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On Statistical Multimode VLF Field Strength Predictions

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13. ABSTRACT

A method has been derived for predicting the time availability of vlf field strengths by using the earth ionosphere waveguide model. For the present calculations the ionospheric parameters β and h of Wait and Spies are condisered to be independent random variables. The field strengths at various distances from the transmitter are calculated as dependent random variables. The results indicate that the statistical distribution of the received field strength will vary as a function of distance from the transmitter.

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ON STATISTICAL MULTIMODE VLF FIELD STRENGTH PREDICTIONS

INTRODUCTION

The time availability of a given field strength during a given time interval is defined as the probability that that field strength will be equaled or exceeded continuously during that interval. In the past many deterministic calculations have been performed on the field strength versus distance from a transmitter under conditions which assume an ionospheric height and an electron density profile. Such calculations have not yet been applied to the production of time availability predictions. This report will show one method by which these deterministic predictions can be used to make statistical predictions.

MATHEMATICAL APPROACH

The approach described here uses linearized expressions for the various propagation parameters as functions of ionospheric profile parameters. A statistical distribution is chosen which may hopefully characterize each ionospheric parameter.

Numerous representative values of the ionospheric parameters are generated according to the selected probability distributions using a random number generator routine. From these sets of ionospheric parameters, sets of propagation parameters are calculated. Next, field strengths are calculated as a function of distance for each set of the ionospheric parameters. Histograms of the field strength distribution are constructed for different distances from the transmitter by counting at each distance the number of times that the calculated field strengths fall within given decibel intervals. When a large number of calculations have been performed, the histograms will be representative of the theoretical probability distribution of the field strength expected at that distance from the transmitter. The time availability of a given signal level is determined by simply counting the total number of counts in each bin or interval, starting from the lowest bin. The lowest bin which has one tenth of the total counts below it, for example, represents a field strength which has approximately ninety percent time availability.

As an example of this technique, the daytime field strength variability as a function of distance was calculated for 24.0 kHz. The ionospheric parameters considered were the ionospheric reference height h and the exponential coefficient of the ionospheric conductivity β, according to Wait and Spies*. Each parameter is assumed to be independently variable, and each is characterized by a Gaussian probability distribution. The mean ionospheric height $\langle h \rangle$ is 70 km, the mean beta $\langle \beta \rangle$ is 0.5 km^{-1}, the standard deviation of the height σ_h is 1 km, and the standard deviation of the beta σ_β is 0.05 km^{-1}. The linearized expressions for the excitation factor Λ_i, the attenuation rate α_i, and the phase velocity v_i, for the i^{th} mode, are

*J. R. Wait and K. P. Spies, "Characteristics of the Earth-Ionosphere Wave-guide for VLF Radio Waves," National Bureau of Standards Technical Note 300, 1964.

$$20 \log_{10} |\Lambda_i(h,\beta)| = 20 \log_{10} |\Lambda_i(70, 0.5)| + (\beta - 0.5) \frac{\partial}{\partial\beta}(20 \log_{10} |\Lambda_i|) + (h-70)\frac{\partial}{\partial h}(20 \log_{10} |\Lambda_i|) \quad (1)$$

$$\alpha_i(h,\beta) = \alpha_i(70, 0.5) + (\beta - 0.5)\frac{\partial\alpha_i}{\partial\beta} + (h-70)\frac{\partial\alpha_i}{\partial h} \quad (2)$$

$$v_i(h,\beta) = v_i(70, 0.5) + (\beta - 0.5)\frac{\partial v_i}{\partial\beta} + (h-70)\frac{\partial v_i}{\partial h} \quad (3)$$

$$\arg \Lambda_i(h,\beta) = \arg \Lambda_i(70, 0.5) + (\beta - 0.5)\frac{\partial}{\partial\beta}(\arg \Lambda_i) + (h-70)\frac{\partial}{\partial h}(\arg \Lambda_i L). \quad (4)$$

The values calculated for the partial derivative using Wait and Spies are given in Table 1. One-thousand independent pairs of parameters β and h were obtained in the

Table
Propagation Parameters and Their Derivatives
(Assumed to be evaluated at $h = 70$ km, $\beta = 0.5$ km^{-1})

Quantities	1st Mode	2nd Mode
$20 \log_{10} \|\Lambda_i\|$	-5.6 dB	2.25 dB
$\frac{\partial}{\partial h}(20 \log_{10} \|\Lambda_i\|)$	-0.33 dB/km	0.40 dB/km
$\frac{\partial}{\partial\beta}(20 \log_{10} \|\Lambda_i\|)$	12.0 dB-km	-1.1 dB-km
α_i	2.05 dB/Mm	3.5 dB/Mm
$\frac{\partial\alpha_i}{\partial h}$	0.01 dB/Mm-km	-0.21 dB/Mm-km
$\frac{\partial\alpha_i}{\partial\beta}$	-5.75 dB-km/Mm	-14.0 dB-km/Mm
$\arg(\Lambda_i)$	11°	1.2°
$\frac{\partial}{\partial h}\arg(\Lambda_i)$	0.37°/km	0.03°/km
$\frac{\partial}{\partial\beta}\arg(\Lambda_i)$	-16.5°-km	16.0°-km
v_i/c	0.9975	1.005
$\frac{1}{c}\frac{\partial v_i}{\partial h}$	-0.00012 km^{-1}	-0.00037 km^{-1}
$\frac{1}{c}\frac{\partial v_i}{\partial\beta}$	-0.0025 km	-0.01 km

above-mentioned probability distributions. From these sets of h and β values, 1000 sets of the propagation parameters were calculated according to Eqs. (1) through (4). Using these sets of propagation parameters, 1000 sets of field strength versus distance calculations were made. From Wait and Spies, the equations used to calculate the field strength are

$$E_r = \frac{e^{-ika\theta}}{a(\theta \sin \theta)^{1/2}} V \tag{5}$$

$$V = \frac{4(\pi x)^{1/2}}{y_0} e^{-i\pi/4} \sum_{n=1}^{\infty} e^{-ixt_n} G_n(y)\, G_n(\hat{y})\Lambda_n, \tag{6}$$

where $x = (ka/2)^{1/3}\theta$, $\hat{y} = (2/ka)^{1/3} k z_0$, and $y = (2/ka)^{1/3} k\,(r - a)$. The symbols are defined as follows:

k	free-space propagation constant of the signal $(k = \omega/c)$
ω	angular frequency of the signal
a	radius of the earth
$a\theta$	great circle distance between the transmitter and the receiver
z_0	height of the transmitter above the earth's surface
$(r - a)$	height of the receiver above the earth's surface
y_0	reference height of the ionosphere
n	mode number
G_n	height gain function of the i^{th} waveguide mode
Λ_n	excitation factor of the i^{th} waveguide mode
t_n	parameter directly related to the attenuation and phase velocity of the i^{th} waveguide mode.

The attenuation rate for the i^{th} mode α_i measured in decibels per unit distance is given by

$$\alpha_i = (20 \log_{10} e)(ka/2)^{1/3} \mathrm{Im}\,(t_i)/a. \tag{7}$$

The phase velocity v_i of the i^{th} mode is given by

$$v_i = \frac{c}{1 + \mathrm{Re}(t_i)(2^{1/2}ka)^{-2/3}}. \tag{8}$$

Field strengths in decibels above 1μ V/m were evaluated at 100-km intervals between 500 km and 5000 km from the transmitter. At each location a histogram was constructed from the calculated field strength. The bin size of the histogram was set equal to 0.1 dB. These histograms are shown in Figs. 1 through 5. Figure 6 shows plots of the field strength which have time availabilities of 50%, 90%, and 99%. Figure 6 also has a field strength versus distance plot for the basic $\beta = 0.5$ km^{-1} and $h = 70.0$ km case and a plot of the variance as determined from the sample distributions.

DISCUSSION

Figures 1 through 6 illustrate two important properties of the field strength. First, the plots of field strength with a given time availability (Fig. 6) show a variation with distance which reflects the modal structure of the basic case ($\beta = 0.5$, $h = 70$). Second, the statistics of the field strength also vary with distance in a complicated way which reflects the basic multimode structure. This distance variation in the statistics is readily evident when the shape of the histograms for different locations is compared.

A significant problem in applying this technique to useable vlf time availability predictions is further knowledge of the correct probability distribution to be chosen for h and β. In this work a Gaussian distribution with certain parameters is arbitrarily chosen. Whether this distribution and set of parameters are correct can only be determined by further experiment and data reduction.

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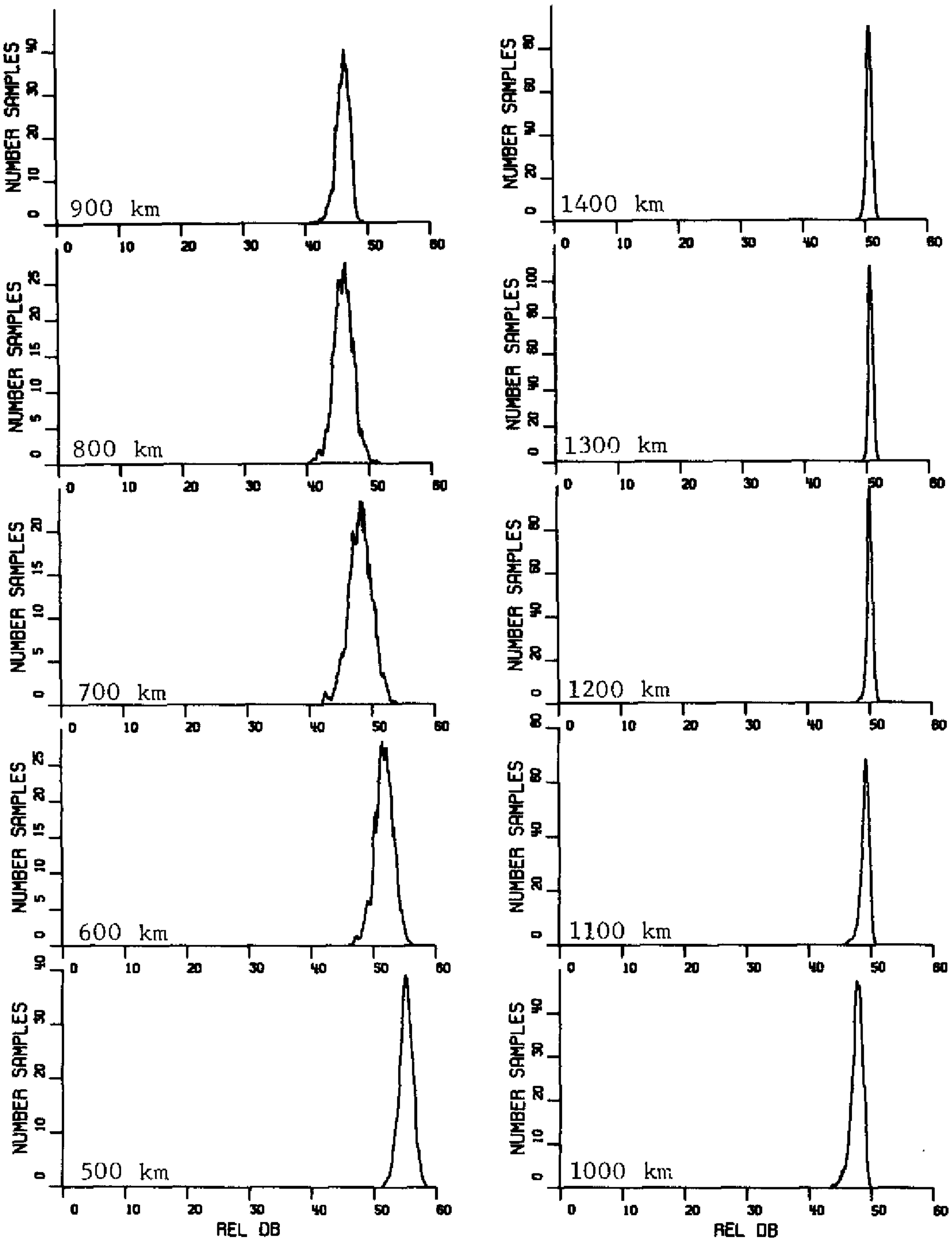

Fig. 1 - Histograms of calculated field strengths (500 to 1400 km)

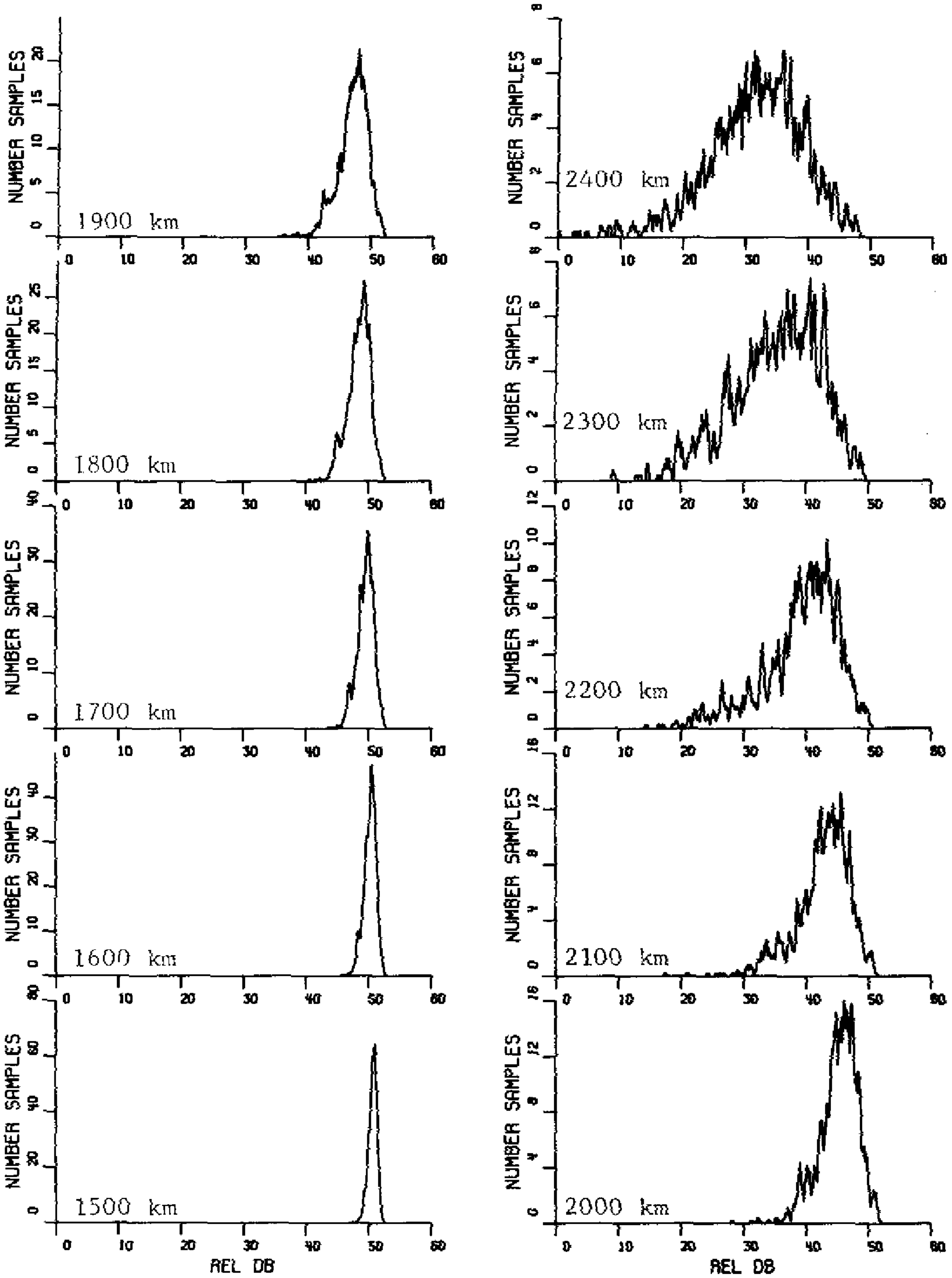

Fig. 2 - Histograms of calculated field strengths (1500 to 2400 km)

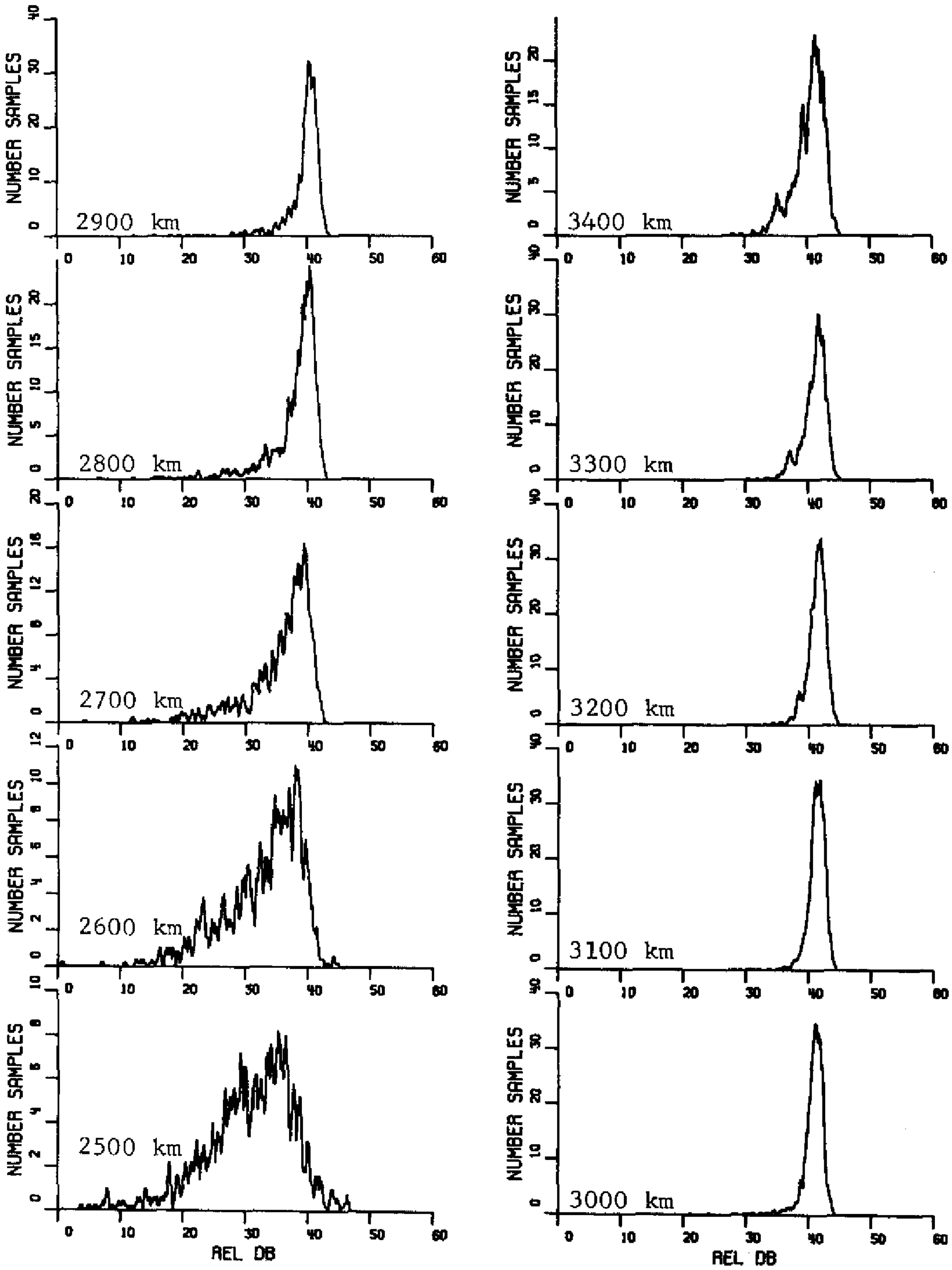

Fig. 3 - Histograms of calculated field strengths (2500 to 3400 km)

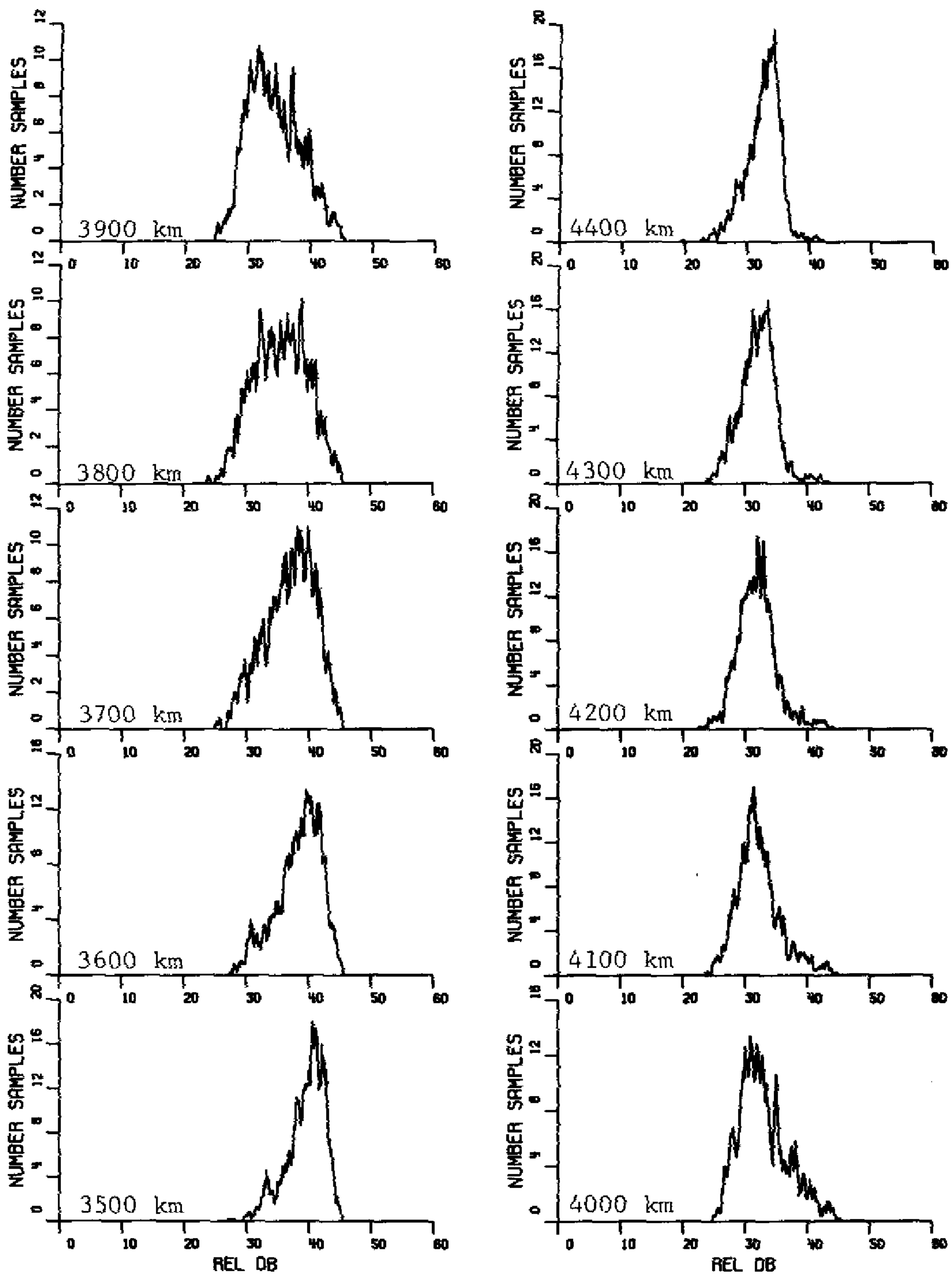

Fig. 4 - Histograms of calculated field strengths (3500 to 4400 km)

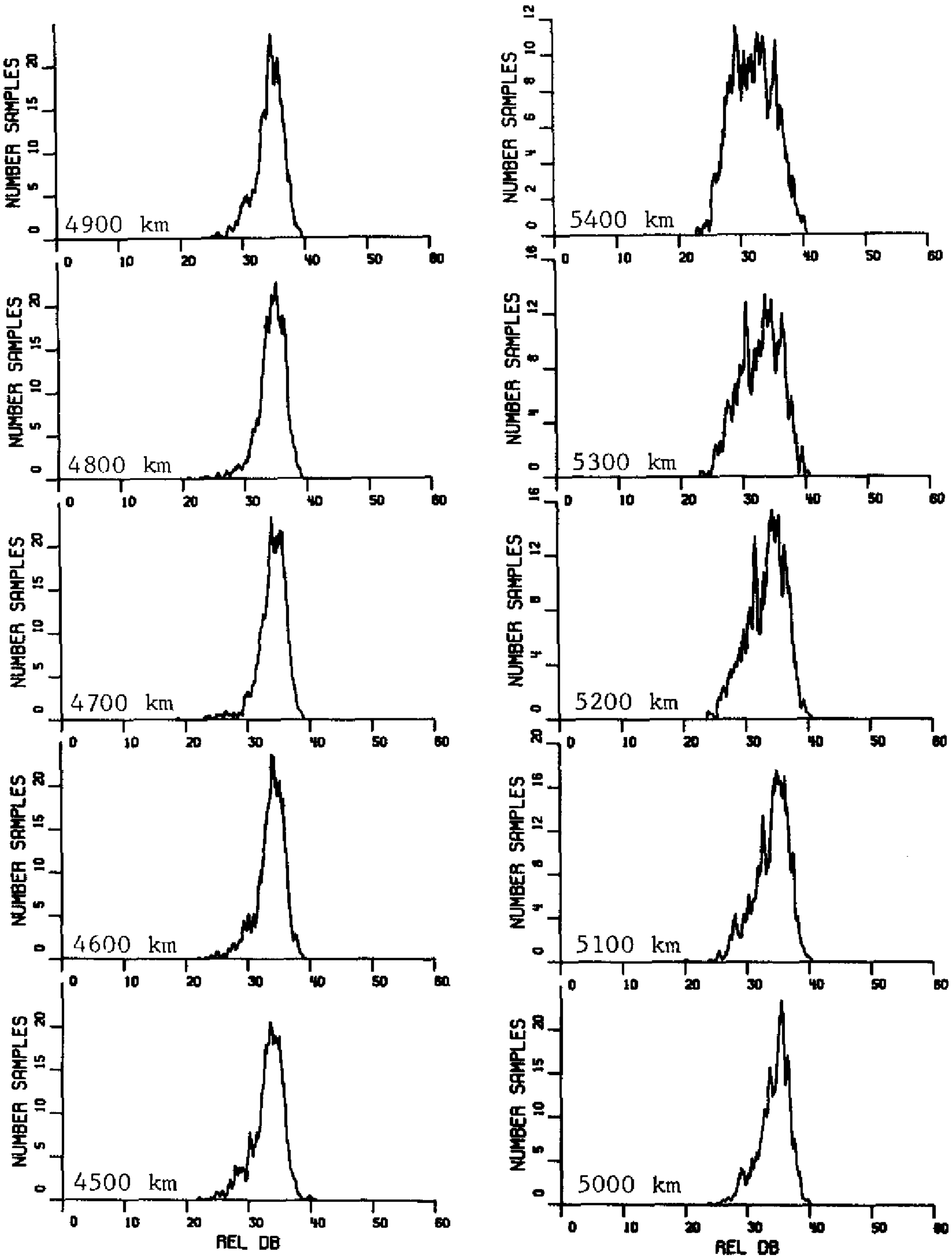

Fig. 5 - Histograms of calculated field strengths (4500 to 5400 km)

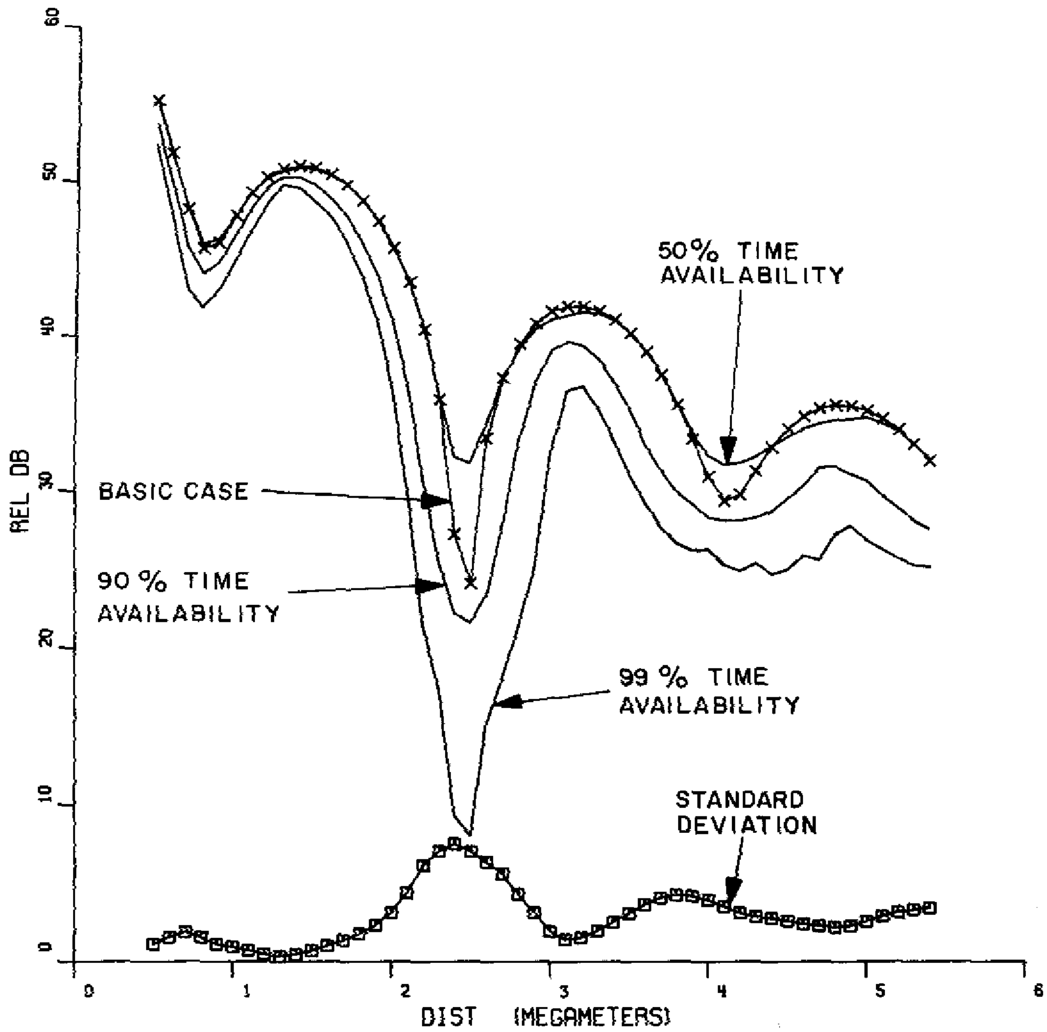

Fig. 6 - Plots of field strengths having 99%, 90%, and 50% time availabilities versus distance and of standard deviation of field strength versus distance; and of basic multimode field strength ($\beta = 0.5$ km^{-1} and h = 70 km) case