

NRL Report 7160

A Test of the Capabilities of CHIEF in the Numerical Calculation of Acoustic Radiation from Arbitrary Surfaces

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November 12, 1970



NAVAL RESEARCH LABORATORY
Washington, D.C.

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)		2a. REPORT SECURITY CLASSIFICATION	
Naval Research Laboratory Washington, D. C. 20390		Unclassified	
3. REPORT TITLE		2b. GROUP	
A TEST OF THE CAPABILITIES OF CHIEF IN THE NUMERICAL CALCULATION OF ACOUSTIC RADIATION FROM ARBITRARY SURFACES			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)			
An interim report on a continuing problem.			
5. AUTHOR(S) (First name, middle initial, last name)			
Van Buren, A. L.			
6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS	
November 12, 1970	26	5	
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)		
NRL Problems S01-28 and S02-12	NRL Report 7160		
b. PROJECT NO.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)		
Projects RR 102-08-41-5225 and			
c. SF 11-121-31, Task 14080			
d.			
10. DISTRIBUTION STATEMENT			
This document has been approved for public release and sale; its distribution is unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY	
		Department of the Navy (Office of Naval Research and Naval Ship Systems Command), Arlington, Va. 22217, Washington, D. C. 20360	
13. ABSTRACT			
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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fortran program CHIEF Integral equations, Helmholtz Elastic wave problems Sound transducers Scalar waves						

CONTENTS

Abstract	ii
Problem Statement	ii
Authorization	ii
INTRODUCTION	1
REVIEW OF THE COMBINED HELMHOLTZ INTEGRAL EQUATION FORMULATION (CHIEF)	1
COMPARISON OF RESULTS	4
GEOMETRICAL OPTIONS	7
SUMMARY	15
ACKNOWLEDGMENT	16
REFERENCES	16
APPENDIX - Listing of the Subroutines CCOORD and CCUNMD and Their Required Input	17

ABSTRACT

The computer program CHIEF was designed by Schenck and Barach of the Naval Undersea Research and Development Center to obtain approximate solutions to exterior steady-state acoustic radiation problems for surfaces of arbitrary shape vibrating with a prescribed normal velocity. To test its capabilities as a research tool, CHIEF was applied to several problems for which accurate answers have been obtained using harmonic expansions. The accuracy and computation time of the results using CHIEF are discussed in terms of the surface subdivision scheme and the number of Gaussian quadrature points used to evaluate the Helmholtz integrals. In addition, new input subroutines to CHIEF which provide numerous geometrical options are also discussed.

PROBLEM STATUS

This is an interim report on a continuing NRL Problem.

AUTHORIZATION

NRL Problem S01-28
Project RR 102-08-41-5225

NRL Problem S02-12
Project SF 11-121-31, Task 14080

Manuscript submitted July 15, 1970.

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IN THE NUMERICAL CALCULATION OF ACOUSTIC
RADIATION FROM ARBITRARY SURFACES

INTRODUCTION

The computer program CHIEF as developed by Schenck (1) and Barach (2) is designed to obtain approximate solutions to exterior steady-state acoustic radiation problems for surfaces of arbitrary shape vibrating with a prescribed normal velocity. To test its capabilities as a research tool, CHIEF was applied to several problems for which accurate answers have been obtained using harmonic expansions.

The first examples involve the acoustic radiation from an oblate spheroid whose top half is vibrating with unit normal velocity and whose bottom half is rigid. The acoustic radiation impedance of the top half and the mutual radiation impedance coefficient between the two halves of the spheroid obtained using CHIEF are compared with results obtained using a harmonic expansion in oblate spheroidal harmonic functions. A Fortran computer program called OBRAD (3) was used to accurately evaluate the necessary spheroidal functions.

An example designed to test CHIEF's ability to handle a multiple surface is that of the radiation of a uniformly pulsating sphere in the presence of a similar stationary sphere. Here the results from CHIEF are compared with those of New (4), who has obtained accurate values for both the near-field and the far-field pressures using a harmonic expansion in terms of spherical functions.

The accuracy and computation time of the results using CHIEF are discussed in terms of the surface subdivision scheme and the number of Gaussian quadrature points used to evaluate the non-self Helmholtz integrals. In addition, new input subroutines to CHIEF which provide numerous geometrical options are discussed. A computer print-out of these subroutines is given in the appendix.

REVIEW OF THE COMBINED HELMHOLTZ INTEGRAL
EQUATION FORMULATION (CHIEF)

Consider a finite region bounded by the regular, closed surface S , as shown in Fig. 1. Let an arbitrary point on the surface be denoted by ξ . The region exterior to S is assumed to be filled with an ideal, homogeneous fluid of density ρ and sound speed c . Let an arbitrary point in this exterior region be denoted by \vec{x} . The surface S is vibrating at an angular frequency ω with a known normal velocity distribution $v(\xi)$. The steady-state pressure $p(\vec{x})$ may be obtained by solving the Helmholtz scalar wave equation,

$$(\nabla^2 + k^2) p(\vec{x}) = 0, \quad (1)$$

where $k = \omega/c$. The time dependence $e^{i\omega t}$ has been suppressed.

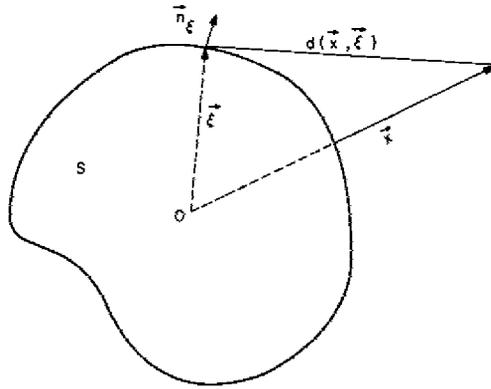


Fig. 1. - An arbitrary surface S

The solution to Eq. (1) must be finite and must satisfy the radiation condition at infinity,

$$\lim_{R \rightarrow \infty} \int_{S_R} \left| \frac{\partial p(\vec{x})}{\partial r} + ikp(\vec{x}) \right|_{r=R}^2 dS = 0, \quad (2)$$

where S_R is the surface of a sphere of radius R surrounding the surface S . In addition, the pressure $p(\vec{x})$ must satisfy the boundary condition on S ,

$$\frac{\partial p}{\partial n_{\xi}} = -i\omega\rho v(\xi), \quad (3)$$

where $\partial/\partial n_{\xi}$ denotes the outward normal derivative evaluated at the surface point ξ . The solution to Eq. (1) which satisfies the boundary conditions, Eq. (2) and Eq. (3), is given by

$$p(\vec{x}) = \frac{1}{4\pi} \int_S \left\{ p(\xi) \frac{\partial}{\partial n_{\xi}} \left[\frac{e^{-ikd(\vec{x}, \xi)}}{d(\vec{x}, \xi)} \right] + i\omega\rho v(\xi) \frac{e^{-ikd(\vec{x}, \xi)}}{d(\vec{x}, \xi)} \right\} dS(\xi), \quad (4)$$

where $d(\vec{x}, \xi)$ is the distance between the exterior point \vec{x} and the surface point ξ .

This expression allows $p(\vec{x})$ to be evaluated when the pressure on the surface $p(\xi)$ is known. In order to obtain $p(\xi)$, the point of observation \vec{x} is allowed to approach the surface. When the limiting process is properly performed, one obtains the surface Helmholtz integral formulation,

$$p(\xi) = \frac{1}{2\pi} \int_S \left\{ p(\xi') \frac{\partial}{\partial n_{\xi'}} \left[\frac{e^{-ikd(\xi', \xi)}}{d(\xi', \xi)} \right] + i\omega\rho v(\xi') \frac{e^{-ikd(\xi', \xi)}}{d(\xi', \xi)} \right\} dS(\xi'). \quad (5)$$

If the field point is taken interior to the surface S , one obtains the interior Helmholtz integral formulation,

$$0 = \frac{1}{4\pi} \int_S \left\{ p(\xi) \frac{\partial}{\partial n_{\xi}} \left[\frac{e^{-ikd(\vec{y}, \xi)}}{d(\vec{y}, \xi)} \right] + i\omega\rho v(\xi) \frac{e^{-ikd(\vec{y}, \xi)}}{d(\vec{y}, \xi)} \right\} dS(\xi), \quad (6)$$

where \vec{y} is the interior point.

The CHIEF program solves for the surface pressure $p(\xi)$ using Eq. (5) and, if necessary, Eq. (6).

In order to solve Eq. (5), the surface is subdivided into small areas. The normal particle velocity is chosen to be constant over each subdivision, and it is assumed that the pressure is constant over each subdivision. The latter assumption is an approximation that will be good only if the true pressure does not vary much over each subdivision. The approximation can be improved, if necessary, by further subdivision of the surface.

The surface integrals in Eq. (5) can now be broken up into integrations over each subdivision S_β , giving

$$2\pi p(\xi') = \sum_{\beta} p_{\beta} \int_{S_{\beta}} \frac{\partial}{\partial n_{\xi}} \left[\frac{e^{-ikd(\xi', \xi_{\beta})}}{d(\xi', \xi_{\beta})} \right] dS(\xi_{\beta}) \\ + i\omega\rho \sum_{\beta} v_{\beta} \int_{S_{\beta}} \frac{e^{-ikd(\xi', \xi_{\beta})}}{d(\xi', \xi_{\beta})} dS(\xi_{\beta}), \quad (7)$$

where p_{β} and v_{β} are the pressure and normal particle velocity for the subdivision S_{β} .

If the observation point ξ' is chosen to be on S_{α} , one obtains a set of simultaneous equations in the unknown pressures,

$$\sum_{\beta} A_{\alpha\beta} p_{\beta} = \sum_{\beta} B_{\alpha\beta} v_{\beta}, \quad (8)$$

where $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are given by

$$A_{\alpha\beta} = 2\pi\delta_{\alpha\beta} - \int_{S_{\beta}} \frac{\partial}{\partial n_{\xi}} \left[\frac{e^{-ikd(\xi_{\alpha}, \xi_{\beta})}}{d(\xi_{\alpha}, \xi_{\beta})} \right] dS(\xi_{\beta}), \quad (9)$$

where $\delta_{\alpha\beta}$ is the Kronecker delta,

and

$$B_{\alpha\beta} = \int_{S_{\beta}} \frac{e^{-ikd(\xi_{\alpha}, \xi_{\beta})}}{d(\xi_{\alpha}, \xi_{\beta})} dS(\xi_{\beta}). \quad (10)$$

The Helmholtz integrals given in Eqs. (9) and (10) are numerically evaluated in CHIEF using a Gaussian quadrature over both surface coordinates. For the so called non-self Helmholtz integrals, when $\alpha \neq \beta$, the user of CHIEF must input the number of quadrature points used to evaluate the integral. The self integrals, i. e., when $\alpha = \beta$,

are automatically evaluated by first subdividing the integrals into four pieces and then evaluating each piece using 16-point Gaussian quadratures over both surface coordinates. Obviously, the choice of the number of quadrature points to be used for the non-self integrals is influenced by the surface subdivision scheme.

If the frequency is near one of the characteristic frequencies for the interior homogeneous Dirichlet problem, the simultaneous equations, Eq. (8), will not yield the correct surface pressures. In this case additional equations are obtained using the interior Helmholtz integral formulation for carefully chosen interior points. These equations add no more unknowns but provide additional equations of constraint for the surface pressures leading to an overdetermined system. Since only the correct set of surface pressures satisfies these additional equations, their addition tends to force the solution to the desired one. The choice of the number and the location of the interior points is a difficult problem. The examples discussed in this paper will not require any interior points. This will allow an uncluttered examination of the more basic question confronting the user of CHIEF: how do both the accuracy and the computation time depend on the surface subdivision scheme and the number of quadrature points?

COMPARISON OF RESULTS

Example I. Oblate Spheroid

The first example is that of radiation from an oblate spheroid, as shown in Fig. 2. Here ξ , the radial coordinate specifying the oblate spheroidal surface, is chosen equal to 0.2. The ratio of major to minor axes is then given by $\sqrt{(\xi^2+1)}/\xi \doteq 5.0$. The parameter h , defined to be π times the ratio of the distance between the foci of the elliptical cross section to the wavelength, is a measure of the acoustical size. Here h is chosen equal to unity to insure that the frequency is well below the lowest characteristic frequency for the interior homogeneous Dirichlet problem. For this example the top half of the surface is specified to be vibrating with unit normal velocity. Of interest are the self acoustic radiation impedance coefficient of the top half and the mutual acoustic radiation impedance coefficient between the two halves. These were first calculated analytically using a harmonic expansion in oblate spheroidal wave functions. Sufficient spheroidal functions were generated to achieve convergence of the series to at least four places of accuracy in both the resistive and the reactive parts of the impedance.



Fig. 2. - Subdivided oblate spheroid

The problem was then input to CHIEF for various subdivision schemes. CHIEF is designed to take advantage of rotational and reflective symmetry in the geometry, greatly reducing the computation time in these cases. In order to use rotational symmetry, the spheroid was first subdivided into strips resembling orange sections. Each strip was then further subdivided into bands. The number of Gaussian quadrature points for the non-self Helmholtz integrals was selected for both surface coordinates. The normal particle velocity was input as unity for the top half and zero for the bottom half of the spheroid.

CHIEF was used to obtain the surface pressures p_n for each subdivision. The normalized self radiation impedance coefficient for the top half of the spheroid, Z_{self} , was then calculated using

$$Z_{\text{self}} = R_{\text{self}} + iX_{\text{self}} = \frac{\sum_{\alpha=1}^n p_{\alpha} S_{\alpha}}{\rho c v \sum_{\alpha=1}^n S_{\alpha}}, \quad (11)$$

where S_{α} is the area of the α th subdivision, and $\alpha = 1$ to n includes only the subdivisions on the top half of the spheroid. R_{self} and X_{self} are the resistive and reactive components of the impedance. Here $v = 1$ m/sec is the normal particle velocity. The normalization factor

$$\rho c \sum_{\alpha=1}^n S_{\alpha}$$

makes Z_{self} dimensionless.

Similarly, the normalized mutual radiation impedance coefficient between the two halves, Z_{mutual} , was calculated using

$$Z_{\text{mutual}} = R_{\text{mutual}} + iX_{\text{mutual}} = \frac{\sum_{\alpha=n+1}^N p_{\alpha} S_{\alpha}}{\rho c v \sum_{\alpha=n+1}^N S_{\alpha}}, \quad (12)$$

where $\alpha = n+1$ to N includes only the subdivisions on the bottom half of the spheroid.

The results for this case are summarized in Table 1. The impedances calculated using the harmonic expansion are given in the last line. The column labeled "time in seconds" indicates the computation time required to determine the surface pressures using the CDC 3800 computer at NRL.

The first CHIEF model had 12 strips, 6 bands, used a 2-point Gaussian quadrature for both surface coordinates (denoted 2×2), and required 36 sec of computation time. The results are good except for the mutual reactance. As the quadrature is increased, the accuracy improves, with the 8×8 quadrature giving surprisingly good results for 76 sec of computer time.

If 24 strips and 12 bands are used, a 2×2 quadrature again gives a poor value for the mutual reactance. As the quadrature increases, the accuracy again improves. In general, the accuracy should be better for increased subdivision for the same quadrature. However, possible random errors may cancel, yielding significantly greater accuracy for the coarser subdivision scheme. This is apparently the case for the model using 12 strips, 6 bands, and an 8×8 quadrature.

Apparently good results can be obtained for this oblate spheroid using rather crude surface subdivision and a small number of quadrature points. It is important to note that the far-field pressure pattern does not depend on the radiation reactance and will be extremely accurate for all of the models except those using a 2×2 quadrature. Because of the smallness of the mutual reactance, the near-field pressures should also be accurate for all but the 2×2 quadrature.

Table 1
Results for the Spheroid with $\xi = 0.2$, $h = 1.0$

Number of Strips	Number of Bands	Number of Quadrature Points	Time (sec)	R_{self}	X_{self}	R_{mutual}	X_{mutual}
12	6	2 x 2	36	0.2364	0.5063	0.1830	0.06148
12	6	4 x 4	49	0.2371	0.5254	0.1760	0.03996
12	6	8 x 8	76	0.2369	0.5285	0.1743	0.03508
24	12	2 x 2	106	0.2361	0.5172	0.1767	0.04543
24	12	4 x 4	154	0.2360	0.5234	0.1751	0.03936
24	12	6 x 6	227	0.2361	0.5246	0.1748	0.03823
24	12	8 x 8	338	0.2361	0.5249	0.1748	0.03791
Harmonic expansion			20	0.2369	0.5287	0.1739	0.03481

Example II. Thin Oblate Spheroid

Next consider the radiation from a thin oblate spheroid whose top half is vibrating with unit velocity and whose bottom half is rigid. Here $h = 1.0$, and $\xi = 0.02$ so that the ratio of major to minor axes is very nearly equal to 50. The radiation impedance of the top half of the spheroid and the mutual radiation impedance coefficients between the two halves of the spheroid were calculated using a harmonic expansion in spheroidal wave functions and using CHIEF with various subdivision schemes. The results are given in Table 2.

Table 2
Results for the Spheroid with $\xi = 0.02$, $h = 1.0$

Number of Strips	Number of Bands	Number of Quadrature Points	Time (sec)	R_{self}	X_{self}	R_{mutual}	X_{mutual}
12	6	4 x 4	42	0.2906	0.4643	0.2900	0.3897
12	6	8 x 8	65	0.2513	0.4247	0.2494	0.3160
24	12	4 x 4	148	0.2460	0.4241	0.2436	0.3048
24	12	8 x 8	334	0.2284	0.4678	0.2163	0.1983
24	18	8 x 8	678	0.2267	0.4726	0.2135	0.1917
48	24	4 x 4	770	0.2285	0.4714	0.1969	0.1890
48	24	8 x 8	2204	0.2288	0.5464	0.1950	0.09471
Harmonic expansion			20	0.2302	0.5732	0.1874	0.06135

The results using a model with 12 strips, 6 bands, and a 4 x 4 quadrature are very poor. Increasing the quadrature to 8 x 8 does not improve the results much. Increased subdivision gradually improves the results. However, even with 48 strips, 24 bands, and an 8 x 8 quadrature, considerable error occurs in the mutual reactance. A good far-field pattern should be obtained for this model, but more subdivisions and increased computer time would be required to obtain very accurate impedances.

Example III. Two Spheres

To determine the capability of CHIEF regarding multiple surfaces, the two-sphere problem, whose geometry is shown in Fig. 3, was considered. The spheres were chosen to have $ka = 1.0$, where a is the radius of each sphere, and were separated by a distance equal to their radii. One sphere was pulsating uniformly, while the other one was rigid. The two-sphere problem has been solved analytically by New (4) using expansions in spherical wave functions.

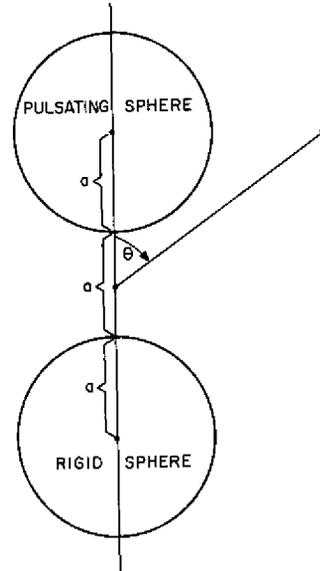


Fig. 3 - Two-sphere geometry

Consider the near-field pressure magnitude on the axis of the system, as shown in Fig. 4. The dashed line represents the $1/r$ dependence to be expected if the rigid sphere were not present. The results of New are represented by the solid line, while the dots represent the results of CHIEF using 6 strips, 6 bands, and a 4×4 quadrature on each sphere. The pressures are normalized to the pressure magnitude that would exist on the surface of the pulsating sphere if the rigid sphere were not present. The maximum error was less than 6%. When 10 strips, 10 bands, and a 4×4 quadrature were used, the maximum error was reduced to less than 1%.

The far-field pressure pattern in the plane bisecting the spheres was also calculated using the CHIEF model with 6 strips, 6 bands, and a 4×4 quadrature on each sphere. Figure 5 gives the pattern as a function of the polar angle θ . Again the solid line represents the results of New, while the CHIEF results are represented by dots. As expected, considering the good accuracy of the near-field results, the far-field is extremely accurate. The total computation time for the two-sphere problem using the coarser subdivision scheme was 126 sec.

GEOMETRICAL OPTIONS

CHIEF is not restricted to a specific coordinate geometry. Any convenient coordinate representation may be used to describe the surface. However, the free-space Green's function and its normal derivative are described internally in CHIEF in terms of a Cartesian coordinate system. Therefore, input subroutines are required which contain conversion formulas giving the Cartesian coordinates of an arbitrary surface point

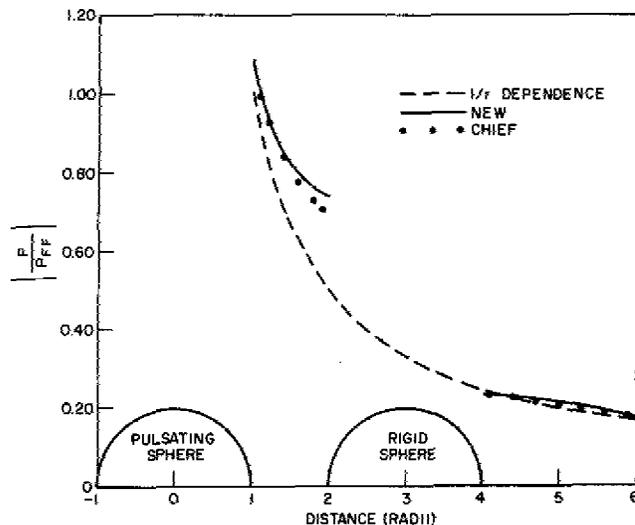


Fig. 4 - Near-field pressure distribution on the axis of two spheres, one pulsating and one rigid

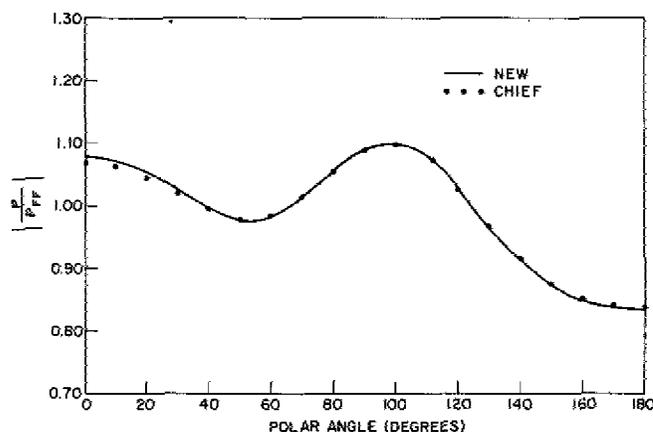


Fig. 5 - The far-field pressure distribution for two spheres, one pulsating and one rigid

as well as the Cartesian components of the normal vector to the surface at that point. The area element associated with the surface coordinates is also required. The entire closed surface is separated into regions such that each region is describable in terms of a single geometry. Each region is assigned an integer index that corresponds to the appropriate conversion formulas in the input subroutines.

In an effort to increase the utility of CHIEF, conversion formulas have been added to the input subroutines so that a wide range of geometrical options is available. These options, which are listed in Table 3, are discussed below. Included are formulas giving the Cartesian coordinates (x, y, z) , the Cartesian components of the unit normal \hat{e}_n , and the magnitude of the area element dS for any surface point (u, v) (Ref. 5). The notation $(u, v) = (a, b)$ means that a and b are the two surface coordinates. Any of the surfaces can be translated or rotated by modifying the formulas giving the Cartesian coordinates. For example, option 15 describes the outside surface of a sphere centered

at $(x, y, z) = (0, 0, 0)$. To describe a sphere centered at $(x, y, z) = (c, 0, 0)$, change the equation for x in terms of the surface coordinates $(u, v) = (\theta, \phi)$ to $x = CC(33) \sin u \cos v + c$.

1. yz plane with Cartesian coordinates, $(u, v) = (y, z)$, normal in $+x$ direction.

$$x = \text{Constant} = CC(1)$$

$$y = u$$

$$z = v$$

$$\hat{e}_n = \hat{e}_x$$

$$dS = du \, dv$$

2. yz plane with Cartesian coordinates, $(u, v) = (y, z)$, normal in $-x$ direction.

$$x = \text{Constant} = CC(2)$$

$$y = u$$

$$z = v$$

$$\hat{e}_n = -\hat{e}_x$$

$$dS = du \, dv$$

3. xz plane with Cartesian coordinates, $(u, v) = (x, z)$, normal in $+y$ direction.

$$x = u$$

$$y = \text{Constant} = CC(3)$$

Table 3
Geometrical Options Added to CHIEF

Surface	Direction of Normal to Surface	Surface	Direction of Normal to Surface
1. yz plane	$+x$	11. Outside of circular cylinder	Outward
2. yz plane	$-x$	12. Inside of circular cylinder	Inward
3. xz plane	$+y$	13. Outside of elliptical cylinder	Outward
4. xz plane	$-y$	14. Inside of elliptical cylinder	Inward
5. xy plane	$+z$	15. Sphere	Outward
6. xy plane	$-z$	16. Oblate spheroid	Outward
7. xy plane, polar coordinates	$+z$	17. Prolate spheroid	Outward
8. xy plane, polar coordinates	$-z$	18. Toroid	Outward
9. xy plane, elliptical coordinates	$+z$		
10. xy plane, elliptical coordinates	$-z$		

$$z = v$$

$$\hat{e}_n = \hat{e}_y$$

$$ds = du dv$$

4. xz plane with Cartesian coordinates, $(u, v) = (x, z)$, normal in $-y$ direction.

$$x = u$$

$$y = \text{Constant} = \text{CC}(4)$$

$$z = v$$

$$\hat{e}_n = -\hat{e}_y$$

$$dS = du dv$$

5. xy plane with Cartesian coordinates, $(u, v) = (x, y)$, normal in $+z$ direction.

$$x = u$$

$$y = v$$

$$z = \text{Constant} = \text{CC}(5)$$

$$\hat{e}_n = \hat{e}_z$$

$$dS = du dv$$

6. xy plane with Cartesian coordinates, $(u, v) = (x, y)$, normal in $-z$ direction.

$$x = u$$

$$y = v$$

$$z = \text{Constant} = \text{CC}(6)$$

$$\hat{e}_n = -\hat{e}_z$$

$$dS = du dv$$

7. xy plane with polar coordinates (r, θ) , $0 \leq r < \infty$, $0 \leq \theta < 2\pi$, $(u, v) = (r, \theta)$, normal in $+z$ direction.

In the xy plane the curve of constant r is a circle of radius r centered at $(x, y) = (0, 0)$, and the curve of constant θ is a semi-infinite line originating at $(x, y) = (0, 0)$.

$$x = u \cos v$$

$$y = u \sin v$$

$$z = \text{Constant} = \text{CC}(7)$$

$$\hat{e}_n = \hat{e}_z$$

$$dS = u du dv$$

8. xy plane with polar coordinates (r, θ) , $0 \leq r < \infty$, $0 \leq \theta < 2\pi$, $(u, v) \equiv (r, \theta)$ normal in $-z$ direction.

In the xy plane the curve of constant r is a circle of radius r centered at $(x, y) = (0, 0)$, and the curve of constant θ is a semi-infinite line originating at $(x, y) = (0, 0)$.

$$x = u \cos v$$

$$y = u \sin v$$

$$z = \text{Constant} = \text{CC}(8)$$

$$\hat{e}_n = -\hat{e}_z$$

$$dS = u \, du \, dv$$

9. xy plane with elliptical coordinates (μ, ψ) , $1 \leq \mu < \infty$, $0 \leq \psi < 2\pi$, $(u, v) \equiv (\mu, \psi)$, normal in $+z$ direction.

In the xy plane the curve of constant μ is an ellipse of interfocal distance $2 \text{CC}(9)$ centered at $(x, y) = (0, 0)$, and the curve of constant ψ is a hyperbola which is orthogonal to the family of ellipses for the same value of $\text{CC}(9)$.

$$x = \text{CC}(9) u \cos v$$

$$y = \text{CC}(9) (u^2 - 1)^{1/2} \sin v$$

$$z = \text{Constant} = \text{CC}(10)$$

$$\hat{e}_n = \hat{e}_z$$

$$dS = \text{CC}(9)^2 (u^2 - 1)^{-1/2} (u^2 - \cos^2 v) \, du \, dv$$

10. xy plane with elliptical coordinates (μ, ψ) , $1 \leq \mu < \infty$, $0 \leq \psi < 2\pi$, $(u, v) \equiv (\mu, \psi)$, normal in $-z$ direction.

In the xy plane the curve of constant μ is an ellipse of interfocal distance $2 \text{CC}(12)$ centered at $(x, y) = (0, 0)$, and the curve of constant ψ is a hyperbola which is orthogonal to the family of ellipses for the same value of $\text{CC}(12)$.

$$x = \text{CC}(12) u \cos v$$

$$y = \text{CC}(12) (u^2 - 1)^{1/2} \sin v$$

$$z = \text{Constant} = \text{CC}(13)$$

$$\hat{e}_n = -\hat{e}_z$$

$$dS = \text{CC}(12)^2 (u^2 - 1)^{-1/2} (u^2 - \cos^2 v) \, du \, dv$$

11. Outside surface of a circular cylinder along the z axis, $(u, v) = (z, \theta)$, $-\infty < z < \infty$, $0 \leq \theta < 2\pi$, normal outward from the cylinder. The radius of the cylinder is a constant given by CC(15).

On the cylindrical surface the curve of constant z is a circle of radius CC(15) centered at $(x, y, z) = (0, 0, z)$, and the curve of constant θ is a line parallel to the z axis.

$$x = \text{CC}(15) \cos v$$

$$y = \text{CC}(15) \sin v$$

$$z = u$$

$$\hat{e}_n = \hat{e}_x \cos v + \hat{e}_y \sin v$$

$$dS = \text{CC}(15) \, du \, dv$$

12. Inside surface of a circular cylinder along the z axis, $(u, v) = (z, \theta)$, $-\infty < z < \infty$, $0 \leq \theta < 2\pi$, normal inward. The radius of the cylinder is a constant given by CC(16).

On the cylindrical surface the curve of constant z is a circle of radius CC(16) centered at $(x, y, z) = (0, 0, z)$, and the curve of constant θ is a line parallel to the z axis.

$$x = \text{CC}(16) \cos v$$

$$y = \text{CC}(16) \sin v$$

$$z = u$$

$$\hat{e}_n = -\hat{e}_x \cos v - \hat{e}_y \sin v$$

$$dS = \text{CC}(16) \, du \, dv$$

13. Outside surface of an elliptic cylinder along the z axis, $(u, v) = (z, \psi)$, $-\infty < z < \infty$, $0 \leq \psi < 2\pi$, normal outward from the cylinder. The interfocal distance of the elliptic cross section is given by $2\text{CC}(17)$. The surface is one of constant $\mu = \text{CC}(18)$, where $1 \leq \mu < \infty$.

On the cylindrical surface the curve of constant z is an ellipse of interfocal distance $2\text{CC}(17)$ centered at $(x, y, z) = (0, 0, z)$, and the curve of constant ψ is a line parallel to the z axis, one of the two lines produced by the intersection of a hyperbolic cylinder and the elliptic cylinder.

$$x = \text{CC}(17) \text{CC}(18) \cos v$$

$$y = \text{CC}(17) [\text{CC}(18)^2 - 1]^{1/2} \sin v$$

$$z = u$$

$$\hat{e}_n = \hat{e}_x \left[\frac{\text{CC}(18)^2 - 1}{\text{CC}(18)^2 - \cos^2 v} \right]^{1/2} \cos v + \hat{e}_y \frac{\text{CC}(18)}{[\text{CC}(18)^2 - \cos^2 v]^{1/2}} \sin v$$

$$dS = \text{CC}(17) [\text{CC}(18)^2 - \cos^2 v]^{1/2} \, du \, dv$$

14. Inside surface of an elliptic cylinder along the z axis, $(u, v) \equiv (z, \psi)$, $-\infty < z < \infty$, $0 \leq \psi < 2\pi$, normal inward. The interfocal distance of the elliptic cross section is given by $2CC(25)$. The surface is one of constant $\mu = CC(26)$, where $1 < \mu < \infty$.

On the cylindrical surface the curve of constant z is an ellipse of interfocal distance $2CC(25)$ centered at $(x, y, z) = (0, 0, z)$, and the curve of constant ψ is a line parallel to the z axis, one of the two lines produced by the intersection of a hyperbolic cylinder and the elliptic cylinder.

$$x = CC(25) CC(26) \cos v$$

$$y = CC(25) [CC(26)^2 - 1]^{\frac{1}{2}} \sin v$$

$$z = u$$

$$\hat{e}_n = -\hat{e}_x \left[\frac{CC(26)^2 - 1}{CC(26)^2 - \cos^2 v} \right]^{\frac{1}{2}} \cos v - \hat{e}_y \frac{CC(26)}{[CC(26)^2 - \cos^2 v]^{\frac{1}{2}}} \sin v$$

$$dS = CC(25) [CC(26)^2 - \cos^2 v]^{\frac{1}{2}} du dv$$

15. Outside surface of a sphere, $(u, v) \equiv (\theta, \phi)$, $0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$, normal outward from the sphere. The radius of the sphere is a constant given by $CC(33)$.

On the spherical surface the curve of constant θ is a circle of radius $CC(33) \sin \theta$ centered at $(x, y, z) = (0, 0, CC(33) \cos \theta)$, and the curve of constant ϕ is a half circle between the poles of the sphere.

$$x = CC(33) \sin u \cos v$$

$$y = CC(33) \sin u \sin v$$

$$z = CC(33) \cos u$$

$$\hat{e}_n = \hat{e}_x \sin u \cos v + \hat{e}_y \sin u \sin v + \hat{e}_z \cos u$$

$$dS = CC(33) \sin u du dv$$

16. Outside surface of an oblate spheroid, $(u, v) \equiv (\eta, \phi)$, $0 \leq \eta \leq \pi$, $0 \leq \phi < 2\pi$, normal outward from the spheroid. The interfocal length of the spheroid is a constant given by $2CC(34)$. The surface is one of constant $\xi = CC(35)$, where $0 \leq \xi < \infty$.

On the spheroidal surface the curve of constant ϕ is a half ellipse containing the poles of the spheroid, and the curve of constant η is a circle of radius $CC(34) [\xi^2 + 1]^{\frac{1}{2}} \sin \eta$ centered at $(x, y, z) = (0, 0, CC(34) CC(35) \cos \eta)$. The z axis is the axis of symmetry.

$$x = CC(34) [CC(35)^2 + 1]^{\frac{1}{2}} \sin u \cos v$$

$$y = CC(34) [CC(35)^2 + 1]^{\frac{1}{2}} \sin u \sin v$$

$$z = CC(34) CC(35) \cos u$$

$$\begin{aligned}\hat{e}_n &= \hat{e}_x \text{CC}(35) [\text{CC}(35)^2 + \cos^2 u]^{-\frac{1}{2}} \sin u \cos v \\ &\quad + \hat{e}_y \text{CC}(35) [\text{CC}(35)^2 + \cos^2 u]^{-\frac{1}{2}} \sin u \sin v \\ &\quad + \hat{e}_z [\text{CC}(35)^2 + 1]^{\frac{1}{2}} [\text{CC}(35)^2 + \cos^2 u]^{-\frac{1}{2}} \cos u \\ dS &= \text{CC}(34)^2 [\text{CC}(35)^2 + 1]^{\frac{1}{2}} [\text{CC}(35)^2 + \cos^2 u]^{\frac{1}{2}} \sin u \, du \, dv\end{aligned}$$

17. Outside surface of a prolate spheroid, $(u, v) = (\eta, \phi)$, $0 \leq \eta \leq \pi$, $0 \leq \phi < 2\pi$, normal outward from the spheroid. The interfocal length of the spheroid is a constant given by $2\text{CC}(42)$. The surface is one of constant $\xi = \text{CC}(43)$, where $1 < \xi < \infty$.

On the spheroidal surface the curve of constant ϕ is a half ellipse containing the poles of the spheroid, and the curve of constant η is a circle of radius $\text{CC}(42) [\xi^2 - 1]^{\frac{1}{2}} \sin \eta$ centered at $(x, y, z) = (0, 0, \text{CC}(42) \text{CC}(43) \cos \eta)$. The z axis is the axis of symmetry.

$$\begin{aligned}x &= \text{CC}(42) [\text{CC}(43)^2 - 1]^{\frac{1}{2}} \sin u \cos v \\ y &= \text{CC}(42) [\text{CC}(43)^2 - 1]^{\frac{1}{2}} \sin u \sin v \\ z &= \text{CC}(42) \text{CC}(43) \cos u \\ \hat{e}_n &= \hat{e}_x \text{CC}(43) [\text{CC}(43)^2 - \cos^2 u]^{-\frac{1}{2}} \sin u \cos v \\ &\quad + \hat{e}_y \text{CC}(43) [\text{CC}(43)^2 - \cos^2 u]^{-\frac{1}{2}} \sin u \sin v \\ &\quad + \hat{e}_z [\text{CC}(43)^2 - 1]^{\frac{1}{2}} [\text{CC}(43)^2 - \cos^2 u]^{-\frac{1}{2}} \cos u \\ dS &= \text{CC}(42)^2 [\text{CC}(43)^2 - 1]^{\frac{1}{2}} [\text{CC}(43)^2 - \cos^2 u]^{\frac{1}{2}} \sin u \, du \, dv\end{aligned}$$

18. Outside surface of a toroid, $(u, v) = (\eta, \phi)$, $-\pi < \eta < \pi$, $0 \leq \phi < 2\pi$, normal outward from the toroid. The toroid is characterized by two radii $\text{CC}(49)$ and $\text{CC}(50)$. The ratio of these two radii $\text{CC}(51)$, $1 < \text{CC}(51) < \infty$, forms an orthogonal system with η and ϕ and is a constant over the toroidal surface. To completely characterize the surface, a second constant $\text{CC}(52) = [\text{CC}(50)^2 - \text{CC}(49)^2]^{\frac{1}{2}}$ is defined.

Thus the surface may be defined by giving either the constants $\text{CC}(49)$ and $\text{CC}(50)$ or the constants $\text{CC}(51)$ and $\text{CC}(52)$. Because the two radii are easier to visualize, $\text{CC}(49)$ and $\text{CC}(50)$ will be the input for CHIEF.

On the toroidal surface the curve of constant ϕ is a circle produced by the intersection of a half plane containing the z axis and the toroid, and the curve of constant η is a circle produced by the intersection of a spherical bowl and the toroid. The z axis is the symmetry axis for the toroid.

$$\begin{aligned}x &= [\text{CC}(50)^2 - \text{CC}(49)^2] \cos v / [\text{CC}(50) - \text{CC}(49) \cos u] \\ y &= [\text{CC}(50)^2 - \text{CC}(49)^2] \sin v / [\text{CC}(50) - \text{CC}(49) \cos u] \\ z &= \text{CC}(49) [\text{CC}(50)^2 - \text{CC}(49)^2]^{\frac{1}{2}} \sin u / [\text{CC}(50) - \text{CC}(49) \cos u]\end{aligned}$$

$$\begin{aligned} \hat{e}_n = & -\hat{e}_x [CC(49) - CC(50) \cos u] \cos v / [CC(50) - CC(49) \cos u] \\ & -\hat{e}_y [CC(49) - CC(50) \cos u] \sin v / [CC(50) - CC(49) \cos u] \\ & +\hat{e}_z [CC(50)^2 - CC(49)^2]^{1/2} \sin u / [CC(50) - CC(49) \cos u] \\ dS = & CC(49) [CC(50)^2 - CC(49)^2]^{1/2} / [CC(50) - CC(49) \cos u]^2 du dv \end{aligned}$$

The input subroutines providing for these geometrical options are listed in the appendix. Note that the two subroutines CCOORD and CCUNMD already exist in CHIEF. CCOORD (U, V, NEQN, A1, A2, A3) is called in CHIEF whenever only the Cartesian coordinates (A1, A2, A3) are desired for a surface point (U, V). The index NEQN refers to the particular geometrical option. The subroutine CCUNMD (U, V, NEQN, AX1, AX2, AX3, AN1, AN2, AN3, RMAGD) is called whenever all of the surface information is desired; i. e., it provides the Cartesian coordinates (AX1, AX2, AX3), the Cartesian components of the unit normal to the surface (AN1, AN2, AN3), and the magnitude of the surface area element RMAGD.

Note that the set of constants C1, C2, C3, ... in CHIEF have been replaced by an array of constants CC(100). The declaration COMMON/ALLC/CC(100) must be substituted in the main program and in CCOORD and CCUNMD for the declaration COMMON/ALLC/C1, C2, C3, ... These constants are quantities that do not vary over the surface and must be input or calculated in the main program. Every combination of constants appearing in the conversion formulas is calculated and stored in CC(100). This reduces the computation time considerably, since the calculation is only performed once instead of every time the subroutines CCOORD and CCUNMD are called.

Most of the input constants refer to a distance and are described in terms of a unit of length called WAVE = $1/k = \lambda/2\pi = c/\omega$. However, some of the input constants in options 13, 14, 16, and 17 are pure numbers and represent ratios of distances. Reference to the previous description of the geometrical options will indicate which parameters the input constants represent.

Option 19 in subroutine CCOORD provides interior points and should be modified to fit the specific geometry whenever interior points are required.

SUMMARY

The following observations may aid the potential user of CHIEF.

1. CHIEF evaluates the solution to a boundary-value problem. This will be a realistic transducer model when the velocities are known; for example, when velocity control exists.
2. CHIEF is a very flexible program, allowing the user a wide range of options. Reducing it to a production program, capable of being used by casual acquaintances, would remove this flexibility. If only a specific geometry is of interest, the user is advised to write a CHIEF-like program taking advantage of the peculiarities of that geometry. This will be more economical in the long run, if the program is to be used extensively. If the frequencies of interest are well below the lowest critical frequency, a simple source method, where the pressure is obtainable in terms of a single integral, might be considered.
3. If rotational or reflective symmetry is not present, the computation times increase significantly. The possibility of using rotational or reflective symmetry should be examined carefully. If both are applicable, it is usually best to use rotational symmetry.

4. The results obtained using CHIEF approach the correct solution as the subdivision is increased. Surprisingly good results can be obtained, especially for the far field, using a relatively crude subdivision scheme. However, objects that are thin may require an unusually fine subdivision. Also, as the acoustic size is increased, the number of subdivisions required to give accurate results will also increase. However, this limitation, which is fundamental to the finite-element method, will be less restrictive in the future as computers are improved.

5. This report describes new input subroutines to CHIEF that provide for a wide range of geometrical options. Most of the standard geometries have been included. Surfaces of complex objects such as a free-flooded ring can be described by combining two or more of the options. The user can easily add additional geometries that are desired.

ACKNOWLEDGMENT

The author gratefully acknowledges the continued support of NAVSHIPS:00V1D.

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Appendix

LISTING OF THE SUBROUTINES CCOORD AND CCUNMD AND THEIR REQUIRED INPUT

```
SUBROUTINE CCOORD(U,V,NEQN,A1,A2,A3)
COMMON/ALLC/CC(100)
GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19) NEQN
1 A1=CC(1)
  A2=U
  A3=V
  RETURN
2 A1=CC(2)
  A2=U
  A3=V
  RETURN
3 A1=U
  A2=CC(3)
  A3=V
  RETURN
4 A1=U
  A2=CC(4)
  A3=V
  RETURN
5 A1=U
  A2=V
  A3=CC(5)
  RETURN
6 A1=U
  A2=V
  A3=CC(6)
  RETURN
7 A1=U*COS(V)
  A2=U*SIN(V)
  A3=CC(7)
  RETURN
8 A1=U*COS(V)
  A2=U*SIN(V)
  A3=CC(8)
  RETURN
9 A1=CC(9)*U*COS(V)
  A2=CC(9)*SQRT(U*U-1.)*SIN(V)
  A3=CC(10)
  RETURN
10 A1=CC(12)*U*COS(V)
  A2=CC(12)*SQRT(U*U-1.)*SIN(V)
  A3=CC(13)
  RETURN
11 A1=CC(15)*COS(V)
  A2=CC(15)*SIN(V)
  A3=U
  RETURN
12 A1=CC(16)*COS(V)
  A2=CC(16)*SIN(V)
  A3=U
  RETURN
13 A1=CC(21)*COS(V)
  A2=CC(22)*SIN(V)
  A3=U
  RETURN
```

```

14 A1=CC(29)*COS(V)
   A2=CC(30)*SIN(V)
   A3=U
   RETURN
15 RSINU=CC(33)*SIN(U)
   A1=RSINU*COS(V)
   A2=RSINU*SIN(V)
   A3=CC(33)*COS(U)
   RETURN
16 CCSINU=CC(39)*SIN(U)
   A1=CCSINU*COS(V)
   A2=CCSINU*SIN(V)
   A3=CC(37)*COS(U)
   RETURN
17 CCSINU=CC(47)*SIN(U)
   A1=CCSINU*COS(V)
   A2=CCSINU*SIN(V)
   A3=CC(45)*COS(U)
   RETURN
18 FAC=CC(54)/(CC(51)-COS(U))
   A1=FAC*COS(V)
   A2=FAC*SIN(V)
   A3=FAC*SIN(U)*CC(55)
   RETURN
19 A1=0.
   A2=U
   A3=CC(60)
   RETURN
   END

```

```

SUBROUTINE CCUNMD(U,V,NEQN,AX1,AX2,AX3,AN1,AN2,AN3,RMAGD)
COMMON/ALLC/CC(100)
GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18) NEQN
1 AN1=1.
  AN2=0.
  AN3=0.
  RMAGD=1.
  AX1=CC(1)
  AX2=U
  AX3=V
  RETURN
2 AN1=-1.
  AN2=0.
  AN3=0.
  RMAGD=1.
  AX1=CC(2)
  AX2=U
  AX3=V
  RETURN
3 AN1=0.
  AN2=1.
  AN3=0.
  RMAGD=1.
  AX1=U
  AX2=CC(3)
  AX3=V
  RETURN
4 AN1=0.
  AN2=-1.
  AN3=0.
  RMAGD=1.
  AX1=U
  AX2=CC(4)

```

```

    AX3=V
    RETURN
5  AN1=0.
    AN2=0.
    AN3=1.
    RMAGD=1.
    AX1=U
    AX2=V
    AX3=CC(5)
    RETURN
6  AN1=0.
    AN2=0.
    AN3=-1.
    RMAGD=1.
    AX1=U
    AX2=V
    AX3=CC(6)
    RETURN
7  AN1=0.
    AN2=0.
    AN3=1.
    RMAGD=U
    AX1=U*COS(V)
    AX2=U*SIN(V)
    AX3=CC(7)
    RETURN
8  AN1=0.
    AN2=0.
    AN3=-1.
    RMAGD=U
    AX1=U*COS(V)
    AX2=U*SIN(V)
    AX3=CC(8)
    RETURN
9  COSV=COS(V)
    COSVS=COSV*COSV
    UU=U*U
    SQ=SQRT(UU-1.)
    AN1=0.
    AN2=0.
    AN3=1.
    RMAGD=CC(11)*(UU-COSVS)/SQ
    AX1=CC(9)*U*COSV
    AX2=CC(9)*SQ*SQRT(1.-COSVS)
    AX3=CC(10)
    RETURN
10 COSV=COS(V)
    UU=U*U
    COSVS=COSV*COSV
    SQ=SQRT(UU-1.)
    AN1=0.
    AN2=0.
    AN3=-1.
    RMAGD=CC(14)*(UU-COSVS)/SQ
    AX1=CC(12)*U*COSV
    AX2=CC(12)*SQ*SQRT(1.-COSVS)
    AX3=CC(13)
    RETURN
11 AN1=COS(V)
    AN2=SIN(V)
    AN3=0.
    RMAGD=CC(15)
    AX1=CC(15)*AN1
    AX2=CC(15)*AN2

```

```

    AX3=U
    RETURN
12 AN1=-COS(V)
    AN2=-SIN(V)
    AN3=0.
    RMAGD=CC(16)
    AX1=-CC(16)*AN1
    AX2=-CC(16)*AN2
    AX3=U
    RETURN
13 COSV=COS(V)
    COSVS=COSV*COSV
    SINV=SQRT(1.-COSVS)
    DENS=1./SQRT(CC(19)-COSVS)
    AN1=CC(20)*DENS*COSV
    AN2=CC(18)*DENS*SINV
    AN3=0.
    RMAGD=CC(17)/DENS
    AX1=CC(21)*COSV
    AX2=CC(22)*SINV
    AX3=U
    RETURN
14 COSV=COS(V)
    COSVS=COSV*COSV
    SINV=SQRT(1.-COSVS)
    DENS=1./SQRT(CC(27)-COSVS)
    AN1=-CC(28)*DENS*COSV
    AN2=-CC(26)*DENS*SINV
    AN3=0.
    RMAGD=CC(25)/DENS
    AX1=CC(29)*COSV
    AX2=CC(30)*SINV
    AX3=U
    RETURN
15 SINU=SIN(U)
    AN1=SINU*COS(V)
    AN2=SINU*SIN(V)
    AN3=COS(U)
    RMAGD=CC(33)*SINU
    AX1=CC(33)*AN1
    AX2=CC(33)*AN2
    AX3=CC(33)*AN3
    RETURN
16 COSV=COS(V)
    SINV=SIN(V)
    COSU=COS(U)
    COSUS=COSU*COSU
    SINU=SQRT(1.-COSUS)
    FAC=1./SQRT(CC(36)+COSUS)
    CCSINU=CC(39)*SINU
    GAC=FAC*CC(35)*SINU
    AN1=GAC*COSV
    AN2=GAC*SINV
    AN3=FAC*CC(38)*COSU
    RMAGD=CC(40)*SINU/FAC
    AX1=CCSINU*COSV
    AX2=CCSINU*SINV
    AX3=CC(37)*COSU
    RETURN
17 COSV=COS(V)
    SINV=SIN(V)
    COSU=COS(U)
    COSUS=COSU*COSU

```