{2}}$, less than 13 degrees.

As an example, the error will be evaluated for a beacon signal that is somewhat weaker than that projected for the TACSAT satellites but may represent the situation under adverse conditions. Table III contains the circuit parameters for determining the received beacon signal to noise power density ratio at a terminal using a three foot diameter antenna.

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* Private communication from W. Judge, Magnavox Research Laboratories, Torrance, California

Table 3

Beacon Circuit Parameters

| | |
|---|---|
| Beacon Effective Radiated Power | + 40 dBm |
| Path Loss | -203 dB |
| Receiving Antenna Gain (3 ft dia.) | + 34 dB |
| Received Carrier Power | -129 dBm |
| Boltzmann's Constant k | -198.6 dBm/$^{o}$K/Hz |
| Receiving System Temperature $T_S = 400^{o}K$ | + 26 dB$^{o}$K |
| $kT_s$ | -172.6 dBm/Hz |
| Carrier Power/Noise Density | + 43.6 dB Hz |
| $B_L$ = 50 Hz | + 17 dB Hz |

The loop bandwidth $B_L$ of 50 Hz is slightly greater than the value of 43 Hz than that computed for a corner frequency of 13 Hz. Insertion of the above parameters into Eq. (16) gives a voltage controlled oscillator (VCO) root mean square phase noise of $(2.2 \times 10^{-3})^{\frac{1}{2}}$ radians or 2.7 degrees. The VCO referred to here would be the combined VCXO, synthesizer, and multiplier chain.

The loop spectral noise distribution would be approximately the same as a bi-phase shift keyed (PSK) system operating at 75 bits per second with the loop designed for a 2 g platform acceleration. At higher data rates, the phase noise due to the phase jitter in the doppler correction is concentrated at the lower frequencies of the loop noise and would be expected to cause less degradation. The total system RMS phase jitter is caused by equal 2.7 degree phase lock noise in the transmitter and the receiver local oscillators. Thus, it would appear that no large degradation in performance should occur when doppler correction is used. This was calculated for an unmodulated beacon. The normal beacon will be bi-phase modulated with 800 bits/second. This will require a phase lock loop which will have slightly greater phase noise for the same C/kT. The whole aspect of the phase lock loop must be examined in greater depth.

Another problem in design of the LO is the high frequency wide band noise components generated in the local oscillator shown in the spectrum of Fig. 8. This is the noise spectrum viewed on a Hewlett-Packard Model 851B/8551B spectrum analyzer of the X band transmitter local oscillator of an existing terminal with the analyzer bandwidth set to 1 kHz. The humps of the noise occur about 6 MHz from the carrier and are about 55 dB below the carrier. A similar though more severe noise situation was experienced at the NRL Microwave Space Research Facility at Waldorf, Maryland. This was corrected by building a tunable 400 MHz transistor oscillator which was phase locked to a synthesizer through a narrow bandwidth loop. Figure 9 is the resulting X band signal taken under the same analyzer settings as Fig. 8. The small cross line at the top indicates the peak of the carrier.

## CONCLUSIONS

It can be concluded that:

1. Even with a stationary satellite significant doppler shift will exist in Naval applications.

2. Doppler compensation derived from the beacon signal is not completely error-free when simple multiplication chains for the local oscillator are used. The error, however, is small making a simple correction method useful for many applications.

3. The proper use of doppler compensation derived from the beacon frequency requires accurate knowledge of the beacon transmitted frequency.

4. Doppler compensation will cause additional terminal design problems with respect to local oscillator phase noise.

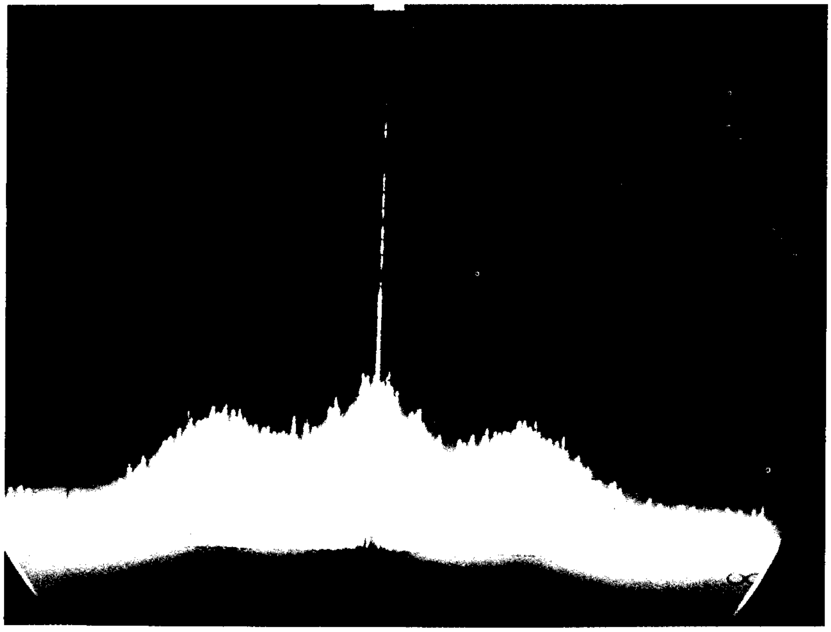

Fig. 8 - Terminal transmitter local
oscillator spectrum

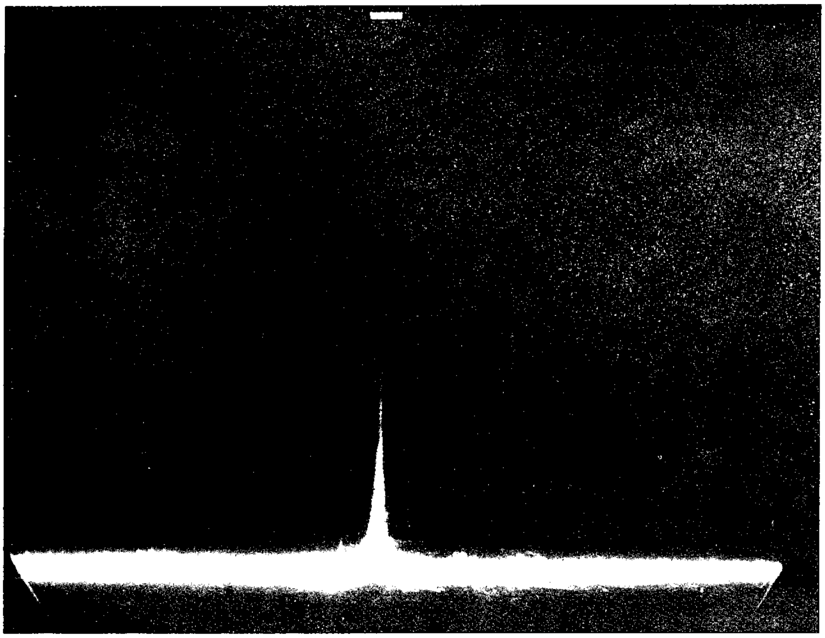

Fig. 9 - Waldorf receiver 7 GHz local oscillator

## RECOMMENDATIONS

It is recommended that theoretical and experimental efforts be made to determine the most feasible method of doppler compensation and the effects on terminal oscillator phase noise, and local oscillator synthesizer design.

## REFERENCES

1. NAVSHIPS 0967-303-7070, "Pedestal Design Handbook for Shipboard Antennas," Jun 1968, pp 2-13
2. Gardner, F.M., "Phaselock Techniques," New York, John Wiley and Sons, Inc., 1966

## LIST OF SYMBOLS (in order of appearance)

| | | |
|---|---|---|
| $v_r$ | = | topocentric radial velocity |
| $v_r(i=o)$, $v_r(\epsilon=o)$ | = | radial velocity evaluated for orbits of zero inclination, and eccentricity respectively |
| $v_{r\ max}$ | = | maximum value of $v_r$ |
| $\epsilon$ | = | eccentricity |
| $\Omega$ | = | argument of perigee |
| $\underline{R}$ | = | geocentric position vector of satellite |
| $\underline{r}$ | = | magnitude of $\underline{R}$ |
| $\psi$ | = | true anomaly |
| $\underline{x}, \underline{y}, \underline{z}$ | = | unit vectors of geocentric Cartesian coordinations |
| $i$ | = | inclination of orbit |
| $a$ | = | semi major axis |
| $\mu$ | = | $5.164 \times 10^{12}$ km$^3$ hr$^{-2}$ |
| $\underline{q}$ | = | unit vector perpendicular to the orbital plane |
| $\underline{p}$ | = | unit vector lying in the orbital plane directed toward the perigee |
| $\underline{q}, \eta$ | = | constants |
| $T$ | = | orbital period |
| $\pi$ | = | 3.141592..... |
| $\omega_o$ | = | angular velocity of Earth |
| $\dot{\underline{R}}$ | = | $\frac{d}{dt} \underline{R}$ |
| $\underline{R}_o$ | = | position vector of Earth terminal with respect to fixed geocentric coordinates |
| $a_e$ | = | radius of the Earth |
| $b$ | = | terminal latitude |
| $\varphi$ | = | terminal (pseudo) longitude |
| $\dot{\underline{R}}_o$ | = | $\frac{d}{dt} \underline{R}_o$ |

| | | |
|---|---|---|
| $\underline{S}$ | = | topocentric position vector of satellite |
| $s$ | = | magnitude of $\underline{S}$ |
| $\dot{\underline{S}}$ | = | $\frac{d}{dt}\ \underline{S}$ |
| $c$ | = | $\varphi - \psi$ relative longitude of satellite |
| $\lvert v_r \rvert_{max}$ | = | maximum absolute value of $v_r$ |
| $\theta$ | = | angular motion of ship due to pitch or roll |
| $\Theta$ | = | maximum excursion of $\theta$ |
| $\tau$ | = | period of pitch or roll motion |
| $t$ | = | time in seconds |
| $\Theta_{roll}$ | = | maximum excursion in roll |
| $\Theta_{pitch}$ | = | maximum excursion in pitch |
| $\tau_{roll}$ | = | period of roll |
| $\tau_{pitch}$ | = | period of pitch |
| $d$ | = | linear displacement of terminal due to ships motion |
| $\dot{d}$ | = | $\frac{d}{dt}\ d$ |
| $h$ | = | height of antenna above the center of motion |
| $\omega$ | = | angular frequency of roll or pitch motion |
| $\theta_B$ | = | bow angle to satellite |
| $v_{total}$ | = | total ship velocity |
| $v_{headway}$ | = | headway velocity |
| $v_{pitch}$ | = | linear velocity due to pitch |
| $v_{roll}$ | = | linear velocity due to roll |
| $v_{design}$ | = | maximum total velocity due to ship motion and headway to be used in terminal design |
| $v_{rs}$ | = | maximum topocentric radial velocity due to relative motion of satellite and Earth |
| $v_t$ | = | total radial velocity, satellite ship motion and headway |
| $\Delta f$ | = | doppler shift |
| $v_c$ | = | velocity of light |
| $f_{be}$ | = | frequency of satellite beacon signal as received at Earth terminal |
| $f_{bs}$ | = | frequency of satellite beacon signal as transmitted |

| | | |
|---|---|---|
| $\alpha$ | = | shift factor, $v_r/v_c$ |
| $f_{rs}$ | = | frequency of down link signal at satellite |
| $f_{te}$ | = | up link frequency at Earth |
| $f_{ts}$ | = | up link frequency at satellite |
| $f$, $fm$, $fn$ | = | general frequencies |
| $f_o$ | = | VCXO frequency, unshifted |
| $m, n, k$ | = | indicies |
| $x_k$, $u_r$, $u_b$, $u_t$ | = | frequency multiplication factors |
| $f_t$, $f_r$, $f_b$ | = | transmit, receiver AM beacon frequencies |
| $f_{ift}$, $f_{ifr}$, $f_{ifb}$ | = | intermediate frequencies for transmit, receive, and beacon channels |
| $\beta$ | = | VCXO shift |
| $f_{if}$ | = | an intermediate frequency |
| $E_r$, $E_t$ | = | error in receive, and transmit frequencies remaining after doppler shift correction |
| $f^*_r$, $f_{2r}$, $f^*_b$, $f_{2b}$ | = | frequencies in an inverting conversion chain |
| $f_{cr}$, $f_{cb}$ | = | constant frequency oscillators |
| $K$, $K'$, $K''$ | = | fixed frequency multiplication factors |
| $\chi$ | = | fractional change in frequency of satellite master oscillator |
| $\Delta f_o$ | = | change in $f_o$ |
| $e$ | = | phase error in radian |
| $f$ | = | rate of change of frequency |
| $f_n$ | = | natural frequency of phase lock loop |
| $g$ | = | acceleration due to gravity |
| $B_L$ | = | phase lock loop bandwidth |
| $\zeta$ | = | camping factor |
| $\theta_{no}$ | = | phase noise |
| $P_s$ | = | input signal power |

| | | |
|---|---|---|
| $\omega_i$ | = | input power spectral density |
| $T$ | = | receiving system temperature |
| $k$ | = | Boltzmann's constant |

APPENDIX A

EQUATIONS OF MOTION OF AN EARTH SATELLITE

This appendix summarizes the derivation of the equations of motion of an Earth satellite that are used in the main text. The gravitational attraction of the Earth on a satellite results in an acceleration

$$\ddot{\underline{R}} = - \mu \frac{\underline{r}}{r^2} \tag{A-1}$$

where $\underline{R}$ is the geocentric position vector, $\underline{r}$ is the unit vector in the $\underline{R}$ direction, r is the magnitude of $\underline{R}$, and $\mu$ is the gravitational constant of Earth. The time derivative of $\underline{Q} = \underline{R} \times \dot{\underline{R}}$ is (A-2)

$$\begin{aligned}
\dot{\underline{Q}} &= \underline{R} \times \ddot{\underline{R}} + \dot{\underline{R}} \times \dot{\underline{R}} \\
&= \underline{R} \times \ddot{\underline{R}} \\
&= - \frac{\mu}{r^2} \underline{R} \times \underline{r} \\
&= 0 \ .
\end{aligned}$$

Hence, $\underline{Q}$ is a constant vector, perpendicular $\underline{R}$, $\dot{\underline{R}}$, and $\ddot{\underline{R}}$, so that the orbit must lie in the plane perpendicular to $\underline{Q}$.

Since $\underline{r} = \underline{R}/r$, taking the cross product with $\underline{Q}$ of both sides of Eq. (A-1) results in

$$\underline{Q} \times \ddot{\underline{R}} + \frac{\mu}{r^3} \underline{Q} \times \underline{R} = 0 ,$$

and from Eq. (A-2)

$$\underline{Q} \times \ddot{\underline{R}} + \frac{\mu}{r^3} (\underline{R} \times \dot{\underline{R}}) \times \underline{R} = 0$$

or

$$\underline{Q} \times \ddot{\underline{R}} + \frac{\mu}{r^3} \underline{R} \times (\dot{\underline{R}} \times \underline{R}) = 0 . \tag{A-3}$$

The vector triple product for the arbitrary vectors $\underline{L}$, $\underline{M}$, $\underline{N}$ is given by the identity:

$$\underline{L} \times (\underline{M} \times \underline{N}) \equiv \underline{M}\,(\underline{L} \cdot \underline{N}) - \underline{N}\,(\underline{L} \cdot \underline{M})$$

so that Eq. (A-3) becomes

$$\underline{Q} \times \ddot{\underline{R}} + \frac{\mu}{r^3}\,\{\dot{\underline{R}}\,(\underline{R} \cdot \underline{R}) - \underline{R}\,(\underline{R} \cdot \dot{\underline{R}})\} = 0 \;. \qquad \text{(A-4)}$$

Using the relationships

$$\frac{d}{dt}\,(\underline{Q} \times \dot{\underline{R}}) = \underline{Q} \times \ddot{\underline{R}} + \dot{\underline{Q}} \times \dot{\underline{R}} = \underline{Q} \times \ddot{\underline{R}}$$

and

$$\underline{R} \cdot \underline{R} = r^2 \;,$$

equation (A-4) becomes

$$\frac{d}{dt}\,(\underline{Q} \times \dot{\underline{R}}) + \frac{\mu}{r^3}\,\{\dot{\underline{R}}\,r^2 - \underline{R}\,(\underline{R} \cdot \dot{\underline{R}})\} = 0 \;. \qquad \text{(A-5)}$$

since

$$\underline{R} \cdot \dot{\underline{R}} = \tfrac{1}{2}\,\frac{d}{dt}\,(\underline{R} \cdot \underline{R}) = \tfrac{1}{2}\,\frac{d}{dt}\,r^2 = r\,\dot{r} \;,$$

equation (A-5) becomes:

$$\frac{d}{dt}\,(\underline{Q} \times \dot{\underline{R}}) + \mu \left\{\frac{r\dot{\underline{R}} - \underline{R}\dot{r}}{r^2}\right\} = 0 \;.$$

Since

$$\frac{d}{dt}\,(\underline{R}/r) = \frac{r\dot{\underline{R}} - \underline{R}\dot{r}}{r^2} \;,$$

the relation becomes

$$\frac{d}{dt}\,(\underline{Q} \times \dot{\underline{R}} + \mu\underline{r}) = 0 \;.$$

Thus $\underline{Q} \times \dot{\underline{R}} + \mu \underline{r}$ is a constant vector which "chosen" to be $-\mu\,\epsilon\,\underline{p}$, where $\epsilon$ and $\underline{p}$ will be shown to be respectively, the orbital eccentricity, and the unit vector directed from Earth's center to the perigee. Hence

$$\underline{Q} \times \dot{\underline{R}} + \mu\,(\underline{r} + \epsilon\,\underline{p}) = 0\ , \tag{A-6}$$

$$\underline{Q} \times \dot{\underline{R}} = -\mu\,(\underline{r} + \epsilon\,\underline{p})\ .$$

Forming the cross product of $\underline{Q}$ with both sides of the equation yields

$$\underline{Q} \times (\underline{Q} \times \dot{\underline{R}}) = -\mu\,\underline{Q} \times (\underline{r} + \epsilon\,\underline{p})\ ,$$

or by applying the identity for the triple vector products

$$\underline{Q}\,(\underline{Q} \cdot \dot{\underline{R}}) - \dot{\underline{R}}\,(\underline{Q} \cdot \underline{Q}) = -Q \times \mu\,(\underline{r} + \epsilon\,\underline{p}) \tag{A-7}$$

Since $\underline{Q}$ is perpendicular to $\dot{\underline{R}}$, $\underline{Q} \cdot \dot{\underline{R}} = 0$ and since $\underline{Q} \cdot \underline{Q} = q^2$ equation (A-7) becomes

$$\dot{\underline{R}}\,q^2 = \mu\,q\,\underline{\underline{q}} \times (\underline{r} + \epsilon\,\underline{\underline{p}})\ .$$

so that the expression used in the text for the geocentric velocity is obtained:

$$\dot{\underline{R}} = \mu\,\frac{\underline{\underline{q}}}{q} \times (\underline{r} + \epsilon\,\underline{p}) \tag{A-8}$$

The equation of the orbit is obtained by forming the scalar product of $\underline{R}$ with the terms of equation (A-6):

$$\underline{R} \cdot \underline{Q} \times \dot{\underline{R}} + \mu\,[\underline{R} \cdot \underline{r} + \epsilon\,\underline{R} \cdot \underline{p}] = 0$$

since

$$\underline{R} \cdot \underline{Q} \times \dot{\underline{R}} = \dot{\underline{R}} \times \underline{R} \cdot \underline{Q} = -\underline{Q} \cdot \underline{Q} = -q^2$$

and $\underline{R} \cdot \underline{r} = r$, the orbit is described by

$$-q^2 + \mu\,r\,(1 + \epsilon\,\underline{r} \cdot \underline{p}) = 0$$

or

$$r = \frac{q^2/\mu}{1 + \epsilon \underline{r} \cdot \underline{p}} \qquad \text{(A-9)}$$

Equation (A-9) is recognized as a conic section expressed in polar form with one focus centered on the Earth. The point of closest approach to the Earth, the perigee, or the point of minimum r occurs at $\underline{r} \cdot \underline{p} = 0$ or when $\underline{r}$ and $\underline{p}$ are parallel so that $\underline{p}$ is the direction from the center of the Earth to the perigee.

If $\psi$ is the angle between $\underline{p}$ and $\underline{R}$

$$r = (q^2/\mu)/(1 + \epsilon \cos \psi) \qquad \text{(A-10)}$$

When $1 > \epsilon > 0$ becomes equation of an elliptical orbit of eccentricity $\epsilon$, where $\psi$ is the "true anomaly" (the geocentric angle between perigee and the satellite position) and where the semi-major axis $a = q^2/\mu \ (1 - \epsilon^2)$. The magnitude q of the constant vector $\underline{Q}$ is given by

$$q = \sqrt{\mu \, a \, (1 - \epsilon^2)} \,. \qquad \text{(A-11)}$$

The magnitude of the position vector $\underline{R}$ is

$$r = a \, (1 - \epsilon^2)/(1 + \epsilon \cos \psi) \,. \qquad \text{(A-12)}$$

This rather elegant derivation is abstracted from notes of lectures by P. Musen of Goddard Space Flight Center and the University of Maryland.