

# A Review of Microwave Switching Schemes and Variable Power-Division Networks Using Phase Shifters

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## ABSTRACT

A mathematical study is made of three basic configurations for steering microwave power by controlling the phase of multiple signals obtained from a single input. The effect of phase errors, coupling errors of hybrid couplers, and component losses on isolation of switches and accuracy of power dividers is determined.

## PROBLEM STATUS

This is an interim report; work continues on other phases of the problem.

## AUTHORIZATION

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# A REVIEW OF MICROWAVE SWITCHING SCHEMES AND VARIABLE POWER DIVISION NETWORKS USING PHASE SHIFTERS

## INTRODUCTION

Modern radar and other microwave systems require components with very fast reaction times. Components such as switches and variable power dividers often must be capable of switching speeds in the microsecond or even nanosecond range. Obviously, mechanical or electromechanical devices are inadequate. The purpose of this report is to review some of the theoretical limitations on some aspects of a particular class of microwave switches and variable power-division networks; namely, that class employing electronic phase shifters as the control element.

## OPERATION OF SPDT SWITCH

Consider the arrangement shown in Fig. 1. An incoming signal is split into two equal parts by a minus 3-dB hybrid coupler (or magic tee). The phase of the signal in one arm is varied relative to the phase of the signal in the other arm by controlling the two adjustable phase shifters. When these two signals recombine in the second hybrid coupler, the phase difference of the phase shifters determines how the total power will be split between the two output ports. By controlling the phase difference over a 180-degree range, all the power can be made to come out of port A, or all out of port B, or in any desired ratio between the two ports.

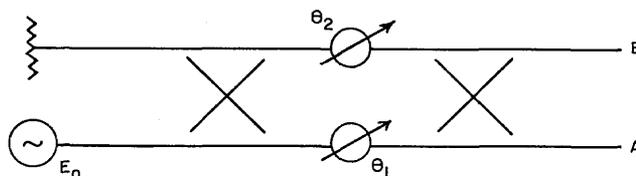


Fig. 1 - Single two-way power divider  
using 90-degree hybrids

The above discussion assumes ideal components; i.e., the hybrid couplers give exactly equal power division, all components are perfectly matched and lossless, and precise settings may be made on the phase shifters. Since physically realizable components are sometimes far from ideal, an analysis of the effect of various types of errors on the performance of this (and other) configurations will be made.

If perfectly matched and lossless components are assumed, the voltages at the two output ports A and B of Fig. 1 will be

$$E_A = [T^2 \exp(j\theta_1) - C^2 \exp(j\theta_2)] E_0$$

and

$$E_B = jTC [\exp(j\theta_1) + \exp(j\theta_2)] E_0 ,$$

where  $T$  and  $C$  are, respectively, the voltage transmission and voltage coupling coefficients of the quadrature hybrid couplers. (It is assumed that the hybrid coupler outputs are in phase quadrature and that all the couplers are identical; i.e., the coupling coefficient is the same for all of the couplers used. Similar equations may be derived based upon a model using magic tees, whose unbalance would correspond to the unbalance of  $T$  and  $C$  in the model employing phase-quadrature couplers.)

Power at the two output ports, A and B, will be

$$P_A = [T^4 + C^4 - 2T^2C^2 \cos(\theta_1 - \theta_2)] P_0$$

and

$$\begin{aligned} P_B &= 2T^2C^2 [1 + \cos(\theta_1 - \theta_2)] P_0 \\ &= 4T^2C^2 \cos^2\left(\frac{\theta_1 - \theta_2}{2}\right) P_0 . \end{aligned}$$

If we assume lossless hybrids, the sum of the power transmission and power coupling coefficients of each hybrid will be unity:

$$T^2 + C^2 = 1 .$$

Loss in the hybrids will give

$$T^2 + C^2 = K , \quad K < 1 .$$

It is easy to see from the equations for output power that loss in the hybrids will not affect the *relative* power of port A compared with port B. If the power divider of Fig. 1 is being used as a single-pole, double-throw (SPDT) switch, a combination of phase error and coupling error may be tolerated while maintaining a given degree of isolation between the two output ports.

For the input power to be switched to port B, the ideal values for phase difference  $\Delta\theta$  ( $\Delta\theta = \theta_1 - \theta_2$ ), and hybrid power coupling coefficient  $C^2$  would be  $\Delta\theta = 0$  and  $C^2 = 0.5$  (-3.01 dB). In Fig. 2, limits of phase error and coupling error allowed while maintaining various degrees of isolation at port A are shown. For example, any phase error-coupling error combination inside the 20-dB curve would result in the power at port A being down by at least 20 dB from the power at port B. The center of the figure represents the ideal value ( $\Delta\theta = 0$ ,  $C^2 = 0.5$ ) for switching the input power to port B.

For the input power to be switched to port A, the ideal values for phase difference and hybrid coupling would be  $\Delta\theta = 180^\circ$  and  $C^2 = 0.5$ . Fig. 3 shows the allowable limits of phase error and hybrid coupling error for maintaining various degrees of isolation at port B. Any combination of phase error and coupling error between the two curves labeled 20 dB, for example, would give power at port B down at least 20 dB from the power at port A. If  $\Delta\theta$  could be maintained precisely at 180 degrees, the isolation of the switch while operating in this mode (output = port A) would be infinite and independent of hybrid coupling.

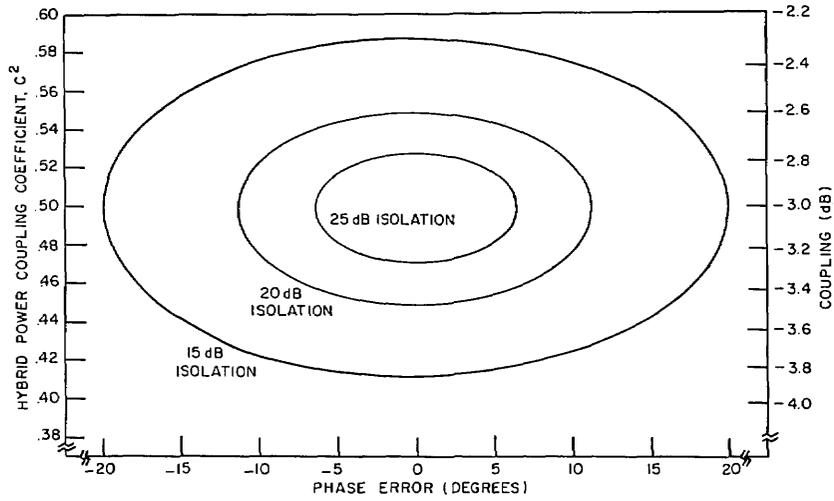


Fig. 2 - Maximum phase and coupling errors allowed to maintain a given isolation for SPDT switch configuration (output at port B)

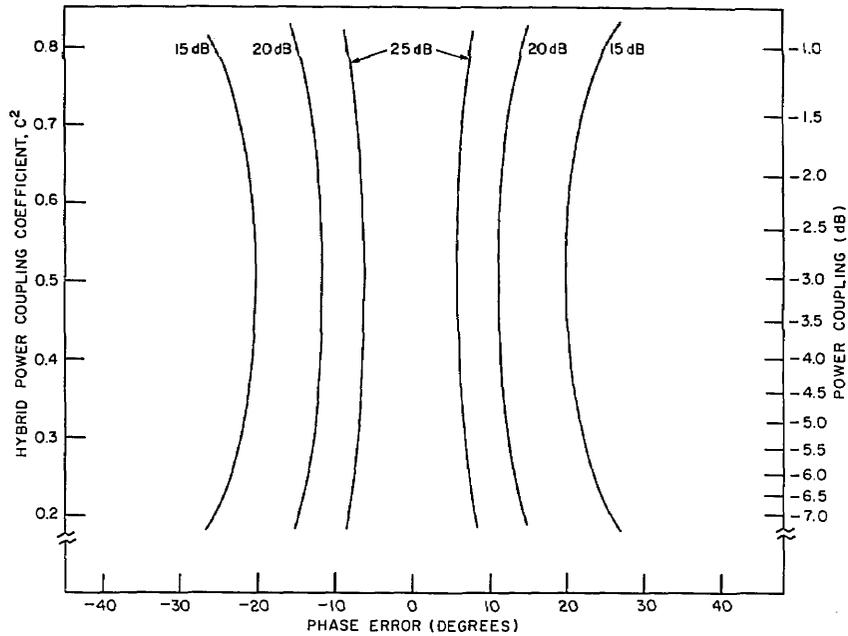


Fig. 3 - Maximum phase and coupling errors allowed to maintain a given isolation for SPDT switch configuration (output at port A)

It is interesting to note the difference in effect of hybrid coupling variations on switch isolation for the two cases (i.e., power switched to output A or output B). When the power is switched to port B, isolation of port A is strongly affected by the coupling coefficient of the hybrids, and in order to maintain a given value of isolation, the coupling error must not exceed a certain limit. When the power is switched to port A, the effect of coupling error on isolation of port B is slight. For a given error in  $\Delta\theta$ , an increase in coupling error would actually increase the isolation of port B by a small amount.

The difference in effect of coupling error on switch isolation for the two outputs (port A or port B) could be significant in the design of a device such as a circulator using nonreciprocal phase shifters.

### OPERATION OF VARIABLE TWO-WAY POWER DIVIDER

When the circuit of Fig. 1 is used as a power divider rather than as a switch, the maximum allowable error in phase and coupling to maintain a power division within given limits is dependent upon the power division. A given error in  $\Delta\theta$  will cause the greatest transfer of power from one output port to the other when the two output powers are equal. If the output power is expressed in dB relative to the input power, the greatest change (in dB) with varying  $\Delta\theta$  and/or  $C^2$  will occur for lower power levels. Figure 4 shows how the power of each output port varies with the difference in phase of the two phase shifters  $\Delta\theta = |\theta_1 - \theta_2|$ . The power at port A is strongly affected by variations in coupling coefficients of the hybrid couplers, especially for low values of  $P_A/P_0$ . Power from port B is affected only slightly by the coupler error. Examination of the equations for power output will explain why the power out of the one port reacts differently to coupling variations than the power from the second port. In the graph of Fig. 4, a coupling coefficient of 0.4 (-3.98 dB) would yield a power curve for port B exactly the same as the one shown for  $C^2 = 0.5$  (-3.01 dB), except it would be displaced downward by 0.18 dB.

Figure 5 shows the variation of output power with changing values of coupling. Notice that the variation of power (in dB) with coupling at port B is the same for all values of differential phase shift  $\Delta\theta$ ; at port A the variation of output power as a function of coupling is determined by the value of  $\Delta\theta$ .

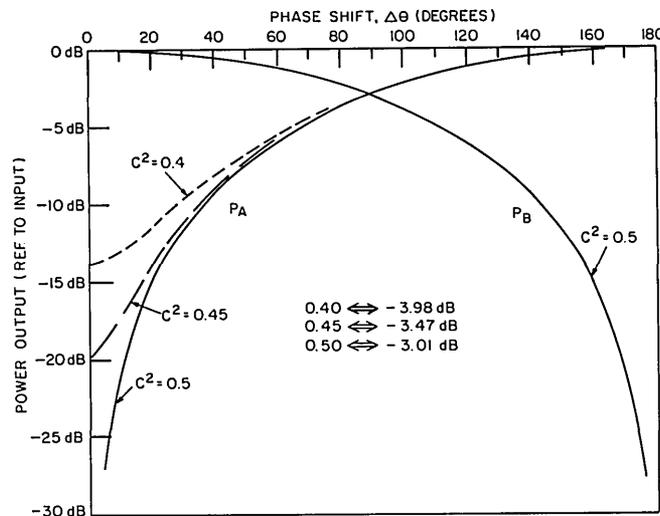


Fig. 4 - Two-way power divider outputs as a function of differential phase shift

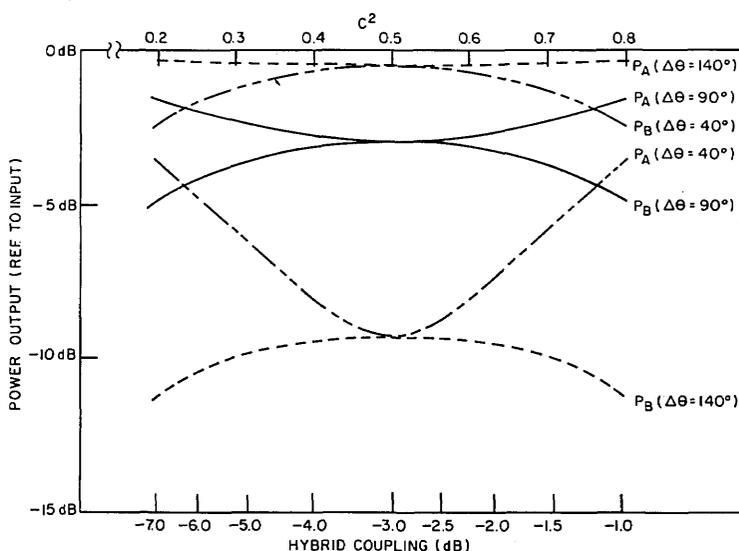


Fig. 5 - Two-way power divider outputs as a function of hybrid coupling coefficients

When using the circuit of Fig. 1 as a variable power divider over a broad frequency band, the coupling factor of the hybrid couplers may deviate greatly from the ideal value of  $C^2 = 0.5$ . The value of  $\Delta\theta$  calculated to give a desired power division, based upon a value of  $C^2 = 0.5$ , might result in a power division different from the desired value by an amount unacceptable for system performance. But for power division and errors of  $C^2$  within certain ranges, exact power division may still be obtained by calculating the necessary value of  $\Delta\theta$  based on the actual value of  $C^2$ . Figure 6 shows the limits of the coupling such that a given power division may be achieved by proper selection of  $\Delta\theta$ . For instance, if an even power division is desired  $P_A = P_B = 0.5 P_o$ , the power coupling  $C^2$  could be any (single) value between 0.15 (-8.24 dB) and 0.85 (-1.41 dB); the "corrected" value of  $\Delta\theta$  will still yield an even power split.

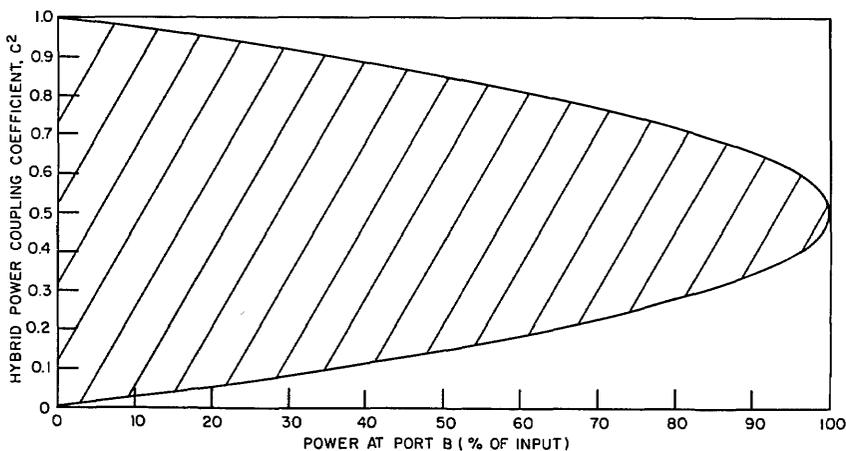


Fig. 6 - Allowable variation in coupling coefficient of hybrids such that a desired power division may be obtained by phase correction in SPDT switch

Of course, for this "correction" technique to be used, the coupling coefficients of the hybrid couplers must be known as a function of frequency, and the computations for  $\Delta\theta$  become more complex. Naturally, when using a single value of  $\Delta\theta$  to obtain a desired power division over a frequency band, correction may be made only for the average deviation of hybrid coupling from the ideal value. Whether or not the increased accuracy of the power divider is worth the additional system complexity is outside the scope of this report.

If the hybrid couplers give ideal coupling of -3.01 dB ( $c^2 = 0.5$ ) and all components are matched and lossless, the equations for the two output signals of the power divider of Fig. 1 reduce to

$$E_A = \left[ j \exp\left(\frac{\theta_1 + \theta_2}{2}\right) \sin \frac{\theta_1 - \theta_2}{2} \right] E_0$$

and

$$E_B = \left[ j \exp\left(\frac{\theta_1 + \theta_2}{2}\right) \cos \frac{\theta_1 - \theta_2}{2} \right] E_0 .$$

As can be seen, the two output signals will be in phase with each other (or 180 degrees out of phase, depending on whether  $\theta_1$  or  $\theta_2$  is larger) regardless of the ratio of the magnitudes. The phase(s) of the output signals may be varied (relative to the input) independently of the power division ratio by the proper selection of  $\theta_1$  and  $\theta_2$ . If only one phase shifter is varied to control the power division, the phase of the outputs (relative to the input) will no longer be independent of the power division. When the power divider is used as a switch, the phase of the output signal when switched to port A will differ by 90 degrees from the phase of the output signal when switched to port B, if only one phase shifter is changed.

Thus far the loss of the phase shifters has been neglected. If each phase shifter has the same loss, say  $X$  dB, the operation of the switch/power divider is the same as though each phase shifter were lossless but the input signal attenuated by  $X$  dB. If one phase shifter has a loss equal to  $X$  dB and the second has a loss equal to  $(X + \Delta X)$  dB, the operation is the same as if the input were attenuated by  $X$  dB with one phase shifter lossless and the other with loss  $\Delta X$  dB. When the circuit of Fig. 1 is used as a switch, and the power is directed to port A, this difference in loss of the phase shifters has a minor effect on the isolation of port B. If coupling errors alone caused the isolation at port B to be 20 dB, a  $\Delta X$  of 0.4 dB would change this value by less than 0.2 dB. The loss at port A varies, depending upon  $c^2$  and which phase shifter ( $\theta_1$  or  $\theta_2$ ) has the extra loss. The average increase in loss at port A is about one-half of  $\Delta X$  (in dB) for reasonably small values of  $\Delta X$ .

If the power is directed to port B and the coupling is other than -3.01 dB, the effect of the phase shifter loss difference ( $\Delta X$ ) upon isolation at port A is quite noticeable. If the error in coupling were such that the isolation at port A were 20 dB when  $\Delta X$  were zero, a value of  $\Delta X = 0.4$  dB would change the isolation by about 3 dB (17 dB or 23 dB, depending on which phase shifter,  $\theta_1$  or  $\theta_2$ , had the greater loss). Essentially, the isolation error graph of Fig. 2 would be shifted up or down the  $c^2$  axis by a nonzero value of  $\Delta X$ . The effect on loss at port B will be practically the same as at port A when the power is switched to port A.

When the circuit of Fig. 1 is used as a variable power divider, the effect of loss difference in the phase shifters will depend on the value of power division, being somewhere between the two effects described above. A "correction" for  $\Delta X$  could be made by

changing  $\Delta\theta$ , as in the case of "correcting" for coupling error. In the expressions for the outputs,  $P_A$  and  $P_B$ , loss in the phase shifters may be taken into account by using complex values for  $\theta_1$  and/or  $\theta_2$ .

#### CORPORATE CONFIGURATION OF SWITCHES OR POWER DIVIDERS

Thus far only a two-output device has been considered. By connecting several power dividers in a corporate system (Fig. 7) an N-port power divider may be constructed. A specialized case of this power divider network is that of a single-pole, N-throw switch. As a switch, the corporate system shown would have a worst-case isolation equal to the isolation of an individual SPDT switch (assuming all switches had identical characteristics). The loss would be proportional to the number of switches traversed between input and output.

If used as a variable power divider, the corporate system would be limited in power division accuracy by the accuracy of each of the component power dividers. Errors at each stage of power splitting would be added to errors from earlier stages. Of course, the magnitude of the error at each output would depend upon the power level at that output. As an example, consider an 8-output power divider used to divide an input signal into eight output signals with the power ratio 1:2:4:8:8:4:2:1. The desired outputs are

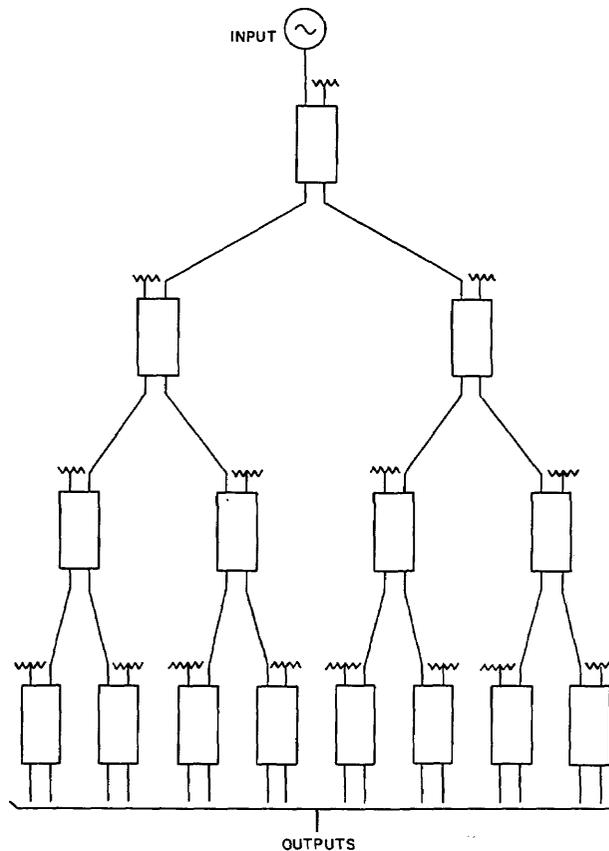


Fig. 7 - SP16T switch (or 16-way power divider) using two-way power dividers in a corporate arrangement

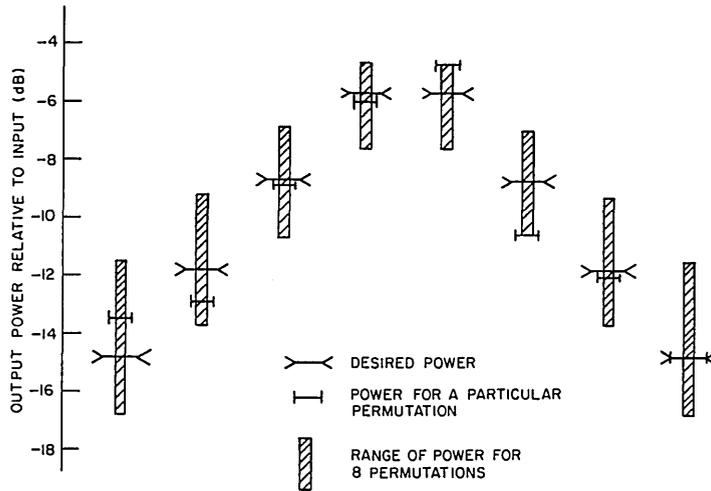


Fig. 8 - Effect of phase and coupling errors on power outputs of 8-port corporate power divider. All values of  $\Delta\theta$  are 10% low; coupling coefficient of hybrids is -3.47 dB.

shown in Fig. 8, along with the outputs that would result if each hybrid coupler had a value of -3.47 dB ( $C^2 = 0.45$ ) and each power divider had a 10% error in the calculated value of  $\Delta\theta$  (assumed are lossless phase shifters and no "correction" for coupling error). Also shown is the range over which each power level would vary as the eight outputs were permuted around (1:2:4:8:8:4:2:1, 1:1:2:4:8:8:4:2, 2:1:1:2:4:8:8:1, etc.), given the same type error (i.e.,  $\Delta\theta$  10% low from the desired value and  $C^2 = 0.45$ ).

#### SERIES CONFIGURATION OF SWITCHES OR POWER DIVIDERS

An alternative network for use as an N-port power divider (or switch) is the arrangement of two-way power dividers in series (Fig. 9). In this arrangement, isolation

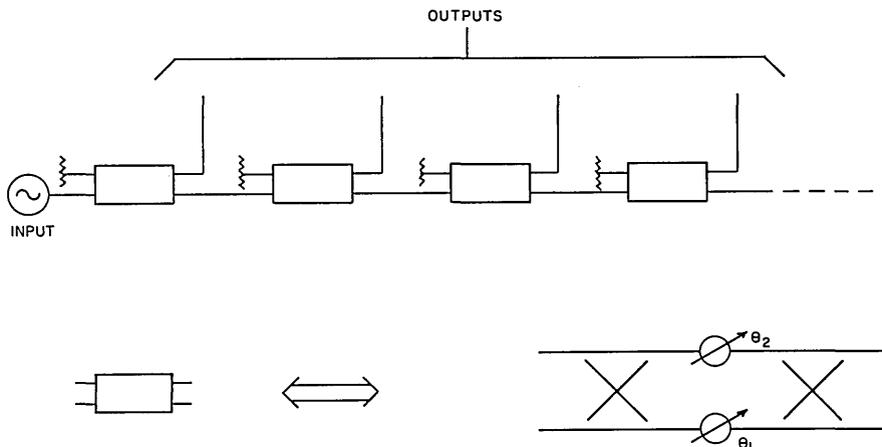


Fig. 9 - Series arrangement of two-way power dividers for use as a phase- and amplitude-controlled feed system

(when used as a switch) and power division accuracy (when used as a power divider) drop off rapidly as the number of output ports increases, due to the compound manner in which the errors add. Also, the worst-case variation in loss is much greater than for the corporate system.

Consider an 8-port switch of the series type. Assume that each individual switch has a loss of 0.5 dB and an isolation of 20 dB. If the power is directed to the eighth output in the series string, the signal will have suffered a total loss of 4 dB, and the power at the first port will be only 16 dB down from the output. Similar individual switches in a corporate arrangement (8-output switch) would result in a loss of 1.5 dB and a worst-case isolation of 20 dB.

If used as a power divider to form eight outputs with power ratios 1:2:4:8:8:4:2:1, and if coupling error and phase error are the same as in the example for the corporate arrangement (i.e.,  $C^2 = 0.45$  and  $\Delta\theta$  is 10% low), the series arrangement will give the results shown in Fig. 10. The variations in the power levels at the outputs are permuted are almost twice (in dB) those for the corporate system.

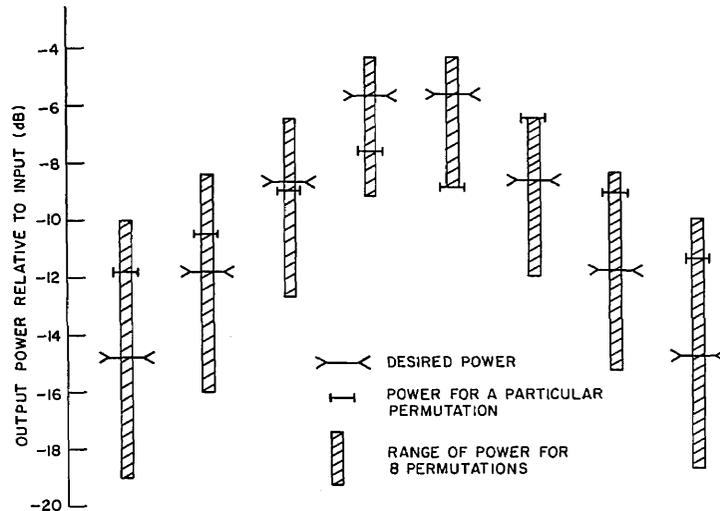


Fig. 10 - Effect of phase and coupling errors on power outputs of 8-port series power divider. All values of 0 are 10% low; coupling coefficient of hybrids is -3.47 dB.

The series arrangement does have one advantage over the corporate arrangement, however. By varying  $\theta_1$  and  $\theta_2$  (but maintaining a fixed difference  $\Delta\theta$ ) of the power dividers, any phase distribution may be obtained for a given power distribution along the series outputs. In the corporate arrangement, partial phase control is possible, but additional phase shifters would be necessary at the outputs if independent phase control of the outputs were required. Nevertheless, for a switch or power divider with a large number of outputs, the corporate arrangement would seem to be preferred generally over the series because of less loss and greater stability with phase and coupling errors.

#### MATRIX SWITCH CONFIGURATION

Another network that has several advantages for use as a single-pole, N-throw switch is the matrix approach (exemplified in Fig. 11 as an SP8T switch). In this

approach, the input signal is fed into a matrix arrangement of hybrid couplers which splits the signal into  $N$  parts of equal magnitude ( $N$  must be an integral power of 2). Each of the  $N$  signals is adjusted to a proper phase setting, and the resulting  $N$  signals are fed into another (identical) matrix of couplers. By setting each phase shifter to the correct value (either 0 or 180 degrees), the total power will emerge from one of the  $N$  output ports.

In the corporate or series arrangement of individual SPDT switches that forms an  $N$ -throw switch, each phase shifter must be capable of handling one-half of the total power, regardless of the number of outputs. In the matrix switch, each phase shifter must handle only  $1/N$ th of the total power. This factor would be an important consideration in the design of a high-power multipole switch.

For the SP8T matrix switch of Fig. 11, the proper phase-shifter settings necessary for the power to emerge from each of the eight output ports are given in Table 1.

Table 1  
Phase Shifter Settings for Matrix Switch of Fig. 11

Output Number	Phase Shifter Settings (degrees)							
	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\theta_8$
1	0	180	180	0	180	0	0	180
2	0	180	0	180	180	0	180	0
3	0	180	180	0	0	180	180	0
4	0	180	0	180	0	180	0	180
5	0	0	180	180	180	180	0	0
6	0	0	0	0	180	180	180	180
7	0	0	180	180	0	0	180	180
8	0	0	0	0	0	0	0	0

An error analysis of the matrix switch of Fig. 11 reveals that in order to maintain a given level of worst-case isolation, the maximum tracking phase error and hybrid coupling error combination that can be tolerated (Fig. 12) is the same as for the SPDT switch of Fig. 1 when the power is directed to port B (Fig. 2), except that the phase error of Fig. 2 is not a tracking phase error. Again it is assumed that all hybrid couplers are identical. "Tracking" phase error is the equal error in all the phase shifters in one of the two states (0 or 180 degrees). As an illustration, suppose the power is to be directed to port 6. The required settings on phase shifters 1 through 8 are 0, 0, 0, 0, 180, 180, 180, and 180 degrees, respectively. A minus 10-degree tracking phase error would mean actual phase settings of 0, 0, 0, 0, 170, 170, 170, and 170 degrees. The assumption that all phase shifters in one state are identical (i.e., same phase shift) seems reasonable for a first approximation, since they are subject to the same operating conditions. Ferrite phase shifters, for example, would have differential phase shifts largely dependent on temperature. Since all the phase shifters are handling the same power level, each would be at the same temperature, and hence would have the same differential phase shift. Of course, there are still small random phase errors in the phase shifters plus phase errors in the hybrid matrixes. A good analysis of this random type of error, plus additional discussion of the matrix type switch, may be found in a report by Schrank, Hooper, and Davis (1).

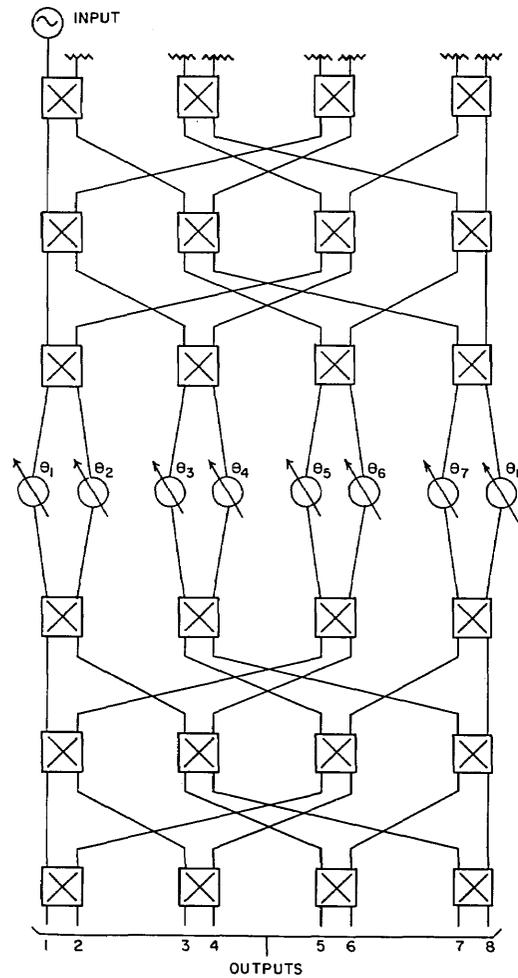


Fig. 11 - SP8T matrix switch

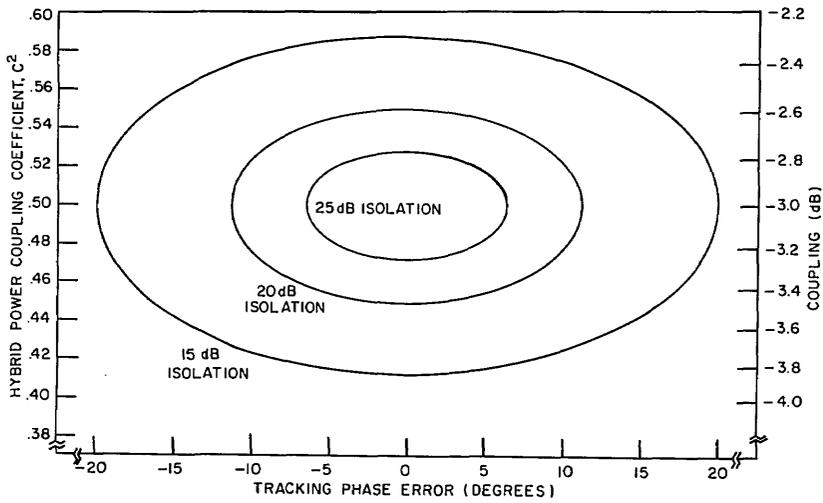


Fig. 12 - Maximum phase and coupling error allowed to maintain a given isolation in matrix switch of Fig. 11

As in the case of the two-way power divider of Fig. 1 when used as a switch, loss in the hybrids will not affect the isolation of the matrix switch (assuming all have the same loss). Likewise, equal loss in the phase shifters does not affect isolation. Unequal loss of the phase shifters will affect isolation only slightly. For example, if the worst-case isolation were 20 dB due to phase errors and coupling errors, a "tracking loss" (i.e., all phase shifters in the same phase state having equal loss) of 0.4 dB would change the worst-case isolation by less than 0.3 dB.

The above analysis of phase error, hybrid coupling error, and component losses affecting worst-case isolation applies to matrix switches with other than eight outputs, but having the type of hybrid matrix arrangement shown in Fig. 11.

One interesting feature of this matrix switch is that the worst-case isolation (i.e., greatest unwanted power) occurs at output port 8 when the phase settings are such to steer the power to any one of the seven other outputs. Of course, if all phase settings, hybrid coupling coefficients, and component losses are ideal, the power at port 8 is zero. But assume equal phase error in the phase shifters in the 180-degree state (i.e., a tracking error). If  $C^2 = 0.5$ , power will appear at port 8 (and at the selected output port, of course), but not at any other port. If one were willing to terminate port 8 in a matched load and have an SP7T switch, isolation would not be affected at all by tracking phase error (i.e., when all phase shifters in any one state are equal and track in phase). Of course, insertion loss of the switched signal would be affected. Also, isolation of all ports would still be limited by hybrid coupling errors and difference in loss among the phase shifters.

The reason that the isolation of output ports other than port 8 in the matrix switch of Fig. 11 is not affected by tracking phase error may be understood by an examination of the phase setting chart in Table 1. At any output port, the total voltage signal is the vector sum of eight component signals, each of which has traversed one of the eight possible paths from the input port to that particular output port. All eight component signals (voltages) are equal in magnitude (for  $C^2 = 0.5$ ), being  $1/8$  times the input voltage, and each has its phase controlled by one of the eight phase shifters. Each row of phase settings in the chart represents the relative phase of the component signals arriving at the corresponding output port when all phase shifters are set to their zero state.

Now define an eight-dimensional vector for each row:

$$\mathbf{v}_n = \frac{1}{\sqrt{8}} [ e^{j\theta_{n1}}, e^{j\theta_{n2}}, \dots, e^{j\theta_{n8}} ] ,$$

where the values of  $\theta$  are those found in the chart and expressed in radians rather than degrees. The eight vectors thus created form an orthonormal set; that is,

$$\mathbf{v}_j \cdot \mathbf{v}_k = \begin{cases} 0 & j \neq k \\ 1 & j = k \quad j, k = 1, 2, \dots, 8 . \end{cases}$$

Also define an eight-dimensional vector to represent the actual settings of the eight phase shifters:

$$\mathbf{v}_\theta = \frac{1}{\sqrt{8}} [ e^{j\theta_1}, e^{j\theta_2}, \dots, e^{j\theta_8} ] ,$$

with all values of  $\theta$  expressed in radians. Call this the theta vector. For any combination of phase shifter settings, the proportion of the input power that will appear at output port  $n$  will be

$$\frac{P_n}{P_0} = |\mathbf{V}_\theta \cdot \mathbf{V}_n|^2 .$$

Thus, for all of the input power to be switched to output port  $n$ ,

$$\mathbf{V}_{\theta_n} = \mathbf{V}_n .$$

For the input power to be switched to any one of the first seven output ports, say port  $n$ ,  $n = 1, 2, \dots, 7$ , half of the phase shifters must be set to 0 degrees and half to 180 degrees. A tracking phase error  $\epsilon$  would result in half of the phase shifters being set to 0 degrees and half to  $(180 + \epsilon)$  degrees. The resulting theta vector  $\mathbf{V}_{\theta_n}$  will be such that

$$(a) \quad \mathbf{V}_{\theta_n} \neq \mathbf{V}_n$$

$$(b) \quad |\mathbf{V}_{\theta_n} \cdot \mathbf{V}_8|^2 = \frac{P_8}{P_0} = \frac{1}{2} - \frac{1}{2} \cos \epsilon$$

$$(c) \quad |\mathbf{V}_{\theta_n} \cdot \mathbf{V}_n|^2 = \frac{P_n}{P_0} = \frac{1}{2} + \frac{1}{2} \cos \epsilon$$

$$(d) \quad |\mathbf{V}_{\theta_n} \cdot \mathbf{V}_m|^2 = \frac{P_m}{P_0} = 0 \quad \text{for } m, n = 1, \dots, 7$$

but  $m \neq n$ .

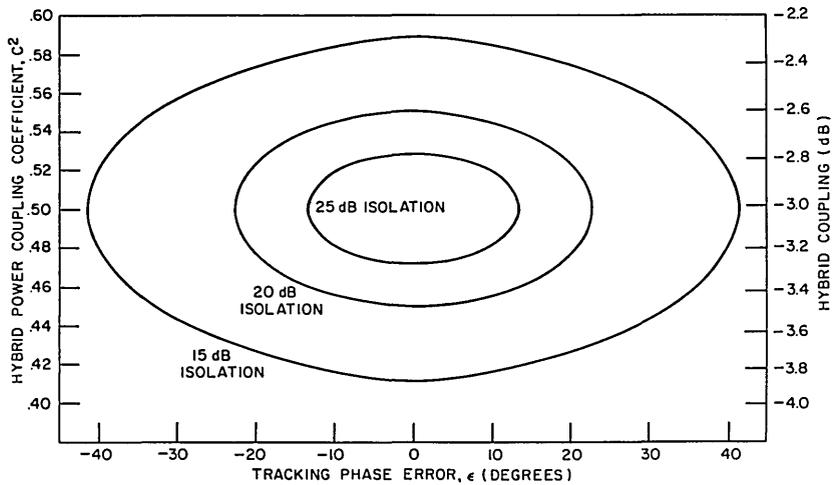
The orthogonality relationship of the vectors in Eq. (d) is due to the fact that  $\mathbf{V}_{\theta_n}$  is a linear combination of  $\mathbf{V}_n$  and  $\mathbf{V}_8$  for any tracking phase error  $\epsilon$ .

Suppose constant 180-degree phase shifters were put in series with any two of the variable phase shifters of Fig. 11, say  $\theta_2$  and  $\theta_3$  (in a waveguide system this could be accomplished by a 180-degree twist in the appropriate waveguide sections). The resulting phase setting chart would be that of Table 2. This is the same as the chart of Table 1, except that the 0- and 180-degree settings are interchanged in the columns corresponding to  $\theta_2$  and  $\theta_3$ . The eight row vectors formed by the phase settings of this chart will constitute an orthonormal set, as in the case of Table 1. For power switched to any output port, a tracking phase error would result in a theta vector such that the isolation of several other ports would be degraded. However, a switch incorporating the phase setting chart of Table 2 could tolerate a larger tracking phase error  $\epsilon$  than could a switch using the phase settings of Table 1, if both switches were required to maintain a certain worst-case isolation at all output ports. The allowable phase and coupling errors for maintaining certain values of isolation in this "modified" matrix switch are shown in Fig. 13.

An increased tolerance of tracking phase error while maintaining any given minimum isolation may be obtained by interconnecting the hybrid couplers in a different arrangement (2), as in Fig. 14. The phase setting chart for this switch is shown in Table 3. Note the similarity with the phase setting chart (Table 2) of the modified matrix switch (i.e., having the two fixed 180-degree phase shifters). The phase and coupling error/isolation characteristics are the same as for the above modified matrix switch (Fig. 13).

**Table 2**  
**Phase Shifter Settings for Matrix Switch Modified by Addition**  
**of Fixed 180-Degree Phase Shifters in Series with  $\theta_2$  and  $\theta_3$**

Output Number	Phase Shifter Settings (degrees)							
	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\theta_8$
1	0	0	0	0	180	0	0	180
2	0	0	180	180	180	0	180	0
3	0	0	0	0	0	180	180	0
4	0	0	180	180	0	180	0	180
5	0	180	0	180	180	180	0	0
6	0	180	180	0	180	180	180	180
7	0	180	0	180	0	0	180	180
8	0	180	180	0	0	0	0	0



**Fig. 13 - Maximum phase and coupling error allowed to maintain a given isolation in the modified SP8T matrix switch**

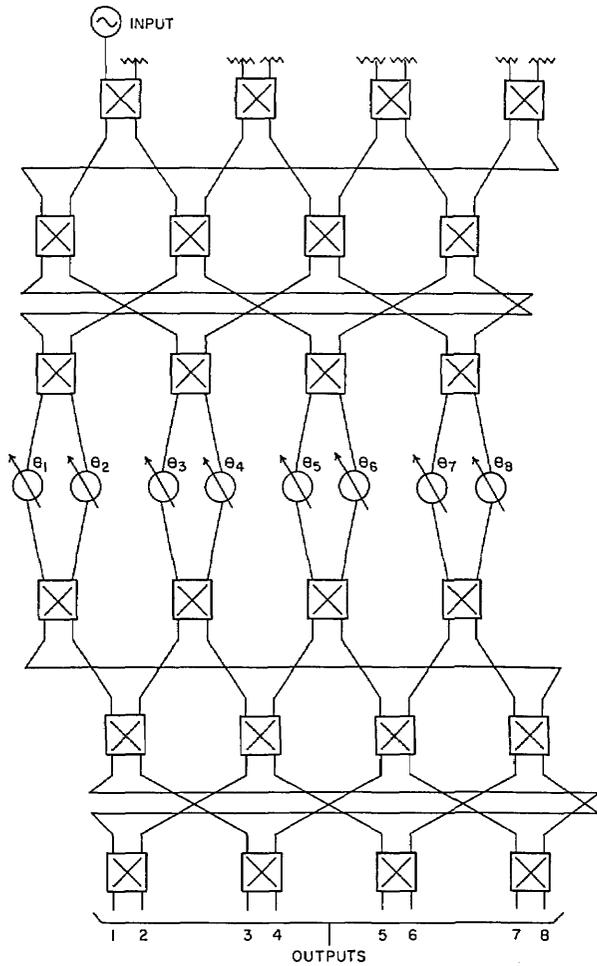


Fig. 14 - SP8T matrix switch with different hybrid interconnections

Table 3  
Phase Shifter Settings for Matrix Switch of Fig. 14

Output Number	Phase Shifter Settings (degrees)							
	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\theta_8$
1	180	180	180	180	0	180	0	180
2	0	0	180	180	0	180	180	0
3	0	180	0	180	0	0	0	0
4	180	0	180	0	0	0	0	0
5	180	180	0	0	0	180	180	0
6	0	0	0	0	0	180	0	180
7	0	180	180	0	0	0	180	180
8	180	0	0	180	0	0	180	180

None of the matrix switches described is suitable for use as a general power divider, since it is not possible to obtain any arbitrary power division at the outputs (see Appendix A).

Thus far, effects of phase error, coupling error, and component losses on switch isolation and power-divider accuracy have been considered. It is assumed that the components are sufficiently well matched such that the effects of mismatch are small compared with the effects of errors discussed. A rigorous mathematical analysis of the effect of mismatch would be very complicated for devices with even a moderately large number of outputs. However, for reasonably well-matched components (VSWR better than 1.2:1), rough calculations indicate that the above assumption is valid.

## SUMMARY

From the preceding analysis, one can conclude that the corporate arrangement would be preferred over the series arrangement for use as a variable power-division network. In general, the corporate network has less insertion loss and greater allowable component tolerances for given accuracy requirements than does a series network with the same number of outputs.

Also, based on the same analysis, the matrix switching approach would be preferable to either the corporate or series approach for use as a single-pole,  $N$ -throw switch, where  $N = 2^k$ . The matrix switch will generally have lower insertion loss and greater isolation for given component errors than either the series or corporate switches. An additional advantage of the matrix switch is that each phase shifter is subject to only  $1/N$  of the total power, whereas the phase shifters in both the corporate and series switches must be capable of handling  $1/2$  of the total power.

## ACKNOWLEDGMENTS

The author would like to thank Dr. M. L. Kales for providing the proof contained in the appendix and Mr. M. L. Reuss, Jr., for providing beneficial suggestions and discussions.

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## Appendix A

### PROOF THAT THE SWITCHING MATRIX MAY NOT BE USED TO OBTAIN AN ARBITRARY POWER DIVISION

**QUESTION:** Can a transfer matrix be designed so that if  $n$  equal inputs  $a_i = 1/\sqrt{n}$  are given, the amplitudes of the outputs  $b_i$  can be arbitrarily determined by properly choosing the phases of the inputs? (The amplitudes of the outputs  $b_i$  are subject to the restriction

$$\sum_{i=1}^n |b_i|^2 = 1.)$$

See Fig. A1.

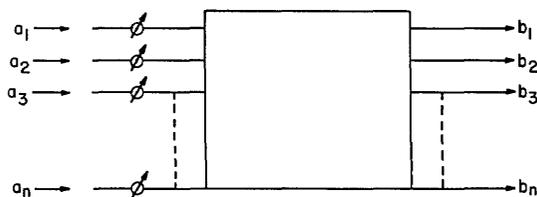


Fig. A1 - Schematic of transfer matrix  $[S]$

The answer to this question is that it cannot be done. The proof follows.

(REMARK: The above conclusion remains valid if the inputs are not assumed to be equal in amplitude. The proof is essentially the same, but the notation becomes a little more involved.)

Let  $[S]$  denote the transfer matrix. Then by hypothesis, if  $b_i$  is an arbitrary set of positive numbers such that

$$\sum_{i=1}^n b_i^2 = 1,$$

there exists angles  $\theta_i$  and  $\psi_i$  such that

$$S \begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\theta_1} \\ \frac{1}{\sqrt{n}} e^{j\theta_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_n} \end{bmatrix} = \begin{bmatrix} b_1 e^{j\psi_1} \\ b_2 e^{j\psi_2} \\ \vdots \\ b_n e^{j\psi_n} \end{bmatrix}.$$

Since the matrix  $[S]$  is unitary, it has an inverse which we denote by  $[T]$ . Hence,

$$\begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\theta_1} \\ \frac{1}{\sqrt{n}} e^{j\theta_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_n} \end{bmatrix} = [S]^{-1} \begin{bmatrix} b_1 e^{j\psi_1} \\ b_2 e^{j\psi_2} \\ \vdots \\ b_n e^{j\psi_n} \end{bmatrix} = [T] \begin{bmatrix} b_1 e^{j\psi_1} \\ b_2 e^{j\psi_2} \\ \vdots \\ b_n e^{j\psi_n} \end{bmatrix}.$$

Denote the elements of  $[T]$  by  $t_{ik}$ . Since the  $b_i$  are arbitrary, let

$$(b_1, b_2, \dots, b_n) = (1, 0, \dots, 0).$$

We then have

$$\begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\theta_1} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_n} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ t_{21} & t_{22} & \dots & t_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n1} & t_{n2} & \dots & t_{nn} \end{bmatrix} \begin{bmatrix} e^{j\psi_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} t_{11} e^{j\psi_1} \\ t_{21} e^{j\psi_1} \\ \vdots \\ t_{n1} e^{j\psi_1} \end{bmatrix}.$$

Hence  $t_{i1} e^{j\psi_1} = 1/\sqrt{n} e^{j\theta_i}$ , and  $|t_{i1}| = 1/\sqrt{n}$ . More generally, if we let  $b_k = 1$ , and  $b_i = 0$  if  $i \neq k$ , we see that

$$\frac{1}{\sqrt{n}} e^{j\theta_i} = \sum_{m=1}^n t_{im} b_m e^{j\psi_m} = t_{ik} e^{j\psi_k}.$$

Thus, we see that  $|t_{ik}| = 1/\sqrt{n}$  for all  $i$  and  $k$ , and hence  $t_{ik} = 1/\sqrt{n} e^{j\alpha_{ik}}$ . (Note: The angles  $\theta_i$  and  $\psi_i$  will depend on the choice of the set  $(b_1, b_2, \dots, b_n)$ . However, to avoid nonessential complication of the notation we have omitted this dependence, since we make use only of the fact that  $|t_{ik}| = 1/\sqrt{n}$ ; i.e., our conclusion involves only the absolute value of  $t_{ik}$ .)

Now let  $b_1 = 1/\sqrt{2}$ ,  $b_k = 1/\sqrt{2}$ , and  $b_i = 0$  for  $i \neq 1$  or  $k$ . Then

$$\begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\theta_1} \\ \frac{1}{\sqrt{n}} e^{j\theta_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_n} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{22} & \dots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \dots & t_{nn} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} e^{j\psi_1} \\ 0 \\ \vdots \\ \frac{1}{\sqrt{2}} e^{j\psi_k} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} (t_{11} e^{j\psi_1} + t_{1k} e^{j\psi_k}) \\ \frac{1}{\sqrt{2}} (t_{21} e^{j\psi_1} + t_{2k} e^{j\psi_k}) \\ \vdots \\ \frac{1}{\sqrt{2}} (t_{n1} e^{j\psi_1} + t_{nk} e^{j\psi_k}) \end{bmatrix}$$

Equating corresponding elements of the first and last column matrix, we have

$$\frac{1}{\sqrt{2}} (t_{i1} e^{j\psi_1} + t_{ik} e^{j\psi_k}) = \frac{1}{\sqrt{n}} e^{j\theta_i} .$$

Squaring the magnitudes of both sides, we get

$$\frac{1}{2} \left( |t_{i1}|^2 + |t_{ik}|^2 + 2 \operatorname{Re} \left[ t_{i1} t_{ik}^* e^{j(\psi_1 - \psi_k)} \right] \right) = \frac{1}{n} .$$

Recalling that  $t_{ik} = e^{j\alpha_{ik}/\sqrt{n}}$ , and hence  $|t_{ik}|^2 = 1/n$ , we get on substituting in the equation above,

$$\frac{1}{2} \left( \frac{1}{n} + \frac{1}{n} + \frac{2}{n} \operatorname{Re} \left[ e^{j(\alpha_{i1} - \alpha_{ik} + \psi_1 - \psi_k)} \right] \right) = \frac{1}{n} .$$

Hence,

$$\operatorname{Re} e^{j(\alpha_{i1} - \alpha_{ik} + \psi_1 - \psi_k)} = 0 .$$

But  $\operatorname{Re} [e^{j\varphi}] = 0$  only if  $\varphi$  is an odd multiple of  $\pi/2$ . Hence  $\operatorname{Re} [e^{j\varphi}] = 0$  implies that

$$e^{j\varphi} = \pm j = \pm e^{j\pi/2} .$$

Thus

$$e^{j(\alpha_{i1} - \alpha_{ik} + \psi_1 - \psi_k)} = \pm e^{j\pi/2} .$$

Hence

$$\begin{aligned} e^{j\alpha_{ik}} &= \pm e^{j(\alpha_{i1} + \psi_1 - \psi_k - \pi/2)} \\ &= \pm e^{j(\psi_1 - \psi_k - \pi/2)} e^{j\alpha_{i1}}. \end{aligned}$$

Now let  $\psi_1 - \psi_k - \pi/2 = \delta_k$ , and in place of  $\pm 1$  write  $(-1)^{N_{ik}}$  where  $N_{ik} = 0$  or  $1$ . Then

$$e^{j\alpha_{ik}} = (-1)^{N_{ik}} e^{j\delta_k} e^{j\alpha_{i1}}.$$

Substituting this last result for

$$t_{ik} = \frac{1}{\sqrt{n}} e^{j\alpha_{ik}},$$

we see that the matrix  $T$  may be written as follows:

$$[T] = \frac{1}{\sqrt{n}} \begin{bmatrix} e^{j\alpha_{11}} & (-1)^{N_{12}} e^{j\delta_2} e^{j\alpha_{11}} & \dots & (-1)^{N_{1n}} e^{j\delta_n} e^{j\alpha_{11}} \\ e^{j\alpha_{21}} & (-1)^{N_{22}} e^{j\delta_2} e^{j\alpha_{21}} & \dots & (-1)^{N_{2n}} e^{j\delta_n} e^{j\alpha_{21}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\alpha_{n1}} & (-1)^{N_{n2}} e^{j\delta_2} e^{j\alpha_{n1}} & \dots & (-1)^{N_{nn}} e^{j\delta_n} e^{j\alpha_{n1}} \end{bmatrix}.$$

or

$$[T] = \frac{1}{\sqrt{n}} \begin{bmatrix} e^{j\alpha_{11}} & 0 & \dots & 0 \\ 0 & e^{j\alpha_{21}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & e^{j\alpha_{n1}} & \dots \end{bmatrix} \begin{bmatrix} 1 & (-1)^{N_{12}} & \dots & (-1)^{N_{1n}} \\ 1 & (-1)^{N_{22}} & \dots & (-1)^{N_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (-1)^{N_{n2}} & \dots & (-1)^{N_{nn}} \end{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{j\delta_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & e^{j\delta_n} & \dots \end{bmatrix}.$$

Since the matrix  $T$  is the inverse of a unitary matrix  $S$ ,  $T$  must also be unitary. In the above equation, each of the diagonal matrixes is unitary. Since the product of any number of unitary matrices is a unitary matrix, it follows that the matrix

$$\frac{1}{\sqrt{n}} \begin{bmatrix} 1 & (-1)^{N_{12}} & \dots & (-1)^{N_{1n}} \\ 1 & (-1)^{N_{22}} & \dots & (-1)^{N_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (-1)^{N_{n2}} & \dots & (-1)^{N_{nn}} \end{bmatrix}$$

is unitary.

Denote this last matrix by  $[R]$ , and let  $[\alpha]$  and  $[\delta]$  represent the two diagonal matrices in the fifth equation on p. 20. We then have

$$[T] = [\alpha][R][\delta]$$

and

$$\begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\theta_1} \\ \frac{1}{\sqrt{n}} e^{j\theta_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_n} \end{bmatrix} = [\alpha][R][\delta] \begin{bmatrix} b_1 e^{j\psi_1} \\ b_2 e^{j\psi_2} \\ \vdots \\ b_n e^{j\psi_n} \end{bmatrix}$$

Multiplying both sides of the above equation by  $[\alpha]^{-1}$  and observing that

$$[\alpha]^{-1} = \begin{bmatrix} e^{-j\alpha_{11}} & 0 & \dots & 0 \\ 0 & e^{-j\alpha_{21}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-j\alpha_{n1}} \end{bmatrix},$$

we get

$$\begin{bmatrix} e^{-j\alpha_{11}} & 0 & \dots & 0 \\ 0 & e^{-j\alpha_{21}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-j\alpha_{n1}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\theta_1} \\ \frac{1}{\sqrt{n}} e^{j\theta_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\theta_n} \end{bmatrix} = [R] \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{j\delta_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{j\delta_n} \end{bmatrix} \begin{bmatrix} b_1 e^{j\psi_1} \\ b_2 e^{j\psi_2} \\ \vdots \\ b_n e^{j\psi_n} \end{bmatrix}$$

Let  $\theta_i - \alpha_{i1} = \gamma_i$ , and  $\psi_i + \delta_i = \beta_i$  ( $\delta_1 = 0$ ). The last equation then becomes

$$\begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\gamma_1} \\ \frac{1}{\sqrt{n}} e^{j\gamma_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\gamma_n} \end{bmatrix} = [R] \begin{bmatrix} b_1 e^{j\beta_1} \\ b_2 e^{j\beta_2} \\ \vdots \\ b_n e^{j\beta_n} \end{bmatrix}$$

Now let  $b_1 = 1/\sqrt{3}$ ,  $b_2 = 1/\sqrt{3}$ ,  $b_3 = 1/\sqrt{3}$ , and the remaining  $b_i$  be zero. Also, denote the elements of  $R$  by  $\sigma_{ik}/\sqrt{n}$ . Referring to the matrix which defines  $R$  at the top of p. 21, we see that

$$\begin{aligned}\sigma_{ik} &= (-1)^{N_{ik}} \\ &= \pm 1,\end{aligned}$$

where  $k \neq 1$  and  $\sigma_{i1} = 1$ . Substituting in the matrix equation above, we get

$$\begin{bmatrix} \frac{1}{\sqrt{n}} e^{j\gamma_1} \\ \frac{1}{\sqrt{n}} e^{j\gamma_2} \\ \vdots \\ \frac{1}{\sqrt{n}} e^{j\gamma_n} \end{bmatrix} = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & \sigma_{12} & \cdots & \sigma_{1n} \\ 1 & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} e^{j\beta_1} \\ \frac{1}{\sqrt{3}} e^{j\beta_2} \\ \frac{1}{\sqrt{3}} e^{j\beta_3} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \frac{1}{\sqrt{3n}} \begin{bmatrix} e^{j\beta_1 + \sigma_{12}} & e^{j\beta_2 + \sigma_{13}} & e^{j\beta_3} \\ e^{j\beta_1 + \sigma_{22}} & e^{j\beta_2 + \sigma_{23}} & e^{j\beta_3} \\ \vdots & \vdots & \vdots \\ e^{j\beta_1 + \sigma_{n2}} & e^{j\beta_2 + \sigma_{n3}} & e^{j\beta_3} \end{bmatrix}.$$

Equating corresponding elements of the first and third column matrixes, and then squaring the magnitudes of both sides yields

$$\frac{1}{3n} \left| e^{j\beta_1 + \sigma_{i2}} e^{j\beta_2 + \sigma_{i3}} e^{j\beta_3} \right|^2 = n$$

or

$$\left| e^{j\beta_1 + \sigma_{i2}} e^{j\beta_2 + \sigma_{i3}} e^{j\beta_3} \right|^2 = 3.$$

Expanding both sides and noting that  $\sigma_{ik}^2 = 1$ , we get

$$1 + 1 + 1 + 2\sigma_{i2} \cos(\beta_1 - \beta_2) + 2\sigma_{i3} \cos(\beta_1 - \beta_3) + 2\sigma_{i2} \sigma_{i3} \cos(\beta_2 - \beta_3) = 3.$$

Let  $\beta_1 - \beta_2 = \varphi$ , and  $\beta_1 - \beta_3 = \psi$ . Then  $\beta_2 - \beta_3 = \psi - \varphi$ . Substituting in the above equation, we get

$$\sigma_{i2} \cos \varphi + \sigma_{i3} \cos \psi + \sigma_{i2} \sigma_{i3} \cos(\psi - \varphi) = 0. \quad (\text{A1})$$

Since the matrix  $R$  is unitary, it follows that

$$\sum_{i=1}^n \sigma_{ij} \sigma_{ik} = 0 \quad \text{if } j \neq k. \quad (\text{A2})$$

Also, since  $\sigma_{i1} = 1$  ( $i = 1, 2, \dots, n$ ), we get on substitution into the preceding equation

$$\sum_{i=1}^n \sigma_{ik} = 0 \quad \text{if } k \neq 1. \quad (\text{A3})$$

If we multiply Eq. (A1) by  $\sigma_{i2}$ , then since  $\sigma_{i2}^2 = 1$ , we get

$$\cos \varphi + \sigma_{i2} \sigma_{i3} \cos \psi + \sigma_{i3} \cos (\psi - \varphi) = 0. \quad (\text{A4})$$

Now by Eqs. (A2) and (A3),

$$\sum_{i=1}^n \sigma_{i2} \sigma_{i3} = 0$$

and

$$\sum_{i=1}^n \sigma_{i3} = 0.$$

Hence, if we sum Eq. (A4) on  $i$ , then

$$n \cos \varphi + \cos \psi \sum_{i=1}^n \sigma_{i2} \sigma_{i3} + \cos (\psi - \varphi) \sum_{i=1}^n \sigma_{i3} = n \cos \varphi = 0. \quad (\text{A5})$$

Multiplying Eq. (A1) by  $\sigma_{i3}$ , we get

$$\sigma_{i2} \sigma_{i3} \cos \varphi + \cos \psi + \sigma_{i2} \cos (\psi - \varphi) = 0. \quad (\text{A6})$$

Summing on  $i$  again gives

$$\cos \varphi \sum_{i=1}^n \sigma_{i2} \sigma_{i3} + n \cos \psi + \cos (\psi - \varphi) \sum_{i=1}^n \sigma_{i2} = n \cos \psi = 0. \quad (\text{A7})$$

Finally, multiplying Eq. (A1) by  $\sigma_{i2} \sigma_{i3}$  yields

$$\sigma_{i3} \cos \varphi + \sigma_{i2} \cos \psi + \cos (\psi - \varphi) = 0. \quad (\text{A8})$$

Again, summing on  $i$ , we get

$$\cos \varphi \sum_{i=1}^n \sigma_{i3} + \cos \psi \sum_{i=1}^n \sigma_{i2} + n \cos (\psi - \varphi) = n \cos (\psi - \varphi) = 0. \quad (\text{A9})$$

But,

$$\sin (\psi - \varphi) = \sin \psi \cos \varphi - \cos \psi \sin \varphi.$$

Substituting from Eqs. (A5) and (A7) yields

$$\sin (\psi - \varphi) = \sin \psi (0) - (0) \sin \varphi = 0.$$

Combining this result with Eq. (A9) gives

$$\sin (\psi - \varphi) = 0 \quad \text{and} \quad \cos (\psi - \varphi) = 0 .$$

But it is impossible for both the sine and cosine of an angle to be zero. Thus, our initial hypothesis has led to a contradiction.

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A mathematical study is made of three basic configurations for steering microwave power by controlling the phase of multiple signals obtained from a single input. The effect of phase errors, coupling errors of hybrid couplers, and component losses on isolation of switches and accuracy of power dividers is determined.			

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	ROLE	WT	ROLE	WT	ROLE	WT
Switches, matrix Microwave phase switching techniques Power-division networks, variable						