NRL Report 7138

Common Right Factors of a Meromorphic Function and Its Derivatives

FRED GROSS AND CHUNG-CHUN YANG

Mathematics Research Center
Report 70-7
Mathematics and Information Sciences Division

September 23, 1970

NAVAL RESEARCH LABORATORY
Washington, D.C.

Common Right Factors of a Meromorphic Function and Its Derivatives

FRED GROSS AND CHUNG-CHUN YANG

Mathematics Research Center
Mathematics and Information Sciences Division

Abstract: The authors investigate common right factors of entire functions and their derivatives. In particular, it is shown that, assuming F to be a meromorphic function, then if F, F'', and $F^{(iv)}$ have the same right factor g, then g is one of the four forms $(az^2 + bz + C)^\delta$, $(Ae^{cz} + B)^\delta$, $[A \cos (cz + d) + B]^\delta$, or an elliptic function of second order, where a, b, c, d, A, B, and C are constants and $\delta = \pm 1$.

INTRODUCTION

In accordance with the definitions in Refs. [1] and [2], a meromorphic function $h(z) = f(g(z))$ is said to have $f(z)$ and $g(z)$ as left and right factors respectively, provided that $f(z)$ is nonlinear and meromorphic and $g(z)$ is nonlinear and entire (g may be meromorphic when $f(z)$ is a rational function). It was shown in Ref. [2] that a meromorphic function f and its derivative f' cannot have a common right factor other than one of the form $e^{cz+b} + d$, where c, b, and d are constants.

In this work we are primarily concerned with common right factors of a meromorphic function F and its second derivative F''. This problem (as well as the analogous problem for F and its nth derivative $F^{(n)}$) seems much more difficult than the F, F' case. In general one would expect the following conjecture to hold:

CONJECTURE. *Let F be meromorphic. Any common right factor of F and F'' must have one of the following forms: $(az^2 + bz + c)^\delta$, $(Ae^{cz} + B)^\delta$, $[A \cos (cz + d) + B]^\delta$, or elliptic functions of second order, where a, b, c, d, A, and B are constants and $\delta = \pm 1$.*

The proof of the above conjecture reduces to the problem of finding meromorphic solutions f, g, and ℓ of the equation

$$g'^2 f''(g) + g'' f'(g) = \ell(g).$$

Though we have not succeeded in solving this problem, we can find simultaneous solutions of pairs of such equations. These simultaneous solutions, as we shall see, lead to some interesting results which are special cases of our conjecture. We shall prove among other things that the assertion of our conjecture holds for common right factors of F, F'', and $F^{(iv)}$.

PRELIMINARY LEMMAS

LEMMA 1 (Briot and Bouquet [3]). *If a solution of an algebraic differential equation of the first order*

$$P\left(\frac{dw}{dz}, w\right) = 0$$

is uniform in the plane, then the solution is a rational function, a rational function of e^{bz} (b a constant), or an elliptic function (second order).

NRL Problem B01-11; Project RR 003-02-41-6153. This is an interim report on the problem. Manuscript submitted June 16, 1970.

LEMMA 2 (Wittich [4]). *Let*

$$p(z, w, w_1, w_2, \ldots, w_p) \equiv \sum a_{n_0 \ldots n_p}(z) w^{n_0} w_1^{n_1} \ldots w_p^{n_p} = 0$$

be an algebraic differential equation, where $w_i(z) = w^{(i)}(z)$, *the ith derivative of* w. *Let* $n = n_0 + n_1 + \ldots + n_p$ *denote the dimension of the term* $w^{n_0} w_1^{n_1} \ldots w_p^{n_p}$. *Then the above equation has no transcendental entire solutions if only one term appears in the equation with a maximal dimension.*

THEOREMS ON COMMON RIGHT FACTORS OF A FUNCTION AND ITS DERIVATIVES

THEOREM 1. *Let* F *and* H *be two nonlinear meromorphic functions with* $F \not\equiv C_1 H + C_2$, *where* C_1 *and* C_2 *are constants. If* F, F'', H, *and* H'' *have a common right factor* g, *then* g *has one of the forms* $(az^2 + bz + C)^\delta$, $(Ae^{cz} + B)^\delta$, $[A \cos (cz + d) + B]^\delta$, *or an elliptic function of second order, where* a, b, c, d, A, B, *and* C *are constants and* $\delta = \pm 1$.

Proof. From the hypotheses of the theorem we have

$$F = \ell_1(g) \quad \text{and} \quad F'' = \ell_2(g) \tag{1}$$

and

$$H = h_1(g) \quad \text{and} \quad H'' = h_2(g), \tag{2}$$

where ℓ_i and h_i $(i = 1, 2)$ are meromorphic and g is entire or ℓ_i and h_i are rational and g is meromorphic. From Eqs. (1) and (2) one obtains

$$\ell_2(g) = \ell_1''(g) g'^2 + \ell_1'(g) g'' \tag{3}$$

and

$$h_2(g) = h_1''(g) g'^2 + h_1'(g) g''. \tag{4}$$

Since $\ell_1'' h_1' - \ell_1' h_1'' \not\equiv 0$ (otherwise it would lead to $F \equiv C_1 H + C_2$, contradicting our hypotheses), one can eliminate g'' from Eqs. (3) and (4) and obtain

$$\begin{aligned} g'^2 &= \frac{h_1'(g)\,\ell_2(g) - \ell_1'(g)\, h_2(g)}{h_1'(g)\,\ell_1''(g) - h_1''(g)\,\ell_1'(g)} \\ &= R(g). \end{aligned} \tag{5}$$

With the aid of a result of Clunie ([5], p. 54) one easily shows (see [3], p. 216) that $R(z)$ must be a rational function.

We have the following cases:

Case A. g is a rational function (but not the form $1/az + b$).

Case B. g is a transcendental function.

Case A. When g is a polynomial, one easily concludes from (5) that g is a second degree. When $g(z) = g_1(z)/g_2(z)$, where the $g_i(z)$ $(i = 1, 2)$ are polynomials and $g_2(z)$ is not a constant, then either $R(z)$ is a polynomial or $R(z) = p_1(z)/p_2(z)$, where p_1 and p_2 are polynomials and $p_2(z) = (z - \alpha)^n$ (n a positive integer) and $g(z) - \alpha \neq 0$. That is, either $R(z)$ is a polynomial or $g(z) = \alpha + [1/h(z)]$, where $h(z)$ is a polynomial. Suppose that $R(z)$ is a polynomial. A count of the poles of $g(z)$ and $g'(z)$ reveals that $R(z)$ is either of degree 3 or 4. Thus, we may write

$$R(z) = (z - \alpha_1)^{n_1}(z - \alpha_2)^{n_2}(z - \alpha_3)^{n_3}(z - \alpha_4)^{n_4} \qquad (\Sigma n_i \leqslant 4).$$

We may assume without any loss of generality that the degree of $g_1 \neq$ degree of g_2. Looking at the orders of the zeros of $g(z) - \alpha_i$ leads to the conclusion that either $n_i = 1$ or $g(z) - \alpha_i \neq 0$ $(i = 1, 2, 3, 4)$. Replacing g in (5) by g_1/g_2 and using a simple degree argument one finds that $n_i = 1$ for at most one i and that in fact $g(z) - \alpha_i \neq 0$. Thus, in any case $g(z)$ has the form $g(z) = \alpha + [1/h(z)]$.

Again using (5) one sees easily that $h(z)$ is a second-degree polynomial. This completes the proof of case A.

Case B. $R(g)$ has the form

$$R(g) = \frac{P_1(g)}{P_2(g)},$$

where $P_1(z)$ and $P_2(z)$ are relatively prime polynomials. If $P_2(z)$ is nonconstant, then g must omit some value, say a, so that $1/(g-a)$ is entire. Thus we may assume in this case without any loss of generality that g is entire. We have

$$g'^2\, P_2(g) = P_1(g).$$

Thus g is entire and has at least one finite Picard exceptional value and at least one other completely ramified value. It follows that g is a constant. This, of course, is not the case. Thus P_2 must be a constant and $R(z)$ must be a polynomial. Since g cannot have more than four completely ramified values, the degree of R is at most 4. Furthermore, if g has a pole, one easily verifies that the degree of $R(z)$ is greater than 2, and if it is entire, it follows by Lemma 2 that the degree $R(z) = 2$. When the degree of $R = 3$ or 4, one can conclude from (5) that g is an elliptic function (second order). When the degree of $R = 2$, we have

$$g'^2 = \alpha(g-a)^2 + b.$$

From this it follows that

$$g = A \cos(cz + d) + B$$

or

$$g = Ae^{cz} + B,$$

where A, B, c, and d are constants. This completes the proof of Theorem 1.

As a special case of Theorem 1 we have the following:

THEOREM 2. *Let F be a meromorphic function. If F, F'', and $F^{(iv)}$ have the same right factor g, then g is one of the four forms in Theorem* 1.

Proof. We set $H = F''$. If $F = C_1H + C_2$, where C_1 and C_2 are constants and $C_1 \neq 0$, then we have $F = C_1F'' + C_2$, so F is entire and has the form

$$\alpha_0 + \beta_1 e^{\alpha_1 z} + \beta_2 e^{\alpha_2 z}, \quad \alpha_1 = \pm\alpha_2 \ (\alpha_i \neq 0,\ i = 1, 2).$$

Thus,

$$F'' = \beta_3 e^{\alpha_1 z} + \beta_4 e^{\alpha_2 z} \quad \text{and} \quad F^{(iv)} = \beta_5 e^{\alpha_1 z} + \beta_6 e^{\alpha_2 z},$$

where β_3, β_i $(i = 1, 2, \ldots, 6)$ are constants. Now F, F'', and $F^{(iv)}$ are pseudo-prime [2]. Therefore two possibilities exist:

Case 1. The left factors are rationals. Then

$$F'' = R_1(g) = \beta_3 e^{\alpha_1 z} + \beta_4 e^{\alpha_2 z} \tag{6}$$

and

$$F^{(iv)} = R_2(g) = \beta_5 e^{\alpha_1 z} + \beta_6 e^{\alpha_2 z}, \tag{7}$$

where R_1 and R_2 are rational functions. One can solve for $e^{\alpha_1 z}$ from Eqs. (6) and (7) and obtain

$$e^{\alpha_1 z} = R_3(g), \tag{8}$$

where $R_3(z)$ is a rational function. Since F is entire, one may assume without any loss of generality that g is also entire. Thus it is easy to conclude that

$$g = a + be^{\alpha z},$$

where a, b, and α are constants.

Case II. The right factors are polynomials. It follows from Ref. [6] that the polynomials must be of second degree. This completes the discussion of the case $F = C_1F'' + C_2$. The case $F \not\equiv C_1F'' + C_2$ follows from the previous theorem. Thus, our proof is complete.

It is easy to verify that g can in fact have any of the forms given in the theorem. For example, let $E(z)$ be an elliptic function of order 2, and let f be any rational function; then both the function $F(z) = f(E(z))$ and its second derivative $F''(z)$ have $E(z)$ as a common right factor.

Our method can be used for more general problems of the same type. For example, one can easily prove the following analog of Theorem 2.

THEOREM 2′. *Let F be a meromorphic function. If F, $F^{(n)}$, ..., $F^{(n^2)}$ have the same right factor g for any integer $n \geqslant 2$, then g is a rational function, a rational function of e^{bz}, or an elliptic function.*

We now return to our original problem and study the common right factors of F and F'' for some interesting special cases.

THEOREM 3. *Let F be a meromorphic function. There do not exist meromorphic functions ℓ and g with g nonlinear such that $F = \ell(g)$ and $F'' = \ell''(g)$.*

Proof. From the hypotheses we have

$$\ell''(g) = F'' = [\ell(g)]'' = \ell''(g)g'^2 + \ell'(g)g''. \tag{9}$$

From this we have

$$\frac{\ell''(g)}{\ell'(g)} = \frac{g''}{1 - g'^2}. \tag{10}$$

As in Theorem 1 we conclude, by a growth argument, that $R(z) \equiv \ell''(z)/\ell'(z)$ is a rational function and

$$R(z) = Q(z) + \sum_{i=1}^{t} \frac{n_i}{z - \alpha_i},$$

where $Q(z)$ is a polynomial, the α_i are poles of R, and the n_i are integers $(i = 1, 2, \ldots, t)$.

We have

$$R(g) = Q(g) + \sum_{i=1}^{k} \frac{n_i}{g - \alpha_i} = \frac{g''}{1 - g'^2} \quad (\text{unless } 1 - g'^2 \equiv 0). \tag{11}$$

If g has a pole of order k at $z = z_0$, then $z = z_0$ is a zero of order k of $g''/(1 - g'^2)$. Thus $Q(z) \equiv 0$ unless g is entire.

We consider two cases.

Case a. $Q(z) \not\equiv 0$. This is the case when ℓ is not a rational function and g is entire.

Case b. $Q(z) \equiv 0$.

In the first case if $1 - g'^2 \not\equiv 0$, we have

$$\frac{P_1(g)}{P_2(g)} = \frac{g''}{1 - g'^2}, \tag{12}$$

where P_1 and P_2 are polynomials and the degree of $P_1 \geqslant$ the degree of P_2. It follows from Lemma 2 that g must be a polynomial and in fact $g \equiv 0$. Hence $1 - g'^2 \equiv 0$ or $g = \pm z + C$.

For case b, $Q(z) \equiv 0$, we have

$$\sum_{i=1}^{t} \frac{n_i}{g - \alpha_i} = \frac{g''}{1 - g'^2}. \tag{13}$$

Multiplying both sides of the above equation by g' and integrating, one obtains

$$1 - g'^2 = C \prod_{i=1}^{k} (g - \alpha_i)^{-2n_i} \tag{14}$$

or

$$g'^2 = R^*(g),$$

where R^* is a rational function. As in the proof of the previous theorem one easily verifies that $R^*(g)$ and thus g'^2 must be a polynomial in g of degree at most 4. Furthermore, from Eq. (10) one gets $\mathcal{L}'(g) = (1 - g'^2)^{-1/2}$. Thus, $\mathcal{L}$ is not meromorphic, contrary to our hypotheses.

Along similar lines we have

THEOREM 4. *Let F be a nonlinear meromorphic function. There do not exist meromorphic functions $\mathcal{L}$ and g (g nonconstant) such that $F = \mathcal{L}(g)$ and $F'' = \mathcal{L}'(g)$ with $\mathcal{L}$ and $\mathcal{L}'$ nonlinear.*

Proof. Let us assume that g is entire. Proceeding as before, we obtain

$$R(g) \equiv \frac{\mathcal{L}''(g)}{\mathcal{L}'(g)} = \frac{1 - g''}{g'^2}, \tag{15}$$

where

$$R(z) = P(z) + \sum_{i=1}^{t} \frac{n_i}{z - \alpha_i},$$

in which $P(z)$ is a polynomial, the n_i are integers, and the α_i are constants. By virtue of Lemma 2 one can again conclude that $P(z) \equiv 0$. Hence, we have

$$R(z) = \sum_{i=1}^{t} \frac{n_i}{z - \alpha_i} = \frac{\mathcal{L}''(z)}{\mathcal{L}'(z)}. \tag{16}$$

Using (15) and (16) and Nevanlinna's theorem on ramification values one easily verifies that $t \leqslant 2$. We will consider two cases: (i) $t = 2$ and (ii) $t = 1$.

Case i, $t = 2$. We have

$$\frac{1 - g''}{g'^2} = \frac{n_1}{g - \alpha_1} + \frac{n_2}{g - \alpha_2} = \frac{\mathcal{L}''(g)}{\mathcal{L}'(g)}. \tag{17}$$

If $g - \alpha_1$ has a zero of order k, then g' has a zero of order $k - 1$. If $1 - g'' \not\equiv 0$, then we have $k = 2(k - 1)$ or $k = 2$. If $1 - g'' \equiv 0$, then $g'' \neq 0$ and $k = 2$ again. Thus $g - \alpha_1 = h_1^2$ and $g - \alpha_2 = h_2^2$ or $h_1^2 + \alpha_1 = h_2^2 + \alpha_2$ or

$$\left(\frac{h_1}{\sqrt{\alpha_2-\alpha_1}}\right)^2+\left(\frac{ih_2}{\sqrt{\alpha_2-\alpha_1}}\right)^2=1.$$

It follows from Ref. [7] or [8] that

$$\frac{h_1}{\sqrt{\alpha_2-\alpha_1}}=\sin\varphi(z)\quad\text{and}\quad\frac{ih_2}{\sqrt{\alpha_2-\alpha_1}}=\cos\varphi(z)$$

or $h_1^2=\lambda\sin^2\varphi(z)$, where $\varphi(z)$ is an entire function and $\lambda=\alpha_2-\alpha_1\neq 0$. So we have

$$g=\lambda\sin^2\varphi(z)+\alpha_1.$$

Substituting this back in (17) and simplifying it, we have

$$(4\lambda n_1+2\lambda)\varphi'^2\cos^2\varphi+(-4\lambda n_2-2\lambda)\varphi'^2\sin^2\varphi-2\lambda\varphi''\sin\varphi\cos\varphi\equiv 1.$$

By a well known theorem of Borel (proved by Nevanlinna) ([9], p. 113) one can conclude that

$$(n_1+n_2+1)\varphi'^2-\frac{\lambda}{2i}\varphi''=0.$$

This implies that φ is a constant, contradicting our hypotheses.

Case ii, $t=1$. In this case ℓ' would have to be of the form

$$\ell'(w)=(w-a)^n;$$

consequently,

$$\ell(w)=\frac{1}{n+1}(w-a)^{n+1}+C.$$

Thus, $F=\ell(g)=[1/(n+1)](g-a)^{n+1}+C$, $F''=(g-a)^n$, and $[(n+1)(F-C)]^n=F''^{n+1}$. If F has a pole of order $k>0$, then we have $nk=(n+1)(k+2)$ or $2n+k+2=0$; so n must be negative. Since ℓ is nonlinear, $|n+1|>1$; consequently $n\leqslant -3$.

From

$$\frac{1-g''}{g'^2}=\frac{n}{g-\alpha}\tag{18}$$

it follows that $g-\alpha=h^2$, where h is an entire function. Thus $g'=2hh'$ and $g''=2(h'^2+hh'')$. Hence (18) becomes

$$1-2(h'^2+hh'')=\frac{4nh^2h'^2}{h^2}$$

or

$$1-(2+4n)h'^2-2hh''=0.\tag{19}$$

To solve (19), let $S=h'$. Then

$$h''=S'=\frac{dS}{dz}=\frac{dS}{dh}\frac{dh}{dz}=\frac{dS}{dh}S.$$

Hence we have

$$1-(2+4n)S^2-2hS\frac{dS}{dh}=0\tag{20}$$

or

$$\frac{2S\,dS}{1-(2+4n)S^2}=\frac{dh}{h}. \tag{21}$$

Let $u = 1-(2+4n)S^2$, so that $du = -2(2+4n)S\,dS$ or $2S\,dS = -du/(2+4n)$. Hence (21) becomes

$$\frac{-du}{u}\,\frac{1}{2+4n}=\frac{dh}{h} \quad \text{and} \quad u^{-1/(2+4n)}=Ch, \tag{22}$$

where C is a constant. Hence we have

$$1-(2+4n)S^2 = C'h^{-(2+4n)}, \tag{23}$$

where C' is a constant. Since $n \leqslant -3$, then $2+4n \leqslant -10$. Thus, Eq. (23) cannot hold unless S and h are both constants. Thus we have a contradiction, and the theorem is proved when g is entire.

Assume that the theorem is false for meromorphic g. One easily shows that for integers n_i $(i=1, 2, 3, 4)$

$$\frac{1-g''}{g'^2}=\frac{n_1}{g-\alpha_1}+\frac{n_2}{g-\alpha_2}+\frac{n_3}{g-\alpha_3}+\frac{n_4}{g-\alpha_4}=\frac{\mathfrak{L}''(g)}{\mathfrak{L}'(g)}. \tag{24}$$

Hence, for some constant C,

$$F''=\mathfrak{L}'(g)=C(g-\alpha_1)^{n_1}(g-\alpha_2)^{n_2}(g-\alpha_3)^{n_3}(g-\alpha_4)^{n_4}.$$

By virtue of what we proved for entire g we may assume that g has a pole z_0 of order t. $F' = g'\mathfrak{L}'(g) = Cg'(g-\alpha_1)^{n_1}(g-\alpha_2)^{n_2}(g-\alpha_3)^{n_3}(g-\alpha_4)^{n_4}$ has a pole at z_0 of order $d = (n_1+n_2+n_3+n_4)t+(t+1)$ whenever $d>0$. Thus F'' has a pole of order $d+1$, and we have

$$(n_1+n_2+n_3+n_4)t=(n_1+n_2+n_3+n_4)t+t+2$$

or $t+2=0$, which is not the case. Thus $d \leqslant 0$. A similar argument shows that $d<0$ is also not possible. Hence $d=0$ and $n_1+n_2+n_3+n_4=-2$. Thus $F''(z)$ has a zero of order $2t \geqslant 2$. However, $F'(z)$ has a zero z_0 of order $2t-(t+1)=t-1$. We must have $t-1>2t$. Since this is impossible for $t \geqslant 1$, our theorem follows.

A similar argument can also be used to show the following:

THEOREM 5. *Let f be a meromorphic function and let n and m be any nonzero integers. If f^n and f'^m have the right factor g, then g is a rational function, a rational function of e^{bz} (b constant), or an elliptic function.*

More generally we might conjecture:

CONJECTURE. *Let R_1 and R_2 be rational functions. If $R_1(f)$ and $R_2(f')$ have the same right factor g, then g has one of the forms stated in Theorem 5.*

Even more generally we state the following:

CONJECTURE. *Let f, h, and k be meromorphic functions and $h(f)$ and $k(f')$ also be meromorphic functions. Then if $h(f)$ and $k(f')$ have the common right factor g, then g has one of the forms stated in Theorem 5.*

REFERENCES

1. Baker, I.N., and Gross, F., "Further Results on Factorization of Entire Functions," Proceedings of Symposia in Pure Mathematics, Vol. XI, Entire Functions and Related Parts of Analysis, pp. 30-35, Providence, R.I., American Mathematical Society, 1968
2. Gross, F., "On Factorization of Meromorphic Functions," *Trans. Amer. Math. Soc.* **131(No. 1)**:215-222 (Apr. 1968)
3. Briot, C.A.A., and Bouquet, J.C., "Theorie des Fonctions Elliptiques," 2nd edition, Paris, 1875, pp. 282-288
4. Wittich, H., "Neuere Untersuchungen über eindeutige analytische Funktionen," Berlin, Springer, 1955, pp. 65-66
5. Hayman, W.K., "Meromorphic Functions," Oxford Mathematical Monographs, Clarendon Press, Oxford, 1964, p. 54
6. Gross, F., "On the Periodicity of Compositions of Entire Functions," *Can. J. Math.* **18**:724-730 (1966)
7. Jategaonkar, A.V., "Elementary Proof of a Theorem of P. Montel on Entire Functions," *J. London Math. Soc.* **40**: 166-170 (1965)
8. Montel, P., "Lecons sur les Familles Normales de Fonctions Analytiques et leurs Applications," Paris, Gauthier-Villars, 1927, pp. 135-137
9. R. Nevanlinna, "Le Théorème de Picard-Borel et al Théorie des Fonctions Méromorphes," Paris, Gauthier-Villars, 1929, pp. 31-36

Security Classification

UNCLASSIFIED

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

COMMON RIGHT FACTORS OF A MEROMORPHIC FUNCTION AND ITS DERIVATIVES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

An interim report on a continuing NRL Problem.

5. AUTHOR(S) (First name, middle initial, last name)

Fred Gross and Chung-chun Yang

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 23, 1970	10	9

8a. CONTRACT OR GRANT NO. NRL Problem B01-11 b. PROJECT NO. RR 003-02-41-6153 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 7138 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) Mathematics Research Center Report 70-7

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Research of Fred Gross was partially supported by National Science Foundation Grant GP-13875	Department of the Navy Office of Naval Research Arlington, Virginia 22217

13. ABSTRACT

The authors investigate common right factors of entire functions and their derivatives. In particular, it is shown that, assuming F to be a meromorphic function, then if F, F'', and $F^{(iv)}$ have the same right factor g, then g is one of the four forms $(az^2 + bz + C)^\delta$, $(Ae^{cz} + B)^\delta$, $[A \cos (cz+d) + B]^\delta$, or an elliptic function of second order, where a, b, c, d, A, B, and C are constants and $\delta = \pm 1$.

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Functional analysis Meromorphic functions Entire functions Factor analysis Complex variables Differential equations Polynomials						