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Radial Heat Conduction in Laser Heating  
of Material Slabs

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13. ABSTRACT  
To quantify the cases in which radial heat conduction is important, this report investigates the heating of metal slabs by laser beams with finite diameters. A two-dimensional approach is taken which assumes cylindrical symmetry, and the method of finite differences is used to determine temperature profiles in the slabs as a function of time and incident flux intensity. The times necessary to induce melting at the front surface of the slabs are obtained in the two-dimensional approach and compared with the times obtained in the one-dimensional approach. This gives a direct indication of the importance of radial heat conduction.

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# RADIAL HEAT CONDUCTION IN LASER HEATING OF MATERIAL SLABS

## INTRODUCTION

It is well known that intense laser beams can heat materials to high temperatures in short times and easily induce melting and vaporization. The incident flux intensity and material properties determine the physical processes involved in the interaction, with the processes becoming more complex as the intensity increases. At relatively low intensities ( $\leq 10^3$  W/cm<sup>2</sup>) the beam induces heating and melting, whereas at the other extreme ( $\geq 10^9$  W/cm<sup>2</sup>) vaporization and ionization occur (see reference). In field applications of laser-induced heating, atmospheric effects and long propagation paths will restrict operation to heating and melting. Therefore, the coupling of the incident energy to the material will be a critical problem.

In theoretically treating the problem of laser heating of a material, the coordinate system for analysis is usually chosen for mathematical expediency. In particular, it is tempting to use a one-dimensional approach which ignores geometry and inhomogeneous heating but allows a high degree of sophistication in the analysis. However, such an approach implies heat conduction in only one direction and irradiation of infinite slabs by beams of infinite diameters. If the objects being irradiated are flat and of the same or smaller size as the beam diameter, or if the beam intensity is sufficiently high to cause very rapid heating, the one-dimensional approach may give meaningful results. However, in realistic applications this is generally not the case, and radial heat conduction and geometric corrections must be considered.

To quantify the cases in which radial conduction is important, this report investigates the heating of metal slabs by laser beams of finite diameters. A two-dimensional approach is taken which assumes cylindrical symmetry, and the method of finite differences is used to determine temperature profiles in the slabs as a function of time and incident flux intensity. The times necessary to induce melting at the front surface of the slabs are obtained in the two-dimensional approach and compared with the times obtained in the one-dimensional approach. This gives a direct indication of the importance of radial conduction.

## ANALYSIS

The heat conduction equation for cylindrical symmetry is

$$\frac{\partial T}{\partial t} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (1)$$

where

- $\kappa = \frac{\rho c}{k}$  = diffusion coefficient,
- $\rho$  = density,
- $c$  = specific heat, and
- $k$  = thermal conductivity.

For laser heating at the front surface, the boundary condition is

$$k \frac{\partial T}{\partial x} = - aP, \quad (2)$$

where

$P$  = incident power per unit area

and

$a$  = absorptance.

If the slab is divided into  $N + 1$  square sections of dimension  $h_x = h_y$ , the above equations become, in finite-difference form,

$$T(x, y, t + h_t) = \frac{1}{4} [T(x + h_x, y, t) + T(x - h_x, y, t) + T(x, y + h_y, t) + T(x, y - h_y, t)] \quad (3)$$

and

$$T(x, y, t) = \frac{aP}{k} h_x + T(x + h_x, y, t), \quad (4)$$

where

$$h_x^2 = h_y^2 = 4\kappa h_t. \quad (5)$$

These equations were programmed and computations made run for the irradiation of aluminum and stainless-steel plates. Other materials could have been easily studied, but it was felt that these two were representative. In all cases it was assumed that the plates were insulated at the back surfaces and at the front surfaces outside the beam diameter. This condition could easily be relaxed by assuming a heat loss due to convection or radiation. Although the edges of the plates were assumed to be insulated, the plate diameters were made sufficiently large to absorb the radially conducted heat. The program is listed in the Appendix.

## RESULTS AND SUMMARY

The results are summarized in the figures. An exhaustive set of cases could have been run, but it was felt that those treated define the regions in which a one-dimensional analysis is not sufficient to predict times to reach melting. It is also felt that this report shows the utility of the finite-differences method in treating realistic problems. Only slight modifications of the program would be necessary to accommodate Gaussian beam profiles (or superposition of such) and heat losses due to convection or radiation at the material surfaces. Also, by using interpolation schemes at the surfaces, the finite-differences method can accommodate curved surfaces and any analytic or step variation with temperature of the material parameters.

## ACKNOWLEDGMENT

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## REFERENCE

M.K. Chun and K. Rose, J. of Appl. Phys. 41, 614 (1970).

## APPENDIX

```

PROGRAM RADCOND
DIMENSION T(8,200,2)
70 FORMAT(15,3E5,2I5,2E5,E10,2E5)
71 FORMAT(15,2F7.3,F5.1,2I5,F5.1,F6.2,F10.1,F7.1,F7.3///)
72 FORMAT(E15,5/(20F5))
73 FORMAT(10(/))
74 FORMAT(20F5)
103 READ 70, N,D,CAP,AMB,LMAX,MMAX,APT,ALPHA,POW,TM,C
IF(EOF,60)101,102
102 PRINT 71,N,D,CAP,AMB,LMAX,MMAX,APT,ALPHA,POW,TM,C
HX = D/(N+1) $ HY = HX $ HT = HX*HX/CAP/4.
J1 = 1 $ J2 = 50 $ I = 1
DO 5 M=1,MMAX
DO 5 L=2,LMAX
5 T(L,M,I) = AMB
7 L = LMAX
DO 10 M=1,MMAX
10 T(L+1,M,I) = T(L,M,I)
M = MMAX
DO 15 L=2,LMAX
15 T(L,M+1,I) = T(L,M,I)
L = 2
DO 25 M=1,MMAX
R = (M-1)*HY
IF(APT-R)22,23,23
22 T(L=1,M,I) = T(L,M,I)
GO TO 25
23 T(L,M,I) = ALPHA*POW*HX/C+T(L+1,M,I)
T(L,M,I+1) = T(L,M,I)
25 CONTINUE
M = 1
DO 30 L=3,LMAX
30 T(L,M,I+1) = (T(L+1,M,I)+T(L-1,M,I)+2.*T(L+2,I))/4.
DO 40 M=2,MMAX
R = (M-1)*HY
IF(APT-R)32,34,34
32 LS = 2
GO TO 35
34 LS = 3
35 DO 36 L=LS,LMAX
36 T(L,M,I+1) = (T(L+1,M,I)+T(L-1,M,I)+T(L,M+1,I)+T(L,M-1,I))/4.
40 CONTINUE
DO 45 M=1,MMAX
DO 45 L=2,LMAX
45 T(L,M,I) = T(L,M,I+1)
IF(TM-T(2,1,1))60,60,50
50 IF(J2-J1)60,60,55
55 J1 = J1+1
GO TO 7
60 TIME = J2*HT
J2 = J2+50
J1 = J1+1
PRINT 72,TIME,(T(2,M,1),M=1,MMAX,10)
PRINT 74, (T(7,M,1),M=1,MMAX,10)
IF(TM-T(2,1,1))100,100,7
100 PRINT 73

```

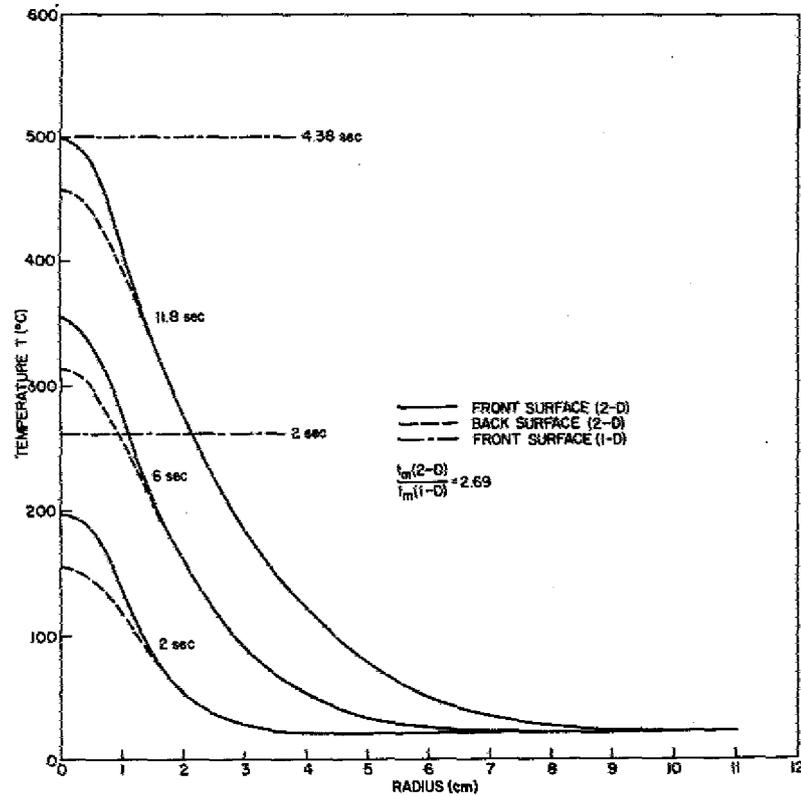


Fig. 1 - A 2024-aluminum disk 0.635 cm thick and 25 cm in radius is irradiated by a 30-kW beam having a radius of 1 cm. The temperature is shown as a function of time and the distance from the center of the beam. The solid curves are for the front-surface temperatures in the two-dimensional (2-D) analysis, as are the dotted curves for the back-surface temperature, and the dash-dot curves are for the front-surface temperatures in the one-dimensional (1-D) analysis. The time for the front surface to reach melting in the two-dimensional analysis is denoted  $t_m(2-D)$  whereas for the one-dimensional analysis it is  $t_m(1-D)$ . In this case it is noted that  $t_m(2-D)/t_m(1-D) = 2.69$ ; that is, it took over two and one half times longer to reach melting in the two-dimensional analysis than in the one-dimensional analysis. Also, the effects of inhomogeneous heating can be seen, and the material remains at ambient temperature ( $20^\circ\text{C}$ ) beyond 10 cm.

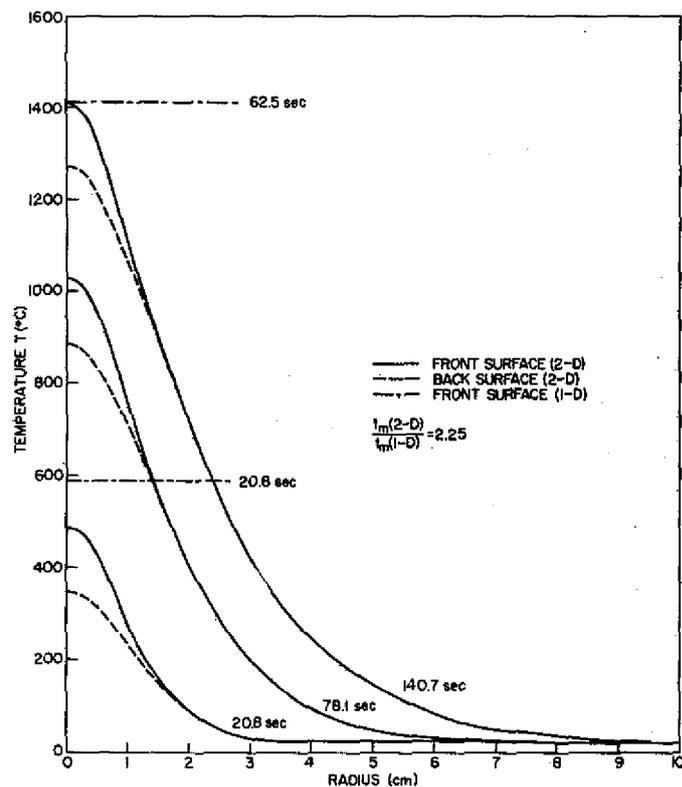


Fig. 2 - A 304-stainless-steel disk with identical dimensions as for the aluminum disk of Fig. 1 is irradiated by a 1,5-kW beam with a radius of 1 cm. The notation of this figure is the same as for Fig. 1. In this case it took two and one quarter times longer to reach melting in the two-dimensional analysis than in the one-dimensional analysis. Again, the effects of inhomogeneous heating can be seen, and the material remains at ambient temperature (20°C) beyond 10 cm.

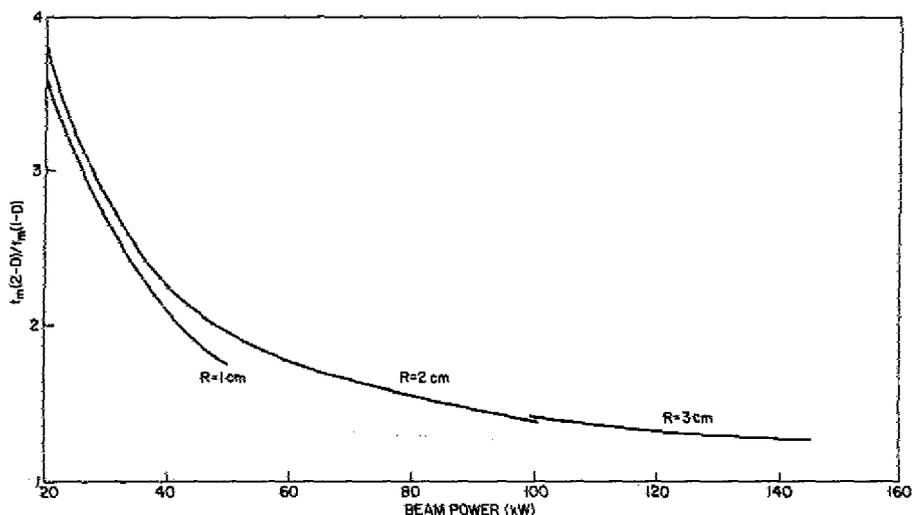


Fig. 3 - A 2024-aluminum disk 0.635 cm thick and 25 cm in radius is irradiated with beams of various powers and radii. The ratios of the two-dimensional-analysis times to reach melting to the one-dimensional-analysis times are plotted as a function of beam power and radius. The curves go asymptotically to 1, with the ratio still about 1.2 at powers as high as 150 kW. Also, below 50 kW the ratios increase rapidly.

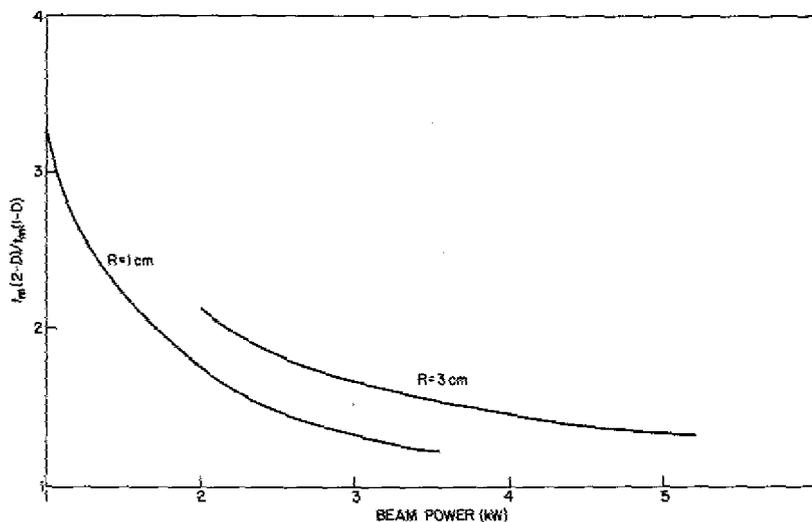


Fig. 4 - With the same notation as for Fig. 3, a 304-stainless-steel disk with identical dimensions as for the aluminum disk is irradiated with beams of various powers and radii. Again the curves go asymptotically to 1, and the ratios increase rapidly below 3 kW.